

TORSIONAL RIGIDITY OF RECTANGULAR WOOD COMPOSITE MATERIALS

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(Received February 2004)

ABSTRACT

The torsional rigidity of wood members is necessary for predicting lateral torsional buckling of laterally unsupported beams, and is useful for estimating the stiffness of two-way floor systems and the natural frequency for wood floors. Current estimations of torsional rigidity of composite wood materials are based upon elastic constant ratios of solid wood. Recently published work has found differences in the elastic constant ratios of solid wood versus structural composite lumber (SCL) materials. These differences in elastic properties may indicate differences in torsional rigidity. Rectangular sections of solid-sawn lumber and various SCL materials were tested to determine values of torsional rigidity. Torsional rigidity of solid-sawn lumber was significantly different ($p < 0.05$) from laminated veneer lumber, while direct comparisons of parallel strand lumber and laminated strand lumber to solid-sawn lumber were not possible due to dimensional differences of test sections. Predictions of torsional rigidity based upon isotropic and orthotropic elasticity and shear moduli derived from bending tests were compared to the experimental results for each material. The solid-sawn lumber torsional rigidity was predicted best by the isotropic elasticity assumptions, while the parallel strand lumber and laminated strand lumber torsional rigidity values were predicted best by the orthotropic elasticity assumptions. The laminated veneer lumber torsional rigidity was predicted well by isotropic elasticity assumptions if shear moduli values derived from torsional testing were used. Torsional rigidity values for both solid-sawn lumber and SCL materials were not predicted well using an E:G ratio of 16:1 and isotropic elasticity assumptions.

Keywords: Torsional rigidity, shear modulus, structural composite lumber, isotropic elasticity, orthotropic elasticity.

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INTRODUCTION

Torsional rigidity (GJ) defines the ability of a material to resist angular distortions or rotational deformations along the effective span of a structural member. While not frequently considered in conventional design of wooden structural members, GJ terms are critical in the determination of lateral torsional buckling behavior relative to unsupported beams (AF&PA 2001), the stiffness of two-way floor systems (Foschi 1982) and the estimation of natural frequency for floor vibration studies (Smith and Chui 1988). As the use of structural composite lumber (SCL) allows longer spans and greater cross-sectional depth compared to solid-sawn lumber, the verification of this design criterion has become more crucial for wood composite material applications.

Current estimation of the GJ term generally relies upon an assumed $E_1:G_{12}$ elastic constant ratio typically associated with sawn lumber and the assumption of isotropic elasticity (AF&PA 2001). Recent research has shown that the elastic constant ratios of solid-sawn lumber and SCL materials are not necessarily equal and that SCL materials behave as more highly orthotropic materials (Hindman et al. 2004). These differences in the elastic constant ratios and characteristic orthotropic elasticity call into question assumptions of GJ equivalency of solid-sawn lumber and SCL materials.

The purpose of this research was to measure the GJ values of solid-sawn lumber and several SCL materials and to evaluate GJ predictions associated with these materials. GJ predictions were based upon both isotropic and orthotropic elasticity assumptions using section properties and planar shear moduli values. Comparisons were made between the experimental GJ values from solid-sawn lumber and SCL testing and between the isotropic and orthotropic predicted GJs for each material.

MATERIALS AND METHODS

Experimental study materials

Study materials included machine stress rated (MSR) lumber, laminated veneer lumber (LVL),

parallel strand lumber (PSL), and laminated strand lumber (LSL) for laboratory evaluation of torsional rigidity. Nominal 2×10 southern yellow pine (*Pinus* spp) members with machine-rated performance were selected because of lower inherent property variability over visually graded structural lumber. LVL and PSL were also uniquely composed of southern pine, while LSL was composed of yellow poplar (*Liriodendron tulipifera*) material. LVL and LSL products were sampled as materials typical to fabrication of SCL material I-joists. Table 1 lists the dimensions of the commercially processed material for use as available test specimens with shear-free modulus of elasticity (MOE) information from quality assurance (QA) data supplied by the manufacturer.

Experimental measurement of torsional rigidity

Figure 1 shows the torsional stress analyzer (TSA) used to measure the torsional rigidity (GJ) values of the MSR lumber and SCL materials. The TSA applied a centric torque to one end of the specimen while the other end was held rigid. A rotary actuator applied the torque, and a LeBow Model 2121-2K torque sensor with 226.0 N-m (2000 in.-lb) capacity and a sensitivity of 0.23 N-m (2 in.-lb) measured the applied torque. An LVDT with a 0.64-cm (0.25-in.) displacement range and 0.00025-cm (0.0001-in.) sensitivity measured the change in arc length during specimen loading. Measurements of the arc length were correlated to the angle of specimen end rotation. LabView software collected data from both instruments to generate a torque vs. angle of rotation curve. Specimens were loaded until an angular rotation of two degrees was attained. Sixteen specimens of each material were tested under three loading repetitions to develop the average torque-rotation curve for each specimen. Figure 2 is a typical torque-angle curve from an MSR specimen.

The stiffness relationship for torsional rigidity of a rectangular section for any material is defined according to Eq. (1). For experimental measure of the GJ term, torsional rigidity of each material was taken as the slope of the applied

torque versus angular rotation curve (T/θ) multiplied by the effective specimen length (L_e). Actual specimen gage lengths between test machine grips were 144.8 cm (57 in.) for the MSR and LVL materials and 147.3 cm (58 in.) for the PSL and LSL materials. Effective length refers to the measured gage length including an adjustment to account for any clamping effect

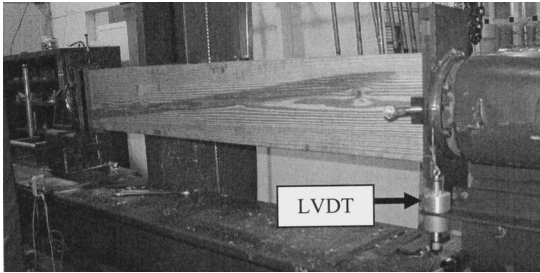


FIG. 1. Torsional stiffness analyzer (TSA) showing placement of LVDT to measure change in arc length.

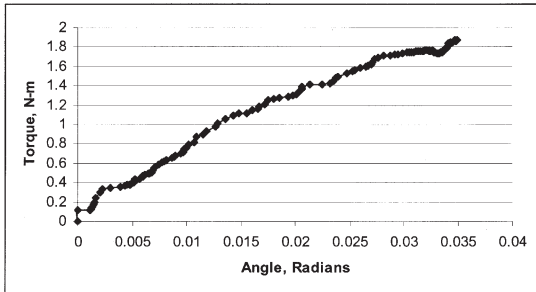


FIG. 2. Typical Torque vs. Twist curve from torsional loading for an MSR Lumber test specimen

imposed by the experimental equipment on the ends of the test specimen.

$$GJ = \left(\frac{T}{\theta} \right) L_e \quad (1)$$

where

T = applied torque, q = resultant angle of rotation

L_e = effective specimen length including ELD adjustment

Timoshenko and Goodier (1970) developed the basic torsional rigidity theory. One of the key assumptions in the derivation was the unrestrained rotation of the specimen. The TSA clamped the ends of the specimen and therefore violated this assumption. An effective length adjustment must be employed to compensate for this clamping effect so that the torsional theory can be applied to the experimental measurements from the TSA. Tarnopol'skii and Kincis (1985) described a simple method to determine the effective length. The effective length difference (ELD) corrective adjustment to the specimen gage length was determined by evaluating the torsional rigidity of a series of reduced length specimens and plotting torque-theta curve vs. the gage length. The intercept of the torque-theta curve vs. the gage length curve yields the resultant ELD value applied to the measured specimen gage length to determine the effective length term used in Eq. (1). Subsequent ELD testing included reduced lengths of 129.5 cm (51 in.), 99 cm (39 in.), and 76.2 cm (30 in.). Proce-

TABLE 1. Properties of the experimental study materials

Material	Description (species)	Width cm (in)	Height cm (in)	Length cm (in)	MOE ¹ GPa (psi)
MSR	MSR 2250f-1.9E (Southern Pine)	3.81 (1.50)	23.5 (9.25)	144.8 (57.0)	13.1 (1.90×10 ⁶)
LVL	Flange Grade (Southern Pine)	3.81 (1.50)	23.5 (9.25)	144.8 (57.0)	14.5 (2.09×10 ⁶)
PSL	2.0E Grade (Southern Pine)	4.44 (1.75)	23.5 (9.25)	147.3 (58.0)	13.8 (2.00×10 ⁶)
LSL	Flange Grade (Yellow Poplar)	3.81 (1.50)	24.1 (9.50)	147.3 (58.0)	11.0 (1.59×10 ⁶)

¹MOE are minimum quality control values as reported from the manufacturer

dures for calculation of the equivalent length term with the clamp restraint correction factor are available in Hindman (2003) and Janowiak and Pellerin (1992).

Analytical prediction methods

Figure 3 shows material axes and the planar shear moduli, G_{12} and G_{13} , given a test specimen subjected to torque about the longitudinal or 1-axis (the 1-axis corresponds to the wood fiber direction). Timoshenko and Goodier (1970) provided the torsional rigidity solution of an isotropic rectangular section (Eq. 2); Lekhnitskii (1963) provided the torsional rigidity solution of an orthotropic rectangular section (Eq. 3). The form of the orthotropic torsional rigidity equations parallels the isotropic torsional rigidity equation with the inclusion of the G_{13} shear moduli. If the assumption of isotropy ($G_{12} / G_{13} = 1.0$) is applied to Eq. (3), the result is nearly equivalent to Eq. (2). Differences in Eqs. (2) and (3) result from the truncation of infinite series terms to solve for Eq. 3 in Lekhnitskii (1963).

$$GJ = G_{12} \left[\frac{b^3}{3} \left(1 - \frac{2}{\pi} \left(\frac{b}{h} \right) \right) \right] \quad (2)$$

$$GJ = G_{12} \left[\frac{b^3 h}{3} \left(1 - \frac{192b}{\pi^5 h} \sqrt{\frac{G_{12}}{G_{13}}} \right) \right] \quad (3)$$

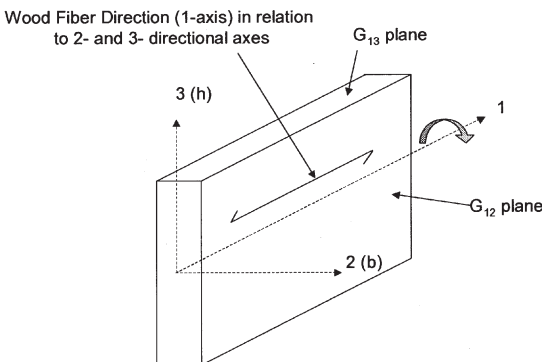


FIG. 3. Definition of material axes with respect to torsional loading.

where

- GJ = torsional rigidity of section
- G_{12}, G_{13} = planar shear moduli (Figure 3)
- b = width of rectangular section
- h = height of rectangular section

The torsional rigidity values of the test materials were predicted based upon previously determined shear moduli from Hindman et al. (2004). Shear moduli were evaluated using the five-point bending test (FPBT) and subsets of the same materials tested herein. The FPBT uses two different bending configurations to simultaneously evaluate the elastic and shear moduli values. FPBT procedures are described in Hindman et al. (2004) and Bradtmueller et al. (1998). These predictions were compared with the experimental torsional rigidity results to determine if satisfactory torsional rigidity terms could be predicted using isotropic or orthotropic elasticity equations. The 95% confidence intervals (CI) of the experimental torsional rigidity terms were compared with the bounds of the predicted torsional rigidity terms resulting from the use of the 95% CI values of the shear moduli terms from Hindman et al. (2004). The torsional rigidity predictions were considered significantly different from the experimental torsional rigidity values if the experimental 95% CI values did not overlap the bounds of the torsional rigidity predictions. Table 2 shows the upper and lower 95% CI values for the G_{12} and G_{13} from FPBT (Hindman et al. (2004).

The isotropic GJ predictions used Eq. (2) and the G_{12} values, whereas the orthotropic GJ predictions used Eq. (3) and both the G_{12} and G_{13} values. To determine the 'upper' and 'lower' isotropic GJ predictions, the upper and lower 95% CI values of the G_{12} terms were used. For the orthotropic GJ predictions, the upper G_{12} and G_{13} 95% CI values were used for the 'upper' GJ and the lower G_{12} and G_{13} 95% CI values were used for the 'lower' GJ .

RESULTS AND DISCUSSION

Summary data collection

Table 3 summarizes the ELD term for the test specimens, the experimentally derived GJ and as-

TABLE 2. Average shear moduli and computed 95% confidence intervals for the study materials from Hindman (2003)¹.

Material	G ₁₂			G ₁₃		
	Lower CI MPa (psi)	Average MPa (psi)	Upper CI MPa (psi)	Lower CI MPa (psi)	Average MPa (psi)	Upper CI MPa (psi)
MSR	663 (9.62×10 ⁴)	751 (1.09×10 ⁵)	841 (1.22×10 ⁵)	384 (5.57×10 ⁴)	433 (6.28×10 ⁴)	482 (6.99×10 ⁴)
LVL	679 (9.85×10 ⁴)	710 (1.03×10 ⁵)	745 (1.08×10 ⁵)	514 (7.46×10 ⁴)	549 (7.96×10 ⁴)	583 (8.46×10 ⁴)
PSL	689 (1.00×10 ⁵)	724 (1.05×10 ⁵)	765 (1.11×10 ⁵)	520 (7.54×10 ⁴)	557 (8.08×10 ⁴)	595 (8.63×10 ⁴)
LSL	965 (1.40×10 ⁵)	1050 (1.51×10 ⁵)	1110 (1.61×10 ⁵)	409 (5.93×10 ⁴)	436 (6.32×10 ⁴)	463 (6.71×10 ⁴)

¹ Shear moduli values were obtained from five-point bending tests.

TABLE 3. Summary of data including actual test values for experimentally-derived torsional rigidity (GJ)¹.

Material	Effective Length cm (in) ²	Torsional Rigidity (GJ) ³		% Difference from MSR Lumber ⁴
		N-m ² (lb-in ²)	COV %	
MSR	120.6 (47.5)	3.07×10 ³ (1.07×10 ⁶)	11.7	—
LVL	121.4 (47.8)	2.43×10 ³ (8.46×10 ⁵)	5.6	-20.8
PSL	128.5 (50.6)	3.73×10 ³ (1.30×10 ⁶)	7.9	+21.5
LSL	125.0 (49.2)	4.16 × 10 ³ (1.45×10 ⁶)	8.8	+35.5

¹ Cross-sectional dimensions are defined in Table 1.

² Effective length procedures are described in Hindman (2003).

³ Sixteen specimens with three loading repetitions were tested for each material.

⁴ % difference = 100 × (GJ - MSR GJ) / MSR GJ.

sociated COV values for the test materials. The grip adjustment lowered the effective length to the range of 120.7 cm (47.5 in.) to 128.5 cm (50.6 in.), showing approximately 5% variation in the equivalent length term for different materials.

Comparative analysis of observed torsional rigidity

The experimental GJs with accompanying coefficient of variation (COVs) and effective length terms for all test materials are shown in Table 3. The PSL and LSL had higher GJ values than either the LVL or the MSR lumber. The COVs of the experimental GJ values ranged from 5.6 to 11.7%. The greatest COV value was 11.7% for the MSR lumber, while all other COV values were less than 10%. All of the SCL GJ values were at

least 20.8% different than the MSR GJ values. The average LVL GJ value was 20.8% less than the MSR lumber GJ value, while the PSL and LSL GJ values were 21.5% and 35.5% greater than the MSR GJ value, respectively.

Because of the different cross-sectional dimensions of the PSL and LSL compared to MSR, the only direct statistical comparison of GJ values was between the LVL and MSR lumber materials. An analysis of variance using Minitab software was performed to compare the GJ values from LVL and MSR using two factors. The factor 'material' examined differences in the MSR and LVL values, while the factor 'specimen' examined differences in the 16 specimens of each material. An alpha factor of 0.05 was used to establish significant difference. The 'material' factor p-value was 0.000, indicating that

the LVL and MSR GJ values were significantly different. The 'specimen' factor p-value was 0.68, indicating there was no significant difference in the GJ values evaluated from specimens of each material. Hindman et al. (2003) reported that the different SCL materials had distinctly different $E_1:G_{12}$ ratios. This trend of different elastic behavior of different SCL materials continues in the torsional rigidity terms.

Experimental vs. predicted torsional rigidity

Table 4 presents the average GJ values from the experimental test results and the isotropic and orthotropic predictions for comparison. The average MSR experimental GJ was 3.6% greater than the isotropic prediction and 6.9% greater than the orthotropic prediction. The average LVL experimental GJ was more than 10% less than both the isotropic and orthotropic GJ predictions. The average PSL experimental GJ was 7.8% less than the orthotropic prediction and 10.2% less than the isotropic prediction. The average LSL experimental GJ was 0.4% less than the orthotropic prediction and more than 10% less than the isotropic prediction. The comparison in Table 4 shows that the assumption of either isotropic or orthotropic elasticity does not have a large effect on the GJ term for rectangular SCL materials except for the LSL material.

Figure 3 shows the 95% CIs of the experimental torsional rigidity values and the lower and upper bounds of the isotropic and orthotropic predictions based upon the 95% CI of the planar

shear moduli. The overlap of the experimental GJ range with the prediction range indicates the values were not significantly different at the 95% CI level. All materials except the LVL overlap at least one of the GJ elasticity predictions. For the MSR material, both the isotropic and orthotropic prediction bounds overlapped the experimental GJ values. The LVL experimental GJ was less than both the isotropic and orthotropic prediction bounds. PSL experimental GJ overlapped the orthotropic prediction only. The LSL experimental GJ overlapped both the isotropic and orthotropic prediction bounds despite the high percentage difference between the isotropic prediction and experimental GJ. For all materials of the corresponding isotropic and orthotropic prediction ranges in Fig. 3, the different elasticity assumptions used did not produce significantly different GJ values. Further understanding of the GJ predictions associated with LVL materials is needed. Previous work by Janowiak et al. (2001) observed that LVL and PSL materials had higher levels of orthotropic behavior compared to LSL and published solid wood values.

The higher orthotropic behavior specific to LVL may influence the shear moduli determination and therefore the torsional rigidity values. Because the ultimate purpose of this research was to examine the lateral torsional buckling of SCL and I-joint beams, the shear moduli values were measured using the five-point bending test (Hindman et al. 2004) rather than a torsional loading. The use of planar shear moduli values evaluated using a torsional loading may provide

TABLE 4. *Percent differences between average experimental and predicted GJ values.*

Material	Experimental GJ N-m ² (lb-in ²)	Isotropic		Orthotropic	
		N-m ² (lb-in ²)	% Difference	GJ N-m ² (lb-in ²)	% Difference ¹
MSR	3.06×10 ³ (1.07×10 ⁶)	2.96×10 ³ (1.03×10 ⁶)	-3.6	2.85×10 ³ (9.92×10 ⁵)	
LVL	2.43×10 ³ (8.46×10 ⁵)	2.77×10 ³ (9.64×10 ⁵)	2.73x10 ³ +14.0	-6.9 (9.52×10 ⁵)	+12.5
PSL	3.74×10 ³ (1.31×10 ⁶)	4.13×10 ³ (1.44×10 ⁶)	+10.2	4.05×10 ³ (1.41×10 ⁶)	+7.8
LSL	4.16×10 ³ (1.45×10 ⁶)	4.59×10 ³ (1.60×10 ⁶)	+10.3	4.33×10 ³ (1.51×10 ⁶)	+0.4

¹ % Difference = 100 × (Predicted-Experimental)/Experimental.

a more accurate relationship with the GJ values compared to shear moduli evaluated using a bending loading.

Simulated isotropic and orthotropic GJ predictions

To examine the differences in GJ prediction caused by the use of shear moduli values from torsion test methods, previously measured LVL and LSL shear moduli values from Janowiak et al. (2001) were used to develop isotropic and orthotropic predictions for comparison to the experimental GJ values. Shear modulus values from Janowiak et al. (2001) were measured using a torsional stiffness measurement test. Figure 4 shows the experimental and predicted GJs for the LVL and LSL materials including isotropic and orthotropic predictions using shear moduli from both bending and torsion test methods. For the LVL, both the isotropic and orthotropic GJ predictions using torsional shear moduli overlap the experimental GJ values. The average experimental GJ value for LVL was 1.5% less than the average torsional isotropic GJ value and 4.0% greater than the average tor-

sional orthotropic GJ value. For the LSL, the average isotropic and orthotropic GJ predictions using torsional shear moduli are both 23% less than the average experimental GJ values. The LVL experimental GJ values were best predicted by the isotropic GJ equation using shear moduli values from a torsional loading, while the prediction of LSL GJ was not improved by using shear moduli from a torsional loading. Predictions of LVL varied significantly compared to other SCL materials studied. The different elasticity behavior reinforces the idea that the different SCL materials have independent elasticity behavior and different elastic constants.

After the best GJ prediction equations for the GJ of each material were determined, the GJ predictions were examined to determine if they represent changes compared to current assumptions of an E:G ratio of 16:1 design specifications such as the National Design Specification (AF&PA 2001). The GJ of a nominal 2 × 12 section for each study material was predicted using the assumed E:G ratio of 16:1 and isotropic elasticity. The E values used to predict the shear moduli were from five-point bending testing conducted by Hindman et al. (2004). Table 5

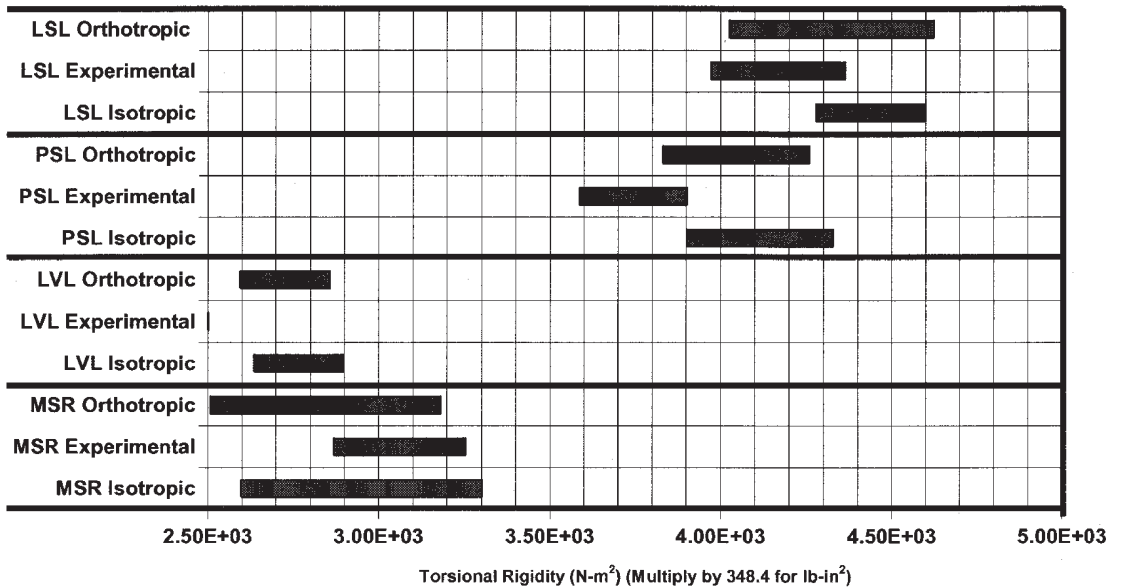


FIG. 4. Torsional rigidity confidence intervals for rectangular materials

TABLE 5. Prediction of torsional rigidity for 2 × 12 members using E:G of 16 and material specific E:G ratios

Material	GJ from E:G = 16:1 ¹ N-m ² (lb-in ²)	Best GJ Predictions ^{2,3} N-m ² (lb-in ²)	% Difference ⁴
MSR	4860 (1.69×10 ⁶)	3620 (1.26×10 ⁶)	-25.5
LVL	5030 (1.75×10 ⁶)	2930 (1.02×10 ⁶)	-41.7
PSL	4140 (1.44×10 ⁶)	3450 (1.20×10 ⁶)	-16.6
LSL	3950 (1.25×10 ⁶)	4780 (1.66×10 ⁶)	+32.9

¹ Measured E:G ratios are from five-point bending as reported in Hindman et al. (2004).

² Best GJ predictions were Isotropic for MSR, Orthotropic for PSL and LSL and Orthotropic Using Torsional Shear Moduli for LVL.

³ G_{ij} values for MSR, PSL and LSL from Hindman et al. (2004) and for LVL from Janowiak et al. (2001).

⁴ % Difference × 100 x (Best Prediction - E:G of 16.0)/E:G of 16.0

shows the resultant GJ predictions and the percent difference of the best GJ predictions compared to those using the assumed E:G ratio of 16:1. Compared to calculated GJs assuming E:G = 16:1, the best fit models using measured shear moduli were 25.5% lower for MSR, 41.7% lower for LVL, 16.6% lower for PSL, and 32.9% greater for LSL. The assumed E:G ratio GJ predictions showed no correspondence with the best fit prediction, demonstrating that the best fit pre-

dictions are different than the assumed E:G ratio for all materials. The three SCL materials again demonstrated independent elasticity terms. Therefore, the GJ predictions of MSR and SCL materials are independent for an identical cross-section.

CONCLUSIONS

The torsional rigidity terms of solid-sawn lumber and various rectangular SCL materials were evaluated and then compared to predictions of torsional rigidity based upon isotropic and orthotropic elasticity using previously measured shear moduli values. The experimental LVL and MSR GJ values were significantly different, while differences in the cross-sectional dimensions of the PSL and LSL confounded direct statistical comparisons to MSR. Comparison of experimental GJ values to isotropic and orthotropic predictions revealed that the MSR GJ followed the isotropic prediction while the PSL and LSL GJ followed the orthotropic prediction. The GJ for LVL materials did not correspond to either isotropic or orthotropic prediction. Com-

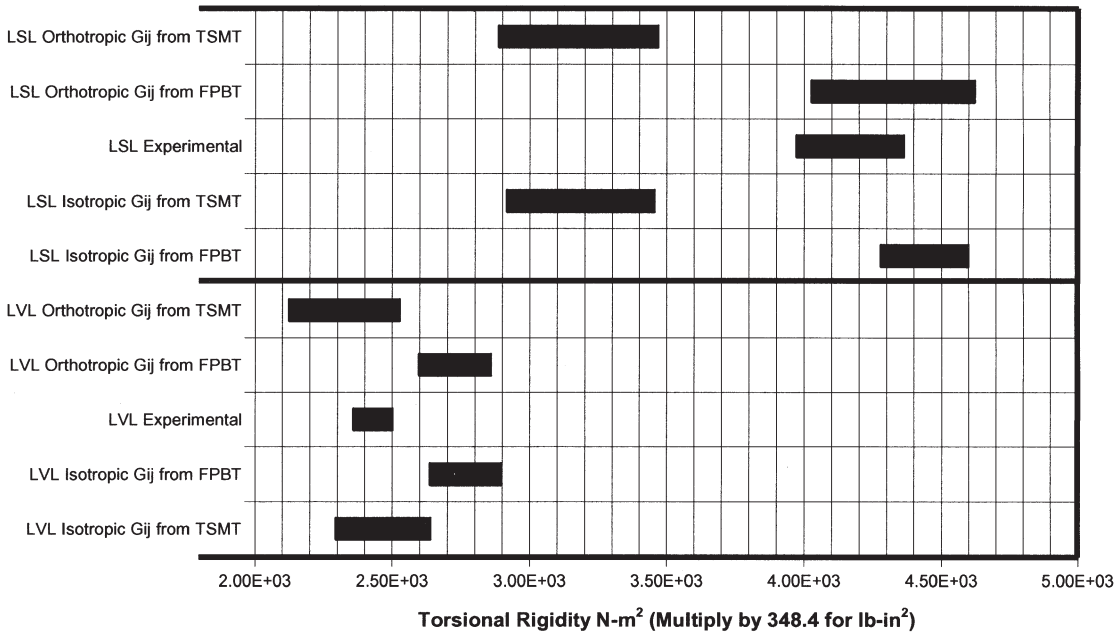


FIG. 5. Torsional rigidity confidence intervals from torsional elastic constants.

parison of LVL experimental GJ values with isotropic and orthotropic predictions formulated using shear moduli derived from a torsional loading demonstrated that LVL followed the isotropic GJ prediction. The torsional rigidity of the MSR lumber and SCL materials are all unique and cannot be predicted adequately by simply assuming isotropic behavior and $E:G = 16:1$.

ACKNOWLEDGMENTS

The authors would like to thank Trus Joist, a Weyerhaeuser company, Rigidply Rafters, the Pennsylvania Housing Research Center, and the Pennsylvania Agricultural Experiment Station for material donations and support for this project.

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