A CHEMICAL KINETICS APPROACH TO THE DURATION-OF-LOAD PROBLEM IN WOOD[1]

D. F. Caulfield

Research Chemist
Forest Products Laboratory,[2] Madison, WI 53705

ABSTRACT

The theory of absolute rates of chemical processes is presented as an appropriate conceptual framework for understanding the creep-rupture phenomena of duration of load (DOL) and rate of loading (ROL). The theory predicts the following experimentally observed phenomena:

(1) The logarithm of the time to failure under constant deadload stress increases linearly as the stress level is decreased.
(2) The rupture strength in a linear-ramp ROL experiment increases with the logarithm of the rate of stressing.

Moreover, the equations derived to describe these phenomena contain the same parameters. These parameters are defined physical quantities that describe the creep characteristics of the material. It is possible to predict how long a material will support a constant deadload stress (DOL behavior) from measurements of apparent rupture strength as a function of the rate of stressing in a linear-ramp loading experiment (ROL behavior).

Rupture of Douglas-fir in bending is selected as an example, and the experimental results from ROL-behavior experiments are used to predict DOL behavior. The theory adequately describes the experimentally observed results.

Keywords: Creep rupture, duration of load, rate of loading, bending rupture, Douglas-fir, absolute reaction rates, stress-strain behavior, tensile strength, modulus of rupture.

INTRODUCTION

Wood (Gerhards 1977), paper (Rance 1953), and textile fibers (Hearle and Morton 1962) all exhibit time-dependent rupture behavior. Any of these materials, loaded to a constant stress level considerably below its normal breaking stress, will nevertheless break—if that stress is maintained over a long enough time. This phenomenon has been called the duration-of-load (DOL) phenomenon. Researchers have empirically found that, for these materials under constant deadload stress, the logarithm of the time to failure varies inversely as the stress level. A second time-dependent rupture phenomenon exhibited by wood (Gerhards 1977), paper (Rance 1953), and textile fibers (Hearle and Morton 1962) can be called the rate-of-loading (ROL) phenomenon. The measured strength of the material increases as the rate at which the material is stressed increases. If the applied stress is increased linearly with time (linear ramp loading), the rupture strength increases with the logarithm of the rate of stressing.

The purpose of this paper is to demonstrate that the theory of absolute rates of chemical processes provides an appropriate conceptual framework for under-

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standing these creep-rupture phenomena. This theory predicts the experimentally observed behavior for both phenomena. It also provides the mathematical formalism connecting DOL and ROL behavior. Using this connection, one can predict how long a material will support a constant deadload stress (DOL behavior) from measurements of strength as a function of rate of stressing in a linear-ramp loading experiment (ROL behavior).

The bending of Douglas-fir is selected as an example, and the experimental results from ROL-behavior experiments alone are used to predict DOL behavior.

THEORETICAL BACKGROUND

A serious mathematical difficulty arises in the interpretation of the stress-strain behavior of a material at the point on the stress-strain curve that corresponds to failure or rupture. Up to that point, one can follow continuously the changes that take place as a result of elastic or viscoelastic response. Rupture, however, represents a discontinuity that cannot be avoided. No continuous set of variables describes both the state before and the state after rupture. One cannot even use thermodynamics (at least not reversible thermodynamics) to describe the rupture process because of the discontinuity and the need to define both starting and final states in consistent terms. Thus one can never describe a breaking process with the same degree of mathematical rigor that one can describe either an elastic deformation or a viscous flow. This mathematical difficulty seriously interferes with a fundamental understanding of rupture phenomena.

THE CHEMICAL KINETICS APPROACH

The chemical kinetics approach is one way of dealing with this mathematical difficulty. The kinetics approach makes the assumption that rupture is determined completely by the magnitude and nature of the deformation preceding rupture and that the elucidation of the role of creep in processes leading to failure is the essential problem.

The guiding principle behind the chemical kinetics approach to an understanding of rupture is the idea that the straining process itself is, or contains within it, a process of failure that becomes unstable at a time (pre)determined by the straining process, thus ending in rupture.

In order to examine the role of creep in fundamental terms, it is helpful to follow the methods used by Eyring and his coworkers (Glasstone et al. 1941; Tobolsky et al. 1943). They extended the theory of absolute rates of chemical reactions to describe the phenomena of viscosity and viscoelasticity.

According to the kinetic theory of matter, all atoms and molecules are in motion. This molecular motion, when analyzed in terms of statistical mechanics, provides a description of the macroscopic motion. In a solid, for example, the motion of its component molecules is more restricted than in a liquid; and in a liquid, more restricted than in a gas. In a solid, each atom or molecule or group of molecules can be pictured as sitting at the bottom of a potential energy well (Fig. 1). At equilibrium, a molecule in its well appears quite satisfied where it is. But that does not mean that it does not move from that site. Because it is in constant motion, it can occasionally surmount the energy barrier and get to a new position of minimum energy. In a solid this jumping of an energy barrier is less frequent than in a liquid, and on the average there are just as many jumps to the left
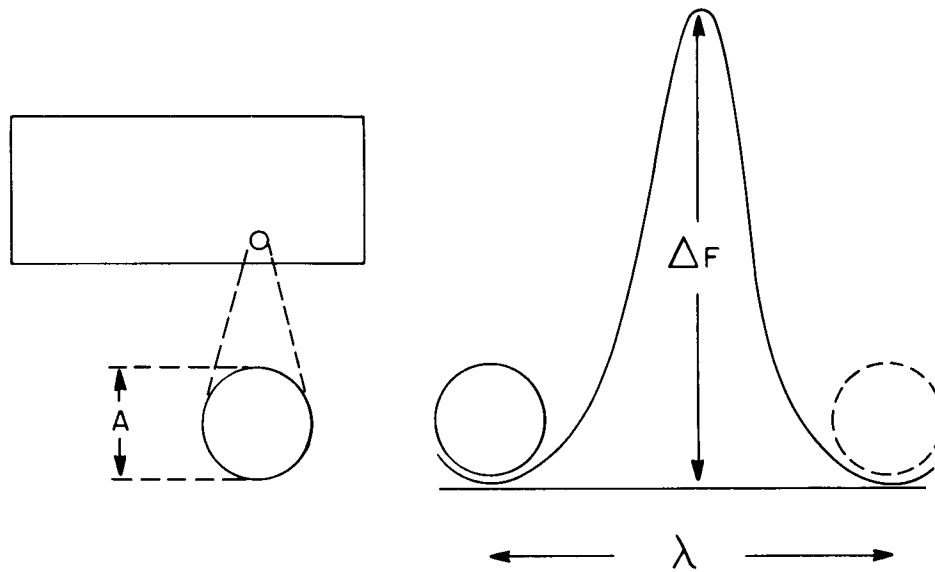
FIG. 1.   For a material with no external stress, the potential energy barrier is symmetrical, and the tendency of the force center to jump the barrier is the same from either direction. The number of jumps right per unit time equals $\dfrac{kT}{h} \exp\left(\dfrac{-\Delta F}{RT}\right)$ and the net number of jumps per unit time equals 0. A = Cross section of the force center. ΔF = Height of the energy barrier. λ = Distance between jumps.

direction as there are to the right, so that there is no net movement of the solid or parts of the solid.

The kinetic theory of absolute rates tells us that the number of jumps per unit time in, say, the right direction, ν_+, is given by

$$\nu_+ = \frac{kT}{h} \exp\left(\frac{-\Delta F}{RT}\right)$$

ΔF is the energy of activation needed for jumping, or the height of the potential energy barrier. Boltzmann's constant is given by k, Planck's constant by h, the absolute temperature by T, and the gas constant by R.

The number of jumps per unit time in the opposite direction

$$\nu_- = \frac{kT}{h} \exp\left(\frac{-\Delta F}{RT}\right)$$

is identical, so that, on the average, there are just as many jumps to the left as there are to the right: so that no net motion occurs.

The situation changes when, superimposed upon this symmetrical energy barrier, we apply an external mechanical stress, f, on the material. The tendency of the molecules to jump the barrier in one direction is different from the tendency to jump the barrier in the reverse direction.

Let us try to generalize by not necessarily calling the jumping objects molecules, because elements larger than molecules might be involved in the motion. They might be groups of molecules, filaments, fibrils, etc. Let us just call them elements
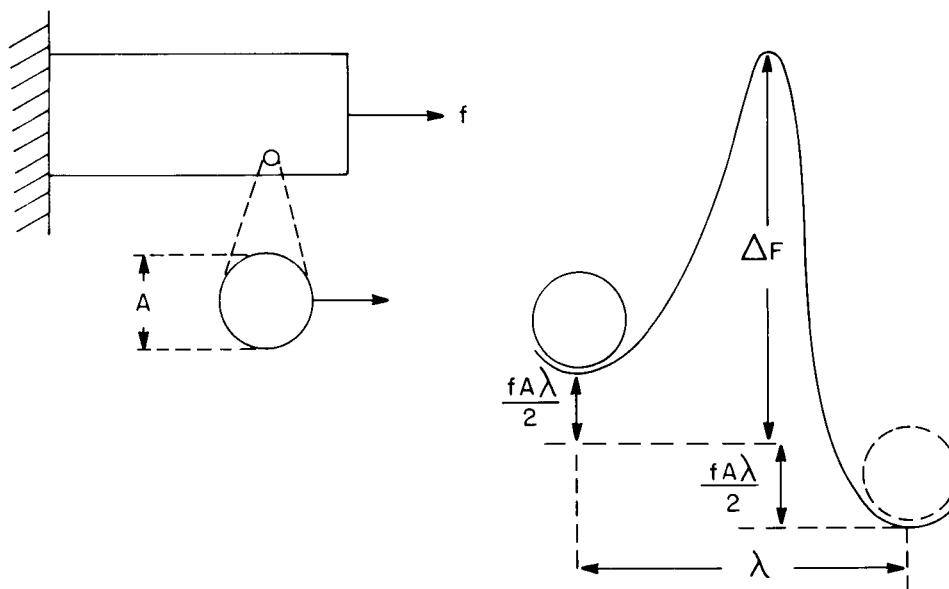
Fig. 2. For a material under external stress, the potential energy barrier is distorted, and the tendency to jump the barrier in the direction of the applied stress is greater than in the opposite direction. The net number of jumps per unit time equals

$$\frac{kT}{h} \exp\left(\frac{-\Delta F}{RT}\right)\left[\exp\left(\frac{fA\lambda}{2kT}\right) - \exp\left(\frac{-fA\lambda}{2kT}\right)\right]$$

A = Cross section of the force center. ΔF = Height of the energy barrier. λ = Distance between jumps. f is the applied stress.

or force centers. The easiest case to envision is a tensile stress. Although a tensile stress is used for simplicity, the treatment is equally applicable to any generalized stress that has the unit's force per unit area and its appropriate response.

The additional force that the molecule or force center feels is the stress on the force center multiplied by A, the cross-sectional area of the force center. And if the distance between minimum potential energy troughs in the direction of the stress is given by λ, then the applied stress contributes an amount of mechanical work towards surmounting the energy barrier equal to

$$\frac{fA\lambda}{2}$$

This is equivalent to a symmetric distortion of the potential energy barrier to give it a form like that shown in Fig. 2. For this unsymmetric energy barrier the number of jumps per unit time in the right direction is given by

$$\nu_+ = \frac{kT}{h} \exp\left(\frac{-\Delta F}{RT}\right) \exp\left(\frac{fA\lambda}{2kT}\right)$$

The number of jumps per unit time in the reverse direction is given by

$$\nu_- = \frac{kT}{h} \exp\left(\frac{-\Delta F}{RT}\right)\exp\left(\frac{-fA\lambda}{2kT}\right)$$

The overall net tendency to jump the energy barrier is the number of jumps per unit time in one direction minus the number of jumps per unit time in the opposite direction;

$$\nu_+ - \nu_- = \frac{kT}{h} \exp\left(\frac{-\Delta F}{RT}\right)\left[\exp\left(\frac{fA\lambda}{2kT}\right) - \exp\left(\frac{-fA\lambda}{2kT}\right)\right]$$

Because the quantities A and λ always appear as a product, let's call that product δ. δ has the units of (m^3) and represents the volume of the force center or moving element.

Then

$$\nu_+ - \nu_- = \frac{kT}{h} \exp\left(\frac{-\Delta F}{RT}\right)\left[\exp\left(\frac{f\delta}{2kT}\right) - \exp\left(\frac{-f\delta}{2kT}\right)\right]$$

$$\sinh X = \frac{e^x - e^{-x}}{2}$$

so

$$\nu_+ - \nu_- = \frac{2kT}{h} \exp\left(\frac{-\Delta F}{RT}\right)\sinh\left(\frac{f\delta}{2kT}\right)$$

The net number of jumps per unit time multiplied by the distance per jump, λ, gives a rate of dislocation of the element. And if we call that rate of local strain dislocation, $\frac{d\gamma}{dt}$, then

$$\frac{d\gamma}{dt} = (\nu_+ - \nu_-)\lambda = \frac{2kT\lambda}{h} \exp\left(\frac{-\Delta F}{RT}\right)\sinh\left(\frac{f\delta}{2kT}\right) \qquad (1)$$

We will assign the function γ the boundary condition that $\gamma = 0$ when $t = 0$.

In this equation γ represents the microscopic strain (in actual displacement) so that $\frac{d\gamma}{dt}$ is the rate of dislocation or rate of creep deformation.

Equation (1) is equivalent to one that is sometimes called the hyperbolic sine law of viscous flow. If $f\delta \ll 2kT$, then the hyperbolic sine is equal to its argument, and the rate of strain is proportional to f, the stress. This is just a description of a Newtonian liquid, where the rate of strain is proportional to stress. Now it is reasonable that we should have an equation that applies to liquids as well as solids because we have not done anything yet to limit our discussion to solids. If we limit our discussion to solids, then we will limit ourselves to only those cases for which $f\delta \gg 2kT$. kT is the measure of thermal energy in the system; $f\delta$, a measure of the mechanical work involved. So we will be dealing with those cases for which the mechanical work involved to obtain movement or dislocation is large compared to the thermal energy of the material. Roughly, this corresponds to a system where large forces are required before movements occur, rather than, for example, in liquids. In ordinary liquids f is of the order of 0.1 Pa, while the molecular volume, δ, is of the order of 1×10^{-27} m^3, i.e., $f\delta \ll 2kT$.

Equation (1) is equivalent in form to an equation used in the damage accumulation model (Gerhards and Link 1983) explanation of time-dependent rupture

phenomena. This equivalence is apparent if the variable of integration, γ, is replaced by its reduced variable, $\dfrac{\gamma}{\gamma_B}$, the ratio of creep deformation to critical creep deformation. The limits of integration in this form are 0 and 1, where unity corresponds to the level of "damage" that causes failure. Unlike the damage accumulation theory, however, all of the parameters used in Eq. (1) have defined physical interpretations. If indeed three characteristics of the material are known— 1) the volume of the moving element, δ, 2) the height of the potential energy barrier, ΔF, and 3) the ratio, $\dfrac{\gamma}{\lambda}$ (the localized strain)—then Eq. (1) contains no adjustable parameters.

In order to integrate Eq. (1), we make use of the critical assumption of the theory. This assumption is the creep-rupture hypothesis: there is an upper limit that the localized strain deformation can reach, γ_B, above which the material can no longer support the stress and the material fails or ruptures. This idea that there is a critical strain that determines rupture has a long history. Probably the first to suggest it was St. Venant (ca. 1855). But the first to suggest this creep-rupture hypothesis in terms of the limits of integration was Coleman (1956), and it is Coleman's formalism and treatment that is followed here with some modifications for our special purposes.

We may integrate Eq. (1) only if we know the functional dependence of the stress on time, f(t). We will consider only two cases in detail. For DOL behavior, we are concerned with the creep deformation that occurs under a stress that is invariant with time, f = constant. For ROL behavior, we are interested in the integration for the case where the stress increases linearly with time, f = αt; a so-called linear ramp, where α is the rate of stressing.

PREDICTION OF DOL BEHAVIOR

If the stress is constant over time, that is, at time t = 0 a constant stress is applied, the material will creep until a localized strain deformation is reached, γ_B; at which point in time, t_B, the specimen fails.

$$\int_0^{\gamma_B} d\gamma = \frac{2kT\lambda}{h} \exp\left(\frac{-\Delta F}{RT}\right) \sinh\left(\frac{f\delta}{2kT}\right) \int_0^{t_B} dt$$

$$\gamma_B = \left[\frac{2kT\lambda}{h} \exp\left(\frac{-\Delta F}{RT}\right) \sinh\left(\frac{f\delta}{2kT}\right)\right] t_B$$

$$\text{let } a = \frac{h\gamma_B}{kT\lambda} \exp\left(\frac{\Delta F}{RT}\right)$$

$$b = \frac{\delta}{2kT}$$

$$\text{then } t_B = \frac{a}{2} \operatorname{csch} fb$$
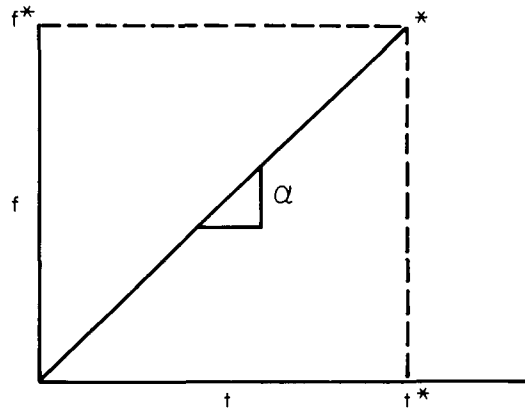
$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

FIG. 3. In a linear-ramp rate-of-loading (ROL) test the applied stress (f) increases linearly with time (t) up to failure (*).

$$= 2e^{-x} \quad \text{if} \quad x \gg 1$$

$$t_B = a \, \exp(-bf)$$

$$f = \frac{1}{b} \ln a - \frac{1}{b} \ln t_B \tag{2}$$

Equation (2) is the well-verified relationship between stress, f, and the logarithm of the time required to break. This equation is most often seen by wood engineers as an empirical relationship between stress (normalized, and expressed as a percent) and the common log t.

This, then, is the first of two important predictions of the chemical kinetics approach, that for a material under constant deadload stress, the logarithm of the time to failure varies inversely as the stress level.

$$f \sim \ln \frac{1}{t_B}$$

PREDICTION OF ROL BEHAVIOR

The result of the integration of Eq. (1) changes if instead of a constant deadload stress we consider a stress that varies with time. The simplest type of time-dependent stress to consider is a linear ramp; one for which the stress is increased linearly with time, at a rate of stress application, α (Fig. 3).

$$f = \alpha t$$

In this case the sample fails at a time, t^*, corresponding to a stress, f^*.

$$\frac{d\gamma}{dt} = \frac{2kT\lambda}{h} \exp\left(\frac{-\Delta F}{RT}\right) \sinh[bf(t)]$$

$$\int_0^{\gamma_B} d\gamma = \frac{2kT\lambda}{h} \exp\left(\frac{-\Delta F}{RT}\right) \int_0^{t^*} \sinh(b\alpha t) \, dt$$

$$\gamma_B = \frac{2kT\lambda}{h} \exp\left(\frac{-\Delta F}{RT}\right)\left[\frac{1}{\alpha b}(\cosh \alpha bt^* - \cosh 0)\right]$$

but $\alpha t^* = f^*$
and if $bf^* \gg 1$

$$\cosh 0 \ll \cosh bf^* = \frac{e^{bf^*}}{2}$$

$$\gamma_B = \frac{kT\lambda}{h} \exp\left(\frac{-\Delta F}{RT}\right)\frac{1}{\alpha b} \exp(bf^*)$$

$$\exp(bf^*) = \left[\frac{h\gamma_B}{kT\lambda} \exp\left(\frac{\Delta F}{RT}\right)\right]\alpha b$$

$$\exp(bf^*) = a\alpha b$$

$$f^* = \frac{1}{b} \ln ab + \frac{1}{b} \ln \alpha \tag{3}$$

Equation (3) is the second important prediction of the kinetic approach amply verified experimentally. That prediction is that the breaking stress (in a linear-ramp ROL experiment) increases as the logarithm of the rate of loading.

SIGNIFICANCE OF THE DERIVED EQUATIONS

We have two equations, Eqs. (2) and (3), that predict 1) the dependence of breaking time on stress in a constant deadload experiment and 2) the dependence of breaking stress on the rate of loading in a linear-ramp ROL experiment.

$$\text{(2)} \qquad\qquad f = \frac{1}{b} \ln a - \frac{1}{b} \ln t_B$$

$$\text{(3)} \qquad\qquad f^* = \frac{1}{b} \ln ab + \frac{1}{b} \ln \alpha$$

The only parameters in Eqs. (2) and (3) are the quantities a and b. These quantities are, in turn, relatable to material properties. For example, since $b = \delta/2kT$, it provides an immediate estimate of the volume of the moving element, δ. On the other hand, because

$$a = \frac{h\gamma_B}{kT\lambda} \exp\left(\frac{\Delta F}{RT}\right)$$

the value of a can provide a measure of the height of the potential energy barrier, ΔF, only if the ratio γ_B/λ, can be estimated from the additional measurement of the critical strain, i.e., the strain at failure.

If the theory is valid, when one plots f versus $\ln t_B$ or plots f^* versus $\ln \alpha$, straight lines result. Moreover, the slope of the line given by Eq. (2) is negative that given by Eq. (3).

We see that f and f^* and t_B and α are all experimental quantities. The only other quantities in these equations are a and b, so that if one could evaluate the quantities

a and *b* in, say, a DOL experiment, one has all the information needed to predict the behavior in an ROL experiment. From Eq. (2), *a* and *b* are easily evaluated: 1/b is negative the slope of the line; and ln a is the value of the intercept of the ln t axis where f = 0. These values, introduced back into Eq. (3), give a prediction of behavior in an ROL experiment.

More importantly, ROL behavior can predict DOL behavior. If an ROL experiment is run and Eq. (3) plotted, *a* and *b* can both be evaluated: 1/b is the slope of the line when stress is plotted versus the logarithm of the rate of stressing, and the intercept on the ln α axis where f* equals zero gives the value of $-$ln ab. These values of *a* and *b,* determined experimentally in an ROL experiment, introduced into Eq. (2), give a prediction of DOL in a constant deadload experiment.

Before we go to some experimental data to see how things fit together, let us look again at Eq. (3) in a form more familiar to wood engineers. It is customary to express Eq. (3) not in terms of ROL, α, but in terms of t*, the time to break. Since $\alpha = f/t$, it can be replaced by the value f*/t* in Eq. (3) to give

$$f^* = \frac{1}{b} \ln abf^* - \frac{1}{b} \ln t^*$$

This is not an equation for a straight line. But it can be shown that if f* is plotted versus ln t*, a straight line results if f*b $\gg$ 1 (see Appendix). The slope of this curve in its linear range is also $-1/b$, i.e., exactly the same slope as the line given by Eq. (2). It should be noted, however, that these lines do not have the same intercepts. The line of f* versus ln t* is shifted upward. A linear plot of f* versus ln t* is familiar to wood engineers in the form of an empirical equation in which ultimate stress level is normalized to the value at a standard time and expressed as a percent.

This means that if one were to plot on the same piece of graph paper f versus ln t_B for a constant deadload experiment and f* versus ln t* for an ROL experiment (for values of f*b $\gg$ 1), two parallel straight lines will result. The one from the ROL experiment will be shifted above the deadload line.

The data obtained in two experiments such as these have to come from different ranges of the variable t. One cannot do a constant deadload experiment at very short times because of the finite time it takes to apply any load, and an ROL experiment does not usually extend to very long times for practical reasons. Therefore, if data from the two different types of experiments are plotted on the same graph, the range of time that both types of experiments overlap is usually not great.

There is no sound reason for drawing a single curve through both sets of data, because f* has a different physical meaning from f. Yet this is what Lyman Wood (1951) did when he fitted his two sets of data to a single curve and got a hyperbolic fit. His only justification for doing this was his belief that, because both sets of data are governed by the same physical properties of the material, they should be represented by one continuous curve. The kinetic theory tells us, instead, that there should be two parallel straight lines when f and f* are plotted versus ln t, with the ROL line above the deadload line.
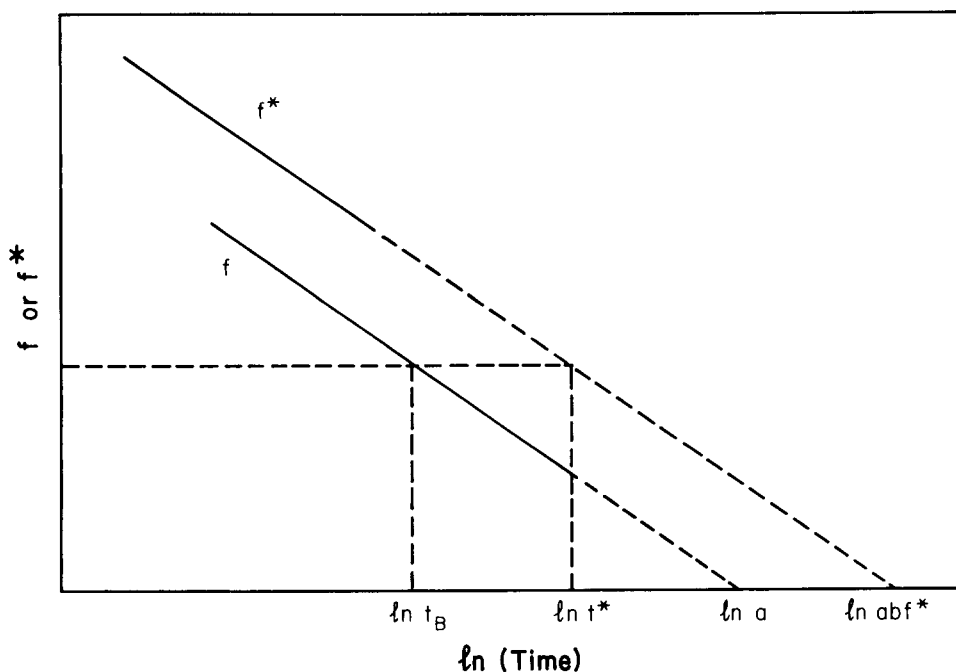
FIG. 4.   Because the rate-of-loading (ROL) line (f^* versus ln t) and the duration-of-load (DOL) line (f versus ln t) are parallel, the horizontal distance between the lines at any value of stress equals the distance between the x intercepts of the extrapolated lines.

From a purely intuitive point of view, one should feel that these lines have to be separate and parallel. Gerhards (1977) pointed out the paradox that results if the lines have different slopes. If the lines are not parallel, they intersect somewhere. If they intersect at any point, it would mean that, at that point in time, the sample would break under a stress whether that stress were applied constantly from t = 0 or applied gradually at ramp loading up to that value of the breaking stress. This seems intuitively unacceptable.

The two parallel straight lines that result from DOL and ROL experiments provide a means for illustrating a simple connection between these phenomena. The mathematical connection is most easily proven graphically. We can extend both lines as straight lines to intersect the ln t axis at a value of f (or f^*) = 0 (Fig. 4). This is obviously beyond the range of experimentally obtainable data and beyond the range where the ROL behavior is linear. The separation between the intercepts of both lines on the ln t axis is given by

$$\ln abf^* - \ln a = \ln bf^*$$

If the lines are indeed parallel, as the theory predicts, that means that the horizontal separation at any value for f (or f^*) is the same as the separation between the intercepts.

$$\ln t^* - \ln t_B = \ln bf^*$$

That means that

$$\ln \frac{t^*}{t_B} = \ln bf^*$$

If one wants to know how long a material will support a constant deadload stress, he can run an ROL experiment and extrapolate to the stress value of interest. That value corresponds to t*; and t_B, the breaking time in a constant deadload test, is related to t* by the simple expression

$$t_B = \frac{t^*}{bf^*}$$

To the best of my knowledge, this simple connection between the duration of time in a constant deadload stress experiment, t_B, with the breaking time in an ROL experiment, t*, has never before been pointed out.

An alternate form of this simple rule is even easier to use in actual practice. The ratio f*/t* defines only one rate of loading. A series of ROL experiments is run, and f* versus ln α is plotted. The stress level of interest, obtained by extrapolation, corresponds to only one rate of stressing, α_1. The duration of time that the sample would support the same stress in a constant deadload experiment is given by

$$t_B = (1/\alpha_1)\,(1/b) \tag{4}$$

RATE OF STRAINING VERSUS RATE OF STRESSING

Historically, it has been more usual to perform experiments at a defined rate of straining rather than at a defined rate of stressing. All of the equations derived so far apply only to rate of stressing; so in order to use data obtained on rate of straining, these data should first be converted to equivalent data on rate of stressing. A simple approximate way of making this transformation is to assume that the material is a Hookian solid over the complete range of the stress-strain curve. That is, we assume that $f = E\epsilon$ where E = Young's modulus and ϵ = strain. We know that this is not true as ϵ increases, but it is a good enough approximation for many purposes. When it is a good enough approximation for the purpose of transforming rate-of-straining data into rate-of-stressing data, it is probably because differences on a logarithmic scale are insignificant. If the rate of straining, α', is given by

$$\epsilon = \alpha' t$$

then

$$\alpha \cong \alpha' E$$

Equation (4) is then modified to

$$t_B \cong \frac{1}{\alpha' Eb} = \frac{t^*}{\epsilon^* Eb} \tag{4a}$$

TEST OF RELATIONSHIPS FOR WOOD IN BENDING RUPTURE

We will now test these relationships (Eqs. 2, 3, and 4a) for the case of wood in bending rupture. Bending is a much more complex process than either tension or compression alone. But the equations derived specifically for a tensile stress and deformation are directly applicable to bending stress and deflection. As an example
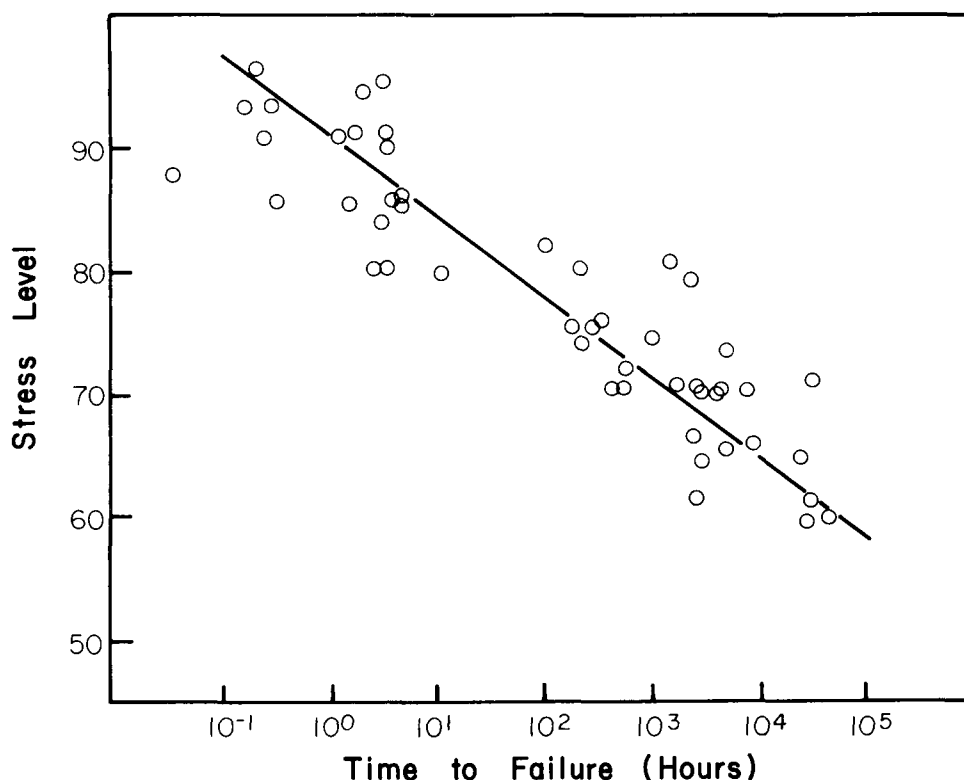
FIG. 5. Duration-of-load (DOL) behavior of Douglas-fir in bending (Youngs and Hilbrand 1963).

we will take the case of Douglas-fir. There is reasonable consistency among the results on bending of several researchers including Wood (1951), Liska (1950), Youngs and Hilbrand (1963), and Schniewind (1967). We will consider in detail the results of Liska and Youngs and Hilbrand as representative.

Youngs and Hilbrand (1963) present two figures of direct interest. In one (see Fig. 5), they present DOL data over a range of times extending from about 5 min to about 50,000 h (about 6 yr). Stress is expressed as a percent of the value chosen as the standard value. One hundred percent (100%) stress level corresponds to the breaking stress measured at a straining rate of 0.0015 min^{-1}. The behavior shows the expected linear relationship between stress and log of time to break.

In another figure (see Fig. 6), Youngs and Hilbrand combine some of their data with earlier data of Liska on strength as a function of rate of strain deflection. Again 100% stress level is chosen as that breaking stress measured at a straining rate of 0.0015 min^{-1}.

According to the Youngs and Hilbrand DOL data (Fig. 5), the slope of the line between stress level and log time to failure equals -6.3.

According to Fig. 6, the measured slope of the rupture line for the rate-of-straining experiment (stress level versus log rate of straining) equals 6.3.

The slope of the DOL line is exactly negative the slope of the ROL line. That is more than just an accident; it is just what Eqs. (2) and (3) predict.

The simple rule expressed by Eq. (4a) can give us an estimate of DOL at a fixed
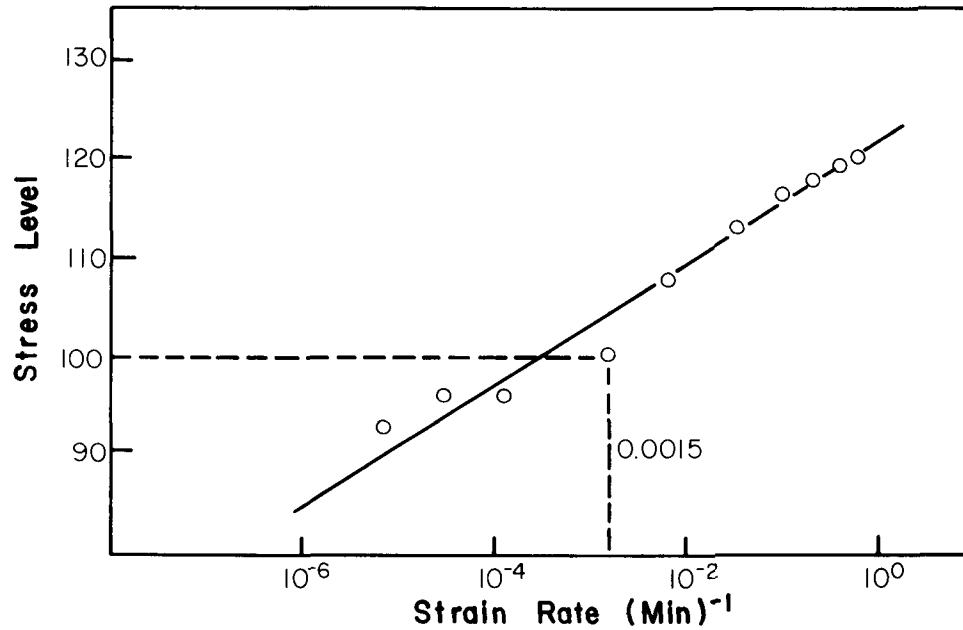
FIG. 6.   Rate-of-loading (ROL) behavior of Douglas-fir in bending rupture (Youngs and Hilbrand 1963). The standard stress level is chosen to be that at straining rate of 0.0015 (min)$^{-1}$.

constant stress level of 65%. This is obtained using only rate-of-straining data. We take Fig. 6 and extend the rupture line down to a value of 65% stress level (Fig. 7). The corresponding straining rate is 6.31×10^{-10} min^{-1}.

The predicted time that a specimen would support a constant deadload stress at a stress level of 65% is given by Eq. (4a);

$$t_B = \frac{1}{\alpha'_t Eb}$$

Young's modulus, E, is introduced as a factor when rate of straining is used, but $1/b$ is still the slope of the experimental line obtained by plotting breaking stress versus ln of the rate of straining. The arithmetic remaining to be done is straightforward. Instead of

$$\frac{df^*}{d \ln \alpha} = \frac{1}{b}$$

we have

$$\frac{d\left(\frac{f^*}{f_{std}} \times 100\right)}{d \ln \alpha'} = 6.3$$

so

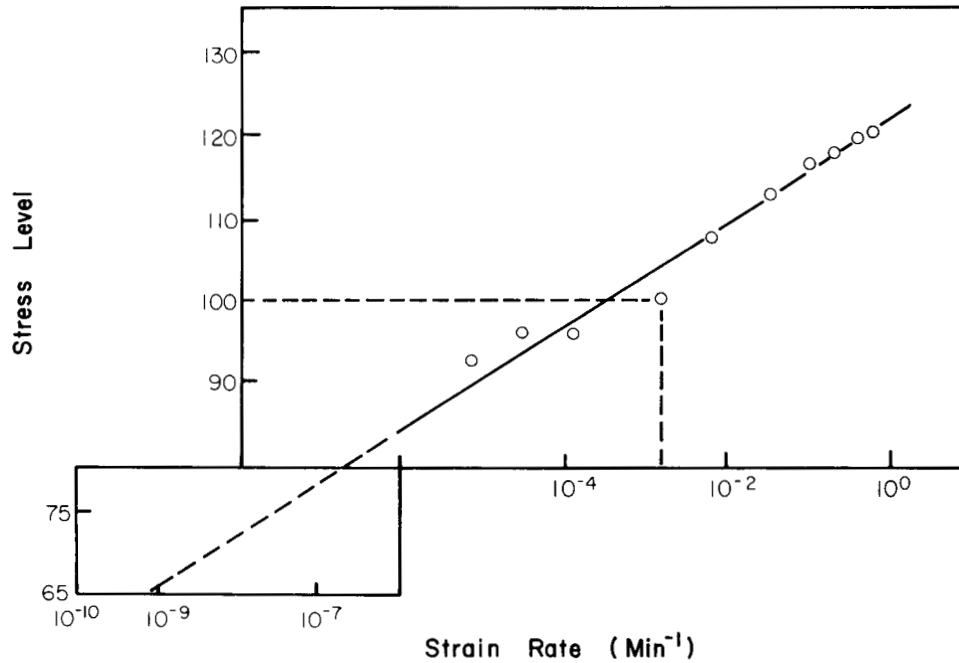$$\frac{1}{b} = \frac{6.3 f_{std}}{(2.303)100}$$

FIG. 7. Extrapolation of Youngs and Hilbrand's (1963) data on rate-of-loading (ROL) behavior gives a strain rate value of 6.31×10^{-10} min^{-1} at a stress level of 65%.

The value used for the modulus of elasticity is 200f(std), 200 times the modulus of rupture, a value consistent with the value given in the Wood Handbook, so that

$$t_b = \frac{1}{\alpha'_1 Eb} = \frac{6.3f_{std}}{100(2.303)[200f_{std}]6.31 \times 10^{-10}}$$

$$t_B = 2.17 \times 10^5 \text{ min} = 3.62 \times 10^3 \text{ hr}$$

The position of this calculated point is shown by the symbol · in Fig. 8.

One can thus show that the ROL data alone are a sufficient predictor of DOL behavior, point by point. More generally, by considering the functional relationships (Eqs. 2 and 3), ROL behavior provides a prediction of DOL behavior. The equation for the ROL line in Fig. 6 is given by

$$\frac{f^*}{f_{std}} \times 100 = 122.5 + 6.25 \log \alpha'$$

We can cast this equation in the form given by Eq. (3)

$$f^* = \frac{1}{b} \ln ab + \frac{1}{b} \ln \alpha$$

using again the approximation that $E\alpha' = \alpha$.

We can evaluate a and b from the slope and intercept of this ROL line. The values of a and b can then be used in Eq. (2) to give a prediction of DOL behavior.
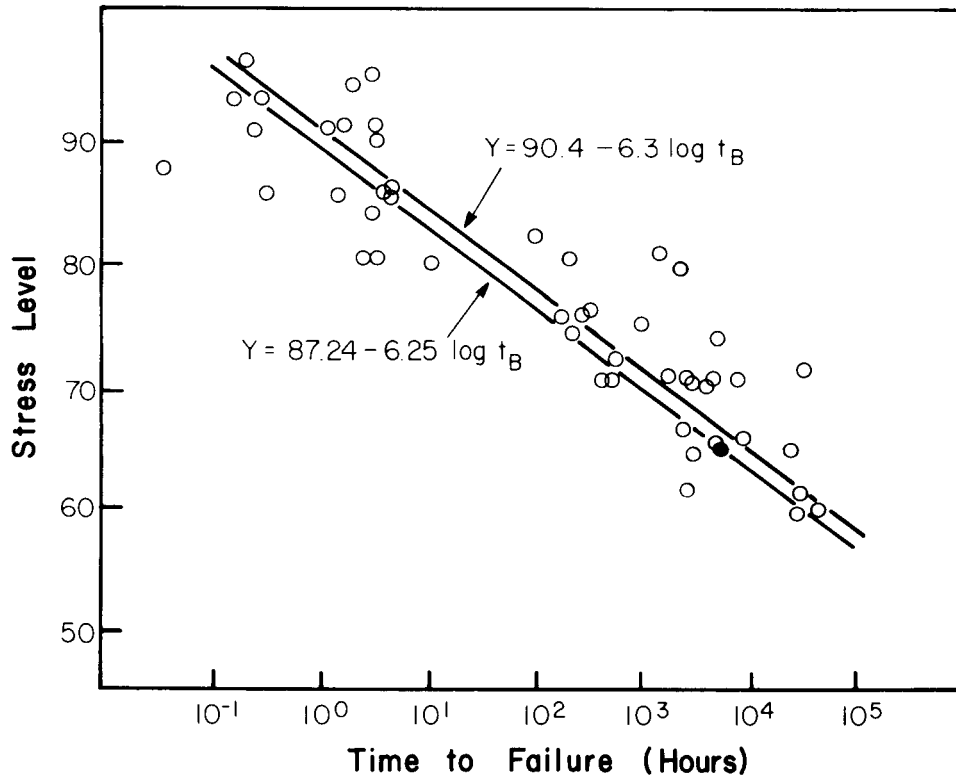
Fig. 8. Duration-of-load (DOL) behavior of Douglas-fir in bending showing both observed dependence of time to failure on stress level (top line) and predicted behavior from rate-of-loading (ROL) experiment (bottom line). The solid dot indicates the calculated time to failure at a stress level of 65%.

If we recast Eq. (2) in the form familiar to wood engineers, expressing stress level as a percent and using common logarithms, we get

$$\frac{f}{f_{std}} \times 100 = 87.24 - 6.25 \log t_B \qquad \text{(calc.)}$$

$$(t_B \text{ in hr})$$

This predicted behavior should be compared with the experimental behavior measured by Youngs and Hilbrand (1963) and reported by Gerhards (1977) in the equation

$$\frac{f}{f_{std}} \times 100 = 90.4 - 6.3 \log t_B \qquad \text{(obs.)}$$

$$(t_B \text{ in hr})$$

This agreement appears quite satisfactory; amounting to only about a 3% to 4% error when measured in terms of stress level. But a 3% to 4% difference in stress level can have large effects in estimated lifetimes. That difference at the 65% stress level calculated earlier, for example, is equivalent to a difference in time amounting to a factor of about 2½. All of that difference, however, can be

explained by an apparent error of about 4% in the determination of the standard stress level. One can see from Fig. 6 that there is a 4% deviation from the straight line of the point occurring at a strain rate of 0.0015 min^{-1} (the stress level at the strain rate chosen as the standard). If, instead, the standard stress level were chosen to be the value where the line crosses 0.0015 min^{-1}, the calculated DOL line in Fig. 8 and the experimental line for DOL behavior would be identical. It seems clear then that DOL behavior calculated only from information obtained from ROL (in this case, rate of straining) can adequately predict experimentally observed DOL behavior. The values of a and b evaluated for Eq. (3) in this way also allow the magnitudes of the creep characteristics of the material to be determined. The size of the moving element, σ, is about 5,600 A^{o3}, and the height of the potential energy barrier, ΔF, is 41.3 kcal.

CONCLUSIONS AND AVENUES FOR FUTURE RESEARCH

The chemical kinetics approach using the theory of absolute rates and a reasonable creep-rupture hypothesis provides a consistent framework for understanding time-dependent rupture phenomena. The formalism connecting DOL and ROL behavior allows one to predict DOL lifetimes from ROL measurements and vice versa. The example chosen of Douglas-fir in bending rupture demonstrates that lifetimes under constant deadloads can be predicted from ROL measurements.

Clearly, the reliability of long-range extrapolations depends upon the accuracy of short-term measurements. For lumber grade specimens, as opposed to the small clear specimens of the example used here, the greater scatter of measured strengths makes long-term extrapolations more questionable. Further experimental data are required to test these methods for lumber grades.

The parameters in the equations derived through the kinetics approach are interpretable in terms of material characteristics. These two characteristics are the size of the moving element and the height of the potential energy barrier impeding motion. For the Douglas-fir example demonstrated, the size of the moving element is about 5,600 A^{o3} (5.6 × 10^{-27}) m^3 and the height of the potential energy barrier is 41.3 kcal/mol (0.173 MJ/mol). These are of the same magnitude as have been reported for paper in tensile rupture (Guthrie and Fulmer 1969). If creep failure involves the breaking of hydrogen bonds, then a potential energy barrier of 41.3 kcal/mol implies that about nine hydrogen bonds must break cooperatively. Nissan (1977) has reported a theory of creep and stress relaxation in hydrogen-bond-dominated solids that involves such a cooperative breaking of hydrogen bonds. More research is needed to establish the full meaning of equation parameters in terms of material characteristics. By so doing, the mechanism by which creep processes lead to rupture will be better understood.

In future reports along these lines, we will try to establish a connection between the kinetic theory of creep rupture and fracture mechanics and to establish a thermodynamic explanation of accelerated creep and accelerated rupture in cyclic humidity environments.

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APPENDIX

Given Eq. (3)

$$f^* = \frac{1}{b} \ln ab\alpha$$

show that

$$\frac{df^*}{d \ln t^*} = -\frac{1}{b} \text{ in the region of interest.}$$

$$f^* = \frac{1}{b} \ln ab\alpha$$

rate of loading $\alpha = \frac{f}{t} = \frac{f^*}{t^*}$

$$f^* = \frac{1}{b} \ln ab + \frac{1}{b} \ln \frac{f^*}{t^*}$$

$$f^* = \frac{1}{b} \ln f^* + \frac{1}{b} \ln ab - \frac{1}{b} \ln t^*$$

$$f^* - \frac{1}{b} \ln f^* = \frac{1}{b} \ln ab - \frac{1}{b} \ln t^*$$

let $f^* - \frac{1}{b} \ln f^* = Y$

$$\frac{dY}{d \ln t^*} = -\frac{1}{b}$$

$$\frac{df^* - \frac{1}{b} \ln f^*}{d \ln t^*} = -\frac{1}{b}$$

$$\frac{df^*}{d \ln t^*} - \frac{1}{b} \frac{d \ln f^*}{d \ln t^*} = \frac{df^*}{d \ln t^*} - \frac{1}{bf^*} \frac{df^*}{d \ln t^*} = -\frac{1}{b}$$

$$\frac{df^*}{d \ln t^*}\left[1 - \frac{1}{bf^*}\right] = -\frac{1}{b}$$

$$\frac{df^*}{d \ln t^*} = -\frac{1}{b} \text{ when } bf^* \gg 1, \text{ i.e., region of interest.}$$

GLOSSARY OF SYMBOLS

Meaning	S.I. Units
$a = \frac{h\gamma_B}{kT\lambda} \exp(\Delta F/RT)$	sec
A = Cross-sectional area of moving element, or force center	m^2
α = Rate of 'loading' in a linear-ramp rate-of-stressing experiment	Pa/sec
α' = Rate of straining in a linear rate-of-straining experiment	sec^{-1}
$b = \frac{\delta}{2kT}$	Pa^{-1}
δ = Volume of moving element or force center	m^3
ϵ = Strain (dimensionless)	—
E = Young's modulus	Pa
ΔF = Height energy barrier; energy of activation	J/mol
f = Stress	Pa
f^* = Breaking stress in rate-of-loading experiment	Pa
γ = Local strain dislocation	m
γ_B = Critical local strain dislocation	m
h = Planck's constant	6.63×10^{-34} Js
k = Boltzmann's constant	1.38×10^{-23} J/°K
λ = Jump distance, separation between positions of minimum potential energy	m
R = Gas constant	8.314 J/mol·K
T = Absolute temperature	K°
t = Time	sec
t_B = Time to break in constant deadload experiment (DOL)	sec
t^* = Time to break in linear rate-of-loading experiment (ROL)	sec