A MODIFIED ELASTIC APPROACH TO THE THEORETICAL DETERMINATION OF THE HYGROSCOPIC WARPING OF LAMINATED WOOD PANELS[1]

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ABSTRACT

This paper deals with the hygroscopic warping of laminated wood and wood composites. The theoretical analysis includes elastic and inelastic approaches. Experiments were conducted on narrow cross-laminated yellow-poplar beams and on two-ply beams constructed from laminas of medium-density fiberboard and particleboard. The elastic strains and the swelling stresses that sustain warp under conditions of increasing moisture content were determined experimentally, using a specially designed restraining device.

In the case of the yellow-poplar laminates, it was found that in the transverse direction (tangential) only about 25% of the free hygroscopic expansion is transformed into elastic strain under conditions of complete restraint. In the longitudinal direction, yellow-poplar behaves elastically. These results were used to modify the inputs for the elastic equation (inelastic approach), which greatly improved the accuracy of the theoretical warping predictions.

In the case of laminate composites, the elastic equation without modification produced good agreements with measured warp.

Keywords: Plywood, restrained swelling, swelling stresses, warping, wood composites.

INTRODUCTION

The center deflection (warp) of a laminated beam due to internal stresses caused by differential hygroscopic expansion of individual layers (laminas) can be calculated using the following equation (Norris 1964; Suchsland and McNatt 1985).

$$W = \frac{L^2}{8}\,\frac{\dfrac{\sum_1^n \alpha_i E_i(S_i^2 - S_{i-1}^2)}{\sum_1^n E_i(S_i^2 - S_{i-1}^2)} - \dfrac{\sum_1^n \alpha_i E_i T_i}{\sum_1^n E_i T_i}}{\dfrac{2\sum_1^n E_i(S_i^3 - S_{i-1}^3)}{3\sum_1^n E_i(S_i^3 - S_{i-1}^3)} - \dfrac{\sum_1^n E_i(S_i^2 - S_{i-1}^2)}{2\sum_1^n E_i T_i}} \quad (1)$$

where W = deflection at center span of laminated beam (mm)

$S_i = \Sigma\, T_i$ (mm)

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α_i = expansion value for given exposure interval of layer i (mm/mm)
E_i = modulus of elasticity at end condition[3] of layer i (kPa)
T_i = thickness of layer i (mm)
L = length of beam (mm)

This equation is based on the assumption that all laminas behave elastically. This assumption is not justified for wood stressed in the cross-grain direction (radial or tangential) (Keylwerth 1962). In those directions, the modulus of elasticity is very low, and the expansion or shrinkage values are very high. In the grain direction (longitudinal), wood is more nearly elastic. In that direction, the modulus of elasticity is very high, and the expansion or shrinkage values are very low.

Since internal stresses in wood cross-laminates undergoing moisture content changes are caused by mutual restraint of adjacent laminas with different expansion or shrinkage characteristics, the above-mentioned combination of properties causes large strains to occur across the grain of the laminas, and small strains to occur along their grain.

There is evidence (Keylwerth 1962; Lang et al. 1995) that such large deformations across the grain are not completely elastic and that associated swelling stresses will not reach their theoretically predicted levels. The above equation may, therefore, produce inaccurate warping predictions. To improve the utility of the above equation, it is necessary to modify the inputs α and E for the cross-grain directions of the laminas.

The objective of this study was to experimentally determine modified input values and to test the modified inputs by comparing the warping predictions of the equation with measured warp of various unbalanced laminated beams. This approach is called here the "modified elastic" or "inelastic" approach.

The above beam equation can, of course, be used for the warping analysis of plates by applying it to both principal directions of the plate (Suchsland and McNatt 1985).

THE CONCEPT OF RESTRAINT SWELLING IN LAMINATED WOOD BEAMS

We will consider the model drawing in Fig. 1, identified as 'Case I'. It shows a laminate consisting of two laminas of different elastic and hygroscopic characteristics, but of equal thicknesses.

Lamina I has a hygroscopic expansion value (α_1) of practically zero, and a very high modulus of elasticity (E_1). Lamina II has a very large expansion value (α_2) and a very low modulus of elasticity (E_2)[4].

During a moisture content increase, lamina II, if free of restraint, would expand by α_2. The length of lamina I would not change. Since the two laminas are glued together, their expansion must be identical. If E_1 was very, very large, the expansion of the laminate would be near zero, which means that the free expansion of lamina II would be completely restrained by lamina I.

The condition of lamina II would thus be equivalent to first allowing it to expand freely, and then compressing it back to its original length. If this large compression strain (ϵ_2) would completely transform into elastic strain, then Eq. 1 would be valid. If only a portion transforms into elastic strain, then this elastic portion ($\epsilon_{2\,elast}$) should be substituted for α_2 in Eq. 1. This is demonstrated in the upper portion of Fig. 1, where the development of the elastic compression strain component is shown on the vertical axis.

Also, instead of a statically determined modulus of elasticity, E_2, at the end condition, a 'deformation modulus', $E'_2 = \sigma'/\epsilon_{2elast}$, should be used. The stress, σ', is the actual stress developed in the expanded lamina II after compressing it to its original length. Both $\epsilon_{2\,elast}$ and σ', and therefore E'_2, can be determined by experiment. The conditions described by 'Case

[3] End condition is equilibrium at end of exposure cycle.

[4] α-values apply to a given exposure interval; E-values apply to condition at end of exposure interval.

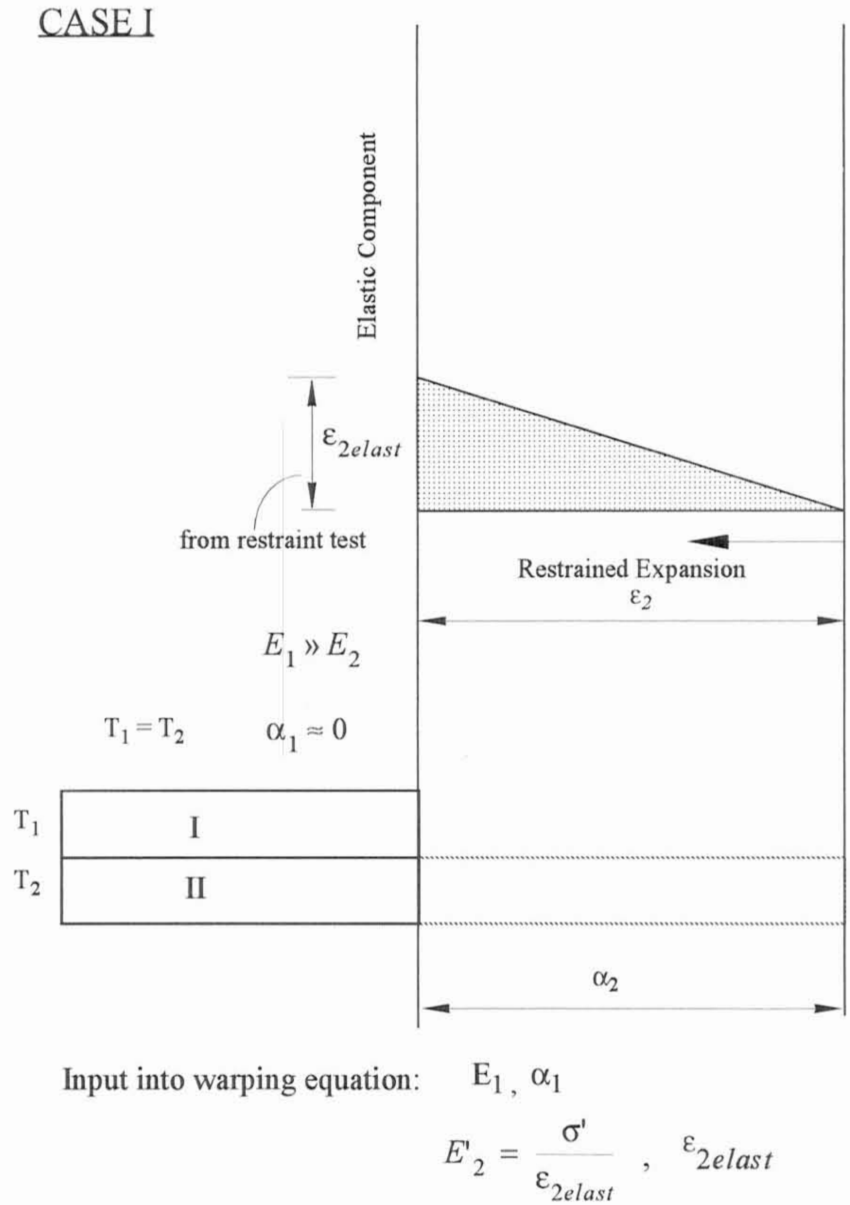


FIG. 1. Model of restrained swelling of laminas in two-layer unbalanced beam. Case I: complete restraint.

I' are approached by a two-layer wood cross-laminate.

A more general case is illustrated by the model drawing in Fig. 2, identified as 'Case II'. Here, compared with 'Case I', both layers have moderate, but unequal, expansion values, and moderate, but unequal, moduli of elasticity.

After a moisture content increase, the laminate would expand by α_{res}, identified by the condition of force balance in the laminate, which would cause tensile stresses to occur in

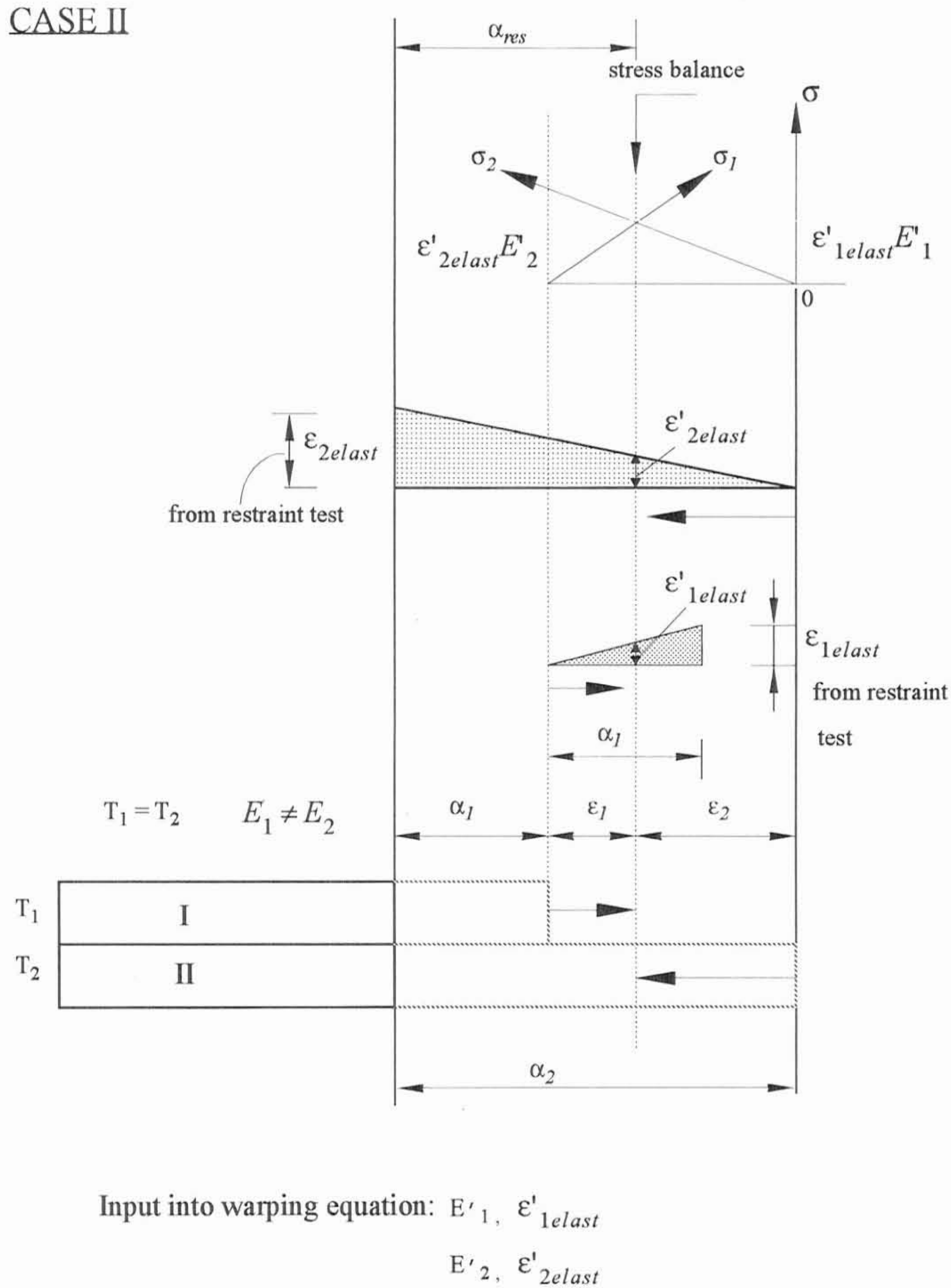


FIG. 2. Model of restrained swelling of laminas in two-layer unbalanced beam. Case II: partial restraint.

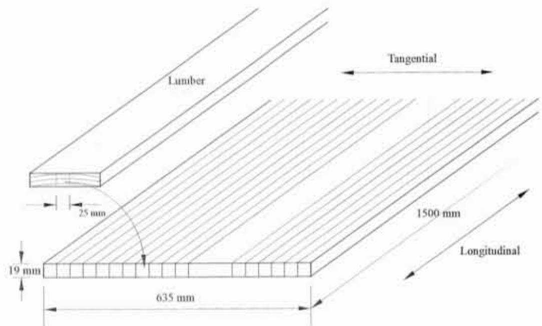


FIG. 3. Design of yellow-poplar test panel.

lamina I and compressive stresses in lamina II. In analogy to 'Case I'

$$\sigma'_1 = \epsilon_{1\ \mathrm{elast}} {*} E'_1$$
$$\sigma'_2 = \epsilon_{2\ \mathrm{elast}} {*} E'_2$$

and at force balance, since $T_1 = T_2$:

$$\sigma'_1 = \sigma'_2$$
$$\epsilon_{1\ \mathrm{elast}} {*} E'_1 = \epsilon_{2\ \mathrm{elast}} {*} E'_2$$

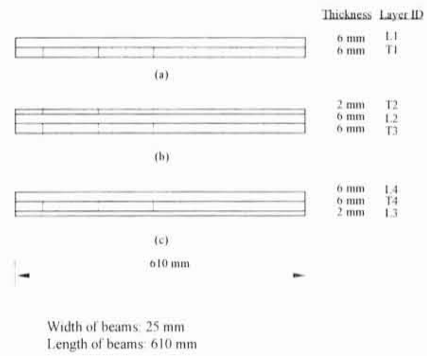


FIG. 5. Design of unbalanced yellow-poplar cross-laminated beams. Increase in relative humidity will cause beams to warp concavely upward. Note: L = longitudinal, T = tangential.

For this condition of partial restraint, these elastic strain components are derived from total restraint conditions by assuming linearity, as indicated in Fig. 2. Since the appropriate tensile tests are much more difficult to conduct than compression tests, the important as-

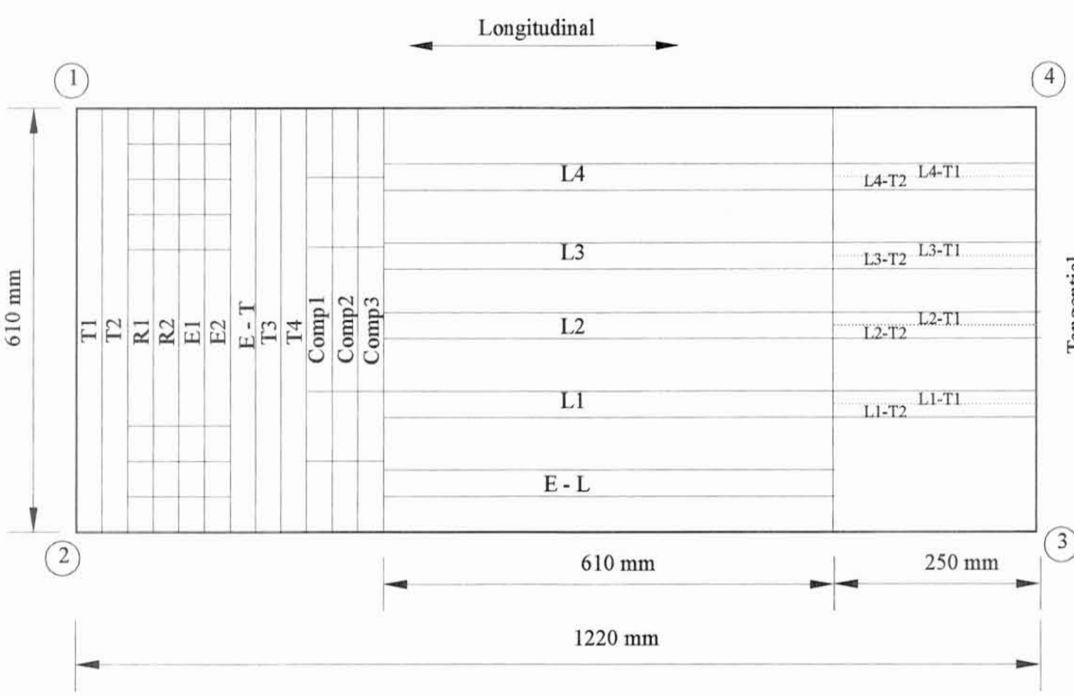


FIG. 4. Specimen arrangement on yellow-poplar test panel. E-L, E-T: longit. and tang. free hygroscopic expansion specimens; L1. . ., T1. . .: longit. and tang. beam laminas; L1T1. . .: modulus of elasticity (tension) specimens; Compl. . .: modulus of elasticity (compression) specimens; R1. . .: restraint test specimens; E1. . .: free expansion specimens.

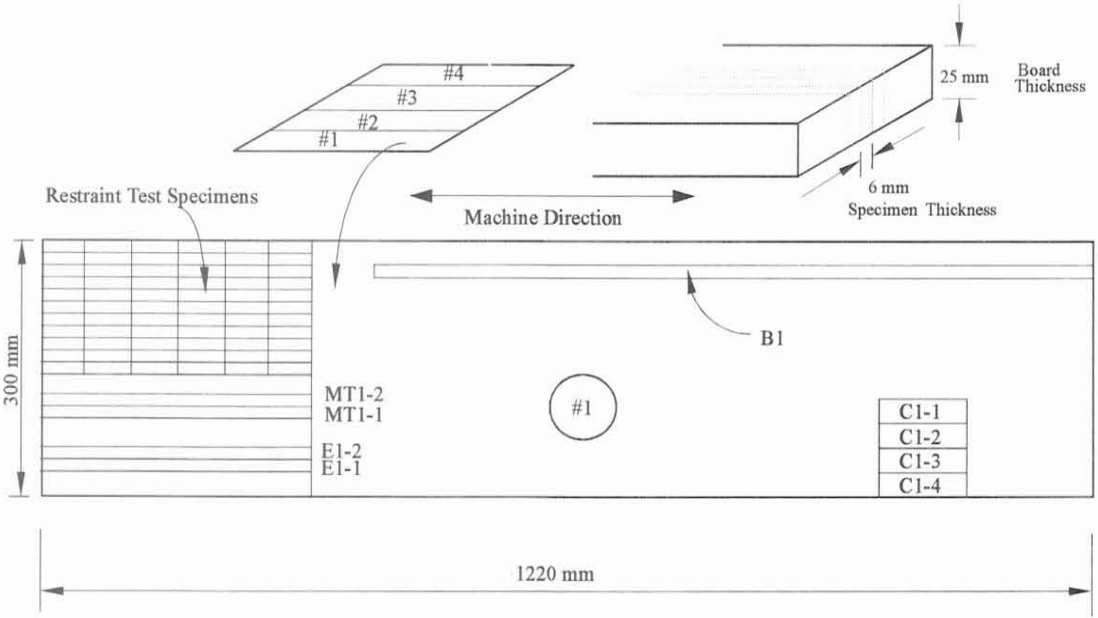
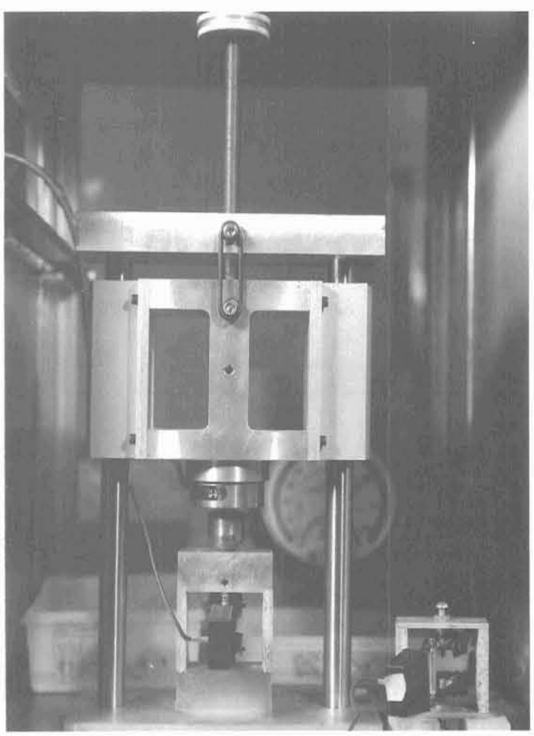


FIG. 6. Specimen arrangement on MDF and particleboard panels. Drawing shows one fourth of one panel. E1-1, E1-2: hygroscopic expansion specimens; B1: beam lamina; MT1-1, MT1-2: modulus of elasticity (tension) specimens; C1-1. . .: modulus of elasticity (compression) specimens.



sumption was made in this study that the stress-strain relations in compression and tension are identical.

Based on the above model considerations, the following sequence of experiments was carried out: (1) determination of moduli of elasticity both in compression and tension at various moisture contents for various wood materials; (2) determination of the elastic components of compression deformations occurring in restrained swelling situations; (3) manufacture of various unbalanced, layered beams from materials characterized by the above tests; (4) measurement of warp of the layered beams after exposure to various humidity changes; and (5) comparison of measured warp with computed warp using the elastic approach and the modified elastic (inelastic) approach.

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FIG. 7. Restraining device for measuring swelling stresses and elastic strain components.

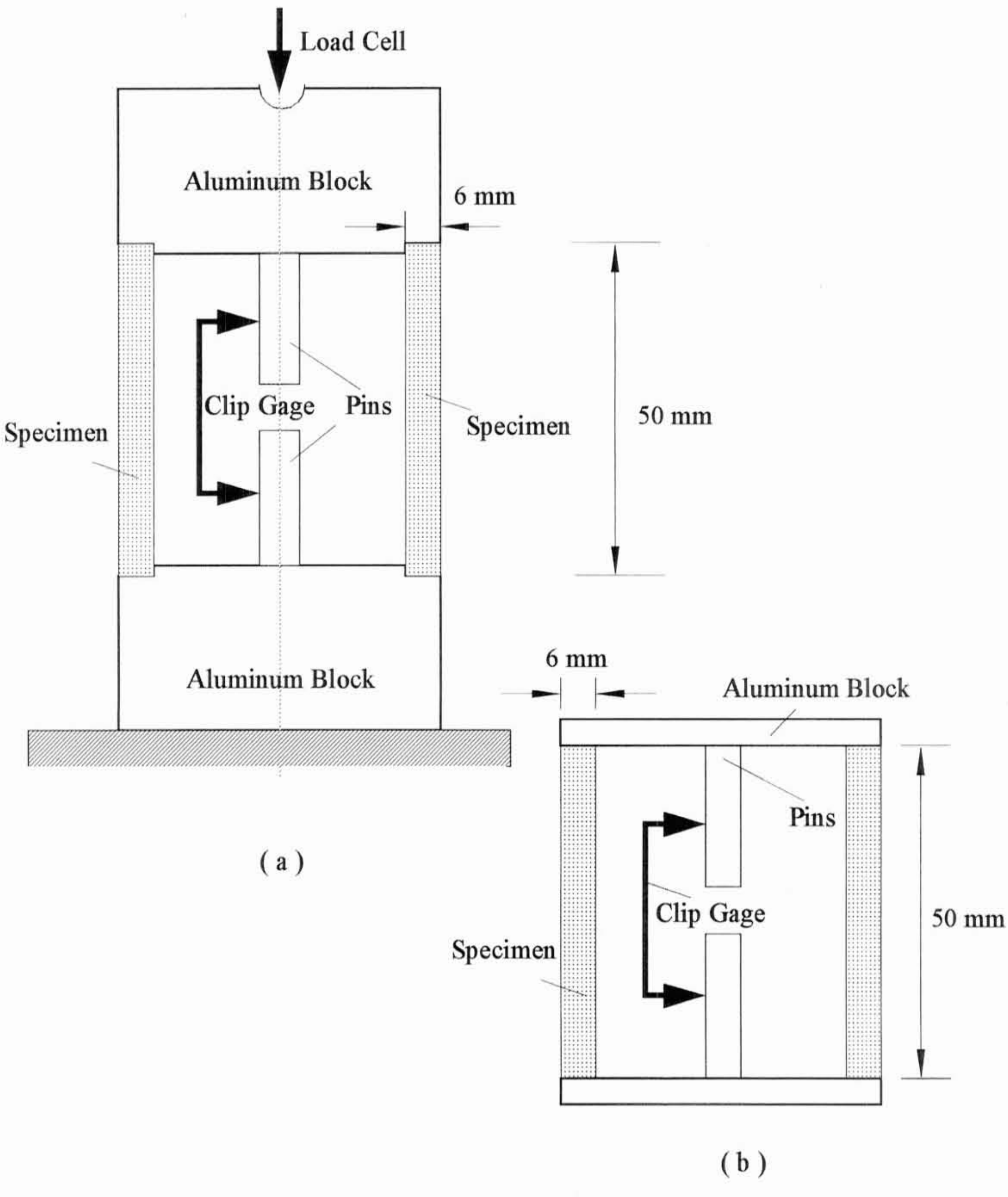

FIG. 8. Specimen placement in restraining device (a), and measurement of free hygroscopic expansion (b).

EXPERIMENTS

Materials

Several panels were made in the laboratory, which yielded sufficient specimens for determination of moduli, free hygroscopic expansion, elastic strain components, and swelling stresses, as well as material for the manufacture of laminated beams. The adhesive in all cases was a commercial epoxy resin.

TABLE 1. *Measured and estimated warp (center deflection, 0.61 m) of two- and three-ply yellow-poplar beams.*

a Exposure (R.H.%)	b ϵ_{elast}/α (tang.)	c Construction	d Layer thickness (mm)	e Meas. warp (mm)	f Estim. elast. warp (mm)	g Estim. inelast. warp (mm)	f/e	g/e	g/f
66–81	0.23	Long.	6.53	7.5	26.0	7.0	3.46	0.93	0.27
		Tang.	6.53						
		Tang.	1.63	7.1	22.7	5.6	3.20	0.79	0.25
		Long.	6.53						
		Tang.	6.53						
		Long.	6.53	1.0	3.7	0.9	3.70	0.90	0.24
		Tang.	6.53						
		Long.	1.63						
66–93	0.26	Long.	6.53	16.4	46.6	15.5	2.84	0.95	0.33
		Tang.	6.53						
		Tang.	1.63	13.9	41.0	13.3	2.95	0.96	0.32
		Long.	6.53						
		Tang.	6.53						
		Long.	6.53	2.1	6.2	2.3	2.95	1.10	0.37
		Tang.	6.53						
		Long.	1.63						

The design of the panels and the removal of test specimens and beam elements from them were such that the test results provided the closest possible representation of the characteristics of the beam elements.

Figure 3 shows a test panel made of narrow yellow-poplar strips, and Fig. 4 shows the cutting schedule for the same panel. Beam elements were removed in both principal directions for cross-lamination. The resulting unbalanced beams (different lamina thicknesses) are illustrated in Fig. 5. Figure 6 shows a cutting schedule for commercial MDF and particleboard panels. MDF and particleboard strips were combined to two-ply beams, the unbalance being caused by the difference in the material properties.

Procedures

The mechanical tests (modulus of elasticity) were conducted after conditioning of the specimens to equilibrium at various relative humidities. The restrained swelling test for the determination of the swelling stress, σ', and of the elastic deformation component, ϵ_{elast}, was conducted on a precision clamping or restraining device, shown in Fig. 7. This device consists essentially of a rigid frame formed by a pair of plates connected by two precision shafts. A ball-bearing-guide-block assembly rides almost frictionless on the two precision shafts. Its position between top and bottom plate is controlled by a screw drive. A miniature load cell is centrally mounted on the lower face of the guide block assembly. Two specimens were placed in between two aluminum blocks and inserted into the restraining device. The guide blocks were so adjusted by means of the screw drive that, at the beginning of the test, the load cell just touched the upper aluminum block.

Upon exposure to a higher humidity level, the ensuing axial expansion of the two specimens was totally restrained by the device, while the load associated with the swelling stresses was measured by the load cell. The change in distance between the aluminum blocks (change in length of specimens) was measured by an extensometer, and during the entire test was held to zero by occasional adjustment of the screw drive (see also Fig. 8).

The exposure conditions were (see also Tables 1 and 2):

TABLE 2. *Measured and estimated warp (center deflection, 0.762 m) of two-ply MDF-particleboard beams.*

a Exposure (R.H.%)	b ϵ_{elast}/α	c Construction	d Layer thickness (mm)	e Meas. warp (mm)	f Estim. elast. warp (mm)	g Estim. inelast. warp (mm)	f/e	g/e	g/f
30–81	0.39	MDF	6.4	20.6	24.5	4.0	1.19	0.19	0.16
	0.26	PBD	6.4						
50–81	0.39	MDF	6.4	17.8	19.5	3.8	1.10	0.21	0.19
	0.26	PBD	6.4						
30–93	0.21	MDF	6.4	42.2	45.0	9.4	1.07	0.22	0.21
	0.20	PBD	6.4						
50–93	0.26	MDF	6.4	33.8	36.8	6.8	1.09	0.20	0.18
	0.21	PBD	6.4						

Yellow-poplar

66–81%RH
and 66–93%RH

MDF, PBD

30–81%RH
50–81%RH
30–93%RH
and 50–93%RH

Upon reaching equilibrium at the higher relative humidity level, the device was opened (guide blocks raised), and the extensometer measured the instantaneous elastic recovery. This recovery is the elastic component of the restrained expansion. The total free expansion was measured in a parallel test, also by means of an extensometer (see Fig. 8). A third specimen was used to measure the weight gain during the test. All tests were run simultaneously

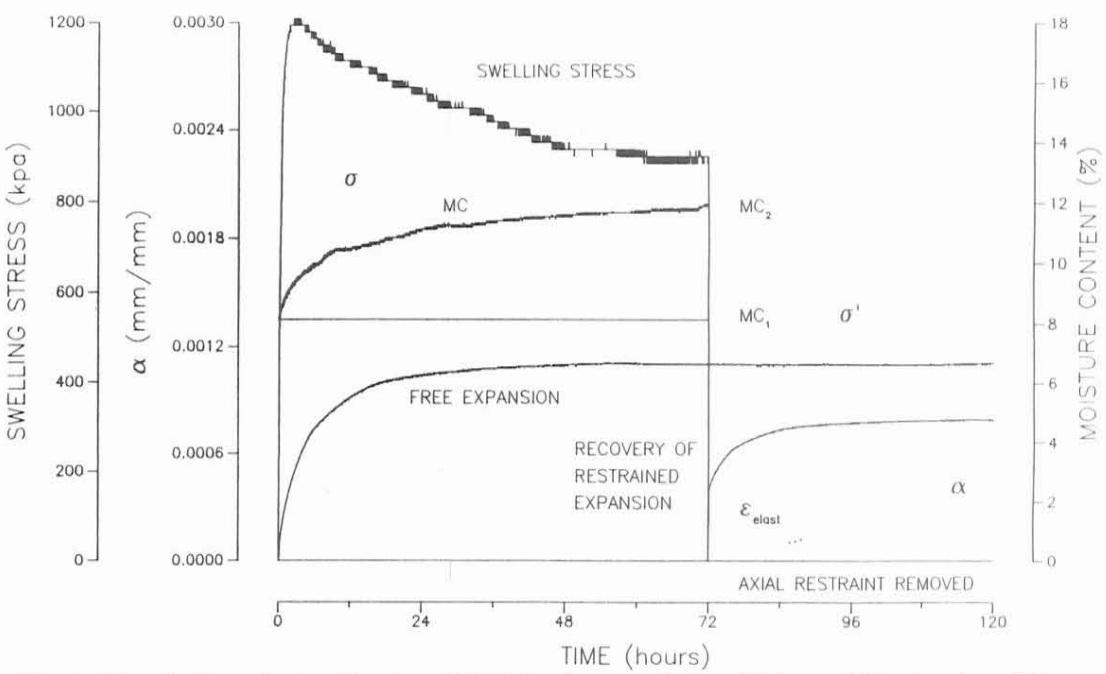

FIG. 9. Record of restrained swelling test of MDF. Specimens reached equilibrium at MC_2 after about 70 hours. Upon removal of restraint, the instantaneous elastic recovery of the free expansion is determined. Swelling stress at that point is σ'.

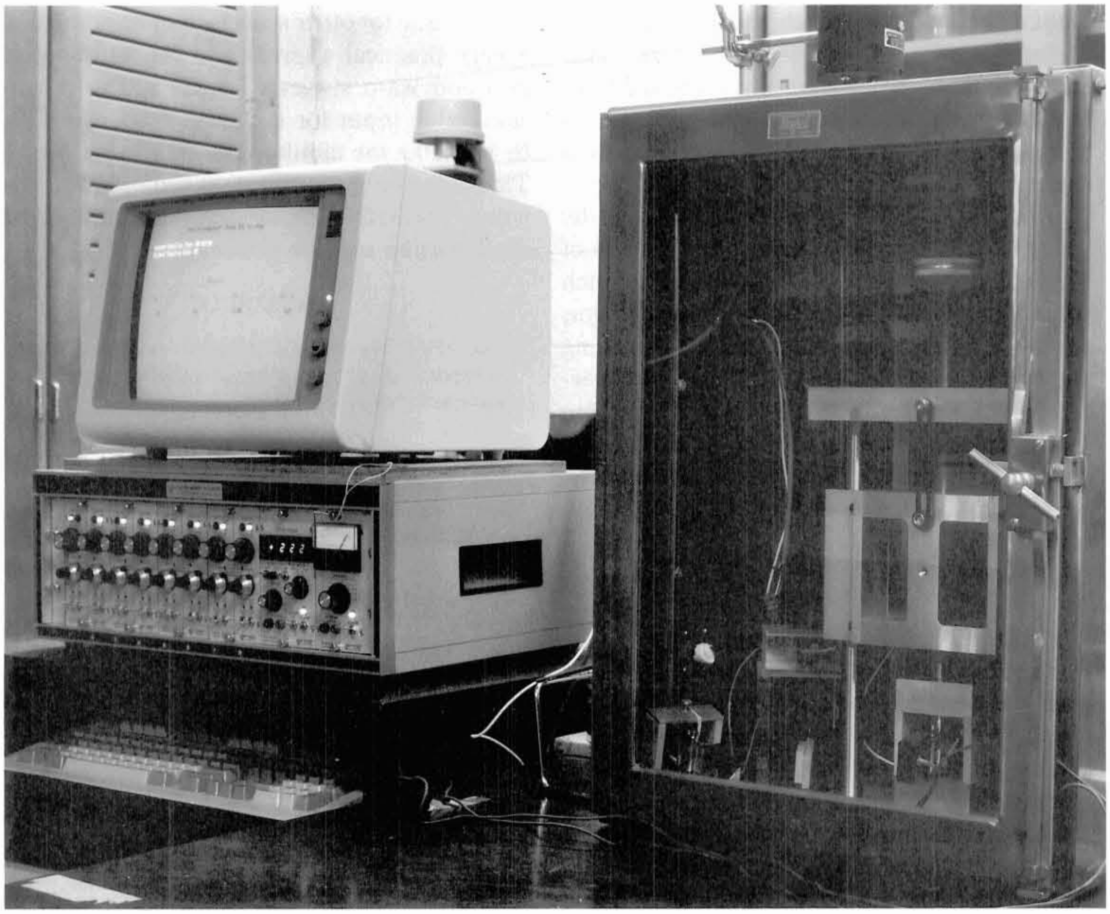

FIG. 10. Restraining device in climate chamber, and electronic data acquisition equipment.

in a small conditioning chamber in which the climate was controlled by means of saturated salt solutions and forced air circulation. A record of such a test run is shown in Fig. 9. Figure 10 shows the conditioning chamber with restraining device and the electronic data acquisition equipment.

The warping of the unbalanced test beams, which were exposed to the same relative humidity changes as were the test specimens, was measured in terms of their center deflection over a given span (0.61 or 0.76 m).

RESULTS

The results are summarized in Tables 1 and 2. Table 1 shows the warp of yellow-poplar beams of three different constructions (columns c and d, see also Fig. 5) exposed to two different relative humidity changes (column a). Compared are the measured warp over 0.61 m span (column e), the estimated elastic warp according to Eq. 1 with elastic inputs (column f), and the estimated inelastic warp according to Eq. 1 with modified elastic inputs (column g).

While the elastic approach severely overestimates the warp of the unbalanced beam, the inelastic approach shows remarkable agreement with the measured values. The ratio of the elastic component to the total restrained expansion in the tangential direction (column b) averages about 25%.

Table 2 shows the corresponding results for the MDF-particleboard laminated beams.

Here, the elastic approach shows the closest agreement with the measured values. This means that the MDF and particleboard laminas behave elastically, even though the ratio of elastic component over the total restrained expansion is similar to that found for yellow-poplar. The explanation lies in the fact that the differences in expansion values and moduli of elasticity of MDF and particleboard are much smaller than was the case with yellow-poplar, so that the actual lamina strains and swelling stresses were relatively small. MDF and particleboard laminas, therefore, do not require modification of inputs into Eq. 1, unless they are paired with materials of very different characteristics (see also Lang et al 1995; Suchsland et al. 1993).

CONCLUSIONS

The described modifications of the elastic warping equation allowed improved estimation of the warping of unbalanced, cross-laminated wood beams. The method can readily be extended to plywoods of any construction. Since the 'deformation modulus', E′, for yellow-poplar turned out to be not too different from the statically measured modulus of elasticity, and since the elastic strain component was approximately 25% of the total restrained expansion, and since there is evidence that this may be true for other species (Keylwerth 1962), a very practical approach to the estimate of plywood warp suggests itself: modify expansion value input for the cross-grain direction by reducing the tabulated value to one fourth. This is the only modification required for plywood. For MDF or particleboard laminas, no modification may be necessary.

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