MODELING AND COMPARING VERTICAL DENSITY PROFILES[1]

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ABSTRACT

The vertical density profile of pressed wood panels is influenced by the manufacturing process and is important to panel end-users. Modeling the vertical density profile and making statistical comparisons among profiles resulting from different manufacturing treatments are critical to understanding and improving panel properties. Nonparametric regression analysis was used to model the vertical density profile of aspen (*Populus tremuloides*) oriented strandboard panels. A methodology is presented to compare vertical density profile curves. Twenty-seven laboratory panels were manufactured at 608 kg/m^3 incorporating three levels of furnish moisture content (4%, 8%, 12%) and three levels of press closure rate (20 s, 60 s, 100 s) in a replicated, experimental design.

The nonparametric regression technique called cubic splines was used to fit the data, R^2 ranged from 0.985 to 0.998. Detailed discussion is presented that describes the method and interpretation of the nonparametric regression analysis. Statistical comparison of vertical density profile curves among treatment levels revealed that the 4% furnish moisture content level was significantly different (P = 0.015) from the 8% and 12% levels; the 8% level was not significantly different ($P > 0.99$) from the 12% level. Vertical density profile curves for all press closure rate treatments were significantly different ($P < 0.003$).

Keywords: Vertical density profile, nonparametric regression, cubic spline, oriented strandboard, furnish moisture content, press closure rate.

INTRODUCTION

For over three decades researchers and industrial producers have described the formation of a density gradient through the thickness of flat-pressed wood panel products as a result of hot-pressing (Suchsland 1962; Winistorfer 1992). A density gradient through the panel thickness is typically reflected by the presence

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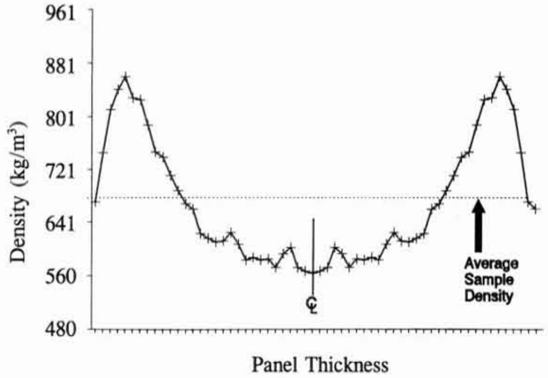

FIG. 1. A vertical density profile plot reflecting the typical high density face layers, low density core layers, and the nearly symmetrical change in density about the panel midpoint.

of high density face layers and low density core layers within the panel. This density gradient has been referred to by many names including, vertical density gradient, vertical density profile, density profile, and vertical density distribution. Vertical density profile (VDP) will be the nomenclature used throughout this paper.

When the density distribution through the panel thickness is plotted on an x–y axis, the resulting plot is frequently referred to as the "shape" of the VDP. Shape is a qualitative, descriptive term used to refer to the relationship between face layer density, core layer density, and panel thickness. The shape of the VDP could be described quantitatively by fitting a mathematical expression to the relationship of density and panel thickness, but there are no such reports in the literature.

The VDP will commonly be nearly symmetrical in shape when viewed about a midpoint that is the centerline representing total panel thickness (Fig. 1). However, variation in manufacturing processes and in product types may result in a variety of VDP shapes that are asymmetric.

The VDP has been of interest to researchers and industrial panel producers because of the influence of the VDP on panel properties. Panel surface quality, edge profiling characteristics, fastener performance, edge-banding performance, bending properties, and internal bond strength are all responsive to changes in the VDP. As well, the VDP is responsive to changes in manufacturing parameters such as furnish moisture content and press closure rate. Hence, it is common for producers to attempt to influence final board properties through manipulation of manufacturing parameters that influence the formation of the VDP. It is generally accepted within the research community and producing industry that high furnish moisture content combined with a fast press closure rate result in greater differences between face and core density than does a low furnish moisture content teemed with a slow press closure rate, although many variations in the shape of the final VDP are possible as a result of manipulating process variables.

While Suchsland (1962) established some of the early principles governing density formation and distribution within a flat-pressed mat of wood particles, recent works are more focused on the rheological properties of the wood mat during pressing (Harless et al. 1987; Kamke and Casey 1988; Bolton et al. 1989; Wolcott et al. 1990; Kamke and Wolcott 1991; McFarland 1992; Winistorfer 1992). Researchers are attempting to describe, model, and manipulate the VDP based on a sound understanding of the physicochemical processes within the mat during pressing. Producers continue to evaluate the VDP from an empirical position, i.e., measuring the VDP after manufacture to assess the influence of changes in process parameters on the formation of the VDP.

Our ability to measure and analyze the VDP of pressed wood panels has improved. The newer, automated, nondestructive gamma densitometry techniques now utilized produce higher resolution measurement with less error than previously employed gravimetric techniques (Laufenberg 1986; Winistorfer et al. 1986). A nondestructive acoustic emission method to measure the density profile has also been reported (LeMaster and Green 1992). Several commercial manufacturers now supply gamma source scanning devices for density measurement, and many panel producers utilize this equipment on a daily basis. Winistorfer and Moschler (1992, 1994) are currently engaged in research to measure the vertical density profile *in situ* during pressing. This research will result in real-time measure of how the VDP changes during the press cycle. The development and application of methods of analysis of VDP data have not kept pace, however, with the development of equipment to measure the VDP.

One technique for modeling curves is regression analysis. One of the objectives of regression analysis is to estimate a curve to describe the relationship between an explanatory variable X (e.g., thickness layer) and a response variable Y (e.g., density). The aim is to produce a reasonable approximation to some unknown function $y = m(x)$. The form of the regression curve may tell us, among other things, something about the location and size of extrema.

The often-used parametric approach to regression analysis is to assume that the function has some prespecified form, e.g., a line with an unknown slope and intercept. The selection of the form of the model is, to a large extent, arbitrary. The usual forms (linear, quadratic, etc.) are very restrictive and could easily lead to missing important features of the curve m. As an alternative, one could try to estimate the function nonparametrically without reference to a specific form. The term *nonparametric* refers to the flexible functional form of the regression curve (Hardle 1990, p. 5).

Given the complicated nature of the form of VDP curves, the parametric approach to modeling such curves seems quite implausible. The nonparametric approach to estimating a regression curve provides a powerful data-analytic technique for exploring the general relationship between two variables. As was mentioned, the main feature of nonparametric regression is to fit a curve "locally," i.e., without imposing a preconceived model. For this reason, nonparametric regression is also called *smoothing.*

Nonparametric regression can provide a great deal of credibility to manufacturers when providing VDP data to customers as an assurance of surface and internal density quality. These techniques may be invaluable in improving the understanding of VDP variation and may help improve panel performance by means of control and manipulation of the VDP.

METHODS

Panel manufacture and data set

The vertical density profile data set used for this analysis originated from a study conducted by McFarland (1992). He produced twenty-seven laboratory oriented strandboard (OSB) panels in a replicated, split-plot experimental design. Three replications of aspen (*Populus tremuloides*) OSB panels were manufactured at three initial levels of furnish moisture content (4%, 8%, 12%—as whole plot treatment) and three levels of press closure rate (20, 60, 100 s—split-plot treatment). Panels were pressed to 12.7 mm at 608 kg/m^3 in a computer-controlled hydraulic hot press. Press temperature was 204°C, with a 5.5-minute press cycle, including a 20 s degas period before culmination of the total press cycle. Time to stops changed according to the closure rate treatment assigned to the panel.

Each of the twenty-seven panels was scanned with a direct scanning gamma densitometry system (Winistorfer et al. 1986) to produce a three-dimensional portrayal of density variation in x, y, and z dimensions within the panel (McFarland 1992). McFarland's whole-panel sampling technique was established on a grid of eight sampling positions in the x dimension (panel width), eight sampling positions in the y dimension (panel length), and sixteen sampling positions in the z dimension (panel thickness), resulting in a total of 1,024 sampling positions of density for each panel. Each successive z-layer represented an incremental step of 0.03 inches through the panel thickness. The mean (N = 64) density value for each of the sixteen z-layers, with 3 replications, was used for this analysis. For elaborate details regarding sampling methodology, including prestudy sampling estimates, the reader is referred to McFarland (1992).

Nonparametric regression

Nonparametric regression or smoothing is a relatively new data-analytic technique; see Hardle (1990) and Altman (1992) for a good review of these techniques. In contrast with parametric regression analysis, nonparametric regression does not assume a priori knowledge of the functional form between the explanatory and response variables. One of the disadvantages of smoothing is that the analysis does not yield an explicit functional form; instead, the resulting smoothed curve can be expressed as a weighted average of the observed responses; see Silverman (1984) and Hardle (1990, p. 162).

There are several smoothing techniques. The more common techniques include kernel smoothing, k-nearest neighbor estimation, orthogonal series estimators, and spline smoothing (Hardle 1990, p. 24; Altman 1992). Smoothing is not a new statistical concept, as noted by Hardle (1990, p. 57); Whittaker (1923) called this mathematical smoothing process a "graduation" or "adjustment" of the observations.

A significant limitation for applying these smoothing techniques at present is the paucity of appropriate algorithms in commercial software. Cubic spline smoothing was used in this analysis and is available on SAS/INSIGHT® (OS/2 and Windows) and JMP® (Macintosh) software.

The cubic spline fit is a "local" cubic polynomial between two successive X-value segments spliced together such that the resulting curve is continuous and smooth at the splices (SAS Institute Inc. 1989). Cubic spline smoothing may also have an advantage relative to other nonparametric techniques in that it quantifies the competition between the aim to produce a good fit to the data and the aim to produce a curve without too much local variation. If a curve were completely unrestricted in functional form, it would pass through every observation and be too wiggly or jagged for a structure-oriented interpretation.

An important feature of cubic splines is the choice of the "roughness penalty." In spline smoothing, this involves the selection of the smoothing parameter λ ($\lambda > 0$). The need to avoid overfitting and to "trade" bias for variance to obtain a better fit is very evident in cubic spline smoothing (Hardle 1990, pp. 76–77). The larger the λ, the more penalty the model gets for being rough, resulting in a smoother curve. The smaller the λ, the less penalty is given to roughness, which results in a more jagged curve. As λ increases, the spline approaches a straight line fitted using least squares. On the other hand, if $\lambda = 0$, the result would be a "curve" that simply connects the points with straight lines, i.e., a very jagged curve. For a more detailed and theoretical discussion of spline smoothing and roughness penalty, refer to Hardle (1990, Chapter 5).

Modeling the vertical density profile

Because of their complicated functional form, cubic splines are ideal for describing VDPs. VDPs usually have sharp peaks at the faces, which makes it difficult to find an explicit functional form for use in parametric regression. Nonparametric regression does not rely on any prespecified functional form required by parametric regression analysis (e.g., linear, quadratic, etc.). In fitting cubic splines, the problem is the selection of the smoothing parameter λ in order to strike a good balance between bias and variance. In the analyses, the estimates of λ were determined using the algorithm available in SAS/INSIGHT®. This algorithm minimizes the generalized cross-validation mean-squared error (see SAS Institute Inc. 1993). For a more detailed and theoretical discussion of the generalized cross-validation mean-squared error, refer to Hardle (1990, pp. 61–62) and Wahba (1979, 1985, 1990).

The data set consisted of three repeated observations at every z-layer (thickness layer). In the case of repeated observations in the X-variable, the spline smoothing algorithm pools the corresponding Y-values by averaging them

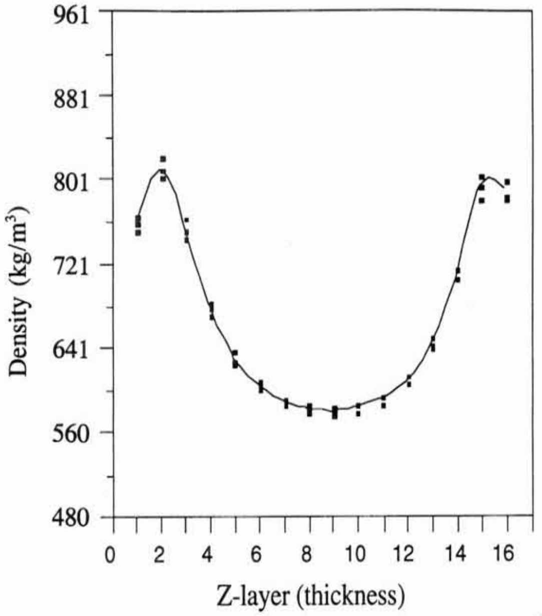

FIG. 2. Spline estimate for mean density (kg/m^3) across panel Z-layers for 4% furnish moisture content treatment.

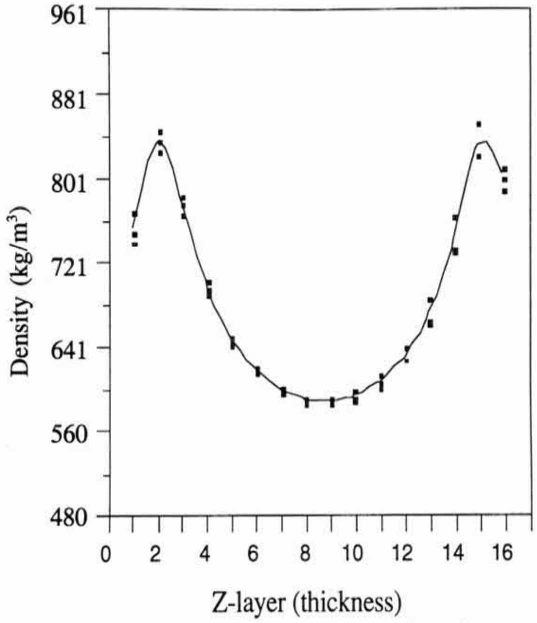

FIG. 3. Spline estimate for mean density (kg/m^3) across panel Z-layers for 8% furnish moisture content treatment.

(Hardle 1990, p. 85). This algorithm is currently available in SAS/INSIGHT® and also in JMP®.

Comparing the vertical density profiles

A frequent objective in experimental work is the comparison of regression curves. Graphical overlay of curves in itself is a useful qualitative tool, but is not reliable for making manufacturing decisions that influence properties and performance.

King et al. (1991) proposed a method for testing the hypothesis that two nonparametric curves are equal. The test method is based on a function of the scaled difference between two nonparametric curve estimates, where each estimate is a smoother. The test requires that each smoother have the same λ. The value of λ in this analysis was the average of the individual λ values, which were obtained by minimizing the generalized cross-validation mean-squared error. Average values of λ were obtained for each response (moisture content and press closure rate).

Because of the complicated form of the test statistic, the P-values for comparing two curves were obtained through Monte Carlo simulation as suggested by King et al. (1991). Given that the test was performed three times, the α risk (probability of type I error) was controlled using Bonferroni's inequality, see Milton and Arnold (1990). The adjusted P-values were computed by multiplying each pairwise P-value by three; see Wright (1992).

RESULTS AND DISCUSSION

Modeling the vertical density profiles

Vertical density profiles were modeled using spline smoothing for each of the three treatment levels of furnish moisture content and press closure rate. These estimates were made from McFarland's (1992) data set that contained three replications of the mean density for each of the sixteen z-layers. The VDP for each treatment is therefore represented by 48 values of density.

Figures 2 through 4 show the cubic splines fitted to the three levels of furnish moisture content, with the measured mean density for

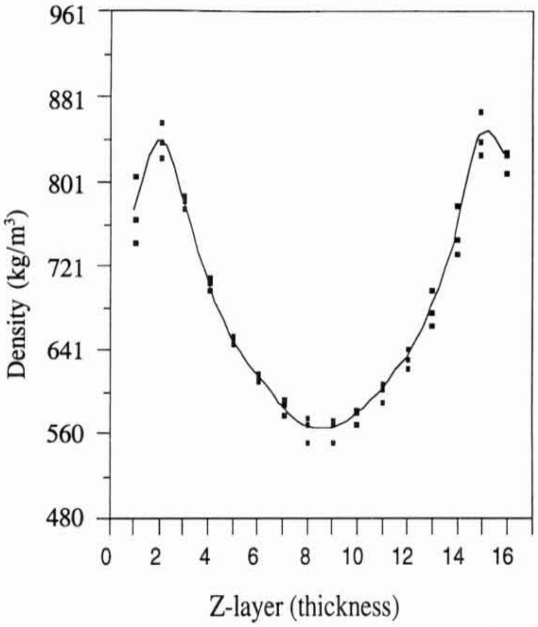

FIG. 4. Spline estimate for mean density (kg/m^3) across panel Z-layers for 12% furnish moisture content treatment.

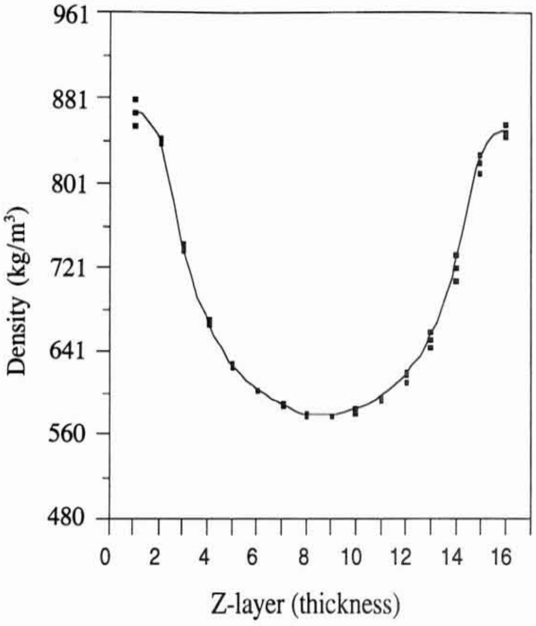

FIG. 5. Spline estimate for mean density (kg/m^3) across panel Z-layers for 20-second press closure rate treatment.

the three replications from McFarland's (1992) study. Figures 5 through 7 show the cubic splines fitted and the actual data for the three levels of press closure rate.

Statistics for the splines shown in Figs. 2 through 4 are given in Table 1. In the table, λ denotes the smoothing parameter (see note in Table 1), R^2 measures the proportion of total variation explained by the cubic spline model. The generalized cross-validation mean-squared error ($MSE_{(GCV)}$) represents the bias and variance of the cubic spline estimate.

All R^2s ranged from 0.985 to 0.998, indicating that the cubic spline models explained almost all of the variation of the response.

Comparing the vertical density profiles

Perhaps the most useful aspect of this work is to present a methodology for comparing VDP curves. McFarland (1992) presented a comparison of the mean density for each z-layer in his analysis of these data, but did not examine the data for potential differences of entire VDP "shapes." To our knowledge, no methodology has been suggested in the literature for such a comparison.

Figures 8 and 9 show the VDP cubic spline estimates overlaid for all levels of furnish

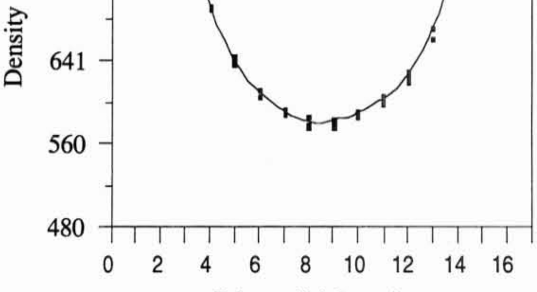

FIG. 6. Spline estimate for mean density (kg/m^3) across panel Z-layers for 60-second press closure rate treatment.

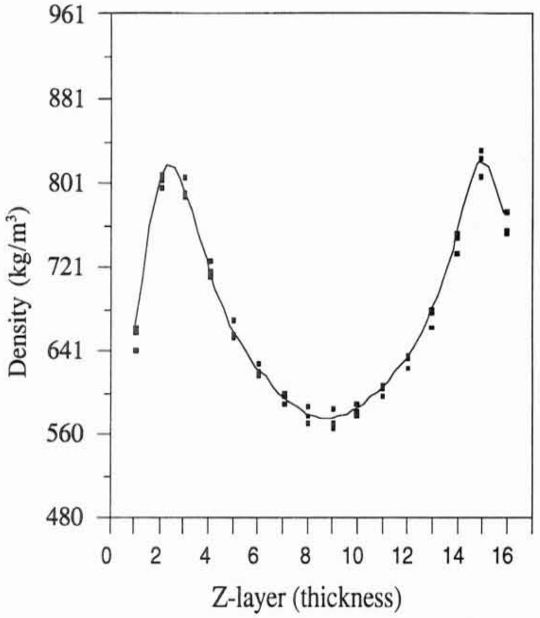

FIG. 7. Spline estimate for mean density (kg/m³) across panel *Z*-layers for 100-second press closure rate treatment.

TABLE 1. *Statistics for nonparametric regression estimates.*

Treatment*	λ**	R^2	$MSE_{(GCV)}$***
4% MC	0.0008	0.996	0.23
8% MC	0.0011	0.991	0.54
12% MC	0.0029	0.985	1.11
PCR 20	0.0008	0.998	0.18
PCR 60	0.0003	0.998	0.16
PCR 100	0.0019	0.993	0.43

* MC denotes furnish moisture content (%) before pressing; PCR denotes press closure rate as time (seconds) to position or stops.
** Denotes smoothing parameter lambda ($\lambda > 0$).
*** Generalized cross-validation mean squared error.

moisture content and press closure rate, respectively. Table 2 shows the stated hypothesis for differences among all treatment levels, resulting test statistics, *P*-values, and average λ. The VDP trends that resulted from all treatment levels were similar—high density faces and a much lower density core. Visual observation of the resulting VDPs would suggest that the profiles are not different from each other, and that normal or expected profile trends resulted from the pressing conditions used. A horizontal line drawn through the average density of each profile would reveal that half the panel density is above the line and half the panel density is below the line, and that basically the shapes of the profiles are not appreciably different with respect to the average density or from each other.

Much has been written about the influence of furnish moisture content and press closure rate effects on the VDP; that is not to be reviewed here. Many researchers and plant personnel have a general experiential knowledge about the supposed differences in the VDP due to changes in these parameters. Results from this study suggest that the VDP curve for the 4% furnish moisture content treatment level is significantly different ($P = 0.015$) from the curve for either the 8% or 12% treatment level. As can be seen in Table 2, even though the test statistics have different values, the *P*-values obtained were the same. This is because the two values fell in the same percentile of the empirical distribution function generated by the Monte Carlo simulation based on 1,000 replications. The 8% treatment level was not significantly different from the 12% level ($P >$ 0.99).

Inspection of the VDP curves for the 8% and 12% treatment levels reveals a gap between the

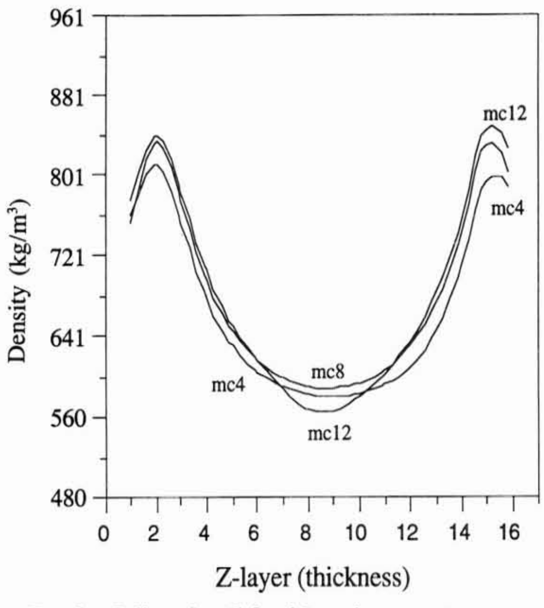

FIG. 8. Splines for all furnish moisture content treatment levels.

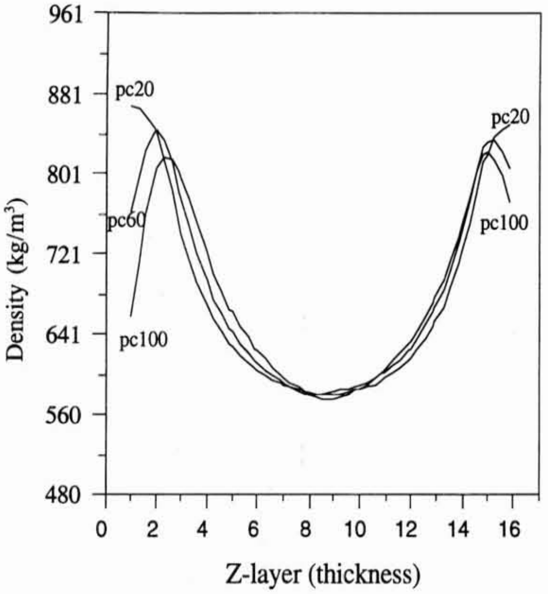

FIG. 9. Splines for all press closure rate treatment levels.

curves near the 7 through 10 z-layers; the curves are very similar for all other z-layers. This visual assessment may lead to speculation that the two curves are different. However, since the test statistic is derived from the predicted and observed densities for all z-layers, there was not enough disparity in the response densities between the 8% and 12% curves to reject the hypothesis that the two curves are equal. On the other hand, except for layers 7 through 10, the 4% treatment curve is consistently under the 8% and 12% treatment curves.

TABLE 2. *Values of the test statistic, T_n and corresponding P-values for cubic spline comparisons.*

Null hypothesis tested	T_n*	*P*-value**	Average λ
Moisture content:			
H_o: $f_{4\%}(x) = f_{8\%}(x)$	3.2	0.015***	0.0016
H_o: $f_{4\%}(x) = f_{12\%}(x)$	3.0	0.015***	0.0016
H_o: $f_{8\%}(x) = f_{12\%}(x)$	0.56	>0.999	0.0016
Press closure rate:			
H_o: $f_{20}(x) = f_{60}(x)$	4.5	<0.003	0.00098
H_o: $f_{20}(x) = f_{100}(x)$	7.5	<0.003	0.00098
H_o: $f_{60}(x) = f_{100}(x)$	6.69	<0.003	0.00098

* Test statistics as defined by King et al. (1991).

** Obtained through Monte Carlo simulation as suggested in King et al. (1991) and adjusted using Bonferroni's inequality.

*** Refer to the third paragraph of the section Comparing the Vertical Density Profiles for an explanation of why the *P*-values are the same.

The curves for the three press closure rate treatments were significantly different ($P < 0.003$) from each other. The difference among these curves is mainly due to the density response in the tails of the vertical density distribution, i.e., panel surface density is responsive to press closure rate treatment.

CONCLUSIONS

In the current manufacturing environment of continuous improvement and statistical process control, measuring and evaluating process and product performance are critical. The nonparametric regression technique and methodology for statistical comparison of vertical density profile curves described in this study may be a useful analytical tool for measuring and comparing product performance in the wood panel industry. We suggest that its practical utility for that purpose be evaluated. Measuring and comparing VDPs resulting from different process treatments can be an important tool to improve the resolution at which we evaluate processes and product performance.

This nonparametric regression analysis technique does not require a given functional form of the model that is required of parametric techniques. A realistic limitation for the application of this analysis technique by practitioners is its lack of availability in commercial software. However, this statistical algorithm is now available in various packages. We are aware of its availability on JMP® (Macintosh), SAS OS/2® and SAS Windows®.

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