

# THE INFLUENCE OF CUTTING-BILL REQUIREMENTS ON LUMBER YIELD USING A FRACTIONAL-FACTORIAL DESIGN PART I. LINEARITY AND LEAST SQUARES

*Urs Buehlmann\**

Associate Professor

*D. Earl Kline†*

Professor

Department of Wood Science and Forest Products  
Virginia Tech  
Blacksburg, VA 24061-0503

*Janice K. Wiedenbeck†*

Project Leader

Northern Research Station  
USDA Forest Service  
Forestry Sciences Laboratory  
241 Mercer Springs Road  
Princeton, WV 24740

*R. Noble, Jr.*

Assistant Professor  
Department of Math/Statistics  
Miami University of Ohio  
Oxford, OH 45056

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**Abstract.** The importance of lumber yield on the financial success of secondary solid wood products manufacturers has been known for quite some time. Various efforts have been undertaken to improve yield, such as inclusion of character marks (defects) in parts, “cookie-cutting” of boards, improved optimization algorithms, or improved cut-up technologies. For a variety of reasons, the relationship between cutting-bill requirements and lumber yield has attracted limited attention. This is Part I of a 2-part examination of this relationship.

The standardized and simplified Buehlmann cutting bill and the Forest Service’s Romi-Rip lumber cut-up simulator were used in this study. An orthogonal,  $2^{20-11}$  fractional-factorial design of resolution V was used to determine the influence of different part sizes on lumber yield. All 20 part sizes contained in the cutting bill and 113 of a total of 190 unique secondary interactions were found to be significant variables in explaining the variability in observed yield. Parameter estimates for the part sizes and the secondary interactions were used to specify the average yield contribution of each variable. Parts 445 mm long and 64 mm wide were found to have the most positive influence on yield. Parts smaller than 445 by 64 mm (such as, for example 254 by 64 mm) had a less pronounced positive yield effect because their quantity requirement is relatively small in an average cutting bill. Thus, the quantity required is obtained quickly during the cut-up process. Parts with size 1842 by 108 mm, on the other hand, had the most negative influence on high yield. However, as further analysis showed, not only the individual parts required by a cutting bill, but also their interaction determines yield. In general, it was found that by adding a sufficiently large number of smaller parts to a cutting bill that required large parts, high levels of yield can be achieved.

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\* Corresponding author: [buehlmann@gmail.com](mailto:buehlmann@gmail.com)

† SWST member

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## INTRODUCTION

Rough mills have limited cutting and sorting capabilities. Therefore, they can cut only a limited number of different part sizes at any given time. The ability to select this limited number of different part sizes to be cut concurrently out of the larger pool of needed sizes, while achieving high yield is of crucial importance to rough mills. Thus, obtaining the highest possible yield from a given set of lumber is the central challenge for every rough mill operation (Buehlmann et al 1998; Buehlmann et al 2003; Wiedenbeck and Thomas 1995; Wengert and Lamb 1994). Planning is important because "*Lumber yield is largely foreordained in the planning process and only secondarily influenced by operations on the cutting room floor* (Moser 1996, as cited in BC Wood Specialties Group, p. 22)." Knowing that a 1–2% lumber yield increase can save \$150,000 to \$300,000 annually in a medium-sized rough mill (Kline et al 1998), the importance of understanding what parameters contribute to high yield is of great importance. Wengert and Lamb (1994) emphasize the close interrelationship of yield to the profitability of a rough mill operation. They estimate that a 1% increase in yield in a furniture rough mill can save up to 2% of manufacturing costs (Wengert and Lamb 1994). Other authors (Buehlmann et al 2003, 1998; Hamilton et al 2002; Wiedenbeck and Thomas 1995; Manalan et al 1980) also emphasize the considerable importance of achieving high yield in a rough mill.

Research into the cutting-bill–lumber yield relationship has been done predominantly in conjunction with work on yield nomograms (Thomas 1965; Englerth and Schumann 1969; Dunmire 1971; Hallock 1980; and Manalan et al 1980). More recently, Buehlmann et al (2003) published a study on the effect of cutting-bill requirements on lumber yield in a rip-first rough mill. Given part quantity distributions as used in industry, this simulation-based study showed the

importance on lumber yield of short parts (shorter than 940 mm), while part widths impact on yield was less. The study also revealed the need for a more refined approach in defining part sizes used in the test cutting bill. In response, Buehlmann et al (2008a, 2008b) addressed the need for a more refined, standardized, and simplified cutting bill.

This study used the standardized and simplified cutting bill (Buehlmann et al 2008a and 2008b) to research the influence of cutting-bill part size and part quantity requirements on lumber yield. It was hypothesized that a better understanding of these relationships would lead to cutting bills that provide higher yields. Two publications describe the methodology and results of this research. Paper I focuses on describing the methodology and the importance of part sizes and part quantities using least squares regression. Paper II shows a more generalized description of the influence on lumber yield of part sizes and part quantities using correlation coefficients. Furthermore, Paper II also discusses the influence of the number of different part sizes cut concurrently on yield.

## METHODS

For this research, methods were specified in respect to: a) the lumber cut-up simulation to derive valid yield results (Thomas and Buehlmann 2002) and b) the statistical analysis tools used to analyze the results. This study used rip-first rough mill lumber cut simulation (Thomas 1995a, 1995b) and digitized representations of red oak lumber (Gatchell et al 1998). Its findings and conclusions therefore do not necessarily apply to other types of rough mills (eg crosscut-first) or lumber species.

### Lumber Cut-up Simulation

To execute the necessary tests, the simulation parameters, the composition of the digital lum-

ber data set, and the cutting-bill requirements had to be defined.

**Rip-first rough mill yield simulation.** ROMI-RIP 1.0 (Thomas 1995a, 1995b) was used to simulate the cut-up of lumber in a rip-first rough mill. ROMI-RIP has been shown to be a valid representation of lumber cut-up in industrial plants (Thomas and Buehlmann 2002). The settings used for this study were as follows: 1) all-blades movable arbor; 2) dynamic exponential cutting-bill part prioritization (Thomas 1996b); 3) smart and unlimited salvage operation (Thomas 1996a; Anderson et al 1992) 4) no random width and no random length parts; 5) no fingerjointed or glued-up parts; 6) continuous updating of part counts; 7) end-and-side trim set at 6 mm on both sides; and 8) only clear-2-side (C2F) parts (Thomas 1995a, 1995b). Preliminary tests indicated that the necessary number of simulation replicates is three (Buehlmann 1998). Unless noted otherwise, yields are given in absolute terms, and include primary and smart salvage yield (Thomas 1995a, 1995b).

The simulation did not substitute parts whose quantity requirement had been met. Thus, the number of part sizes to be cut in a given simulation run would decline toward the end, when some part size quantity requirements were met. However, since the simulation used dynamic exponential cutting-bill part prioritization (Thomas 1996b), this decline in number of different part sizes to be cut did occur only toward the very end of fulfillment of all parts requirements.

**Lumber.** Digital lumber representations created by Gatchell et al (1998) that are contained in “The 1998 kiln-dried red oak data bank” were used for this research. No. 1 Common lumber collections with board size and quality distributions according to Wiedenbeck et al (2003) were composed using the “custom datafile creation” feature of ROMI-RIP (Thomas 1995a, 1995b). The cutting-bill quantity requirements were set such that a minimum of 150 boards were necessary to be cut to achieve the required quantities (Buehlmann et al 1998).

**Cutting bills.** Cutting-bill requirements vary greatly depending on the company or the industry sector. To obtain a better tool for conducting research involving cutting bills, Buehlmann et al (2008a and 2008b) created a standardized and simplified cutting bill (often referred to as the “Buehlmann” cutting bill), which was used in this research. Table 1 shows the part-size and -quantity requirements of the cutting bill used. The part quantity was set such that a minimum of 150 boards was required for all tests (Buehlmann et al 1998). Since each of the parts in this cutting bill represents a part group midpoint representing a range of potential part lengths and widths (Buehlmann et al 2008a), the term part group or part is used interchangeably to refer to a specific part in this cutting bill.

### Statistical Analysis

To obtain data to create a model indicating the yield contribution of different part sizes, a fractional-factorial design had to be designed and executed to obtain the data to derive the least squares estimates for the parameters as a proxy

Table 1. Part sizes and part quantity requirements of the Buehlmann cutting bill.

Part no.	Part name	Quantity	Length	Width
		–units–	–mm–	
1	L <sub>1</sub> W <sub>1</sub>	341	254	38
2	L <sub>2</sub> W <sub>1</sub>	742	445	38
3	L <sub>3</sub> W <sub>1</sub>	1083	699	38
4	L <sub>4</sub> W <sub>1</sub>	608	1207	38
5	L <sub>5</sub> W <sub>1</sub>	258	1842	38
6	L <sub>1</sub> W <sub>2</sub>	379	254	57
7	L <sub>2</sub> W <sub>2</sub>	746	445	57
8	L <sub>3</sub> W <sub>2</sub>	1200	699	57
9	L <sub>4</sub> W <sub>2</sub>	654	1207	57
10	L <sub>5</sub> W <sub>2</sub>	246	1842	57
11	L <sub>1</sub> W <sub>3</sub>	114	254	89
12	L <sub>2</sub> W <sub>3</sub>	254	445	89
13	L <sub>3</sub> W <sub>3</sub>	365	699	89
14	L <sub>4</sub> W <sub>3</sub>	221	1207	89
15	L <sub>5</sub> W <sub>3</sub>	142	1842	89
16	L <sub>1</sub> W <sub>4</sub>	123	254	108
17	L <sub>2</sub> W <sub>4</sub>	248	445	108
18	L <sub>3</sub> W <sub>4</sub>	395	699	108
19	L <sub>4</sub> W <sub>4</sub>	213	1207	108
20	L <sub>5</sub> W <sub>4</sub>	100	1842	108

for individual part contributions to yield. Since a 2-level fractional-factorial design, which assumes linearity between the minimum and maximum quantity setting, was the preferred design of experiments to minimize the necessary simulation runs, linearity had to be verified.

**Fractional-factorial design.** Assuming linearity of the part quantity–yield relationship, the study’s fractional-factorial design was a 1/2048 replicate of a 2-level 20-factor fractional-factorial design with resolution V, ie a  $2^{20-11}$  fractional-factorial design (Box et al 1978). The complete factorial design for this study would consist of  $2^{20} = 1,048,576$  experiments. In a resolution V fractional-factorial design, main effects are free of secondary and tertiary degree interactions, and secondary interactions are free of other secondary interactions. Hence, according to the “sparsity of effects principle” (Montgomery 2005; Box et al 1978), both the main effects and the secondary interactions can be reliably estimated. Details about the  $2^{20-11}$  fractional-factorial design used can be found in Buehlmann (1998). Analysis of variance (ANOVA,  $\alpha = 0.05$ ) of the data was performed to establish the importance of individual part groups on yield. Since both main effects and secondary interactions are free of the same order effects, the importance of all the 20 main effects and the 190 unique secondary interactions can be established.

**Validation of the within-part group linearity assumption.** The within-part group linearity assumption describes the relationship between part quantity required by a specific part size (also called a part group as explained in Buehlmann et al 2008a and 2008b) and yield. When part quantity of one part size changes, yield also changes in most cases. However, it is unknown if the change in yield is a linear function of the part quantity. Figure 1 displays two hypothetical part quantity–yield relationships. The dashed line represents a linear, whereas the solid line represents a nonlinear relationship. If the part quantity–yield relationship is found to be approximately linear, a 2-factor factorial design will suffice to capture the effect of a part quantity

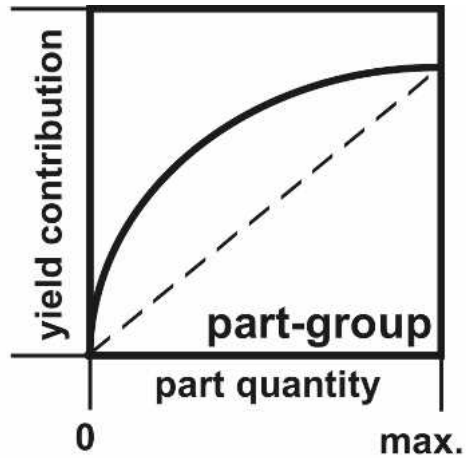


Figure 1. Example of a nonlinear and a linear part quantity–yield relationship.

change on yield over the entire part quantity range, ie from 0- to maximum-part quantity. If this relationship is found to be non-linear, more than 2 factors will be needed, since information about the curvature between 0- and maximum-part quantity will have to be obtained. Increasing the numbers of factors, however, will require an increased number of tests to be performed under the fractional-factorial design (512–6561 from a 2- to 3-factor fractional-factorial design). The linearity assumption only applies for the within-part group quantity–yield relationship, but is not required between part groups.

To test for the assumed within-part group linearity, each of the part groups was tested as follows: 1) Set all part groups at maximum quantity; 2) Run ROMI RIP simulation (3 replicates); 3) For  $i = 1, j = 1$ , set part quantity in part group ( $L_iW_j$ ) at 75% of maximum, with all other part groups remaining at maximum quantity; 4) Repeat step 2; 5) Repeat steps 3 and 4 but set part quantity for the part group under investigation to 50, then to 25 and, subsequently, to 0%; 6) Repeat steps 2–5 for all lengths ( $i = 1-5$ ), and all widths ( $j = 1-4$ ). Overall, 5 tests with 3 replicates of each were necessary for each part group. The design of experiments for part group  $L_1W_1$  is displayed in Table 2.

Table 2. Experiments to research whether the within-part group linearity assumption holds for part group  $L_1W_1$ , eg only the part quantities in  $L_1W_1$  are adjusted.

Part group	Length (mm)	Width (mm)	Test 1 quantity 100%	Test 2 quantity 75%	Test 3 quantity 50%	Test 4 quantity 25%	Test 5 quantity 0%
$L_1W_1$	254	38	341	256	171	85	0
$L_2W_1$	445	38	742	742	742	742	742
$L_3W_1$	699	38	1083	1083	1083	1083	1083
*	*	*	*	*	*	*	*
*	*	*	*	*	*	*	*
$L_3W_4$	699	108	395	395	395	395	395
$L_4W_4$	1207	108	213	213	213	213	213
$L_5W_4$	1842	108	100	100	100	100	100

A purely linear relationship would produce yield results that lie on a straight line between the yield obtained for maximum quantity and yield obtained for 0 quantity for a particular part group. Nonlinearity is measured as the distance between the linear line and the actual yield-point obtained (see Fig 1). Paired t-tests were employed to detect if there is a significant deviation ( $\alpha = 0.05$ ) from linearity. The standard deviation of all the tests conducted for the determination of linearity was used as the population standard deviation.

There may be some part groups that behave nonlinearly under the tests described above, but the error they introduce may be rather small, therefore, the following thresholds to conclude linearity were defined: 1) no more than 20% of all points tested are allowed to be nonlinear at the 95% level of significance; and 2) no single parts group nonlinear deviation shall exceed 1% absolute yield.

**Least squares parameter estimate for each part group.** Simple linear least squares parameter estimates can be derived as proxy for individual parts group contribution to yield if within-part group linearity is found to be true and the data derived using the fractional-factorial design are found to show that at least one of the effects is different from the others (ANOVA,  $\alpha = 0.05$ ). For such least-squares models to be valid, the factor levels have to be known constants, the observed responses must be random variables, and the random error terms have to be independently, identically, and normally-distributed with mean 0 and common variance  $\sigma^2$  (Ott 1993).

## RESULTS

Before the resolution V 2-factor fractional-factorial design could be executed, the within-part group linearity assumption had to be tested. Only after verifying that a 2-level fractional-factorial design could derive the data for a linear least squares model, could the number of factors be set.

### Validation of the Within-part Group Linearity Assumption

One hundred tests, each with three replicates, were conducted to obtain the data necessary to test the within-part group linearity assumption. Table 3 shows the results obtained. Test 1 (column 4, representing the yield with maximum part quantity specified in a part group) and test 5 (column 8, representing the yield with 0 part quantity specified in a part group) were used to set the starting and ending point of the linear yield-line for each part group. This line was then compared with the deviation of the yield data obtained for test numbers 2, 3, and 4. Using paired t-tests, this deviation was tested for significance ( $\alpha = 0.05$ ) of the obtained yield point and the linear line calculated previously.

As can be seen in Table 3, the largest nonlinearity observed was 0.62% for part group  $L_2W_2$  when requiring 50% of the maximum quantity. In fact, part group  $L_2W_2$  was the only part group that resulted in significant nonlinearity ( $\alpha = 0.05$ ) for all three points tested (ie part quantities at 75, 50, and 25%). Part group  $L_2W_4$  had two

Table 3. Results of the experiments testing the within-part group linearity assumption.

Part group	Length (mm)	Width (mm)	Yield difference (% absolute)				Yield
			Yield	Test 1 quantity 100%	Test 2 quantity 75%	Test 3 quantity 50%	
L <sub>1</sub> W <sub>1</sub>	254	38	70.19	0.19*	0.06	0.01	69.55
L <sub>2</sub> W <sub>1</sub>	445	38	70.19	0.15	0.11	0.06	68.89
L <sub>3</sub> W <sub>1</sub>	699	38	70.19	0.12	0.20	0.35**	69.48
L <sub>4</sub> W <sub>1</sub>	1207	38	70.19	0.02	0.13	0.08	70.28
L <sub>5</sub> W <sub>1</sub>	1842	38	70.19	0.12	0.18	0.08	70.39
L <sub>1</sub> W <sub>2</sub>	254	57	70.19	0.00	0.01	0.02	69.25
L <sub>2</sub> W <sub>2</sub>	445	57	70.19	0.33**	0.62**	0.38**	68.55
L <sub>3</sub> W <sub>2</sub>	699	57	70.19	0.04	0.01	0.11	70.36
L <sub>4</sub> W <sub>2</sub>	1207	57	70.19	0.02	0.04	0.04	70.25
L <sub>5</sub> W <sub>2</sub>	1842	57	70.19	0.09	0.21*	0.19	70.33
L <sub>1</sub> W <sub>3</sub>	254	89	70.19	0.03	0.05	0.06	69.85
L <sub>2</sub> W <sub>3</sub>	445	89	70.19	0.13	0.05	0.16	69.52
L <sub>3</sub> W <sub>3</sub>	699	89	70.19	0.11	0.11	0.04	70.05
L <sub>4</sub> W <sub>3</sub>	1207	89	70.19	0.11	0.18*	0.05	70.22
L <sub>5</sub> W <sub>3</sub>	1842	89	70.19	0.09	0.06	0.17	70.12
L <sub>1</sub> W <sub>4</sub>	254	108	70.19	0.05	0.12	0.03	69.68
L <sub>2</sub> W <sub>4</sub>	445	108	70.19	0.25*	0.34**	0.01	69.14
L <sub>3</sub> W <sub>4</sub>	699	108	70.19	0.08	0.03	0.18*	70.32
L <sub>4</sub> W <sub>4</sub>	1207	108	70.19	0.13	0.09	0.04	70.33
L <sub>5</sub> W <sub>4</sub>	1842	108	70.19	0.09	0.10	0.06	70.23

notation: \* = significant at 95% level  
 \*\* = significant at 99% level

observations that were significant ( $\alpha = 0.05$ ), however, its maximum deviation from linearity was about one-half of that found for part group L<sub>2</sub>W<sub>2</sub>, -0.34%. Overall, however, only 10 out of 60 observations were found to be significant ( $\alpha = 0.05$ ). Since the results of the tests were below the threshold set forth to conclude nonlinearity, a two-level resolution V fractional-factorial design could be employed.

### Fractional-factorial Design

The resolution V two-level fractional-factorial design required the execution of 512 tests, each with 3 replicates. Details of the design can be found in Buehlmann (1998), Appendix D. The maximum yield response difference between minimum and maximum yield observed from these 512 tests was 22.94% yield (70.81 vs 47.87%). The average yield found for these tests was 65.09% yield, with a standard deviation of 3.59%. The standard deviation between replicates for the cuttings that resulted in low yield was higher than the one observed for the cuttings

that resulted in high yield. The standard deviation between replicates for the 10 lowest yielding cutting bills was found to be, on average, 0.48% compared with an average standard deviation for the 10 cutting bills resulting in highest yield of 0.29%. However, this heteroscedasticity was still below levels that would require transformation (Ott 1993), given that an equal sample size was maintained for all tests.

Analysis of variance performed on the 20 main effects and 190 secondary interactions returned an F-Value of 123.5 ( $p > 0.0001$ ), showing that at least one of the effects is different from the others. All 20 main effects were found to be significantly different ( $\alpha = 0.05$ ) from each other (19 at  $p < 0.01$  and 1 L<sub>5</sub>W<sub>3</sub> at  $p < 0.05$ ). Also, 113 of the 190 secondary interactions were found to be significant ( $\alpha = 0.05$ ). The coefficient of determination, R<sup>2</sup>, which indicates how much of the variability of the data can be explained by the variables tested, was found to be 0.95.

### Contribution of Individual Part Groups on Yield

Since the main effects and 113 of the secondary interactions were significantly different from each other, parameter estimates could be calculated that account for the influence of individual part groups on yield. These estimates are an indicator of the average yield contribution of a specific part group to yield, for the 512 tests performed. However, to better understand their contribution to yield, some explanation is necessary as to how they were derived. When the least squares estimation procedure (Proc GLM in SAS Institute (1996)) was run, the amount reflecting 0-part quantity in a given part group was encoded as  $-1$ , and maximum part quantity was encoded  $+1$ . Hence, 50% part quantity was encoded as 0. The yield contribution of a part group as measured by the parameter estimate therefore is multiplied by a negative value (between 0 and  $-1$ ) when 0 or less than 50% of the maximum part quantities are required, 0 when 50% quantity is required, and positive (between 0 and  $+1$ ) when more than 50% of the maximum part quantity is required. Figure 2 displays the

yield slopes of 2 part groups, namely part groups  $L_2W_2$  and  $L_5W_4$  to illustrate the concept explained above.

The intercept (ie the average yield of all 512 cutting bills tested) was found to be 65.09%, while the parameter estimates (ie the slopes) of part groups  $L_2W_2$  and  $L_5W_4$  were found to be 1.60 and  $-0.32$ , respectively. Thus, when part group  $L_2W_2$  requires no parts to be cut, (part quantity is 0) yield will decrease by 1.60%. If  $L_2W_2$  requires 50% of the maximum part quantity to be cut, this part group will have no impact on yield. However, when  $L_2W_2$  asks for maximum part quantity, this part group will increase yield by 1.60%. Thus, the total possible contribution to yield of a particular part group between 0 and maximum part quantity is twice the value of the parameter estimate. Hence, on average for the 512 tests performed, the part group  $L_2W_2$  total contribution to yield is 3.20% ( $2 \times 1.60\%$ ) and the part group  $L_5W_4$  contribution to yield is  $-0.64\%$  ( $2 \times -0.32\%$ ). However, this observation holds only when all other 19 part groups ask for 50% of maximum quantity. Only then are all secondary interactions zero (and thus do not in-

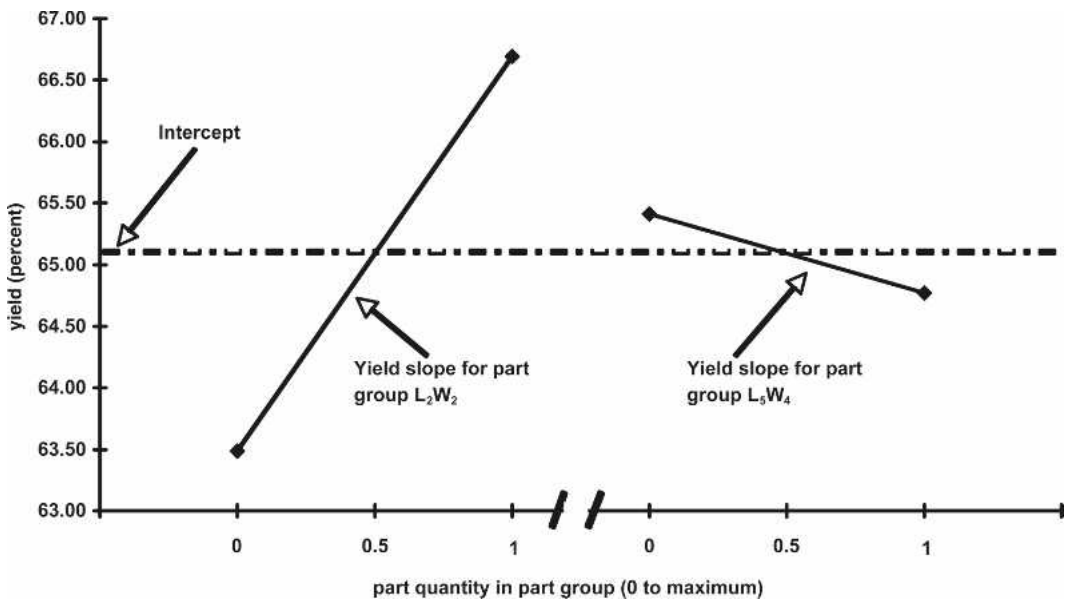


Figure 2. Intercept and yield slopes of part groups  $L_2W_2$  and  $L_5W_4$ .

fluence the yield from the part group observed). In the case in which 1 or more of the other 19 part groups ask for a part quantity other than 50%, the secondary interaction terms (and other higher interaction terms that could not be quantified but must be of small magnitude) alter the influence of part groups  $L_2W_2$  and  $L_5W_4$  on yield (positively or negatively). Basically the higher interaction terms describe how the various part sizes interact to compete for lumber resources by appropriately altering the part group slopes. Table 4 reveals the parameter estimates (slopes) for all 20 part groups and the average for each length and width group.

### DISCUSSION

Marginal yield contributions of each part group under base conditions (eg all part quantities at 50% of maximum quantity requirements) are shown in Table 4. According to these parameter estimates, length is more influential than width on yield. This can be concluded from the observation that the parameter estimates vary more over length (ie columns) than over width (ie rows). Width group  $W_1$ , ranging 25–51 mm, is the group that contributes most positively to yield followed by width groups 2, 3, and 4. This is consistent with generally accepted knowledge. The greater the width of the required parts, the more difficult it is to find a clear area within the board from which these parts can be cut. Also, when a board is cut into wider strips, the probability of having defects in these strips increases, thus reducing yield. However, as Table 4 shows, the difference in positive contribution to yield between width groups 1 and 2 is only 0.04. Thus, adding parts to a cutting bill with either widths between 25 and 51 mm (group  $W_1$ ) or 51

and 76 mm (group  $W_2$ ) has nearly the same effect on yield, on average for the 512 cutting bills researched. Adding parts wider than 76 mm, however, has a less favorable impact on yield.

As to length groups, Table 4 shows that group  $L_2$ , ranging 381–508 mm has the most positive effect on yield (average yield slope estimate 1.13, Table 4). Thus, it is not the shortest length group that has the most favorable influence on yield, but the longer parts belonging to group  $L_2$ . This observation is consistent with findings by Buehlmann et al (2003) that, given a cutting bill that requires a finite, restricted number of parts of various sizes, the shorter parts do not contribute the most to yield. Given the ability to rapidly attain short parts from group  $L_1$ , the required quantities are obtained after few boards are processed. Thus this does not leave short parts (from group  $L_1$ ) that could be cut from the remaining boards later in the production cycle. Given the restricted need for short parts in  $L_1$ , yield therefore declines. Since more parts are demanded or required for group  $L_2$  according to the research of Araman et al (1982; also see Buehlmann 2003 et al and Buehlmann 2008a et al), group  $L_2$  is more favorable for achieving high yield than group  $L_1$ . Group  $L_5$ , on the other hand, is a negative contributor to yield on average of the 512 cutting bills tested. This contradicts the often-heard rule-of-thumb that long lengths do not influence yield. Even though the average parameter estimate for group  $L_5$  is not much below 0 (Table 4), it is by far the most negative contributor to yield compared with all other length groups. The negative influence of long length (ie  $L_5$ ) is further amplified when the part required is also wide. Therefore, part group  $L_5W_4$  was found to be the one group that influenced yield most negatively.

The above inferences about the contribution of part groups give an indication of the marginal yield influences under a base case when all part groups require 50% part quantity, as explained earlier. The more of a departure from this case, secondary interactions, ie the mutual influence of two part groups together on yield, need to be considered to understand the contribution of in-

Table 4. *Parameter estimates and average parameter estimate of each length or width group.*

Width/length	$L_1$	$L_2$	$L_3$	$L_4$	$L_5$	Average
$W_1$	0.48	1.24	1.20	0.58	0.14	0.73
$W_2$	0.83	1.60	0.95	0.21	-0.16	0.69
$W_3$	0.30	0.78	0.64	0.24	-0.04	0.38
$W_4$	0.46	0.89	0.32	-0.06	-0.32	0.26
Average	0.52	1.13	0.78	0.24	-0.09	



dividual part groups. Assessing the effects of secondary interactions is not only complicated by the fact that there are 190 secondary interactions that have to be taken into account, but also because their effect on yield changes depending on the part quantities associated with the two main effects (ie part groups) involved. Interaction-slopes are no longer unidimensional lines as is the case for the main effects, but they are in fact twisted planes. The interpretation of the secondary interactions for part groups and their influence on yield is therefore more complicated than it is for main effects. Also, as was to be expected according to the scarcity-of-effects principle (Montgomery 2005; Box et al 1978), the magnitude of the secondary interaction parameters was found to be lower than the one for the main effects. The absolute average parameter estimate for all the 190 secondary interactions was 0.08 compared with a value of 0.57 for the main effects (Buehlmann 1998).

Based on the ANOVA tests performed, 113 of the 190 secondary interactions were found to be significantly different from 0. With a parameter estimate of  $-0.39$ , the interaction between part groups  $L_3W_1$  and  $L_4W_1$  (significant at  $\alpha = 0.01$ ) had the most negative impact on yield of all secondary interactions (Table 5). The influence of this secondary interaction is best explained for the case when the two part groups,  $L_3W_1$  and  $L_4W_1$ , require maximum part quantity and all the other part groups require 50% part quantity. The parameter estimates of the main effects of part groups  $L_3W_1$  and  $L_4W_1$  are  $+1.20$  and  $+0.58$ , respectively. Their secondary interaction parameter estimate of  $-0.39$  indicates that a loss will occur when both of these part groups are added above the 50% level. For this scenario, the yield contribution above the intercept of the two main effects of part groups  $L_3W_1$  and  $L_4W_1$  is 1.39% yield ( $1.20 + 0.58 - 0.39$ ). Theoretically, this loss in yield could be avoided by not including part groups  $L_3W_1$  and  $L_4W_1$  in the same cutting bill, but rather separating them into two cutting bills.

Only two other secondary interactions had parameter estimates of  $-0.30$  or lower (significant

Table 5. Secondary interactions with parameter estimates smaller than  $-0.20$  and larger than  $+0.10$ .

Interaction between		Parameter estimate	T for $H_0$	Probability (p)
Part group	Part group			
Negative secondary interactions below $-0.20$				
$L_3W_1$	$L_4W_1$	$-0.39$	$-17.98$	0.0001
$L_3W_1$	$L_3W_2$	$-0.35$	$-16.02$	0.0001
$L_2W_2$	$L_3W_1$	$-0.32$	$-14.59$	0.0001
$L_3W_2$	$L_4W_2$	$-0.27$	$-12.49$	0.0001
$L_3W_2$	$L_3W_3$	$-0.27$	$-12.43$	0.0001
$L_3W_2$	$L_4W_1$	$-0.26$	$-11.82$	0.0001
$L_2W_2$	$L_3W_2$	$-0.25$	$-11.58$	0.0001
$L_2W_1$	$L_3W_1$	$-0.25$	$-11.33$	0.0001
$L_2W_2$	$L_4W_1$	$-0.25$	$-11.28$	0.0001
$L_2W_4$	$L_3W_2$	$-0.23$	$-10.63$	0.0001
$L_3W_2$	$L_3W_4$	$-0.23$	$-10.60$	0.0001
$L_3W_3$	$L_3W_3$	$-0.23$	$-10.54$	0.0001
Positive secondary interactions above $+0.10$				
$L_2W_3$	$L_4W_3$	0.11	4.99	0.0001
$L_3W_2$	$L_5W_4$	0.11	5.05	0.0001
$L_1W_3$	$L_5W_3$	0.11	5.22	0.0001
$L_1W_1$	$L_1W_3$	0.12	5.42	0.0001
$L_2W_2$	$L_4W_2$	0.13	5.85	0.0001
$L_3W_4$	$L_5W_4$	0.15	6.87	0.0001
$L_2W_4$	$L_4W_4$	0.15	7.08	0.0001
$L_2W_2$	$L_5W_2$	0.16	7.51	0.0001
$L_4W_4$	$L_5W_2$	0.17	7.97	0.0001
$L_2W_3$	$L_5W_3$	0.19	8.92	0.0001
$L_2W_4$	$L_5W_4$	0.27	12.53	0.0001

at  $\alpha = 0.01$ ). Another nine secondary interaction parameter estimates were found to be between  $-0.20$  and  $-0.30$  (significant at  $\alpha = 0.01$ ). Of the 190 secondary interactions, 120 were found to have negative parameter estimates (not all significant at  $\alpha = 0.05$ , Buehlmann 1998). Table 5 shows the 12 secondary interactions with parameter estimates smaller than  $-0.20$ . It is interesting to note that all of the part group combinations, whose parameter estimates are below  $-0.20$ , are part groups that are adjacent, or near each other (Table 5). Adjacent part sizes are similar and thus there is a small probability of fitting one of these parts into a clear area in a board when a similar-sized part does not fit. As this and the following observation shows, secondary interactions are generally negative for similar part sizes, and positive for dissimilar part sizes. In general, dissimilar part sizes allow the clear areas in boards to be used more efficiently.

Table 5 also displays the 11 positive secondary interaction terms with values above 0.10 (all significant at  $\alpha = 0.01$ ). The largest secondary interaction parameter estimate between part groups  $L_2W_4$  and  $L_5W_4$  was found to be +0.28 (significant at  $\alpha = 0.01$ ). Thus, the yield of a cutting bill that requires the maximum part quantity from both part groups  $L_2W_4$  and  $L_5W_4$  will be higher, as compared with one where these two part groups are not both required. That this is the highest positive secondary interaction can be explained by the fact that if parts from the largest part group  $L_5W_4$  must be cut, strips of width  $W_4$  must be produced. With such long lengths in length class  $L_5$ , the clear areas in the  $W_4$  widths cannot be used efficiently. The most geometrical complementary use of the remaining areas in a strip of width  $W_4$ , after parts  $L_5W_4$  are cut, is thus to cut parts from  $L_2W_4$ . Therefore, the secondary interactions that can be used as a tool to guide which parts added to a cutting bill are complimentary or not in terms of yield.

In the second part of this publication, entitled "*The influence of cutting bill characteristics on lumber yield using fractional factorials Part II: Correlation and number of part sizes*," the viewpoint of the contribution of different part groups to yield will be different. The impact of requiring parts from a particular part group will be researched under the assumption of having cutting bills where no parts are added or removed. In other words, based on the 512 different cutting bills researched, which part groups have been most positively correlated with high yield? The second part will also look at the impact on yield of the number of part sizes to be cut concurrently.

#### SUMMARY AND CONCLUSIONS

Assuming that there is a linear relationship between part quantity and lumber yield, a 2-level, 20-factor resolution V fractional-factorial design was employed to research the influence of cutting-bill part size and part quantity requirements on lumber yield. Analysis of variance performed on the data obtained showed that all 20 main

effects and 113 of the 190 secondary interactions were significant ( $\alpha = 0.05$ ). Least squares parameter estimates indicated that the highest positive impact on yield can be achieved by adding parts 445-mm long and 57-mm wide to the cutting bill. Part sizes 1842-mm long and 108-mm wide had the most negative impact on yield when added to a cutting bill. The findings demonstrate that by carefully selecting which part sizes are added to a cutting bill, yield can be increased.

Knowing which part sizes can help to increase lumber yield in a rip-first rough mill will permit practitioners to more carefully design their cutting schedules. For example, based on the knowledge gained in this research it can be hypothesized that higher yield may be achieved if a large requirement of a small- to medium-sized parts can be broken up and assigned to two different cutting bills. By so doing, the overall yield of two runs both requiring some of the small- to medium-sized parts to be cut may be higher than requesting this part size in only one cutting bill. However, while this research has quantified the impact of different sized parts in a cutting bill, more research is needed to learn about the impact of a wide variety of cutting bill scheduling schemes. Further insight of the cutting-bill requirements and lumber yield relationship will be presented in the second part of this publication, where the correlation of different part sizes with high yield and the influence of the number of different part sizes to be cut concurrently are discussed.

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