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What is Mathematics and Why Won't It Go Away

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ABSTRACT. We report on a seminar for first-year college students that weaves mathematical proof and problem-solving together with discussions of cultural, philosophical, and aesthetic issues surrounding mathematics.

1. Introduction

While mathematics is a fundamental school subject, readers of this journal know that it does not routinely ignite the intellectual passions of most students. The resulting loss for the discipline is obvious and undeniable. Moreover, there is a greater problem, namely, that the general educated public, even the scientifically literate public, views mathematics as an inscrutable, inhuman tool useful only for quantitative tasks. That is a loss for everyone.

Views about mathematics outside the discipline are so entrenched that it is very difficult for collegiate mathematics faculty to begin to address this problem. The technical mathematics required for client disciplines necessitates jam-packed syllabi and does not allow much flexibility to explore meta-issues or questions about mathematics as a human undertaking. Students do not expect a focus on the humanistic side of mathematics either. However, they very much value the opportunity to engage ideas and interrogate their own thinking and learning. And they do find outlets for their intellectual needs outside of their mathematics classes. As mathematics faculty, we need to recapture some of this enthusiasm and academic energy. One opportunity for a different sort of mathematical experience rests with the first-year seminar, a type of discussion-oriented course that has appeared on more and more campuses. While it is certainly too ambitious to expect a single class to realign ingrained views of mathematics, a course that combines some challenging mathematical ideas together with discussion of the place of the discipline within the intellectual landscape can begin to help students explore mathematics and related issues in a fresh manner.

In this article, I report on a first-year seminar I have taught three times. My goal is for students to learn some mathematics, and—just as important—to think about the nature of mathematics, its significance as a vibrant intellectual discipline, and the culture of its practitioners, and also to confront a variety of "affect issues." I want to expose students to some of the ideas and issues that I found compelling when I was a student and

that helped to draw me into mathematics. This is an auspicious time to teach such a course, given both the golden age of mathematical research that is and has been underway, and the degree to which mathematics has popped up in popular culture recently. I believe that providing a more complete picture of mathematics and mathematicians is the intellectually honest thing to do. Whether students pursue the study of mathematics or not, I want them to choose to do so with an accurate appreciation of the subject.

2. Oberlin's First-Year Seminar Program

The First-Year Seminar Program began at Oberlin College in the fall of 2001 with several objectives. Chief among these is to provide new college students with an introduction to learning in the context of a liberal arts and sciences curriculum and to acquaint students with some of the values that sustain a community of learning. A more specific goal of each seminar is to develop and hone students' skills in critical and creative thinking, discussion, writing, and, as appropriate, quantitative work. Thus, each seminar enables students to satisfy part of Oberlin's writing proficiency requirement for the B.A. degree; the more mathematically intensive seminars (including mine) enable students to satisfy part of the quantitative proficiency requirement as well. Each seminar is also intended to provide new students with the experience of working actively in a small class; consequently enrollment is limited to 14. How class time is used can include discussion, fieldwork, archival work, as well as occasional lectures; although too much lecturing defeats much of the purpose of the seminar. The topics for the seminars vary with the instructor and range broadly through the arts and sciences curriculum.

The seminars are *not* designed to serve as introductory courses in any particular discipline or even to particular interdisciplinary work. They are taught by regular faculty under the aegis of the Oberlin's First-Year Seminar Program, which has a faculty director and provides oversight through a faculty committee. The seminars are described in the Oberlin Course Catalog in their own section. With an entering class in the College of Arts and Sciences of approximately 600–620 students, some 40–45 seminars are offered each year, the vast majority during the fall semester. Some titles of recent seminars include:

- Satire and the Uses of Laughter (taught by English faculty)
- Freud's Vienna: Artists, Intellectuals, and Anti-Semites at the Fin de Siècle (taught by history faculty)
- Peace, Conflict, and Violence (taught by psychology faculty)
- What's in a Name? Understanding the World through the Names of Its Places (taught by anthropology faculty)
- The Brain Is Wider than the Sky: Neurobiology of the Mind (taught by biology/neuroscience faculty)

First-year seminars are *not* required of entering students, although the faculty strongly urges new students to enroll in one. Approximately 90% of new students elect to take a seminar, so the courses are very popular.

3. My Course

Here is the 2007–08 course catalog listing of my seminar:

FYSP 177 – What is Mathematics and Why Won't It Go Away?

Semester Offered:	First Semester
Credits:	4 Hours
Attributes:	**4 NS, QPh, WR**

This seminar will provide opportunities to engage in various activities (problem-solving, conjecture, and proof) and to explore the nature of mathematical thinking and discourse. Works of both non-fiction and fiction will be discussed and issues such as problem-solving vs. theory-building, the nature of mathematical truth and proof, aesthetic qualities in mathematics, mathematics and madness, cognition and mathematics will be considered. Intended for students without extensive background beyond high school mathematics.
Enrollment Limit: 14.

The "attributes" indicate that the course offered four credit hours in the Division of Natural Sciences and Mathematics, that it satisfied half of the Arts and Sciences quantitative proficiency requirement, and that it served as one of two required courses that offer writing certification.

The seminar's title was intended to catch the student's eye, of course. It was also meant to be taken in multiple ways: mathematics as an important subject that will never go away, mathematics as a fundamental activity, and the sometimes painful—yet apparently inescapable—compulsion of mathematical thinking and problem-solving. Unlike some liberal arts mathematics courses, the seminar was not designed to illustrate how mathematics aids other scientific or technical fields, but rather how it behaves and coheres internally. I was also trying to use mathematics as a vehicle for thinking about more general issues relevant to new college students, e.g., how to commit to an intellectual life, how much obsession for that life to allow oneself, and when to avoid too much compulsive behavior.

To date, the audience has consisted primarily of students with some background in calculus and an interest in the sciences. Given the wide range of seminars available to students, I had expected that this would be the case. Nonetheless, some humanities students, curious to look at mathematics in a new way, have elected the course. Occasionally, a few very math-averse people have taken the class; they have enjoyed varying levels of success, including one who had expected that I was going to "make the mathematics go away."

In order to have time to view videos and films, as well as to encourage free-flowing and relaxed discussion, the seminar met twice a week in two-hour blocks, although not every class meeting ran the full period. I varied class time among mathematical work (including the occasional lecture), discussion of reading, and viewing and discussion of films. Major assignments for the seminar consisted of six mathematical problem sets and three five-page essays; each essay was submitted twice. The student's final grade was based on the problem sets (45%), the essays (45%) and class participation and other small assignments (10%). There were no examinations—given the wide range of mathematical expertise that students brought to the course, written examinations would have made a poor tool for evaluation and could have disturbed the atmosphere of the class. Fortunately, Oberlin College enjoys a robust honor code that applies to all academic work, so that traditional testing was not essential.

4. Outline of Syllabus

I divided the thirteen-week semester into four (loosely constructed) units, with a theme for each unit. The content for each unit was as follows:

Doing Mathematics (5 weeks)

- Mathematical topics: general problem-solving, elementary number theory
- Reading:
  A. Doxiadis, *Uncle Petros and Goldbach's Conjecture* [6]
  D. Auburn, *Proof* [1]
  P. Davis and R. Hersh, *The Mathematical Experience* [5]
- Viewing:
  Proof [18]
  The Proof (NOVA video) [14]

About Mathematicians (3 weeks)

- Mathematical topics: induction, working with infinity
- Reading:
  C. Henrion, *Women in Mathematics: The Addition of Difference* [12]
  S. Nasar, *A Beautiful Mind* [17]
- Viewing:
  N is a Number [4]
  A Beautiful Mind [10]
  A Brilliant Madness (PBS video) [15]

Mathematical Theories: Two Examples (2.5 weeks)

- Mathematical topics: Topology/geometry, elementary group theory

- Reading:
    J. Weeks, *The Shape of Space* [22] (especially Parts I and II)
- Viewing:
    The Shape of Space videos [9]
    Flatland [3]
    Not Knot [8]

Mathematics in Culture and Society (3 weeks)

- Reading:
    T. Stoppard, *Arcadia* [20]
    H. M. Enzensburger, *The Number Devil* [7]
    C. P. Snow, *The Two Cultures* [19]
    G. H. Hardy, *A Mathematician's Apology* [11]
- Viewing:
    Mathematics in Arcadia [16]

I expected students to read all ten books listed above, in their entirety. Fortunately, all but *The Shape of Space* [22] were available in paperback editions; low-cost used copies were often available as well. In addition to the videos and films listed above, I also arranged a few optional, out-of-class screenings of works such as *π: Faith in Chaos* [21], *Good Will Hunting* [2], and *Fermat's Last Tango* [13].

During the first phase of the seminar ("Doing Mathematics"), I began with some mathematical work, in part to give students the opportunity to assess the level of technical facility I was expecting. Thus, during the very first class I posed some puzzles, and discussed some elementary results involving binomial coefficients so that students could observe mathematical arguments that were different from routine calculation and formula manipulation. For example, after introducing the binomial coefficient $\binom{n}{k}$ as the number of different k-element subsets of an n-element set X, we would establish the well-known identity

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n$$

by counting the number of possible subsets of X in two ways: first by counting the number of subsets of a particular size k to obtain the left side of the identity, and, second, by considering each element of X individually and recognizing that it may be included in the subset or not (thus obtaining the right side). Similarly, we obtained Pascal's formula

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

by recognizing that every k-element subset of X must be one of two types: those that include a particular element x_0 in X (there are $\binom{n-1}{k-1}$ of these) and those that do not

$\left(\dbinom{n-1}{k}\right.$ of these). These types of combinatorial arguments, particularly their descriptive, nonalgebraic nature, were quite new, and occasionally jarring, to some students. Students also worked on exercises in class involving basic propositional logic and truth tables to prepare them for the arguments and proofs of elementary number-theoretic results (e.g., results about primes, irrationality of $\sqrt{2}$). This mathematical work set them up well for reading *Uncle Petros* [6] and *Proof* [1], as they found the proofs simultaneously tantalizing and frustrating and could more readily respond to some of my discussion prompts, such as:

- What does it mean to be "the best" in mathematics? Is this notion unique to mathematics? Or is this just about ambition in general? Is it like musical performance or professional sports?

- Do we carry the belief in a genetic predisposition for mathematics too far?

- Collaboration *vs.* solo efforts: Which is best? Does it matter?

Davis and Hersh's *The Mathematical Experience* [5] was one of the most challenging readings of the semester and led to some very thoughtful and eye-opening discussions regarding different fields of mathematics and schools of philosophy of mathematics such as Platonism and formalism. Indeed, this was the first time that most students had considered the mathematical landscape as a whole, or anything regarding the nature of mathematical reality.

During the second part of the seminar, the focus gradually shifted to the lives of actual mathematicians. Of course, we discussed extreme personalities, such as John Nash and Paul Erdős (and, previously, the fictional characters Petros Papachristos of *Uncle Petros* [6] and the mathematician Robert of *Proof* [1]), but we also considered the "normal" mathematician Andrew Wiles and his extraordinary achievement. In this regard, Claudia Henrion's *Women in Mathematics* [12] was very helpful both in giving a sense of the sociology of mathematics, and in examining the various myths concerning the mathematical and personal life courses of mathematicians. After a thorough indoctrination of G. H. Hardy's view of mathematics as "a young man's game," which is echoed in nearly every reading that we had previously discussed, Henrion's many counterexamples provided some good and needed balance. Henrion's book also offered a natural entré for talking about personal identity (e.g., gender, ethnicity) and the pursuit of mathematics. Collectively, the various stories about mathematicians prompted discussion questions such as:

- Are mathematicians different from other scientists? Other scholars? In what ways?

- Does mathematical achievement require a measure of insanity?

- Is there really a "community of mathematicians"?

- How do portrayals of mathematicians (and scientists) in popular media affect the public's expectations about mathematics? (Consider who is constructing the portrayal.) What are the possible impacts on the learning of mathematics?

After focusing on issues related to identity and mathematical careers, it seemed appropriate to return to mathematics proper. Thus, in the third part of the seminar, I presented introductions to two parts of pure mathematics I especially like: topology and group theory. To discuss geometry and topology, students read the only mathematical textbook I assigned for the semester: *The Shape of Space* [22]. This book is written for nonmathematicians and assumes nothing beyond high school mathematics, but does require active attention from the reader. Although I asked students to read the whole work, during class we focused on Parts I and II (roughly the first half of the book) as these parts involved concepts and arguments that were easier to visualize. We played Tic-Tac-Toe, chess, and solved jigsaw puzzles on different 2-manifolds, courtesy of Jeff Weeks's website of downloadable topological games [23]. I also prepared a PowerPoint file to illustrate various topological and geometric constructions. The group theory, as well as all other mathematical content (besides geometry and topology), was handled through class time instruction and handouts I had written.

For the final part of the seminar, we returned to mathematics in a humanistic and societal context. Together we read Stoppard's play *Arcadia* [20] and discussed the exhalting of scientific and rational thinking in an environment of emotional entanglements. Mathematically, *Arcadia* also provided a brief opportunity to consider the behavior of chaotic dynamical systems and its philosophical implications. Enzensberger's children's novel *The Number Devil* [7], charming in its own right, gave rise to interesting discussions about mathematics education, which we continued in more global and political terms when we focused on *The Two Cultures* [19]. We finished the semester with Hardy's *A Mathematician's Apology* [11]. Hardy had already appeared to students in *Uncle Petros* [6], was quoted in *The Mathematical Experience* [5] and *Women in Mathematics* [12], and mentioned significantly by C. P. Snow [19]. Thus it seemed fitting to have students read his influential work in its entirety to conclude the seminar.

5. Assignments

The six problem sets involved, in order:

- General problem-solving
- Elementary number theory
- Mathematical induction
- Working with infinity
- Problems from *The Shape of Space* [22]
- Elementary group theory

For each assignment, I asked students to submit solutions to six problems; there was some choice available in all but the number theory assignment. (After the second assignment, students could submit additional problems for extra credit.) Each set involved problems that varied in difficulty. For example, in the induction assignment, I had questions such as:

Show that $\dfrac{n^5}{5} + \dfrac{n^4}{2} + \dfrac{n^3}{3} - \dfrac{n}{30}$ is an integer for $n = 0, 1, 2, \ldots$.

But I also included more challenging questions, such as

> The plane is divided into regions by drawing n straight lines. Show that it is possible to color each of these regions either red or yellow in such a way that no two adjacent regions have the same color. (Hint: Draw some pictures first: Take a piece of paper and follow the instructions with just one line, then draw a second line and revise your picture, then a third line, etc.)

and

> The **Fibonacci numbers** are the sequence 1, 1, 2, 3, 5, 8, 13, 21,…. This sequence is generated (after the first two initial 1's) by adding two successive numbers to get the next one. (Thus $F_3 = F_1 + F_2 = 1 + 1 = 2$ and $F_4 = F_2 + F_3 = 1 + 2 = 3$.) To be more formal, we take $F_1 = F_2 = 1$ and, for $n \geq 2$, define $F_{n+1} = F_{n-1} + F_n$. Show that every positive integer greater than 2 can be written as a sum of *distinct* Fibonacci numbers. (Just to be clear: the first two Fibonacci numbers F_1 and F_2 are *not* distinct.)

Each problem was scored out of a total of six points, 0–5 points for the mathematical content and 0 or 1 point for the written quality of the solution.

Insofar as possible, I attempted to have the mathematics coordinate and interact with the readings. Thus, for example, as the mathematics in *Uncle Petros* [6] and *Proof* [1] centers on number theory, students worked on number-theoretic questions, such as showing $\sqrt{p}$ is irrational for any prime p and providing a (directed) proof of the Fundamental Theorem of Arithmetic. *The Mathematical Experience* exposed them to some set theory and mathematical and logical issues involving infinite sets [5, pp. 152–157]. Thus it was natural for me to talk about ways to work with infinity, including some simple series computations, arguments involving countable and uncountable sets, and to assign problems such as:

> Without attempting to supply all the details, argue convincingly that the union of a countably infinite family of countable sets is countable. Do *not* assume that the sets are pairwise disjoint. (A reasonable solution to this problem would be to outline a procedure for listing the elements of the union under consideration.)

The Mathematical Experience also mentions group theory [5, pp. 203–209], which is one of the reasons we worked through some ideas from this area, although, admittedly, not at time moment when we first read about it. I asked students to work some standard elementary problems, such as:

Prove that every Cayley table of a finite group is a **Latin square**; that is, each element of the group appears exactly once in each row and each column of the main body of the table. (Note: Latin squares are useful in the design of statistical experiments. They are also closely related to finite geometries.)

At the same time, I wanted students to get some feeling for group theory's use in understanding symmetry. Thus, they were also given problems such as:

Pictured are three figures: a square, and two other figures derived from a regular octagon. (See Figure 1.) The figures look different, of course, but can we use their symmetry groups in order to distinguish them? See if you can describe and identify the corresponding groups of symmetries (i.e., rotations and reflections). By doing so, you should either conclude that (1) group theory allows you to differentiate among some or all of the figures or (2) group theory does *not* enable you to differentiate all of the figures.
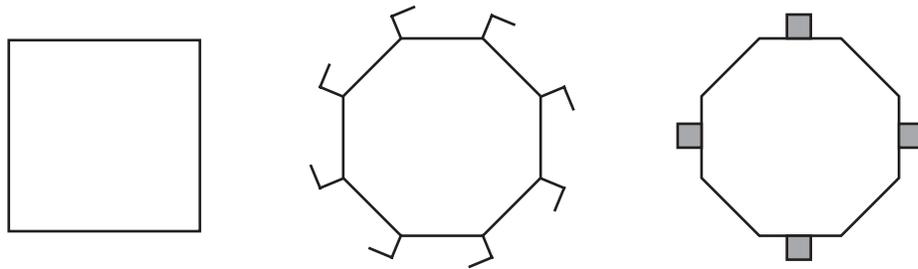

Figure 1. A square and two figures derived from a regular octagon.

The three essay topics were:

- Reflections on mathematics and doing mathematics
- Images and representations of mathematicians
- Mathematical aesthetics

For each of these topics I offered some elaboration and prompting questions, although students were free to respond to the general topic in whatever manner they wished. For example, the prompting questions for the third essay on aesthetics were:

- Is mathematics beautiful? In what ways?
- Discuss aesthetic qualities in mathematics that are wholly internal to mathematics. For instance, you might describe some especially compelling proofs or theorems. What about your examples are beautiful and why? (You might also consider contrasting mathematical arguments you find particularly pleasing with some that you do *not* find especially aesthetic.)

- Provide examples of aesthetic qualities of mathematics that are related to other modes of thought or expression. Again, how does the particular elegance/beauty arise and why?
- Describe any analogies (or lack thereof) between elegance in mathematics and elegance in other intellectual activities.
- Is the beauty one finds in mathematics relative to one's expertise in the field?

What was crucial in every essay was that students provide coherent observations and arguments supported with specific examples and evidence from a variety of sources, along with proper citation of sources.[1] As mentioned above, each essay was submitted to me twice: once as a draft on which I commented liberally, and a second time for final evaluation.

6. Challenges and Rewards

Although it has been extremely gratifying to teach this course, a seminar like mine nonetheless presents some pedagogical challenges. Perhaps chief among these are the disparate mathematical levels which students present. I attempted to mitigate this by being clear from the beginning about the mathematical expectations, by not having any tests, and by allowing students to collaborate on their homework. This seems to have worked in my local context. However, such an approach might not be feasible in a different institutional culture, and it is not invariably successful in mine. Indeed, anonymous end-of-semester comments from students suggest that the mathematical work was received with varying degrees of appreciation:

I loved learning (& proving) tidbits of information from so many different areas of mathematics. (Fall 2006)

The handouts and books read in class were great, but the problem sets were way too difficult. (Fall 2006)

I think I have learned more about writing real proofs and gotten a good basic introduction into fields I never knew existed (group theory, number theory, etc.). (Fall 2007)

Somewhat less problematical, but still requiring special attention, were the varied writing abilities of students. This was especially an issue with some international students whose command of English and, sometimes, experience writing essays were relatively weak. I found that I could deal with this situation by offering additional critiques and individual assistance. In addition, there were and are very generous campus resources to assist students with their writing. Again, students reacted in different ways to the writing assignments and my critiques:

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[1] I note that proper citation and bibliographic work, as well as some appreciation of information literacy, is also one of the goals of the First-Year Seminar Program. I did not place heavy emphasis on this in my course, however.

The writing in this course was not very rigorous. The papers should be more heavily graded. (Fall 2007)

My writing skills have changed for the better. I'm much more efficient than I used to be. (Fall 2007)

I've progressed in my essay writing from the three papers. (Fall 2007)

While my writing skills haven't changed all that much, I did learn something important about writing papers in general—choosing a compelling topic and picking a thesis that genuinely interests me makes my writing more pleasurable to both read & write. (Fall 2007)

Additional challenges arose from the need to teach in ways in which I was never trained as a mathematician. Leading an effective classroom discussion is as much art as skill. It depends on the instructor's having identified not only key points to be considered, but also in finding the means to enable the students to make relevant, sometimes unanticipated, observations. To be nimble enough to bring out the best in each group of students, and to be able to recover quickly when that doesn't happen in a particular class meeting is not an easy skill to learn. I certainly do not claim to have mastered it completely. Here are some student comments:

Group discussions helped a great deal. (Fall 2004)

I got absolutely nothing out of discussions…. (Fall 2004)

I felt that some of the discussions we got into were not prolonged enough to get really deep into any particular issue…. (Fall 2006)

Some of the uneven reactions to class discussion no doubt arose from the variety of personal experiences of students have had with mathematics and their own education. This was particularly in evidence when we talked about Henrion's *Women in Mathematics* [12], when we heard minority students tell of their struggles for recognition, or when we read Snow's *The Two Cultures* [19] and international students gave first-hand evaluations of their education systems and offered comparison with what they had experienced thus far in the United States. Such unexpected classroom moments certainly enriched the seminar, and it was important that I let students articulate their thoughts regardless of what my plan may have been. In addition, at times it felt unnatural to respond to and to evaluate student writing. As a result, it took me quite a lot of time to read and react to student work. Unfortunately, I have no magic advice to offer regarding student writing, other than to suggest that one find some minimally efficient system that is reliable.

On the other hand, teaching my seminar has been a thoroughly invigorating experience. For one thing, the small class size and emphasis on discussion have meant that I was able to get to know all of my students very well, and not only in terms of their mathematical skills. While running open-ended discussions and focusing on student writing were daunting tasks, teaching in these and other new ways was refreshing and, I found, had positive carry-over to my "regular" courses and general interactions with

students. Simply being liberated from a standard technical syllabus has enabled me to take the time to talk seriously with students about many of the aspects of mathematics that I love. Many of the students appear to have benefited from the experience:

> I can see that there is a lot of room for myself & my own interests to be pursued in math. I had given up in high school. I'm less hostile towards it. (Fall 2004)

> I enjoyed every aspect of this class. I hope it will be offered next year because it truly opened my eyes to the beautiful world of mathematics. (Fall 2004)

> I was compelled by the differences between the mathematical, artistic, & social science worlds. I also enjoyed learning about the dynamic of the math world itself. (Fall 2007)

Overall, I enthusiastically urge others to develop similar classes.

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Biographical Sketch

Susan Colley received her S.B. and Ph.D. degrees in mathematics from MIT. Since 1983 she has been a member of the faculty of Oberlin College, where she is the Andrew and Pauline Delaney Professor of Mathematics. Her research focuses on enumerative problems in algebraic geometry and she teaches a wide range of courses in undergraduate mathematics. She is the author of the *Vector Calculus* (3rd edition, Pearson Prentice Hall, 2006). Hobbies include watching old movies, serving her various felines, and attending college committee meetings.