ORTHOTROPIC STRENGTH AND ELASTICITY OF HARDWOODS IN RELATION TO COMPOSITE MANUFACTURE. PART I. ORTHOTROPY OF SHEAR STRENGTH

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ABSTRACT

The orthotropy of apparent shear strength of three Appalachian (aspen, red oak, and yellow-poplar) and two East European (true poplar and turkey oak) hardwood species was investigated. The experimental approach included shear force applications in planes parallel to the grain so that the annual ring orientation and the orientation of the grain relative to the applied force direction were systematically rotated. Statistical analyses of results demonstrated significant effects of grain and ring orientation on the shear strength for all species. Furthermore, interaction between these two factors was detected. Three models, developed to appraise the orthotropic nature of shear strength, were fitted to experimental data demonstrating acceptable to good agreement between predicted and experimental values. A combined model based on tensor theory and a modified version of Hankinson's formula provided the best fit by r^2 analysis. The information obtained and the models developed might be used to explore the shear strength of structural composites in which the constituents are systematically or randomly aligned.

Keywords: Hardwood, shear strength, orthotropy, composites.

INTRODUCTION

Under social, political, and economic pressures, the wood-based composite manufacturing industry is facing imminent problems regarding raw material supply. Worldwide resource utilization trends and the year-by-year decreasing quality and quantity of the available resources are forcing the industry to use smaller wood elements such as veneers, strands, flakes, and fibers. Also, the increasing demand for structural, wood-based composites

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triggers more intensive utilization of fast-growing species previously neglected because of unfavorable physical and mechanical properties.

The inherent orthotropic, physical, and mechanical properties, which govern the performance of these structural composites, are dependent on the physical and mechanical properties of the constituents. Furthermore, the manufacturing processes used to consolidate individual wood elements into a contiguous composite also intrinsically affect the final performance of the product. To achieve the ultimate goal, fully engineered structural composites, a thorough understanding of the origin and nature of orthotropic elasticity and strength of the raw materials is necessary. Once these material properties are described, they can be related to the properties of composites with incorporation of the effect of manufacturing parameters. The information gained will provide a basis for further product development.

Wood-based composite materials have been developed largely through empirical studies. During the past decade, researchers have realized the need for fundamental understanding of composite manufacture. Our ability to establish universal guidelines for wood-based composite design is limited by the lack of knowledge regarding raw material properties and the vast array of interacting processing variables. One can easily realize that the unpredictable laws of nature govern many of these material properties and interactions. These facts prevent researchers from developing completely deterministic design procedures. As a result, combined stochastic/deterministic models have been developed over the years for predicting one or more properties of composite products. To fully understand the complex interaction between the raw material properties, manufacturing parameters, and the performance of composite products, further research is needed. This is particularly important when new material resources are introduced into the composite manufacturing processes.

Wood always was and will be the primary construction material in the United States. There are probably more buildings constructed with wood and wood-based composites than with any other construction material. These structures include residential dwellings, apartment complexes, and commercial constructions (Breyer 1980). Full exploration of the material properties of the structural composite elements eventually will lead to better design, saving costs, energy, and material resources.

Although several centuries ago wood was the major construction material in Central and Eastern Europe, now inorganic construction materials such as brick, concrete, etc. are dominant. Wood is used as raw material for manufacturing doors and windows and is utilized as structural elements in roof structures. The lack of softwood supply in this region and the ever-increasing price of quality softwoods imported from Scandinavia or Russia initiated research projects to explore the potential of using Central and East European hardwood species for manufacturing structural composites. A typical project, involving academic units and industrial partners from France, Hungary, Poland, Slovenia, and United Kingdom, investigated the feasibility of using such species for structural composite manufacture (Kovacs et al. 1997). The project was dedicated to evaluating the performance of several species: alder (*Alnus glutinosa*), beech (*Fagus silvatica*), birch (*Betula pendula*), turkey oak (*Quercus cerris*), and true poplar (*Populus* spp.) as structural veneer for laminated veneer lumber (LVL) manufacture. All five species demonstrated significant potential for composite manufacture.

Realizing the economic importance of such research, the Hungarian National Science Foundation (OTKA, stands for HNSF) provided financial support to further investigate the possibility of developing structural composites made out of locally grown hardwood species. The project was launched at the University of Sopron in 1998. Researches in Hungary and West Virginia University recognized the similarities between the two countries regarding raw material supply, needs, and research objectives and successfully applied for financial aid for international cooperation. The North Atlantic Treaty Organization (NATO) granted sufficient financial aid to fulfill the combined objectives and goals of this project through international cooperation.

OBJECTIVES

Envisioning the ultimate goal of fully engineered design of wood-based, structural composites this research is aimed at identifying the anisotropic characteristics of raw materials, exploring the effect of manufacturing parameters on the constituents' properties, and relating these results to the performance of composite products. The project is divided into four phases with specific goals as follows:

Phase I includes the development of an adequate database for validation of different models that describe the orthotropic strength and elasticity of underutilized Appalachian hardwoods and species grown in Hungary. Phase II is aimed at investigating the effect of manufacturing parameters on the properties of veneer/strand/flake constituents obtained from the above-mentioned resources. Phase III consists of the assessment of key mechanical properties of the composites such as Laminated Veneer Lumber (LVL), Parallel Strand Lumber (PSL), and Laminated Strand Lumber (LSL). Phase IV is devoted to exploring the relationship between final product performance, properties of constituents, and manufacturing practices via deterministic and stochastic model development.

As a part of Phase I, the investigation of the orthotropic strength of the above-mentioned species has started with the exploration of shear strength. The focused objectives of this segment of the research were to experimentally validate existing models and to develop new approaches for assessing the orthotropic nature of the shear strength of possible raw materials for composite manufacture.

Although the shear strength of the raw materials may not be directly related to the shear strength of the composite products, we believe that the information gained during this investigation will help to develop a reliable method for predicting and/or assessing the shear strength of structural composites.

LITERATURE REVIEW

True shear strength is one of the most difficult characteristics to measure. Creation of the pure stress state is a real challenge. Furthermore, the always present normal stresses, combined with the inherent anisotropy of wood, make the strength determination uncertain. Several publications have dealt with the improvement of shear strength assessment. One of the most comprehensive studies on this topic was provided by Yilinen (1963). The author investigated and critically reviewed several standardized shear testing methods. He concluded that the majority of block shear tests usually underestimate the true shear strength of solid wood.

The standard ASTM block shear test has received much criticism for not providing pure shear load on the specimens. A number of researchers addressed this problem and some also proposed alternative solutions. Norris (1957) recommended the panel shear test, and Liu (1984) suggested the adaptation of a device, proposed by Arcan et al. (1978), for wood. The drawback of these tests is that they involve complicated specimen preparation and testing procedures. Lang (1997) proposed a new device for shear strength assessment of solid wood. The advantages of the described testing apparatus are the smaller specimen size, alleviation of normal stresses, and acceptable agreement with shear strength values obtained by the ASTM method.

The majority of previous research projects have focused on the shear strength of solid wood parallel to the grain. Limited publications are available that address the anisotropy of wood in shear strength assessment. The first formula that described the strength anisotropy of wood is the well-known Hankinson's formula (Hankinson 1921). It was developed empirically from compression tests. This equation describes the effect of grain-orientation changes on the measured properties. Many researchers examined the validity of this formula finding that it fits experimental data well (Goodman and Bodig 1972; Bodig and Jayne 1982). However, the equation was deemed to provide adequate predictions only for compression and tension strength as well as moduli of elasticity. Kollman and Côté (1968) proposed some changes to the formula. They stated that using an experimentally determined power will provide better approximation of the direction dependent strength and elastic properties. The first attempt to describe the orthotropy of shear strength was made by Norris (1950). He applied the general Henky–von Mises theory to orthotropic materials. Although in his study the predicted shear strength values agreed reasonably well with experimental data for structural plywood, the approach has received criticism from others (Wu 1974; Cowin 1979). Over the decades, with the advancement of man-made composites, ample research has been devoted to explore the strength and elasticity of anisotropic materials. Many of these results and theories developed can be applied to wood with care.

Ashkenazi (1976) used the tensor theory for describing the anisotropy of wood and wood-based composites. In an earlier work, he measured the shear strength of pine at various grain angles (Ashkenazi 1959). His results were unusual in that shear strength showed maximum values at approximately 15° grain orientation, rather than in the longitudinal direction. Cowin (1979) stated that a quadratic form of the Hankinson's formula describes Ashkenazi's data reasonably well. The proposed model, however, can not describe the shear strength maximum at 15° grain orientation. Liu and Floeter (1984) measured the shear strength of spruce at 0°, 30°, 60°, and 90° grain angles with the special device described by Arcan et al. (1978) designed to provide uniform plane stress. Their results agreed well with the theory of Cowin (1979).

Some other researchers incorporated the effect of ring orientation in their works. The experiment of Bendsten and Porter (1978) included ring-angle, but only as a blocking factor; its effect was not of interest. Okkonen and River (1989), among other factors, examined the effect of radial and tangential ring orientation on the shear strength in the longitudinal direction. They concluded that Douglas-fir had higher strength when the orientation of the sheared plane was radial, while oak and maple were stronger in the tangential direction. Riyanto and Gupta (1996) tried to establish a relationship between ring angle and shear strength parallel to the grain. Using a completely randomized design, they found that ring angle had very little effect on the shear strength of Douglas-fir and Dahurian Larch. Rather, the specific gravity, the percentage of latewood, and the number of rings per inch were much more deterministic factors. Szalai (1994) provided an integrated approach that tackles both ring and grain angle orientation. A general equation, derived from tensor analysis, can determine the shear strength at any given ring and grain angle combination.

THEORETICAL BACKGROUND

The orthotropic nature of solid wood is usually depicted in a three-dimensional Cartesian coordinate system as shown in Fig. 1. The principal directions of the material coordinate system are noted as L, R, and T, longitudinal, radial, and tangential directions, respectively. If an aligned global coordinate system (x_i; i = 1, 2, 3) is systematically rotated around R and L axes, the angles between the axes of L, R, T and x_i' (i = 1, 2, 3) systems denote the grain and ring orientation of solid wood relative to the global coordinate system as marked in Fig. 1. Note, that the x_1' x_3' plane is always parallel to the grain. If shear forces are acting in the above-mentioned plane and the direction of the applied forces is x_1', the orthotropy of shear strength can be investigated as a function of grain and ring angle. Using the described rotation, block shear specimens can be machined and tested. Such specimens are shown on Fig. 2 representing the shear strength (τ) measurements in the principal material directions. The first subscript of τ marks the normal direction of the sheared plane, while the second denotes the direction of shear forces. Specimens in Fig. 2 a and c represent the standard shear application parallel to the grain, while shear strength measured on specimens b and d are sometimes referred to as rolling shear of solid wood.

Because of the inherent duality of shear stresses, the failure of the specimens may not be manifested in the theoretically sheared plane. Furthermore, the unavoidable normal stresses may induce and propagate cracks along the weakest interface within the volume of the specimen. Such out-of-sheared-plane failure may occur with certain grain and ring angle combinations at the earlywood–latewood boundary or along the ray tissues. Consequently, the experimentally determined values can be considered as apparent shear strength only.

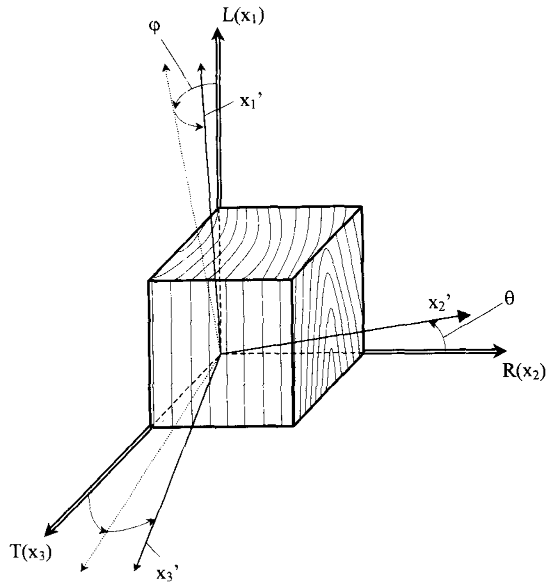


FIG. 1. The orthotropy of solid wood shown in the principal material and global coordinate systems. Interpretation of grain angle (φ) and ring angle (θ).

MODELS PREDICTING THE ORTHOTROPY OF SHEAR STRENGTH

The Orthotropic Tensor Theory

In a comprehensive work, Szalai (1994) used the orthotropic tensor theory to describe the direction-dependent strength and elasticity of wood. Based on Ashkenazi's (1976) strength criteria, he applied a four-dimensional tensor approach to predict the shear strength of wood in any oblique plane and direction of shear forces. Substituting the tensor components with the appropriate strength values and eliminating the zero components, resulting from the constraint that shear is applied only in the planes parallel to the grain, the equation takes the following form:

$$\frac{1}{\tau_{\varphi,\theta}} = \frac{4}{\tau_{90^\circ}^{45^\circ}}\cos^2\theta\sin^2\theta\sin^2\varphi + \frac{1}{\tau_{RT}}\cos^2 2\theta\sin^2\varphi + \frac{1}{\tau_{TL}}\sin^2\theta\cos^2\varphi + \frac{1}{\tau_{RL}}\cos^2\theta\cos^2\varphi \quad (1)$$

where

φ = grain angle

θ = ring angle

$\tau_{\varphi,\theta}$ = shear strength at grain angle φ and ring angle θ.

τ_{ij} = shear strength in the main anatomical planes, (i = R, T; j = T, L) where i is the direction normal of the sheared plane and j is the direction of the applied load.

$\tau_{90^\circ}^{45^\circ}$ = shear strength at 90° grain and 45° ring angle (φ = 90°, θ = 45°)

Note that this solution requires four experimentally predetermined strength values: three obtained in the principal anatomical planes such as τ_{RL}, τ_{RT}, and τ_{TL} shown in Fig. 2 a, b, and c, respectively, and a strength value at 90° grain and 45° ring angle ($\tau_{90^\circ}^{45^\circ}$). The advantages of this model are that it has a firm theoretical basis, uses only four experimentally determined data points for prediction, and is very straightforward.

Quadratic model

Cowin (1979) demonstrated that the shear strength of wood may follow the Hankinson-type strength criterion in a quadratic form. Liu

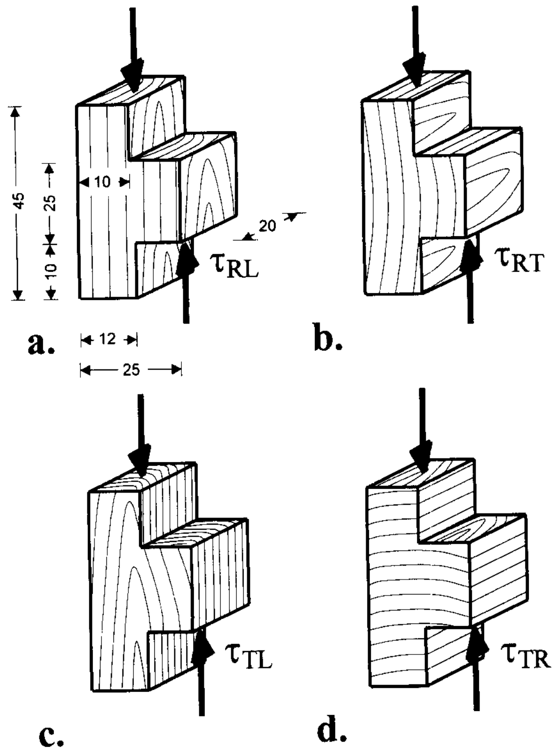


FIG. 2. The applied shear forces in the principal anatomical planes and the notation of corresponding shear stresses. Specimen target dimensions are in millimeters. a, c—traditional, shear parallel to the grain; b, d—rolling shear.

and Floeter (1984) used a tensor polynomial theory, developed by Tsai and Wu (1971), to re-derive the formula for predicting shear strength in a principal material plane of solid wood. The equation in general form is given as follows:

$$\tau_\varphi^2 = \frac{\tau_{0°}^2 \tau_{90°}^2}{\tau_{0°}^2 \sin^2\varphi + \tau_{90°}^2 \cos^2\varphi} \quad (2)$$

where

τ_φ = estimated shear strength at grain angle φ
$\tau_{0°}$ = shear strength at grain angle $\varphi = 0°$
$\tau_{90°}$ = shear strength at grain angle $\varphi = 90°$

Like Szalai's approach, this formula has a well-defined theoretical basis. However, it does not include the effect of ring orientation and has been verified experimentally in the LT plane only using Sitka spruce specimens.

Modified Hankinson's formula

Kollman and Côté (1968) modified the original Hankinson's formula replacing the power 2, to which the trigonometric terms are raised, by an arbitrary power n:

$$\tau_\varphi = \frac{\tau_{0°}\tau_{90°}}{\tau_{0°}\sin^n\varphi + \tau_{90°}\cos^n\varphi} \quad (3)$$

The authors claimed that this equation provides better fit than the original Hankinson's formula for predicting tensile strength and modulus of elasticity. Although this model is purely empirical, it has the capability to describe peak shear stresses at inclined grain, by using a higher power (i.e., n > 2). Beside the lack of theoretical basis, this model is probably very species-specific and requires a significant database for accurate determination of the value of n. Like the quadratic formula, it can handle only fixed ring orientation in its present form.

Combined models

So far, the orthotropic tensor theory was the only model that could handle both grain and ring angle changes. Researchers addressed the effect of ring orientation on the shear strength parallel to the grain and usually found it negligible. The apparent low degree of orthotropy of shear strength between the LT and LR main anatomical planes (i.e., $\tau_{RL} \approx \tau_{TL}$) did not trigger extensive model development to describe the phenomenon. The only available model was published by Szalai (1994). It includes two equations derived from tensor analysis as follows:

$$\tau_{0°;\theta} = \frac{1}{\left(\dfrac{\cos^2\theta}{\tau_{RL}} + \dfrac{\sin^2\theta}{\tau_{TL}}\right)} \quad (4)$$

$$\tau_{90°;\theta} = 1 \Bigg/ \left[\frac{\cos^4\theta}{\tau_{RT}} + \frac{\sin^4\theta}{\tau_{TR}} + \left(\frac{1}{\tau_{90°}^{45°}} - \frac{1}{4\tau_{RT}} - \frac{1}{4\tau_{TR}}\right)\sin^2 2\theta\right] \quad (5)$$

where

$\tau_{0°,\theta}$ = shear strength at θ ring angle, φ = 0°;
$\tau_{90°,\theta}$ = shear strength at θ ring angle, φ = 90°;
and the other symbols are as given at Eq. (1).

Equation 4 approximates the shear strength of traditional, parallel to the grain specimens as a function of ring orientation. It requires two experimentally predetermined strength values. The rolling shear strength variations are given by Eq. (5) where three predetermined strength values are needed. Note that τ_{RT} and τ_{TR} represent the maximum stresses (i.e., shear strength) values. Due to the duality, the stresses in these two directions are identical. However, it is not necessarily true for the strength values of wood because of the unpredictable failure mode as discussed earlier. Although these equations have not been experimentally verified, theoretically they should describe the effect of ring orientation on the shear strength of orthotropic materials.

One can realize that these equations can provide predetermined strength data for the quadratic model and for the modified Hankinson's formula for predicting the effect of grain orientation. Consequently, combining Eqs. (4) and (5) with Eqs. (2) or (3), we can obtain two additional models for estimating the orthotropy of shear strength as a function of grain and ring orientation. This combination for the quadratic model is given in a shorthand form as follows:

$$\tau_{\varphi,\theta}{}^2 = \frac{\tau_{0°;\theta}{}^2\tau_{90°;\theta}{}^2}{\tau_{0°;\theta}{}^2\sin^2\varphi + \tau_{90°;\theta}{}^2\cos^2\varphi} \tag{6}$$

Furthermore, using the modified Hankinson's formula we obtain:

$$\tau_{\varphi,\theta} = \frac{\tau_{0°;\theta}\tau_{90°;\theta}}{\tau_{0°;\theta}\sin^n\varphi + \tau_{90°;\theta}\cos^n\varphi} \tag{7}$$

Figure 3 gives a graphical explanation of these combined models. Note that both of these approximations require five experimentally predetermined strength values, and Eqs. (6) or (7) should be solved m times where m is the resolution (i.e., m = (1 + 90/ring angle increment)). During this research, these two

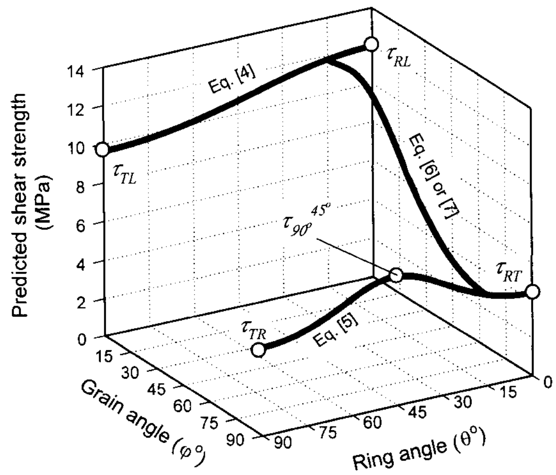


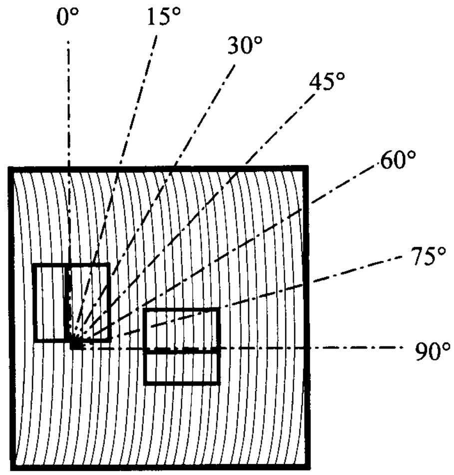
FIG. 3. Interpretation and the principle of prediction process of the combined models.

models along with the orthotropic tensor theory (Eq. (1)), were fitted to experimental data and statistically analyzed.

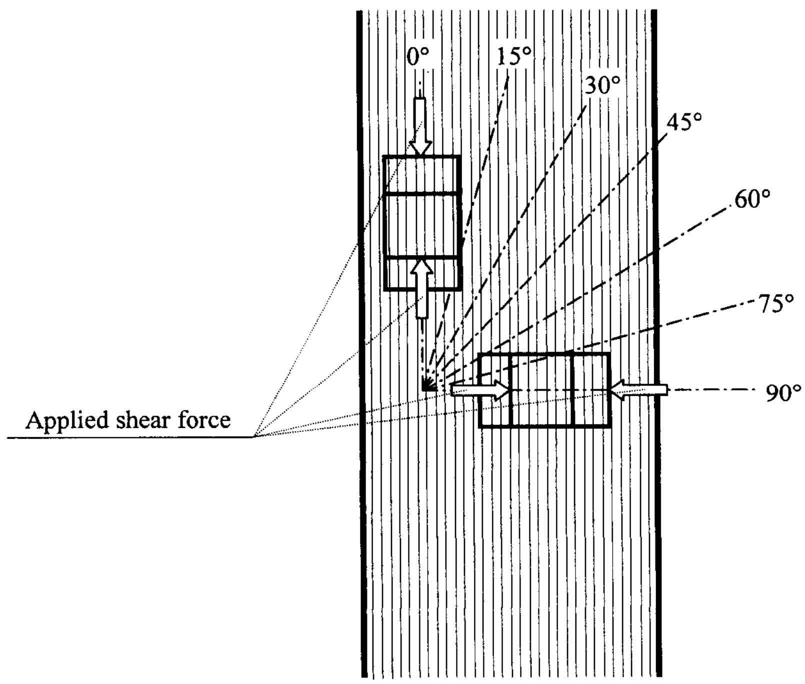
MATERIALS AND METHODS

Experimental determination of shear strength values included three Appalachian hardwood species: quaking aspen (*Populus tremuloides*), red oak (*Quercus rubra*), and yellow-poplar (*Liriodendron tulipifera*); and two European hardwood species: true poplar (*Populus x. Euramericana cv. Pannonia*) and turkey oak (*Quercus cerris*). Figure 2a shows the specimen shape and target dimensions, which differed from those specified by the ASTM D 143-94 standard (ASTM 1996a). The double-notched shear blocks were prepared from blanks having varying ring and grain angle between 0° and 90° with 15° increments. Figure 4 demonstrates this specimen preparation practice. Test series included sets for all combinations of the above angles, for all the examined species. The sample size for each set varied between six and fifteen.

Prior to testing, specimens were conditioned to approximately 12% moisture content in a controlled environment (i.e., 21°C and 65% RH). Representative samples of specimens (n = 10) were prepared for moisture content and



a. Top View



b. Side View

FIG. 4. Schematic of the specimen manufacturing practice from straight-grained, prepared blanks.

TABLE 1. *Summary statistics of the measured physical properties.*

Species	n	Moisture content [%] Mean value	Moisture content [%] COV [%]	Specific gravity Mean value	Specific gravity COV [%]
Aspen	10	11.4	8.11	0.39	3.57
Red oak	10	11.1	2.71	0.63	5.25
Yellow-poplar	10	11.3	4.50	0.39	3.41
True poplar	10	10.9	11.01	0.37	9.51
Turkey oak	10	11.5	7.83	0.70	5.84

specific gravity determination. The evaluation of these physical properties followed the specifications of the relevant ASTM standards such as ASTM D 4442-92 (ASTM 1996c) and ASTM D 2395-93 (ASTM 1996b). Table 1 contains the summary statistics of these measured properties.

Shear forces were applied through a special device providing a single plane of shear within the specimens. The area of the sheared section was approximately 500 mm^2 according to the target dimensions shown on Fig. 2a. The advantages of this alternative shear strength assessment and the description of the device were discussed in details in a separate publication (Lang 1997). Figure 5 shows the principal and schematic of the shear testing apparatus along with the experimental setup. The justification of this alternative testing method lies in the smaller specimen dimensions for which the grain and ring orientations are better controlled. Furthermore, it requires significantly less volume of raw material, and waste is minimized when machining more than 1,800 specimens.

Tests were conducted at two locations. At West Virginia University, Division of Forestry, Morgantown, WV, we used an MTS universal servo-hydraulic testing equipment mounted with 10 kN ± 1 N load cell for assessing the shear strength of yellow-poplar, red oak, and aspen species. The machine operated under displacement control with a rate of speed of 0.6 mm/min required by the ASTM D 143-94 standard. The Hungarian partners used the same test setup on a screw-driven universal testing machine for measuring the strength values of local hardwoods (turkey oak and true poplar). Because of machine constraints, the applied crosshead speed was 2 mm/min. Other testing parameters, including specimen conditioning, were the same.

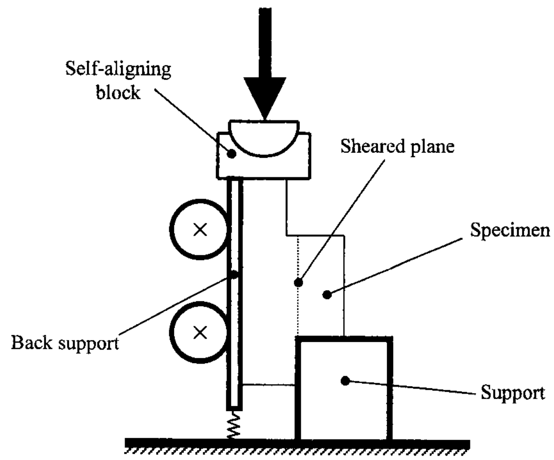
RESULTS AND DISCUSSION

Table 2 compiles the basic statistics of all the experimentally obtained shear strength data by species. The mean values were used to create anisotropy diagrams in three-dimensional, Cartesian coordinate systems as shown in Figs. 6a to 10a. The intermediate grid data points were generated by inverse distance interpolation using a commercial software SigmaPlot® (SPSS Inc. 1997) for better viewing.

In general, shear strength decreased significantly with the increase of grain angle for all species involved in the study. At zero degree grain angle (traditional shear, parallel to the grain) the shear strength decreased slightly as ring angle increased from 0 to 90 degrees. However, this tendency was not observed at fixed 90° grain angle (i.e., rolling shear). Either a slight increase or local maximum was experienced. It does appear that shear strength at this grain orientation might be species-specific as demonstrated by the similarities in strength variations of the two oak species (Figs. 7a and 10a).

Maximum shear strength values (MSS) were not consistently measured at 0° grain orientation. In fact, out of 35 species/ring angle combinations, 23 times the maximum shear strength was observed at 15° grain angle. At 0° ring angle for all the species–except turkey oak—the MSS was measured at 15° grain orientation. No further specific trend or pattern could be detected as demonstrated in Table 2 by the bold and italic set MSS values. Other researchers reported the same phenomenon (Ashkenazi 1959; Szalai 1994). This characteristic may be explained by the study of stress distribution function along the length of the sheared plane. Yilinen (1963) demonstrated

a.



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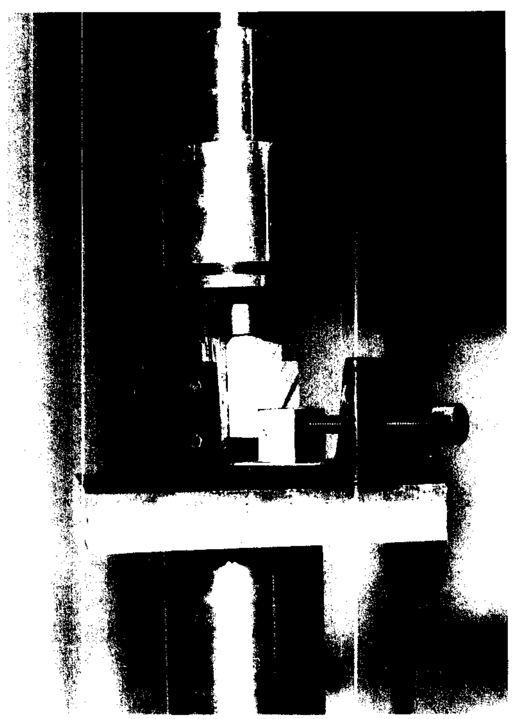


FIG. 5. Schematic of the testing apparatus and the experimental setup.

TABLE 2. *Summary and basic statistics of the experimentally determined shear strength values.*

ASPEN

Grain angle (φ°)	Ring orientation (θ°) 0			15			30			45 (Shear strength)			60			75			90		
	n[a]	$\bar{x}$[b]	COV[c]	n	$\bar{x}$	COV	n	$\bar{x}$	COV	n	$\bar{x}$	COV	n	$\bar{x}$	COV	n	$\bar{x}$	COV	n	$\bar{x}$	COV
0	15	7.87	2.92	15	7.30	6.85	15	6.77	6.79	15	5.42	14.57	15	6.16	10.39	15	6.17	4.38	15	6.30	12.70
15	9	***8.39***	4.53	8	***8.16***	12.87	9	***7.98***	11.65	8	***7.30***	14.11	7	***6.67***	12.59	8	5.78	6.92	9	5.99	10.02
30	10	7.16	4.47	8	6.37	14.13	7	7.21	11.10	8	5.74	8.19	7	5.62	18.33	8	4.99	29.06	7	5.62	14.23
45	6	4.98	7.63	6	4.29	7.93	6	4.75	6.32	6	4.27	5.62	6	4.44	5.41	6	5.02	11.35	6	4.42	4.98
60	6	3.18	9.74	6	3.45	13.04	6	3.43	6.41	5	3.25	11.69	5	2.54	19.69	4	2.61	10.34	5	3.13	14.06
75	6	1.92	6.25	6	2.52	14.28	6	2.81	12.81	6	1.93	7.25	6	1.90	4.74	6	2.73	4.40	6	2.45	9.80
90	10	2.24	3.13	10	1.43	9.09	11	2.24	4.02	11	2.28	3.95	11	3.11	2.89	10	2.98	3.02	10	2.54	4.72

OAK

Grain angle (φ°)	Ring orientation (θ°) 0			*15*			30			45 (Shear strength)			60			75			90		
	n	$\bar{x}$	*COV*	*n*	$\bar{x}$	COV	n	$\bar{x}$	COV	n	$\bar{x}$	COV	n	$\bar{x}$	COV	n	$\bar{x}$	COV	n	$\bar{x}$	COV
0	15	11.97	8.69	15	12.01	7.91	15	11.6	4.14	15	11.95	8.37	15	12.77	5.01	15	11.36	5.72	15	10.62	4.24
15	8	***11.98***	7.51	9	11.28	5.93	9	11.55	8.57	9	***12.84***	2.57	9	***13.04***	6.83	9	***12.16***	4.11	9	***12.36***	2.91
30	8	11.85	6.24	9	11.60	15.17	8	11.48	10.10	8	11.36	8.01	8	11.32	9.89	8	11.17	9.67	8	11.33	4.41
45	5	9.99	3.40	6	9.34	8.03	5	6.64	11.30	6	9.94	8.65	5	8.59	4.31	5	8.38	5.97	5	8.82	2.61
60	7	7.01	2.14	7	8.07	4.83	7	8.14	3.47	6	7.69	5.98	7	7.20	2.92	5	7.80	2.69	7	6.81	4.85
75	7	6.06	4.95	6	5.64	4.61	6	5.65	9.73	7	6.41	6.71	7	6.12	2.12	7	6.84	6.58	6	6.57	3.35
90	11	4.44	29.28	11	5.55	15.50	11	5.72	12.76	10	6.88	3.63	11	7.06	7.22	10	4.62	18.83	10	5.37	4.66

YELLOW-POPLAR

Grain angle (φ°)	Ring orientation (θ°) 0			15			30			45 (Shear strength)			60			75			90		
	n	$\bar{x}$	COV	n	$\bar{x}$	COV	n	$\bar{x}$	COV	n	$\bar{x}$	COV	n	$\bar{x}$	COV	n	$\bar{x}$	COV	n	$\bar{x}$	COV
0	15	7.91	6.19	15	7.13	8.42	15	7.13	6.03	15	7.32	6.69	15	7.23	10.37	15	9.06	9.71	15	6.18	4.85
15	9	***10.59***	5.10	9	***8.37***	15.05	9	***7.31***	10.94	9	6.65	8.42	9	6.74	5.19	10	6.77	5.17	10	***6.42***	4.67
30	6	6.94	5.33	8	6.70	11.04	9	6.57	20.85	7	6.10	26.89	9	5.91	14.38	8	5.61	5.88	8	5.95	25.04
45	5	4.85	3.92	4	4.98	6.63	6	4.85	8.04	6	5.74	14.29	5	5.05	10.89	6	4.35	6.21	5	4.73	19.45
60	6	2.81	13.87	5	3.85	10.91	6	2.81	3.56	6	3.53	28.05	6	3.60	17.5	6	3.13	9.90	6	3.79	6.33
75	6	2.75	15.64	6	2.96	14.19	6	2.28	7.02	6	2.28	6.58	6	1.94	3.09	6	1.94	4.12	6	2.87	9.06
90	3	2.43	27.98	4	2.60	20.08	9	2.86	27.27	11	3.52	15.34	11	3.42	5.84	11	3.35	17.01	11	3.17	13.56

[a] Sample size.
[b] Mean value of τ [MPa].
[c] Coefficient of variation (%).

TABLE 2. *Continued.*

TRUE POPLAR

Grain angle ($\varphi°$)	Ring orientation ($\theta°$) 0: n	$\bar{x}$	COV	15: n	$\bar{x}$	COV	30: n	$\bar{x}$	COV	45: n	$\bar{x}$	COV	60: n	$\bar{x}$	COV	75: n	$\bar{x}$	COV	90: n	$\bar{x}$	COV
										Shear strength											
0	9	7.05	6.10	6	8.16	16.42	6	7.46	5.63	6	6.08	7.89	6	6.17	4.38	6	6.28	8.28	9	5.89	5.43
15	6	***7.07***	14.14	6	6.52	11.20	6	7.12	9.83	6	6.04	6.95	6	***6.52***	5.52	6	***6.51***	6.14	6	***5.93***	6.58
30	6	6.28	10.99	6	6.59	12.59	6	5.89	5.60	6	6.25	22.56	6	5.25	11.81	6	5.90	11.02	6	5.64	12.41
45	6	5.63	11.01	6	5.76	12.33	6	4.79	2.92	6	4.34	1.15	6	4.83	4.76	6	4.73	4.86	6	4.24	3.54
60	6	3.52	16.48	6	3.87	13.18	6	3.56	8.71	6	3.60	9.17	6	3.71	8.63	6	3.92	5.87	6	3.46	5.49
75	6	2.50	2.80	6	2.95	5.76	6	2.90	3.79	6	3.26	9.82	6	2.72	3.68	6	3.58	6.42	6	2.83	4.59
90	10	2.17	9.22	6	2.53	6.72	6	2.55	8.63	10	2.86	11.89	5	2.76	13.77	6	2.87	6.97	10	2.79	4.66

TURKEY OAK

Grain angle ($\varphi°$)	Ring orientation ($\theta°$) 0: n[a]	$\bar{x}$[b]	COV[c]	15: n	$\bar{x}$	COV	30: n	$\bar{x}$	COV	45: n	$\bar{x}$	COV	60: n	$\bar{x}$	COV	75: n	$\bar{x}$	COV	90: n	$\bar{x}$	COV
										Shear strength											
0	10	15.32	3.39	6	13.74	4.44	6	12.44	3.53	6	13.51	4.96	6	12.88	5.67	6	11.29	8.33	10	12.56	8.12
15	6	14.94	8.37	6	***13.92***	3.23	6	***13.93***	3.30	5	***14.24***	9.27	6	***14.15***	6.29	5	***12.22***	7.61	6	12.51	7.67
30	5	10.99	6.64	5	11.34	1.94	5	12.14	5.27	6	12.34	6.32	6	13.18	5.39	6	11.75	10.38	6	9.79	7.66
45	6	9.77	4.61	6	9.72	4.84	5	11.26	4.00	6	11.47	4.36	6	11.56	4.07	6	10.79	5.65	6	9.56	3.45
60	6	9.44	5.40	6	9.21	3.26	6	8.79	1.82	6	8.90	6.85	6	8.95	7.60	6	8.96	8.48	6	8.44	7.11
75	6	8.53	5.98	6	7.87	5.84	6	8.26	7.99	6	9.24	9.96	6	8.66	10.05	6	6.50	38.92	6	8.78	6.61
90	10	7.09	6.91	6	7.59	3.16	6	7.95	9.06	10	8.36	6.94	6	9.56	10.04	6	10.18	3.24	10	9.37	10.89

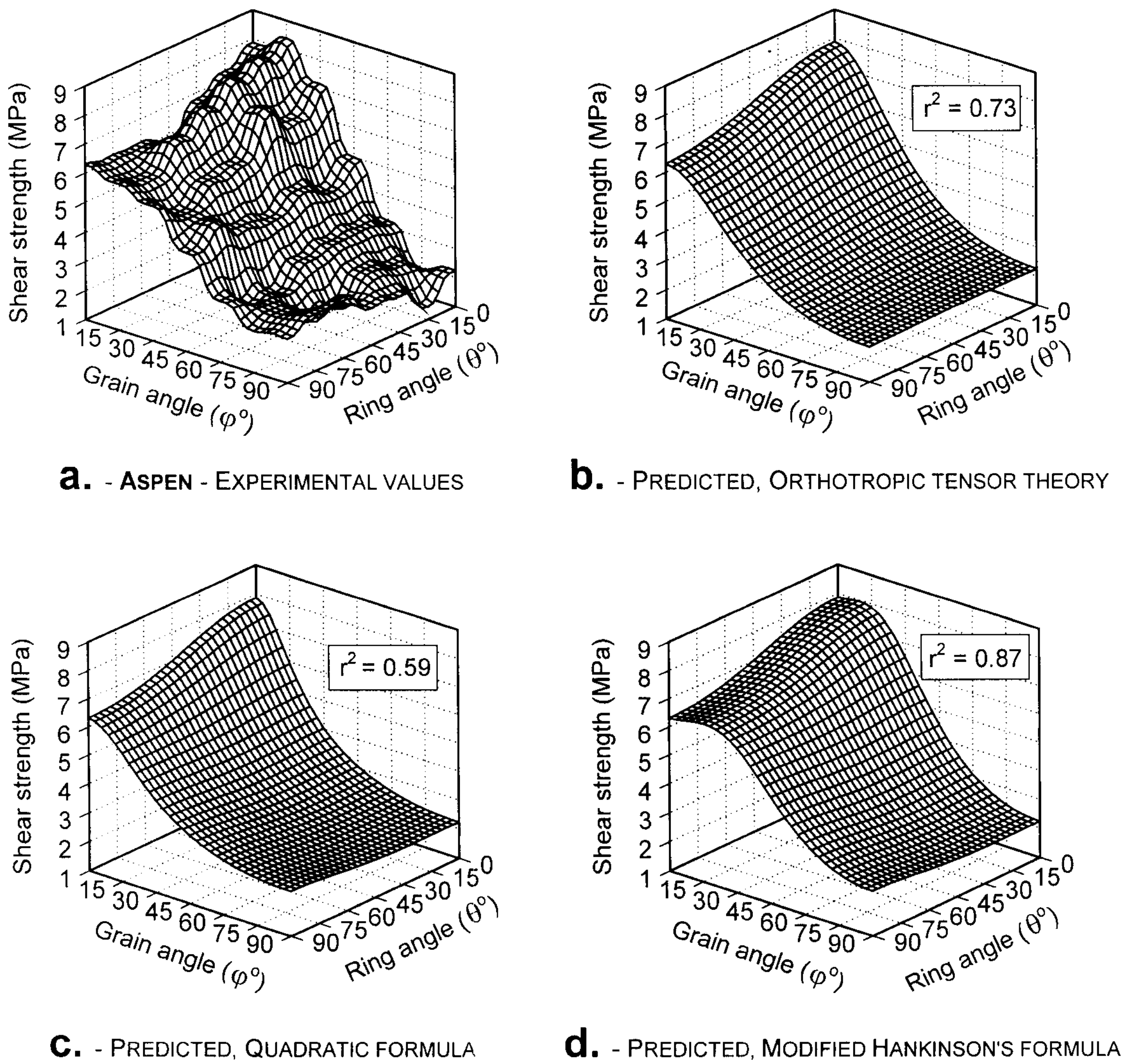


FIG. 6. Comparison of experimental and three model predicted shear strength data by orthotropy diagrams. Species: aspen (*Populus tremuloides*). The coefficients of determination (r^2) values are listed.

that this function depends on the length to width ratio of the sheared plane and other factors including force application method, etc. It might be suspected that the stress distribution at 15° grain orientation becomes more uniform along the length of the sheared plane, while possible stress peaks near the entrance notch at 0 degree grain angle accelerate the failure. This problem needs further investigation.

The failure mode experienced during this study was not always pure shear. Over 45°, grain angle ring-porous wood (oak) inclined to fail along the earlywood/latewood interface or along the ray parenchyma. The same tendency was encountered regarding the other species as the grain angle approached 90°. Liu and Floeter (1984) observed similar failure modes when testing Sitka spruce specimens in pure shear in the LT plane. They concluded that the shear strength depends only on the initiation of failure and not on the direction of fracture propagation. Consequently, we

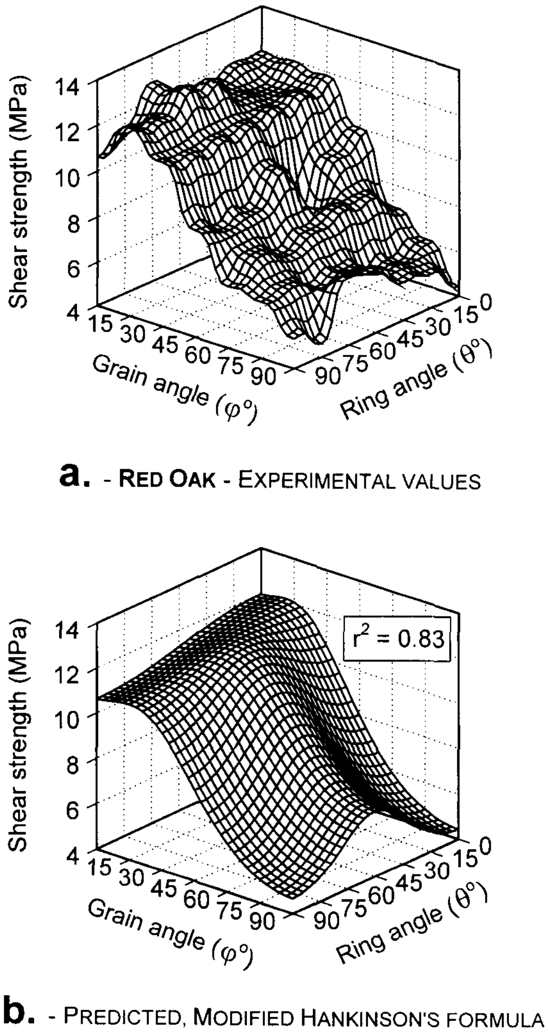


FIG. 7. Orthotropy diagrams of experimental and the best model predicted shear strength of red oak (*Quercus rubra*). The coefficient of determination (r^2) is listed.

deemed the obtained data as apparent shear strength. All of the measured shear strength values were kept, even if the specimen failed in a plane that was out of the theoretically sheared plane.

Inferences based on statistical evaluation

Standard statistical evaluation of the data included a two-way ANOVA procedure at 95% confidence level. The two factors were the grain and ring orientations, both with seven levels concurring with the 15° angle increments. For all species, the procedure revealed statistically significant differences among the levels of both factors. Furthermore, significant interaction was detected between the two factors. These results justify the applicability of prediction models that account for the effect of both ring and grain angle on the shear strength. Table 3 contains a typical ANOVA outcome.

TABLE 3. *ANOVA results—shear strength of aspen.*

Source	df	Sum of squares	Mean square	*P* value
Model	48	1,727	36.0	<0.0001
Grain angle	6	1,530	255.0	<0.0001
Ring angle	6	40	6.7	<0.0001
Grain × Ring	36	125	3.5	<0.0001
Error	364	111	0.3	
Total	412	1,838		

It should be noted, however, that four out of five data sets by species had lack of normality and demonstrated unequal variances. The violation of these statistical assumptions originated from the limited sample size and specimen manufacturing practice. Based on the standard deviations of the measurements, robust statistical analyses would have required approximately two hundred specimens for each factor and level combination. Moreover, most the specimens were cut from the same stem or lumber, and the specimens in a group were machined from blanks consecutively. Thus, complete randomization could not be achieved. More extensive testing and the fulfillment of completely randomized design were beyond the limitations of this research.

In the next step, the models discussed above, including the orthotropic tensor theory and the two combination models, based on the quadratic formula and the modified Hankinson's equation, were evaluated for the accuracy of their estimation. The necessary input data (τ_{RL}; τ_{TL}; τ_{RT}; τ_{TR}; $\tau_{90°}^{45°}$) were the average measured strength values. The power (n) for the modified Hankinson's equation was determined by curve fitting, using the entire experimental database. Each species had its n value

TABLE 4. *Coefficients of determination provided by the various prediction models.*

Species	Orthotropic tensor theory r^2	Quadratic formula r^2	Modified Hankinson's formula	
			n	r^2
Aspen	0.73	0.59	2.72	0.87
Oak	0.61	0.57	2.62	0.83
Yellow-poplar	0.68	0.62	2.47	0.76
True poplar	0.63	0.55	2.70	0.86
Turkey Oak	0.74	0.76	2.05	0.77

n – the power in the modified Hankinson's formula.

as listed in Table 4. The model generated strength values were plotted as orthotropy diagrams for visual evaluation. Figure 6 demonstrates the comparison between experimental and the three model predicted results.

Due to the deficiency of complete randomization, conservative statistical fitting procedures resulted in lack of fit for all of the cases. Thus, we selected the r^2 analysis to evaluate and rank the performance of the fitted models. The coefficient of determination (r^2) is a measure of how well the model describes the data. Larger values, close to 1, indicate that the model describes the relationship between independent and dependent variables well. The value of r^2, by definition, equals one minus the proportion of variability unexplained by the model (Dowdy and Wearden 1991). Numerically it is given by the following equation:

$$r^2 = 1 - \frac{\sigma^2_U}{\sigma^2_T} = 1 - \frac{\frac{\sum (\tau_{\varphi\theta i} - \hat{\tau}_{\varphi\theta})^2}{N - 1}}{\frac{\sum (\tau_{\varphi\theta i} - \bar{\tau})^2}{N - 1}}$$

$$= 1 - \frac{\sum (\tau_{\varphi\theta i} - \hat{\tau}_{\varphi\theta})^2}{\sum (\tau_{\varphi\theta i} - \bar{\tau})^2} \quad (8)$$

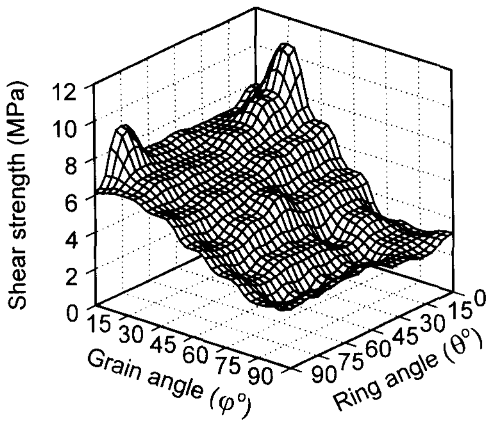
where

σ^2_T = total variance (or total sum of squares, as provided by the ANOVA);
σ^2_U = variance unexplained by the model;
N = the total number of shear strength measurements on the given species;
$\tau_{\varphi\theta i}$ = the i^{th} measurement at grain angle φ and ring angle θ;
$\bar{\tau}$ = average of the N measurement points (grand average);
$\hat{\tau}_{\varphi\theta}$ = predicted shear strength at grain angle φ and ring angle θ.

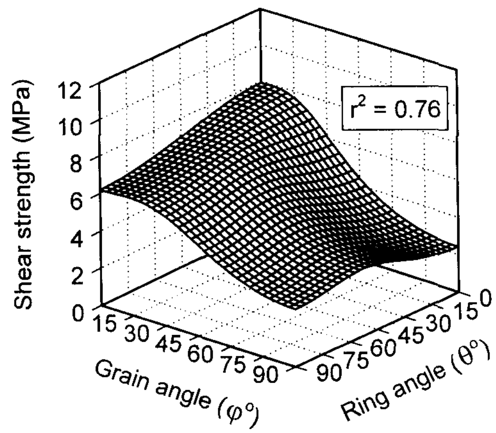
Table 4 compiles the results of the coefficients of determination analyses by species and model types. For all species, the combination model based on the modified Hankinson's equation resulted in the closest agreement with experimental data. Figures 6 to 10 show the comparison of experimental data and best model predictions by orthotrophy diagrams with the listed r^2 values. The good performance of this model was expected because the power determination was based on the entire experimental data set. Furthermore, only this model can mathematically estimate the peak stresses at small grain angle deviations.

Calculated r^2 values indicate that Eq. (1), derived from a 4-dimensional tensor analysis, can predict the orthotropy of shear strength reasonably well. The consistency of this model regarding the quality of the predictions and its strong theoretical background encourage its use, although the model can not predict the peek stresses other than at 0° grain orientation.

Conversely, the combined model, using the quadratic formula, did not provide good fit for four out of five species. Although the calculated r^2 values were over 0.55 that are acceptable for biological materials, compared to the other models, the accuracy of the predictions was significantly lower. Results imply that the derived equation may not be valid in all the oblique directions other than the principal an-

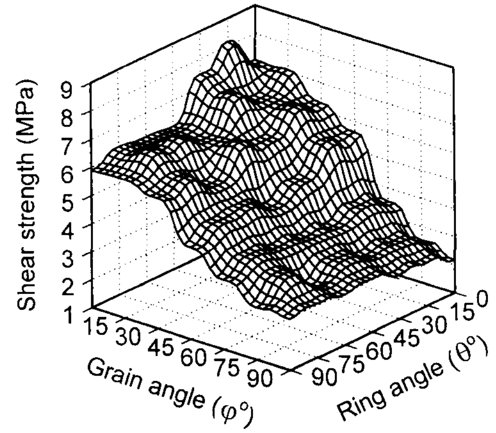


a. - **YELLOW-POPLAR** - EXPERIMENTAL VALUES

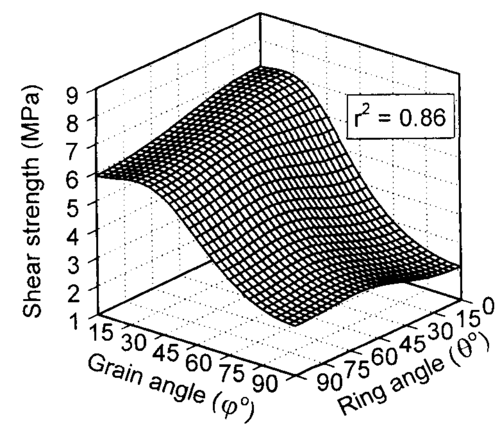


b. - PREDICTED, MODIFIED HANKINSON'S FORMULA

FIG. 8. Orthotropy diagrams of experimental and the best model predicted shear strength of yellow-poplar (*Liriodendron tulipifera*). The coefficient of determination (r^2) is listed.



a. - **TRUE POPLAR** - EXPERIMENTAL VALUES



b. - PREDICTED, MODIFIED HANKINSON'S FORMULA

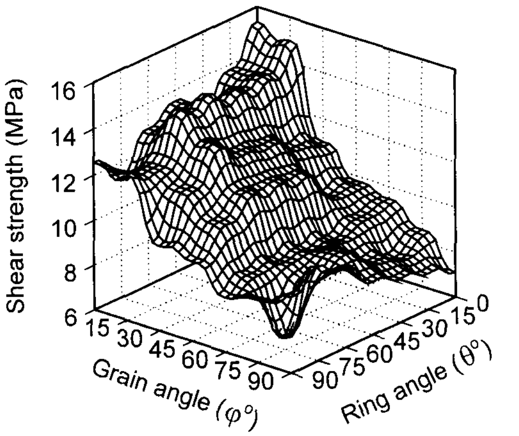
FIG. 9. Orthotropy diagrams of experimental and the best model predicted shear strength of true poplar (*Populus x. Euroamericana*). The coefficient of determination (r^2) is listed.

atomical planes because of the unique composite structure of solid wood.

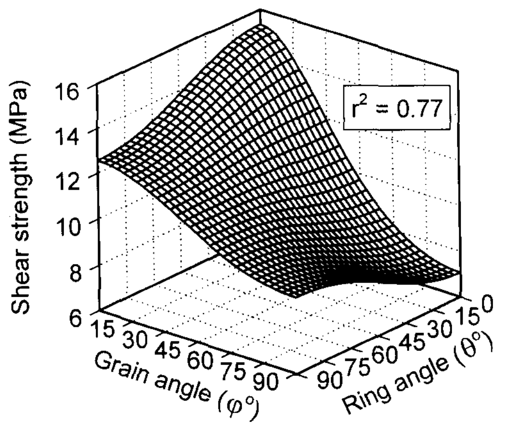
Test results of turkey oak (*Quercus cerris*) revealed high degree of orthotropy as a function of ring angle especially around 0° grain orientations (Fig. 10a). The r^2 values were about the same for all three models regarding this species, and the power of the modified Hankison's formula approached 2 as originally proposed. However, compared to other hardwoods, the quality of the prediction of this model decreased. On the contrary, the performance of the orthotropic tensor theory and the quadratic formula improved significantly.

SUMMARY AND CONCLUSIONS

Three North American and two European hardwood species were investigated for shear strength orthotropy. Apparent strength data were measured in 15° increments of grain and ring angle. Shear forces were applied in planes always parallel to the grain direction of the



a. - TURKEY OAK - EXPERIMENTAL VALUES



b. - PREDICTED, MODIFIED HANKINSON'S FORMULA

FIG. 10. Orthotropy diagrams of experimental and the best model predicted shear strength of turkey oak (*Quercus cerris*). The coefficient of determination (r^2) is listed.

specimens. Three models were fitted to experimental data. Although the strict randomization criterion of statistics has been violated, experimental data agreed reasonably well with the predictions.

A combined model, including two equations derived from tensor analysis and a modified version of the Hankinson's formula, proved to be the best predictor of apparent shear strength. This mostly empirical model, however, requires significant experimental database for power determination and may be very species specific.

The orthotropic tensor theory (Eq. (1)) has an advantage in that it uses only four experimentally predetermined data points. Furthermore, the apparent flexibility and its strong theoretical background encourage the application of this model for estimating the orthotropy of shear strength not only for solid wood but wood-based composites as well.

Based on the findings of this study and the above discussions, the orthotropic tensor theory appears to be a good tool for exploring the direction-dependent shear strength of structural composites including laminated veneer lumber (LVL) laminated strand lumber (LSL), and parallel strand lumber (PSL). The orthotropic shear strength of these composites may be critical in applications where the structural elements contain notches, angle cuts, and different types of connectors.

We developed and validated the models based on solid wood experimental data because of less expensive and more available raw materials. However, the shear strength values of solid wood in different anatomical directions may not be easy to relate to the shear strength of composites manufactured from the same species.

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