CHARACTERIZATION OF FLAKE ORIENTATION IN FLAKEBOARD BY THE VON MISES PROBABILITY DISTRIBUTION FUNCTION[1]

Robert A. Harris

Assistant Professor
Department of Forestry, Clemson University, Clemson, SC 29631

and

Jay A. Johnson

Scientific Specialist
Composite Products R & D Department
Weyerhaeuser Company, Tacoma, WA 98477

(Received 27 October 1981)

ABSTRACT

The orientation of flakes in thin experimental wood composite boards was characterized by the von Mises distribution function. An *in situ* measuring procedure was developed for acquiring wood flake grain angle data within the board. Parameters characterizing the extent of orientation for a variety of boards with prespecified degrees of alignment were verified using the measurement procedure.

Keywords: Wood particle orientation, degree of alignment, flakeboard, directional data, von Mises distribution, angular measurements.

INTRODUCTION

Flakeboard is manufactured by pressing small, flat particles of wood coated with an adhesive into a dense mat. The arrangement, shape, and orientation of the anisotropic flakes determine, to a great extent, the mechanical behavior of the board. Since the principal material direction changes from point to point inside the board in a nonregular manner, this type of composite material can be classified as a statistically heterogeneous material. A theoretical treatment of the physical behavior of this type of material has been given by Beran (1968). It is apparent from his work that probability density functions will be indispensable with regard to modeling the performance of these materials.

The mechanical behavior of flakeboard is affected by many variables, including: density, resin content, pressing time, species used, and the size, shape, and arrangement of the particles. Since the wood strands are anisotropic, with an elastic modulus ratio between the longitudinal and transverse directions on the order of 20:1, it is apparent that the orientation of the principal material directions of the flakes will influence greatly the mechanical and physical properties of the board. Indeed, research dealing with the effects of orientation have shown it to be of prime importance. Klauditz (1960) doubled the stiffness of particleboard by orienting the grain of the particles in the direction of the load, and Brumbaugh

[1] This work was made possible by a grant from the National Science Foundation, NSF grant number ENG74-16368. The work was done at Virginia Tech.

© 1982 by the Society of Wood Science and Technology

(1960) tripled the stiffness of specially fabricated flakeboards when particle orientation was coincidental with load direction. Steinmetz and Polley (1974) reported that with highly oriented fiberboards, the elastic modulus was in the range of many clear lumber specimens. They also reported that by orienting only the surface mat, the elastic modulus and the modulus of rupture were significantly increased. These and other studies indicate that the particle orientation within particleboard is a variable whose manipulation can produce drastic changes in the mechanical behavior of particleboard.

Past studies dealing with orientation primarily consisted of manufacturing particleboards with various degrees of orientation, varying from nonoriented to a highly oriented system. The extent of orientation, however, was not measured. Without a measure of orientation, it is not possible to establish a mathematical link between the extent of orientation and changes in the board properties. One way to measure orientation is to use a directional distribution function that specifies the probability of finding a given particle orientation at each point in the board. This approach was used in this study.

OBJECTIVES

The objectives of this study were to: 1) explore the use of directional distribution functions to characterize the state of orientation of the wood particles within flakeboard, 2) develop a sampling technique for measuring the grain directions of the flakes that are needed to estimate the parameters of the distribution function, and 3) verify this procedure by constructing boards with known orientations and reproducing the values of the distribution function parameters using the sampling technique.

Although somewhat ideal and simplistic by design, this study was undertaken to help establish the role of orientation in a very controlled system. It was felt that if success was achieved, the information gained in this study would lead to methods that could be used to characterize the orientation of commercial flakeboards and consequently help predict the mechanical and physical properties of these boards.

THEORETICAL CONSIDERATIONS

The orientation of the principal material directions (longitudinal, radial, and tangential) of wood particles within a sheet of particleboard is an excellent example of a directional random variable. As such, the analysis of this variable can be handled by the statistical methods outlined by Mardia (1972). These methods were developed to treat the particular class of angular data in order to avoid certain anomalies. One such anomaly can occur when the arithmetic mean is used to characterize the central tendency of angular data. Consider, for example, the disappearance angle of migrating birds (Emlen 1975). If a bird disappears into the horizon at a bearing of 350° and another at a bearing of 10°, it is intuitively obvious that the central tendency of the bird's flight would be a bearing of 0° rather than 180°, the arithmetic average of the angles.

Since the values of the measurements are restricted to a finite interval (0 to 2π radians or 0° to 360°), any continuous distribution over an infinite interval will not apply. Moreover, the values and derivatives of the distribution at both ends of the interval are required to be equal to maintain the cyclic nature of the phenomena; hence additional constraints are placed on the distributions for use with angular data. The following discussion will acquaint the reader with some basic aspects of handling directional data. A more detailed discussion is given in the Appendix.

A collection of angular measurements can be presented as a circular histogram, (Fig. 1). Each measurement, θ_i, is represented as a point on a unit circle whose coordinates are given by $x_i = \cos\theta_i$ and $y_i = \sin\theta_i$. The entire set of data can be characterized by an orientation vector whose angular position relative to a preselected reference is given by a mean angle θ_0 and whose length r_0 is related to the central tendency of the set. When $r_0 = 1$, the system is totally aligned; when $r_0 = 0$, the system is completely nonaligned (it is a uniform or isotropic distribution). The mean direction and vector length of a set of n angular observations are determined from the coordinates of the points on the unit circle by:

$$\theta_0 = \arctan(y_0/x_0) \tag{1}$$

$$r_0 = \sqrt{x_0^2 + y_0^2} \tag{2}$$

where

$$x_0 = \frac{1}{n}\sum_{i=1}^{n} \cos\theta_i \tag{3}$$

$$y_0 = \frac{1}{n}\sum_{i=1}^{n} \sin\theta_i \tag{4}$$

For certain systems, namely those in which directional data are obtained from the orientation of line elements, it is generally impossible to distinguish a positive or negative sense in direction. Each sample could be assigned to two angles—one differing from the other by π radians (180°). It is customary to measure only the angle that falls in the 0–π radian or 0°–180° interval. This type of data is referred to as *axial data*. The angles representing axial data, θ^*, can be transformed to ordinary directional data by doubling the angles, $\theta = 2\theta^*$, and then Eqs. 1–4 can be used to characterize the system. Grain angles of wood particles or fiber directions in composite materials must be treated as axial data.

A collection of angular data, like all measured data, is a finite sample from a presumed infinite population described by an underlying probability density function (pdf). Mardia (1972) discusses a simple, but elegant function for characterizing directional axial data called the von Mises pdf whose functional form is

$$g(\theta;\mu_0,\kappa) = \frac{1}{\pi I_0(\kappa)} e^{\kappa\cos 2(\theta-\mu_0)} \tag{5}$$

where $0 < \theta < \pi$ and I_0 is the modified Bessel function of order zero. The most probable direction of the population relative to a fixed reference frame is given by the angle $\theta = \mu_0$. The degree of concentration of the angles about μ_0 is specified by the concentration parameter κ. For example, if $\kappa = 0$, the distribution of angles is uniform over 0 to π; if $\kappa = \infty$, all angles are concentrated at $\theta = \mu_0$. The extent of orientation, or degree of alignment, is specified by a value of κ.

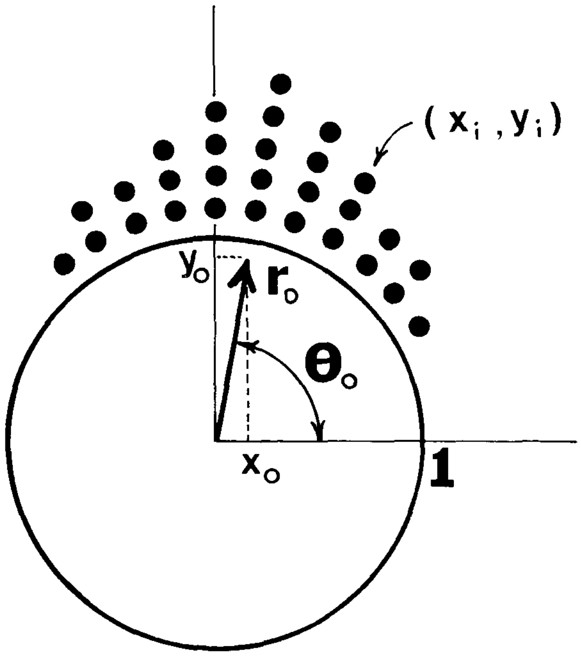

FIG. 1. Example of a circular histogram representing angular data ($x_i = \cos\theta_i$, $y_i = \sin\theta_i$).

The length of the orientation vector can be used to estimate the value of the concentration parameter of a von Mises pdf. The relationship between r_0 and κ has been studied by Batschelet (1965) and is shown in Fig. 2. Hence, once r_0 is determined from a set of data, an estimate of the concentration parameter κ can be found.

In the early stages of our work, we used a truncated normal pdf to characterize the orientation distribution which had the form

$$f(\theta;\mu_0,\beta) = \frac{\beta e^{-\beta^2(\theta-\mu_0)^2}}{\sqrt{\pi}\,\mathrm{erf}(\beta\pi/2)} \tag{6}$$

where $0 < \theta < \pi$, erf(x) is the error function and μ_0 is the mean angle. With this pdf, the shape factor β controls the distribution. Although the truncated normal and von Mises pdf's are similar (Fig. 3), it was felt that the von Mises distribution is better suited to characterize flake orientation. We have calibrated the parameters of the two distributions by minimizing the square of the differences between them and the calibration curve is shown in Fig. 4. It will be noticed that the curve, for $\beta > 1.5$, is approximately given by

$$\kappa = \frac{1}{2}(1 + \beta^2) \tag{7}$$

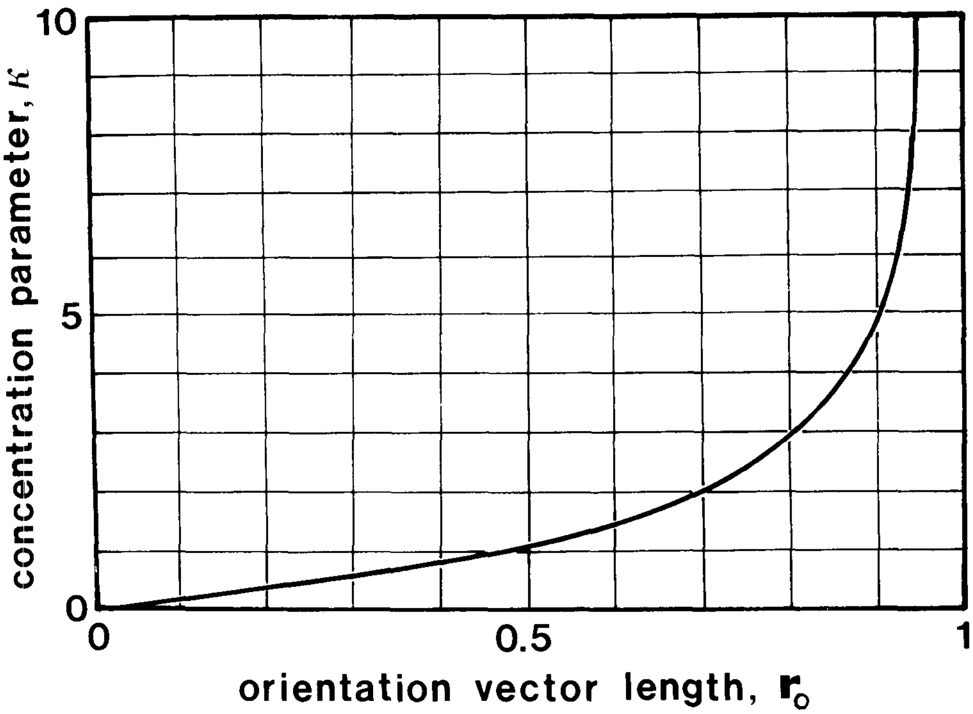

FIG. 2. Relationship between the orientation vector length r_o and the concentration parameter of the von Mises distribution κ.

From this discussion it is seen that three indexes, all related, are available for characterizing the degree of orientation: the length of the mean vector r_0, the von Mises concentration parameter κ, or the truncated normal shape factor β.

MATERIALS AND METHODS

Particleboard manufacture

White pine (*Pinus strobus* L.) was selected for use in making the experimental flakeboards because of its fine texture, gradual transition from earlywood to latewood, as well as its good slicing characteristics. Flakes for the boards were cut from green blocks using a sliding microtome. The dimensions were 38 mm × 13 mm × 0.38 mm (1.5 in. × 0.5 in. × 0.015 in.) in the L, R, and T directions, respectively. Care was taken to insure that the grain direction was coincident with the long axis of the strands. The flakes were pressed dried to prevent curling. After drying, they were weighed and placed in a container with a pulverized two-step phenolic glue manufactured by Varcum Chemicals (V6661-I-EX). The container was agitated thoroughly, and upon removal from the container, the flakes were coated uniformly with the powdered glue, approximately 6% on a weight basis.

Thin flakeboards were manufactured by individually laying down the coated particles in the form of a mat and then pressing at approximatey 2.1 MPa (300 psi), with heat, 163 C (325 F), for 5 min to a thickness of 2 mm (0.08 in.). The

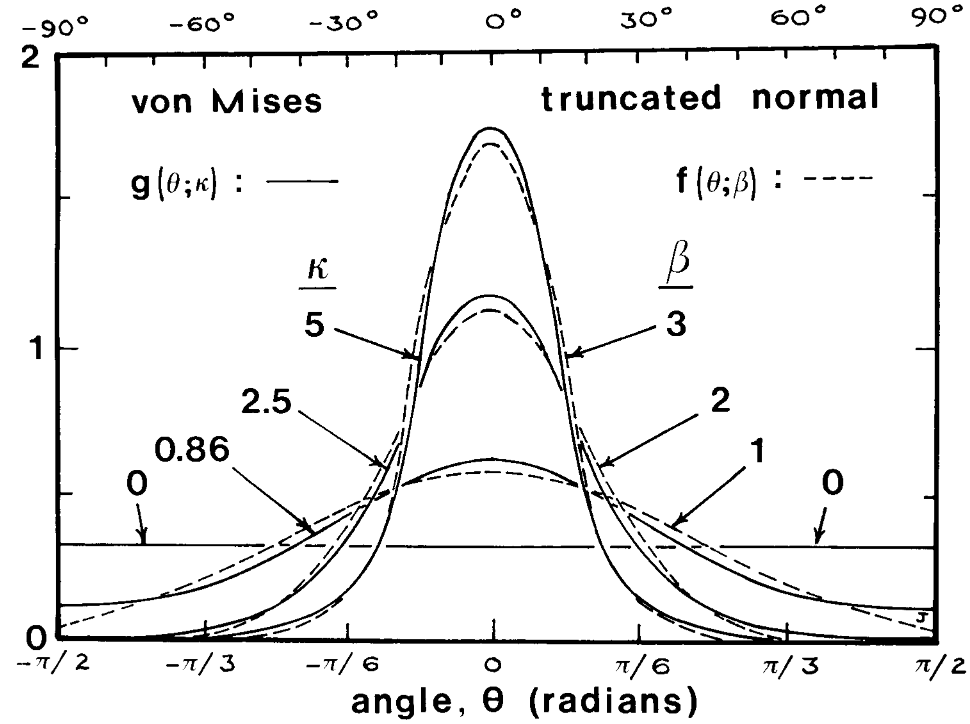

FIG. 3. Comparison of truncated normal and von Mises probability density functions.

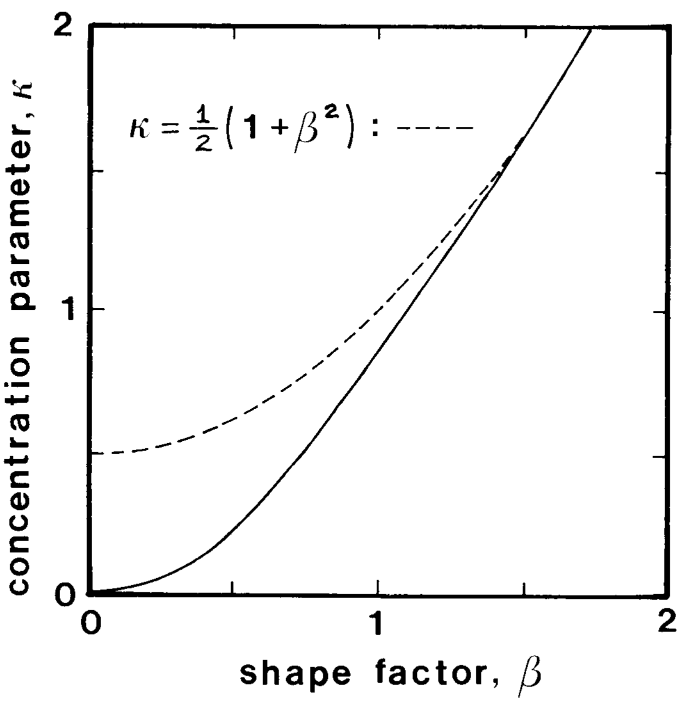

FIG. 4. Calibration curve (solid line) between the concentration parameter of von Mises distribution and shape factor of the truncated normal distribution.

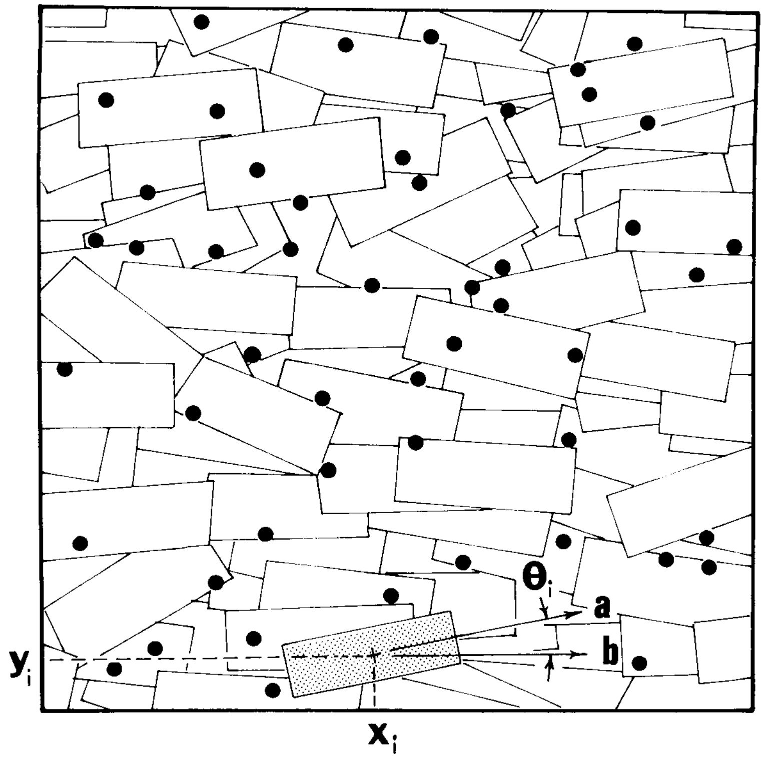

FIG. 5. An example of positioning a strand in a flakeboard mat (a: long axis of the strand, b: principal direction of the board) and fifty random sampling points superimposed on a portion of a flakeboard sample.

location of the particles in the mat was determined by two random coordinates, each being generated from a uniform distribution function. The angular orientation of the long axis of the strands relative to the principal board direction as shown in Fig. 5, was specified by a random angle generated from the truncated normal distribution, Eq. 6. The angle was obtained by first selecting a random number, uniform on the unit interval, and using this value to solve for a corresponding angle given by the inverse of the cumulative truncated normal distribution. This process is represented schematically in Fig. 6. Newton's method of successive approximations was used during this process. Five experimental boards were constructed with varying degrees of orientation: $\beta = 0.1$, 1.0, 2.0, 3.0, and 10.0.

Reconstruction of the orientation distribution function

An attempt was made to reconstruct the known orientation distribution function of the flakes for the experimental boards as follows. Photographic images, black and white slides, of the 152 mm × 152 mm (6 in. × 6 in.) flakeboards were projected onto a rear-view screen. Fifty random points, previously marked on the screen, served as sampling locations for taking measurements of the orientation

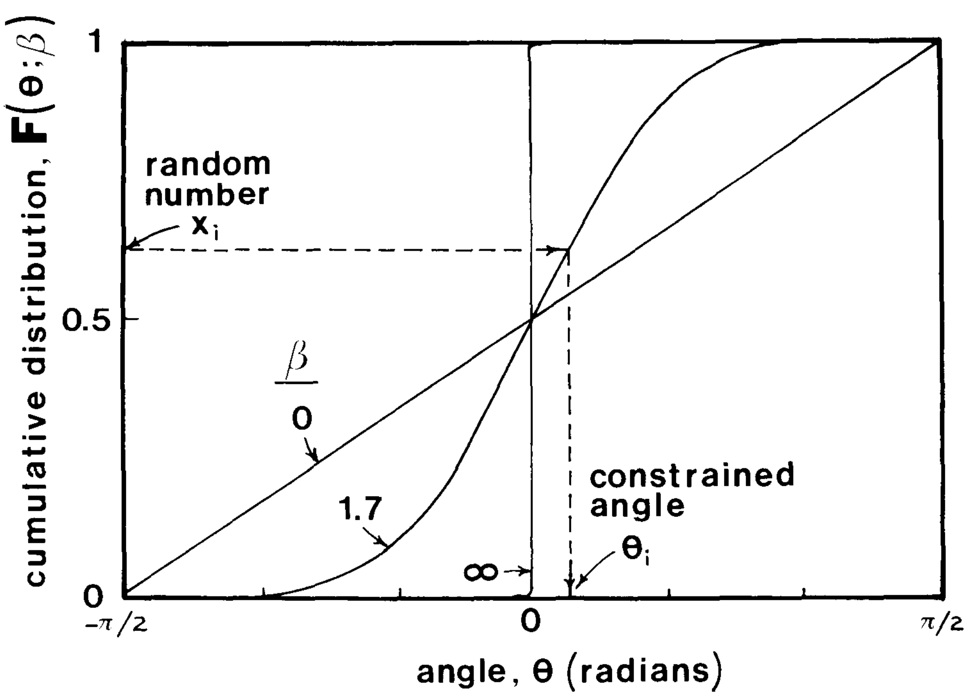

FIG. 6. Method of generating random angles conforming to the cumulative truncated normal distribution function (see Appendix for definition of $F(\theta; \beta)$).

of the exposed wood particles as illustrated in Fig. 5. Data were collected on both sides of the board yielding 100 angular measurements for each board.

The orientation was recorded as the angle between the longitudinal direction of the wood material in the neighborhood of the sampling point and the principal board direction. This angle was determined by placing the rotatable straight edge of a drafting machine parallel to the longitudinal (grain) direction and reading the angular deviation from a scale at the base of the straight edge. Angles were recorded to the nearest 0.0175 radian (1°).

RESULTS AND DISCUSSION

Degree of orientation

The purpose of this portion of the study was to experimentally determine the state of strand orientation in flakeboards specifically manufactured with known orientations. It was felt that a match between the measured degree of orientation and the degree of orientation used to construct the flakeboards would help establish the validity of the sampling method.

The orientation of randomly sampled strands within the experimental boards is shown in the circular histograms of Fig. 7. The frequency of measurements with 0.175 radian (10°) intervals is represented by different size dots. The largest of the four dot sizes represents four measurements; the smallest, one. It is clear that the distributions tend to cluster about a mean direction as the

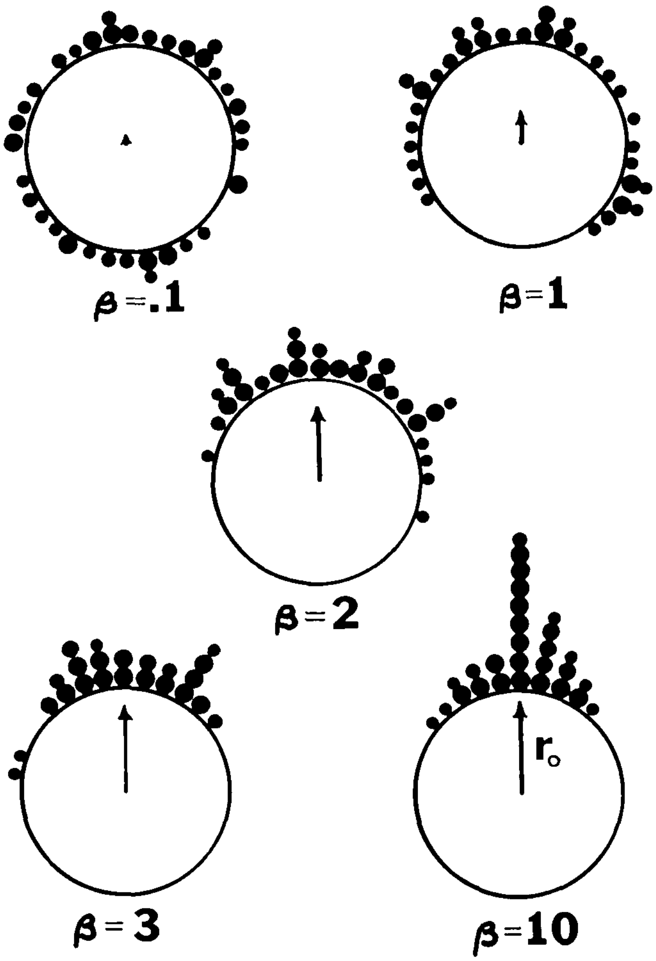

FIG. 7. Circular histograms showing the degree of orientation of the experimental boards (the measured angles were doubled).

shape factor β increases. A measure of this clustering is provided by the length of the orientation vector r_0 calculated according to Eq. 2 after doubling the angles.

In order to compare the measured results with the known distributions used to manufacture the flakeboards, the von Mises distribution was assumed to be the underlying pdf for the angular orientations. Since the truncated normal and von Mises distributions are similar, a value of the concentration parameter corresponding to the fixed value of the shape factor used to manufacture the boards can be found using the calibration curve of Fig. 4. Furthermore, an estimate of the concentration parameter, denoted $\hat{\kappa}$, can be determined by computing r_0 and using the relationship shown in Fig. 2.

The known values of β and the experimentally determined r_0 are shown in Table 1. Also shown are the calibrated values of κ and the estimates. In all but one case, the calibrated values are very close to the estimated values. The greatest

TABLE 1. *Comparison of calibrated and estimated values for the concentration parameter of the von Mises distribution for the experimental boards.*

Shape factor β	Length of mean vector r_0	Concentration parameter[1]	
		Calibrated κ	Estimated $\hat{\kappa}$
0.1	0.056	0.03	0.11
1.0	0.403	0.90	0.88
2.0	0.766	2.50	2.51
3.0	0.893	5.00	4.99
10.0	0.960	50.50	12.80

[1] Paired comparison t-test; No significant difference between κ and $\hat{\kappa}$ at the 5% level, i.e. t(calc) = 0.996 < t(0.05,4) = 2.78.

discrepancy occurred for the most oriented system. This is understandable. Referring to Fig. 2, it is seen that in the range of $0.8 < r_0 < 1.0$, small changes in r_0 will cause substantial changes in κ. Consequently, the difference between κ and $\hat{\kappa}$ for the highly aligned case should not be viewed with too much alarm.

Despite the difference for the highly oriented flakeboards, a paired comparison using a t-test indicated that there was no difference between the two sets of concentration parameters at the 5% confidence level. It is felt, therefore, that the sampling method for determining the degree of orientation in flakeboards is consistent, and it is hoped that it will provide a means of specifying the degree of orientation in boards in which the angular distribution is not known.

SUMMARY

The use of the von Mises distribution to characterize the angular distribution of flakes in flakeboard was developed and would appear to be useful in establishing the degree of orientation in wood composites. The concentration parameter provides a measure of flake alignment and can be easily determined from a set of measured angular values. The use of the concentration parameter for characterizing the extent alignment may lend itself to modeling as the von Mises distribution can be used to predict, in a probablistic sense, various mechanical and physical properties.

REFERENCES

ABRAMOWITZ, M., AND I. A. STEGUN. 1965. Handbook of mathematical functions. Dover, New York. 1046 pp.

BATSCHELET. 1965. Statistical methods for the analysis of problems in animal orientation and certain biological rhythms. American Institute of Biological Sciences, Washington, D.C. 57 pp.

BERAN, M. J. 1968. Statistical continuum theories. John Wiley and Sons, New York. 424 pp.

BRUMBAUGH, J. 1960. Directional properties possible in flakeboards. The Lumberman 87(7):46–47.

CORTE, H., AND J. KALLMES. 1962. Statistical geometry of a fibrous network. Pages 13–52 *in* F. Bolam, ed. The formation and structure of paper; transactions of the symposium held at Oxford in September 1961, Vol. 1. Tech. Sec. of the British Paper and Board Assoc., London.

EMLEN, S. T. 1975. The stellar-orientation system of a migratory bird. Sci. Am. 233(2):102–111.

KLAUDITZ, W. 1960. Manufacture and properties of wood chip materials with directed strength. Holz Roh-Werkst. 18(10):377–385.

MARDIA, K. V. 1972. Statistics of directional data. Academic Press, New York. 357 pp.

STEINMETZ, P. E., AND C. W. POLLEY. 1974. Influence of fiber alignment on stiffness and dimensional stability of high density dry-formed hardboard. For. Prod. J. 24(5):45–50.

APPENDIX—DIRECTIONAL DATA

It is convenient to assume a complex number representation of an angular measurement θ_k in which the angle is associated with a point z_k on the unit circle in the complex plane centered at the origin. Then

$$x_k + iy_k = z_k = e^{i\theta_k} = \cos\theta_k + i\sin\theta_k \tag{A1}$$

where $i^2 = -1$ and $k = 1, 2, \ldots n$. In this form, the arithmetic mean of the set of associated complex points $\{z_k\}$ provides the information needed to characterize the extent of orientation of the set of angles $\{\theta_k\}$, i.e.,

$$z_o = \frac{1}{n}\sum_{i=1}^{n} z_k = x_o + iy_o = r_o e^{i\theta_o} \tag{A2}$$

Obviously, Eqs. 1–4 in the text follow from this interpretation. Hence, anomalies inherent in the averaging of angles are resolved if the associated complex points are averaged rather than the angles themselves.

A measure of dispersion, known as the circular variance, S_o, can be defined in terms of the complex averaging process

$$S_o = 1 - r_o \tag{A3}$$

This measure is similar to an "entropy" measure in that it attains a maximum, $S_o = 1$, for a completely nonaligned system (one in which there is no preferred direction; the arrangement of features portraying directionality is totally chaotic) and vanishes, $S_o = 0$, for a totally aligned system (one in which there is no doubt whatsoever about the direction of the features; they are all the same and point in only one direction). The term "random" is commonly used for a completely nonaligned system ($S_o = 1$) but, strictly speaking this is a misnomer. An angular measurement is random whether it is from a non- or partially aligned system. Axial data can be processed by the methodology discussed in the text if the angles are doubled prior to calculating the complex average. An estimate of the circular variance for axial data, according to Mardia (1972), is given by

$$S_o^* = 1 - (1 - S_o)^{1/4} \tag{A4}$$

In the work reported in this article, we have preferred to use r_o directly as an index of orientation rather than use the circular variance.

Since directional data are restricted to a finite interval (the end points of which must have equal values as well as equal derivatives), most traditional probability density functions (pdf) are not applicable. Mardia (1972) has shown how traditional pdf's can be modified by "wrapping" them around the interval. Unfortunately, the resulting formulas are rather complicated. There is, however, a particular distribution for characterizing angular measurements, which appears to be quite versatile. This is the von Mises pdf which, when centered about $\mu_o = 0$, is given by

$$g(\theta;0,\kappa) = \frac{1}{2\pi I_o(\kappa)} e^{\kappa\cos\theta} \tag{A5}$$

The form of the cumulative distribution function (df) is given by

$$G(\theta;0,\kappa) = \frac{1}{2} + \frac{1}{2\pi}\left\{\theta + 2\sum_{n=0}^{\infty}\left(\frac{I_n(\kappa)}{I_o(\kappa)}\right)\frac{\sin(n\theta)}{n}\right\} \tag{A6}$$

where $I_n(\kappa)$, the modified Bessel Function of the first kind, order n, and is defined by

$$I_n(x) = \left(\frac{x}{2}\right)^n \sum_{m=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{2m}}{(m+n)!m!} \tag{A7}$$

but can be computed using the recurrence relation

$$I_n(x) = I_{n-2}(x) - \frac{2(n-1)}{x} I_{n-1}(x) \tag{A8}$$

Obviously $I_0(x)$ and $I_1(x)$ are needed to "prime" this relationship for $n \geq 2$. We have found the polynomial approximations, six digit accuracy, given on page 378 (formulas 9.8.1 to 9.8.2) of the Handbook of Mathematical Functions, Abramowitz and Stegun (1965), very useful. Values of the modified Bessel functions can also be obtained from standard computational packages available at most computing facilities.

An interesting feature of the von Mises density function is that an approximation to it for small values of the concentration parameter has the form of a cardioid density function

$$g(\theta;\mu_0,\kappa) = \frac{1}{2\pi}\{1 + \kappa\cos(\theta - \mu_0)\} \tag{A9}$$

The cardioid pdf has been used by Corte and Kallmes (1962) for characterizing the orientation of fibers in a paper web. For large values of κ, the density function is approximated by the normal density function

$$g(\theta;\mu_0,\kappa) = \frac{\sqrt{\kappa}}{2\pi}e^{-(\kappa/2)(\theta-\mu_0)^2} \tag{A10}$$

The axial form of the von Mises pdf (Eq. 5 in the text) is

$$g_2(\theta;\mu_0,\kappa) = \frac{1}{\pi I_0(\kappa)}e^{\kappa\cos 2(\theta-\mu_0)} \tag{A11}$$

and the axial df, centered about $\mu_0 = 0$, is

$$G_2(\theta;0,\kappa) = \frac{1}{2} + \frac{1}{\pi}\left\{\theta + \sum_{n=0}^{\infty}\left(\frac{I_n(\kappa)}{I_0(\kappa)}\right)\frac{\sin(2n\theta)}{n}\right\} \tag{A12}$$

As mentioned in the text, during the preliminary stages of this investigation the authors were unaware of the von Mises distribution. Consequently, a truncated normal pdf was used to specify the orientation of the principal material directions of the wood strands in flakeboard mates. The functional form selected for this pdf was

$$f(\theta;\mu_0,\beta) = \frac{\beta e^{-\beta^2(\theta-\mu_0)^2}}{\sqrt{\pi}\,\mathrm{erf}(\beta\pi/2)} \tag{A13}$$

and the cumulative distribution function, centered about $\mu_0 = 0$, was

$$F(\theta;0,\beta) = \frac{1}{2}\left\{1 + \frac{\mathrm{erf}(\beta\theta)}{\mathrm{erf}(\beta\pi/2)}\right\} \tag{A14}$$

where the error function is defined as

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt \tag{A15}$$

A useful polynomial for this function is also given by Abramowitz and Stegun (1965) on page 299 (formula 7.1.26).

Although the truncated normal distribution served us well in the initial stages of this work, we feel that the von Mises pdf is probably the better distribution to characterize orientation. For one thing, there is no discontinuity in derivatives at the end points of the interval as there is with the truncated normal (see Fig. 3). For another, there is a large body of literature dealing with the von Mises pdf (hypothesis testing, theoretical development, etc.) whereas this does not exist for the truncated normal.

The calibrated relationship between the shape factor β of the truncated normal distribution and the concentration parameter κ of the von Mises distribution (Fig. 4) is approximately given by the following expression:

$$\kappa = \kappa_1 - \kappa_2 + \kappa_3 \tag{A16}$$

where

$$\kappa_1 = \frac{1}{2}(1 + \beta^2)$$

$$\kappa_2 = \begin{cases} \frac{1}{2}\left(1 - \frac{2}{3}\beta\right) & : 0 \leq \beta \leq 1.5 \\ 0 & : \beta > 1.5 \end{cases}$$

$$\kappa_3 = \begin{cases} 0.348\beta(\beta - 0.83) & : 0 \leq \beta \leq 0.83 \\ 0.035 - 0.312(\beta - 1.165)^2 & : 0.83 < \beta \leq 1.5 \\ 0 & : \beta > 1.5 \end{cases}$$

Continued from page 245

who served as part of the publication apparatus over the past thirteen years; editors, editorial boards, reviewers and authors. It also is a reflection of a devoted professional, Carol Ovens, the one person who has been a part of *Wood and Fiber* since its inception and the willingness of Joe McCarthy to provide the venture capital with no thought of personal gain.

I never believed I would join those who wrote nostalgia—my interest being the future, not the past; however, my association with the beginning of *Wood and Fiber* was the most rewarding of any extra-curricular activity. It's good to both look back at its beginning and forward to its growth in a new form.

WILLIAM NEARN

Weyerhauser Company
Seattle, WA