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Fundamental approach to exchange rate modeling: Toward an augmented theory of purchasing power parity

Jianzhou Zhu

FUNDAMENTAL APPROACH TO EXCHANGE RATE MODELING: TOWARD AN AUGMENTED THEORY OF PURCHASING POWER PARITY

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Jianzhou Zhu, M.E.

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Doctor of Business Administration

Dwight C Anderson
Supervisor of Dissertation Research

Dwight C Anderson
Head of Department

Finance
Department

Recommendation concurred in:

Committee

Approved:
Director of Graduate Studies

Dean of the College

Approved:
Terry McConathy
Director of the Graduate School

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ABSTRACT

In this dissertation, I test the homogeneity and symmetry conditions of PPP by applying the Johansen multivariate cointegration methodology to quarterly data for six countries. I perform the tests in the framework of both a traditional version and an augmented version of PPP. The results of tests on the traditional version of PPP reveal that in all cases the theoretical PPP-vector [1, 1, –1] is not contained in the cointegrating space. This finding is consistent with that of existing literature and indicates the empirical failure of the homogeneity and symmetry conditions of PPP. However, when the traditional PPP is augmented with several non-price variables (real interest rate differential, relative growth rate of real GDP, relative current account balance as a percentage of GDP, and relative terms of trade), the theoretical PPP-vector [1, 1, –1] exists in all the instances except Germany. The fact that the theoretical PPP-vector exists in the augmented model but not in the traditional model indicates that the empirical failure of the PPP is caused by misspecification as a result of missing variables. The true relationship between exchange rates and prices, i.e., the homogeneity and symmetry conditions of PPP, is revealed once those 'missing variables' are added to the model. One potential reason for the failure to find the theoretical PPP-vector [1, 1, –1] in the case of Germany is the structural break caused by monetary reunification between Eastern and Western Germany in 1992.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

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CHAPTER 1

INTRODUCTION

This dissertation presents an empirical investigation of fundamental determinants of the long run equilibrium exchange rate. The choice of this particular topic is motivated by my skepticism about the prevailing interpretation of recent empirical findings concerning the validity of the purchasing power parity (PPP) as a long run equilibrium relationship. Recent empirical tests of PPP have been conducted in one of two ways. One approach involves examining whether nominal exchange rates are cointegrated with relative prices, while the other seeks to determine if real exchange rates contain a unit root. Findings of these studies generally indicate the existence of some form of long run equilibrium exchange rate. In particular, many currencies are found to have a unique cointegrating relationship between exchange rate and relative prices, and real exchange rates display mean reverting behavior. These two pieces of evidence have been interpreted by many researchers as supportive of PPP in the long run. If the findings are reliable and the interpretation of the findings is appropriate, it would be unnecessary to search for fundamental economic factors that potentially determine long run equilibrium exchange rates, since PPP posits that the exchange rate between two currencies is determined by, and only by, the relative prices in the two issuing countries. Therefore, this dissertation starts with an assessment of the validity of PPP as a long run equilibrium relationship.

As a theory of exchange rate determination, PPP has been a major preoccupation of international finance since it was first put forward by Cassel in 1918. Studies of the theoretical and empirical validity of PPP have been both intense and extensive. Despite the persistent effort, the issue remains unresolved. While most researchers now accept that PPP does not hold well in the short run, such agreement says little about the validity of the theory. PPP, as originally formulated by Cassel, was never assumed to be a steady state; rather, it states that there is an equilibrium level of exchange rate toward which the actual exchange rate gravitates. Since the parity relationship is not a steady state, deviations from parity at some periods of time cannot invalidate the theory. The validity of PPP is more appropriately assessed by examining exchange rate behavior in the long run. Unfortunately a definite conclusion cannot be drawn from existing literature of PPP as a long run equilibrium relationship. Studies on time series behavior of real exchange rates during the recent float period (1973 onward) generally fail to reject the null hypothesis that the series contains a unit root, implying deviations from PPP are permanent rather than transitory. There is no tendency for the real exchange rate to return to a long run average. Under these conditions, PPP as a long run equilibrium relationship would not be expected to hold. Consistent with studies on time series behavior of real exchange rates, cointegration studies based on the Engle-Granger procedure generally fail to establish a cointegrating relationship between the nominal exchange rate and relative price indices for the recent float period, meaning that the two series could depart from one another without bound. Thus, it seems that PPP as a long run equilibrium hypothesis has finally been knocked out. However, in a seminal work, Frankel (1986) showed that the traditional Dickey-Fuller type unit root tests are likely to have low power in discriminating nonstationary and near nonstationary series, such as slowly mean reverting real exchange rates. Frankel's analysis casts considerable doubt on PPP tests based on small sample periods. To improve the test power, researchers follow two broad directions. One group uses long-horizon data (usually more than one hundred years of annual observations) while the other group employs panel data to take advantage of cross sectional variation. Results from these more recent studies have been much more supportive of PPP as a long run equilibrium relationship. The prevailing view is that real exchange rates are, indeed, slowly mean-reverting, and that given a sufficiently long time span, the hypothesis that they are nonstationary, can often be rejected. In addition, tests based on cointegration analysis tend to confirm the existence of a stable long run relation between nominal exchange rates and relative prices.

Given that many open macroeconomic models are built on the assumption that PPP holds as a long run equilibrium relationship, it is understandable that the recent supportive findings have been warmly accepted by many researchers. However, the great theoretical implications of PPP also require that any skepticism about its validity be erased before a celebration can be held. There are some troubling facts that make findings from previous studies based on long-horizon or panel data suspicious. The data sets used in the long-horizon studies usually span more than one hundred years. Since the components and their weights of price index in each country changes over time as new products and services are added to the consumption mix, the components of price index at the start and end of such a long sample period might have changed beyond recognition. Also, the long-span sample periods cover different nominal exchange rate regimes. It is well-established that real exchange rates are more volatile under a flexible exchange rate regime than they are under a fixed exchange rate regime. Given the different behavior of real exchange rates under different nominal exchange rate regimes, the possibility of structural shifts during the sample periods that cover both flexible and fixed nominal exchange rate regimes cannot be easily ruled out. Tests ignoring this possibility may lead to biased estimation and misleading conclusions. The problem with previous tests using panel-data is that the potential cross sectional correlations have been largely ignored in these tests. In the increasingly integrated world economy, worldwide shocks inevitably cause co-movements in some variables among a group of highly interdependent countries. If cross sectional correlations are indeed present in the data set and are ignored, the test results could be biased or even completely reversed. Also, Papell (1997) cautions that the inferences based on the panel method can be sensitive to sample selection, in particular, to the size of the panel as well as the grouping of countries. The existence of these weaknesses requires the findings from long-horizon and panel tests be treated with caution. However, even if one assumes that the test results are reliable, they are not strong enough to validate PPP as a long run equilibrium relationship. Findings of mean reversion and cointegration merely indicate that a long-term relation exists among the exchange rate and relative prices. However it is very difficult for one to believe that this long run relationship is what is implied by PPP. Since the symmetry and homogeneity restrictions implied by PPP are strongly rejected and the mean reversion of the real exchange rate is unreasonably slow, there would appear to be more to exchange rates than simply relative prices. Thus, the key to resolving the failure of PPP lies in understanding the forces that keep a nominal exchange rate away from a PPP equilibrium. One traditional explanation for the failure of PPP is that market frictions, such as transport costs and tariffs, impede international goods arbitrage, allowing departures from PPP to grow. If this theory is correct, then one would expect large deviations from PPP to be less persistent than small differences. O'Connell (1998) investigates this hypothesis over the current floating period. His findings are strongly negative. Specifically, it appears that large deviations from PPP can be more persistent than small deviations. This implies that market frictions alone cannot explain the failure of PPP during the recent float period (1973 onward). Another frequently cited explanation for the deviations from PPP is the rigidity of prices in the face of monetary shocks. This explanation could be convincing if departures from PPP were relatively short. However, the consensus from studies that obtain mean-reversion results is that the half-life of PPP deviations is between three to five years. For example, Abauf and Jorion (1990) use 1901-1972 data for eight currencies and find strong rejections of the random walk model. Their estimates suggest a half-life for PPP deviations of 3.3 years. Lothian and Taylor (1995) test the random walk hypothesis on two centuries of data (1791-1990) for the dollar-pound exchange rate. Evidence of mean-reversion is found and the half-life of PPP deviations is estimated to be 4.7 years. Rogoff (1996) points out:

> It would seem hard to explain the short-term volatility without a dominant role for shocks to money and financial markets. But given that such shocks should

> be largely neutral in the medium run, it is hard to see how this explanation is consistent with a half-life for PPP deviations of three to five years. (p.664)

Another justification for continuing the research on PPP is the potentially great importance of PPP as both a theoretical tool for analyzing exchange rate changes and a practical policy guide for foreign exchange market intervention. Modern models of exchange rate dynamics, such as Dornbusch (1976), hinge on the validity of long-run PPP theory, while many other macroeconomic models often use PPP to link domestic and foreign developments. It has been frequently proposed that monetary authorities adopt a role of intervening in the foreign exchange markets whenever observed deviations from PPP exceed a certain amount. Underlying this suggestion is the presumption that exchange rate changes eventually offset fully all changes in relative national prices, so that deviations from PPP are temporary departures from long-run equilibrium that the market will eventually "correct". Then authorities may, under such circumstances, be able to reduce the variability of the real exchange rate by countering PPP deviations through intervention. The concept of PPP is also used as a conversion factor to translate data from denomination in one national currency into another. This is important in cross country income comparisons and settlements of international claims.

One reason for the general empirical failure of PPP could be the misspecification of the model, in the sense that PPP defines the price ratio as the self-sufficient and independent determinant of the exchange rate between the two currencies. If, in fact, other variables are involved in the determination of the exchange rate and are excluded from the model, the estimates for the coefficients in the empirical PPP model would be biased and inconsistent. Therefore the symmetry conditions between the effect of home and foreign prices on the exchange rate and the homogeneity condition between changes in the exchange rate and the price ratio could be rejected even though they are in fact true. On the other hand, the interpretations of the time series properties of the real exchange rate that have been found in previous studies would be greatly complicated if variables other than relative prices are also present in the determination of the real exchange rate. Suppose the long-run equilibrium level of the real exchange rate is jointly determined by several factors including the price ratio, an exogenous shock that knocks the real exchange rate out of equilibrium would trigger an adjustment process simultaneously in all the variables that jointly define the equilibrium. The time needed for the real exchange rate to return to its long-run equilibrium level would be a result of interactions among the adjustments in all these variables. Therefore the persistence of deviation from its long-run equilibrium in the real exchange rate would not measure the simple relations between the exchange rate and the relative prices. In this situation, the only way to explain the behavior of the exchange rate correctly is to explicitly specify other variables that are involved in the determination process.

The goal of this dissertation is to augment the conventional theory of purchasing power parity with several fundamental economic variables that potentially have systematic impact on the level of the real exchange rate. The variables considered include (1) relative growth rate in gross domestic product (GDP), as a proxy for the productivity bias proposed by Balassa (1964) and Samuelson (1964); (2) relative accumulated current account balance as a percentage of GDP; models of exchange rate determination proposed by Dornbusch (1976) and Dornbusch and Fisher (1980) suggest that a relative increase in current account balance will raise the exchange value of domestic currency against the foreign currency through wealth and portfolio effects; (3) the real interest rate differential; many monetary models predict that a relative increase in real interest rates will appreciate the domestic currency; and finally (4) the relative changes in terms of trade; some researchers argue that a relative improvement in terms of trade tends to increase the exchange value of domestic currency through a wealth effect. All these economic arguments can be formulated into testable hypotheses. Given the exclusive nature of PPP as it is explicitly specified, the empirical validity of PPP should be examined to clear the way for a more general model of exchange rate determination. Therefore the hypotheses that will be tested in this dissertation are:

H1: The exchange rate changes proportionately to the ratio of the prices in the two countries (PPP)

H2: The currency of a relatively fast-growing (slow-growing) country tends to appreciate (depreciate).

H3: A relative increase (decrease) in the current account balance leads to appreciation (depreciation) of the domestic currency.

H4: A relative rise (fall) in the real interest rate leads to appreciation (depreciation) of the domestic currency.

H5: A relative improvement (deterioration) in the terms of trade leads to an appreciation (depreciation) of the domestic currency.

The above five hypotheses are tested using data for 10 OECD countries: Australia, Japan, Germany, Canada, United Kingdom, France, Italy, Netherlands, Norway and the United States. The sample period is 1973:Q1 though 1998:Q4, corresponding to the recent floating exchange rate period. Quarterly data are used to gain sufficient degrees of freedom and to avoid introducing "noise" by overly disaggregating the data. All the data are taken from the IMF International Financial Statistics CD ROM issued July 2000. Since the data series are potentially nonstationary, the multivariate cointegration methodology developed by Johansen and Juselius (1990) is adopted for the empirical testing. The first step of the procedure is to discover the number of potential cointegrating relationships among all the variables discussed above as well as relative prices, and the second step is to test the five hypotheses by imposing restrictions on the cointegrating vectors. This procedure is not subject to the limitations associated with the single equation approach, such as Engle and Granger procedure. While single equation estimation is convenient and often efficient, in some instances only the estimation of a system can provide sufficient information. If there are $n > 2$ variables in the model, and if $n - 1$ of these variables are not weakly exogenous, the single equation approach can be misleading, particularly if more than one cointegration relationship is present.

The remainder of this study is structured as follows: Chapter 2 provides a detailed description of recent empirical work on long run PPP. The empirical failure of PPP and the need for a more general model of equilibrium exchange rate are highlighted. Details of the data, variable construction and the methodologies used in this study are discussed in chapter 3. Empirical evidence is presented in Chapter 4. Chapter

5 concludes the study by summarizing the major findings and pointing out limitations of this study and directions for future research.

CHAPTER 2

DOES PPP REALLY HOLD? EMPIRICAL EVIDENCE

In this chapter I present an evaluation of recent evidence on the empirical validity of long run equilibrium purchasing power parity. Recent studies in the area have followed one of two broad lines. One line of the studies uses various cointegration techniques to investigate the long run equilibrium relationship between the nominal exchange rate and the relative prices. Studies in this line are direct tests on the long run equilibrium purchasing power parity. The existence of a cointegration relationship between the nominal exchange rate and the relative prices is generally interpreted as supportive evidence for PPP, while the failure to find the cointegration is regarded as negative evidence for PPP. The other line of study uses unit root or variance ratio tests to examine the time series property of real exchange rates. Studies in this line constitute indirect tests of PPP. If a real exchange rate series is found to be stationary, a long run PPP relationship is considered to exist between the two currencies concerned. On the other hand, a real exchange rate series that demonstrates random walk would deny the existence of PPP. In the last section of this chapter, I will argue that the prevailing interpretations of findings in both lines of the studies are inappropriate.

2.1: The Theory of Purchasing Power Parity

The theory of purchasing power parity has two major variants: absolute PPP and relative PPP. The absolute PPP is a direct extension of the law of one price. Assuming zero information and transaction costs, the exchange-adjusted prices of identical tradable goods and financial assets must be equal worldwide in competitive markets. An equivalent formula expression is:

$$p_{it}^{d} = S_t p_{it}^{f}, \qquad (1)$$

where p_{it}^{d} denotes the domestic price of good i at time t, p_{it}^{f} denotes the foreign price of good i at time t, and S_t denotes spot exchange rate at time t defined as the home currency price of a unit of foreign currency. Equation (1) is maintained by international arbitrage. Thus, if for some reason the left-hand side of (1) is greater than the right-hand side, it would be profitable to ship the good from the foreign country to the domestic country thereby forcing the domestic currency value of the foreign good up (by a rise in S_t and/or p_{it}^{f}) and the domestic price of the good down, until the equity between the two prices is restored. Since Equation (1) holds for each and every one of the goods, it must also hold for any bundle of the goods assuming the same goods in each country enter the bundle with a equal and constant share α_i. Therefore the following equation holds:

$$\sum_{i=l}^{n} \alpha_i p_{it}^{d} = S_t \sum_{i=l}^{n} \alpha_i p_{it}^{f} \qquad (2)$$

Let $P_t^d = \sum_{i=1}^{n} \alpha_i p_{it}^d$, $P_t^f = \sum_{i=1}^{n} \alpha_i p_{it}^f$ and rearrange equation (2), the condition of absolute PPP can be obtained:

$$S_t = P_t^d / P_t^f \tag{3}$$

Equation (3) states that the long run equilibrium exchange rate between two currencies is determined by the ratio of price levels in the two countries. An increase in the domestic price level, generated, say, by a monetary expansion, should result in an proportionate depreciation of the exchange rate (a rise in S). However, it is clear from the above derivation process that absolute PPP is based on a set of extremely restrictive assumptions. For example, it requires zero information and transaction costs, no trade barriers such as tariffs and quotas, each country produces the same bundle of goods and services, and all the goods and services are tradable, thereby subject to international arbitrage. Any violations of these assumptions will prevent PPP from holding exactly. Nevertheless, proponents of PPP argue that if such factors are assumed constant over time, then either condition (1) or (2) would be expected to hold up to a constant factor Ψ.

$$S_t = \Psi P_t^d / P_t^f \tag{4}$$

or in logarithm form:

$$s_t = \psi + p_t^d - p_t^f, \tag{4'}$$

where lower case letters now indicate that the level of the variable has been transformed using the natural logarithm operator.

The relative version of PPP has a different message from the absolute version. It says that the change in the exchange rate from base period to current period equals the change in the relative purchasing power of the two currencies. An expression for relative PPP can be obtained from Equation (4) by dividing the nominal exchange rate and the price levels by their respective base period values. Since the fixed wedge Ψ is eliminated in relative PPP, this yields:

$$S_t / S_0 = \left(P_t^d / P_0^d \right) / \left(P_t^f / P_0^f \right) = PI_t^d / PI_t^f, \tag{5}$$

where PI_t^d and PI_t^f are the price indices for the domestic and foreign countries, respectively. Taking the log of both sides of Equation (5), the following is obtained:

$$s_t = s_0 + pi_t^d - pi_t^f, \tag{6}$$

where s_t and s_0 are the logs of the nominal exchange rate at time period t and zero, respectively, and pi_t^d and pi_t^f are the logs of the domestic and foreign price indices.

Several points should be noted about the theory of PPP as outlined above. First, the theory is derived by assuming that price indices in both countries are constructed using a common basket of goods and an equal weighting scheme. It is clear that the price measures implied in the theory are not available in the real world. Therefore the first problem facing an empirical researcher is to choose appropriate price indices to be used in the tests. Between the two popular price indices used in PPP testing, the consumer price index (CPI) or the wholesale price index (WPI), many researchers prefer to use WPI, on the grounds that the WPI contains relatively a larger share of traded goods than the CPI, and therefore will be more likely to yield a supportive result. But the appropriateness of using the WPI in testing PPP is questionable. Although free international trade is the required mechanism through which the purchasing power parity between two currencies is formed, absolute PPP is defined by general price levels, representing prices of all goods and services available for purchasing. The theory accepts the fact that there are nontraded goods but notes that the prices of traded and nontraded goods are closely related through various links. As Haberler indicates:

The Proposition that general price levels in different countries are connected through the prices of internationally traded goods is the foundation of the purchasing power parity doctrine. (1975, p.24)

In conducting an empirical test, a researcher should not be biased for or against any particular results. The appropriately chosen price indices should meet two requirements: (1) reflect the implications of the theory; (2) minimize the differences across countries with regard to components and weighting scheme of the indices. Based on these two requirements, CPI is used in this study.

Second, PPP is exclusive. The theory accepts that several non-price factors such as trade restrictions, transportation costs and speculation in foreign exchange markets can affect exchange rates and cause deviations from PPP in the short run. But the ratio of price levels between two countries is the self-sufficient and independent explanation of the long run equilibrium exchange rate. Although several authors, Officer (1976) for example, argue that PPP is not closed to other potential explanatory variables, this "open version" of PPP has never been explicit and is clearly not the same as the theory generally understood as purchasing power parity. One implication of this exclusive nature is that it makes modeling real exchange rates a pointless effort. Another implication is that PPP can be tested indirectly. If only one variable is identified as a significant explanatory variable of the long run equilibrium exchange rate, PPP would be invalidated.

Third, the causation between the exchange rate and the relative prices could be bidirectional. Many previous studies intentionally or unintentionally imposed a causality assumption that runs unidirectionally from the ratio of price indices to exchange rates. This implicit assumption has neither a theoretical nor an empirical basis. The proposition of PPP simply states that percentage changes in the exchange rate should equal percentage changes in the ratio of price indices. It does not necessarily imply that relative-price movements cause exchange rate fluctuations. It is quite possible for the causation to run in the opposite direction. Starting from initial equilibrium, suppose there is a one-shot capital outflow from the home country, the nominal exchange value of the home currency will depreciate. According to Dornbusch (1976), the real exchange value of the home currency will also depreciate due to the stickiness of commodity prices in the short run. How will this disequilibrium be corrected? It is clearly possible that this adjustment takes place, at least in part, if not wholly, by prices reacting to the initial change in the exchange rate. This simultaneous relationship between exchange rate and price has important implications in empirical testing of PPP. Procedures that ignore this potential simultaneity could result in biased and inconsistent estimates.

Finally, the economic conditions during the sample period in an empirical test of PPP should be reasonably stable. This is because the relative version of PPP is derived from the absolute PPP under the assumption that economic conditions have been constant since the base period. The factors that are assumed to be constant include, among others, the severity of trade restrictions, the level of transport costs, the mobility of capital flows across countries and the internal relative prices. Any change in these factors will cause a deviation of computed relative parity from the long run equilibrium exchange rate. For example, capital flows may have become increasingly mobile over time. If the affected balance of payment flows have shifted in direction or magnitude since the base period, part of the changes in the long run exchange rate would not be captured by the relative-parity computation. Since the beginning of the sample implicitly serves as the base period for the following periods within the sample in the tests using time series data, the implication is that the sample period should not be so long that the economic conditions are significantly different between the beginning and the end of the sample. This presents a dilemma for empirical researchers. The assumption of constant economic conditions requires the sample period to be reasonably short, while tests of long run relationships require a considerable amount of data. Balancing the two requirements is important for a reliable test result.

2.2: Studies Based on Nominal Exchange Rates

Recent empirical research on the theory of PPP is largely driven by new developments in time series methodologies, particularly those related to cointegration and unit root testing. From the early 1980's, applied economists working with time series data became aware of certain difficulties that arise when unit roots are present in the data, that is, when the time series of the variables are nonstationary. The estimates obtained from applying traditional regression procedure to nonstationary variables may be true or spurious representation of the long run economic relationships, depending on whether or not there exists at least one linear combination of these nonstationary variables that will yield a stationary series. The regression coefficients represent true long run equilibrium relationships among the nonstationary variables only when there exists at least one such linear combination. The long run equilibrium (stationary) relationships among two or several nonstationary variables are described in the literature as cointegration. If two or more series are linked by some economic mechanism to form an equilibrium relationship spanning the long run, then even though the series themselves may be nonstationary, they will nevertheless move together over time and the difference between them will be stable. Such a set of variables are said to be cointegrated. In statistical terms, cointegration can be defined as follows: If a series must be differenced d times before it becomes stationary, then it contains d unit roots and is said to be integrated of order d, denoted as $I(d)$. Consider time series Y_t and X_t, which are both $I(d)$ series. If the series of disturbance terms obtained from regressing Y_t on X_t is of a lower order of integration, $I(d\text{-}b)$, where $b>0$, then Y_t and X_t are defined as cointegrated of order $(d\text{-}b)$, denoted as $CI(d\text{-}b)$.

Since the theory of PPP represents the long run equilibrium relationship between nominal exchange rates and relative prices, and both exchange rates and price indices are usually found to be nonstationary $I(1)$ series, cointegration methodologies can be neatly applied in testing of PPP. It is not surprising that all the new developments in cointegration methodology quickly found their applications in empirical studies of PPP. The following discussion of recent literature is organized by the testing procedure and presented largely in chronological order. Namely, single equation cointegration tests, multivariate cointegration tests, panel cointegration tests, and finally, fractional cointegration tests.

2.2.1: Single equation cointegration tests

Engle and Granger (1987) proposed a two-step procedure for a test of cointegration relationships among integrated variables of the same order. In the first step, the parameters of the cointegrating vector are estimated by running the static regression in the levels of the variables. In the second step, the series of residuals from the cointegration regression is tested to see if it contains a unit root. If the null hypothesis that the residual series contains a unit root is rejected, i.e., the residual series is stationary, then the variables are said to be cointegrated, and the cointegrating vector is the true representation of the long run equilibrium relationship among the variables.

To apply the Engle-Granger two-step procedure in testing of PPP, the first step is to estimate the following equation by ordinary least squares:

$$s_t = \alpha + \beta_0 p_t^d + \beta_1 p_t^f + e_t. \quad (7)$$

Equation (7) is the regression equation analogous to Equation (4'). In order to support the existence of long run PPP, three conditions should be satisfied. First, the error series e_t from the estimated version of equation (7) must be stationary, or equivalently, a *I(0)* process. This is the most important condition. For in its absence, the set of estimated parameters in Equation (7) would not be the true representation of the long run relationships among s_t, p_t^d, and p_t^f. Equation (7) would be misleading. The second condition that PPP requires is $\beta_0 = -\beta_1$, implying domestic and foreign prices affect the exchange rate in equal magnitude but opposite directions. This is usually described in the literature as the condition of symmetry. The third condition, usually referred to as the condition of proportionality, is $|\beta_0| = |\beta_1| = 1$, implying a rise in a country's price level will cause an equiproportionate depreciation of the country's currency. It is clear that the theoretical implications of PPP is reflected in the conditions of symmetry and proportionality, while the role of the first condition, i.e., the condition of cointegration, is simply to ensure that the relationships as described in the set of the estimated parameters truly reflect the underlying long run economic linkage among the variables in the equation.

The second step in the Engle-Granger procedure is to conduct a unit root test on the residual series, $\hat{e}_t$. The most popular method to accomplish the task is the Augmented Dickey-Fuller (ADF) test, which consists of regressing the first difference of the variable under consideration on its own lagged level, a constant, and to control for autocorrelation, an appropriate number of lagged first differences. Formally, the equation of the following form is estimated by ordinary least squares:

$$\Delta \hat{e}_t = \psi \hat{e}_{t-1} + \sum_{i=1}^{p-1} \psi_i \Delta \hat{e}_{t-i} + \mu + \eta_t \qquad \eta_t \sim IID\,(0,\sigma^2) \qquad (8)$$

The coefficient estimate on the lagged level ψ is crucial. If ψ is not significantly different from zero, then the residuals e_t are a *I(1)* process and the null of no cointegration could not be rejected. The test statistic, denoted τ, is simply the estimate of the coefficient ψ divided by its standard error. Since this test statistic does not have the usual *t*-distribution, the significance test of ψ is obtained by comparing the test statistic with a critical value calculated from nonstandard distribution, which is tabulated in Fuller (1976).

Taylor (1988) applies the Engle-Granger two-step procedure to five U.S. dollar based bilateral exchange rates (UK pound, West German mark, French franc, Canadian dollar, and Japanese yen) over the period from June 1973 through December 1985. The relative price series are constructed from manufacturing output price indices to correspond broadly to the price of tradables. The model is specified to allow for possible measurement errors and/or transportation costs. Although the slope coefficients in every case are correctly signed and in some cases (for the UK pound and Japanese yen) are reasonably close to unity, the null hypothesis of no cointegration could not be rejected. Therefore the linear regressions involving the price levels of the two countries concerned, as well as the bilateral exchange rate, can only be interpreted as spurious regressions in the sense of Granger and Newbold (1974). The results are extremely unfavorable to the PPP hypothesis considering that the model specification and data construction are biased toward supporting it as a long run equilibrium condition. Corbae and Ouliaris (1988) apply the same procedure to six bilateral exchange rates (Canadian dollar/U.S. dollar, French franc/ U.S. dollar, Italian lira/U.S. dollar, UK pound/U.S. dollar, West German mark/U.S. dollar, and Japanese yen/U.S. dollar) over the sample period beginning July 1973 and ending September 1986. Their relative price indices are constructed from consumer price indices. Similar to Taylor (1988), they could not reject the null of no cointegration between the exchange rates and relative prices for any one of the six pairs of currencies. Using wholesale price indices (WPI), Enders (1988) adopts Engle-Granger cointegration methodology to examine whether PPP holds for the U.S. dollar vis-à-vis the West German mark, Canadian dollar, and Japanese yen over two sample periods: January 1960-April 1971 (representing a period of fixed exchange rate regime) and January 1973-November 1986 (representing a period of floating exchange rate regime). The slope coefficients in all three cases are far from the theoretical value (unity). Except for the cases of Japanese yen / U.S. dollar during the Bretton Woods period and Canadian dollar/U.S. dollar after 1973, the null hypothesis of no cointegration could not be rejected for all the other instances. Mark (1990) uses the same procedure to investigate a number of OECD bilateral exchange rates based, respectively, on the U.S. dollar, UK pound, and Japanese yen as the home currency for the period of June 1973 to February 1988. He chose CPI as the relevant price index. Like other researchers, he finds little support for PPP as a long run equilibrium relationship. In twelve of thirteen instances, the null of no cointegration could not be rejected. Layton and Stark (1990) examine the cointegration relationship between the U.S. inflation rate (as measured by the consumer price index) and an effective-exchange-rate adjusted inflation series computed from its six major trading partners, namely, Canada, France, Germany, Italy, Japan, and the United Kingdom. The Engle-Granger two-step procedure is applied to the sample period from January 1963 to December 1987. In no instances, regardless of whether the sampling frequency is monthly or quarterly, could the null of no cointegration be rejected. When the procedure is applied to two subsample periods, namely, 1963-1973 inclusive (fixed exchange rate regime) and 1974-1987 inclusive (flexible exchange rate regime), they find that the data appears to be more favorable to PPP during the fixed exchange rate period than in the flexible exchange rate period.

Since the critical values for t-test with nonstandard distribution vary with the number of regressors, and in their initial paper Engle and Granger only computed critical values for τ for a regression equation with two variables, the studies by Taylor (1988), Corbae and Ouliaris (1988), Enders (1988) and Mark (1990) constrain the coefficient on the relative price terms to be equal and in opposite signs (that is they impose symmetry). A paper by Engle and Yoo (1987) tabulates critical values for τ from a regression of up to five variables and these are used by Patel (1990) to estimate equation (7) in unconstrained fashion. Patel uses a quarterly data base spanning the period 1974-86 for Canada, Germany, Japan, the Netherlands, and the United States (a variety of bilateral exchange rate combinations are considered for these countries) and reports that the null is rejected in only 4 instances out of a total of 15. MacDonald (1995) estimates equation (7) using a data set consisting of bilateral U.S. exchange rates and relative consumer and wholesale prices for nine currencies (Canada, France, Germany, Italy, Japan, Netherlands, Sweden, Switzerland, United Kingdom)) over the period March 1973 to December 1992. Although the slope coefficients are correctly signed in most of the cases, their values are far from the theoretical value (1 and –1). More importantly, whether WPI or CPI is used as the relevant price measure, the null hypothesis of no cointegration could not be rejected in any instances.

Not all studies using the Engle-Granger two-step procedure have failed to find cointegration between the exchange rate and the relative prices. Using monthly data over the period of October 1950 through May 1961, Choudhry, McNown and Wallace (1991) find evidence of cointegration for both WPI and CPI series between the United States and Canada. Some evidence of cointegration between U.S. and UK data are also reported.Using annual data over the period 1900 to 1987, Kim (1990) applies a single equation cointegration test to the bilateral exchange rates of the United States vis-à-vis five currencies: Canada, France, Italy, Japan, and the United Kingdom. He finds the nominal exchange rate is cointegrated with both the WPI-ratio and CPI-ratio for the French Franc and Italian lira. For Japanese yen and British pound, cointegration is found for the WPI-ratio but not for the CPI-ratio. Only for the Canadian dollar no cointegration is found for either price ratios. When cointegration is confirmed, the cointegrating coefficient between the exchange rate and the price ratio is close to 1. Kim's explanation for his failure to find cointegration for the Canadian dollar is limited variability in the data. The general conclusion obtained from Kim's study is that the exchange rate and price ratio are cointegrated in the long run and this cointegration relationship is stronger between the exchange rate and WPI ratio than between the exchange rate and CPI ratio.

2.2.2: Multivariate cointegration test

The Engle-Granger two-step cointegration test is popular for its simplicity and its "super-consistency". According to the 'super-consistency' property, if Y_t and X_t are both nonstationary $I(1)$ variables, and $e_t \sim I(0)$, then as sample size, T, becomes larger, the ordinary least square estimate of β converges to its true value at a much faster rate than the usual ordinary least square estimator with stationary $I(0)$ variables (Stock, 1987). But the methodology possesses certain potential limitations. First, large finite-sample biases can arise in static single equation OLS estimates of cointegrating vectors. While these estimates will be super-consistent, Banerjee, Dolado, Hendry and Smith (1986), through Monte Carlo experiments, find that large sample sizes (relative to those found in economics) may be required before the biases become small. On the other hand, the use of dynamic regression may mitigate the bias in the static single equation approach and dynamic regressors may be more robust to a range of data generating processes. In particular, tests of cointegration with a vector autoregressive framework, such as that of Johansen and Juselius (1990), often possesses not only better statistical properties but the power of the cointegration test is also generally higher.

Second, while single equation estimation is convenient and often efficient, in some instances only the estimation of a system can provide sufficient information. For example, a system approach is necessary where the estimation of multiple cointegration vectors is desired. Finally, if weak exogeneity is absent, the single equation approach can have a detrimental effect on the bias and efficiency of OLS estimators and system approaches to estimation have typically been used in these instances.

Partly because the limitations of a single equation approach and partly because of a lack of convincing economic interpretations for the failures to find cointegration between the nominal exchange rate and the ratio of price indices, a number of researchers have argued that the failure to find a cointegrating relationship between relative prices and an exchange rate may be due to the econometric method used, rather than the absence of a long run relationship. The multivariate testing procedure developed by Johansen and Juselius (1990) provides researchers in the area of PPP with more sophisticated methodology to unearth the potential cointegrating relationships between the exchange rates and the relative prices.

Johansen and Juselius (1990)'s maximum likelihood procedure for testing and modeling cointegrated processes is based on the vector error correction representation. Defining a vector $\boldsymbol{X_t}$ of n potentially endogenous variables, it is possible to model $\boldsymbol{X_t}$ as an unrestricted vector autoregression (VAR) involving up to k-lags of $\boldsymbol{X_t}$:

$$\boldsymbol{X}_t = \boldsymbol{A}_1 \boldsymbol{X}_{t-1} + \ldots + \boldsymbol{A}_k \boldsymbol{X}_{t-k} + \theta_t , \qquad (8)$$

where $\boldsymbol{A}_i$ is an $(n \times n)$ matrix of parameters, and θ_t is $IN(0,\Sigma)$ distributed. Equation (8) can be reformulated into a vector error correction (VECM) form:

$$\Delta \boldsymbol{X}_t = \Gamma_1 \Delta \boldsymbol{X}_{t-1} + \ldots + \Gamma_{k-1} \Delta \boldsymbol{X}_{t-k-1} + \Pi_{t-k} \mathbf{X}_{t-k} + \theta_t , \qquad (9)$$

where $\Gamma_i = -(I - A_1 - ... - A_i)$, ($i=1$, ... , $k-1$), and $\Pi = -(I - A_1 - ... - A_k)$. This way of specifying the system contains information on both the short run and long run adjustment to changes in X_t, via the estimates of $\hat{\Gamma}_i$ and $\hat{\Pi}$ respectively. Johansen and Juselius (1990) show that Π can be factorized into $\Pi = \alpha\beta'$ assuming cointegration exists, where α represents the speed of adjustment to disequilibrium, while β is a matrix of long run coefficients. Thus the term $\beta' X_{t-k}$ embedded in equation (9) represents cointegration relationships in the multivariate model which ensure that X_t converge to their long run steady-state solutions. Johansen (1988) adopts the procedure known as reduced rank regression to obtain the estimates of α and β'. First, rewriting Equation (9) as:

$$\Delta X_t + \alpha\beta' X_{t-k} = \Gamma_1 \Delta X_{t-1} + ... + \Gamma_{k-1} \Delta X_{t-k-1} + \theta_t, \tag{10}$$

and regressing ΔX_t and X_{t-k} separately on the right-hand side of Equation (10) to produce the residuals R_{0t} and R_{kt}:

$$\Delta X_t = P_1 \Delta X_{t-1} + ... + P_{k-1} \Delta X_{t-k-1} + R_{0t}, \tag{11}$$

$$X_{t-k} = T_1 \Delta X_{t-1} + ... + T_{k-1} \Delta X_{t-k-1} + R_{kt} \tag{12}$$

Then R_{0t} and R_{kt} are used to form cross product matrices:

$$S_{ij} = \sum_{i=1}^{T} R_{it} R'_{jt} \Big/ T \qquad i, j = 0, k \tag{13}$$

The maximum likelihood estimate of $\boldsymbol{\beta}$ is obtained as the eigenvectors corresponding to the r largest and significant eigenvalues from solving the equation

$$\left| \lambda S_{kk} - S_{k0} S_{00}^{-1} S_{0k} \right| = 0 \tag{14}$$

which gives n eigenvalues $\hat{\lambda}_1 > \hat{\lambda}_2 > \ldots > \hat{\lambda}_n$ and their corresponding eigenvectors $\hat{V} = (\hat{v}_1, \ldots, \hat{v}_n)$. The significance of eigenvalues can be tested using the follow two maximum likelihood test statistics

$$\lambda_{trace} = -T \sum_{i=r+1}^{n} log(1 - \hat{\lambda}_i) \qquad r = 0, 1, 2, \ldots, n\text{-}2, n\text{-}1. \tag{15}$$

and

$$\lambda_{max} = -T\, log(1 - \hat{\lambda}_{r+1}) \qquad r = 0, 1, 2, \ldots, n\text{-}1, n\text{-}2. \tag{16}$$

The first test statistic is called λ-trace statistic and it is used to test the null hypothesis that there are at most *r* cointegration vectors. The second statistic is called λ-max statistic, which tests the null hypothesis that there are *r* cointegration vectors against the alternative that $r + 1$ exist. The critical values for both test statistics are tabulated in Osterwald-Lenum (1992). Once $\hat{\beta}$ is obtained, $\hat{\alpha}$ can be solved out from the relation $\Pi = \alpha\beta$.

Kugler and Lenz (1993) apply the procedure outlined above to fifteen bilateral exchange rates of the German mark against the Swiss franc, French franc, Italian lira, UK pound, U.S. dollar, Japanese yen, Austrian schilling, Dutch guilder, Belgian franc, Spanish peseta, Swedish krone, Danish krone, Canadian dollar, Portuguese escudo and Norwegian krone. The price levels are measured by the consumer price index and the monthly data cover the period from January 1973 through November 1990. The authors include a constant in their model. To check the robustness of the results with respect to the lag length, they experiment with two different lag structures defined by multivariate criterion and multivariate Akaike criterion respectively. Ten currencies (Italian lira, UK pound, U.S. dollar, Austrian schilling, Belgian franc, Spanish peseta, Danish krone, Canadian dollar, Portuguese escudo and Norwegian krone.) are found to be cointegrated while the results for the remaining five currencies (Swiss franc, French franc, Japanese yen, Dutch guilder and Swedish krone) are mixed, depending on which lag structure is used. The French franc, Japanese yen and Dutch guilder pass the cointegration test when the shorter Hannan-Quinn Lag length is adopted but fail the test when Akaike lag length is used. The reverse is true for Swiss franc and Swedish krone. Of the ten currencies that pass the cointegration test under both lag structures, only in the case of six of currencies (Italian lira, UK pound, Austrian schilling, Spanish peseta, Portuguese escudo and Norwegian krone) can the hypothesis that the PPP vector is an element of the cointegrating space not be rejected. For the other four currencies (U.S. dollar, Canadian dollar, Belgian franc and Danish krone), PPP clearly does not hold, although the exchange rates and the price levels are cointegrated.

MacDonald (1993) tests PPP in its strong-form as well as weak-form using data for five U.S. dollar based bilateral exchange rates. The strong-form PPP requires not only the existence of cointegration but also the theoretical vector (1, 1, -1) be included in the cointegrating space, while the weak-form requires only the existence of the cointegration. The currencies involved, except the U.S. dollar, are Canadian dollar, French franc, German mark, Japanese yen, and UK pound. Two alternative price series are used, namely, the wholesale price index and consumer price index. The data period runs from January 1974 to June 1990. His model specification includes eleven seasonal dummies. Twelve lags in the underlying VAR are used to ensure residual whiteness and account for any seasonality not captured by the seasonal dummies. The test results provide supportive evidence for the weak-form PPP as defined by MacDonald but no evidence of the strong-form is find. Specifically, when wholesale price indices are used, all five currency-price combinations reveal evidence of one unique cointegrating vector. Cointegration could not be found for two out of five currency-price combinations if consumer price indices are used. There is an absence of cointegration for the Canadian dollar and Japanese yen when consumer prices are used but not when wholesale prices are used. Thus the null hypothesis of no cointegration could not be rejected in only two of ten instances. The test results change dramatically when the homogeneity hypothesis is imposed that a one percent increase in relative prices results in an equiproportionate increase in the dollar bilateral exchange rate. The null of no cointegration could not be rejected in any cases. The rejection of the homogeneity hypothesis is supported by the estimated cointegrating vectors, which are numerically quite far from their prior values although most of them are correctly signed.

As mentioned, MacDonald (1995) failed to find any evidence of cointegration when he applied the Engle Granger two step procedure to a data set consisting of bilateral U.S. exchange rates and relative consumer and wholesale prices for nine currencies (Canada, France, Germany, Italy, Japan, Netherlands, Sweden, Switzerland, United Kingdom)) over the period March 1973 to December 1992. In that study, the author also performs the Johansen and Juselius (1990) procedure on the same data set and the results obtained are quite different. Based on the λ-trace and λ-max statistics, there is evidence of at least one cointegrating vector for each currency apart from the Sweden franc and German mark. But similar to findings from his 1993 study, the proportionality and symmetry hypothesis are convincingly rejected for most currency-price combinations. Not only are all the estimated coefficients numerically quite far away from their hypothesized values, some of them are even wrongly signed. Nevertheless, the exercise shows that the test results of PPP could be sensitive to the procedures used.

Using monthly data from January 1975 to December 1991, Cochran and DeFina (1995) apply the Johansen and Juselius (1990) cointegration methodology to eleven sets of nominal exchange rates and price indices. The U.S. dollar is used as the 'pricing' currency and the other currencies involved are the Austrian shilling, Canadian dollar, Danish krone, French franc, German mark, Italian lira, Japanese yen, Norwegian krone, Swedish krone, Swiss franc, and UK pound. The authors include a constant in their VAR model and the lag structure is defined based on the multivariate Akaike (1974) criterion. For the unrestricted model, one or two cointegrating vectors are found for all currencies with the exception of the Danish krone. Two cointegrating relationships between nominal exchange rates and price indices exist for the Japanese yen, Swiss franc, and UK pound. When a further test on the restricted model is conducted for the set of currency-price combinations that contains one or two cointegrating vectors, the hypothesis that the theoretical vector is contained in the cointegration space is rejected for all instances. This is consistent with the findings that the estimated coefficients for the unrestricted models differ substantially from the theoretical values.

In summary, a considerable amount of evidence from the Johansen and Juselius (1990) cointegration approach supports the contention that there is indeed a long run equilibrium relationship between exchange rates and the relative price indices. However, often the restrictions of symmetry and proportionality implied by the theory of PPP are rejected in these studies, indicating that the cointegration unearthed in this group of studies could not be the same relationship as implied in PPP.

2.2.3: Panel cointegration tests

Multivariate cointegration methodology developed by Johansen and Juselius (1990) is successful in uncovering the cointegration between nominal exchange rates and relative prices over the recent float period, and thereby is generally regarded as more favorable to the theory of PPP than the Engle-Granger bivariate cointegration procedure. Nevertheless the fact that the empirical values of the cointegrating vector are far away from their theoretical priors and some times even wrongly signed is troubling. Many researchers blame the failure to find the homogeneity and symmetry implied in PPP on trade frictions, measurement errors, as well as the low power of the test methodology. To improve empirical results, several researchers suggest that panel data should be used in the test to take advantage of the cross sectional variation. The unrestricted empirical model of PPP in a panel data context can be specified as the following form:

$$s_{it} = \alpha_i + \beta_i^d p_{it}^d + \beta_i^f p_{it}^f + \varepsilon_{it}, \tag{17}$$

where i represents the set of countries 1, 2, ... , N, t represents the time period 1, 2, ... , T, ε_{it} is the residual term, s_{it} is a vector of exchange rates, and domestic and foreign prices are denoted by p_{it}^d and p_{it}^f, respectively. Since cointegration has been found for the recent float period in the Johansen and Juselius (1990) procedure, the role of the panel test is to examine some forms of restricted models. The most restricted model requires $\alpha_i = 0$ for i =1, 2, ... , N, and the symmetry and homogeneity restriction, $\beta_i^d = 1$ and $\beta_i^f = -1$. In recognizing the real world frictions, studies of this type usually impose only the symmetry restriction, $\beta_i^d = -\beta_i^f$. Equation (17) becomes:

$$s_{it} = \alpha_i + \gamma_i(p_{it}^d + p_{it}^f) + \varepsilon_{it}. \tag{18}$$

Equation (18) is a heterogeneous model. It will be a homogeneous model if γ_i is constrained to be equal for all i = 1, 2, ... , N. Since there is no reason to believe all the parameters are the same across countries, the heterogeneous model like Equation (18) is usually employed in the tests.

Pedroni (1995) develops asymptotic and finite-sample properties of test statistics to test the null hypothesis of no cointegration in panel data. He argues that in addition to the "spurious regression" problem, one needs to handle the difficulties introduced by the off-diagonal terms in the long run covariance of the residuals. Under the null of no cointegration, the "spurious regression" affects the asymptotic distributions in the panel, and this effect is likely to be more severe in the heterogeneous model. While consistent long run estimates can be obtained for the homogeneous model when the sample is large, the same results cannot be obtained under the heterogeneous model. Convergent panel statistics can be transformed into nonconvergent ones in the heterogeneous model. In order to cope with these problems, Pedroni (1995) suggests that the following test statistics should be used:

$$\rho \text{ statistic} \quad Z_\rho = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} L_{11i}^{-2}(\varepsilon_{it-1}\Delta\varepsilon_{it} - \eta_i)}{\sum_{i=1}^{N}\sum_{t=1}^{T} L_{11i}^{-2}\varepsilon_{it-1}^{2}}, \tag{19}$$

$$\text{Parametric t statistic} \quad Z_t = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} L_{11i}^{-2}\varepsilon^*_{it-1}\,\Delta\varepsilon^*_{it}}{\sqrt{\sigma^2\sum_{i=1}^{N}\sum_{t=1}^{T} L_{11i}^{-2}\varepsilon^{*2}_{it-1}}}, \tag{20}$$

$$\text{Nonparametric t statistic} \quad Z_{pp} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} L_{11i}^{-2}(\varepsilon_{it-1}\Delta\varepsilon_{it} - \eta_i)}{\sqrt{\sigma^2\sum_{i=1}^{N}\sum_{t=1}^{T} L_{11i}^{-2}\varepsilon_{it-1}^{2}}}, \tag{21}$$

where L_i is the ith component of the lower-triangular Cholesky decomposition of the long run asymptotic covariance matrix, Ω_i, which can be obtained by ε_{it}. Therefore, L^2_{11i} can also be constructed using the off-diagonal elements of Ω_i ($L^2_{11i} = \sqrt{\Omega_{11i} - \Omega^2_{21i}/\Omega_{22i}}$). Since the off-diagonal of Ω_i is non-zero, it allows idiosyncratic feedback effects in the statistics. The term ε_{it} is the residual of Equation (18) and is obtained from the nonparametric method, and ε^*_{it} is the residual obtained from the parametric model. Other terms are obtained as follows: σ^2 is a pooled, long run variance for the nonparametric model ($\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} L^{-2}_{11i}\sigma^2$), and $\eta_i = 1/2(\sigma^2_i - s^2_i)$. The term η_i is used to adjust for autocorrelation in parametric models, and σ^2_i and s^2_i are long run and contemporaneous variances for individual i, which can, in turn, be obtained from v_{it}, where $\varepsilon_{it} = \rho_i\varepsilon_{it-1} + v_{it}$. Similarly, σ^{*2} is the long run variance for the parametric model and can be obtained as $\sigma^{*2} = 1/N\sum_{i=1}^{N} s^{*2}_i$ and s^*_i is the individual contemporaneous variance obtained using the residuals of the parametric model. The critical values for these statistics are tabulated in Pedroni (1995), and a large negative value from ρ, Z_t and Z_{pp} suggests the rejection of the null of no cointegration.

Nagayasu (1998) applies the procedure outlined above to a panel of sixteen African countries. Since the exchange rates of many countries in the panel were not market determined during the sample period. The parallel market exchange rates are used in the study. The countries involved include Botswana, Burundi, Cameron, Gambia, Ghana, Kenya, Malawi, Mauritius, Mozambique, Nigeria, Sierra Leone, South

Africa, Tanzania, Uganda, Zambia, and Zimbabwe. The relative prices are measured by the consumer price indices, and the annual data cover the period from 1981 to 1994. The results clearly demonstrate improvement in the power of the panel test over the Johansen and Juselius (1990) procedure. All three statistics are negative and large enough to reject the null hypothesis of no cointegration. When the same data set is tested in individual multivariate methodology, the null could not be rejected for any one of these African countries. Considering the fact that the symmetry restriction is imposed on the test model, the result represents the strongest evidence in favor of PPP. Nevertheless, an important restriction implied in PPP, namely, the homogeneity restriction, could not be tested in the specification of the model, though the estimated coefficients, like those from conventional tests, are far from their priors. The estimated α's for Equation (18) range from 0.785 (Botswana) to 7.8 (Mozambique), with the average of 3.909 (the theoretical value is zero). The estimated γ's for Equation (18) ranges from 0.13 (Cameron) to 2.591(Burundi), with the average of 1.1(the theoretical value is 1).

2.3: Studies Based on Real Exchange Rates

Another broad category of empirical research on long run PPP follows the line of investigating the time series property of real exchange rates. The doctrine of PPP states that any change in relative national prices is offset by a change in the nominal exchange rate between the two currencies. This implies the real exchange rate is constant as long as PPP holds and changes in the real exchange rate represent a measure of deviations from the parity relationship. As indicated in the introduction, PPP has never been assumed to be a steady state and therefore changes in the real exchange rate are compatible with the theory. The relevant question concerning the empirical validity of PPP is whether the theory holds up as a long run equilibrium relationship. If PPP holds in the long run, then the parity relation between currencies will be re-established eventually even if there are deviations from purchasing power parity in the short run. In this case, the deviation from PPP is only transitory. On the other hand, if PPP as a long run equilibrium relationship does not hold, then the deviation from purchasing power parity is not transitory and will not disappear even in the long run. Thus, whether PPP holds in the long run is equivalent to whether the deviations from PPP are transitory. Since changes in the real exchange rate measure the deviation from PPP, the same question translates to whether the real exchange rate is stationary or nonstationary. If the real exchange rate follows a nonstationary process, then the deviations from purchasing power parity are permanent and PPP does not hold in the long run. On the other hand, if the real exchange rate follows a stationary process, then the deviations from purchasing power parity are only transitory and will eventually disappear in the long run. In this case, the parity condition will eventually be restored.

A variety of econometric methodologies have been developed for testing stationarity or nonstationarity of time series. The following discussion of the literature on the behavior of real exchange rates is organized by the type of the methodologies used in the studies. Namely, univariate unit root tests, panel unit root tests, and alternatives to unit root tests.

2.3.1: Univariate unit root tests

The most commonly used unit root test in an univariate context is the Augmented Dickey Fuller (ADF) test. It first regresses the first difference of the variables under consideration on a constant, its lagged level and a number of lagged first differences. Other deterministic terms may also be included in the regression if it is regarded as necessary. Formally, a testing model of the following form is estimated first:

$$\Delta y_t = \mu + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \varepsilon_t \qquad (22)$$

then a t test is constructed for the estimated coefficient on the lagged level $\hat{\alpha}$. If $\hat{\alpha}$ is not significantly different from zero then the null hypothesis of unit root cannot be rejected and the variable is said to be a nonstationary process. The test statistic, denoted τ, is simply the estimate of the coefficient $\hat{\alpha}$ divided by its standard error. Since this test statistic does not have the usual t-distribution, the significance test of $\hat{\alpha}$ is obtained by comparing the test statistic with a critical value calculated from a nonstandard distribution, which is tabulated in Fuller (1976).

Another popular unit root test in univariate context is Phillips-Perron (1988) test. From one perspective, the effect of the Phillips-Perron test is the same as that of the ADF-type tests. Both procedures are designed to handle the autocorrelation problem in the residuals that would be present when the underlying data generating process follows a higher-order autoregression process. ADF-type tests solve the problem by adding an appropriate number of the lagged first differences to the model to whiten the residual series. The Phillips-Perron procedure acts instead to modify the statistics after estimation in order to take into account the effect that the autocorrelated errors will have on the results. Asymptotically, the statistic is corrected by the appropriate amount and so the same limiting distributions apply. To conduct Phillips type tests one needs first to apply OLS to one of the following three models:

$$y_t = \rho_a y_{t-1} + u_{at} \tag{23}$$

$$y_t = \mu_b + \rho_b y_{t-1} + u_{bt} \tag{24}$$

$$y_t = \mu_c + \gamma_c(t - T/2) + \rho_c y_{t-1} + u_{ct} \tag{25}$$

where T is sample size less one. It is easy to calculate from these regressions the coefficient estimates and the 't-statistics' for each. To test the null hypothesis of $\rho_i = 1$ (where $i = a, b, c$) in the above models, the statistics are then adjusted to reflect autocorrelation in the corresponding u_{it} series. The adjustments are made according to the following formulas:

for Equation (23)

$$Z(t(\hat{\rho}_a)) = (S_u / S_{T\ell})t(\hat{\rho}_a) - \frac{1}{2}(S_{T\ell}^2 - S_u^2)\left[S_{T\ell}(T^{-2}\sum_{t=2}^{T} y_{t-1}^2)^{1/2}\right]^{-1}, \tag{26}$$

for Equation (24)

$$Z(t(\hat{\rho}_b)) = (S_u / S_{T\ell})t(\hat{\rho}_b) - \frac{1}{2}(S_{T\ell}^2 - S_u^2)\left\{S_{T\ell}\left[T^{-2}\sum_{t=2}^{T}(y_{t-1} - \bar{y}_{-1})\right]^{1/2}\right\}^{-1}, \tag{27}$$

for Equation (25)

$$Z(t(\hat{\rho}_c)) = (S_u / S_{T\ell})t(\hat{\rho}_c) - \left[T^3 /(4\sqrt{3})\right]\left(D_x^{-1/2} S_{T\ell}^{-1}\right)(S_{T\ell}^2 - S_u^2), \tag{28}$$

where $$S_u^2 = T^{-1}\sum_{t=1}^{T}\hat{u}_t^2, \tag{29}$$

$$S_{T\ell}^2 = T^{-1}\sum_{t=1}^{T}\hat{u}_t^2 + 2T^{-1}\sum_{j=1}^{\ell}\sum_{t=j+1}^{T}\hat{u}_t\hat{u}_{t-j}, \tag{30}$$

$$\begin{aligned} D_x &= \left[T^2(T^2-1)/12\right]\sum_{t=2}^{T} y_{t-1}^2 - T\left(\sum_{t=2}^{T} t y_{t-1}\right)^2 \\ &\quad + T(T+1)\sum_{t=2}^{T} t y_{t-1}\sum_{t=2}^{T} y_{t-1} - \left[T(T+1)(2T+1)/6\right]\left(\sum_{t=2}^{T} y_{t-1}\right)^2 \end{aligned} \tag{31}$$

$t(\hat{\rho}_i)$ is the t-statistic associated with testing the null hypothesis $\rho_i = 1$, and ℓ is the lag truncation parameter. Finally, the *Z(t)*-statistics so obtained are compared with critical values tabulated by Fuller (1976). A set of $Z(\Phi)$ statistics transformed from F-statistics are also available for joint tests from Perron (1988).

Empirical results from univariate unit root tests are largely divided by length of the data series. Tests for the recent floating period generally fail to reject the null of unit root, implying that the real exchange rate series follows a random walk and purchasing power parity does not hold for the period. Studies by Meese and Rogoff (1988), Edison and Pauls (1993), and MacDonald (1995, 1996) all fall into this group, though not all specifications of the test equations in these studies are exactly same. As part of their effort to investigate the long run relationship between real exchange rates and real interest rate differentials, Meese and Rogoff (1988) apply the ADF procedure on three US dollar based bilateral real exchange rates, namely, dollar/mark, dollar/yen, and dollar/pound rates. Consumer price index is used in calculating the real exchange rate series. The study employs monthly data for the period from February 1974 through March 1986. The econometric specification considered in their study includes a constant and seasonal dummies, two lags of the dependent variable are included to whiten the residuals in all three cases. Their results show that all three sets of real exchange rates are sufficiently nonstationary that one cannot reject the null hypothesis that there exists a unit root in the series. In a similar context, Edison and Pauls (1993) apply the ADF procedure on the real effective exchange rate between US dollar and G-10 currencies as well as four sets of US dollar based bilateral real exchange rates, which include US dollar/German mark, US dollar/Japanese yen, US dollar/British pound, and US dollar/Canadian dollar rates. The data are sampled in the quarterly interval for the period 1974 Q3 through 1990 Q4 (the data ended in 1990 Q3 for the trade weighted real effective exchange rate series). As did Meese and Rogoff (1988), the authors use CPI as a price index in calculating all the effective and bilateral real exchange rate series. The econometric specifications, however, are different from Messe and Rogoff (1988). A time trend is included in the model of Edison and Pauls (1993) and only one lag of the dependent variable is used to whiten the residuals. Despite the different model specification employed, Edison and Pauls (1993) fail to change the results obtained by Meese and Rogoff (1988): the null hypothesis that the real exchange rate series are nonstationary cannot be rejected in all five cases.

There is a wide spread perception that the results of unit root tests on real exchange rates could be sensitive to the price index used in constructing the real exchange rate series [see MacDonald (1995) and Kim (1990)]. Because of the existence of non-traded goods and the fact that the wholesale price index (WPI) contains a relatively large traded goods element, many researchers suggest that it could be more likely to reject the null hypothesis of unit root in WPI-based real exchange rate series. While the view intuitively makes sense, it has been shown in the empirical literature that the choice between WPI and CPI makes no meaningful difference for the vast majority of the real exchange rate series. MacDonald (1995) tests for a unit root in real bilateral exchange rates between the U.S. dollar and nine other currencies (Canadian dollar, French franc, German mark, Italian lira, Japanese yen, Dutch guilder, Swedish krona, Swiss franc and UK pound) over the period from March 1973 to December 1992. Two series are constructed for each real bilateral exchange rate: one based on the CPI, the other based on the WPI (except for the U.S. dollar/French franc rate, for which only CPI-based real exchange rate is constructed due to the data limitation). He tries two specifications of deterministic terms: one includes a constant, the other includes both a constant and a time trend. The results strongly indicate the presence of a stochastic unit root in all instances, regardless of the choice of the price measures in calculating the real exchange rate series. Ironically, almost all the test statistics for the WPI-based real exchange rates are smaller than their counterparts for the CPI-based real exchange rates.

The only exception is for the U.S. dollar/Japanese yen rate when the model's deterministic term includes only a constant. In another study, MacDonald (1996) tests for PPP in OECD countries using annual data for the period from 1973 to1992. As in MacDonald (1995), two econometric specifications of the testing models (one includes a constant and the other includes a constant plus a time trend) are exercised. For the consumer price based real exchange rate series, the null hypothesis of unit root is rejected in only 2 of 48 individual tests. While for the wholesale price based real exchange rate series, the null hypothesis of unit root is rejected only in 3of 48 individual tests. The results of MacDonald (1995,1996) are consistent with studies mentioned above, and more importantly, they show that the results of unit root tests on real exchange rates are not meaningfully sensitive to the choice of the two popular price measures. They also seem to suggest that there is no empirical significance to sampling the data at different frequencies (monthly, quarterly, or annually) as long as the sample period is fixed.

The failure to reject the null of unit root by studies over the recent floating period is not accepted by most researchers as evidence that the purchasing power parity does not hold. Instead, it is widely believed that the failure is caused by the low power of statistic tests in discriminating nonstationary and near nonstationary series over a short sample period. To improve the power of the statistic test, researchers adopt three broad strategies: (1) apply a conventional univariate test procedure to a much longer sample period; (2) concentrate on the recent float period but employ a panel unit root test; (3) apply a more sophisticated individual test (as oppose to a panel test) to the recent floating period. The rest of this subsection is devoted to the studies following the first test strategy. Studies using panel data and studies using alternative univariate testing methodology will be dealt with in the following two subsections, respectively.

Two influential early studies using long-span data series are Adler and Lehmann (1983) and Edison (1987), although they do not fall exactly into the category of univariate unit root tests on real exchange rates. Adler and Lehmann (1983) specify a martingale model of real exchange rates in the form of the following equation:

$$y_t = \sum_{i=1}^{n} b_i y_{t-i} + v_t \,, \tag{32}$$

where y_t is the percentage change in log of the real exchange rate. The authors argue that if the long run PPP hypothesis is true, real exchange rate changes should be positively, serially correlated owing both to the cumulative nature (due to PPP not holding in the short run) of the initial deviation and to their systematic tendency to revert to parity thereafter. In contrast, the martingale model implies that these increments should be serially independent, or equivalently, the sum of the coefficients on the autoregressive terms are jointly equal to zero. Equation (32) is estimated using eight sets of WPI-based U.S. dollar bilateral real exchange rates over the period 1900-1972 (other currencies involved are the Canadian dollar, French franc, Italian lira, Japanese yen, U.K. pound Netherlands guilder, German mark, and Swiss franc), five sets of CPI-based U.S. dollar bilateral real exchange rates over the period 1915-1972 (other currencies involved are Canadian dollar, French franc, Italian lira, Japanese yen, and U.K. pound.), and CPI-based U.S. dollar/Canadian dollar real rate over the period 1870-1975, respectively. The null hypothesis that the coefficients for the autoregressive terms in Equation (32) are jointly equal to zero cannot be rejected in almost all instances. The only exception is the ten-lag result for France when the1915-1972 sample is used. Based on these findings, Adler and Lehmann (1983) conclude that the behavior of real exchange rates can be better described by a martingale model than by a mean reverting model, regardless of the length of sample period or the frequency of data interval used in the test. Edison (1987) tests PPP in a dynamic error correction model using a GDP-deflator based annual real exchange rate between the U.S. dollar and pound sterling over the period 1890 to 1978. She finds that the symmetry and homogeneity hypothesis implied by PPP cannot be rejected. Interestingly, though, even over this long time span, the estimated coefficient on the error correction term suggests that only about 9 percent of any deviation from PPP is distinguished within a year. She is unable to reject the condition of exclusiveness when a enlarged, monetary model is estimated. Thus permanent deviations from PPP cannot be ruled out.

Kim (1990) employs the Phillips-Perron (1988) procedure to test the null hypothesis that real exchange rates follow a unit root process. Annual data for five sets of U.S. dollar bilateral real exchange rates are used in the study. The other currencies involved in the study are the Canadian dollar, French franc, Italian lira, Japanese yen, and U.K. pound. To check the robustness of the results to the choice of price indices, both the WPI-based and the CPI-based real exchange rates are constructed. Since data on CPI are not available before 1914, the period of estimation is 1901-1987 for WPI-based rates and 1915-1987 for CPI-based rates. The results of the unit root tests indicate that the null hypothesis of random walk can be rejected in seven out of ten instances. A unit root is found for CPI-based real Canadian dollar, yen and pound. Kim's results suggest that the choice of price measure does matter in a unit root study. This is inconsistent with findings by MacDonald (1995). On the apparent inconsistency of his findings and those of Adler and Lehmann (1983), Kim argues that the autoregressive model as specified in Adler and Lehmann (1983) implicitly imposes a set of restrictions which refutes the error correction mechanism and cointegration which otherwise might be detected.

Some researchers suggest that PPP as a long run hypothesis cannot legitimately be tested with monthly data. They argue that with monthly observations, the pure noise, or the part of the serial correlations due to random deviations in the short run, may obscure longer run tendencies. Using annual or less frequent data may mitigate this risk. The empirical significance of data frequency has not been proved. For example, Adler and Lehmann (1983) cannot reject the martingale model even if annual data over a lengthy sample period are employed. On the other hand, Grilli and Kaminsky (1991) apply the Phillips-Perron (1988) procedure to the monthly data for the WPI-based real exchange rate between the U.S. dollar and U.K. pound over the sample period from January 1885 to December 1986. Their results indicate that the null of unit root is rejected for the full sample period but not for a variety of subsamples.

The study on time series properties of real exchange rates that employs the longest data series is provided by Lothian and Taylor (1996). They apply the Phillips-Perron (1988) procedure to the real dollar-sterling exchange rate and the real franc-sterling exchange rate over the period 1791-1990 and the period 1803-1990, respectively. Both sets of the real rates are WPI-based. Their results reject the unit root null and suggest that sterling real exchange rates against the franc and the dollar over the past 200 years are adequately characterized as realizations from stationary AR(1) processes. It is also interesting to note that, like many other studies, the estimation results again indicate deviations from PPP are highly persistent. Specifically, the point estimates for the coefficients indicate that shocks to the real exchange rate are corrected at the rate of 23 percent per year for franc sterling and only 11 percent per year for dollar-sterling, implying a half-life of real exchange rate shocks of about 6 years for dollar-sterling and a little under three years for franc-sterling.

Based on the cited studies above, it seems that the balance of empirical evidence from unit root tests using long-span data series is weighted toward the hypothesis that real exchange rates follow a mean-reverting process, despite a painfully slow adjustment process. But, this is far from conclusive. In addition to the findings from Adler and Lehmann (1983), supporting evidence for the random walk model of real exchange rates is also reported by Rogers (1998). He applies both the standard ADF procedure and a relatively new procedure proposed by Kwiatowski, Phillips, Schmidt, and Shin (1992, hereafter KPSS). Contrary to the standard ADF test, the KPSS procedure tests the null hypothesis of stationarity against the alternative of nonstationarity. Using annual data for the real exchange rate between the U.S. dollar and the U. K. pound over the period of 1889 to 1992, Rogers reports both a failure to reject the unit root null hypothesis of ADF and a rejection of the trend-stationary null in the case of the KPSS test. This result holds whether WPI, GNP deflator or log ratio of the government expenditure deflator to the GNP deflator (U.K. less U.S.) is used as the price proxy.

2.3.2: Panel unit root tests

One potential hazard in using long runs of time series data is that the behavior of the real exchange rate could vary substantially across historical periods. Many researchers therefore caution against econometric analysis that uses long time series of the real exchange rate without properly controlling for institutional changes, the development of commodity and financial markets, and fiscal and monetary regimes. The conflicting findings from some studies using long data series suggest that the regularities of the real exchange rate behavior are likely to be specific to a particular sample period and, therefore, cannot be generalized. Thus it is not surprising to see that most recent empirical studies on the time series property of real exchange rates concentrate on the current floating period, but adopt panel unit root testing procedure to exploit the information content of cross sectional variation to increase the power of the statistic tests.

The best-known panel unit root tests have been developed by Levin and Lin (1992, hereafter LL) and Im, Pesaran, and Shin (1997, hereafter IPS). The LL test is designed to test the null hypothesis that each individual series contains a unit root against the alternative hypothesis that all the series considered as a panel are stationary. The test statistic is based on the conventional t-statistic. The procedure is motivated by considering the following equation:

$$q_{it} = \alpha + \beta t + \eta_i + \omega_t + \rho q_{it-l} + \varepsilon_{it} \tag{33}$$

where q denotes a real exchange rate, i denotes a currency, η_i and ω_t are the country specific and time specific fixed effects, respectively. and ε_{it} is the idiosyncratic disturbance. Assume for the moment that the disturbance term ε_{it} is i.i.d. with E(ε_{it}) = 0, E(ε_{it}^2) = σ^2 and $E|\varepsilon_{it}|^{2+\lambda} < \infty$ for some $\lambda > 0$. First, the following panel transformation can be performed to remove from the data the common intercept term, the time trend, and the individual specific and time specific effects from the data:

$$\hat{q}_{it} = q_{it} - \overline{q}_i, \tag{34}$$

$$\overline{q}_{it} = \hat{q}_{it} - \overline{\hat{q}}_t, \tag{35}$$

where $\overline{q}_i = \frac{1}{T}\sum_{t=1}^{T} q_{it}$, and $\overline{\hat{q}}_t = \frac{1}{N}\sum_{i=1}^{N} \hat{q}_{it}$. The model to be estimated becomes

$$\overline{q}_{it} = \rho\overline{q}_{it-1} + \overline{\varepsilon}_{it}. \tag{36}$$

The least-squares estimator of ρ is given by

$$\hat{\rho} = \sum_{i=1}^{N}\sum_{t=1}^{T}\overline{q}_{it}\overline{q}_{it-1} \Big/ \sum_{i=1}^{N}\sum_{t=1}^{T}\overline{q}_{it-1}^{2}, \tag{37}$$

and the t-statistic to test for a unit root is defined as

$$t_{\rho} = \left[\sum_{i=1}^{N} \sum_{t=1}^{T} \bar{q}_{it-1}^{2} \right]^{1/2} \frac{(\hat{\rho}-1)}{\hat{\sigma}}, \tag{38}$$

where $\hat{\sigma} = \left[\frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} (\bar{q}_{it} - \hat{\rho}\bar{q}_{it-1})^2 \right]^{1/2}$. Levin and Lin (1992) prove that as both the time periods and number of individuals in the panel increase to infinity under the null of $\rho = 1$, the t-statistic so obtained has a standard normal distribution and the critical values of the standard normal distribution can be used to test the null hypothesis of $\rho = 1$. These results are independent of whether a constant, a time trend, or time specific effects are included in the model. In the more general case when the disturbance term ε_{it} is serially correlated, the serial correlation can be appropriately corrected either by including the lagged difference terms, $\sum_{L=1}^{p} \Delta\bar{q}_{it-L}$ in equation (36), following the ADF rationale, or by a nonparametric method, following the PP rationale. The above asymptotic distributions still obtain after correcting for the serial correlation of the disturbance term.

The IPS procedure tests the null hypothesis that all series in the panel contain a unit root against the alternative hypothesis that some of these series are stationary. The test statistic is based on individual ADF test statistics. To conduct the IPS test, one first runs ADF regressions for N countries, selects the ADF regression lag length based on some information criterion such as Akaike information criterion, obtains the cross-sectional average of the ADF statistics $\bar{t}_{NT}$, and then calculates $\bar{\Psi}$ statistic according to the formula:

$$\overline{\Psi} = \sqrt{N}\left(\bar{t}_{NT} - \frac{1}{N}\sum_{i=1}^{N} E(t_{iT})\right)\left(\frac{1}{N}\sum_{i=1}^{N} VAR(t_{iT})\right)^{1/2}, \tag{39}$$

where $E(t_{iT})$ and $VAR(t_{iT})$ are the empirical first and second moments of the ADF test statistics under the null, which IPS have computed through simulations for various combination of sample length and ADF lag order and tabulated in IPS (1997). The test statistic $\overline{\Psi}$ so obtained has a standard normal distribution for large N, and thus the critical values of the standard normal distribution can be used to test the null hypothesis. Monte Carlo simulations in the IPS study (1997) show that the size and power properties of the IPS test are superior to those of the LL test.

After failing to reject the null hypothesis of unit root in 91 out of 96 individual tests, Macdonald (1996) demonstrated that when he moved from the univariate ADF test to the LL method, for the annual real exchange rate series of OECD countries over the period 1973-1992, the panel LL tests comfortably reject the null that all real exchange rates are nonstationary, regardless of the chosen deterministic specification and the price index used to construct real exchange rate series. Wu (1996) applies the LL test to 18 sets of CPI-based and 16 sets of WPI-based U.S. bilateral real exchange rates. The countries involved in the CPI-based test are Austria, Belgium, Canada, Germany, Denmark, Spain, Finland, France, Britain, Greece, Italy, Japan, Luxembourg, Netherlands, Norway, Portugal, Sweden, and Switzerland. For the WPI-based series, Italy, Luxembourg and Portugal are excluded as observations on WPI are unavailable for these countries, while Ireland is added since only WPI observations are available for this country. The test is implemented for monthly, quarterly and annual data, respectively. For the monthly data, the sample covers the period from January 1974 to April 1993, with 232 observations. For the quarterly data, the sample period runs from 1972 Q1 through 1993 Q1, with 77 observations. The annual data cover the period from 1974 to 1992. While the ADF tests confirm the standard result that on a univariate basis, and for the recent float period, real exchange rates are nonstationary series. LL tests on both CPI-based and WPI based series result in strong rejection of the null of a unit root for all data frequencies. The estimated ρ suggests a half-life of one-time-shock to the WPI rates is 2.7 years for monthly data, 2.6 and 2.3 years for quarterly and annual data, respectively. Using annual data from 1973 through 1993, Bayoumi and MacDonald (1999) rejects the null of unit root in a panel of 20 real exchange rates, whether CPI or WPI is used as the price proxy. The estimated ρ translates into a two-year half-life deviation from the mean.

Coakley and Fuertes (1997) use the IPS method to test PPP in a panel of G-10 countries plus Switzerland, relying on monthly bilateral real exchange rates for the period from July 1973 to June 1996. The tests are carried out on a real exchange rate series constructed from wholesale as well as consumer price indices. They conclude that while the nonstationary null cannot be rejected from conventional ADF tests, it can be rejected at a conventional significance level. Specifically, the WPI-based panel is found to be stationary with or without a trend at the 5 percent level, while a CPI-based panel is only stationary around a constant at the 10 percent level.

Some researchers, for example O'Connell (1998), warn that findings from panel unit root tests could be misleading. They argue that panel data typically contain some degree of cross-section correlation, usually reflecting the presence of common factors, which may affect both the result of unit root tests and long run parameter estimates. The importance of controlling for cross sectional dependence in the data is highlighted in a study by Habermeier and Mesquita (1999). In this study the authors apply the IPS procedure to a panel of 51 of the largest market economies (except Hong-Kong for data limitation) for the sample period from 1971-1997. The CPI is used to construct real exchange rate series. in the model including a constant, but no trend term. The test on the original data indicate a rejection of the null, while the test based on de-meaned data indicates that the null of nonstationarity cannot be rejected. Only in the model including a constant and a time trend is the unit root null rejected with both the original and the de-meaned data.

2.3.3: Alternative tests

Despite the general failure to reject the nonstationary null in the context of univariate unit root tests for the recent floating period, the poor power property of the statistical procedure makes the findings less conclusive. The failure to reject the null could be a reflection that real exchange rates do in fact follow a random walk, as researchers on one side would argue. But the possibility should not be downplayed that the real exchange rate is in fact a weak stationary process which could not be discriminated from a true random walk process given the l

ow power of the statistical methodologies. The relatively positive findings from most studies using long-span or panel data are questionable for their own problems. For example, it would be inappropriate to use long-span data series if the behavior of the real exchange rates changes over the historical periods. On the other hand, some researchers indicate that results of the panel test could be sensitive to the size and components of the panel. More importantly, even if the results of tests using long-span or panel data set are reliable, it is still interesting to test PPP for current floating period on individual basis. Thus a more appropriate strategy to improve the test results is to adopt more efficient methodologies, rather than expanding the data set in either temporal or spatial dimensions. In this subsection, I introduce several important studies along this line.

Sims (1988) argues that the classical DF type tests are biased toward accepting the null of unit root. This is because the priors implicit in the classical tests give excessive weight to the unit root null. He suggests using a Bayesian posterior odds ratio test as an alternative. To conduct this test, one first obtains the estimated ρ by applying OLS to the autoregressive model of the variable under consideration. Then assigning a prior distribution for ρ which spreads probability α ($0 < \alpha < 1$) uniformly on the interval (0, 1), and gives the unit root ($\rho = 1$) probability ($1-\alpha$). Finally define $T \equiv (1-\hat{\rho})/\sigma_\rho$, where $\sigma_\rho \equiv \sqrt{\left(\sigma^2/\sum y_{t-1}^2\right)}$, to be the conventional t statistic for testing $\rho = 1$, $\Phi(x)$ to be the cumulative distribution function for the standard normal distribution evaluated at x, and $\phi(x)$ to be its probability density function. Sims shows that in large samples the Bayesian posterior odds ratio favors the null hypothesis $\rho = 1$ if

$$\text{Bayesian posterior odds ratio} = \frac{(1-\alpha)\phi(T)}{\sigma_\rho[\alpha\Phi(T)]} > 1 \qquad (40)$$

In actual application, Sims suggests that for annual economic data the alternative hypothesis can reasonably be limited to the value of ρ between 0.5 and 1, For more frequent data, the interval associated with this alternative hypothesis has a lower bound closer to 1. In the case of quarterly data, the interval is approximately (0.84, 1) because 0.84 to the fourth power is equal to the lower bound for annual data; the interval is (0.94, 1) for monthly data. Therefore, he proposes the following revised criterion: the null hypothesis $\rho = 1$ is favored if

$$\gamma \equiv 2\log\left(\frac{1-\alpha}{\alpha}\right) - \log\left(\sigma_{\rho}^{2}\right) + 2\log(1-2^{-1/s}) - T^{2} > 0 \tag{41}$$

where s is the number of periods per year.

Whitt (1992) adopts the Sims-Bayesian approach to test PPP with monthly data over the period June 1973 to December 1989. Two sets of U.S. dollar based real exchange rates are constructed for each of the five countries: the United Kingdom, France, Germany, Switzerland, and Japan. One based on wholesale prices and the other based on consumer prices. Using the value of 0.8 for α, Whitt is able to reject the null hypothesis of a unit root for each real exchange rate.

A commonly used nonparametric methodology in discriminating random walk series from long memory stationary ones is the variance ratio test due to Cochrane (1988). This test exploits the fact that if the series under consideration follows a random walk, then the variance should be proportional to the differencing horizon. The variance grows less than proportionally to the differencing horizon for negatively autocorrelated series while more than proportionally to positively autocorrelated series. Therefore the time series property of the variable under consideration can be decided according to the value of the variance ratio,

$$VR(y) = \frac{VAR(y_t - y_{t-k})}{kVAR(y_t - y_{t-1})}, \tag{42}$$

where, y is a random walk process if $VR(y) = 1$, a stationary, or mean-reverting, process if $VR(y) < 1$, and a explosive process if $VR(y) > 1$. Lo and Mackinlay (1988) have derived two test statistics that facilitate testing the null hypothesis that $VR(y)$ equals one. The first test statistic is calculated under the assumption that the error terms in the y series are independently and identically distributed, while the second statistic is robust to the more general case of uncorrelated but weakly dependent and possibly heteroskedastic innovations.

Grilli and Kaminsky (1991) calculate variance ratios for the real dollar/pound rate over seven different historical periods. Their results indicate that most of the variance ratio statistics based on post-WWII data are larger than one regardless of the lag length used. For the period of March 1973 though December 1986, the ratio at the lag of 6 months is 1.77 and continues to increase with the lag length until it reaches 48 months, where the ratio is 2.89 and significant at a 5 percent level. The ratio falls below one only when the lag length increases to 96 months and it is not significantly different from unity. On the other hand, all the variance ratios based on the pre-WWII data are less than one when lag length is 36 months or longer. Two important implications can be drawn from this study: first, the real exchange rate (between the dollar and pound, at least) displays very little mean-reverting behavior in the period of current float.

Although the deviations from PPP are not actually permanent, they are so persistent that the practical use of PPP is seriously challenged. Second the behavior of the real exchange rates could change dramatically across historical periods. This confirms the skepticism that many researchers expressed about the reliability of findings from studies using long data series.

Glen (1992) applies the variance ratio test to the monthly data for a wide range of U.S. dollar based real bilateral exchange rates over the current floating period. He increases lag length up to 32 months and finds all the ratios are significantly larger than one. He concludes that the real exchange rates follow a super-persistence or explosive process instead of random walk. However, evidence of mean-reversion is found when he moves to an annual data set for the period 1900-1987. Glen's result is largely consistent with findings by Grilli and Kaminsky (1991) in the sense that the real exchange rates demonstrate much more mean-reverting behavior during the pre-WWII period. This might explain why mean-reversion is found when he includes the annual observations before WWII. The seeming inconsistency in findings from monthly data for the current floating period in the two studies may reflect the fact that Glen's lag horizon is simply not long enough to pick up the very weak mean-reverting behavior.

In a more recent study, Wu (1996) calculates the variance ratio using quarterly data for real exchange rates of four countries: the United States, the United Kingdom, Japan and Germany. The sample period runs from 1973 Q1 to 1994 Q3 with 87 observations. The statistically significant mean-reverting behavior is found in all the rates at the lag length of 20 quarters. This result is close to Grilli and Kaminsky (1991), where mean-reverting behavior is found at the lag length of 96 months.

An alternative to the non-parametric measures of persistence is to explicitly model the time series properties of the data using the fractionally integrated ARMA or ARFIMA model due to Granger and Joyeux (1980) and Hosking (1981).The concept of fractional integration is a generalization of the more familiar integer-based time series methodology such as testing for unit root nonstationarity. Fractional integration allows for the possibility that exogenous shocks may affect variables for more than just a few time periods, while still allowing the effect of shocks to eventually dissipate. In contrast, the integer restricted methodology requires that shocks either fully die out within very few periods or persist into the infinite future. Fractional integration can be viewed as a more flexible framework within which to model long run behavior. To test time series properties of the real exchange rate using fractional integration method, the ARFIMA (*p, d, q*) model of the following type can be fitted using maximum likelihood method (ML).

$$\Phi(L)(1-L)^d R_t = \delta + \Psi(L)\varepsilon_t \tag{43}$$

where $\Phi(L)$ and $\Psi(L)$ are lag operator polynomials of order p and q respectively, δ is a constant, the error is an independently and identically distributed Gaussian process and $(1-L)^d$ is the fractional differencing filter and defined by

$$(1-L)^d = \sum_{j=0}^{\infty} \frac{\Gamma(k-d)L^j}{\Gamma(-d)\Gamma(j+1)} \tag{44}$$

where $\Gamma(\cdot)$ is the gamma function. The ML procedure simultaneously estimates all of the parameters in Equation (43). The persistence of exogenous shocks to the data can be quantified though the estimation of *d*. If *d* equals to zero, Equation (43) simplified to a standard ARMA (*p, q*) model, the series is stationary. If d equals to unity, Equation (43) simplifies to a standard ARIMA (*p, 1, q*) model and the effect exogenous shocks to the variable is permanent. When *d* is a fractional value between zero and one, the series displays slow mean-reverting and it takes a long time for the effect of an exogenous shock to die out.

Baum, Barkoulas, and Caglayan (1999) use a fractional integration approach to test PPP over the recent floating period. They construct seventeen sets of CPI-based and twelve sets of WPI-based U.S. dollar bilateral real exchange rates. The monthly data run from August 1973 to December 1995. Their findings strongly support the presence of a unit root in the autoregressive polynomial of the real exchange rate series. Regardless of the information criterion employed to choose the final ARFIMA specification, there is no evidence of mean-reverting in any of the series at the 5 percent level. PPP cannot hold as a long run relationship based on these findings. In most cases, a pure martingale model appears to be an appropriate characterization of the dynamic behavior of the series.

2.4: Summary

The following conclusions can be tentatively drawn from the empirical findings discussed in this chapter. First, it appears that a long-run equilibrium relationship does exist between the exchange rates and the relative prices. Although the studies using the Engle-Granger procedure generally fail to reject the null hypothesis of noncointegration for the period of recent float, evidence of the cointegration is found when the more sophisticated Johansen procedure is adopted. Studies on time series properties of real exchange rates follow the same pattern. Conventional unit root tests in univariate context fails to reject the null hypothesis that real exchange rates follow a random walk process over the recent floating period, while the mean-reverting behavior in the real exchange rates over this period is detected when more efficient univariate tests, such as the variance ratio test, are employed. I do not have confidence in the findings from those studies using long-span or panel data set for reasons that I have indicated previously, though these findings seem to be consistent with the existence of a long run equilibrium relationship between exchange rates and relative prices.

Second, long-run equilibrium between exchange rates and relative prices could be reached at a level different from the parity implied by PPP, and the adjustment process to disequilibrium is extremely slow. Considering the measurement errors, transaction costs, and the different adjustment speed between assets and commodity prices, the theory of PPP can hold well if the real exchange rates are stationary around a range that is not too far from parity, and deviations from this range die out within a reasonable period. Unfortunately, this is not the case. Of studies that report cointegration results, not only are the symmetry and homogeneity conditions usually rejected, the estimated coefficients even have wrong signs in some cases. The mean-reverting behavior that is detected in the real exchange rates is extremely weak. For example, the variance ratio tests using data for recent float indicate that deviations from parity tend to grow over time and do not revert their course until seven or eight years after the initial shock. Even studies using long-span data series indicate that the half-life of the deviations from parity is as long as three to five years. Several studies using panel data for the recent float period find the half-life of the deviations for the panel as a whole to be two to three years. These findings are not necessarily inconsistent with those from univariate studies. Since deviations from equilibrium could occur in both directions, the magnitude and persistence of the deviations measured for a group of countries could be much smaller than those measured individually.

Finally, the exchange rate is not a pure monetary process; its behavior could not be reasonably explained without considering some real factors. O'Connell (1998) shows that large deviations from PPP do not revert to parity more quickly than small deviations, implying that deviations from PPP cannot be explained by transaction costs alone. The overshooting model of Dornbusch (1976) might explain why the exchange rate departs from its parity level in the short run. But as mentioned above, it takes three to five years to correct deviations from the equilibrium by only 50 percent, implying ten or more years could be needed for most effects of an exogenous shock to die out. A period as long as ten years could not be taken as a 'short run'. The rejection of symmetry and homogeneity conditions implied by PPP and the very persistent deviations from the equilibrium level indicate that there is more to exchange rates than relative prices alone. Edison (1987) is unable to reject the condition of exclusiveness implied by PPP. Officer (1976) admits that PPP is not closed to other variables, although only the price ratio is explicitly specified. Given the empirical failure of relative prices in explaining exchange rate behavior, a more productive research direction is to study the forces that keep exchange rates away from parity levels and explicitly specify other factors in addition to relative prices that have a systematic impact on exchange rates.

CHAPTER 3

AUGMENTING PPP: MODELLING METHODOLOGY

In this chapter, a general model of exchange rate determination is obtained by augmenting the conventional theory of purchasing power parity with several fundamental economic variables that potentially have systematic impact on the level of exchange rates. Although the relationship between exchange rates and relative prices as implied by PPP does not hold empirically within the simple framework of PPP, it is possible that the symmetry and homogeneity conditions implied by PPP might not be rejected in a more general empirical framework. This chapter discusses the variables that are used to augment PPP and the data and econometric methodology that are employed to test the more general model of exchange rate determination.

3.1: The Variables

The first candidate of explanatory variables is provided by the productivity bias hypothesis. Balassa (1964) and Samuelson (1964) conjecture that currencies of relatively fast-growing countries tend to appreciate in real terms and vice versa for currencies of relatively slow-growing countries. The reason for this phenomenon is the asymmetric advances in productivity between traded and non-traded sectors. Productivity advances usually occur in traded sectors. Prices of traded goods are tied to the world price level through international arbitrage. A rise in productivity in one country cannot affect the price of traded goods. Instead, the initial effect of productivity advance is to increase wages in the traded sector. As wages in the traded sector increase, the wage level in the non-traded sector will match up though a unified labor market. Assuming there is no change in productivity in the non-traded sector, prices of non-traded goods must increase to keep profitability from falling as the wage level increases. With a constant price level of traded goods and an increased price level of non-traded goods, the general price level as measured by the CPI will rise. Since the rise in the general price level is not accompanied by a corresponding adjustment in the nominal exchange rate, the currency of the country with relatively fast growing productivity appreciates in real terms. The productivity bias hypothesis may be tested by relating a country's real exchange rate to the relative growth of its aggregated productivity, or per capita real GDP.

Another potential explanatory variable for the exchange rate is the relative accumulated current account balance. In early theories the causation between the exchange rate and the current account runs unidirectionally from the former to the latter. Dornbusch (1976) and Dornbusch and Fisher (1980) introduce wealth and portfolio effects into their models of exchange rate determination, thereby opening channels through which changes in the current account balance can affect the exchange rate. Starting from an initial equilibrium state, a rise in the current account balance increases wealth, which in turn increases demand for domestic goods and services. To maintain the equilibrium, an appreciation of the domestic currency is required to shift the expenditure from domestic to foreign goods and services. Since the rise in the current account balance represents excess supply of foreign assets, a portfolio adjustment in the asset market also indicates an appreciation of the domestic currency.

A monetary approach focuses on the role of interest rates in the exchange rate determination. The flexible-price monetary model assumes that PPP holds continuously. Monetary shocks change prices and interest rates simultaneously. Changes in interest rates are deemed as a reflection of changes in inflation expectation. A relative rise in domestic interest rates reduces the demand for domestic money stock. As agents try to get rid of their excess money balances, they increase their expenditure and prices rise until money market equilibrium is achieved. As prices rise, PPP ensures a depreciation of the domestic currency. In the sticky-price monetary model, since prices do not respond fully to monetary shocks in the short run, changes in the nominal interest rate usually represent real changes in the interest rate. Therefore a rise in domestic interest rates will attract capital inflows and appreciate the domestic currency. Since the flexible monetary model is based on the continuous holding of PPP and the fact that PPP does not hold continuously has been well established, the prediction of the flexible monetary model is generally taken as unreliable.

Finally, an improvement in the terms of trade may bring about an appreciation of the real exchange rate. The wealth effect associated with such an improvement will tend to lead to an increase in consumption, which at least in a small country for which traded goods prices are given by the world market, will raise the prices of non-traded goods and hence domestic price levels.

Based on the above arguments, an augmented PPP model can be specified in the following way:

Let: S denote spot exchange rate,

P denote domestic price level,

R denote domestic real interest rate,

G denote domestic growth rate in real GDP,

C denote domestic accumulated current account balance as a percentage of GDP,

TT denote domestic terms of trade,

and finally let superscription f denote a corresponding foreign measurement, the relationship between the exchange rate and other variables can be summarized in the following functional form:

$$S = \delta\left(P/P^f, R - R^f, G/G^f, C - C^f, TT/TT^f\right) \tag{45}$$

3.2: The Data

Data for 10 OECD countries: Australia, Japan, Germany, Canada, United Kingdom, France, Italy, Norway, Netherlands and the United States are used. The sample period runs from 1973:Q1 through 1998:Q4, corresponding to the recent floating exchange rate period. Quarterly data are used to gain sufficient degrees of freedom and to avoid introducing "noise" by overly disaggregating the data at the same time. All the data are taken from IMF International Financial Statistics CD-ROM issued July 2000.

The spot exchange rate S is measured as the quarterly average of the home currency price of one unit foreign currency. The United States is defined as the foreign country while all other countries involved in the study are defined as home countries. Price P and P^f are measured by a consumer price index. The justification for the choice of this index is given in section **2.1**. Real interest rates for all countries are measured by the nominal yield on 10-year government bonds adjusted by 4-quarter moving average inflation rate, which in turn is measured by changes in the consumer price index. Measurements for G, C, and their foreign counterparts G^f, C^f, are straightforward in their definitions given in the last section. Finally terms of trade TT and TT^f are constructed as the ratios of export unit value to import unit value for domestic and foreign countries, respectively. Except domestic and foreign accumulative current account balance C and C^f, which are expressed as the percentage point of domestic and foreign GDP respectively, all the variables are transformed using the natural logarithmic operator. Using lower case to denote the logarithmic transformed variables (except interest rate), Equation (45) can be rewritten as:

$$s = \delta'(p - p^f, r - r^f, g - g^f, C - C^f, tt - tt^f) \tag{46}$$

3.3: The Methodology

Define $\boldsymbol{X}$ as a (9×1) non-stationary $I(1)$ vector $(s, p, p^f, r, r^f, g, g^f, C^*, tt^*)$, where $C^* = C - C^f, tt^* = tt - tt^f$. The relationship among these variables can be described by the following vector autoregression (VAR) representation:

$$\boldsymbol{X}_t = \sum_{i=1}^{k} \boldsymbol{A}_i \boldsymbol{X}_{t-i} + \boldsymbol{\Phi D} + \boldsymbol{e}_t \tag{47}$$

where each of A_i is an $(9{\times}9)$ matrix of parameters, the deterministic terms D_t contain a constant and centered seasonal dummies, e_t is a $(9{\times}1)$ vector of white noise disturbances, with mean zero and covariance matrix Σ. Letting Δ represent the first difference operator, Equation (47) can be reformulated into an equivalent vector error correction (VECM) form:

$$\Delta X_t = \Pi X_{t-k} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Phi D_t + e_t \tag{48}$$

where $\Gamma_i = -(I - A_1 - \ldots - A_i)$, $(i = 1, 2, \ldots, k-1)$, $\Pi = -(I - A_1 - \ldots - A_k)$. This specification of the system contains information on both the short run and long run adjustment to changes in X_t, via the estimates of $\hat{\Gamma}_i$ and $\hat{\Pi}$ respectively. Under the assumptions that X is a nonstationary $I(1)$ vector and e_t is a vector of white noise disturbances, the first difference terms $\Gamma_i \Delta X_{t-i}$ must be stationary and Π must be a zero or reduced rank matrix. If Π is a reduced rank matrix, it can be factorized as $\Pi = \alpha\beta'$, where β is the cointegrating vectors that define the long run equilibrium relationship among the variables included in X, α represents the speed of adjustment to disequilibrium. Since α and β cannot generally be obtained by an ordinary least squares method, Johansen (1988) suggests using the reduced rank regression method. Rewriting Equation (48) as:

$$\Delta X_t + \alpha\beta' X_{t-k} = \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Phi D_t + e_t\ , \tag{49}$$

Regress ΔX_t and X_{t-k} separately on the right-hand side of Equation (47) to produce the residuals R_{0t} and R_{kt}:

$$\Delta X_t = \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Phi D_t + e_t \ , \qquad (50)$$

$$\Delta X_t = \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Phi D_t + e_t \qquad (51)$$

Then R_{0t} and R_{kt} are used to form cross product matrices:

$$S_{ij} = \sum_{i=1}^{T} R_{it} R'_{jt} \Big/ T \qquad i, j = 0, k \qquad (52)$$

The maximum likelihood estimate of $\Delta X_t = \Pi X_{t-k+1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Phi D_t + e_t$ is obtained as the eigenvectors corresponding to the r largest eigenvalues from solving the equation

$$\left| \lambda S_{kk} - S_{k0} S_{00}^{-1} S_{0k} \right| = 0 \ , \qquad (53)$$

which gives n eigenvalues $\hat{\lambda}_1 > \hat{\lambda}_2 > \dots > \hat{\lambda}_n$ and their corresponding eigenvectors $\hat{V} = (\hat{v}_1, \dots, \hat{v}_n)$. The estimate of β is the eigenvectors associated with the r $(r \leq n)$

largest and statistically significant eigenvalues, $\hat{\beta} = (\hat{v}_1, \ldots, \hat{v}_r)$. The likelihood ratio test statistic for the null hypothesis that there are at most r cointegration vectors is given by

$$\lambda_{trace} = -T \sum_{i=r+1}^{n} log(1 - \hat{\lambda}_i) \qquad r = 0, 1, 2, \ldots, n\text{-}2, n\text{-}1. \tag{54}$$

Another likelihood ratio test statistic, which tests the null hypothesis that there are r cointegration vectors against the alternative that $r + 1$ exist, is given by

$$\lambda_{max} = -T\, log(1 - \hat{\lambda}_{r+1}) \qquad r = 0, 1, 2, \ldots, n\text{-}2, n\text{-}1. \tag{55}$$

The asymptotic distributions of these test statistics are not standard normal, and depend on the deterministic terms present in the model. The critical values for both test statistics are generated through Monte Carlo methods and are tabulated in Osterwald-Lenum (1992).

Since the Johansen reduced rank regression procedure only determines how many unique cointegration vectors span the cointegration space, and since any linear combination of the stationary vectors is also a stationary vector, the estimates produced for any particular column in β are not necessarily unique. Therefore it is necessary to impose restrictions motivated by theories on β and then test whether the restricted columns of β are identified. One difficulty in formulating restrictions on β is that economic theory is not usually explicit about the structural relationships among the variables under consideration. In the current context, except for the sub-vector of (s, p, p^f), for which the theory of the purchasing power parity clearly indicates that the expected values are *(1, 1, -1)*, there is no theory that provides any specific expected values for the other variables included vector *X*. Even the expected signs of some of these variables are subject to debate. Therefore only a restriction of *(1, 1, -1)* is imposed in this study for the sub-vector of (s, p, p^f) based on the homogeneity and symmetry conditions of PPP. The sub-vector of other variables used to augment the theory of PPP is unconstrained. There is another reason for this way of formulating the restrictions on the cointegrating vectors. The primary objective of this study is to test the homogeneity and symmetry conditions of PPP in an augmented model. If the restrictions were imposed based on more than one theory and if the restricted cointegrating vectors were rejected by the data, it would be difficult to tell whether the rejection is caused by the failure of PPP or by the failure of other theories. Although the restriction is imposed only on the sub-vector of (s, p, p^f), information about the nature of the relationship between the exchange rate and other variables can be gained from the signs of the identified cointegrating vectors.

Based on the above arguments and the five hypothesis listed in the introduction part of this study, the restricted cointegrating vectors are formulated and presented in Table 3.1:

Table 3.1

Hypotheses and Their Restrictions on Cointegrating Vectors

Hypotheses	Testing Restriction
H1: PPP alone forms a cointegrating vector	β=(1, 1, -1, 0, 0, 0, 0, 0, 0)
H2: H1 is augmented by an interest rate differential	β=(1, 1, -1, *, *, 0, 0, 0, 0)
H3: H2 is augmented by productivity bias	β=(1, 1, -1, *, *, *, *, 0, 0)
H4: H3 is augmented by current account balance	β=(1, 1, -1, *, *, *, *, *, 0)
H5: H4 is augmented by terms of trade	β=(1, 1, -1, *, *, *, *, *, *)

Where '*' denotes an unconstrained value. It is clear from Table 3.1 that the homogeneity and symmetry conditions of PPP are tested directly by imposing their theoretical values on the cointegrating vectors, while for the other variables in X, there are no theoretical values are imposed. Follow Johansen (1988) and Johansen and Juselius (1990), each of the above tests can be conducted by forming a linear restriction on β as:

$$\beta = H\varphi \tag{56}$$

where H is a known matrix of dimension $n \times s$ ($s = n$ - the number of restrictions on the column), φ is a $s \times n$ parameter matrix to be estimated. The maximum likelihood estimator of β is obtained by first solving:

$$\left|\lambda H' S_{kk} H - H' S_{k0} S_{00}^{-1} S_{0k} H\right| = 0 \tag{57}$$

for $\lambda_1^* > \cdots > \lambda_s^* > 0$ and $V = (v_1, \cdots v_s)$, which is then normalized by $V'H'S_{kk}HV = I$.

Then these new eigenvalues λ_i^* are used to calculate the following likelihood ratio test statistic:

$$-2\log(Q) = T\sum_{i=1}^{r}\log\left[\frac{1-\lambda_i^*}{1-\hat{\lambda}_i}\right] \tag{58}$$

This test statistic is compared with the χ^2-distribution with $[r \times (n - s)]$ degrees of freedom in order to obtain the significance level for rejecting the null hypothesis. The estimate of φ is the eigenvectors associated with the r $(r \leq n)$ largest and statistically significant eigenvalues, $\hat{\varphi} = (\hat{v}_1, \ldots, \hat{v}_r)$, and $\hat{\beta} = H\hat{\varphi}$.

CHAPTER 4

AUGMENTING PPP: EMPIRICAL EVIDENCE

4.1: Testing for the Order of Integration of the Variables

Before performing the cointegration procedure outlined in Chapter 3, I first test for the order of integration of all the variables included in the model. Although it is not necessary for all the variables in the model to have the same order of integration unless the number of the variables included in the model is two, it is important to understand and take account of the implications when all the variables are not $I(1)$. For example, if the model contains $I(2)$ variables, some or all of the I(2) variables may cointegrate down to I(1) space and then further cointegrate with other I(1) variables to obtain a cointegration vector(s). Thus, the presence of variables that must be differenced twice to induce stationarity does not preclude the possibility of stationary relationship in the model. However, applying the standard Johansen approach, which is designed to handle I(1) and I(0) variables, will not provide the necessary stationary vectors. When there are I(2) variables in the model, one must either replace them with I(1) alternatives, or it will be necessary to use the approach developed by Johansen (1994) for I(2) model. Knowing there are I(2) variables in the model can help in formulating the right approach to estimating the cointegration relationship in such situations.

The Philips-Perron (1988) unit root tests are used to test for the order of integration of all the variables included in the model. The tests are robust to a wide variety of serial correlation and time-dependent heteroskedasticity. The tests involve estimating the following regression model:

$$y_t = \mu + \beta t + \alpha y_{t-1} + u_t \qquad (62)$$

where y_t is any variables under examination, μ is the drift, t is the time trend, and u_t is the error term that could be an ARMA process with time-dependent variances. The null hypothesis is $\alpha = 1$ against the alternative $|\alpha| < 1$. The results of applying the Philips-Perron unit root tests are reported in table 4.1. The test statistics are computed using up to 10 lags in the regression residuals based on Akaike Information Criterion.

Table 4.1

Philips-Perron Unit Root Tests

Model: $y_t = \mu + \beta t + \alpha y_{t-1} + u_t$

Country	Variable	X	ΔX	Country	Variable	X	ΔX
AUS	s [2, 2]	-1.8008	-48.0176***	JAP	s [3, 2]	-2.3147	-34.9146***
	p [3, 2]	-0.5588	-35.9228***		p [9, 8]	-1.7229	-101.7515***
	r [2, 2]	-3.6343	-56.7018***		r [6, 4]	-6.4060	-57.6698***
	g [6, 2]	-0.4242	-44.4150***		g [3, 2]	-0.8954	-59.8157***
	C [3, 2]	-1.0076	-13.2787*		C [11, 10]	-0.0171	-7.3410
	tt [4, 3]	-2.9117	-39.6351***		tt [2, 2]	-3.6761	-34.7282***
CAN	s [5, 4]	-5.3189	-39.0390***	U. K.	s [3, 2]	-4.4155	-42.8591***
	p [10, 10]	-1.1928	-13.1463*		p [7, 10]	-1.1398	-62.6472***
	r [2, 2]	-5.3559	-45.1397***		r [2, 2]	-4.8205	-48.6529***
	g [3, 2]	-0.9641	-28.9061***		g [5, 4]	-0.4058	-69.2436***
	C [10, 6]	-4.7386	-19.5329***		C [10, 10]	-0.1291	-11.0547*
	tt [3, 3]	-4.5249	-37.5764***		tt [3, 3]	-8.9730	-34.5256***
FRA	s [9, 2]	-4.2804	-30.2302***	NETH	s [3, 2]	-3.3787	-35.1622***
	p [3, 8]	-1.5066	-5.7204		p [6, 6]	-1.7893	-49.3194***
	r [6, 5]	-3.9017	-30.3702***		r [5, 4]	-4.4218	-47.4933***
	g [10, 10]	-0.2100	-65.1359***		g [5, 10]	-0.8818	-80.1940***
	C [10, 10]	-7.4340	-36.4646***		C [10, 10]	-0.3345	-41.0214***
	tt [2, 2]	-5.1972	-42.2219***		tt [3, 6]	-15.43**	-33.8138***
GER	s [3, 2]	-3.4063	-34.5857***	NOR	s [10, 2]	-3.8483	-38.0618***
	p [3, 2]	-1.0767	-29.0683***		p [10, 5]	-0.7572	-38.2411***
	r [10, 4]	-6.0730	-43.3032***		r [2, 2]	-4.1656	-37.8803***
	g [2, 2]	-5.5000	-58.4343***		g [10, 10]	-1.3616	-42.9016***
	C [10, 10]	-0.5893	-30.2207***		C [10, 4]	-1.4161	-21.7127***
	tt [2, 2]	-6.3484	-40.1415***		tt [2, 3]	-5.1046	-46.5132***
ITA	s [10, 2]	-3.5791	-28.7850***	U. S.	s ------	------	------
	p [10, 10]	-1.2482	-11.5767*		p [5, 4]	-1.3899	-11.9686*
	r [3, 2]	-2.7858	-37.8702***		r [3, 2]	-4.8843	-39.2325***
	g [2, 2]	-0.3227	-35.4813***		g [10, 10]	-0.6360	-35.6874***
	C [10, 10]	-9.3083	-22.4001***		C ------	------	------
	tt [6, 5]	-2.9846	-49.4535***		tt ------	------	------

s is the log of the exchange rate expressed as national currency price of the U.S. dollar. p is the log of consumer price index. r is the log of annualized (real) government bond yield expressed in percentage points. g is the log of the real GDP index. C is a country's accumulated current account balance as a proportion of nominal GDP relative to the same measurement of the United States. tt is constructed as the ratio of domestic export unit value to import unit value relative to the equivalent ratio of the United States and is expressed in logarithmic terms. The first number in brackets following a variable is the lag order in level test and the second number is the lag order in first difference test. The order of lags chosen for serial correlation correction in all cases is determined by Akaike Information Criterion.

* 10 percent level of significance.
** 5 percent level of significance.
*** 1 percent level of significance.

Generally speaking, the results listed in table 4.1 are consistent with the hypotheses that most economic time series are nonstationary since there is no tendency for them to return to an average value over time. For the level of the variables, the null hypothesis of a unit root cannot be rejected in 56 of 67 cases at any conventional significance level. The only exception is the case for the relative terms of trade *tt* of Netherlands, for which the null hypothesis of a unit root can be easily rejected at 5 percent level of significance. In contrast to the test results involving the levels of the variables, the test results involving the first differences of the variables reject the null hypothesis of a unit root in 54 of 57 cases. The three exceptions are p (log of Consumer Price Index) of France, *C* (accumulative current account balance as a percentage of nominal GDP in a country relative to the same measurement of the United States) of Japan and United Kingdom. The null hypothesis of a unit root could not be rejected for these three variables at any conventional significance level. These results suggest that (1) *tt* for Netherlands follows a stationary process; (2) *p* for France is likely a second order integrated series; and (3) *C* for Japan and UK are also likely *I(2)* series. As stated previously, since the standard Johansen cointegration procedure requires the main variables in the model be nonstationary in levels but stationary in first differences, and since the main purpose of this study is not to test PPP as it stands, but to augment it with other variables including *tt* and *C*, all the variables for Netherlands, France, Japan and United Kingdom are dropped in the following analysis.

4.2: Testing for Reduced Rank

The results obtained from applying the Johansen reduced rank regression to the augmented PPP model are presented in Table 4.2 - Table 4.6. The various hypotheses to be tested, from no cointegration (i.e., $r = 0$ or alternatively $n - r = 9$) to increasing number of cointegration vectors, are presented in the first column. The eigenvalues associated with the combinations of the $I(1)$ levels of X_t are in the second column, ordered from the highest to the lowest. Next come the λ_{trace} statistics which test the null that $r = q$ ($q = 1, 2, \ldots, n - 1$) against the unrestricted alternative that $r = n$. The λ_{max} statistics test whether $r = 0$ against $r = 1$ or $r = 1$ against $r = 2$, etc. The adjusted λ_{trace} and λ_{max} statistics are presented in column 5 and column 8 through Table 4.2 to Table 4.6. These adjusted statistics were first suggested by Reinsel and Ahn (1992) for small sample correction. They show that in small samples the standard Johansen procedure has a tendency to over-reject when the null hypothesis is true. Thus, they suggest taking account of the number of parameters to be estimated in the model and making an adjustment for degree of freedom by using the factor of $(T - nk)$ instead of the sample size T in the calculation of the test statistics for cointegrating rank, where n is the number of variables in the model and k is the lag-length in the reduced rank regression. This idea has been investigated by Reimers (1992) and it is found that the approximation to the limit distribution is better with the corrected sample size. The critical values are provided in Osterwald-Lenum (1992, Table 1).

Table 4.2

Reduced Rank Tests of the Augmented PPP Using Australian Data

$H_0 : r$	n-r	$\hat{\lambda}_i$	λ_{trace}	Adj. λ_{trace}	$\lambda_{trace}(.95)$	λ_{max}	Adj. λ_{max}	$\lambda_{max}(.95)$
0	9	0.54549	313.59**	200.70**	192.89	78.85**	50.47	51.72
1	8	0.44363	234.74**	150.23	156.00	58.63**	37.53	51.42
2	7	0.40244	176.11**	112.71	124.24	51.49**	32.95	45.28
3	6	0.35816	180.28**	79.75	94.15	44.34**	28.38	39.37
4	5	0.29659	124.61**	51.38	68.52	35.18	22.52	33.46
5	4	0.16558	45.09**	28.86	47.21	18.10	11.59	27.07
6	3	0.16180	26.99**	17.27	29.68	17.65	11.30	20.97
7	2	0.07013	9.34	5.98	15.41	7.27	4.65	14.07
8	1	0.02048	2.07	1.33	3.76	2.07	1.33	3.76

* 10 percent level of significance.
** 5 percent level of significance.

Table 4.3

Reduced Rank Tests of the Augmented PPP Using Canadian Data

$H_0 : r$	n-r	$\hat{\lambda}_i$	λ_{trace}	Adj. λ_{trace}	$\lambda_{trace}(.95)$	λ_{max}	Adj. λ_{max}	$\lambda_{max}(.95)$
0	9	0.58759	346.97**	222.06**	192.89	88.57**	56.69*	51.72
1	8	0.49097	258.39**	165.37**	156.00	67.43**	43.12	51.42
2	7	0.40465	190.97**	122.22	124.24	51.86*	33.19	45.28
3	6	0.34589	139.10**	89.03	94.15	42.86*	27.43	39.37
4	5	0.26943	96.24**	61.60	68.52	31.39	20.09	33.46
5	4	0.24535	64.85**	41.50	47.21	28.15	18.02	27.07
6	3	0.22134	36.70**	23.49	29.68	25.02	16.01	20.97
7	2	0.00118	11.68	7.48	15.41	11.67	7.47	14.07
8	1	0.02048	0.01	0.01	3.76	0.01	0.01	3.76

* 10 percent level of significance.
** 5 percent level of significance.

Table 4.4

Reduced Rank Tests of the Augmented PPP Using German Data

$H_0 : r$	n-r	$\hat{\lambda}_i$	λ_{trace}	Adj. λ_{trace}	$\lambda_{trace}(.95)$	$\lambda_{\max}$	Adj. $\lambda_{\max}$	$\lambda_{\max}(.95)$
0	9	0.67011	377.32**	222.96**	192.89	97.59**	57.67*	51.72
1	8	0.56567	279.73**	165.29**	156.00	73.39**	43.37	51.42
2	7	0.47460	200.34**	121.93	124.24	56.64**	33.47	45.28
3	6	0.42743	149.70**	88.46	94.15	49.07**	29.00	39.37
4	5	0.38106	100.63**	59.46	68.52	42.22**	24.95	33.46
5	4	0.22151	58.41**	34.52	47.21	22.04	13.02	27.07
6	3	0.18356	36.38**	21.50	29.68	17.85	10.55	20.97
7	2	0.11212	18.53**	10.95	15.41	10.47	6.18	14.07
8	1	0.08756	8.06**	4.77**	3.76	8.06	4.77**	3.76

* 10 percent level of significance.
** 5 percent level of significance.

Table 4.5

Reduced Rank Tests of the Augmented PPP Using Italian Data

$H_0 : r$	n-r	$\hat{\lambda}_i$	λ_{trace}	Adj. λ_{trace}	$\lambda_{trace}(.95)$	$\lambda_{\max}$	Adj. $\lambda_{\max}$	$\lambda_{\max}(.95)$
0	9	0.59517	341.14**	217.09	192.89	88.65**	56.41**	51.72
1	8	0.44982	252.49**	160.68	156.00	59.55**	37.64	51.42
2	7	0.42700	193.34**	123.03	124.24	55.13**	35.08	45.28
3	6	0.33436	138.21**	87.95	94.15	40.29**	25.64	39.37
4	5	0.29499	97.92**	62.31	68.52	34.61**	22.02	33.46
5	4	0.23402	63.31**	40.29	47.21	26.39	16.80	27.07
6	3	0.17829	36.91**	23.49	29.68	19.44	12.37	20.97
7	2	0.15684	17.47**	11.12	15.41	16.89	10.74	14.07
8	1	0.05876	0.58	0.37	3.76	0.58	0.37	3.76

* 10 percent level of significance.
** 5 percent level of significance.

Table 4.6

Reduced Rank Tests of the Augmented PPP Using Norwegian Data

$H_0: r$	n-r	$\hat{\lambda}_i$	λ_{trace}	Adj. λ_{trace}	λ_{trace} (.95)	$\lambda_{\max}$	Adj. $\lambda_{\max}$	$\lambda_{\max}$ (.95)
0	9	0.79837	430.62**	188.40*	192.89	102.48	44.84	51.72
1	8	0.78097	328.14**	143.56	156.00	97.19	42.52	51.42
2	7	0.72138	230.95**	101.04	124.24	81.79	35.78	45.28
3	6	0.48517	149.17**	65.26	94.15	42.49	18.59	39.37
4	5	0.45355	106.68**	46.67	68.52	38.69	16.92	33.46
5	4	0.34572	68.00**	29.75	47.21	27.15	11.88	27.07
6	3	0.26068	40.85**	17.87	29.68	19.33	8.46	20.97
7	2	0.22675	21.52**	9.41	15.41	16.46	7.20	14.07
8	1	0.07603	5.06**	2.21	3.76	5.06	2.21	3.76

* 10 percent level of significance.
** 5 percent level of significance

From Table 4.2 to Table 4.6, it is obvious that the conclusions about the number of cointegrating vectors could be very different based on different test statistics. For example, in the case of Australia, it is possible to accept that there are seven cointegrating vectors based on λ_{trace}, while only four cointegrating vectors can be accepted based on $\lambda_{\max}$. The null hypothesis of no cointegration cannot be rejected at the 5 percent significance level if adjusted $\lambda_{\max}$ is used, whereas the adjusted λ_{trace} statistic would lead us to accept one cointegration vector. This apparent contradiction in the tests for cointegration rank is not uncommon. Johansen and Juselius (1992), among others, encountered similar problems when they applied the procedure to PPP and UIP models using U.K data. Their explanation for this ambiguity is the low power of the test in cases when the cointegration relation is quite close to the nonstationary boundary. Also, the inclusion of dummy or dummy type variables in D_t affects the underlying distribution of the test statistics, such that the critical values for these tests are different depending on the number of dummies included.

In this study the final determination of the number of cointegrating vectors is based on the adjusted λ_{trace} statistics. There are at least two reasons for the choice of this decision rule. First, this study examines the behavior of exchange rates during the recent floating period using quarterly data; the sample size is relatively small for all the countries involved. The number of observations is 100 for Australia and Canada, 94 for Italy, 88 for Germany, and only 77 for Norwegian data, respectively. Since the system includes nine variables with four lags, three seasonal dummy variables and a constant, each equation is fitted with $nk + d = 40$ parameters, the degrees of freedom for each country varies form 60 to 37. Therefore, the small sample bias toward over-rejection could be significant in present application, suggesting that the adjusted statistics should be used. Secondly, the Monte Carlo experiments reported in Cheung and Lai (1993) suggest that '...between Johansen's two Likelihood Ratio tests for cointegration, the λ_{trace} test shows more robustness to both skewness and excess kurtosis in the residuals than the $\lambda_{\max}$ test'. Since data diagnoses indicate that many variables in the Augmented PPP model suffer from excess kurtosis, it is preferable to place greater weight on the λ_{trace} test. Therefore, the adjusted λ_{trace} statistics are used as the decision rule in this study. Based on this decision rule, one cointegrating vector is found for Australia and Norway. Two cointegrating vectors are found for Canada, Germany and Italy. Thus, for these five countries investigated, the Johansen multivariate cointegration tests show that there is a long run equilibrium relationship among the variables included in the augmented PPP model.

Tables 4.2 to 4.6 are also informative about the small sample bias of the λ_{trace} and $\lambda_{\max}$ statistics as originally proposed by Johansen. It has been pointed out that these two statistics have a tendency to over reject the null hypothesis when it is in fact true. This over-rejection bias is particularly significant when sample size is small. Since the sample sizes for all the countries in this study are relatively small, there are significant differences between the results based on the original test statistics and the results based on the adjusted test statistics. What is more instructive is the fact that this difference is related to the sample size. For example, based on original λ_{trace} statistics, the number of cointegrating vectors for Germany and Norway is nine, which is equal to the number of the variables included in the model. This clearly cannot be true since it requires all the variables in vector $\boldsymbol{X}$ to be stationary, but the unit root tests conducted in the previous step indicate this is not the case. Since the sample sizes for these two countries are the smallest ones in this study, the severity of bias toward over rejection is apparently related to the sample size.

4.3: Testing for Constrained Cointegration Vectors

In Chapter 3, I indicate that the Johansen reduced rank regression procedure only determines how many unique cointegration vectors span the cointegration space, and since any linear combination of the stationary vectors is also a stationary vector, the estimates produced for any particular column in β are not necessarily unique. This poses difficulties in the interpretation of the unrestricted cointegration vectors. Fortunately researchers have found that the imposition of testable restrictions on the space spanned by the vectors can reveal more meaningful relationships. Based on the model outlined in Chapter 3, five nested hypotheses about the nature of the long run relationship can be imposed on the cointegration space. The first one is the hypothesis that the Purchasing Power Parity by itself may form a cointegrating relationship, that is, s, p and p^f enter into the cointegration relations with coefficients proportional to (1, 1, -1) and the coefficients for all the other variables in the model are zeros. In this case, the normalized restricted cointegration vector will be $\beta' = (1,1,-1,0,0,0,0,0,0)$. If the first hypothesis is rejected, as it was in many previous studies, it would be interesting to test the second through fifth hypotheses, each of them augments the conventional PPP with additional variables. The logic for this exercise is this: the rejection of hypothesis 1 is not sufficient to deny the existence of the homogeneity and proportionality between the bilateral exchange rates and the relative prices. The relationship could be distorted by other variables that have systematic impacts on the exchange rates but were excluded from the conventional PPP model. If the effects of these variables are taken account of by explicitly including them in the augmented PPP model, the true relationship between nominal exchange rate and the relative prices could be revealed. The five nested hypotheses and their implied restrictions on the cointegrating vectors have been presented in Table 3.1. It is reproduced here for convenience,

Table 3.1

Hypotheses and Their Restrictions on Cointegrating Vectors

Hypotheses	Testing Restriction
H1: PPP alone forms a cointegrating vector	β=(1, 1, -1, 0, 0, 0, 0, 0, 0)
H2: H1 is augmented by an interest rate differential	β=(1, 1, -1, *, *, 0, 0, 0, 0)
H3: H2 is augmented by productivity bias	β=(1, 1, -1, *, *, *, *, 0, 0)
H4: H3 is augmented by current account balance	β=(1, 1, -1, *, *, *, *, *, 0)
H5: H4 is augmented by terms of trade	β=(1, 1, -1, *, *, *, *, *, *)

To implement the likelihood ratio test, an H matrix is formulated for each of the five hypotheses:

$$H1=\begin{vmatrix}1\\1\\-1\\0\\0\\0\\0\\0\\0\end{vmatrix};\qquad H2=\begin{vmatrix}1&0&0\\1&0&0\\-1&0&0\\0&1&0\\0&0&1\\0&0&0\\0&0&0\\0&0&0\\0&0&0\end{vmatrix};\qquad H3=\begin{vmatrix}1&0&0&0&0\\1&0&0&0&0\\-1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\\0&0&0&0&0\\0&0&0&0&0\end{vmatrix}$$

$$H4=\begin{vmatrix}1&0&0&0&0&0\\1&0&0&0&0&0\\-1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{vmatrix};\qquad H5=\begin{vmatrix}1&0&0&0&0&0&0\\1&0&0&0&0&0&0\\-1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&1&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\end{vmatrix}.$$

Each of the five hypotheses is tested by first plugging its corresponding H matrix into Equation (57), $|\lambda H' S_{kk} H - H' S_{k0} S_{00}^{-1} S_{0k} H| = 0$, with S_{kk}, S_{k0} and S_{00} as defined in Equation (52). The solution to the eigenvalue problem of Equation (57) is the s new eigenvalues λ_i^* for the restricted model, where $s = n -$ the number of the restrictions imposed on the coefficients. The new eigenvalues λ_i^* are then used to obtain the likelihood ratio test statistic given by Equation (58), $-2\log(Q) = T\sum_{i=1}^{r}\log\left[\frac{1-\lambda_i^*}{1-\hat{\lambda}_i}\right]$, which is then compared with the χ^2 distribution with $[r \times (n - s)]$ degrees of freedom in order to obtain the significance level for rejecting the null hypothesis. As with the testing procedure for reduced rank, it has been suggested that the LR statistic given by Equation (58) should be corrected for degrees of freedom, which involves replacing T, the sample size, by T - *(l/n),* where l is the number of parameters estimated in the reduced rank regression model (i. e., l =[$(k \times n)$ + number of deterministic components)] $\times$ n.). Psaradakis (1994) found, on the basis of Monte Carlo testing, that such a modification improved the small sample behavior of the LR statistics. The results of testing for the five linear hypotheses on cointegrating relations are given in Table 4.7. The LR statistics reported in the table are corrected for degrees of freedom.

Table 4.7

LR Tests for Linear Constraints on Cointegrating Vectors

Country	Hypothesis	1st r Eigenvalues	Test Statistics	$\chi^2_{r\times(n-s)}(0.95)$
Australia [r = 1]	H1	0.27153	28.3036***	15.51
	H2	0.37094	19.5004***	12.59
	H3	0.48825	**7.1170**	9.49
	H4	0.48841	**7.0982**	7.81
	H5	0.53705	**1.1040**	5.99
Canada [r = 2]	H1	0.28416	33.0863***	26.30
	H2	0.34654, 0.31075	45.8016***	21.03
	H3	0.45420, 0.36688	29.9033***	15.51
	H4	0.46650, 0.37589	27.6756***	12.59
	H5	0.54947, 0.45885	**8.9758**	9.49
Germany [r = 2]	H1	0.20296	42.3430***	26.30
	H2	0.63667, 0.25222	30.7131***	21.03
	H3	0.63847, 0.39110	20.6129***	15.51
	H4	0.63849, 0.41856	18.3952***	12.59
	H5	0.63862, 0.48828	12.2469**	9.49
Italy [r = 2]	H1	0.20206	39.5131***	26.30
	H2	0.38781, 0.24968	42.1836***	21.03
	H3	0.51289, 0.33582	21.1836***	15.51
	H4	0.51325, 0.35041	20.1500***	12.59
	H5	0.56933, 0.44966	**3.1454**	9.49
Norway [r = 1]	H1	0.41867	25.5696***	15.51
	H2	0.43985	24.5225***	12.59
	H3	0.51901	20.8659***	9.49
	H4	0.53078	20.2713***	7.81
	H5	0.79368	**0.5519**	5.99

** 5 percent level of significance.
*** 1 percent level of significance.

Several points can be made from Table 4.7: (1) the restricted cointegrating vector for conventional PPP is uniformly rejected at a 1 percent significance level for all five countries. This result is consistent with the evidence documented in existing literature. Although cointegration among the exchange rate and the domestic and foreign price levels has been found in many previous studies, the relationship is not the same as implied in the homogeneity and symmetry conditions of PPP. There are only two interpretations for the failure to find supportive evidence for H1: the first interpretation is simply that PPP is wrong about the specific relationships among the exchange rate and the domestic and foreign price levels. The second interpretation is some other non-price variables are also involved in the determination of the exchange rate and the true relationship among the exchange rate and the price variables are distorted due to the omission of these non-price variables from the model. (2) When all the non-price variables are included in the augmented PPP model, the restricted cointegrating vector cannot be rejected at any conventional significance levels in 4 out of 5 countries. The only exception is the case for Germany, where the data still reject the restricted cointegrating vectors even though all the price and non-price variables are included in the model. One potential reason for the failure to find the theoretical PPP-vector [1, 1, −1] in the case of Germany is the structural break caused by monetary reunification between the Eastern and Western Germany in 1992. In the case of Australia, the restricted cointegrating vectors are identified as soon as the interest rate differential and relative GDP growth rate are included in the model. These findings are supportive of my conjecture that the reason for the empirical failure of the homogeneity and symmetry conditions of PPP is not because such conditions do not exist, as many previous studies suggest, but because other variables in addition to the relative price are involved in the determination of the exchange rate. When these non-price variables are missing from the model, the true relationship between the exchange rate and the price variables is distorted. Once these variables are explicitly included in the model, the homogeneity and symmetry conditions as implied in PPP are revealed.

Table 4.8 presents the restricted cointegrating vectors that are accepted by the data. In addition to confirming the homogeneity and symmetry conditions of PPP, the

Table 4.8

Restricted Cointegrating Vectors

Variables	Australia	Canada (1)	Canada (2)	Italy (1)	Italy (2)	Norway
s +	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
p +	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
p^f -	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000
r -	1.7916	-0.8918	-1.7794	-5.1467	-11.6874	-7.9205
r^f +	-1.0993	1.6718	3.0566	0.6678	14.3091	6.2237
g -	28.8714	-3.0943	-6.8134	27.2159	-29.2617	-27.9015
g^f +	-32.0587	3.8542	-10.3391	-21.3334	30.8166	17.9993
C -	0.1215	0.0279	0.0007	0.0251	0.1890	0.0105
tt -	-6.0833	3.3944	-3.5320	-9.8531	-41.5563	7.6977

Note: the '+' and '-' signs following the variables indicate the theoretical signs for the variables.

signs and values of these restricted cointegrating vectors are also indicative of the nature of the relationship between the exchange rate and the non-price variables. The signs for the interest rate variables are consistent with their theoretical priors in ten of twelve cases, with the case for Australia as the only exception. This is supportive of the hypothesis that a relative increase in the domestic real interest rate will cause an appreciation of the domestic currency (a decrease in *s*), while a relative increase in the foreign real interest rate will cause a depreciation of the domestic currency (an increase in s). Also, the impact of the domestic and foreign real interest rates on the exchange rate is reasonably symmetrical in the cases of Italy and Norway. Table 4.8 also shows correct signs for the growth variables in 8 of 10 cases. A relatively faster growth in domestic GDP tends to appreciate the domestic currency (a decrease in s), while a relatively faster growth in foreign GDP tends to depreciate the domestic currency (an increase in s). These results are consistent with the productivity bias hypothesis that the currency of a relatively faster growing economy tends to appreciate in real terms. This study does not provide supportive evidence for the hypothesized relationship between the exchange rate and current account balance. The signs are wrong in all cases. Finally, Table 4.8 provides supportive evidence for the hypothesized relationship between the exchange rate and terms of trade. A relative improvement in terms of trade tends to appreciate the domestic currency (a decrease in s), while deterioration in terms of trade tends to depreciate the domestic currency (an increase in s).

I plot the restricted cointegrating relationships ($\hat{\beta}'\mathbf{X}$) discovered above in Figure 4.1 – Figure 4-6. As expected, a visual observation indicates that the linear combinations of these variables as defined by the restricted cointegrating vectors are stationary. This is consistent with the results of the formal tests.

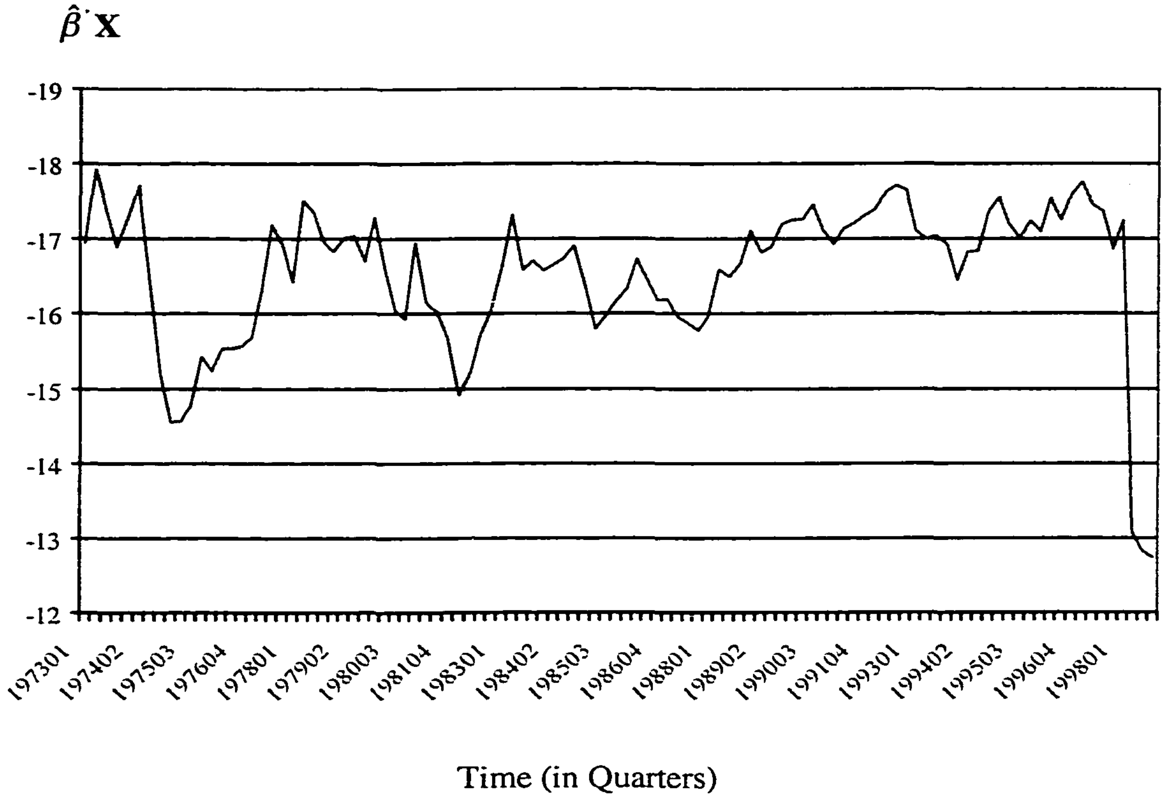

Figure 4.1

Cointegrating Relationship for Australian Data

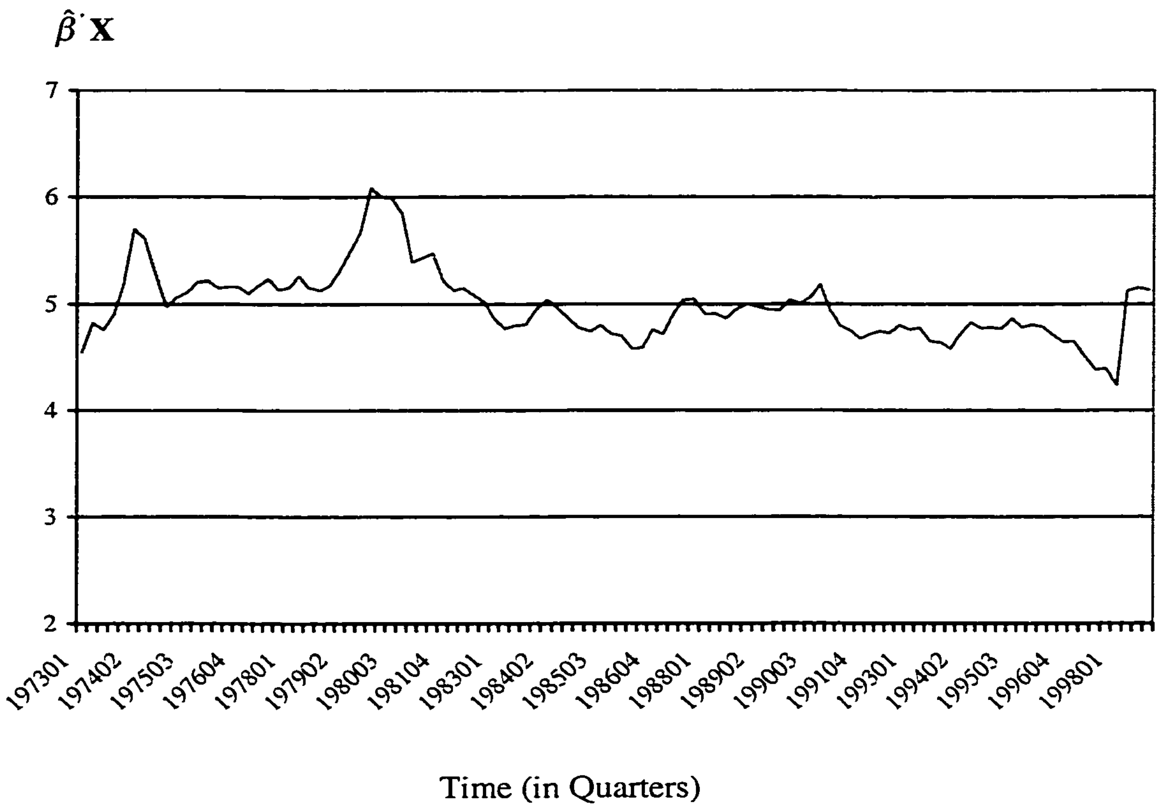

Figure 4.2

Cointegrating Relationship for Canadian Data (1)

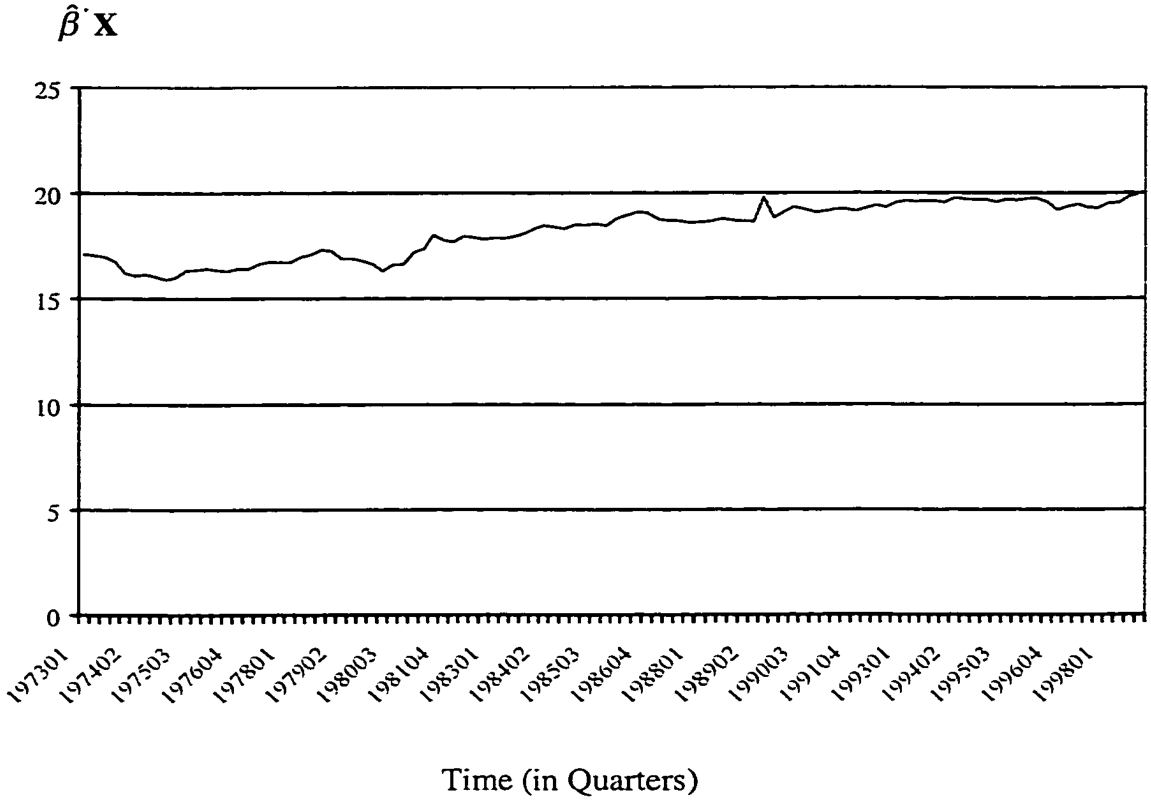

Figure 4.3

Cointegrating Relationship for Canadian Data (2)

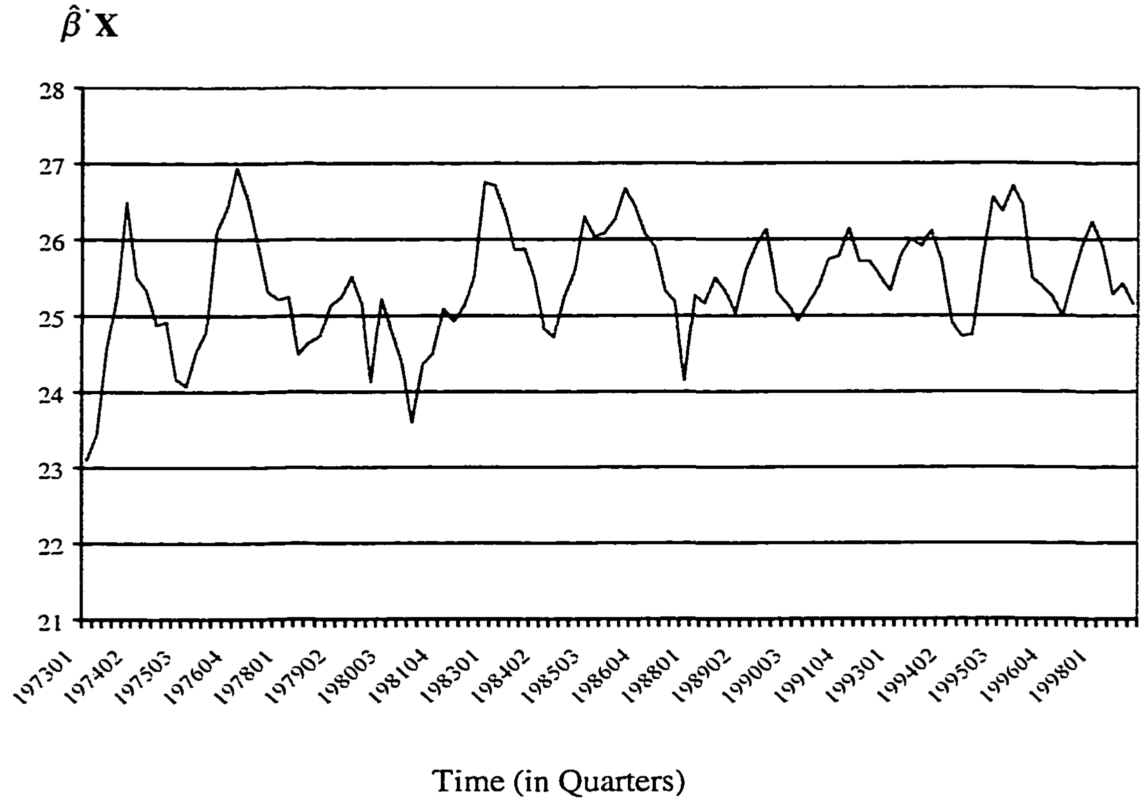

Figure 4.4

Cointegrating Relationship for Italian Data (1)

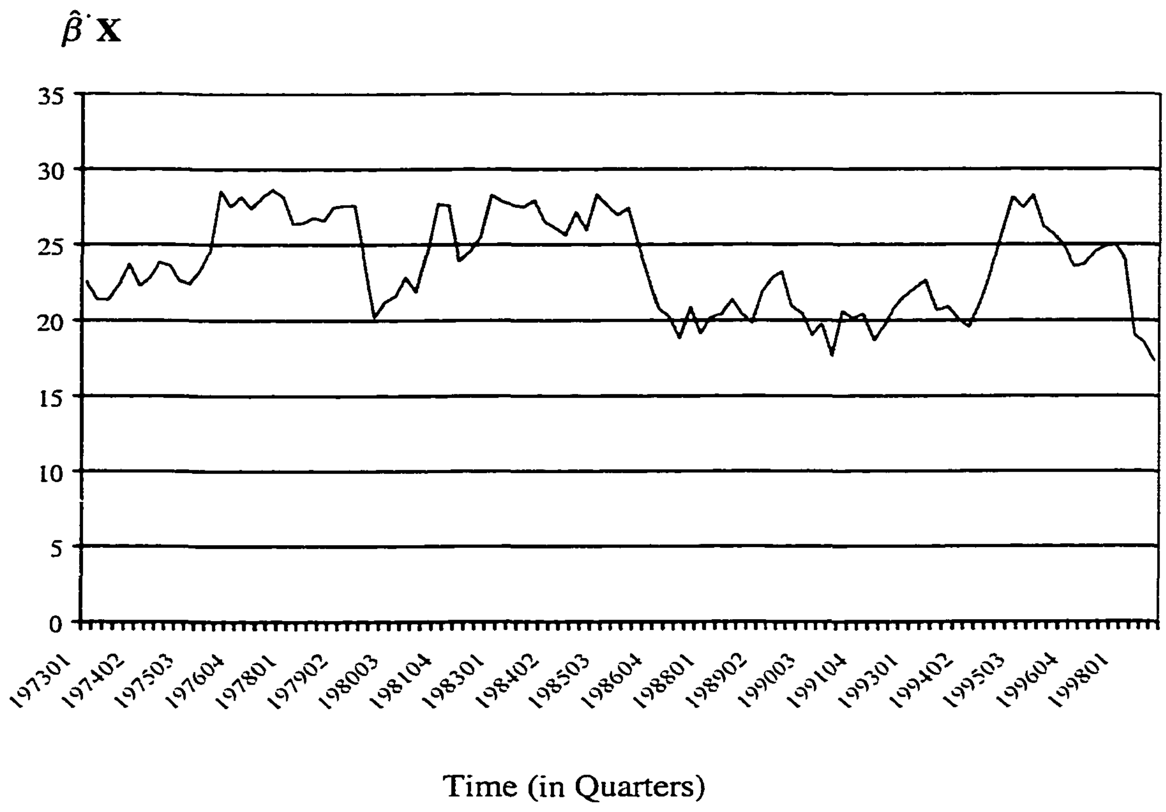

Figure 4.5

Cointegrating Relationship for Italian Data (2)

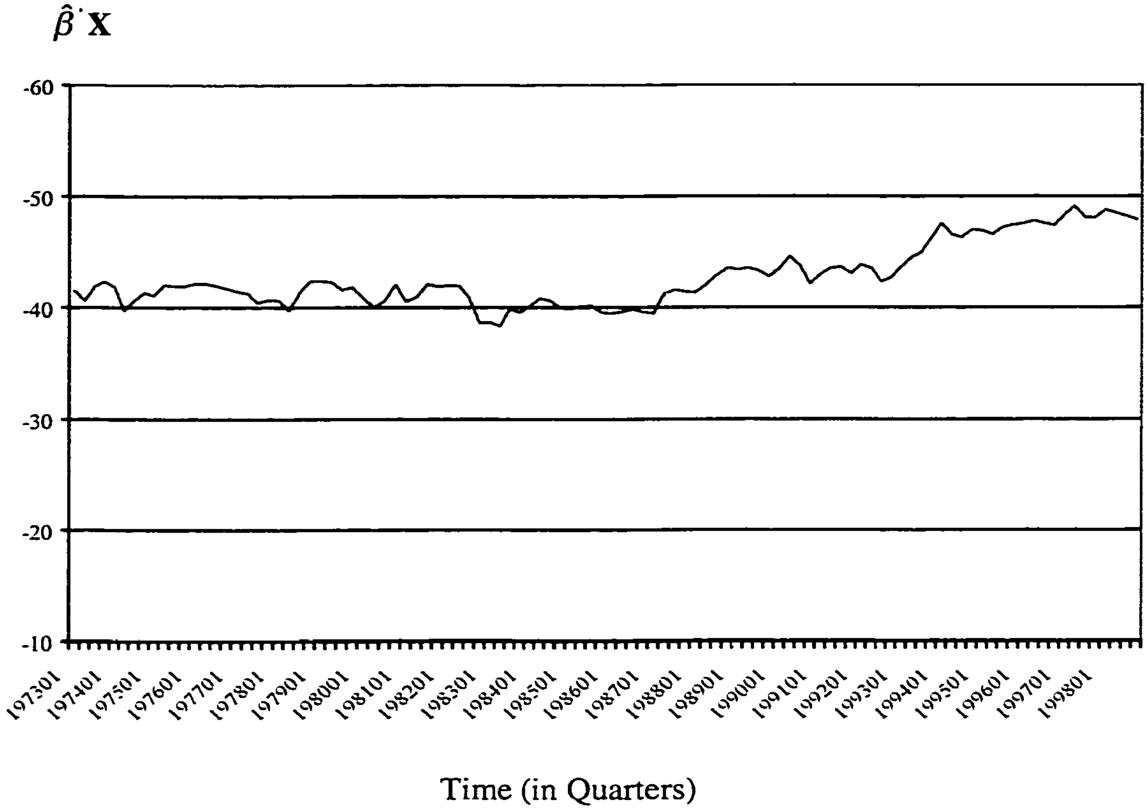

Figure 4.6

Cointegrating Relationship for Norwegian Data

CHAPTER 5

CONCLUSIONS

Purchasing Power Parity, or PPP, as it now stands, says that exchange rate is a function of only relative prices between the two countries concerned, and this functional relationship is simple and stable. An x% change in the price ratio will be accompanied by an x% change in the exchange rate. The impact of domestic and foreign price on the exchange rate is the same magnitude but in opposite direction. These hypothesized functional relationships between the exchange rate and the domestic and foreign prices are usually called homogeneity (or proportionality) and symmetry conditions of PPP. They are based on the law of one price, a direct consequence of international arbitrage. They are logical and intuitively make sense. But these two conditions have almost consistently failed in empirical tests. Three reasons have been provided in the existing literature for the empirical failure of these two conditions implied in PPP: measurement errors, transaction costs, and the different adjustment speed between assets and commodity prices. All these three explanations argue that the homogeneity and symmetry conditions implied in PPP simply do not exist.

In this study, I argue that the two conditions implied in PPP could be true and one reason for the empirical failure of PPP could be the misspecification of the model. PPP is very restrictive in the sense that it defines the price ratio as the self-sufficient and independent determinant of the exchange rate between two currencies. If, in fact, some non-price variables are also involved in the determination of the exchange rate and are excluded from the model, the estimates for the coefficients in the empirical PPP model would be biased and inconsistent. Therefore, the homogeneity condition between changes in the exchange rate and the price ratio, and the symmetry conditions between the effect of home and foreign prices on the exchange rate could be rejected even though they are in fact true. To investigate this possibility I augment the conventional theory of purchasing power parity with several fundamental economic variables that potentially have a systematic impact on the level of the real exchange rate. The variables considered include (1) relative growth rate in gross domestic product (GDP); (2) relative accumulated current account balance as a percentage of GDP; (3) the real interest rate differential; and finally, (4) the relative changes in terms of trade. The Johansen multivariate cointegration procedure is adopted to test whether the parity relationship between the exchange rate and the relative prices holds in the augmented model.

My conjecture is confirmed by the empirical results from the Johansen procedure. When the restricted cointegrating vectors based on the homogeneity and symmetry conditions of PPP are tested in the conventional PPP model, the restrictions are decisively rejected by the data. But once the non-price variables are included and the constrained cointegrating vectors are tested in the augmented model, the data accept the restrictions for four of five countries, with Germany as the only exception. These results indicate that the reason for the empirical failure of the homogeneity and symmetry conditions of PPP is not because such conditions do not exist, as many previous studies suggest, but because other variables in addition to the relative price are involved in the determination of the exchange rate. When these non-price variables are missing from the model, the true relationship between the exchange rate and the price variables are distorted. Once these variables are explicitly included in the model, the homogeneity and symmetry conditions as implied in PPP are revealed.

The findings in this study are also consistent with previous studies on the relationship between exchange rate and some of the non-price variables considered. For example, I find that the domestic currency tends to appreciate (decrease in s) if there is a relative increase in domestic real interest rate, a relative faster growth in domestic GDP, or a relative improvement in the terms of trade. These findings are consistent with theoretical priors. I find no evidence, however, that a relative increase in the current account balance will cause an appreciation in the domestic currency.

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