EVALUATION OF MODULUS OF RIGIDITY BY DYNAMIC PLATE SHEAR TESTING

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ABSTRACT

The modulus of rigidity of wood-based panels was measured by the dynamic plate shear method. This involves measuring the torsional vibration of free, square plates. The proposed equation to calculate modulus of rigidity was found to agree within ±5% of the experimentally determined values. The experimentally determined modulus of rigidity values were very close to moduli determined by other techniques.

Keywords: Modulus of rigidity, dynamic plate shear, wood-based panels, torsional vibration.

INTRODUCTION

The modulus of rigidity (G) is important for wood and wood-based panel products and is usually measured by static methods, that is, plate shear and panel shear methods. These methods are usually not convenient and take a considerable amount of time to evaluate G. Furthermore, there exists a considerable difference between the G values determined by each method. On the other hand, a torsional vibration method of free-free beams was recently proposed as a method of measuring anisotropic G (Nakao et al. 1984, 1985a). The dynamic method provides an accurate value of anisotropic G and is easier to do than the static methods. Other than the sample dimensions and density, only the resonant frequency of a sample is required to evaluate the modulus. The system for the vibration method of free-free beams is simple because the beams are supported by threads and the measurement apparatus consists of an acoustical pick-up, and a frequency counter coupled to an FFT analyzer.

In this paper, the moduli of rigidity for several kinds of wood-based panels were measured by the torsional vibration of free, square plates method. These values were compared to values obtained using conventional methods as well as the flexural vibration method based on Timoshenko's theory (Hearmon 1958).

THEORY

From anisotropic plate and energy method theory (Hearmon 1960), the following equation is obtained for the torsional vibration of a free, rectangular plate:

$$D_1 \frac{\alpha_1}{a^4} + D_2 \frac{\alpha_2}{b^4} + 2D_{12} \frac{\alpha_3}{a^2b^2} + 4D_{66} \frac{\alpha_4}{a^2b^2} = \rho h \omega^2 \quad (1)$$

where:

$$D_1 = \frac{E_x h^3}{12(1 - \nu_x \nu_y)}, \quad D_2 = \frac{E_y h^3}{12(1 - \nu_x \nu_y)}, \quad D_{12} = \nu_x D_2, \quad \text{and} \quad D_{66} = \frac{Gh^3}{12}$$

E_x and E_y are the moduli of elasticity in the x- and y-directions, respectively (1 dyne/cm^2 = 10^{10} GPa),
ν_x and ν_y are Poisson's ratios, in which the subscript denotes the direction of the applied stress,
a, b are the lateral dimensions of the plate (cm),
h is the plate thickness (cm),
ρ is the sample density (g/cm^3),
$\omega = 2\pi f_r$ = angular frequency, and
f_r is the resonant frequency in Hertz.

The α coefficients can be determined from vibration modes and boundary conditions. Many researchers (Hearmon 1960; Leissa 1968; Nakao and Okano 1985) indicated that the values of α_1, α_2, and α_3 are close to zero for the torsional vibration mode. Therefore, Eq. (1) can be rewritten as follows:

$$G = \left[1 + \left(\frac{D_1 b^2}{4D_{66} a^2 \alpha_4}\alpha_1 + \frac{D_2 a^2}{4D_{66} b^2 \alpha_4}\alpha_2 + \frac{D_{12}}{2D_{66}\alpha_4}\alpha_3\right)\right]^{-1} \frac{12\pi^2}{\alpha_4} \rho \left(\frac{ab}{h} f_r\right)^2$$
$$= \beta\rho\left(\frac{ab}{h} f_r\right)^2 \quad (2)$$

Hearmon (1960) showed that values of α_1, α_2, and $\alpha_3 = 0$ and $\alpha_4 = 144$ can be obtained by assuming that the bending slope of a plate is expressed by linear lines. In this case, the value of beta is 0.822. Leissa (1968) collected a large amount of theoretical and experimental data for the vibration of plates. Based on his data, a beta value of 0.9 is suitable for the torsional vibration of isotropic plates.

Since plywood is an orthotropic material, the value of beta should be determined from anisotropic theory. Based on the anisotropic, elastic moduli of plywood (Tsuzuki et al. 1972), the torsional vibration resonant frequency was calculated by the Rayleigh-Ritz method (Nakao and Okano 1985) with the results for various cases shown in Table 1. For all three cases, beta is approximately 0.86. This is about 5% smaller than Leissa's value for an isotropic material.

Furthermore, the influence of shear deflection on the torsional vibration of plates was evaluated using Mindlin's technique (1951). Numerical results for beta were obtained for particleboards, the thickest of our materials, and the most likely to be influenced by shear. The results of this evaluation are shown in Fig. 1. Assuming 25-cm-square boards, the beta value is estimated to be only 5% greater than Leissa's value. Therefore, when we substitute $\beta = 0.9$ in Eq. (2), the calculated value of G for any type of wood-based panel is expected to be accurate within $\pm 5\%$ since the value of beta is increased by 5% for plate anisotropy and is decreased by 5% for shear. Therefore the following equation is proposed to estimate the modulus of rigidity (G) for wood and wood-based panels:

TABLE 1. *Effect of anisotropy on the value of β in Eq. (2).*

	MOE[a]	MOE[a]	Modulus of rigidity (GPa)	fr (Hz)	β
Case 1[b]	10.0	3.0	0.5	123.26	0.862
Case 2	8.5	3.5	0.5	123.37	0.860
Case 3	8.0	5.0	0.5	123.77	0.855

[a] Modulus of elasticity in the x- and y-direction, respectively.
[b] Poisson's ratio, $\nu_{yx} = 0.1$, $\rho = 0.55$ g/cm³, and dimensions are 25 × 25 × 0.75 cm.

$$G = 0.9\rho\left(\frac{ab}{h}f_r\right)^2 \tag{3}$$

EXPERIMENTAL METHODS

Resonant frequency measurement

Samples of plywood, particleboards, and fiber-based panels were supported by threads along the nodal lines as shown in Fig. 2. Specific sample dimensions and densities are given in Table 2. Torsional vibration was generated by hitting a corner of the plate. An FFT analyzer recorded vibration frequencies. As in any such vibrating method, other vibration modes are also generated. However, we can easily find the resonant frequency since it is generally lowest for square plates and can be verified experimentally by the shape of the vibration mode when the nodal lines are struck.

Flexural vibration method

It is well known that values of G obtained from static bending tests are decreased by the occurrence of shear stresses when the thickness of the beam is small when

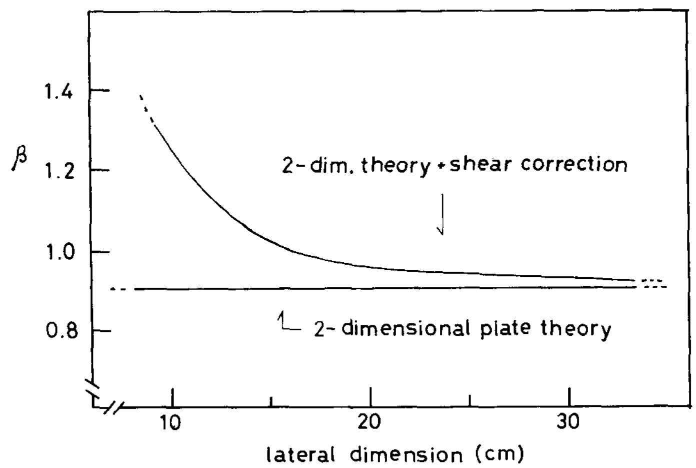

FIG. 1. Effect of lateral dimensions of a square plate on the values of β in Eq. (3). In this calculation, the following values were used: thickness = 1.5 cm, modulus of elasticity = 5 GPa, modulus of rigidity = 1.8 GPa, Poisson's ratio = 0.39, modulus of rigidity on thickness direction = 0.3 GPa, and ρ = 0.8.

TABLE 2. *Dimensions and density of specimens.*

Specimen[a]	Thickness (cm)		Lateral dimensions (cm)	Density (g/cm³)
Plywood (5-ply, Lauan)	1	0.78	25 × 25	0.534
	2	0.94		0.526
	3	1.28		0.545
Particleboard (3-layered, Urea Melamine resin)	1	1.24	25 × 25	0.844
	2	1.53		0.721
	3	1.54		0.775
Fiberboards:				
Hardboard		0.73	25 × 25	0.917
MDF		1.52		0.751
Insulation board		1.24		0.302

[a] Three specimens were used for each thickness.

compared to the span. In the case of flexural vibration of beams, E is decreased by the shear effect if the depth/span ratio is relatively large or if the beams are excited at high frequencies. Timoshenko's theory explicitly explains the effect (Nakao et al. 1985b). Hearmon (1958) proposed a measuring method for G for beams which are rather short and are influenced by the shear effect. The Timoshenko-Hearmon (TH) flexural vibration method was used to determine G for beams that were 5 cm deep and 40 cm long. Materials were as shown in Table 2.

The measuring apparatus was similar to that shown in Fig. 2. The beams were supported at the nodal points and measurements were taken with the plane of vibration parallel to the breadth of the cross section. The resonant frequencies of the 1st, 2nd, and 3rd modes were obtained by giving a blow to an edge of the beam and recording the results with the FFT analyzer.

Static panel methods

G was also determined for the panels by the plate shear method and the panel shear method. The plate shear method tests were done using the 25 × 25-cm

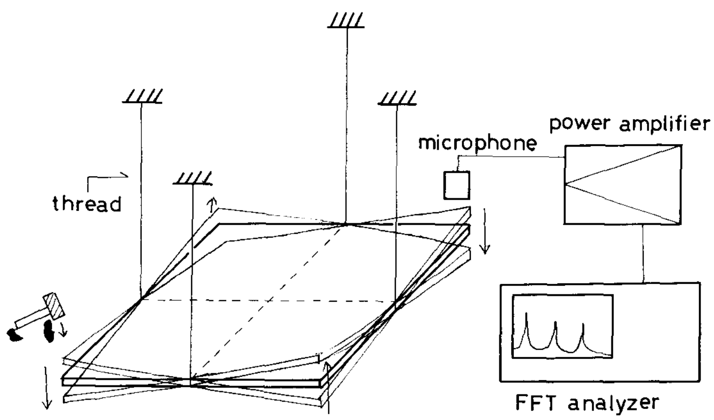

FIG. 2. Schematic diagram of apparatus. Dotted lines indicate the nodal lines of the torsional vibration mode of a free, square plate.

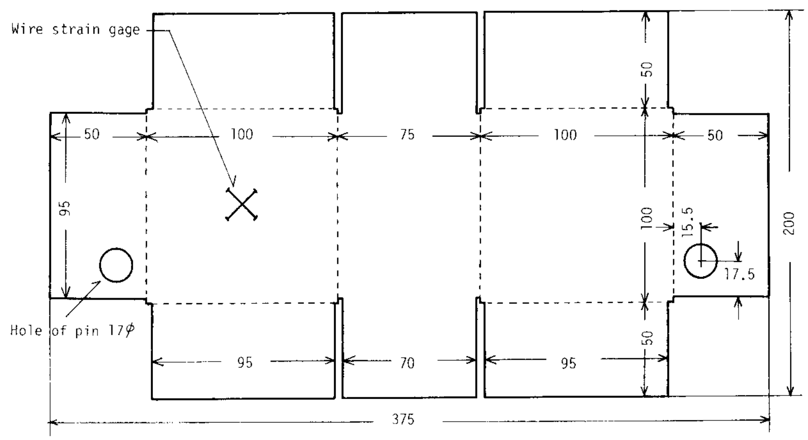

FIG. 3. Test specimen for LW-improved method. (Unit: mm.)

plates used in the dynamic plate shear tests. For the 12-mm thick plywood, the three particleboards, the medium density fiberboard, and the insulation board, 45 × 45-cm plates were also tested to evaluate the effect of panel size. Almost equal values were obtained. Okuma's (1966) LW-improved method was used as the panel shear method since it provides more accurate results than does the ASTM D805-52 test method. The specimen shape and experimental apparatus are shown in Figs. 3 and 4, respectively. The modulus of rigidity is obtained from:

$$G = \frac{1}{2Lt}\left(\frac{P}{|\epsilon - \epsilon'|}\right) \tag{4}$$

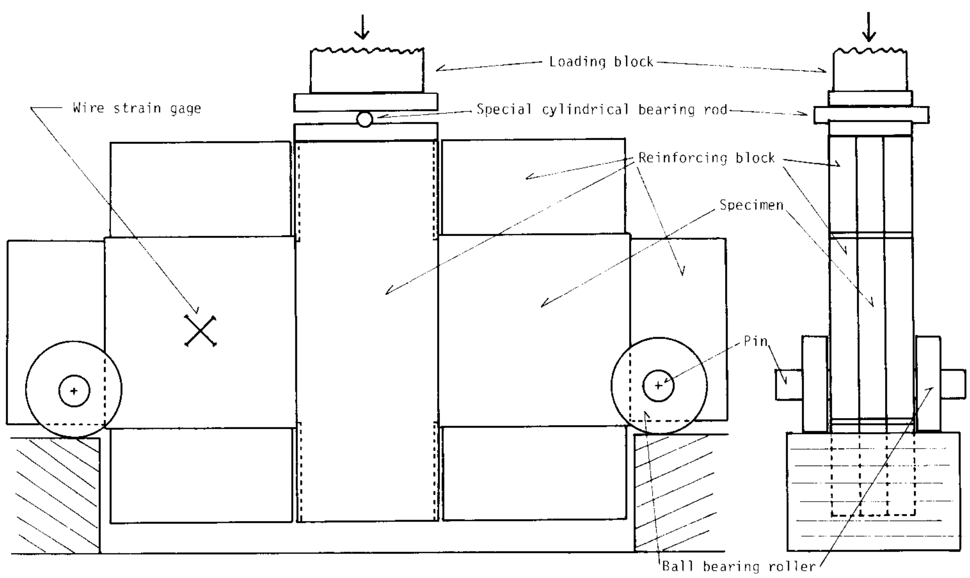

FIG. 4. Experimental arrangement of LW-improved method.

TABLE 3. *Modulus of rigidity of various specimens.*

Specimen		Dynamic method		Static method	
		Dynamic plate shear	Timoshenko-Hearmon method	Okuma's LW-improved method	Plate shear (GPa)
Plywood	1[a]	0.41	0.42		0.40
	2	0.50	0.50	0.4–0.5[b]	0.44
	3	0.46	0.41		0.50
Particleboard	1	1.73	1.53	1.21	1.98
	2	1.33	1.13	1.03	2.10
	3	1.70	1.82	1.33	2.60
Fiberboards:					
Hardboard		1.68	1.93	1.45	1.42
MDF		1.72	1.30	1.22	2.40
Insulation board		0.23	0.22	—	0.26

[a] See Table 2.
[b] Okuma 1966.

where ϵ and ϵ' are the strains in the two orthogonal directions; P is the load; t, the specimen thickness; and L, the side length of the specimen (10 cm).

RESULTS AND DISCUSSION

The results of the four methods of measuring G are shown in Table 3. For the three plywoods, the hardboard and the insulation board, the results are very similar regardless of the method. However, for the three particleboards, the two dynamic test results were in relatively close agreement. Both the dynamic plate shear results and the Timoshenko-Hearmon method results were greater than results from Okuma's LW-improved method and considerably less than the results of the static plate shear test. While the agreement between the two dynamic tests was not as close for the MDF, the overall trend was the same.

In both static test procedures, the test information is obtained from only a few discrete points on each sample, that is, displacements on diagonal lines for the plate shear method and surface strain at the panel's midpoint for the LW-improved method. In samples like the particleboards, the mechanical and physical properties of the samples change from face to core. The dynamic test methods sample the entire thickness of the board during the procedure and, we assume, automatically account for these differences. The dynamic plate shear method, that is, the torsional vibration method of free, square plates, is simpler than any of the other methods described in this paper, and provides accurate values for G, the modulus of rigidity.

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