COMPARISON OF THREE EQUATIONS FOR PREDICTING STRESS WAVE VELOCITY AS A FUNCTION OF GRAIN ANGLE[1]

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ABSTRACT

Assessment of a nondestructive test system for detecting defects in the gluelines of edge-glued hardwood panels required development of a mathematical relationship for predicting stress wave velocity as a function of grain angle. This relationship was necessary to understand better how stress waves propagated around gaps or flaws in a glueline. In addition, the relationship was needed to assess the influence of specimen geometry upon the effectiveness of the stress wave technique.

Equations were generated by a statistical regression analysis software package and compared to Hankinson's equation. Equations were based upon measured velocity of stress waves traveling at angles between 0 and 90 degrees to the grain at 15 degree intervals in birch, black cherry, red oak, yellow-poplar, and western white pine boards. Regression analyses indicated that the best correlations were found with second order hyperbolic and parabolic equations. The two equations were compared to Hankinson's equation and to each other by using Absolute Average Error (AAE) for each equation for each species and for all species combined at each of the grain angles for which data were collected. Hankinson's equation produces the least AAE of the three equations although the hyperbolic and parabolic equations must also be considered reasonable predictors of stress wave velocity at most angles to the grain.

Keywords: Nondestructive testing, Hankinson's equation, stress wave velocity, hardwood properties, softwoods.

INTRODUCTION

The application of measuring stress wave velocity for estimating mechanical properties and locating defects in wood and wood products is well documented (Gerhards 1982; Ross and Pellerin 1988; Kaiserlik 1978; McDonald 1978; Szymani and McDonald 1981). A West Virginia University research program (Mallory 1988) involves evaluating a stress wave technique for detecting flaws in the gluelines of edge-glued hardwood panels. It is hoped that this research could further

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the acceptance of edge-glued panels as a raw material substitute for rough lumber in the cabinet and furniture industries.

The concept of marketing "standard-size blanks," a type of edge-glued hardwood panel, was developed to produce high value products from small diameter, low quality trees (Araman et al. 1982). The panels are made by edge-gluing narrow, clear cuttings into panels of a predetermined size that can be readily used by furniture and cabinet manufacturers.

For this idea to be commercially feasible, furniture and cabinet manufacturers must be willing to substitute edge-glued panels for rough lumber in their manufacturing process. Thus, they must be guaranteed that the panels they purchase will be of equivalent or better quality than panels produced in their own rough mill. A nondestructive test for evaluating the integrity of the gluelines in the panels would provide manufacturers with a quality control tool that will help them gain acceptance of standard-size panels. The intent of the larger research program of which this study is a part is to assess a quality control technique that may be employed in the manufacture of standard-size hardwood panels.

The purpose of the research described in this paper was to develop a means for mathematically estimating the influence of grain angle upon measured stress wave velocity in four hardwood species: red oak (*Quercus* spp.), birch (*Betula* spp.), black cherry (*Prunus serotina* Ehrh.), and yellow-poplar (*Liriodendron tulipifera* L.), and one softwood species, western white pine (*Pinus monticola* Dougl.). Stress wave velocities were measured in samples of each species at various angles to the grain. Through linear regression analysis of the data, two equations with the highest correlation coefficients were selected for further examination and evaluation as estimators of transit time in edge-glued panels. Also, a relationship in the form of Hankinson's equation was derived empirically from the experimental data and compared to the equations generated by regression analysis.

The need for this study became evident when trying to understand behavior of stress waves propagating across panels containing gaps or skips in a glueline. Test samples containing a single, centered glueline were tested by measuring transit times of impact-induced stress waves propagating between a "sending" and "receiving" transducer located on opposite edges of the panel. In panels with solid gluelines, propagation of the initial stress wave reaching the receiving transducer occurs in a straight line between transducers as shown by path "A" in Fig. 1. In this example, the stress wave velocity is the distance between transducers divided by the measured transit time.

When gaps or skips were encountered in the glueline, transit times were observed to increase. At first, it was not known whether the first signal recorded by the receiving transducer was from waves diverted around gaps in the glueline as illustrated by wave path "B" in Fig. 1 or whether they reflected from the ends of the panel, as indicated by path "C." In both cases the distance of the path taken from the sending to the receiving transducer would increase. However, the anisotropic properties of wood cause stress wave velocity to increase as the angle between the grain and the stress wave path is decreased. The velocity of stress waves propagating perpendicular to the grain was reported as approximately 30 to 50% of the velocities parallel to the grain (Elvery and Nwokoye 1970; Gerhards 1980, 1982; Jung 1979; Lee 1958; McDonald 1978). This means that the stress waves reflecting from the ends of the panel would travel at greater velocities than

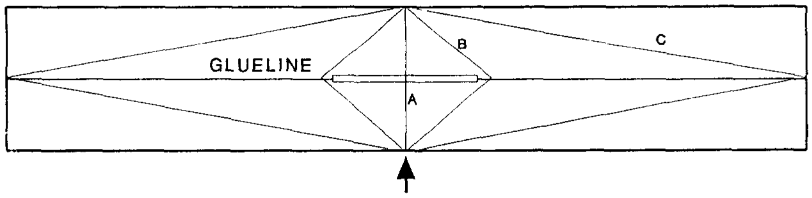

FIG. 1. Possible principal pathways for stress waves traveling from edge to edge in an edge-glued panel containing a gap in the glueline: (A) directly across the gap, (B) around the ends of the gap, and (C) reflected from the panel ends.

stress waves propagating around the gap in the glueline or directly across the panel as illustrated by path "A" of Fig. 1.

This raised an additional concern that measured transit times for panels with peculiar dimensions (short, wide panels) might actually be from end-reflected waves as opposed to waves traveling directly across the panel or around a gap in a glueline.

To understand better how the panel geometry and the anisotropic properties of wood affect the sensitivity of the stress wave technique, a mathematical relationship for estimating stress wave velocities at various angles to the grain was needed. Such a relationship would allow a comparison of observed results with an estimated transit time based upon the known properties of the panel.

PROCEDURE

Stress waves were generated and their propagation velocities determined using the system illustrated in Fig. 2 (Mallory 1988). The system consists of an impact device for generating stress waves, two piezoelectric transducers and charge amplifiers for monitoring propagation of the stress wave, and a digital oscilloscope to display and determine the transit time output for the system. In addition, output from the oscilloscope could be read directly into a personal computer.

Piezoelectric transducers are small, rugged, hermetically sealed units that work on the piezoelectric effect of quartz. Quartz, when stressed along its primary axis, produces an electric charge that is proportional to the applied force. In combination with a charge amplifier, such transducers can have a measurement axis sensitivity of 100 mV/lb. These transducers are designed to measure the short duration dynamic tensile and compressive stresses in machinery and other mechanical constructions.

The two transducers are mounted directly on the panel edges so that the transit time of the initial stress wave traveling between the two transducers can be determined. They are attached using screw or adhesive secured mounting bases so that stress excitation to be measured is transmitted normal to the base of the transducer. The transducers are by design extremely insensitive (0.01%) to transverse excitations. In this study, the mounting bases were secured to the wood with an isocyanate adhesive.

The sending or impact-side transducer begins timing the stress wave transit time upon impact, and the receiving or opposite-side transducer detects the arrival of the initial stress wave at the end of the stress wave's path of travel. Figures 3 and 4 show examples of the measured transducer signals at both the sending and

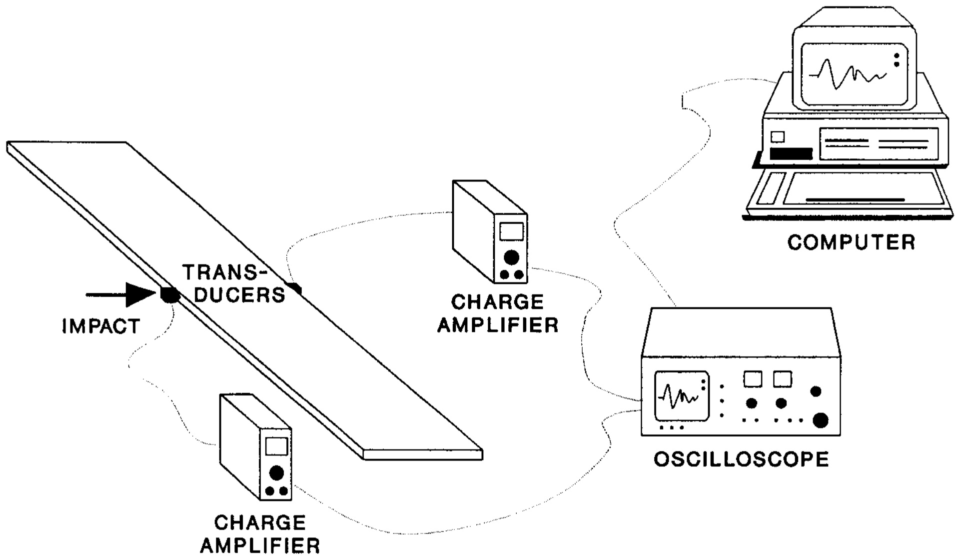

FIG. 2. Diagram of the stress wave nondestructive test system set up for testing in the direction perpendicular to the grain.

receiving transducers. Since the sending transducer acts as a trigger for the oscilloscope display, a flat line appears at the beginning of the receiving waveform until the time that the receiving transducer detects the arrival of the first wave. Since both transducers were similarly calibrated and essentially set to trigger at

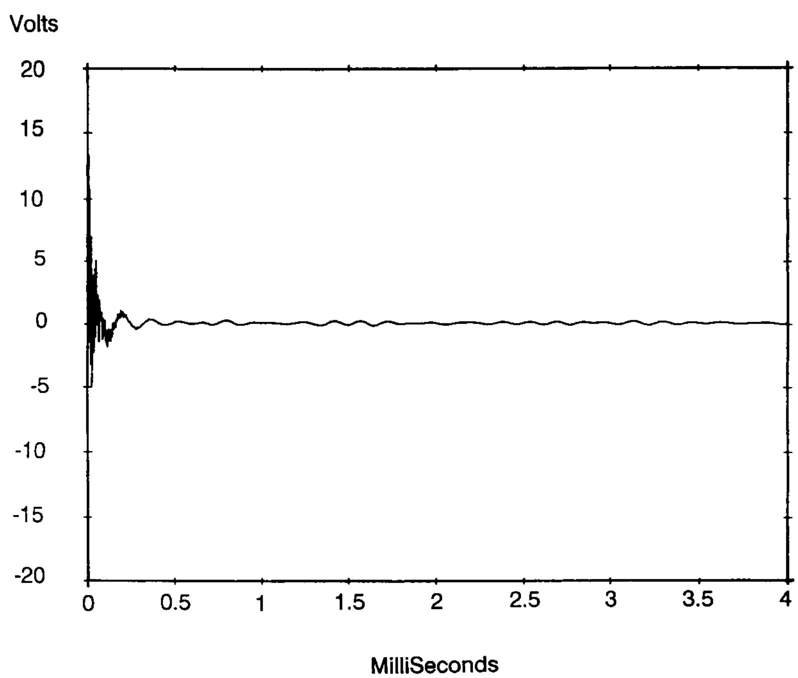

FIG. 3. Typical stress wave history detected by the sending or impact-side transducer for a wave propagating longitudinally (Mallory 1988).

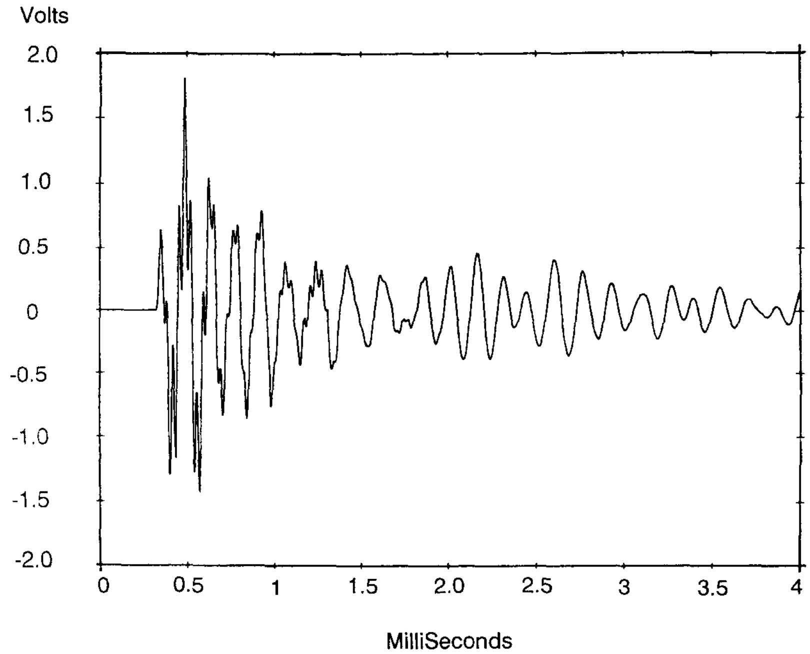

FIG. 4. Typical stress wave history detected by the receiving or opposite-side transducer illustrating a transit time of 622 microseconds for a wave propagating longitudinally (Mallory 1988).

the same stress wave level, this flat-line time is a measure of the transit time for the generated stress wave to propagate across the panel.

Tests were conducted on solid, clear ¾- by 10- by 60-inch boards of red oak, birch, black cherry, yellow-poplar, and western white pine. Four boards of each species were tested. The transducers were arranged from 0 (parallel to the grain) to 90 degrees (perpendicular to the grain) at intervals of 15 degrees in the pattern illustrated in Fig. 5. At least three tests were conducted for each transducer position. Thus, a total of at least 24 tests were conducted per angle per species. Statistical analysis of the data indicated that the sample size, although small, was sufficient.

All samples were flat-sawn, which means that the measured transverse stress wave propagation occurred in the longitudinal/tangential plane. This configuration was chosen since standard-size blanks are typically constructed of flat-sawn ma-

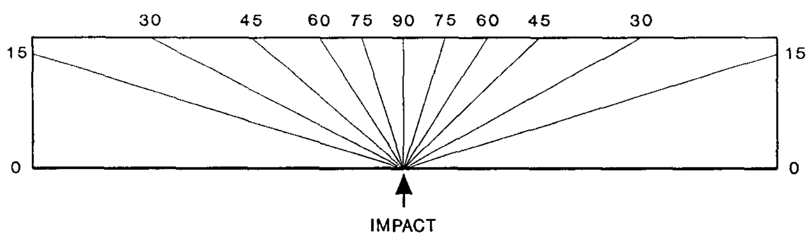

FIG. 5. Load cell positions for measurement of stress wave velocity at angles to the grain of 0 to 90 degrees.

terial (Araman et al. 1982). Different results would be expected from tests in a longitudinal/radial plane since differences in propagation velocities have been observed between the radial and tangential directions (Lee 1958; McDonald 1978).

Tests were conducted at a constant temperature and relative humidity so that the equilibrium moisture content of the samples was approximately 5%. Following tests, the specific gravity and moisture content of each specimen were determined using ASTM standard procedures. These tests confirmed that average moisture contents of all samples ranged between 4.5 and 5.5%. Average specific gravities of the samples were 0.63 for red oak, 0.65 for birch, 0.55 for black cherry, 0.52 for yellow-poplar, and 0.38 for western white pine.

Following testing, the relationship of stress wave velocity to grain angle was analyzed using a statistical regression analysis package that determined best fit for twenty-five polynomial equations. The results of the analysis indicated that a second order parabolic and second order hyperbolic equation provided the best correlation of the data.

It is generally assumed that since stress wave velocity is a function of the dynamic modulus of elasticity (MOE), the relationship between velocity and grain angle would conform to Hankinson's equation as does MOE. Therefore, a relationship in the form of Hankinson's equation was determined empirically. The two polynomial equations were then compared with the derivation of Hankinson's equation by calculating Absolute Average Error (AAE) of the equations.

RESULTS AND DISCUSSION

Stress wave velocity

Results of the stress wave velocity tests are given in Table 1. For each species, the stress wave velocity perpendicular to the grain was approximately 30% of the longitudinal velocity. When moisture content differences are taken into account, these results are relatively consistent with those reported in the literature for the same species (Jung 1979; McDonald 1978; Wen and Mohsenin 1970).

Results of regression analyses

Regression analyses indicate that second order parabolic and hyperbolic equations provide the best fit of the data with R-squares ranging between 0.97 and 0.99 as reported in Table 2. The parabolic equation is of the form:

$$V = A + B\theta + C\theta^2 \tag{1}$$

and the hyperbolic takes the form:

$$V = A + \frac{B}{\theta} + \frac{C}{\theta^2} \tag{2}$$

where V is the stress wave velocity, θ is the angle to the grain in degrees, and A, B, and C are the constants of the regression. The constants are given in Table 2.

Neither regression equation completely fits the data as illustrated in Fig. 6. The hyperbolic equation, because of its very nature, deviates greatly from the data in the 15 to 0 degree range by predicting exceedingly high values as the angle is reduced. Because the constant "C" is negative, as the angle goes from 1 to 0 degrees, the values start to approach negative infinity.

TABLE 1. *The mean and standard deviations of stress wave velocity measured at 0, 15, 30, 45, 60, 75, and 90 degrees to the grain of five species.*

Species	Stress wave velocity (meters per second) Grain angle (degrees from longitudinal 0	15	30	45	60	75	90
Birch							
Mean	5,210	4,280	2,810	2,120	1,710	1,530	1,440
SD	260	230	210	160	140	60	20
Yellow-Poplar							
Mean	5,460	4,320	2.760	2,070	1,730	1,570	1,500
SD	150	300	160	100	60	80	100
Black Cherry							
Mean	5,150	4,030	2,770	2,110	1,750	1,600	1,530
SD	140	320	250	150	100	60	40
Red Oak							
Mean	5,040	4,090	2,800	2,190	1,860	1,690	1,650
SD	320	310	220	110	60	40	50
Western White Pine							
Mean	5,450	4,270	2,720	1,880	1,580	1,410	1,400
SD	250	180	150	100	60	40	40
All Species							
Mean	5,270	4,200	2,770	2,070	1,730	1,560	1,510
SD	280	280	190	160	120	110	100

The parabolic equation predicts its lowest value in the 70 to 75 degree range and then, the values increase as the angle approaches 90 degrees. This does not reflect the data where the lowest stress wave velocity occurs in the direction perpendicular to the grain.

TABLE 2. *Results of regression analysis for the second order hyperbolic and the second order parabolic equations for predicting stress wave velocity from grain angle in five species and for all five species combined.*

Species	Hyperbolic equation (meters/second)	R^2	Parabolic equation (meters/second)	R^2
Birch	$900 + \frac{52,000}{\Theta} - \frac{5,160}{\Theta^2}$	0.98	$5,290 - 95\Theta + 0.59\Theta^2$	0.98
Yellow-poplar	$920 + \frac{52,000}{\Theta} - \frac{5,150}{\Theta^2}$	0.99	$5,520 - 106\Theta + 0.69\Theta^2$	0.98
Black Cherry	$1,040 + \frac{46,300}{\Theta} - \frac{4,590}{\Theta^2}$	0.98	$5,180 - 94\Theta + 0.60\Theta^2$	0.98
Red Oak	$1,150 + \frac{45,200}{\Theta} - \frac{4,490}{\Theta^2}$	0.98	$5,100 - 89\Theta + 0.56\Theta^2$	0.97
Western White Pine	$750 + \frac{54,000}{\Theta} - \frac{5,350}{\Theta^2}$	0.99	$5,520 - 110\Theta + 0.73\Theta^2$	0.98
All Species	$950 + \frac{49,900}{\Theta} - \frac{4,950}{\Theta^2}$	0.98	$5,320 - 99\Theta + 0.64\Theta^2$	0.98

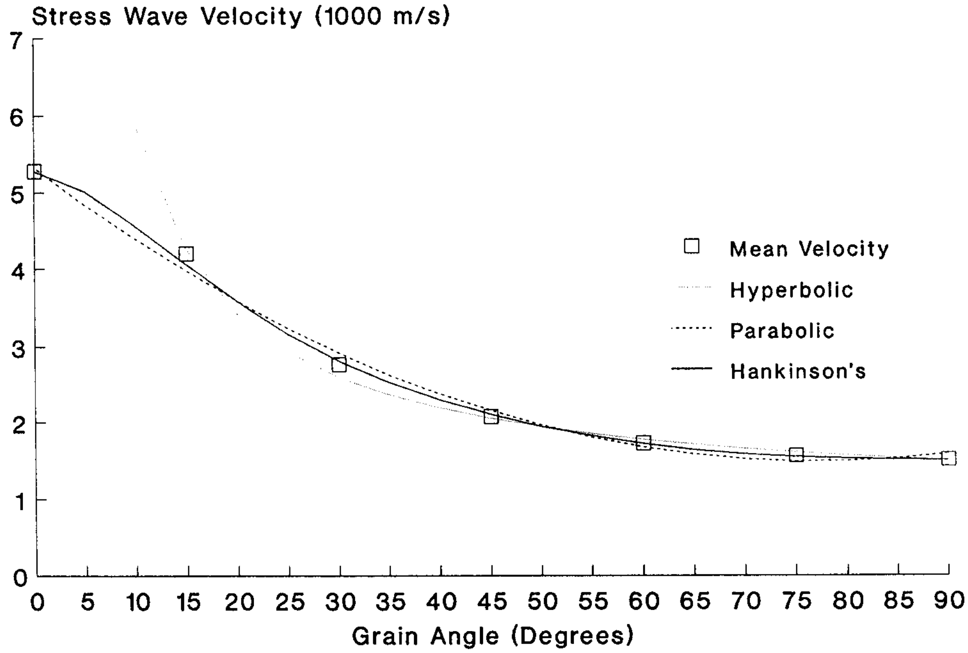

FIG. 6. Mean values of stress wave velocities for all five species tested as a function of grain angle compared with the predicted values derived from (A) the second order hyperbolic equation, (B) the second order parabolic equation, and (C) Hankinson's equation.

Hankinson's equation

The relationship between many mechanical and physical properties of wood and grain angle may be expressed as a form of Hankinson's equation (Forest Products Laboratory 1987):

$$N = \frac{PQ}{P \sin^n(\theta) + Q \cos^n(\theta)} \quad (3)$$

where: N is the property at an angle θ from the grain direction, P is the property parallel to the grain, Q is the property perpendicular to the grain, and n is an empirically determined exponent. Thus, the usefulness of Hankinson's equation depends upon accurately determining the optimal exponent,"n," for the material property in question—in the present case, stress wave velocity. The *Wood Handbook* (Forest Products Laboratory 1987) gives a range of values for the exponents of 1.5 to 2.5 for various properties, with the variation depending upon the property and the Q/P ratio.

The data for stress wave velocity for all species were combined to determine the optimal exponent for Hankinson's equation. The data for angles between 15 and 75 degrees, inclusive, were used since values for 0 and 90 degrees are independent variables in Hankinson's equation. Using the means (M) of the combined data for each angle, five equations were developed of the form:

$$P \sin^n(\theta) + Q \cos^n(\theta) = \frac{PQ}{M} \quad (4)$$

TABLE 3. *The Average Absolute Error (AAE) in meters per second for predicting the stress wave velocity from 15 to 75 degrees to the grain by species and equation.*

	Equation		
Species	Hyperbolic	Parabolic	Hankinson's
Birch	86	116	116
Yellow-Poplar	48	144	66
Black Cherry	86	90	24
Red Oak	68	112	44
Western White Pine	88	140	82
All species	64	116	50
Average Absolute Error	73	120	64

The exponent was increased by set increments through its range of values. The value of the left side of the equation was compared to the value of the right side of the equation. It became obvious that no single exponent results in all five equations having the left side equal the right. It was decided that the best exponent was one that resulted in two equations having the right side greater than the left, two equations having the left side greater than the right, and the remaining equation having both sides equal. The exponent meeting this criterion was 1.69.

Comparison of polynomial equations and Hankinson's

Each of the three equations derived in this study has practical limitations. Hankinson's equation requires prior knowledge of stress wave velocities for 0 and 90 degrees in order to predict velocities at any other angle. The hyperbolic equation appears to accurately predict velocity only at angles greater than 15 degrees (Fig. 6). The parabolic equation appears to accurately predict velocity only at angles less than 75 degrees.

Because Hankinson's equation is not statistically derived, its relative accuracy cannot be determined by R-squared values. Therefore, Average Absolute Error (AAE) was used to determine the best equation for predicting stress wave velocities for the intermediate grain angles.

In the AAE method, the mean is calculated for the absolute values of the errors (the absolute value of the difference between the predicted value from the equation and the mean of the experimental data for a given grain angle). Thus "perfect fit" is indicated by AAE = 0. With five angles for five species plus the combined data, there were 30 errors averaged for each equation. Table 3 contains the AAE for each equation by species. The equation with the lowest AAE was Hankinson's (64 meters per second), although it had the highest single absolute error (310 meters per second at an angle of 15 degrees for birch). The hyperbolic equation had the next lowest AAE (73 meters per second), and the parabolic equation had the highest AAE (120 meters per second).

The hyperbolic equation had the lowest AAE for birch and yellow-poplar. Hankinson's equation had the lowest AAE for the remaining three species and for all species combined. The hyperbolic was also close to Hankinson's equation for western white pine.

Table 4 contains the AAE for each equation by grain angle. The greatest errors for Hankinson's occurred at 15 degrees (153 meters per second). The greatest

TABLE 4. *The Average Absolute Error (AAE) in meters per second for predicting the stress wave velocity by grain angle and equation for all species combined. The figures in parentheses represent AAE as a percentage of the mean stress wave velocity.*

	Equation		
Grain angle (degrees)	Hyperbolic	Parabolic	Hankinson's
15	57 (1.4)	218 (5.2)	153 (3.6)
30	162 (5.8)	152 (5.5)	67 (2.4)
45	38 (1.8)	78 (3.8)	55 (2.6)
60	55 (3.2)	60 (3.5)	17 (1.0)
75	55 (3.5)	90 (5.8)	27 (1.7)

error for the hyperbolic was observed at 30 degrees (162 meters per second). For the parabolic equation, large errors were observed at 15 and 30 degrees (218 and 152 meters per second respectively).

AAE analysis was conducted on Hankinson's equation using three other exponents, n = 1.50, 1.60, and 1.80, in order to assess the sensitivity of the accuracy of the prediction when the exponent is changed slightly. Figure 7 illustrates the calculated AAE at each value of "n." In addition, the AAE for both the hyperbolic and parabolic equations are included for comparison. With an exponent of n = 1.50, the AAE was 179 meters per second; with n = 1.60, the AAE was 103 meters per second; and with an exponent of n = 1.80, the AAE was 86 meters per second. The AAE for the derived exponential term of 1.69 was 64 meters per second. These results indicate that if the exponent were changed by even less than 0.10 from its optimum value, the hyperbolic equation was a better predictor since its AAE was 73 meters per second.

SUMMARY AND CONCLUSIONS

Stress wave velocity in the tangential (perpendicular to grain) direction is approximately 30% of the longitudinal velocity. The mathematical relationship between grain angle and velocity may be expressed in the form of a second order hyperbolic equation, a second order parabolic equation, or Hankinson's equation at most angles to the grain with reasonable accuracy.

The second order parabolic and the second order hyperbolic equations each possess a characteristic limitation. The parabolic equation produces an increase in predicted velocity from 75 to 90 degrees to the grain. This does not confirm to the data where the least velocity occurs at 90 degrees. The hyperbolic equation, due to its very nature, cannot be used to predict the stress wave velocity at 0 degrees although it appears to be better predictor of velocity than the parabolic equation at angles greater than zero.

Hankinson's equation provides the best fit of the data generated in this study. However, it requires prior knowledge of stress wave velocities both parallel and perpendicular to the grain in order to estimate velocity at some other angle. Therefore, it may not always be appropriate for explaining deviations from expected transit times in edge-glued hardwood panels containing defects as proposed in this study.

Table 4 illustrates that the AAE for any of the equations at any angle to the grain never exceeds 6% of the mean of the data for all species. For the hyperbolic equation, the AAE ranges from 1.4% of the mean at 15 degrees to 5.8% at 30

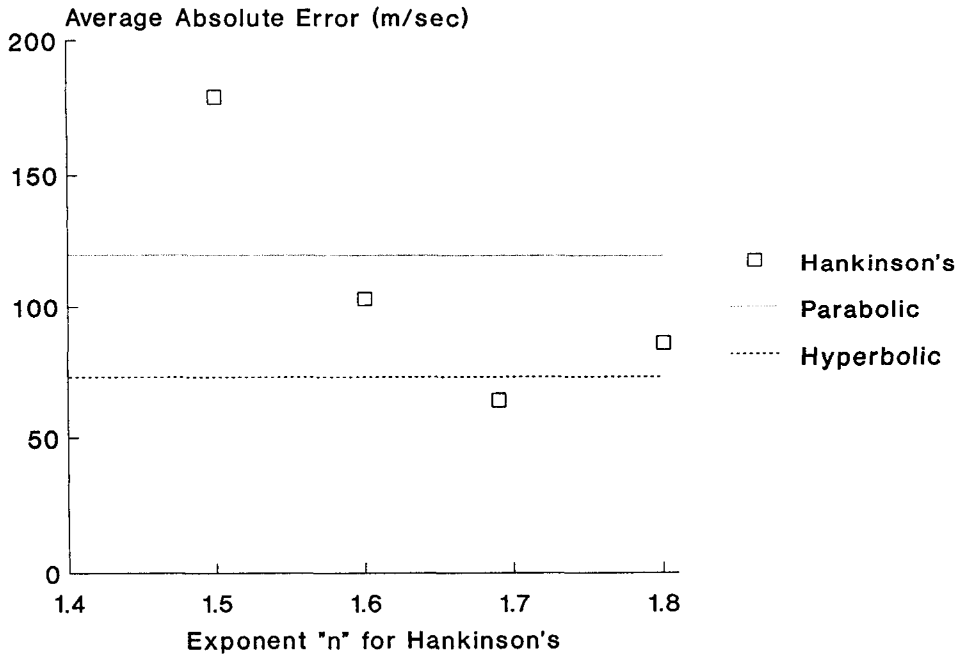

FIG. 7. Average Absolute Error of Hankinson's equation with the exponential term "n" = 1.50, 1.60, 1.69, and 1.80 and of the second order hyperbolic and parabolic equations.

degrees. Ranges for the parabolic are 3.5% at 60 degrees to 5.8% at 75 degrees. For Hankinson's equation, the range is from 1.0% at 60 degrees to 3.6% at 15 degrees. Thus in discussing the limitations of the equations, it must be kept in mind that the errors are all probably within a reasonable margin of the experimental data.

The equations derived in this paper may be used to better explain transit times of a stress wave traveling across an edge-glued panel containing a gap of known length in its glueline as illustrated in Fig. 1. By measuring the distance of travel around the gap (path "B" in Fig. 1) and calculating the angle to the grain of that path, the transit time can be estimated using either the hyperbolic or parabolic equation. Comparing to test data gives the researcher a better idea of the paths of propagation in panels of varying geometries and with gaps of various dimensions in the glueline. The effectiveness of the stress wave technique for detecting flaws in the glueline is the subject of a subsequent paper.

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