THE COMBINED OPTIMIZATION OF LOG BUCKING AND SAWING STRATEGIES

Thomas C. Maness
Assistant Professor
Faculty of Forestry
270-2357 Main Mall
University of British Columbia
Vancouver, BC V6T 1W5, Canada

and

Darius M. Adams
Professor
College of Forest Resources AR10
University of Washington
Seattle, WA 98195

(Received March 1990)

ABSTRACT

Determination of optimal bucking and sawing policies is linked in a common model. The core of this model is a linear program (LP) that selects stem bucking and log sawing policies to maximize profits given an input distribution of raw material. Product output is controlled by price-volume relationships that simulate product demand curves. The model uses a three stage solution process performed iteratively until identical solution bases are obtained. A variation of the Dantzig-Wolfe decomposition principle is used, linking the three models through the use of the Lagrange multipliers from the LP. The procedure is demonstrated for a sample sawmill. The revenue gain from using the policies suggested by the integrated model over those found by the bucking and sawing programs working separately was found to be 26%–36%.

Keywords: Sawmill planning, linear programming, decomposition, process control.

INTRODUCTION

The production of dimension lumber at a sawmill involves a complex sequence of decisions at the various manufacturing stations as a log is processed into component products. These manufacturing stations include log bucking, which reduces log stems into shorter logs; primary sawing at the headrig, which cuts the bucked logs into rough lumber; and secondary processing, which edges, resaws, and trims the rough lumber into its final green dimensions. In a modern sawmill, many of these decisions are automated by using computer controlled optimization algorithms working from a set of known or "guessed" final product values. The objective of the mill is to produce a mix of products that maximizes net revenue to the mill.

Two problems are inherent in this approach. First, the set of product values used in the algorithms is not necessarily the correct set to maximize net revenue. Determining this set of product values is a complex and time-consuming problem in itself. Second, each manufacturing station typically operates in isolation from those preceding and following it. Optimizing a set of individual manufacturing stations is not the same problem as optimizing the production of the mill as a

Wood and Fiber Science, 23(2), 1991, pp. 296–314

whole. This paper develops a planning model to allow a random length softwood dimension lumber mill to target its production from week to week based on its restricted processing technology, current marketing data, and available raw material. The technique used in this study integrates the log bucking, sawing and allocation problems into a single model, building from the work of many previous researchers.

Log allocation models that help balance mill inventory levels and assign logs to competing facilities have been available for some time. Linear programming (LP) applications were developed by Row et al. (1965), Jackson and Smith (1961), and Pearse and Sidneysmith (1966). Sampson and Fasick (1970) extended these models to allocate logs to competing conversion stations within a single mill facility. These models suffer from two principal problems. First, each possible sawing pattern at the headrig uses a separate column in the LP. The same is true of the bucking patterns. To enumerate all of the possible sawing and bucking patterns would create a matrix of technical coefficients so large as to make the resulting model computationally infeasible. However, the ability to select between alternative bucking and sawing patterns is one of the principal benefits of the log allocation model. Therefore, the full set of possible patterns must be included if the model is to perform correctly. In the models discussed above, a very small subset of the possible patterns could be included. The second problem deals with the formulation of the mill's marketing conditions. These models essentially maximized the revenue associated with producing a fixed level of output. This formulation bypasses the determination of the output mix that will maximize the revenue to the mill, one of the most interesting and pressing concerns of the mill manager.

An early stem bucking model was developed by Pnevmaticos and Mann (1972), using dynamic programming to maximize the net value of a series of logs cut from a single stem. The value of the individual log segments themselves must be known to use this model, an interesting and useful problem in itself. Briggs (1980) and later Faaland and Briggs (1984) integrated a log sawing algorithm into the bucking model, which eliminated the need to value the individual log segments and allowed the sawing pattern to change, depending on the value of the actual boards sawn from the bucked logs. However, since this model operates on a single stem at a time, it is not capable of considering the problem of determining optimal output from the sawmill as a whole.

The first full integration of the bucking, sawing, and allocation model was proposed by Gluck and Koch (1973). McPhalen (1978) developed a log allocation model for a mill complex that made a stem bucking problem based on dynamic programming (DP) auxiliary to the allocation LP, and used the Dantzig-Wolfe decomposition principle (Dantzig and Wolfe 1960) to link the models. A subroutine to the stem bucking algorithm allowed the DP to choose between four different sawing patterns for determining the value of the resulting bucked logs. Mendoza (1980) created a similar model for a regionwide planning system, using regional yield tables as the basis for the matrix of log sawing technical coefficients. Eng et al. (1986) used the same principle to develop a planning model for optimal wood bucking practices. All three of these models assumed constant final product prices over a fixed level of output.

The model developed in this paper follows the example of McPhalen in that linear and dynamic programming are employed using the Dantzig-Wolfe decomposition principle, with two innovations. A log sawing algorithm is included as a sub-model to the LP to generate all possible sawing patterns in the solution space. A system of price-volume relationships is used to control output rather than fixed output and fixed prices.

Three individual models are presented in this paper to address each aspect of the production planning model. Since these models are constructed for a particular manufacturing facility, their application is mill, rather than region specific. The individual models are:

1. A cutting pattern optimizer that simulates the cant sawing of a log with a given diameter, taper and length, and determines the optimal sawing pattern given lumber values by dimension and length.
2. A stem bucking model that solves the optimal bucking policy.
3. A log allocation model that introduces production constraints and optimizes the output of a sawmill based on input from 1 and 2 above.

The three models are linked and solved simultaneously, the first two programs as sub-problems to the LP. Suggested cant breakdown patterns and log bucking solutions are generated for the LP as a function of the current marginal value of both primary and secondary products. The LP then chooses between the set of "suggested" patterns rather than the full set of feasible patterns. The set of "suggested" patterns continues to grow until no new pattern increases the objective function of the LP. At this point the problem is solved.

THE CUTTING PATTERN OPTIMIZER

Limitations on the types of patterns possible for a given mill are imposed primarily by the headrig and to a lesser extent by downstream processing centers. The cutting pattern optimizer outlined below is designed for a chipper-canter QUAD bandmill. Figure 1 shows the placement of saws and chipper heads in the headrig, and Fig. 2 shows the resulting placement of boards in the sawn pattern. The chipper heads open the face of the log and provide a smooth edge for the outside side boards. The pattern is symmetric about a vertical axis running through the center of the log. The bandsaws remove up to 4 side flitches, which drop off the chain at the headrig and are carried to secondary processing centers, such as an optimizing edger, which determines the width of the board that is to be taken from each flitch. After processing at the headrig, the cant is rotated 90 degrees, and laid flat on a conveyor. One edge of the cant is set against a fixed fence and the cant is run through a gang saw. The top and bottom boards from the cant may then also travel to the edger. All boards are transported to a trimmer, which trims them to the best length. The four stations where optimization occurs in this mill are the headrig, the gang, the edger, and the trimmer.

Mathematical structure

The optimal pattern for a given cant width is built in four steps:

1. Build the largest full length cant possible.
2. Maximize the value added due to side boards. This step represents a subproblem utilizing a recursive procedure.

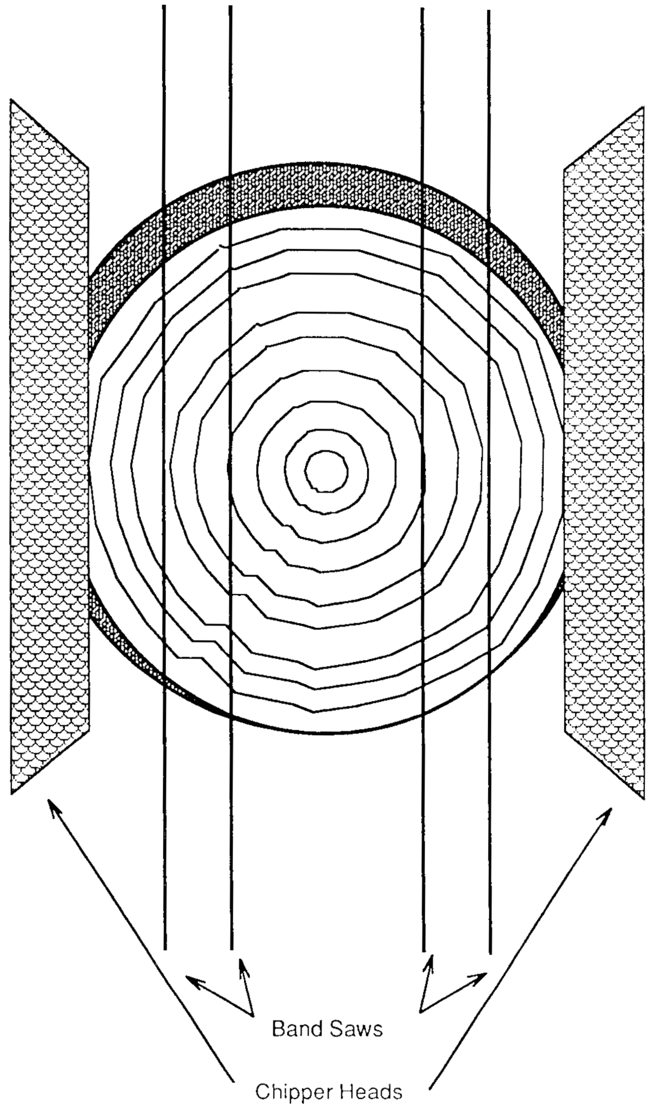

FIG. 1. Placement of saws and chipper heads in the chipper-canter QUAD bandmill.

3. Maximize the value added due to top and bottom boards. This step is also a sub-problem similar to number 2.
4. Remove one board from the cant and repeat step 3. If total value goes down, the pattern is complete; if value goes up repeat step 4.

These steps are performed sequentially. All four steps are performed for each possible cant width. The highest valued cant width is then chosen as the optimal pattern.

Step 1.—Using the Pythagorean theorem, the largest cant height (highest rectangle of width CW) that can be fit into a circle representing the small end diameter

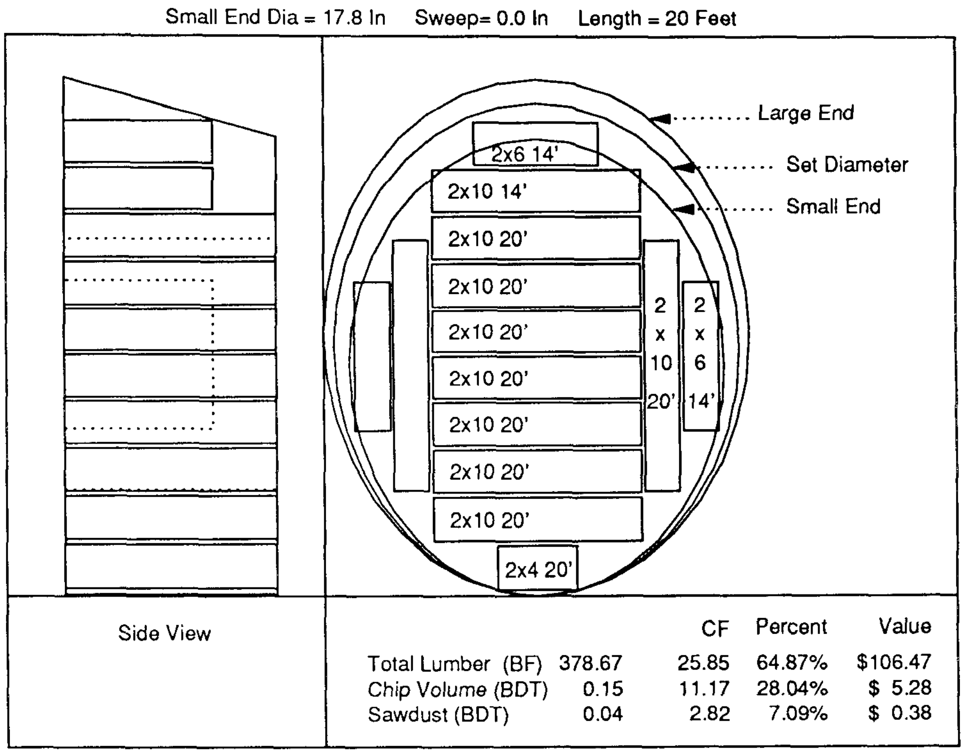

FIG. 2. A sawing pattern for the QUAD bandmill developed by the cutting pattern optimizer.

of the log is given by the relationship:

$$CH = 2*\{r^2 - (0.5*CW)^2\}^{0.5} \tag{1}$$

where:

CH = maximum cant height in inches
r = radius of the log at the small end
CW = green target width of cant.

The maximum number of cant boards with nominal thickness i that can fit inside this cant is:

$$NB = INT\{(CH + SK)/(TS_i + SK)\} \tag{2}$$

where:

NB = max number of boards of a given thickness that will fit in a log of small end diameter 2r
TS_i = target size of nominal thickness i
SK = saw blade kerf.

NB must be an integer. The actual cant height used is then determined by:

$$CH = NB*TS_i + (NB - 1)*SK. \tag{3}$$

A feasibility restriction is placed on the minimum allowable cant height as compared to the diameter of the log to ensure that it will ride correctly on a conveyor after sawing. If the maximum possible cant height falls below this level, the cant pattern is discarded without further optimization.

Step 2.—This step determines the optimal side boards to be included in the pattern and is solved using a recursive relationship. For a log with small end diameter (sed) d, length K, and taper t, the problem is expressed mathematically as:

Find the n, i, j, and k so as to:

$$\text{Max} \sum_{n} \{p(n, i_n, j_n, k_n) + p_f(LV_{d,t,k}, n, i_n, j_n, k_n)\} \quad (4)$$

subject to:

$$\{(0.5 * CW + n * SK + i_{n-1} + i_n)^2 + (0.5 * j_n)^2\}^{0.5} < r_{d,t,k} \quad (5)$$

$n = 1, 2$ Board number (1 is inside board, 2 is outside)

where:

$p(n, i, j, k)$ = price for board n in thickness class i, width j, and length k
$p_f(LV, n, i, j, k)$ = resulting value of fiber when board n is removed from log
$LV_{d,t,k}$ = cubic volume of a log of sed d, length k, and taper t
i_n = green target thickness (in.) for board number n
j_n = green target width (in.) for board number n
k_n = length of board n
CW = cant width (given by step 1)
$r_{d,t,k}$ = radius of log of sed d and taper t, at k ft from the large end
SK = saw kerf.

The problem is to maximize the sum of the value of the side boards plus the resulting fiber, subject to the constraint that the outside corner of the board is within the radius of the log at that length (i.e., no wane present). The algorithm is made more efficient than an exhaustive search by organizing all boards into thickness classes. This is because the only factor that has an effect on the feasability of outside boards to be added is the thickness of the inside side board. The width of the inside board only affects whether or not it itself is feasible. The problem is then to find the highest valued product in each thickness class that is feasible for the inside side board, and pair this with the highest valued feasible board for the outside side board. Also, the algorithm is more efficient if the length of the board (k) varies on the inner loop. As soon as a given length is found to be infeasible, it is not necessary to check longer lengths since they, too, will be infeasible.

Step 3.—The third step in the program is solved in a manner similar to step 2, except that the presence of the bottom board makes the structure of the algorithm slightly more complicated. The mathematical structure of the problem is to maximize (4) above subject to (5) with n = 1, 2, 3 (where n = 1 is the bottom board in the pattern), and:

$$k = \begin{cases} 8, 10, 12, \ldots K & \text{if } n > 1 \\ K & \text{if } n = 1 \end{cases}$$

The restriction on k when n = 1 holds because the bottom board is not affected by log taper, i.e., they must be full length because the cant is sawn with a fixed fence. The algorithm used for step 3 is essentially the same as that for step 2, except that potential bottom boards are not organized into thickness classes since both the thickness and width of this board affect the remaining decisions to be made. When $n > 1$, the boards are classed in thickness groups to reduce the number of iterations in the algorithm as above. Also, since the removal of the side boards occurs first, a restriction is placed on the feasibility of top and bottom boards that the maximum width of these boards must be less than or equal to the cant width. Any potential product with width greater than the cant width is not examined in step 3.

The stem bucking model

The stem bucking model's function is to determine the optimal combination of bucked logs that should be cut from a long length stem at the mill.

Mathematical structure.—The problem for a given stem is to find the x_{ij} that:

$$\text{Max} \sum_i \sum_j x_{ij}\pi_{ij} \tag{6}$$

Subject to:

$$\sum_j x_{ij}j \leqq L \tag{7}$$

$$x_{ij} \geqq 0, \text{ integer}$$

where:

x_{ij} = number of logs of sed i and length j
π_{ij} = value of log with sed i and length j
L = stem length

This problem is solved in a DP framework using the knapsack problem and fixed intervals of cummulative log length termed cutpoints, measured from the small end of the log. The optimal value function is:

f(n) = maximum value of stem from cut point n back to cut point zero.

The fundamental recurrence relationship is:

$$f(n) = \max_j\{\pi_{ij} + f(n - j)\} \tag{8}$$

with the boundary condition,

$$f(0) = 0.$$

Implementation.—To allow for log trim, cut points are set on 25-inch intervals. Thus, a resulting 16-ft log would have 8 inches of trim allowance. The values for the log segments (the π_{ij}'s) are obtained as the Lagrange multipliers from the master problem (which is developed below).

The log allocation model

The log allocation model is the master linear programming model which uses the cutting pattern optimizer and the stem bucking model to generate columns. The purpose of the allocation model is to distribute materials to various manufacturing stations, and to select the appropriate bucking and sawing strategies to maximize the objectives of management. The key variables in the maximization are the lumber produced (LUM_SALES_{ilq}), the stem bucking solutions (STM_BUCK_{SB}), and the log sawing solutions ($LOGS\_SAWN_{LC}$).

Mathematical structure.—The general structure of the log allocation model used is to maximize net returns subject to four types of constraints. Net returns would be defined as the proceeds received from the sale of products at the mill level less the cost of producing them. The objective function used is piecewise linear to model decreasing sales prices as production increases.

The mathematical formulation of the model is:

Maximize:

$$\begin{aligned}
&\sum_i \sum_l \sum_q \{LUM_SALES_{ilq} * LUM_PR_{ilq})\} \\
&\quad + \sum_L \{LOG_SALES_L * LOGS_PR_L\} \\
&\quad + \{RNDWD_FIBER + GRN_CHIPS\} * FIBER_PR \\
&\quad - \{STEM_PRCH * STEMS_PR\} - \sum_K \{HOURS_k * COST_HR_k\} \\
&\quad - TOT_OPER_COST - TOT_FIN_COST \qquad (9)
\end{aligned}$$

Subject to:

$$STEM\_PRCH * \%\_STEM\_DIST_S - \sum_B \{STM\_BUCK_{SB}\} = 0 \quad (10)$$

$$\sum_S \sum_B \{STM\_BUCK_{SB} * LOG\_RECOV_{LSB}\} - \sum_C LOGS\_SAWN_{LC} - LOG\_SALES_L = 0 \quad (11)$$

$$\sum_L \sum_C \{LOGS\_SAWN_{LC} * LUM\_RECOV_{ilLC}\} - \sum_q \{LUM\_SALES_{ilq}\} = 0 \quad (12)$$

$$\sum_L \sum_C \{LOGS\_SAWN_{LC}\} * HRS\_PER\_CCF_L - HOURS_1 = 0 \quad (13)$$

$$\sum_i \sum_l \sum_q \{LUM\_SALES_{ilq}\} * HRS\_PER\_MBF_{ilq} - HOURS_2 = 0 \quad (14)$$

$$\sum_S \sum_B \{STM\_BUCK_{SB} * RNDWD\_RECOV_{SB}\} - RNDWD\_FIBER = 0 \quad (15)$$

$$\sum_C \sum_L \{LOGS\_SAWN_{LC} * FIBER\_RECOV_{LC}\} - GRN\_CHIPS = 0 \quad (16)$$

$$\sum_i \sum_l \sum_q \{LUM\_SALES_{ilq}\} * OPER\_COST_{ilq}\} - TOT\_OPER\_COST = 0 \quad (17)$$

$$\sum_i \sum_l \sum_q \{LUM\_SALES_{ilq}\} * FIN\_COST_{ilq}\} - TOT\_FIN\_COST = 0 \quad (18)$$

$$LUM\_SALES_{ilq} \leqq SALES\_LIMIT_{ilq} \quad (19)$$

$$HOURS_K \leqq AVAIL\_HRS_K \quad (20)$$

$$STEM\_PRCH \leqq AVAIL\_STEMS \quad (21)$$

Nonnegativity on all variables

where:

$AVAIL\_HRS_K$ = Total available hours for machine center K.
$AVAIL\_STEMS$ = Maximum cubic volume of raw material available over the planning period.
$COST\_HR_k$ = Cost of an hour of time for process K.
$FIBER\_PR$ = Price received for wood fiber in $/ton.
$FIBER\_RECOV_{LC}$ = Recovery of chip fiber in percent when sawing a log in class L using sawing policy C.
FIN_COST_{ilq} = Cost in $/MBF for finishing, marketing and shipping lumber of dimension i, length l and demand group q.
GRN_CHIPS = Total amount of chip fiber produced as a by-product of sawing logs into lumber.
$HOURS_K$ = Total hours used in process K, where K = 1 for the sawmill, K = 2 for the planer mill.
$HRS\_PER\_CCF_L$ = Hours used to saw one cunit of logs in log class L.
$HRS\_PER\_MBF_{ilq}$ = Hours used to finish one MBF of lumber of dimension i, length l and demand group q.
LOG_RECOV_{LSB} = Percent of volume in stem class S which will be allocated to log class L when using bucking policy B.
$LOGS\_PR_L$ = Price received for logs in log class L if sold on open market.
LOG_SALES_L = Cubic volume of logs sold on market in log class L.
$LOGS\_SAWN_{LC}$ = Cubic volume of logs in class L which are sawn using sawing policy C.
LUM_PR_{ilq} = Price of lumber in $/MBF by dimension i, length l, and demand group q.
LUM_RECOV_{ilLC} = Recovery in MBF/CCF of lumber with dimension i and length l obtained when sawing a log in class L using sawing policy C.
LUM_SALES_{ilq} = Amount of lumber sold of dimension i, and length l in demand group q.
$OPER\_COST_{ilq}$ = Operating cost in dollars per MBF for overhead in producing lumber of dimension i, length l and demand group q.
$RNDWD\_FIBER$ = Total amount of roundwood fiber produced by bucking logs to segments smaller than 8 ft.
$RNDWD\_RECOV_{SB}$ = Recovery of roundwood fiber in tons per CCF when bucking a stem in class S using policy B.

$SALES\_LIMIT_{ilq}$	=	The maximum amount of lumber of dimension i, length l that can be sold in demand group q.
STEM_PRCH	=	Cubic volume of stems purchased (raw material) by the sawmill.
STEMS_PR	=	Price per cunit of raw material purchased.
STM_BUCK_{SB}	=	Cubic volume of stems in stem class S which are bucked using policy B. Stem class S refers to a small end diameter and length.
$\%\_STEM\_DIST_S$	=	Percent of raw material found to be in stem class S when purchasing a cunit of STEM_PRCH.
TOT_FIN_COST	=	Total finishing cost in dollars.
TOT_OPER_COST	=	Total sawmill labor and overhead cost in dollars.

The objective function maximizes the value of all products produced minus production costs. Lumber products are divided into demand groups (q), each with a separate price. The volume of sales in each demand group is limited by the upper bound on sales $SALES\_LIMIT_{ilq}$. In this formulation the price received for a product in demand group q must be higher than that received for demand group q + 1. This provides the price driven controls on lumber production. Fiber logs are sold as outside sales through the variable LOG_SALES_L. Production costs come in as a cost per hour for the sawmill and finishing mill. Other costs which are not included in this hourly charge can be charged at a rate per MBF through the operating cost and finishing cost. Raw material costs are handled with the variable STEM_PRCH.

Constraints (10) ensure that no more stems are bucked than are input into the mill. Constraints (11) allocate each bucked log to one and only one process. For a complex mill, valid processes might be a Quad mill line, a Chip-N-Saw mill line and outside sales. These equations contain the matrix of technological coefficients that convert stems into logs (LOG_RECOV_{LSB}), which are constructed by the stem bucking program. Constraints (12) contain the matrix of technological coefficients that convert logs into lumber (LUM_RECOV_{ilLC}), the columns of which are constructed by the cutting pattern optimizer. These constraints may also be broken into subgroups, with each subgroup handling a separate manufacturing line. Constraints (12) also perform the function of allocating the lumber produced into each demand group. Upper bounds (constraint (19)) are placed on each of the demand groups as necessary to mimic pseudo-demand curves.

Constraints (13) and (14) are mill production and capacity constraints that ensure that no more than the maximum amount of sawing hours and finishing hours are used by the mill. Constraint (15) accumulates chip fiber recovered from the sawmill process, and (16) accumulates roundwood fiber recovered from the bucking station. Coefficients in these two constraints convert chip volume to weight. Constraints (17) and (18) accumulate operating and finishing costs for producing lumber. These can contain items such as drying costs, overhead, or any other costs not explicitly charged out on an hourly basis. Note also that these charges vary by dimension, length and demand level. Thus, if the mill exhibits increasing costs these may be input in a manner similar to the lumber price curves.

The resulting matrix for the LP model is shown in Fig. 3. The constraints run down the page from (10) to (18) in the groups discussed. Constraint types (19)

Constraint Name	STEMS PRCH	STEM_BUCK	RNDWD FIBER	LOGS_SAWN	LUM_SALES	GRN CHIPS	LOG_SALES	HOURS1	HOURS2	TOT OPER COST	TOT FIN COST		RHS
OBJECTIVE FUNCTION	STEMS PR		FIBER PR		LUM PR	FIBER PR	LOG PR	COST HR1	COST HR2	-1	-1		
UPPER BOUNDS	AVAIL STEMS				SALES LIMITS			AVAIL HRS1	AVAIL HR2				
STEMS (10)	% STEM DIST	-1 -1 -1 .. -1										=	0
LOGS (11)		LOG RECOV		-1 -1 -1 ... -1			-1 -1 -1 ... -1					=	0
LUMBER (12)				LUM_RECOV	-1.. -1... -1.. .. -1..							=	0
HRS1 (13)				HRS_PER_CCF				-1				=	0
HRS2 (14)					HRS_PER_MBF2				-1			=	0
RNDWD (15)		RNDWD RECOV	-1									=	0
CHIPS (16)				FIBER_RECOV		-1						=	0
OPER COST (17)					OPER COST					-1		=	0
FIN COST (18)					FIN COST						-1	=	0

FIG. 3. Matrix for the LP model.

through (21) are handled as upper bounds on variables (row 2 in the diagram). The objective function is located in row 1.

Integration of the three models

Using the cutting pattern model as an example, note that the objective function is:

Maximize:

$$\sum_{i} \sum_{l} \mathrm{LUM\_SALES}_{il} * \pi_{il} \qquad (22)$$

This model chooses a sawing policy for a given log class that maximizes the value of the lumber produced. The π_{il}'s are the Lagrangian multipliers for the associated constraint (12) in the LP model. When the sawing submodel is called, the LP passes the current marginal values for each lumber product. Therefore,

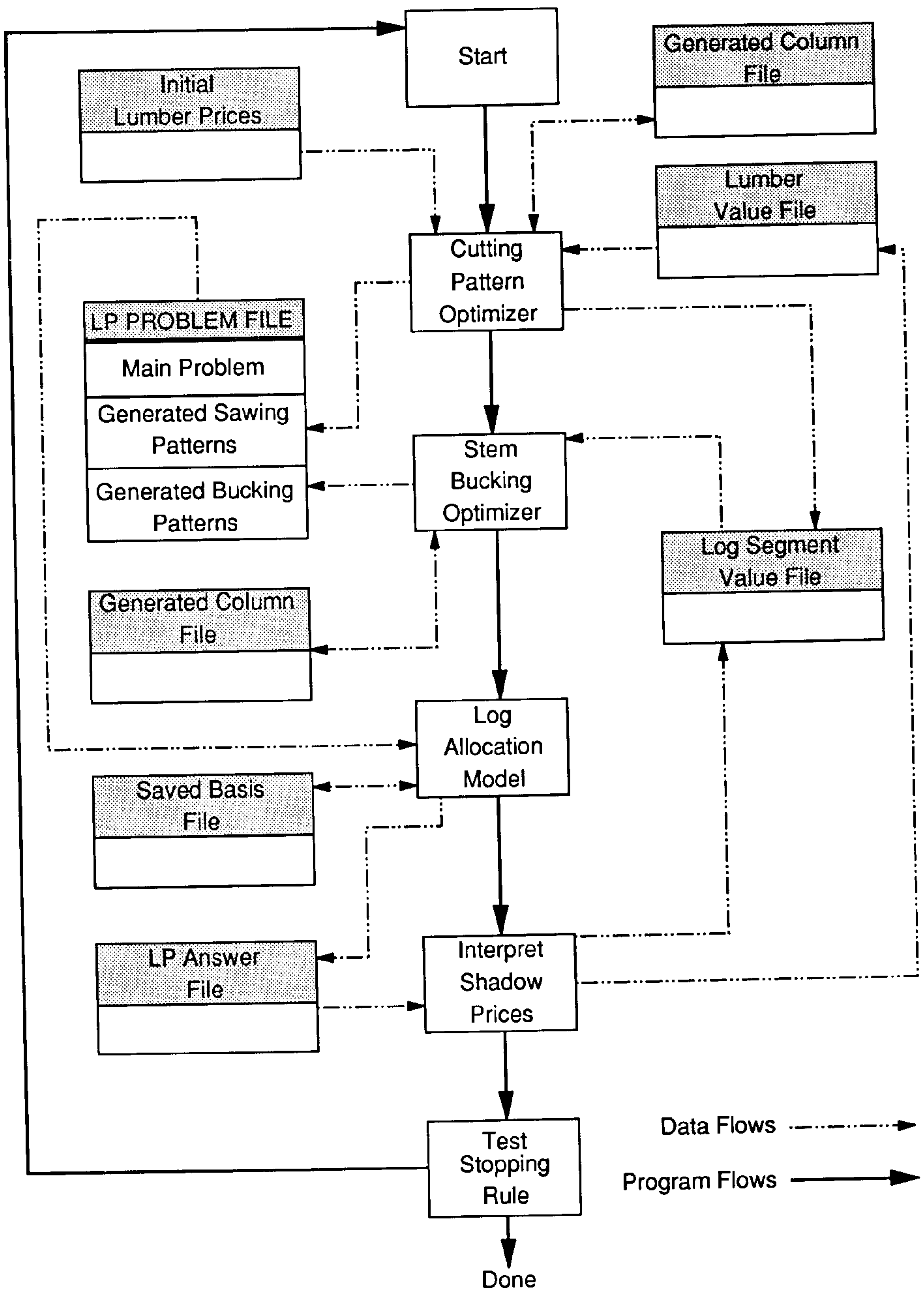

FIG. 4. A flow chart of the integrated model.

the cutting pattern optimizer is able to determine the sawing policy which will have the largest positive impact on the objective function of the LP. The current best policy is then added to the LP tableau as a new column.

A flow chart of the integrated model is shown in Fig. 4. The procedure is as follows: First the cutting pattern generator reads the current marginal value of each lumber product by dimension and length, which is output from the LP. The program then develops one sawing pattern for each log class using these lumber values. The cutting pattern generator then builds a file containing the generated columns that will be passed to the LP and maintains a list of the columns that has been generated so far (to prevent duplicate columns). Next, the stem bucking model reads the shadow prices for log segments generated by the LP, and uses these to generate a bucking pattern for each stem class. Again a list of all columns added is kept to prevent duplicate columns. The log allocation model then reads in all available columns to this point and obtains an optimal solution. If this value has increased by less than a specified value, the problem is solved. Otherwise the program execution repeats.

APPLICATION

The methodology outlined above was used to build a model supporting the production planning function of a medium sized QUAD sawmill producing roughly 80 MMBF of dimension lumber annually. The model is used for production control and inventory management. The marginal values for lumber and logs developed by this model are used to control the process control logic of the automated log bucking (merchandising) station, headrig setworks, trimmer and edgers.

Background information

This sawmill produces Douglas-fir and HEM-FIR dimension lumber consisting of 2 × 4's, 2 × 6's, 2 × 8's, and 2 × 10's in random lengths varying from 8 to 20 ft. The market for these products is variable so that consistent production of a set product mix from week to week results in a lopsided inventory with numerous shortages and surpluses. The raw material is in the form of trucked woods bucked stems ranging from 8 to 44 ft long, and 4 to 16 inches in diameter at the small end. The average stem has a small end diameter of roughly 7 inches and a 30-ft length.

Stems travelling into the sawmill are optically scanned at a computer controlled bucking station where the dynamic programming model outlined above determines the best bucking policy. The logs then travel to a QUAD headrig, where they are scanned again, and the optimum sawing strategy is executed. Outside flitches travel to an optimizing edger, which determines the best board dimension and length to take from the flitch. All boards then travel to an optimizing trimmer.

The optimization stations require a table of values by dimension and length for the different lumber or log segment products produced at that station. However, as discussed earlier, the values used for these products must reflect a whole array of complex issues such as the strength of the market, the raw material available, inventory levels and sawmill technology. Typically, using fixed prices results in the overproduction of products with shallow markets, because the prices of such products will in fact fall as output expands. Fixed prices in general do not reflect the market realities facing a sawmill. The solution used in this example is to use the shadow prices from the log allocation model as the input to the three optimization stations.

The model is designed as a weekly planning model. Each week the raw material input is adjusted to reflect the following week's logs. Lumber prices in the model are adjusted to reflect current markets and inventory levels. For example, if too many 8-ft long 2 × 4's were produced the previous week, the price structure is altered to inhibit their production. The model is then employed and the resulting forecasted output distribution examined. If management detects a problem with the proposed production, the price structure is altered appropriately and the process repeated. Otherwise the new shadow prices are obtained and put into the decision logic of the optimization stations. The production forecasts for the coming week can then be used by marketing to develop a sales plan.

Inputs

The raw material input mix used by the model is obtained by segregating the actual raw material received by the mill into stem classes. The actual stem distribution was determined from output by the merchandizing station. Stem classes are defined by three attributes of the stem: small end diameter, length, and taper class. For this example problem, all stems were assumed to have a constant taper of 1% (roughly 1 in. in 8 ft).

The model was configured to accept up to four levels of demand for each lumber product sold, each stating a price and maximum sales level at that price. The fourth demand level was given a very high sales level to prevent infeasible solutions. It is important to note that the price volume relationships used in the model should not be considered as absolute "demand curves" for the various products. There are many markets and grades of products that this model does not consider, and there is no demand for generic, largely grouped products. Rather, these price volume relationships should be thought of as controls on production that offer more insight into the production problem than would strict upper bounds on output.

Price-volume relationships are illustrated in Table 1 for 2 × 4 and 2 × 10 dimensions. Similar tables are employed for 2 × 6 and 2 × 8 dimensions but are not shown here to conserve space. There is a limited market for lumber in the shorter lengths (8 and 10 ft), and practically no market for 2 × 10's in this range. In 12 and 14 ft lumber, the 2 × 10 market is very strong with high prices that hold up very well. However, this is not a good length for 2 × 4's. These prices start well but fall off sharply. Sixteen, 18 and 20 ft lumber exhibits very high prices, with 2 × 10 prices falling sharply at first but holding up well at higher sales levels.

An exhaustive presentation of the actual input data used in the model is not given in this paper as it is quite lengthy. Interested readers should refer to Maness (1989) for a complete description of the inputs used in the model.

Solution efficiency

The sample problem was run on a 386 based microcomputer, taking 7 iterations of the integrated model with a run time of approximately 1 hour to convergence. The stopping rule used for the model was to converge to identical objective

TABLE 1. *Table of lumber price relationships used in the model for 2 × 4 and 2 × 10.*

Table of lumber prices								
		Lumber length in feet						
Demand group	Category	8	10	12	14	16	18	20
				2 × 4				
1	Price	$187	$195	$215	$215	$250	$235	$240
	Max sales	38.3	41.8	62.7	52.3	59.2	62.7	31.4
2	Price	$159	$166	$183	$183	$213	$200	$204
	Max sales	19.7	21.5	32.3	26.9	30.5	32.3	16.2
3	Price	$140	$146	$161	$161	$188	$176	$180
	Max sales	19.2	20.9	31.4	26.1	29.6	31.4	15.7
4	Price	$94	$98	$108	$108	$125	$118	$120
	Max sales	9,999	9,999	9,999	9,999	9,999	9,999	9,999
				2 × 10				
1	Price	$175	$180	$295	$315	$310	$310	$290
	Max sales	2.6	15.5	49.2	77.6	28.5	36.2	49.2
2	Price	$158	$162	$266	$284	$279	$279	$261
	Max sales	1.1	6.7	21.1	33.3	12.2	15.5	21.1
3	Price	$140	$144	$236	$252	$248	$248	$232
	Max sales	1.5	8.9	28.1	44.4	16.3	20.7	28.1
4	Price	$131	$135	$221	$236	$233	$233	$218
	Max sales	9,999	9,999	9,999	9,999	9,999	9,999	9,999

function values. This was achieved when the shadow prices for lumber stabilized and the cutting pattern and stem bucking subprograms no longer brought in new columns.

Figure 5 summarizes the convergence of the objective function in terms of the number of iterations and LP problem size. Note that the objective function improves steeply during the first two iterations and then tapers off quite rapidly. Practically no improvement in the objective function can be seen after iteration 4 of the integrated problem. The integrated problem starts out with roughly 800 columns, adds columns rapidly until iteration number 5, and then stabilizes at nearly 1600 columns. The number of iterations necessary to solve the LP within each iteration of the integrated problem is related to the number of new columns added at every step.

Model results

Model output consists of the optimal bucking and sawing patterns for each log and stem class, the final distribution of lumber output, and the ending shadow prices for stems, bucked logs, and sawn lumber. Each of these serves a distinct purpose in analyzing the performance of the model.

Lumber distribution produced.—The lumber distribution produced in this sample problem is given in Table 2. The production of 2 × 4's fills demand group 1 at its upper limit for every length, demand group 2 is also filled for the 12–20-ft lengths, and demand group 3 for the 18- and 20-ft lengths. The 10-ft length has a very small volume in demand group 2. The production of 2 × 10 lumber stretched beyond the second demand group in the 12–18-ft lengths, and was

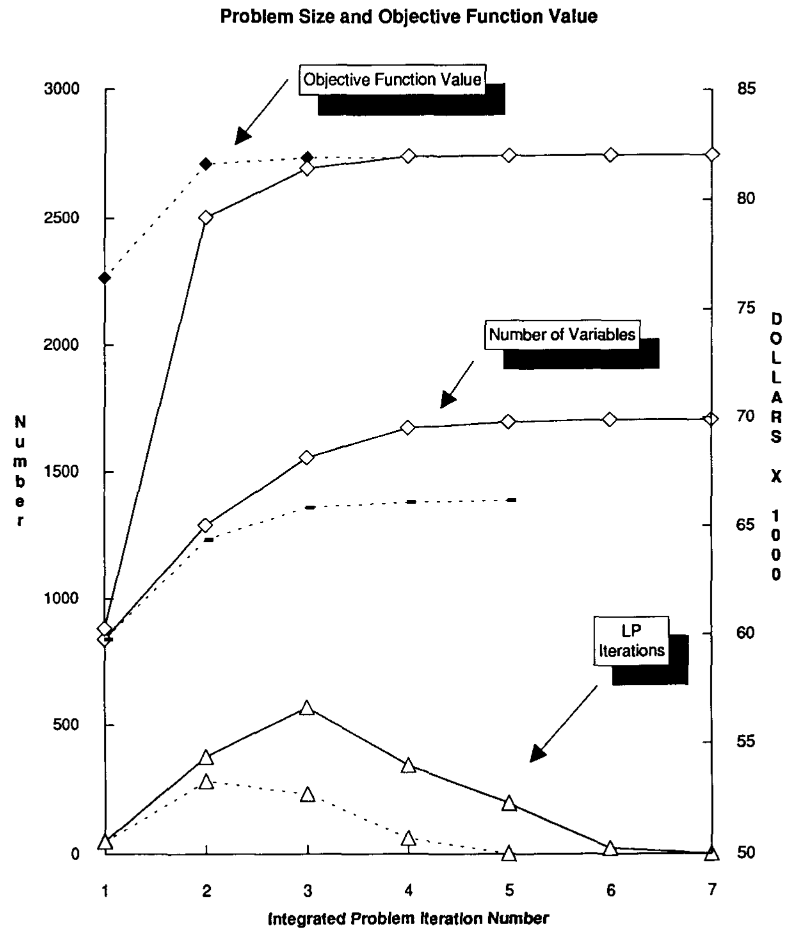

FIG. 5. A comparison of the efficiency of solving the sample problem between using final shadow prices (dashed lines) and demand group 1 prices (solid lines) as first iteration inputs.

particularly heavy in the 14-ft length, a product for which there is a very high demand.

Using the output.—The question remains as to how a sawmill could use this information to advantage. The suggested bucking and sawing patterns form a basis for investigating whether or not the mill is functioning correctly to produce desired output. However, in actual practice it would be very difficult to implement these suggested patterns. Decisions made at each machine center typically rely on various intermediate or final product values. The stem bucking model is a good example of this, as it relies on log segment values to make bucking decisions. Another problem is that the integrated model abstracts from reality by grouping stems into classes with similar small end diameters, lengths and tapers. In reality, stem taper varies all along its length so it is difficult to assign an actual stem to a particular class.

What is needed is a way to translate the information obtained from running the model into a convenient form to be used by the systems making the production

TABLE 2. *Lumber distribution produced from the sample problem for 2 × 4 and 2 × 10.*

Product and demand group	MBF produced in each length and demand group: Lumber length 8	10	12	14	16	18	20
2 × 4							
1	38.33	41.82	62.73	52.27	59.24	62.73	31.36
2		3.00	32.31	26.93	30.52	32.31	16.16
3					29.62	31.36	15.68
4							
2 × 10							
1	2.59	14.40	49.16	77.62	28.46	36.22	49.16
2			21.07	33.26	12.20	15.52	21.07
3			5.71	34.86	16.26	20.70	
4							

decisions in the mill. The final shadow prices from the integrated model are a means to perform this translation. To demonstrate this, the final shadow prices obtained from the sample problem were used as initial prices for the sample problem and the model started again. All other aspects of the run were identical to the initial run. Examining the first iteration of the integrated model when run under these conditions simulates what would happen in a mill if a single set of prices were used for the stem bucking model and cutting pattern generator. This is because the LP cannot choose between opposing patterns as it is presented with only one pattern for each log and stem class in the first iteration. The function of the LP in this case is simply to add up the products obtained from implementing the sawing and bucking decisions made by those programs.

The results of this test are shown in Fig. 5. In the first iteration, the value of the objective function was $76,450 as opposed to $60,277 obtained in the first iteration of the sample problem using the initial lumber prices (demand group 1 prices) and log values generated from the sum of lumber values in the log. This represents an increase of roughly 27% due to using the shadow prices as inputs. This is a particularly high increase when it is considered that a gain of only 36% was obtained in the sample problem when it was run to optimality at seven full iterations. Thus, the value obtained in one iteration when using shadow prices as inputs to the model is only 9% less than the theoretical maximum possible value.

Note also that in this case the model converged in 5 iterations, using significantly fewer variables and iterations. The model was solved within 0.5% of the maximum possible value after only 2 iterations, as opposed to the 3 or 4 iterations required in the sample problem to reach this level. The total number of LP iterations to optimality was 628 when using the shadow prices versus 1,558 in the sample problem, a 60% decrease.

CONCLUSIONS

The solid wood forest products industry has undergone major changes in the past decade. The declining raw material base (both in size and amount) on the West Coast, shifting sources of demand, and rising costs have forced many adjustments in the industry. In many cases the effect has been to move the sawmill closer to its customers' needs, attempting to produce products that take advantage of specific economic opportunities. To capture these opportunities, sawmills must ensure that their bucking and sawing practices are flexible and responsive. Many mills have turned to operations research models and process control technology to manage this process in an effective manner.

Economic uncertainties and years of poor profit levels have caused the solid wood industry to lag far behind other manufacturing industries in terms of technology advancements. At present, however, the industry is quickly catching up in terms of the installed technology base of image scanners, automated sawing equipment, and similar improvements. The challenge of the future is to manage this vast array of technology effectively to produce the most profitable mix of products. Automated equipment that routinely makes poor decisions will lead the way to financial disaster.

A procedure was demonstrated by means of a sample problem that modeled a QUAD sawmill with a set of hypothetical price-volume relationships. This model was solved in seven iterations of the integrated model, yielding an optimal set of bucking and sawing strategies to maximize mill profits. The revenue gain from using the policies suggested by the integrated model over those found by the bucking and sawing programs working separately was found to be roughly 36%.

These findings indicate that in general no single set of lumber and log values determined in isolation can solve the sawing and bucking problem to global optimality in one iteration. Under the best conditions, it requires at least two iterations to closely approximate an optimal solution. Optimality cannot be obtained using separate dynamic programming models as the drivers for process control equipment (which require a single set of values). However, results here indicate that solutions within roughly 9% of the theoretical maximum can be obtained if the final shadow prices are correctly used in these dynamic programming models.

REFERENCES

Briggs, D. G. 1980. A dynamic programming approach to optimizing stem conversion. Ph.D. dissertation, University of Washington, Seattle, WA.

Dantzig, G. B., and P. Wolfe. 1960. Decomposition principle for linear programs. Operations Research 8:101–111.

Eng, G., H. G. Dallenbach, and A. G. D. Whyte. 1986. Bucking tree-length stems optimally. Canadian J. Forest Res. 16:1030–1035.

Faaland, B., and D. G. Briggs. 1984. Log bucking and lumber manufacturing using dynamic programming. Management Sci. 30(2):245–257.

Gluck, P., and W. Koch. 1973. Die optimale rohholzausformung. Centralblatt fur des Gesampte Forstwesen 90(4):1193–1228.

Jackson, N. D., and G. W. Smith. 1961. Linear programming in lumber production. Forest Prod. J. 11:272–274.

Maness, T. C. 1989. A technique for the combined optimization of log sawing and bucking strategies. Ph.D. dissertation, University of Washington, Seattle, WA.

McPhalen, J. C. 1978. A method of evaluating bucking and sawing strategies for sawlogs. M.S. thesis, University of British Columbia, Vancouver, B.C., Canada.

Mendoza, G. A. 1980. Integrating stem conversion and log allocation models for wood utilization planning. Ph.D. dissertation, University of Washington, Seattle, WA.

Pearse, P. H., and G. Sydneysmith. 1966. Method for allocating logs among several utilization processes. Forest Prod. J. 16(9):87–98.

PNEVMATICOS, S. M., AND S. H. MANN. 1972. Dynamic programming in tree bucking. Forest Prod. J. 22(2):26–30.

ROW, C., C. A. FASICK, AND S. GUTTENBERG. 1965. Improving sawmill profits through operations research. USDA Forest Service Research Paper SO-20, Southern Forest Experiment Station. 26 pp.

SAMPSON, G. R., AND C. A. FASICK. 1970. Operations research application in lumber production. Forest Prod. J. 20(5):12–16.