# COMPARISONS OF SHEAR STRESS/SHEAR STRAIN RELATIONS OF WOOD OBTAINED BY IOSIPESCU AND TORSION TESTS


*Hiroshi Yoshihara*

Associate Professor,
Department of Natural Products Resource Engineering
Shimane University
Matsue, Shimane
Japan

*Hisashi Ohsaki*

Researcher,
Hokkaido Forest Products Research Institute
Asahikawa, Hokkaido
Japan

*Yoshitaka Kubojima*

Researcher,
Forestry and Forest Products Research Institute
Tsukuba, Ibaraki
Japan

and

*Masamitsu Ohta*

Professor
Graduate School of Agricultural and Life Sciences
The University of Tokyo
Bunkyo-ku, Tokyo
Japan



(Received November 1999)



ABSTRACT

In this paper, we compared the shear stress/shear strain relations of wood obtained by Iosipescu and torsion tests.

Quartersawn boards of Sitka spruce (*Picea sitchensis* Carr.) and shioji (Japanese ash, *Fraxinus spaethiana* Lingelsh.) provided the specimens. Iosipescu tests were conducted with specimens loaded in the radial direction, and the shear stress/shear strain relations were obtained. Shear stress/shear strain relations were obtained independently of the Iosipescu tests by torsion tests of rectangular bars. The following results were obtained:

(1) The shear moduli, shear yield stresses, and shear strengths obtained from both methods showed good agreement with each other, except for the shear strength of ash.

(2) As for spruce, the difference between the shear stress/shear strain relations obtained by Iosipescu and torsion tests was significant in the 5% significance level, whereas that for the ash was not significant.

(3) Although the Iosipescu test can derive the shear stress/shear strain relation directly, it has the drawback that failure occurs earlier than with the torsion test. In contrast, the torsion test has the drawback that the procedure for obtaining the stress/strain relation is quite complicated. In determining the shear stress/shear strain relation of wood properly, shear stress/shear strain data should be measured more frequently by these methods.



*Keywords*: Shear stress/shear strain relation, Iosipescu test, torsion test.

*Wood and Fiber Science*, 33(2), 2001, pp. 275–283

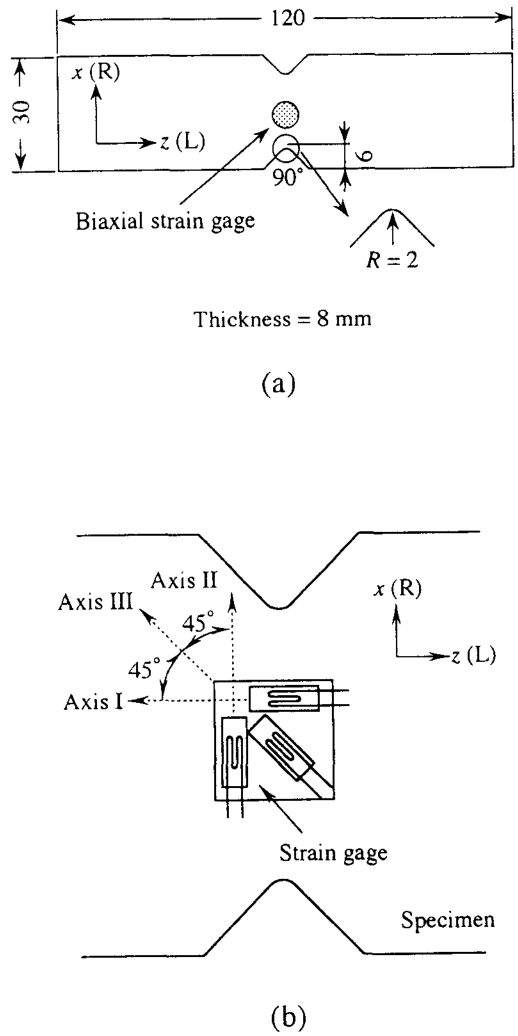


FIG. 1. Iosipescu test specimen and triaxial strain gage arrangement in the Iosipescu test.
Notes: Unit: mm. L, R represent the longitudinal and radial directions, respectively.

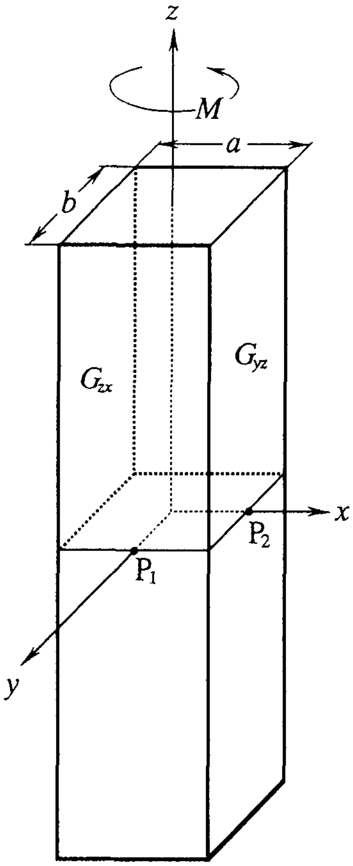


FIG. 2. Diagram of the rectangular specimen subjected to the torsional force.

## INTRODUCTION

In determining the shear stress/shear strain relation of isotropic or transversely isotropic materials such as metals, concrete, and fiber-reinforced plastics (FRP), a torsion test of the specimen with a cylindrical cross section is often conducted because a rigorous equation representing the shear stress/shear strain relation exists (Hill 1950). In a torsion test of a rectangular bar, a pure shear stress condition is expected in the specimen. In the torsion of an orthotropic material like wood, however, the shear stress in the rectangular section of the specimen is a function of aspect ratio and shear modulus, which is derived by Prandtl's membrane analogy, and is not distributed uniformly in the surface (Lekhnitskii 1963). This phenomenon causes a difficulty in determining the shear stress/shear strain relation of wood by torsion (Yoshihara and Ohta 1995, 1997). The Iosipescu test is another promising method because of the simplicity in characterizing the shearing properties (Janowiak and Pellerin 1991; Yoshihara et al. 1999; Kubojima et al, 2000; Dumail, et al. 2000). In determining the shear/stress/shear strain relation by an Iosipes-

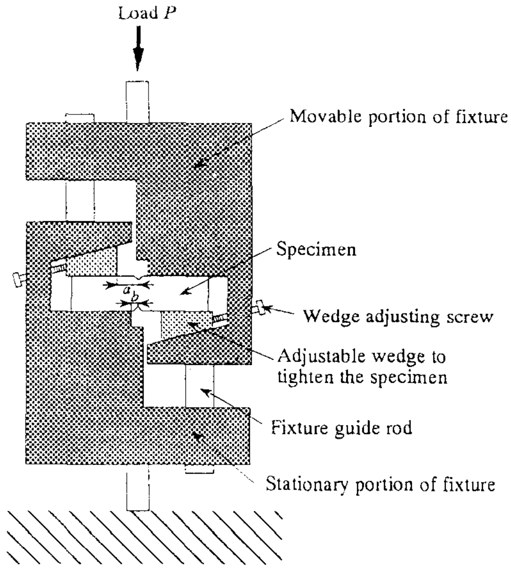


FIG. 3. Iosipescu shear test fixture.
Notes: $a$ = 22 mm, $b$ = 12 mm.

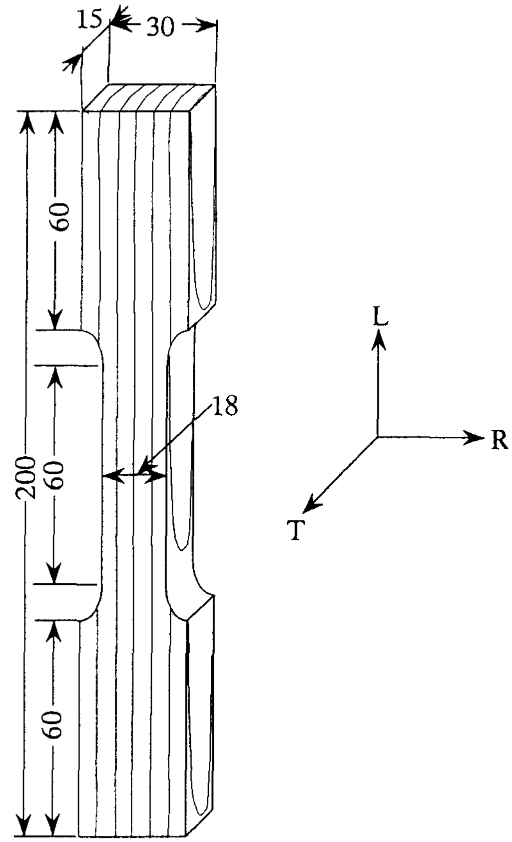


FIG. 4. Torsion test specimen (unit: mm).

cu test, however, there is concern that the cracks that occur at the notch roots of the specimen could have a serious influence on the stress distribution.

Although Iosipescu and torsion tests are effective in determining shear stress/shear strain properties, they have been rarely applied to wood, and there are few examples comparing the shear stress/shear strain properties obtained by these methods. Here we compared the shear stress/shear strain relations obtained by these methods and examined the applicability of these methods for determining the shear stress/shear strain relations of wood.

## THEORIES

### *Shear stress/shear strain relations obtained by Iosipescu test*

Figure 1a shows the schematic diagram of an Iosipescu test specimen. The $z$- and $x$- axes are defined as those parallel and perpendicular to the long axis of the specimen. When rigidly-clamped halves are displaced relative to each other, a pure shear stress condition is thought to be obtained in the mid-length of the specimen. In this loading condition, the bending moment is equal to zero in the middle section of the specimen, whereas the shearing force equals $P$ in the middle section. Thus, the shear stress $\tau_{zx}$ can be obtained by the following equation (Seerat-Un-Nabi and Derby 1990):

$$\tau_{zx} = \frac{P}{td} \tag{1}$$

where $t$ is the thickness of the specimen, $d$ is the distance between the notches. The engineering shear strain at the midpoint of the notches, $\gamma_{zx}$, can be measured by bonding the strain gages between the notches, and the shear stress/shear strain relation can be determined.

The shear stress/shear strain relation in the elastic condition is represented by Hooke's law, whereas it is approximated by the Ludwik's power function (Chakrabarty 1989) in the plastic strain range. Thus, the shear stress/shear strain relation all over the strain range is represented as follows:

$$\gamma_{zx} = \frac{\tau_{zx}}{G_{zx}} \qquad (\tau_{zx} \leq S_{zx}) \quad (2a)$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G_{zx}} + K\left(\frac{\tau_{zx} - S_{zx}}{S_{zx}}\right)^n \qquad (\tau_{zx} > S_{zx}) \quad (2b)$$

where $S_{zx}$ is the yield shear stress, and $K$ and $n$ are the material parameters with no dimensions.

*Shear stress/shear strain relation predicted from the torsion testing data*

Figure 2 shows the torsion test diagram. When the rectangular bar is twisted around the $z$-axis by the torsional moment of $M$, the shear stress at the center of $zx$-plane $\tau_{zx}$ would be approximated as follows (Yoshihara and Ohta 1998):

$$\tau_{zx} = p_{zx}M = G_{zx}\gamma_{zx} \qquad (\tau_{zx} \leq S_{zx}) \quad (3a)$$

$$\tau_{zx} = p_{zx}\left[M + 0.2\cdot\frac{a^2+b^2}{ab}\cdot(\gamma_{zx}{}^{p})^2\frac{d}{d\gamma_{zx}{}^{p}}\left(\frac{M-M_y}{\gamma_{zx}{}^{p}}\right)\right] \qquad (\tau_{zx} > S_{zx}) \quad (3b)$$

where $a$ and $b$ are the length of the $x$- and $y$-directions, respectively, $M_y$ is the torsional moment at the occurrence of yielding, $G_{zx}$ and $S_{zx}$ are the shear modulus and the yield shear stress on the $zx$-plane, respectively, and $\gamma_{zx}$ and $\gamma_{zx}{}^{p}$ are the total shear strain and the plastic shear strain at the center of $zx$-plane, respectively. In this equation, $p_{zx}$ is written as:

$$p_{zx} = \frac{1}{a^2bk}\cdot\left[-\frac{8}{\pi^2}\sqrt{\frac{G_{zx}}{G_{yz}}}\sum_{j=1}^{\infty}\frac{(-1)^{j-1}}{(2j-1)^2} \times \tanh\frac{(2j-1)\pi b}{2a}\sqrt{\frac{G_{zx}}{G_{yz}}}\right] \quad (4)$$

where $G_{yz}$ is the shear modulus on the $yz$-plane, and

$$k = \frac{1}{3} - \frac{2a}{b}\sqrt{\frac{G_{yz}}{G_{zx}}}\left(\frac{2}{\pi}\right)^5\sum_{j=1}^{\infty}\frac{1}{(2j-1)^5} \times \tanh\frac{(2j-1)\pi b}{2a}\sqrt{\frac{G_{zx}}{G_{yz}}} \quad (5)$$

The yield shear stress is represented by the yield torsional moment as follows:

$$S_{zx} = p_{zx}M_y \quad (6)$$

The torsional moment/plastic strain relation is represented in the range of $\tau_{zx} > S_{zx}$ by a power function as:

$$\gamma_{zx}{}^{p} = \alpha\left(\frac{M - M_y}{M_y}\right)^m \quad (7)$$

where $\alpha$ and $m$ are the material parameters with no dimensions. The second term in the braces of Eq. (3b) is derived by substituting Eq. (7) into Eq. (3b) as:

$$0.2\cdot\frac{a^2+b^2}{ab}\cdot(\gamma_{zx}{}^{p})^2\cdot\frac{d}{d\gamma_{zx}{}^{p}}\left(\frac{M-M_y}{\gamma_{zx}{}^{p}}\right) = 0.2\cdot\frac{a^2+b^2}{ab}\cdot\frac{1-m}{m}\cdot M_y\cdot\left(\frac{\gamma_{zx}{}^{p}}{\alpha}\right)^{1/m} \quad (8)$$

From Eqs. (3b), (6), (7), and (8), $M$ and $M_y$ are eliminated and the shear stress in the range of $\tau_{zx} > S_{zx}$ is represented as follows:

$$\tau_{zx} = p_{zx}\left[\left(\frac{\gamma_{zx}{}^{p}}{\alpha}\right)^{1/m}M_y + M_y\right] + 0.2\cdot\frac{a^2+b^2}{ab}\cdot\frac{1-m}{m}\cdot p_{zx}M_y\cdot\left(\frac{\gamma_{zx}{}^{p}}{\alpha}\right)^{1/m} = S_{zx} + \left(1 + 0.2\cdot\frac{a^2+b^2}{ab}\cdot\frac{1-m}{m}\right)\cdot S_{zx} \times \left(\frac{\gamma_{zx}{}^{p}}{\alpha}\right)^{1/m} \qquad (\tau_{zx} > S_{zx}) \quad (9)$$

Hence the plastic strain in the shear stress/shear strain relation can be obtained as follows:

$$\gamma_{zx} = \frac{\tau_{zx}}{G_{zx}} \qquad (\tau_{zx} \leq S_{zx}) \tag{10a}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G_{zx}} + K'\left(\frac{\tau_{zx} - S_{zx}}{S_{zx}}\right)^m \qquad (\tau_{zx} > S_{zx}) \tag{10b}$$

where

$$K' = \frac{\alpha}{\left(1 + 0.2 \cdot \frac{a^2 + b^2}{ab} \cdot \frac{1 - m}{m}\right)^m} \tag{11}$$

When the $M - \gamma_{zx}{}^{p}$ relation in the plastic region is measured and regressed into Eq. (7), the shear stress/shear strain relation can be represented by Eq. (10), which has a similar formula derived for the Iosipescu test, Eq. (2).

## EXPERIMENT

### Materials

Sitka spruce (*Picea sitchensis* Carr.) and shioji (Japanese ash, *Fraxinus spaethiana* Lingelsh.) were used for the specimens. The density of spruce was 0.43 g/cm$^3$, whereas that of ash was 0.59 g/cm$^3$. For both species, all specimens were cut from the same lumber, and were conditioned at 20°C and 65% relative humidity before and during the tests.

In this experiment, we defined the radial, tangential, and longitudinal directions as *x*-, *y*-, and *z*-axes, respectively.

### Iosipescu tests

The dimensions of the Iosipescu test specimen are shown in Fig. 1a. The long axis of the specimen coincided with the longitudinal direction, whereas the notch direction coincided with the radial direction. To measure the strain condition between the opposed notches, a triaxial-strain gauge (gauge length = 2 mm, YFRA-2-11, Tokyo Sokki Co., Ltd.) was bonded at the midpoint between the notches. Figure 1b shows the triaxial-strain gauge arrangement. The shear strain $\gamma_{zx}$ was obtained from the following equation:

$$\gamma_{zx} = 2\epsilon_{III} - \epsilon_{I} - \epsilon_{II} \tag{12}$$

where $\epsilon_I$ and $\epsilon_{II}$ are the strains in the longitudinal and the radial directions, respectively, and $\epsilon_{III}$ is the direction inclined at 45 degrees with respect to the longitudinal direction.

Figure 3 shows the diagram of the Idaho test fixture (Conant and Odom 1995). The loading surfaces begin at points of 22 and 12 mm away from the center of the test specimen, and the load with the velocity of 1 mm/min was applied on the specimen. From the load-strain relations recorded by an XY-recorder, the shear stress/shear strain relation was obtained by Eqs. (1) and (2). Six specimens were used for each species.

### Torsion tests

Figure 4 illustrates the torsion test specimen. To measure the strains at the centers of the side surfaces, same strain gages used in the Iosipescu tests were bonded on the LR- and LT-planes. The specimen was twisted by a manual torsion-test device, and the torsional moment/shear strains relations on both planes were obtained. The shear moduli $G_{zx}$ and $G_{yz}$ were obtained from the following equation (Lekhnitskii 1963; Yoshihara and Ohta 1993);

$$G_{zx} = \frac{k_{zx}}{a^2bk}\left[-\frac{8}{\pi^2}\sqrt{\frac{G_{zx}}{G_{yz}}}\sum_{j=1}^{\infty}\frac{(-1)^{j-1}}{(2j-1)^2} \times \tanh\frac{(2j-1)\pi b}{2a}\sqrt{\frac{G_{zx}}{G_{yz}}}\right]$$

$$G_{yz} = \frac{k_{yz}}{a^2bk}\left[1 - 2\left(\frac{2}{\pi}\right)^2\sum_{j=1}^{\infty}\frac{1}{(2j-1)^2} \times \left\{\cosh\frac{(2j-1)\pi b}{2a}\sqrt{\frac{G_{zx}}{G_{yz}}}\right\}^{-1}\right] \tag{13}$$

where $k_{yz}$ and $k_{yz}$ are the initial inclinations of the torsional moment/shear strains relations and the plastic strain $\gamma_{zx}{}^{p}$ was calculated by subtracting the elastic strain as shown in Fig. 5a. Then, the torsional moment/plastic shear strain relation on the LR-plane was regressed into Eq. (7), and the yield torsional moment $M_y$ was determined as in Fig. 5b. In a previous paper, however, we pointed out that the value

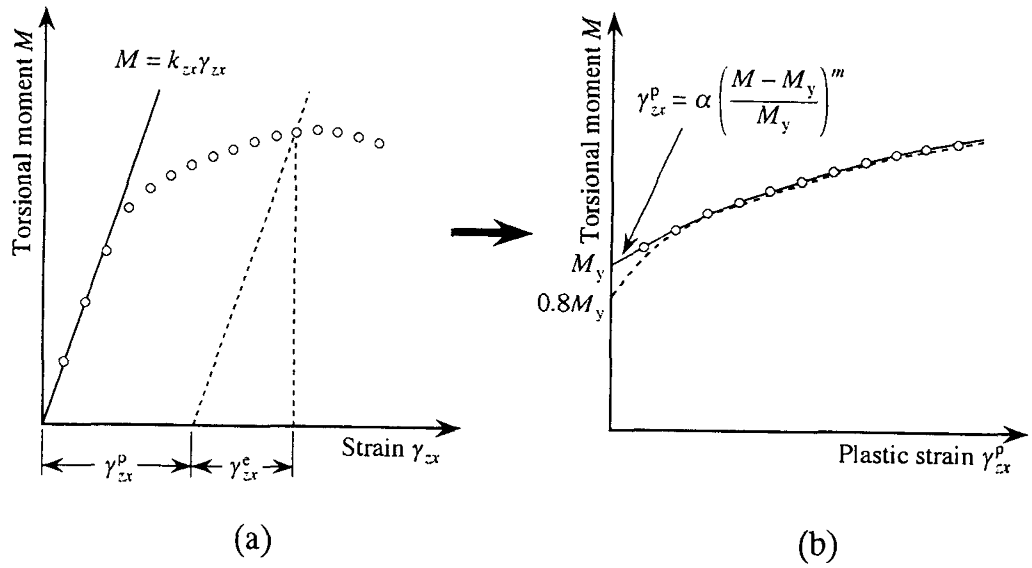


FIG. 5. Transformation of torsional moment/shear strain relation into moment-plastic shear strain relation.
Note: (a): Torsional moment/shear strain relation, (b): Torsional moment/plastic shear strain relation.
Legend: Circles indicate the measurement point. Solid and dashed lines in (b) are obtained by the direct regression of the experimental data into Eq. (7) and by the regression using the reduced value of yield moment, respectively.

of $M_y$ tends to be measured as higher than the real value by the torsion test because the elastic region is widely distributed soon after the occurrence of yielding, which initiates locally at the wider surface of specimen in the torsion of a rectangular bar, and the torsional moments/shear strain relation tends to be linear after the initiation of yielding. According to the numerical calculations based on the strain incremental theory, we found that the shear stress/shear strain relation can be derived properly by torsion when $M_y$ is evaluated as 80% of the measured one (Yoshihara and Ohta 1995). Using the reduced value of $M_y$, the tor-

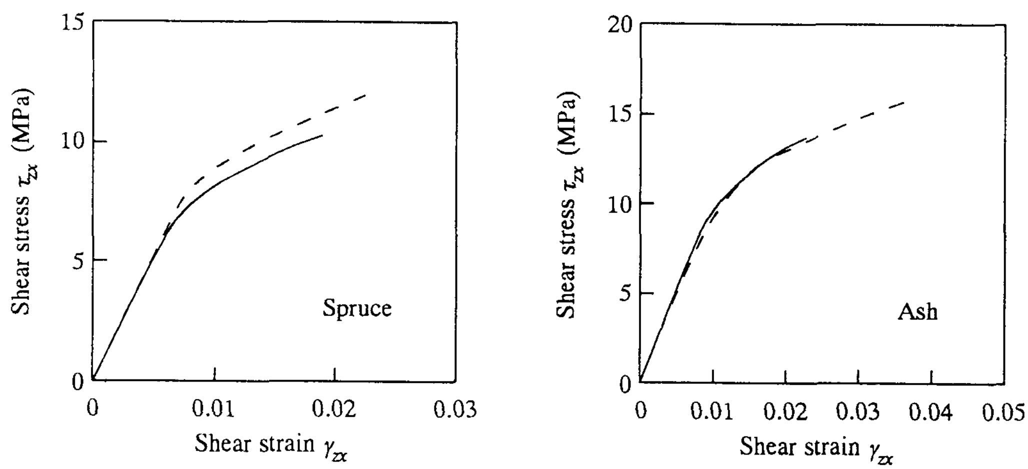


FIG. 6. Typical shear stress/shear strain relations obtained from the Iosipescu and torsion tests.
Legend: Solid and dashed lines are obtained from the Iosipescu and torsion tests, respectively.

TABLE 1. *Shear moduli obtained by the Iosipescu shear test and the torsion tests.*

| | Iosipescu $G_{zx}$ (GPa) | | Torsion $G_{zx}$ (GPa) | | |
|---|---|---|---|---|---|
| Species | Ave. | SD | Ave. | SD | $t$ |
| Spruce | 1.04 | 0.38 | 1.07 | 0.14 | 0.181 (NS) |
| Ash | 1.05 | 0.13 | 1.00 | 0.11 | 0.719 (NS) |

Notes: Ave. and SD represent the average and standard deviation, respectively. The $t$ values were obtained by $t$-tests between the Iosipescu and the torsion test. NS = not significant.

TABLE 3. *Shear strengths obtained by the Iosipescu and torsion tests.*

| | Iosipescu $F_{zx}$ (MPa) | | Torsion $F_{zx}$ (MPa) | | |
|---|---|---|---|---|---|
| Species | Ave. | SD | Ave. | SD | $t$ |
| Spruce | 10.3 | 1.6 | 11.9 | 2.1 | 1.484 (NS) |
| Ash | 13.7 | 1.2 | 15.7 | 0.8 | 3.397 (1%) |

Notes: Ave., SD, $t$ and NS: same as in Table 1. 1% = significant at the 1% significance level.

sional moment/plastic shear strain relation was regressed into Eq. (7) again as in Fig. 5b, and the material parameters α and $m$ were determined. The yield shear stress $S_{zx}$ was calculated from Eq. (6) by substituting the reduced value of $M_y$, whereas the value of $K'$ was calculated by substituting α and $m$ into Eq. (11). With the mechanical properties, $G_{zx}$, $S_{zx}$, $K'$, and $m$, the shear stress/shear strain relations were obtained from Eq. (10), and were compared with those obtained from the Iosipescu tests. The shear strength, $F_{zx}$, was determined by substituting the strain $\gamma_{zx}$ at the occurrence of failure into Eq. (10b). Six specimens were used for each species.

## RESULTS AND DISCUSSION

Tables 1, 2, and 3 show the shear moduli, yield shear stresses, and shear strengths obtained from the Iosipescu and torsion tests. Comparing these properties by using $t$-tests reveals a difference between the shear strength of ash. The other properties are in good agreement with each other for both species.

The values of α and $m$ obtained by regressing the torsional moment/shear strain relation into the power function of Eq. (7) are shown in Table 4. With these values, the torsional moment/shear strain relation was transformed into the shear stress/shear strain relation. Table 5 shows the comparisons of the parameters determining the shear stress/shear strain relation. For both species, the values of $K$ and $n$ obtained from the Iosipescu tests were smaller than those of $K'$ and $m$ obtained from the torsion tests, respectively. This tendency indicates that the plastic strain region predicted by the torsion test is larger than that predicted by the Iosipescu test. By substituting the average values of the parameters, the shear stress/shear strain relations were calculated, and are shown in Fig. 5. For examining the coincidence, we conducted the $t$-tests between the stress-strain curves obtained from the Iosipescu test and the torsion test in the stress range where both stress/strain relations existed. In the $t$-tests, ten pairs of shear strains obtained by substituting the shear stresses into Eqs. (2) and (10) were used. The substituted stress values were 1.03 to 10.3 GPa at the intervals of 1.03 GPa (1/10 of $F_{zx}$ by the Iosipescu tests) for spruce and 1.37 to 13.7 GPa at the intervals of 1.37 GPa for ash. The statistical analyses showed that there was a difference in the 5% significance level for spruce, whereas the difference was not significant for ash.

As shown in Fig. 5, the failure occurred ear-

TABLE 2. *Yield shear stresses obtained from the Iosipescu shear tests and the torsion tests.*

| | Iosipescu $S_{zx}$ (MPa) | | Torsion $S_{zx}$ (MPa) | | |
|---|---|---|---|---|---|
| Species | Ave. | SD | Ave. | SD | $t$ |
| Spruce | 6.2 | 1.4 | 7.0 | 1.6 | 0.922 (NS) |
| Ash | 7.8 | 2.1 | 7.9 | 1.0 | 0.105 (NS) |

Note: Ave., SD, $t$ and NS: same as in Table 1. Yield stresses obtained from the torsion tests are calculated by using the 80% reduced value of yield torsional moment.

TABLE 4. *Parameters determining the torsional moment/shear strain relation,* α and m.

| | α (× 10 $^{-2}$) | | $m$ | |
|---|---|---|---|---|
| Species | Ave. | SD | Ave. | SD |
| Spruce | 1.39 | 0.48 | 2.36 | 0.11 |
| Ash | 0.98 | 0.11 | 2.53 | 0.64 |

TABLE 5. *Parameters determining the shear stress/shear strain relation obtained from the Iosipescu shear test data,* K *and* n, *and those obtained from the torsion test data,* K′ *and* m.

| | Iosipescu | | | | Torsion | | | |
|---|---|---|---|---|---|---|---|---|
| | $K$ ($\times 10^{-2}$) | | $n$ | | $K'$ ($\times 10^{-2}$) | | $m$ | |
| Species | Ave. | SD | Ave. | SD | Ave. | SD | Ave. | SD |
| Spruce | 1.60 | 0.40 | 1.42 | 0.13 | 2.60 | 0.91 | 2.36 | 0.11 |
| Ash | 1.63 | 0.51 | 1.91 | 0.62 | 2.10 | 0.69 | 2.53 | 0.64 |

lier in the Iosipescu test than in the torsion test for both species. In the Iosipescu test, there is a concern that the cracking initiated at the notch roots would have a serious influence on the stress uniformity, and that the distorted stress condition might accelerate a catastrophic failure. The experimental results proved the acceleration of failure in the Iosipescu test. Torsion is free from the concern because pure shear stress condition is assured until failure occurs in the specimen. Nevertheless, the procedure determining the stress/strain relation is complicated, and the several hypotheses introduced in the determination such as the reduction of yield stress might have a serious influence on the torsion testing results. In determining the shear stress/shear strain relation of wood properly, shear stress/shear strain data should be measured more frequently by these methods and the disadvantages existing in these methods should be reduced effectively.

## CONCLUSIONS

Using the specimens of spruce and ash, we tried to determine the shear stress/shear strain relation by Iosipescu and torsion tests, and obtained the following results:

(1) The shear moduli, shear yield stresses, and shear strengths obtained from both methods showed good agreement with each other, except for the shear strength of ash.
(2) As for spruce, the difference between the shear stress/shear strain relations obtained by the Iosipescu and torsion tests was significant at the 5% significance level, whereas that for ash was not significant.
(3) Although the Iosipescu test can derive the shear stress/shear strain relation directly, it has the drawback that the failure occurs earlier than with the torsion test. In contrast, the torsion test has the drawback that the procedure in obtaining the stress/strain relation is quite complicated. In determining the shear stress/shear strain relation of wood properly, these tests should be conducted more frequently.

## ACKNOWLEDGMENTS

This research was supported partly by a Grant-in-Aid for Scientific Research (No. 09460072) from the Ministry of Education, Science and Culture of Japan.

## REFERENCES

CHAKRABARTY, J. 1987. Theory of plasticity. McGraw-Hill, New York, NY. 9 pp.

CONANT, N. R., AND M. ODOM. 1995. An improved Iosipescu shear test fixture. J. Comp. Technol. Res. 17:50–55.

DUMAIL, J. F., K. OLOFSSON, AND L. SALMÉN. 2000. An analysis of rolling shear of spruce wood by the Iosipescu method. Holzforschung 54(4):420–426.

HILL, R. 1950. The mathematical theory of plasticity. Oxford Clarendon Press, London, UK, 84 pp.

JANOWIAK, J. J., AND R. F. PELLERIN. 1991. Iosipescu shear test apparatus applied to wood composites. Wood Fiber Sci. 23:410–418.

KUBOJIMA, Y., H. YOSHIHARA, H. OHSAKI, AND M. OHTA. 2000. Accuracy of shear properties of wood obtained by simplified Iosipescu shear test. J. Wood Sci. 46(4): 279–283.

LEKHNITSKII, S. G. 1963. Theory of an anisotropic elastic body. Holden-Day, San Francisco, CA. Pp. 197–205.

SEERAT-UN-NABI, A., AND B. DERBY. 1990. Iosipescu in-plane shear tests of SiC-Pyrex composites. J. Mater. Sci. Lett. 9:63–66.

YOSHIHARA, H., AND M. OHTA. 1993. Measurement of the shear moduli of wood by the torsion of a rectangular bar. Mokuzai Gakkaishi 39(9):993–997.

———, AND ———. 1995. Shear stress-shear strain relation of wood in the plastic region. Mokuzai Gakkaishi 41(6):529–536.

———, AND ———. 1997. Analysis of the shear stress/shear strain relations in wood obtained by the torsion tests. Mokuzai Gakkaishi 43(6):457–463.

———, AND ——— 1998. Formulation of the shear stress/shear strain relation of wood. Bull. Tokyo Univ. Forest 99:11–17.

———, H. OHSAKI, Y. KUBOJIMA, AND M. OHTA. 1999. Applicability of the Iosipescu shear test on the measurement of the shear properties of wood. J Wood Sci 45(1):24–29.