PRODUCTION PLANNING FOR INTEGRATED PRIMARY AND SECONDARY LUMBER MANUFACTURING

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ABSTRACT

This paper describes two linear programming models that were developed for production planning in value-added lumber manufacturing facilities. One model is designed for nonintegrated value-added facilities; the other is designed for value-added facilities integrated with a sawmill. The models were then used to explore the financial benefits for a sawmill to integrate a value-added lumber manufacturing facility at the back end of the mill. Net revenues are compared from the sawmill's point of view for two experimental cases. In Case 1 the sawmill sells its entire lumber production to the market (including to an independent value-added facility). In Case 2, the sawmill sells only the lumber that it is not directed to the value-added facility for further processing. Net revenue for Case 2 exceeds the net revenue of Case 1 by 10%. Results shown demonstrate that production decisions in the value-added facility had a significant influence on production decisions in the sawmill.

Keywords: Value-added lumber, production planning, linear programming, optimization.

INTRODUCTION

In recent years, the secondary wood products industry has received increasing attention and study in North America by both governments and industry. As harvest volumes have declined due to increased land set-asides and an increased environmental awareness, many people have looked to the secondary manufacturing industry to maintain employment in forest-dependent communities.

Secondary wood processing facilities employ a variety of equipment to add value to primary wood products, such as: resaws, chop-saws, finger-jointers, edge-gluers, and molders. Because of the increased complexity of this equipment, numerous production programs can be developed for adding value to the raw material mix. These production programs have to provide increasing flexibility for meeting market demands for forest products.

For years independent lumber remanufacturers have taken advantage of opportunities for reworking the wood produced by primary lumber manufacturers. Successful entrepreneurs in this business have a keen awareness of the market and the ability to buy lower

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grade lumber. They are typically small companies with high manufacturing flexibility and a low capital cost structure.

Manufacturing decisions should be made earlier in the primary production process so that companies respond promptly to the needs of the remanufacturing facilities. Many integrated forest manufacturing companies are now closely examining the methods and benefits of adding value to their current products (Cohen 1992).

However, the decision complexity makes it difficult to fully use production flexibility. Managers have to make several difficult planning decisions for the overall production process, such as: what lumber mix they need in order to produce the current order file, and how much of each product is to be further manufactured by which process. At the same time, they must ensure they are using maximum capacity while minimizing the production of products with low market value.

As a whole, the planning problem becomes too large to be solved effectively with a spreadsheet. One advanced technique that has been used with success is linear programming.

There have been a number of applications of linear programming in the wood products industry (McKillop and Hoyer-Nielson 1968; McPhalen 1978; Mendoza 1980; Yaptenco and Wylie 1970; Wellwood 1971). Most of these applications are proposals of methodologies to deal with problems in the industry, demonstrating their effectiveness with realistic scenarios. Dynamic programming has also been used to deal with manufacturing optimization. For example, Faaland and Briggs (1984) proposed a method for optimizing log bucking and lumber manufacturing using dynamic programming.

For complete optimization in a facility that integrates both primary and secondary processing, a decision support system is required that will model the production process from stem to finished product. This is because decisions that are made early in the primary manufacturing process can directly impact the decisions made in the secondary manufacturing facility. As a result, the overall optimization model should include stem bucking and log sawing optimization in the sawmill, and also take into account the production processes in secondary manufacturing. Ideally, this model should be easy to use by managers in day to day operations.

For a remanufacturing facility, raw material purchasing is one of the most important decisions since it is always constrained by what is available on the market. If an integrated decision support system were applied, the sawmills could consider the raw material constraints, and the system could provide recommendations on what materials should be produced and in what quantity.

Therefore, integrated remanufacturing operations have control over the volumes available of the desired raw materials produced in the primary operations. Although constrained by the characteristics of their timber supply, they still have significant control in making more of the desired intermediate lumber products. This can be best accomplished by manipulating the sawmill's process control equipment.

Modern sawmills apply process control equipment at all stages of the manufacturing process: log bucking, primary breakdown, edging, trimming, and sorting. Computer programs controlling the equipment generate processing solutions by:

1) using piece scan data and generating a sawing solution for the piece based on the value or volume of the products it is capable of yielding, or

2) using a predetermined solution from a look-up table for the corresponding piece dimensions.

Adjusting the volume of a particular product can be done by modifying the values of the desired products or by adjusting the solutions in the look-up table. In either case, generating an acceptable product mix is a trial and error process that is time-consuming and costly. Moreover, once an acceptable product mix is achieved, it may not be the best acceptable mix.

This paper provides a method that, when applied to a mill's process control equipment, will produce the most profitable product mix that will also benefit the remanufacturing processes.

PRODUCTION PLANNING SYSTEM

Two linear program (LP)-based production planning models were developed in this research to respond to this need. The first model is a production planning model designed for nonintegrated value-added facilities, the VAF model. This model demonstrates how linear programming can be used by nonintegrated lumber remanufacturing companies to optimize their production planning process.

The second model is an integrated production planning system that optimizes the production from the sawmill log yard through secondary manufacturing. This is achieved by combining a revised version of the Sawmill Production Control Model (SPCM) developed by Maness and Adams (1991) with the VAF model. This combined model encompasses the entire integrated value-added manufacturing process from long-log to finished product and includes the key benefits of the Sawmill Production Control Model: the combined optimization of bucking and sawing. The resulting integrated model could be used to illustrate the benefits of integrating primary and secondary manufacturing as opposed to using separate models for each purpose. The model formulations are described in the appendix.

The two models were field-calibrated to model the operations of an existing integrated value-added manufacturing facility in western Canada (hereafter called the Study Plant).

STUDY PLANT

The raw material source for the Study Plant is lodgepole pine (*Pinus contorta,* Doug. ex. Loud). The mill processes logs with small end diameters ranging from 3.5 in. (9 cm) to approximately 16 in. (40.5 cm). The operation's sawmill runs a long-log merchandiser, a large log primary breakdown line, a small log primary breakdown line, two optimizing board edgers, a trimmer, and a multi-bin sorter.

Both sawing lines break down logs with the split taper log, split taper curve sawing cant method (Wilson 1992). In both cases, logs are rotated "horns-down" prior to canting. The small line sawing patterns are determined by an optimization system, while the large line sawing patterns are selected from a set table.

After kiln-drying, rough sawn lumber is graded, trimmed, and sorted into lumber products. These lumber products are the raw material for the value-added facility.

The value-added facility (VAF) consists of a resaw and a molder. Both machines may share three sorts. The resaw and the molder may run in combination with the resaw feeding the molder or they may run separately, both being fed from their own respective infeeds. Provided that the combined number of sorts required is not more than three, the molder and the resaw may be run separately or simultaneously.

Generally, one sort is dedicated to a finished premium grade value-added product, another sort is dedicated to a mid-grade value-added product, and the last sort is dedicated to subgrade products. With resawing capabilities and the ability to rerun products through the VAF, the possibility exists for numerous complex VAF production schedules. This creates the problem of planning efficient production schedules.

The value-added facility produces the following product dimensions: 24×72 mm, 24×89 mm, 25×110 mm, 25×89 mm, 30×125 mm, 30×150 mm, 30×72 mm, $5/4 \times 5$ in, and $5/4 \times 6$ in, with lengths of 72, 84, 96, 108, 120, 144, 156, 168, and 192 inches. These products correspond to architectural millwork, parts for the Japanese housing construction, door and window parts, flooring, and furniture parts.

APPLICATION OF THE MODEL

The primary focus of this research was to develop a comprehensive production planning model that encompassed the entire integrated value-added manufacturing process. The purpose was to ensure that production policies of the value-added process are allowed to influence the decisions of bucking and sawing in the sawmill in producing rough stock for the value-added facility. After the model was developed, the senior author spent 6 months at the Study Plant's site collecting model input data and validating that the model successfully predicted production. Validation was done according to Williams' (1990) method for linear programming models.

The remainder of this paper illustrates the use of both the independent VAF model and the Integrated Model, and investigates the influence of the value-added production decisions on the bucking and sawing decisions. This will demonstrate how the two models can be used by integrated forest products companies to determine if adding a value-added production facility would be more beneficial to their operations.

Procedures and methodology

In investigating this question, analyses were made using the revised SPCM and the VAF model separately (Case 1) and then combined (Case 2). By comparing the results of the two cases, it is possible to gauge the effect that the value-added process can have on bucking and sawing decisions in the study plant.

The revised SPCM was applied first. The results of this run were a bucked log distribution and a lumber production distribution. The controlling input parameters were a set of lumber values and production targets for some products. Overproduction (inventory) and underproduction (shortage) costs were applied such that a feasible lumber product mix was achieved with minimum amounts of over- and underproduction.

The resulting lumber mix was then used as the maximum possible input for the stand-alone VAF model. The VAF model run was made to generate the value-added product mix and the associated production costs and revenues. The costs and revenues were then combined with the raw material and production costs incurred with the SPCM run to produce an overall financial statement for the two runs combined with the two facilities working independently.

Next, the combined model was run. In this case, value-added production was controlled by applying production targets with over-and underproduction costs to the value-added products themselves, rather than the lumber products as with the SPCM run alone.

Input parameters

Model input parameters are consistent with the mill's current operating parameters. These parameters include stem raw material mix, product dimensions, saw kerfs, machine production rates, value-added process options and yields, operating costs, and product values.

The lumber values used were provided by the mill personnel. The planning period for both cases was one month. This was defined by limiting the operating hours of the two sawmill breakdown lines to the number of available operating hours in one month.

Full details of the inputs used in this research project are available in Donald (1996).

RESULTS AND DISCUSSION

The results of the two cases are summarized in the series of tables below. Table 1 provides operating statements for the two cases. In Case 1, the sawmill produces lumber (12,714 m^3) and sells 2,516 m^3 of its production to the independent value-added facility at market prices. Based on market prices, 2,516 m^3 is the optimal amount of lumber, relative to the sawmill, to be sold to the value-added facility. The value-added facility then processes the 2,516 m^3 of lumber into 2,449 m^3 of value-added products. Total profit over the entire production run from the sawmill's lumber output is \$1.925 million.

In Case 2, however, the sawmill is integrated with the value-added facility. Therefore, there are no raw-material costs of producing

TABLE 1. *Operating statements.*

Operating statement	Case 1 Volume	Case 1 Revenues	Case 2 Volume	Case 2 Revenues
SPCM				
Lumber sales (m^3)	12,714	$3,244,452	7,382	$1,859,345
Chips (tons)	11,931	$ 125,275	12,069	$ 126,724
Total revenues		$3,369,727		$1,986,069
	Volume	Costs	Volume	Costs
Raw material (m^3)	28,143	$ (844,288)	28,384	$ (851,508)
Sawmill costs (hrs)	720	$ (600,120)	720	$ (600,120)
Total costs		$(1,444,408)		$(1,451,628)
Net revenue		$1,925,320		$ 534,441
VAF				
VAP sales (m^3)	2,449	$ 817,471	5372	$1,810,804
Chips (tons)	0	$—	0	$—
Total revenues		$ 817,471		$1,810,804
	Volume	Costs	Volume	Costs
Raw material-lumber (m^3)	2,516	$ (651,998)	5,444	
Drying (hrs)	605	$ (12,094)	1,353	$ (27,054)
Dry-sorting (hrs)	93	$ (79,339)	209	$ (177,649)
Molding (hrs)	160	$ (12,024)	305	$ (22,879)
Total costs		$ (755,456)		$ (227,583)
Net revenue		$ 62,015		$1,583,221
			Increase	
Sawmill Net Revenue		$1,925,320	10%	$2,117,662

value-added products; the value-added facility simply takes whatever products it wants from the sawmill's output, and the VAFs profit is added to that of the sawmill. It is very important to see how, in Case 2, the VAF model influences the SPCM model in producing more lumber that is destined for the value-added facility (5,444 m^3), and less is sold to outside customers.

Table 2 lists the bucked log distributions by length for Cases 1 and 2. It is clear that in this case there are no significant differences in the bucking strategies (see also Fig. 1). This is expected since there are no major differences in prices for different lengths in the Study Plant. However, in a typical sawmill where prices vary a great deal by length, there is expected to be even larger differences than those found here.

The distributions of lumber products produced in the sawmill for both cases are listed

TABLE 2. *Bucked log distributions, volumes (m^3).*

Length (cm)	120	144	156	168	192	Trim loss
Case 1	3,293	4,561	6,396	3,915	8,967	1,011
Case 2	3,364	4,344	6,138	3,744	9,795	997

(Column heads 120–192 fall under: Bucked Log Distributions by Length (cm))

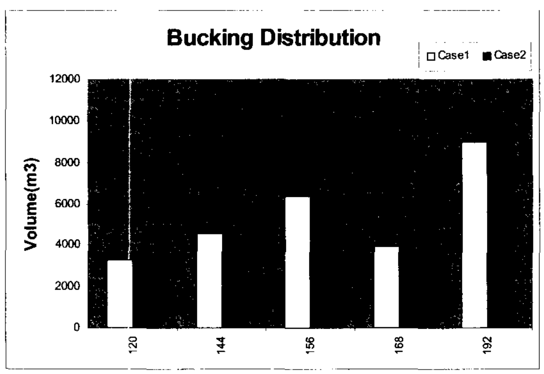

FIG. 1. Bucked Log Distribution, Volumes (m^3)

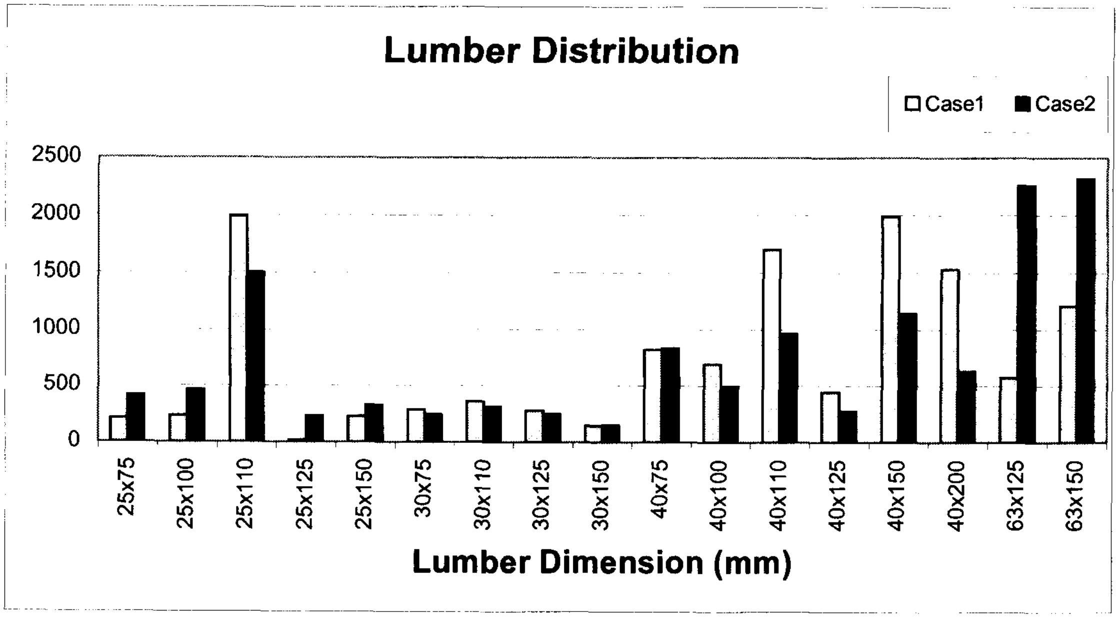

FIG. 2. Sawmill Lumber Distribution by Volume (m^3)

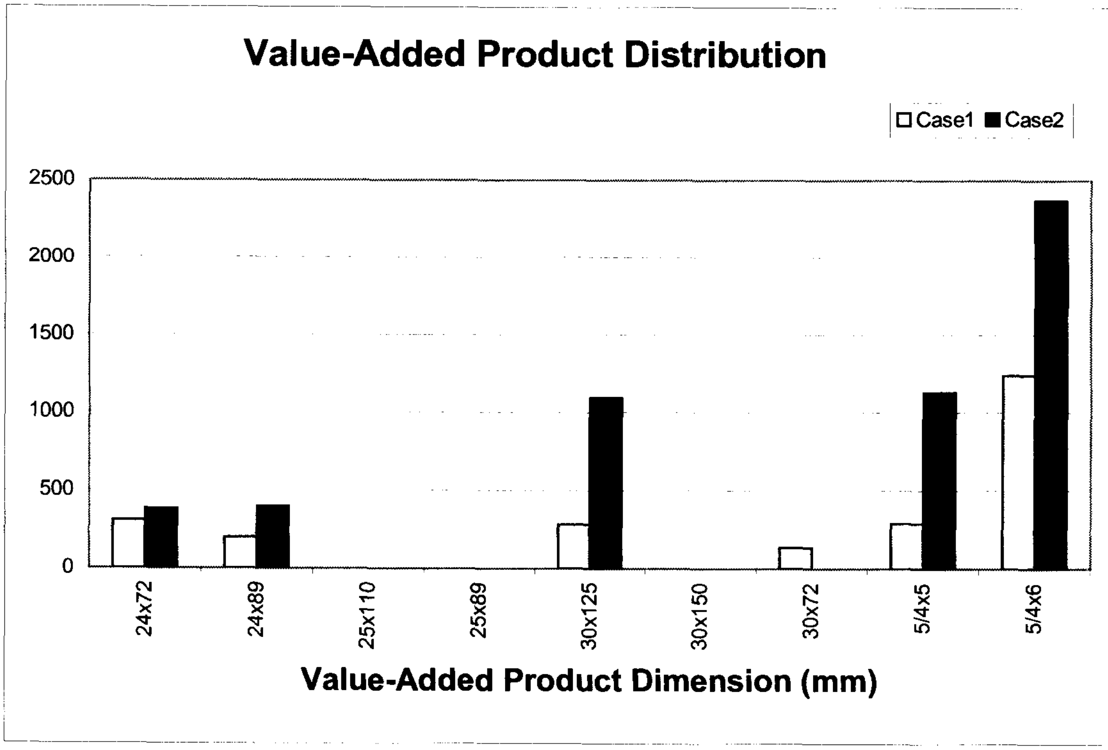

FIG. 3. Value-Added Product Distribution by Volume (m^3)

TABLE 3. *Sawmill lumber production by volume (m^3).*

Lumber dimensions	Lumber volume (m^3) Case 1	Case 2
25 × 75	210	414
25 × 100	232	464
25 × 110	1,984	1,502
25 × 125	16	233
25 × 150	228	331
30 × 75	288	245
30 × 110	362	320
30 × 125	279	251
30 × 150	146	152
40 × 75	823	839
40 × 100	690	494
40 × 110	1,702	963
40 × 125	447	277
40 × 150	1,987	1,143
40 × 200	1,534	633
63 × 125	576	2,252
63 × 150	1,209	2,314
Total	12,714	12,827

in Table 3. This table shows that in Case 2, the sawmill produces significantly more lumber in 63-mm thickness (4,566 m^3 vs. 1,785 m^3) and less in the 40-mm and 30-mm thickness (5,317 m^3 vs. 8,258 m^3).

Table 4 shows the differences between the two cases in terms of the value-added products manufactured. The most notable difference between the two cases is the large increase in the production of 5/4 stock in the integrated case (3,500 m^3 vs. 1,528 m^3), and a fourfold increase in the production of 30 × 125-mm stock (1,094 m^3), vs. 280 m^3).

Most of the 10% uplift form Case 1 to Case 2 is attributable to increases in these two product groups. This stands to reason in that the 5/4 stock and the 30 × 125 are generated at the VAF by resawing and molding 63-mm lumber. There are three factors that contribute to the uplift as a result of this. First, the 63-mm stock is a good productivity stock for the VAF as the volume per piece is high relative to all other stock products. Second, the average prices for the 5/4 and 30 × 125-mm stock are high relative to the other VAF products. These two factors drive the sawmill to produce 63-mm stock in the Case 2 model. The third factor is that 63-mm is a good volume recovery product to produce at the sawmill as less wood is lost to saw kerf. In Case 1 the volume recovery associated with the sawing 63-mm products is offset by the value recovery associated with sawing 40-mm products.

TABLE 4. *Value added production distribution.*

VAF products	Case 1 m^3	Case 1 Percent	Case 2 m^3	Case 2 Percent
24 × 72	304	12.4%	381	7.1%
24 × 89	198	8.1%	396	7.3%
25 × 110	0	0.0%	0	0.0%
25 × 89	0	0.0%	0	0.0%
30 × 125	280	11.4	1,094	20.4%
30 × 150	0	0.0%	0	0.0%
30 × 72	138	5.6%	0	0.0%
5/4 × 5	289	11.8%	1,130	21.0%
5/4 × 6	1,239	50.6%	2,370	44.1%
Total	2,448	100.0%	5,371	100.0%

From this it can be seen that the VAF yield information in the combined model has considerable influence on the sawing decisions in the sawmill for the rough products that are remanufactured into these particular VAF products. When the two models are working separately, these types of opportunities cannot be capitalized upon.

SUMMARY AND CONCLUSIONS

The value-added sector of the forest products industry continues to evolve. The forest industry seeks out opportunities to pull more value from a resource, which is growing in cost. Along with these value-added strategies comes the complexity of management decisions in the manufacturing process. As demonstrated here, this complexity can hinder the performance of the industry.

Two models were developed to assist managers in the value-added sector in their production planning decisions. The first model is designed for non-integrated value-added facilities and assumes that raw materials are purchased from outside sources. The second model is designed for integrated value-added facilities with the ability of producing their own VAF raw materials from their primary operations. This second model is an extension of the revised SPCM model described by Maness and Adams. The significance of this model is that it is a comprehensive model encompassing the entire value-added manufacturing process from long-log to finished products, thereby allowing production policies in VAF manufacturing to affect the combined optimization of bucking and sawing in sawmilling.

In exploring the influences that VAF policies may have on sawmilling decisions, an analysis was conducted that compares the use of the integrated model with the SPCM model and the independent VAF model used separately. The results of this analysis indicated that VAF production policies have a significant influence on production decisions in the sawmill, and yield a 10% increase in revenue from the sawmill's perspective if the two facilities are integrated.

The results of this study should now be validated by practical testing and field use of the model. Practical testing will answer the questions of how useful the models are in real applications and how easily they can be used and understood by mill personnel with little or no background in mathematical programming.

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APPENDIX: MODEL DESCRIPTIONS

Sawing pattern generator

The SPCM sawing pattern generator and other commercially available packages were incapable of modeling the complex gang saw configurations of the Study Plant. Thus, it was necessary to develop a pattern generator that could adequately model the Study Plant's primary breakdown.

The sawmill's curve sawing primary breakdown system is capable of sawing logs with up to 10.2 cm (4 in.) of sweep in 4.9 m (16 ft) (Wilson 1992). Almost all the logs coming into the mill have less sweep than this degree of sweep. Therefore, modeling the sawmill's sawing capabilities was simply dealt with by assuming that logs arriving at the mill were straight truncated cones.

Sawing pattern optimizer.—The algorithm is an ad hoc procedure, which is best described as an exhaustive search algorithm. Generalized procedures of the algorithm are described as follows.

1. For the cross section at the distance equal to the minimum lumber length measured from the large end of the truncated cone, find the largest cant width that fits in the log and the corresponding cant height. The cant width is obtained in the manner described by Zheng et al. (1989).

2. Within that cant height, generate a two-dimensional sawing pattern for every conceivable lumber thickness combination. And for each combination apply the pattern throughout the entire log length to generate a three-dimensional cant sawing pattern. Select the most valuable three-dimensional cant pattern.
3. With this cant pattern, for each possible combination of side boards, add the side board value to the cant value. Select the side board combination that yields the greatest value when added to the cant value. This will be the most valuable sawing pattern for the given cant width.
4. Repeat steps 1 to 3 for all remaining cant widths that are possible to fit in the log. Select the most valuable sawing pattern as the optimal sawing pattern.

VAF LP formulation

The VAF LP was formulated to model the Study Plant's manufacturing processes subsequent to the sawmilling process. This is the manufacture of rough sawn green lumber into finished value-added products. The manufacturing steps include kiln-drying, dry-sorting, resawing, molding, sorting, rerunning, and packaging. The LP model is formulated as follows.

Maximize:

$$\begin{aligned} -\sum_k cRM_k \times RM_k - \sum_j cCGrp_j \times CGrpHrs_j \\ - \sum_m \{prDS_m \times DS_m - cODS_m \times ODS_m - cUDS_m \times UDS_m\} \\ + \sum_n \{prVA_n \times VA_n - cOVA_n \times OVA_n - cUVA_n \times UVA_n\} + prChips \times Chips \\ + prResidue \times Residue \end{aligned} \tag{1}$$

subject to:

$$RM_k - KlnChrg_k \times VolKln_k = 0 \quad \text{(for all raw material products } k\text{)} \tag{2}$$

$$\sum_k \{RM_k \times DSrate_k\} - DSHrs = 0 \tag{3}$$

$$\sum_l \{VAOp_l \times Eqrate_{il}\} - EqHrs_i = 0 \quad \text{(for all } i \text{ pieces of equipment)} \tag{4}$$

$$\sum_k \{KlnChrg_k \times HrsKlnChrg_k\} - KlnHrs = 0 \tag{5}$$

$$EqHrs_i - CGrpHrs_j \leq 0 \quad \text{(for all } i \text{ pieces of equipment if belonging to cost group } j\text{)} \tag{6}$$

$$\sum_k \{RM_k \times RMDSyld_{km}\} - \sum_l \{VAOp_l \times VADSyld_{lm}\} - DS_m = 0 \quad \text{(for all } m \text{ dry sorted products)} \tag{7}$$

$$\sum_l \{VAOp_l \times VAyld_{ln}\} - VA_n = 0 \quad \text{(for all } n \text{ value added products)} \tag{8}$$

$$\sum_l \{RM_k \times RMChpyld_k\} + \sum_l \{VAOp_l \times VAOpChpyld_l\} - Chips = 0 \tag{9}$$

$$\sum_k \{RM_k \times RMRsdyld_k\} + \sum_l \{VAOp_l \times VAOpRsdyld_l\} - Residue = 0 \tag{10}$$

$$DS_m - ODS_m + UDS_m = DSdemand_m \quad \text{(for all } m \text{ dry sorted products)} \tag{11}$$

$$VA_n - OVA_n + UVA_n = VAdemand_m \quad \text{(for all } n \text{ value-added products)} \tag{12}$$

$$RM_k \leq Avail\_RM_k \quad \text{(for all } k \text{ raw material products)} \tag{13}$$

$$KlnHrs \leq Avail\_KlnHrs \tag{14}$$

$$DSHrs \leq Avail\_DSHrs \tag{15}$$

$$EqHrs_i \leq Avail\_EqHrs_i \quad \text{(for all } i \text{ pieces of equipment)} \tag{16}$$

All variables are non-negative.
Where

$Avail\ DSHrs$ = the available number of Dry Sorter production hours;
$Avail\ EqHrs_i$ = the available number of production hours for piece of equipment i.
$Avail\ KlnHrs$ = the available number of kiln hours;
$Avail\ RM_k$ = the available procurable volume of raw material product k;
$cCGrp_j$ = the cost per hour of cost group j;
CEq_i = the cost per hour of using piece of equipment i;
$CGrpHrs_j$ = the number of hours used of cost group j;
$Chips$ = volume of chips produced;
$cKlns$ = the cost per hour of operating the dry kilns;
$cODS_m$ = the cost of overproducing dry sorted product m;
$cOVA_n$ = the cost of overproducing value-added product n;
cRM_k = the cost per unit of volume of procuring raw material product k;
$cUDS_m$ = the cost of underproducing dry sorted product m;
$cUVA_n$ = the cost of underproducing value-added product n;
$cVAOp_l$ = the cost of using value-added option l;
$DSdemand_m$ = the demand for dry sorted product m;
$DSHrs$ = the number of hours the dry sorting system operates;
DS_m = the volume of dry sorted product m produced;
$Dsrate_k$ = the rate raw material product k can be dry sorted;
$EqHrs_i$ = the number of hours used of piece of equipment i;
$Eqrate_{il}$ = the rate piece of equipment i processes material under value-added option l;
$HrsKlnChrg_k$ = number of hours per kiln charge of raw material product k necessary to dry raw material product k;
$KlnChrg_k$ = the number of kiln charges of raw material product k;
$KlnHrs$ = the number kiln hours used;
ODS_m = the volume of dry sorted product m overproduced;
OVA_n = the volume of value-added product n overproduced;
$prChips$ = the sales price for chips;
$prDS_m$ = the sales price of dry sorted product m;
$prResidue$ = the sales price of wood residue products;
$prVA_n$ = the sales price of value-added product n;
$Residue$ = the volume of wood residue produced;
$RMChpyld_k$ = the yield of chips as a result of dry sorting of raw material product k;
$RMDSyld_{km}$ = the yield of dry sorted product m as a result of dry sorting raw material product k;
Rm_k = the volume of raw material product k procured;
$RMRsdyld_k$ = the yield of wood residue as a result of dry sorting raw material product k;
UDS_m = the volume of dry sorted product m underproduced;
UVA_n = the volume of value-added product n underproduced;
$VAdemand_n$ = the demand for value-added product n;
$VADSyld_{lm}$ = the yield of dry sorted product m from the volume of material processed by value-added option l;
VA_n = the volume of value-added product n produced;
$VAOpChpyld_l$ = the yield of chips from the volume of material processed by value-added option l;
$VAOp_l$ = the volume of material processed by the value-added option l;
$VAOpRsdyld_l$ = the yield of wood residue from the volume of material processed by value-added option l;
$VAyld_{ln}$ = the yield of value added product n from the volume of material processed by value-added option l;
$VolKln_k$ = the volume of raw material k per kiln charge;

Objective function.—The objective function is a maximization function whereby the raw materials costs, processing costs, and production penalties are subtracted from the revenues from dry sorted products, value-added products, chips, and residues.

The first statement of the objective function sums the raw material costs.

The second statement sums the processing costs. The equipment of the VAF facility was divided up into cost groups. In each cost group, the piece of equipment that operates for the most hours determines the number of operating hours for the cost group. The processing costs are determined by the summation of multiplying the cost per hour of each equipment cost group j by the number of hours utilized of equipment cost group j.

The second and third statements of the objective function sum the net revenues for the dry sorted products and the value-added products, respectively. The revenues are net of over and underproduction penalties.

The last two statements of the objective function determine the revenues for chips and residues respectively.

Key decision variables.—The *VAOp* variables are the key decision variables. These variables indicate the volume of input material to be processed by value added option l. Value-added options consist of a specific input material (either a dry sorted product or a previously produced value-added product), equipment requirements, production rates, and the resulting value-added product yields, residue yields, and chip yields.

Constraint definitions.—Constraint (2) composes the raw material into kiln charges.

Constraints (3), (4), and (5) are VAF capacity constraints, which ensure that the available operating hours for the dry-sorter, kilns and value-added processing equipment are not exceeded.

Constraints (6) allow the equipment to be grouped together into cost groups with respective operating costs (for the reason described in the discussion of the objective function).

Constraints (7), (8), (9), and (10) are material balance constraints that ensure the volume of products and by-products produced equals the volume of sorted products. Constraints (7) recognize that dry-sorted products may be produced from two sources: directly from the raw material, or indirectly from other dry-sorted products as a result of the application value added policies. Constraints (8) are material balance constraints that contain the yield coefficients for value-added products from the dry-sorted products as a result of applying a value-added policy. Constraints (9) and (10) are material balance constraints that contain the yield coefficients for the by-products chips and residue.

Constraints (11) and (12) are the dry-sorted and value-added product market demand constraints. Over-and underproduction are controlled by the application of inventory carrying costs and shortage costs to over-and underproduction of each product.

Constraints (13), (14), (15), and (16) are raw material and production capacity constraints that impose upper bounds on the available volume of raw material, kiln hours, dry sorter hours, and VAF equipment hours respectively.

Revised SPCM LP formulation

The Sawmill Production Control Model is a large-scale optimization model that combines linear and dynamic programming routines to model the production at a sawmill. The mathematical formulation of the model is described by Maness and Adams (1991). The model has been used in industrial applications to determine the optimal bucking and sawing strategies for given raw material availability in the form of logs, sawing technology, and market considerations.

The VAF model was linked to the SPCM model through a simple change in the formulation of the market constraints in the SPCM model. In SPCM the marketing constraints are as follows:

$$\sum_{N} \{Lum\_Prod_{ilN}\} - \{Lum\_Sales_{il}\} = 0 \qquad (17)$$

This constraints ensures that all lumber production (Lum_Prod) from the SPCM model is sold (Lum_Sales). This constraint was changed to allow SPCM to either sell lumber directly, or to process it through the value-added facility as shown in the following constraint:

$$\sum_{N} \{Lum\_Prod_{ilN}\} - \{Lum\_Sales_{il}\} - \{RM_{il}\} = 0 \qquad (18)$$

In this formulation, the SPCM model provides the raw material input for the VAF and the two models are linked through the *RM* variables. This constraint now allows rough lumber to be sold as *Lum_Sales* or to proceed through the VAF as raw material input *RM*. The *RM* variables from the independent VAF model represent the volume of each raw material product procured for VAF processing.

This linkage changes the independent VAF formulation slightly in that the *RM* variables in the independent model were constrained by upper bounds on the procurable volume available (constraint (13)). In the combined model, the sawmill supplies the raw material; therefore, the *RM* variables are now constrained by the SPCM constraints on raw material availability and primary manufacturing. Consequently, in the combined model the upper bounds on the *RM* variables are not necessary.

Other minor changes to the Sawmill Production Control Model's log sawing model were made to ensure the system adequately modeled the Study Plant's primary breakdown system. Changes of this type are typical whenever SPCM is introduced to a sawmill to ensure compatibility. Full details of the changes to SPCM can be found in Donald (1997).