ANALYSIS OF RACKED WOOD PALLETS

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ABSTRACT

A rational analysis procedure for designing wood stringer pallets for use in warehouse storage racks was developed for manufacturers and pallet users and is part of a computerized automatic design and analysis program called the Pallet Design System (PDS). The procedure uses simplified analog models of pallets and matrix structural analysis methods to compute the stress and deflection of critical structural elements. Semi-rigid nail joints are modeled as spring elements. Pallets with 2, 3, 4, or 5 stringers and up to 15 deckboards can be analyzed with a variety of load types including distributed and concentrated loads. The strength and stiffness of experimental pallets were compared to predicted values and showed good agreement.

Keywords: Wood pallets, matrix structural analysis, analog models, stress, deflection, semi-rigid joints, spring elements.

INTRODUCTION

The pallet, combined with fork-trucks or hand-jacks, is the basis of an efficient materials handling system for storing and transporting products and manufactured goods. Currently in North America most pallets are made of wood; over 370 million were manufactured in 1986 (NWPCA 1987). Until recently there were no widely accepted engineering procedures for comparing competing pallet designs and materials on a performance basis. The Pallet Design System (PDS) (NWPCA 1984) was developed to enable the user or manufacturer to optimize wood pallet performance and cost in specific service environments. An overview of the PDS methodology for stringer-type pallets is presented elsewhere (Loferski and McLain 1987). The objective of this paper is to explain the analytical methodology used in PDS for computing the stresses in and deflections of loaded pallets stored in warehouse racks.

In warehouses, multiple story racks provide efficient space utilization and access to individual unit loads. Supported along two opposite edges a racked pallet bridges the free span as shown in Fig. 1. The pallet must be designed to satisfy two limit states, strength and deflection. Catastrophic failure of a racked pallet can be costly,

a)Racked Across
Stringers
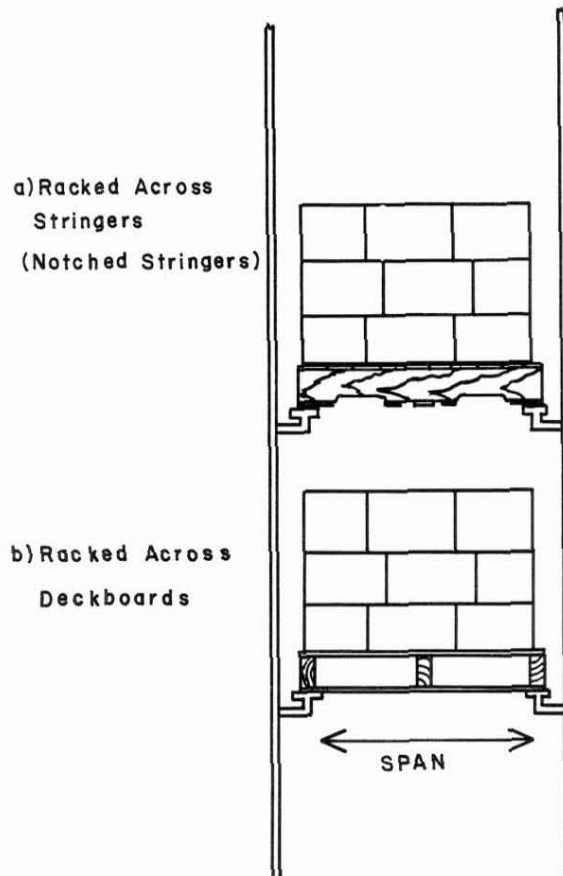(Notched Stringers)

b)Racked Across
Deckboards

SPAN

FIG. 1. Pallets supported in storage racks. a) Racked across stringers, b) Racked across deckboards.

especially if the load is highly valued or a hazardous material. In some cases, failure may be life-threatening to warehouse personnel.

Users may also specify deflection limits to accommodate automatic handling equipment. Such equipment may be unable to adjust to the shape of a deflected pallet. Also, a fragile unit load may be damaged by excessive deflection.

Two principal racking modes are common, racked across the stringers (RAS) and racked across the deckboards (RAD) (Fig. 1). In the RAS mode the stringers act as multiple parallel beams. In the RAD mode both the top and bottom deckboards form a composite structure connected by semi-rigid nailed joints. A generalized mathematical model based on the matrix displacement method of structural analysis was developed for each support mode.

BACKGROUND

Previous investigators have developed limited analysis methods for racked pallets. Wallin et al. (1976) developed a simplified procedure for computing the uniform and concentrated load capacity and deflection for both RAD and RAS support modes. Pinned or rigid deckboard-stringer joint fixity was assumed. However, strength or deflection predictions from this procedure are approximate

because of assumptions incorporated into the model. Mack (1975) modeled three-stringer pallets (RAD), loaded with a single concentrated load, as a two-dimensional frame with semi-rigid joints. Kyokong (1979) used matrix structural analysis methods for both RAD and RAS support modes and treated the joints as pinned connections constrained by rotational springs. His method requires mainframe computer capacity, which limits its practical use by pallet manufacturers.

Urbanik (1985) developed a theory for analyzing and designing three-stringer RAD pallets loaded with a fractional uniform load or a concentrated center load. The pallet was modeled as a plane frame with nonrigid connections. A joint modulus ratio was used to characterize the effects of the joints, deckboard stiffness, and loading.

Because of the limitations and the difficulty of applying these analysis methods, pallet manufacturers have relied on empirical field testing and occasional laboratory tests to develop acceptable pallet designs. Without efficient and effective design methods, structural optimization is a difficult and time-consuming task.

OBJECTIVE AND SCOPE

The objective of this research was to develop a method for predicting the structural behavior of stringer pallets stored in warehouse racks. Generalized mathematical models were developed to analyze any rectangular pallet design with 2, 3, 4, or 5 stringers and up to 15 deckboards in each deck. The procedure considers both notched or solid stringers and nonrigid joints with any degree of lateral and rotational stiffness. Studies using three-dimensional pallet models (Mulheren 1982) showed that the influence of joint stiffness on structural performance was only significant in RAD, and that only two of the six possible stiffness components were important: lateral slip parallel to the longitudinal axis of the deckboard and out of plane rotation of the deckboard relative to the stringer. A model for predicting the joint rotational stiffness from fastener and wood characteristics was developed by Loferski (1985) and will be presented elsewhere.

The proposed models consider many support modes including RAD, RAS, and a "winged" mode to represent sling handling equipment. For the RAD and RAS modes, the supports can be placed virtually anywhere under the bottom deck, limited only by stability considerations. This is an improvement over previous approaches, which restricted the support placement to the vicinity of the pallet edges.

Virtually any arbitrary loading can be analyzed with the simplified models, but solutions for five specific load types were developed for use in PDS. These five generic loads represent a broad range of products such as boxes, bags, barrels, or coils and are modeled as full and partially distributed uniform loads, and 1, 2, or 3 symmetrically placed concentrated loads.

The simplified method extended Kyokong's (1979) work by treating the joints as zero length springs. Simplified, two-dimensional structural models, based on half and quarter symmetry, were developed to reduce the degrees of freedom in the models; this simplification allowed rapid yet accurate solutions to be obtained even on microcomputers.

The procedure was "packaged" in user-friendly software as an overall design system so that manufacturers can optimize designs for specific service conditions by selective placement of lumber with different species or sizes as well as changing

Index Array:

$$
\begin{bmatrix}
G_1 & G_2 & G_4 & -G_1 & -G_2 & G_4 \\
 & G_3 & G_5 & -G_2 & -G_3 & G_5 \\
 & & G_6 & -G_4 & -G_5 & G_7 \\
 & \text{Symmetric} & & G_1 & G_2 & -G_4 \\
 & & & & G_3 & -G_5 \\
 & & & & & G_6
\end{bmatrix}
$$

Where:

Beam Element	Rotational Spring	Axial Spring
$G_1 = \alpha(\beta c_1^2 + 12 c_2^2)$	$G_1 = \gamma_1$	$G_1 = 0$
$G_2 = \alpha c_1 c_2 (\beta - 12)$	$G_2 = 0$	$G_2 = 0$
$G_3 = \alpha(\beta c_2^2 + 12 c_1^2)$	$G_3 = \gamma_2$	$G_3 = \gamma_4$
$G_4 = -\alpha 6 L c_2$	$G_4 = 0$	$G_4 = 0$
$G_5 = \alpha 6 L c_1$	$G_5 = 0$	$G_5 = 0$
$G_6 = \alpha 4 L^2$	$G_6 = \gamma_3$	$G_6 = 0$
$G_7 = \alpha 2 L^2$	$G_7 = -\gamma_3$	$G_7 = 0$

Where:

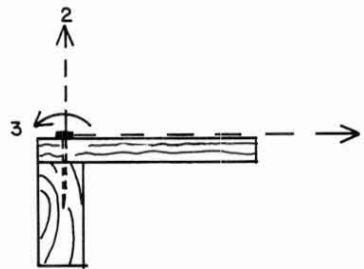$$\alpha = \frac{EI}{L^3} \; ; \; \beta = \frac{AL^2}{I}$$

C_1, C_2 = direction cosines,   L = element length,

γ_1 = lateral slip stiffness,   E = modulus of elasticity,

γ_2 = withdrawal stiffness,   I = moment of inertia,

γ_3 = rotational stiffness,   A = area,

γ_4 = axial spring stiffness.

FIG. 2. Element stiffness matrices for beam elements, zero length rotational springs, and zero length axial springs.
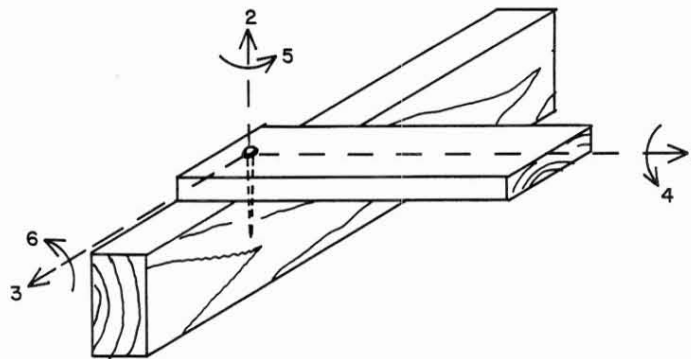
a) 3-D JOINT WITH 6 DOF

b) 2-D JOINT WITH 3 DOF

FIG. 3. Possible stiffness components for joints. a) Three-dimension system with 6 degrees of freedom, b) Two-dimension system with three degrees of freedom.

connector characteristics. In particular, the models allow the selective placement of parts because the properties of the top and bottom deckboards and the stringers are treated separately. Variability of material and joint properties and loads are considered by a reliability-based safety checking procedure (Loferski 1985; Loferski and McLain 1987).

MATRIX STRUCTURAL ANALYSIS

The displacement or stiffness method was used for pallet design to take advantage of its suitability for automatic computer-aided analysis. The structure is represented by a model consisting of discrete elements connected only at nodes. The stiffness properties of each element are formulated into a set of simultaneous equations that relate the element end forces to the unknown end displacements. From these equations an element stiffness matrix is developed for each type of element in the structure (Holzer 1985).

For example, the RAD model included 1) beam elements, 2) rotational springs, and 3) axial springs. The stiffness matrices for these elements are shown in Fig. 2. Beam elements have axial and bending stiffness and represent wooden members such as stringers or deckboards. Rotational and axial springs are fictitious elements of zero length and model the stiffness of nail joints between stringers and deck-

boards. The spring stiffness in a given direction is selected relative to a local reference frame shown in Fig. 3, and is established from tests of representative pallet fasteners.

For example, in a two-dimensional system (Fig. 3b) lateral stiffness parallel to the deckboard grain is assigned to the local 1 direction and rotational stiffness is assigned to the local 3 direction.

After establishing the localized element matrices, a global system stiffness matrix (K) is assembled by imposing conditions of compatibility and equilibrium. This matrix defines the interaction between all members in the structure and relates the known applied forces to the unknown joint displacements. Conditions of compatibility ensure that the model reflects the geometric properties and boundary conditions of the members. For example, continuity between element and joint displacements including joint constraints are defined by the compatibility conditions. Equilibrium is satisfied by balancing the applied joint forces with the member end forces. An efficient computer algorithm utilizing a "member code matrix" was used to formulate the system matrix from element matrices and is discussed by Holzer (1985). Note that constitutive laws for material behavior are included in the element models. The governing matrix equation is:

$$Q = K*D \tag{1}$$

where

Q = applied external force vector
K = system stiffness matrix
D = joint displacement vector

The solution of Eq. (1) is the displaced configuration of the entire structure. From the joint displacements, any measure of structural response, such as member stresses, can be determined (Holzer 1985).

Some assumptions inherent in the formulation of the matrix method are common to many structural analysis methods. A linearly elastic, homogeneous material is assumed. Plane sections are assumed to remain plane after deformation, and small rotations at joints are assumed. The neutral axis of a beam is the longitudinal plane of symmetry of the element. Additional assumptions regarding the use of pallets were made to keep the design procedure focused and manageable. For example, the deckboards are assumed to carry the uniformly distributed load in proportion to their widths. Symmetry around centerlines is assumed for both the pallet geometry and loading. This assumption allowed use of quarter and half symmetry, thereby reducing the size of the models and minimizing the computation time. Limits are also placed on the number of components from which the pallet is constructed.

DEVELOPMENT OF PALLET MODELS

A mathematical analog model of a pallet has an assembly of members, joints, constraints, and supports placed so that the analog behaves in the same manner as the real structure. The analog members and joints are assigned stiffness properties and geometries that correspond to those in the real structure. The matrix method uses the analog model to estimate the stresses and deformations of the components in the real pallet.
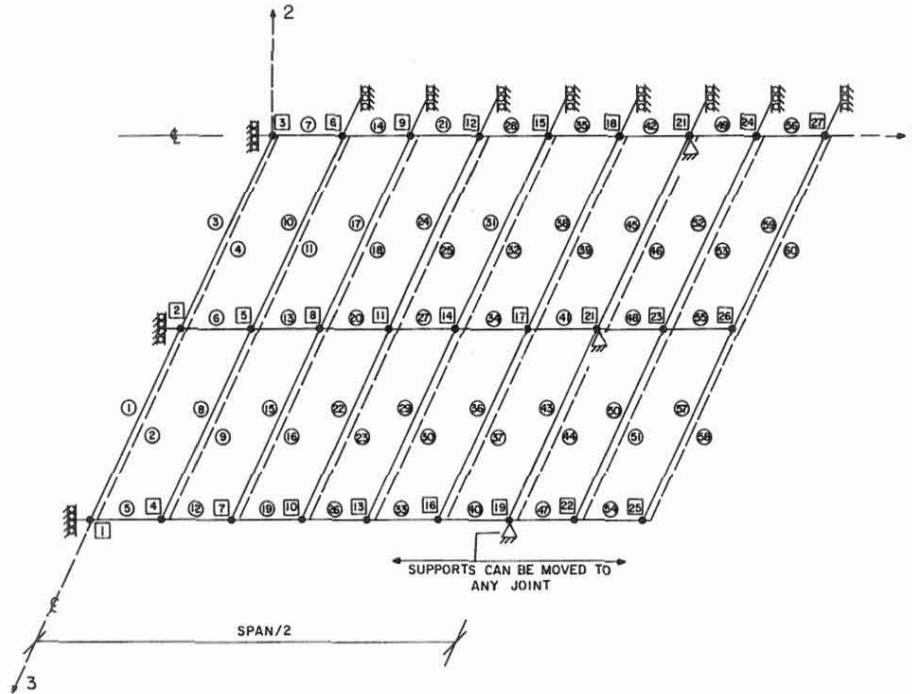
FIG. 4.   Matrix grid model for pallets racked across stringers.

Although it is possible to develop a single three-dimensional analog model for RAD and RAS support modes, solving such a model would require large computer capacity. This is incompatible with the need for accessibility to the design procedure by many diverse users or manufacturers since most of these people own microcomputers that are used for other purposes. Therefore, two different simplified models were developed to represent RAD or RAS pallets. The models require minimal computer memory and are based upon two-dimensional representation of one quarter or one half of the pallet for RAS or RAD, respectively. These models are shown in Figs. 4 and 5.

To ensure that the symmetric model behaves identically to a full model (and the real structure), a special type of support called a shear release is used to represent the cut ends of the members. A shear release permits bending moment to be transmitted into the support while allowing the member end to translate vertically; this simulates the mid-span action of a symmetrically loaded simply supported beam (i.e., vertical deflection without rotation).

RAS model

RAS pallets were modeled using a grid or planar framework of elements loaded normal to its plane. An unconstrained joint can rotate only about the global 1 and 3 axes and translate only in the global 2 direction, resulting in three degrees of freedom for a joint. The RAS model consists of only one basic element type, but since all elements are parallel to either the global 1 or 3 axis, the element stiffness matrix was specialized for deckboards and stringers relative to the global reference system (Fig. 6).
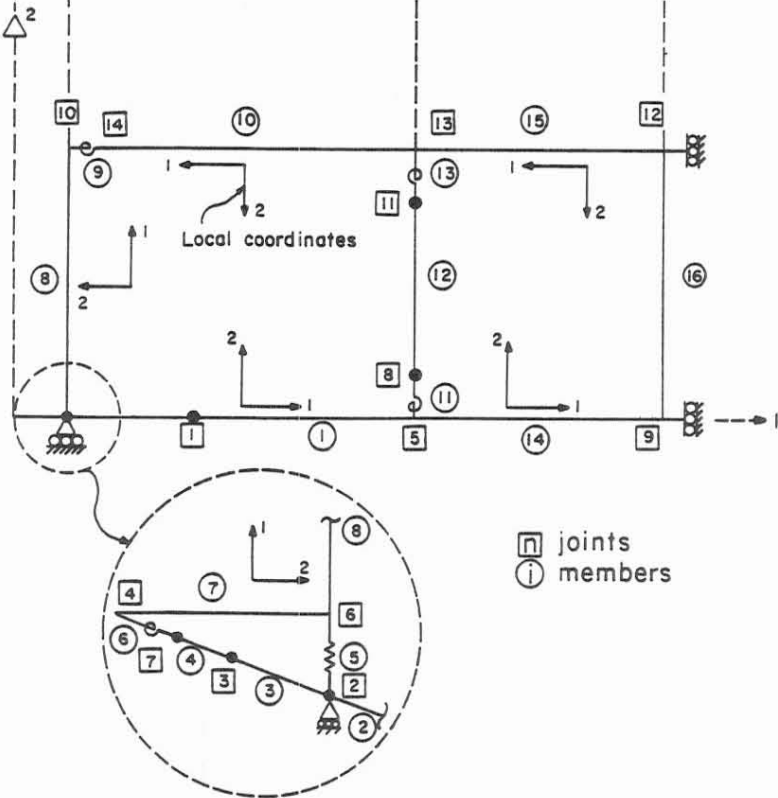
a) Cross-section of real pallet
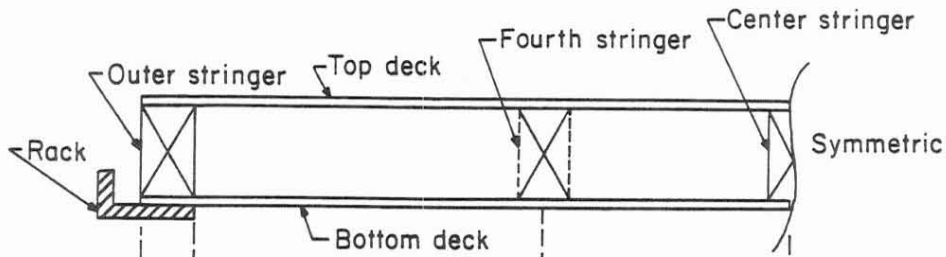
FIG. 5. Analog model of RAD pallet. a) Cross-section of real pallet, b) RAD model.

In the full model (Fig. 4), deckboard elements are parallel to the 3 axis and stringer elements are parallel to the 1 axis. The complete model has 63 degrees of freedom and represents a pallet with 5 stringers and 15 top and bottom deckboards. For pallets with fewer elements, a special numbering scheme was used to remove unnecessary elements from the model. This reduces the number of degrees

Index

	1	2	3	4	5	6
	G_1	G_2	G_4	G_8	$-G_2$	G_4
		G_3	G_5	G_2	$-G_3$	G_5
			G_6	$-G_4$	$-G_5$	G_7
	Symmetric			G_1	$-G_2$	$-G_4$
					G_3	$-G_5$
						G_6

Where:

Stringer	Deckboard
$G_1 = \delta$	$G_1 = 4L^2\alpha$
$G_2 = 0$	$G_2 = 6L\alpha$
$G_3 = 12\alpha$	$G_3 = 12\alpha$
$G_4 = 0$	$G_4 = 0$
$G_5 = 6L\alpha$	$G_5 = 0$
$G_6 = 4L^2\alpha$	$G_6 = \delta$
$G_7 = 2L^2\alpha$	$G_7 = -\delta$
$G_8 = -\delta$	$G_8 = 2L^2\alpha$

$$\alpha = \frac{EI}{L^3} \ ; \ \delta = \frac{GJ}{L}$$

E = elastic modulus (psi),
I = moment of inertia (in.^{4}),
L = length (in.),
G = modulus of rigidity (E/16 assumed) psi,
J = polar moment of inertia (in.^{4}).

FIG. 6. Element stiffness matrix index array used to reference deckboard or stringer stiffness coefficients.

of freedom and consequently the time required to solve Eq. (1). Figure 7 illustrates the reduced model for several pallet geometries. Sensitivity studies showed that joint stiffness has an insignificant influence on RAS pallet response. Therefore, to further reduce degrees of freedom in the model, the deckboard stringer joints were modeled as rigid connections instead of springs. (In the RAD mode joint, stiffness was very important and was modeled differently as described later.)
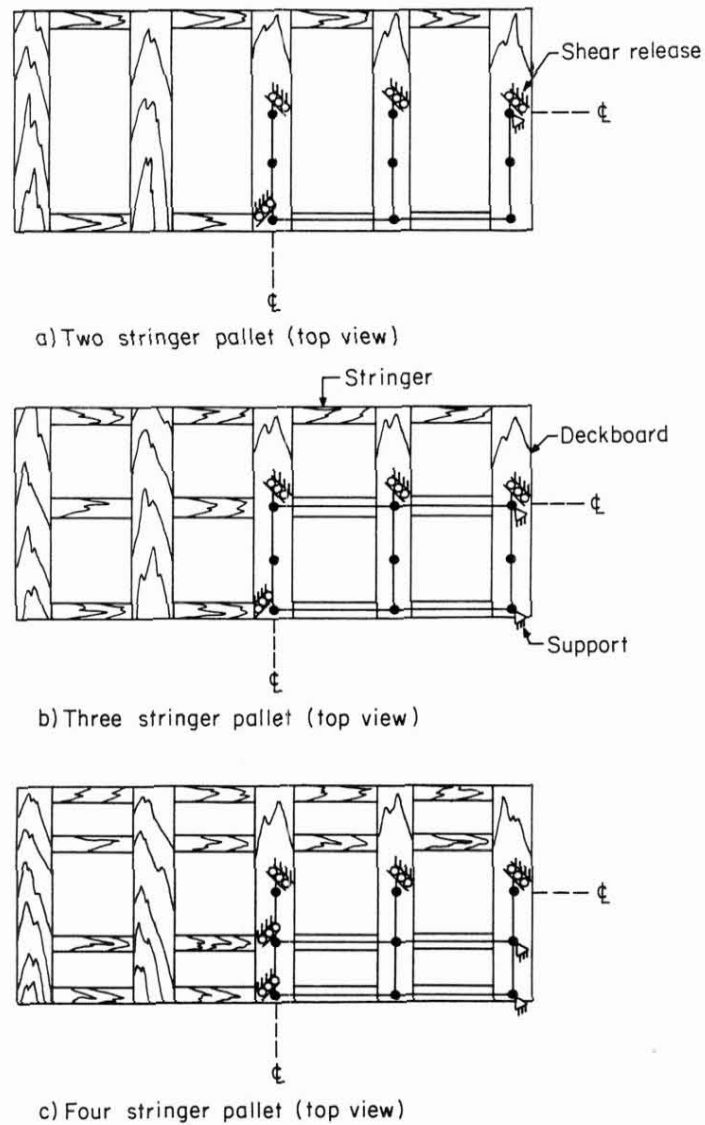
a) Two stringer pallet (top view)

b) Three stringer pallet (top view)

c) Four stringer pallet (top view)

Fig. 7. Example of grid model representing 2, 3, and 4 stringer pallets with ¼ symmetry (other elements in grid model are "dummied out" by assigning zero properties).

An automatic assembly algorithm was used to formulate the RAS system stiffness matrix. This process uses input information to automatically define the location of supports, the number of elements and joints, and the material properties and geometries (these include moduli of elasticity and rigidity, section properties, and element lengths). The process also selectively assigns zero properties to elements that will not influence the structural response of the model. For example, with a two-stringer pallet the members representing the inner stringers (i.e., in line with elements 7, 14, 21, and 6, 13, 20 in Fig. 4) are given zero stiffness.

After solving Eq. (1) the stress in each stringer element in the model is computed

from the joint displacements using the equations discussed by Holzer (1985). Since the stringers are the critical elements in the RAS mode, other member stresses are not computed. (Deckboard stresses are investigated in the RAD support mode.)

For an unnotched stringer pallet, the maximum stringer element stress is determined and is used in the probability-based safety checking procedure (Loferski 1985; Loferski and McLain 1987). Likewise, the maximum vertical joint displacement (usually located on the span centerline) is compared to serviceability criteria.

Notched stringers allow fork entry to a pallet from four directions. The notch causes a stress concentration at its corners and greatly reduces the pallet load capacity. The stress concentration effect is computed from relations developed by Gerhardt (1984). These equations are based on finite element analysis and account for the geometry of the notch and stringer. The moment and shear at the interior notch corner are computed by linear interpolation of moment or shear found at ends of the member containing the notch. Gerhardt's (1984) equations are used to compute the stress at the notch corner, which is compared to a "critical-stress" for the species using the safety checking procedure. The critical-stress is a material property defined as the stress which causes an unstable crack to propagate from the notch toward the span centerline. For common pallet notch geometries, the critical stress is typically in the range of 40% to 60% of the MOR of an unnotched stringer (Gerhardt 1984).

The deflection of a notched stringer pallet is computed by modifying the centerline deflection of a corresponding unnotched pallet. The deflection modification was also developed by Gerhardt (1984) and is a function of both the stringer and notch geometry.

RAD model

Observations of in-service and test RAD pallets showed that there is negligible bending of the inner stringers of three, four, and five stringer pallets. Hence, there is nearly equal deflection across the width of the pallet. From this observation, bending in two directions was judged not significant in RAD pallets, and a two-dimensional analog model was justified.

A challenge in modeling RAD pallets was describing the action of the deckboard-stringer joints. Experimental observations of loaded pallets revealed that the top deckboards pivot around the inside edge of the outer stringers (Fig. 8). However, the actions of bottom deckboards depend on the location of the support relative to the outer stringer-deckboard joint, the flexural stiffness of the bottom deck, and the magnitude of the unit load. For example, if the support is located under the stringer (Fig. 8a), the bottom joint tends to open about a point located on the outer edge of the stringer. If the support is located in the span between the outer and inner stringers (Fig. 8b), the bottom joint remains closed and appears to stiffen with increased load. If the support is extremely wide (Fig. 8c), the joint's actions are similar to those of a fixed end condition instead of a pinned support.

To simulate this behavior, zero length spring elements are incorporated into the RAD model. Figures 2 and 5 show that elements 6, 9, 11, and 13 are rotational-translational springs, and element 5 is an axial spring. Elements 9 and 13 represent the accumulated joint stiffness of all the top deck stringer-deckboard joints; ele-
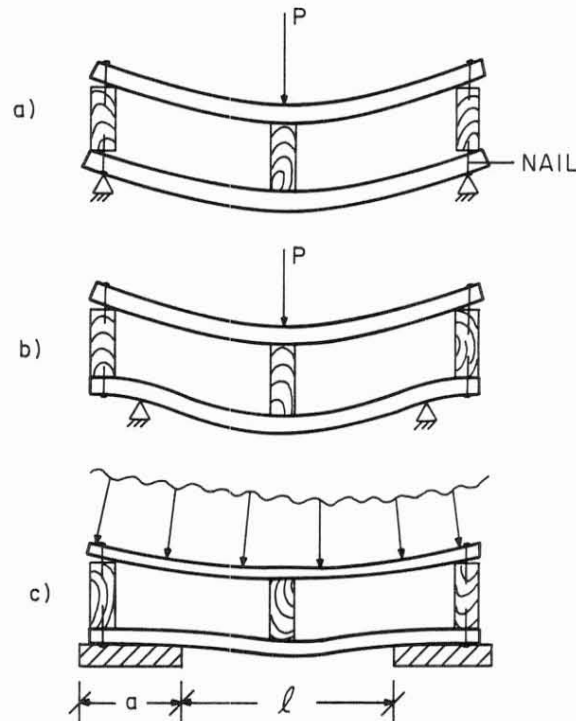
FIG. 8. Pivot points for RAD support mode.

ments 6 and 11 correspond to the bottom deck joints. The spring elements mimic the actions of nonrigid joints on structural performance. Assuming symmetry, member 16 is located on the pallet centerline and joints 9 and 12 may not rotate or translate laterally. Therefore, spring elements are not used here.

The rack support is always represented as a pinned support and can be placed at nodes 1, 2, or 3 depending on the geometry of the rack and pallet. Nodes 1 and 3 are mobile and can be moved to correspond to the required geometry.

Member 5 is a zero length axial spring with initial stiffness of zero. After solving Eq. (1) for displacements, the movement of joint 2 relative to joint 6 is checked. If joints 2 and 6 have reversed positions (a physical impossibility in a real pallet), member 5 is assigned a high stiffness, the original system stiffness matrix is adjusted accordingly, and a new solution is obtained. This dynamic adjustment enables the model to simulate either the closing action or opening of the bottom joints (Fig. 8).

The RAD model shown in Fig. 5 represents the left half of a symmetric pallet. A wide variety of structures can be simulated with this model by selectively assigning material properties to the elements. For example, for two stringer pallets, members 12 and 16 are assigned zero stiffness; member 12 is "dummied out" to represent a three stringer structure; and member 16 is dummied for four stringer pallets.

By inverting the model, the action of a sling-supported winged pallet is simulated as shown in Fig. 9. In this mode member 5 is given the stiffness values of a rotational-lateral spring (with high stiffness in withdrawal) and is used to represent
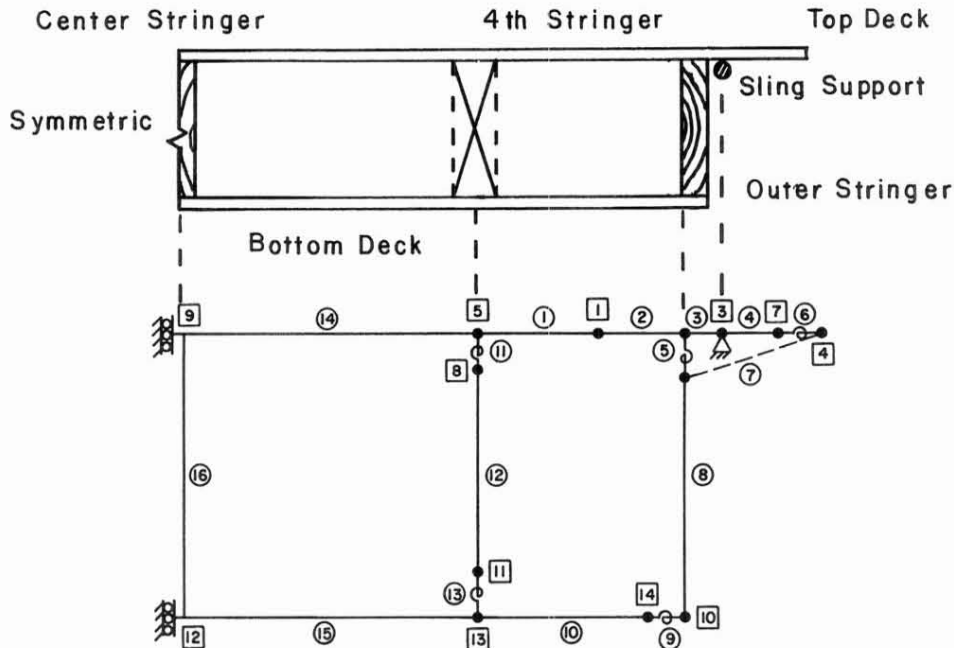
FIG. 9.   Model for sling-supported winged pallet.

the stiffness of the top deckboard-stringer nail joints. Member 6 is given zero stiffness in each direction to represent a free connection between the deckboard wing and the stringer edge. The wing can then deflect independently of the stringer edge. Other elements are assigned properties and geometries that correspond to those of the real pallet and the analysis proceeds as described above.

Each deck is analyzed as if it were one deckboard whose width is equal to the sum of the widths of the individual boards. The possible critical members are the top or bottom decks. Using the joint displacements found in the solution of Eq. (1), the force or elastic stress for each element in the model can be computed using relationships discussed by Holzer (1985). The maximum stress and maximum vertical deflection in the model are used in the safety checking procedure (Loferski 1985).

VERIFICATION

An experimental program with multiple objectives was used to verify the theoretical models by comparison with test results. Fourteen different pallet designs for RAS and ten designs for RAD with 5 replications of each design were constructed and tested. The pallets were constructed of green, mixed oak species randomly sampled from a local pallet mill. A variety of common commercial designs, with both three or four stringers, were selected for testing. The combined top and bottom deckboard coverage ranged from 58% to 165%. These designs, detailed by Collie (1984), resulted in a range of pallet strength and stiffness that covers most commercial pallet designs. The pallets were assembled with either pneumatically or hand-driven threaded pallet nails, and were tested to failure (or to the testing machine capacity) with a uniformly distributed load applied by a
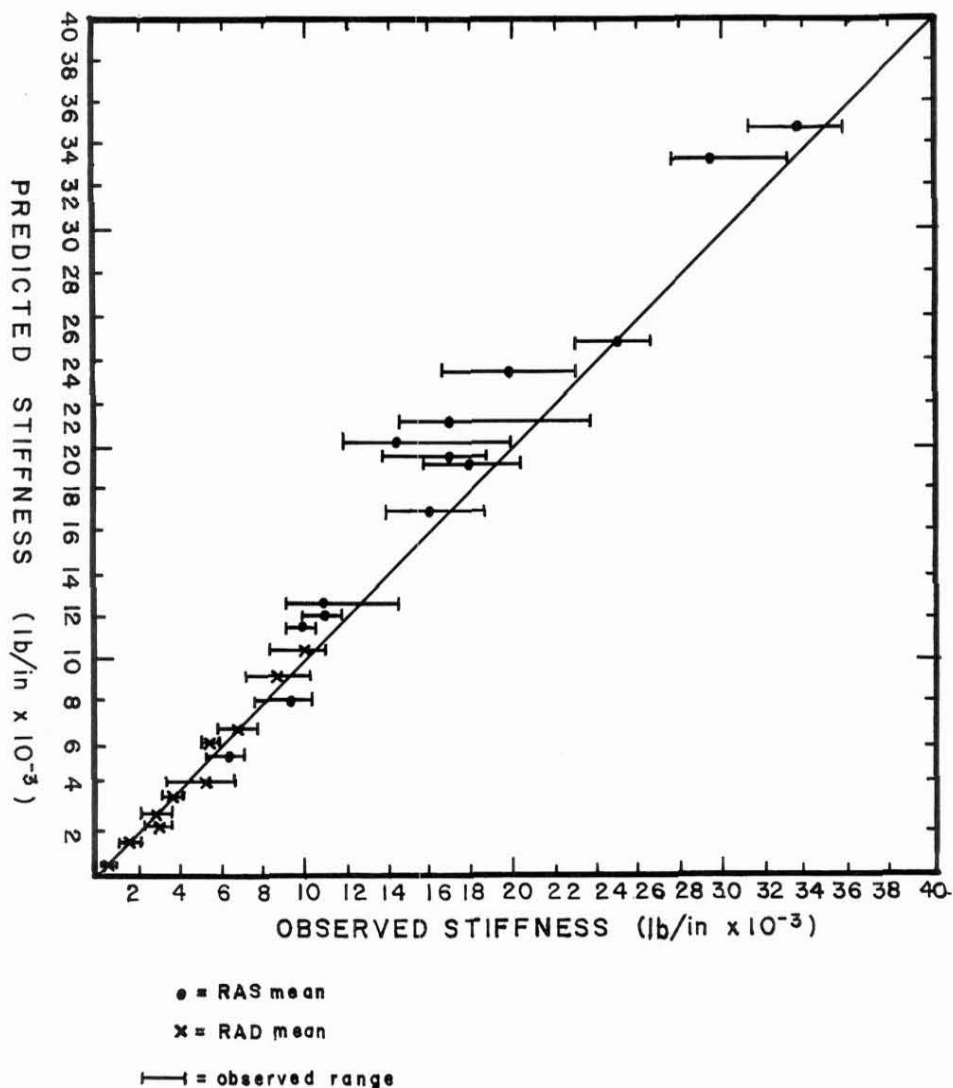
FIG. 10.   Observed vs. predicted stiffness for RAS and RAD modes. Each mean observed stiffness represents at least five test pallets.

constrained air-bag in a specially constructed testing machine. Test spans ranged from 33 to 54 inches. Load was measured by load cells located under each of the four pallet corners and deflection was measured with potentiometers at three places, the center of the pallet, and the center of each unsupported edge. A microcomputer recorded and plotted the load and deflection measured by the transducers during testing.

Figures 10 and 11 show the observed stiffness and failure load for each class of test pallet plotted against predicted values obtained from the RAS and RAD analog models. Pallet stiffness was computed by dividing the load at a point in the linear region of a test curve by the corresponding deflection. For RAD pallets
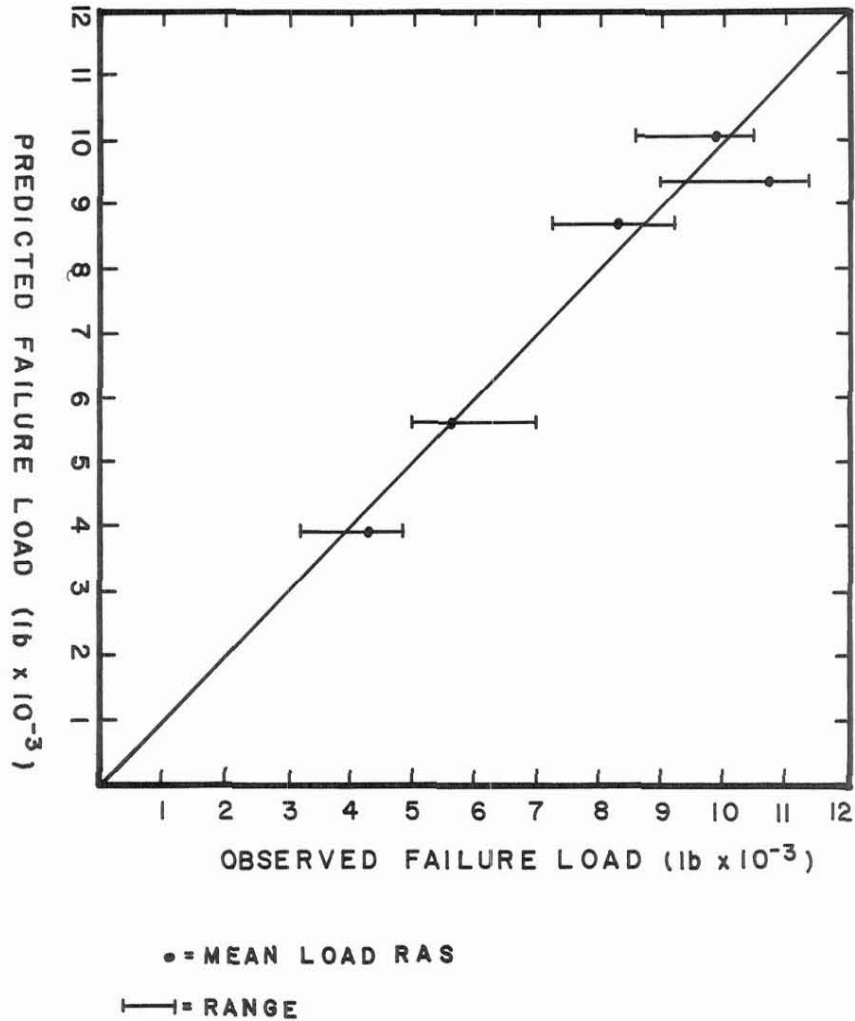
FIG. 11.   Observed vs. predicted load capacity for RAS pallets. Each mean presents at least five test pallets.

stiffness was used as the main parameter for verification because green oak pallets RAD rarely fail in bending without undergoing enormous deformations. The magnitude of these deformations violates some of the assumptions inherent in the analog models. Additionally, excessive pallet deflection usually exceeded the deflection-measuring capacity of the testing machine so a "true" failure could not be recorded. Similarly some RAS pallets often exceeded the load capacity of the test machine, and only five pallet designs could consistently be used for strength verification.

The material properties input to the analog models was the mean MOE and MOR obtained from bending tests of lumber randomly sampled from the material used to make the test pallets. Joint properties were estimated from the developed joint stiffness prediction model. Comparison between predicted and actual stiff-

ness is seen in Fig. 10 in both RAS and RAD support conditions. Figure 11 shows good agreement between the predicted and observed failure loads for RAS pallets.

SUMMARY

A theoretical method, based on matrix structural analysis, was developed to analyze and design pallets stored in warehouse racks. Simple generalized two-dimensional models that consider unique joint characteristics were developed for racked across stringers and racked across deckboards support modes. Pallets constructed with 2, 3, 4, or 5 stringers and up to 15 deckboards can be modeled. Semi-rigid nail joints are modeled as springs. The analysis methods were incorporated into an overall pallet design program to provide manufacturers with a powerful design tool. The analytical models were experimentally verified and showed good agreement between predicted and observed strength and stiffness.

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