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# Independent Monophonic Sets in Graphs

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## Abstract

In this paper, we obtain two variables for the connected $(p, q)$-graphs $G$ which is the minimum size of an independent monophonic set and an outer independent monophonic set of $G$, termed as an independent monophonic number $m_\alpha(G)$ and an outer independent monophonic number $m_\alpha^\perp(G)$ of the connected $(p, q)$-graphs $G$, respectively.

**Keywords:** Chordless Path, Monophonic Number, Independence Number, Geodetic Number, Independent Monophonic Number, Outer Independent Monophonic Number

Mathematics Subject Classification (2010): 05C05, 05C12, 05C69

## 1. Introduction

All graphs considered in this paper are finite, connected, simple and undirected. For the basic graph theoretical terms, refer to [**? ? ?** ]. A graph $G$ is an ordered couplet of $V$ and $E$, denoted by $G = (V, E)$ where $V$ is called vertex set and $E$ is an edge set of $G$ respectively. A graph $G$ is said to be a $(p, q)$-graph if $n(V) = p$ and $n(E) = q$. The counts $p$ and $q$ are termed as order and size of a graph $G$ respectively. A path $P$: $[x_1, x_2, \ldots, x_n]$ in a connected $(p, q)$-graph $G$ is a sequence of adjacent vertices $x_1, x_2, \ldots, x_n$ in $G$. The distance $d(u, v)$ between two vertices $u$ and $v$ is the length of a shortest $u - v$ path in a connected graph $G$. A $u - v$ path $P$ is called geodesic if it is a shortest $u - v$ path and $E(P) = d(u, v)$.

A subset $S \subseteq V$ in a connected graph $G$ is called a geodetic set if every vertex in $V - S$ lies on a shortest path between two vertices from

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$S$.The minimum cardinality among all the geodetic set is termed as the geodetic number $g(G)$ of $G$. The geodetic number of a graph was introduced in [**? ? ?** ].

An edge $e = \{x_s, x_t\} \in E(P)$ in a connected graph $G$ is said to be a chord if $t \geq s+2$. A path $P$ is a monophonic path if it is a chordless path (or written as m-path). The monophonic path is special subgraph of a connected graph $G$. All the variables related to the distance in a connected $(p, q)$-graphs $G$ can be defined by the monophonic path, see [**? ?** ].
The concept of monophonic path first seemed at [**?** ] and was initiated by Ignacio M. Pelayo et al. [**?** ]. An interval $J : V \times V \longrightarrow \mathcal{P}(V)$ is a mapping from the vertex set $V$ to the power set $\mathcal{P}(V)$ such that

$$J(x, y) = \{ \text{ all the vertices lying on some x-y m-path } \}.$$

$J(x, y)$ is known as a monophonic interval of $x$ and $y$. The closed monophonic interval of $x$ and $y$ is $J[x, y] = J(x, y) \cup \{x, y\}$. For any set $M \subseteq V$, monophonic closure of $M$ is denoted by $M^c$ and it is defined as

$$M^c = \bigcup_{x,y \in M} J[x, y].$$

If $M^c = V$, then we say that $M$ is a monophonic set of $G$. In other words, a set of vertices $M \subseteq V$ is said to be a monophonic set of $G$ if each vertex $v \in V$ lies in an $x - y$-monophonic path in $G$ for some $x, y \in M$. The minimum cardinalities of all monophonic sets in $G$ is the monophonic number of $G$. It is notated as $m(G)$ and written as

$$m(G) = \min\{n(M) : \text{M, monophonic set of G}\}.$$

A vertex $v \in V$ is said to be a monophonic vertex of $G$ if $v$ belongs to every minimum monophonic sets of $G$. If $G$ has a unique monophonic set $M$, then all the vertices of $M$ are monophonic.

The monophonic distance in a connected graph was studied by A. P. Santhakumaran et.al [**?** ]. The concept of connected monophonic number and upper monophonic number of a graph was introduced by J.John et.al[**? ? ?** ]. But the monophonic number of a graph have not yet been fully explored and discussed in details. For the latest literature work on the topic monophonic number and the combined variable monophonic domination number of a graph, refer to [**? ? ? ? ? ?** ].

A vertex $v \in V$ in $G$ is said to be an extreme vertex if induced subgraph $< N(v) >$ is complete. The set of all extreme vertices of $G$ is denoted by $Ext(G)$ and the set of all end vertices of $G$ by $End(G)$.

An independent vertex set $S \subseteq V$ is a set of pairwise non-adjacent vertices in $G$.The maximum cardinality of an independent set of vertices is called an independence number of $G$. It is denoted by $\beta = \beta(G)$ [Bollobas 1981],refer [**?** **?** ] and it is inscribed as

$$\beta(G) = \max\{n(S) : S,\ \text{independent set of G}\}$$

Independent sets were introduced into the communication theory on noisy channels [**?** ]. The independence number of a connected graph is difficult to compute. Finding a maximum independent set is an NP-hard problem. Many upper and lower bounds for the independence number of a graph appear in the available Mathematical literature. The common upper bounds of $\beta(G)$ are $\beta(G) \leq p - \frac{q}{\Delta}$ (known as KWOK bound) and $\beta(G) \leq p - \delta$ (known as Minimum degree bound) where $\delta$ and $\Delta$ are the minimum and maximum degrees of $(p, q)$-graphs $G$.

For any connected graph $G$, some subsets of vertex set $V$ are monophonic but not an independent set in $G$ and conversely. This clear-cut motivation help us to study the subsets of vertex set $V$ which is both monophonic and independent.

In the rest of the paper, we propose two graph variables, namely an independent monophonic number and outer independent monophonic number of connected $(p, q)$-graphs $G$ respectively.

## 2. Independent Monophonic Number of a Graph

**Definition 2.1.** An independent monophonic set $M \subseteq V$ in a connected $(p, q)$-graph $G$ is a set of vertices which is both monophonic and independent. The minimum size of an independent monophonic set is called an independent monophonic number of $G$. It is denoted by $m_\alpha(G)$ and it is inscribed as

$$m_\alpha(G) = \min\{n(M) : M,\ \text{independent monophonic set of G}\}$$

An independent monophonic set of size $m_\alpha(G)$ is known as $m_\alpha$-set.

An independent monophonic set is always a monophonic set but the converse need not be true. An independence number $\beta(G)$ is the maximum size of an independent set in $G$, so that the graph variables $m_\alpha(G)$ and $\beta(G)$ are different in the general sense. It follows that $m_\alpha(G) \leq \beta(G) \leq p$.

**Definition 2.2.** A monophonic vertex set $M \subseteq V$ is said to be an outer independent monophonic set if the vertices of $V - M$ are non adjacent. The minimum size of an outer independent monophonic set is called an outer independent monophonic number of $G$. It is denoted by $m_\alpha^\perp(G)$ and it is inscribed as

$$m_\alpha^\perp(G) = \min\{n(M) : M, \text{ outer independent monophonic set of G}\}$$

An outer independent monophonic set of size $m_\alpha^\perp(G)$ is known as $m_\alpha^\perp$-set.

**Example 2.3.** Consider a graph $G$ given in the Figure 1. The sets $M_1 = \{v_1, v_3\}$ and $M_2 = \{v_3, v_5\}$ are the minimum monophonic sets of $G$, also they are independent. Therefore $m(G) = 2$ and $m_\alpha(G) = 2$.

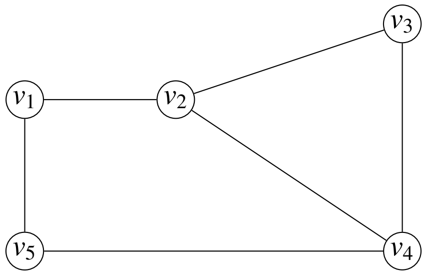


**Figure 1:** A graph $G$ with $m(G) = m_\alpha(G) = 2$ and $m_\alpha^\perp(G) = 0$

It follows that the variables $m(G)$ and $m_\alpha(G)$ are coincide and $v_3$ is the only monophonic vertex of $G$. $V - M_1 = \{v_2, v_4, v_5\}$ or $\{v_1, v_2, v_4\}$. Since the vertices of $V - M_1$ are adjacent, $M_1$ is not an outer independent monophonic set of $G$.
Hence graph $G$ has an independent monophonic set, but does not have an outer independent monophonic set, that is $m_\alpha^\perp(G) = 0$

**Example 2.4.** Consider the graph $G$ given in the Figure 2.

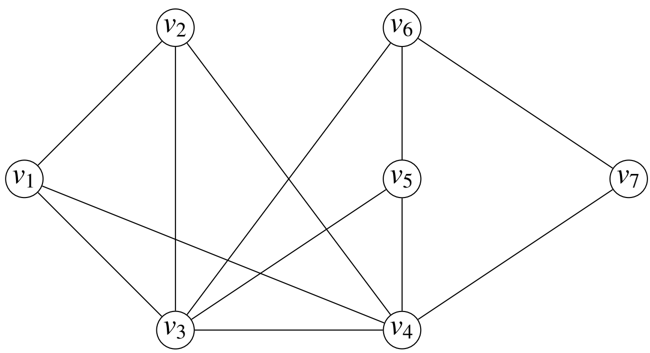


**Figure 2:** A graph $G$ with $m(G) = 5$, $m_\alpha(G) = 0$ and $m_\alpha^\perp(G) = 2$

The set $M = \{v_1, v_2, v_3, v_5, v_7\}$ is the minimum monophonic set with some adjacent vertices. Clearly see that $M$ is not an independent monophonic set of $G$ and $m_\alpha(G) = 0$. But $V - M = \{v_4, v_6\}$ is independent. It follows that $m_\alpha^\perp(G) = 7 - 5 = 2$.

**Note 2.5.** In a cycle graph $C_6$, the two monophonic variables are coincide but an independence number of $C_6$ is different, that is $m(C_6) = m_\alpha(C_6) = 2$ and $m_\alpha^\perp(C_6) = 0$ but the independence number of $C_6$ is $\beta(C_6) = 3$.

For the connected $(p, q)$-graphs $G$, the trivial lower and upper bounds of $m(G)$ is $2 \leq m(G) \leq p$. Since complete graphs $G$ has no non-adjacent vertices, the trivial lower and upper bounds of an independent monophonic number $m_\alpha(G)$ becomes $0 \leq m_\alpha(G) \leq \beta(G) \leq p$.

Next example shows that all the variables of some graphs $G$ are lying in the strict boarder, that is $0 < m_\alpha(G) < \beta(G) < p$

**Example 2.6.** Consider a graph $G$ given in the Figure 3. Here $p = 7$ and $M_1 = \{v_3, v_6, v_7\}$ is the minimum monophonic set of $G$ so that $m(G) = 3$. But $M_2 = \{v_1, v_3, v_6, v_7\}$ is the maximum independent set in $G$ so that $\beta(G) = 4$. It follows that $M_1$ is both monophonic and independent set with minimum cardinality, that is $m_\alpha(G) = 3$. But the set $V - M = \{v_1, v_2, v_4, v_5\}$ is not an independent set of $G$.
Hence we obtain the strict inequality $0 < m_\alpha(G) < \beta(G) < p$ in the given graph $G$.

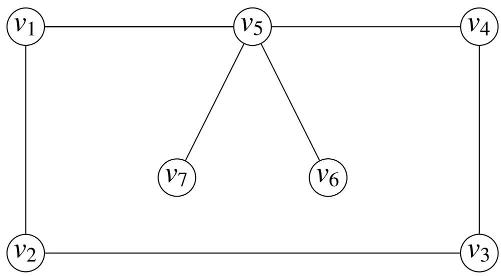


**Figure 3:** A graph $G$ with strict inequality $0 < m(G) < \beta(G) < p$

For the common terms and symbols, refer to [**? ? ? ?** ]. Based on the Definitions 2.1, 2.2, we present some preliminary observations which are to be used wherever required.

1. All extreme vertices of a connected $(p, q)$- graph $G$ are included in all independent monophonic sets of $G$.

2. If the set $Ext(G)$ is an independent monophonic set of connected graphs $G$, then it is unique .

3. An extreme vertex in a connected graph $G$ is also a monophonic vertex of the connected graph $G$.

4. All the vertices of the complete graph $K_p$ are known as extreme vertices and also known as monophonic vertices of $K_p$.

5. All end vertices of a connected graph $G$ are included in all independent monophonic sets of $G$.

6. If both the sets $m_\alpha$- sets and $m_\alpha^\perp$ -sets are non-empty sets in $V$, then
$$m_\alpha^\perp(G) = |V| - m_\alpha(G) = p - m_\alpha(G).$$

7. The graph variables $m_\alpha(G)$ and $m_\alpha^\perp(G)$ are may or may not be exists in a connected graph $G$.
In the case of a complete graph $K_p$, $m_\alpha(K_p) = 0$ and for the Star graph $S_6$ on 6 vertices, $m_\alpha^\perp(S_6) = 1$.

## 3. Basic Results

**Proposition 3.1.** For a complete graph $G = K_p$, $m_\alpha(G) = 0$.

*Proof.* Let $V = \{v_1, v_2, \ldots v_p\}$ be the vertex set of $G$. Then we see that $V$ is the unique monophonic set of $G$. Since all the vertices are adjacent, graph $G$ does not have an independent monophonic set so that $m_\alpha(G) = 0$. □

**Proposition 3.2.** For an edge deleted graph $G = K_p - \{e\}$, $m_\alpha(G) = 2$ where $K_p$ is a complete graph on $p$ vertices and $e \in E(K_p)$, $p \geq 3$.

*Proof.* Consider an edge $e = \{u, v\} \in E(K_p)$. Choose a subset $M = \{u, v\}$ of $V(G)$. For each vertex $w \in V(G) - M$, there exists a $u - v$ monophonic path of length 2 containing $w$. Since $d(u) = d(v) = p - 2$, $M$ is both monophonic and an independent set of $G$. It gives that $m_\alpha(G) = 2$. □

**Proposition 3.3.** For an edge deleted graph $G = K_p - \{e_1, e_2\}$,

$$m_\alpha(G) = \begin{cases} 2 & \text{edges } e_1 \text{ and } e_2 \text{ are non-adjacent} \\ 3 & \text{edges } e_1 \text{ and } e_2 \text{ are adjacent} \end{cases}$$

where $K_p$ is a complete graph and edge set $\{e_1, e_2\} \subseteq E(K_p)$, $p \geq 4$.

*Proof.* Consider the edge set $\{e_1, e_2\} \subseteq E(K_p)$ where $e_1 = \{u_1, v_1\}$ and $e_2 = \{u_2, v_2\}$ for the vertices $u_1, u_2, v_1, v_2 \in V(K_p)$.

Case(i) ($e_1$ and $e_2$ are non-adjacent edges ): Let $M = \{u_1, v_1\}$ be a subset of $V$. Clearly, $d(u_1) = d(v_1) = p - 2$. By the Proposition 3.2, the set $M$ is a minimum independent monophonic set of $G$ and its cardinality is $m_\alpha(G) = 2$ .

Case(ii) ($e_1$ and $e_2$ are adjacent edges ): In this case $e_1$ and $e_2$ have a common vertex say $v_1 = u_2$. Let $M = \{u_1, v_1, v_2\} \subseteq V$. Then every vertex in $V - M$ lies in a $u_1 - v_1$ monophonic path of length 2. Thus $M$ is an independent monophonic set of $G$ and $2 \leq m_\alpha(G) \leq 3$.

Finally, we have to show that $m_\alpha(G) = 3$. Let $M = \{x, y\}$ be an independent monophonic set of $G$. Since $p \geq 4$, vertices $x$ and $y$ are not adjacent in $G$. Clearly $M$ is either $\{u_1, u_2\}$ or $\{u_2, v_2\}$. In all cases, there is a vertex in $\{u_1, v_1, v_2\} - M$ which does not lie in a monophonic path of some vertices from $M$. Therefore, the vertex set $M$ of $G$ is an independent monophonic set of $G$. Hence $m_\alpha(G) = 3$. □

**Proposition 3.4.** For a graph $G = K_p - \{e_1, e_2\}$ obtained from $K_p$ by removing non-adjacent edges $e_1$ and $e_2$, $m_\alpha^\perp(G) = p - 2$, where $p \geq 5$.

*Proof.* For the vertices $u, v, x, y \in V$, we choose two non-adjacent vertices $e_1 = \{u, v\}$ and $e_2 = \{x, y\}$. Clearly $M = \{u, v\}$ or $\{x, y\}$ is the monophonic set with minimum size and $V - M$ is not independent. If $M_1 = M \cup N$ where $N \subseteq V - M$ having $p - 4$ elements and $\max deg(v) \leq p - 1$, for $v \in N$. Therefore the closure $M_1^c = V$ and $V - M_1$ is an independent set of $G$. Hence $M_1$ is an outer independent monophonic set of $G$. It gives that $n(M_1) = n(M \cup N) = 2 + p - 4 = p - 2$. Hence $m_\alpha^\perp(G) = p - 2$. □

**Proposition 3.5.** Let $\xi_\alpha(G) = \{G : m_\alpha(G) \geq 1\}$ be the family of connected graphs. Then $m_\alpha(G) = 2$ if and only if $m(G) = 2$

*Proof.* Assume that $m_\alpha(G) = 2$. By the definition , it follows that $m(G) = 2$. Conversely assume that $m(G) = 2$ , there exists a minimum monophonic set $M = \{x, y\}$ such that $n(M) = 2$. Clearly we have $V - M \neq \phi$ and every $x - y$ monophonic path contains one more vertex and $d_m(x, y) \geq 2$. Hence $M$ is independent and $m_\alpha(G) = 2$. □

**Proposition 3.6.** For connected $(p, q)$-graphs $G$ with maximum degree $\Delta$,

$$m_\alpha(G) \geq \frac{\sum_{i=1}^{k}(1 + deg(v_i))}{1 + \Delta}$$

*Proof.* The vertex set $V$ may or may not be an independent set of $G$, so we may assume without loss of generality that $m_\alpha(G) = k$ where $k \leq p$. Then there exists a monophonic set $M = \{v_1, v_2, \ldots v_k\}$ such that $n(M) = k$. For $i = 1, 2, 3, \ldots k$, since $deg(v_i) \leq \Delta$, sum of all degrees,

$$deg(v_1) + deg(v_2) + \cdots + deg(v_k) \leq k\Delta.$$

This implies that $(1+deg(v_1))+(1+deg(v_2))+\cdots+(1+deg(v_k)) \leq k+k\Delta$ and $\sum_{i=1}^{k} 1 + deg(v_i) \leq k(1+\Delta)$. Hence we obtain the required bound

$$k \geq \frac{\sum_{i=1}^{k} 1 + deg(v_i)}{1+\Delta} \Rightarrow m_\alpha(G) \geq \frac{\sum_{i=1}^{k} 1 + deg(v_i)}{1+\Delta}$$

**Remark 3.7.** The trivial upper bound of $m_\alpha(G)$ is $p = n(V)$.

**Remark 3.8.** If $T$ is a tree of order $p \geq 3$ and $|End(T)| = k$, then $m_\alpha(T) = k$ and independence number $\beta(T) \geq |End(T)|$.

□

## 4. Some Realizations

**Theorem 4.1.** For any $k \in Z^+$ with $2 \leq k \leq p-1$, there exists connected graphs $G$ such that $m_\alpha(G) = k$ and $n(V) = p$.

*Proof.* Choose a positive integer $k \in Z^+$ such that $2 \leq k \leq p-1$. Let $P : [v_1, v_2, \ldots, v_{p-k+1}]$ be a path graph of length at most $p-k$. Adjoin the $k-1$ non-adjacent vertices $\{u_1, u_2, \ldots, u_{k-1}\}$ to an end vertex $v_{p-k+1}$ of the path graph $P$.

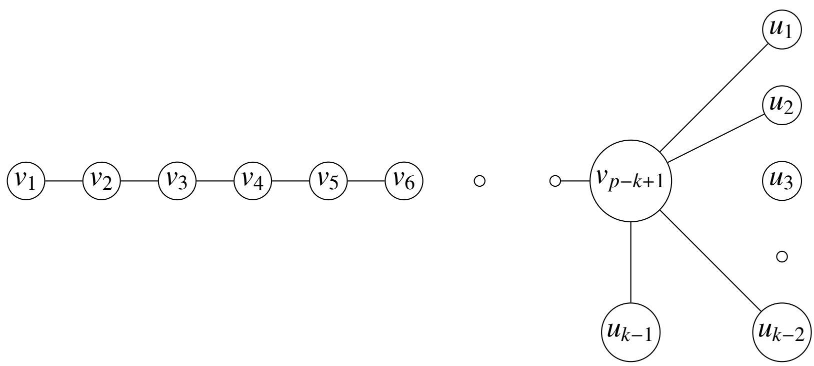


**Figure 4:** A graph $G$ with $m_\alpha(G) = k$ and $n(V) = p$

We obtain a graph $G$ with $|End(G)| = k$ is given in Figure 4. Clearly $\{u_1, u_2, \ldots, u_{k-1}\}$ is a unique monophonic set of $G$ and all of its vertices are non-adjacent. It follows that $m_\alpha(G) = k$ and total vertex count $n(V) = p-k+1+k-1 = p$. □

**Remark 4.2.** For $k = p$, there is no connected graph $G$ with the requirements of the Theorem 4.1.

**Remark 4.3.** For any connected $(p,q)$-graph, $m_\alpha(G) = 0 \Leftrightarrow G = K_p$

**Theorem 4.4.** For $\lambda, \mu \in Z^+$ with $2 \leq \lambda \leq \mu$, there exists a connected graph $G$ such that $m_\alpha(G) = \lambda$, $g(G) = \mu$ and $\beta(G) = \mu + 1$.

*Proof.* When we consider the case $2 \leq \lambda = \mu$, choose any tree with $\lambda$-end vertices has the required properties. Next we assume that $2 \leq \lambda < \mu$.

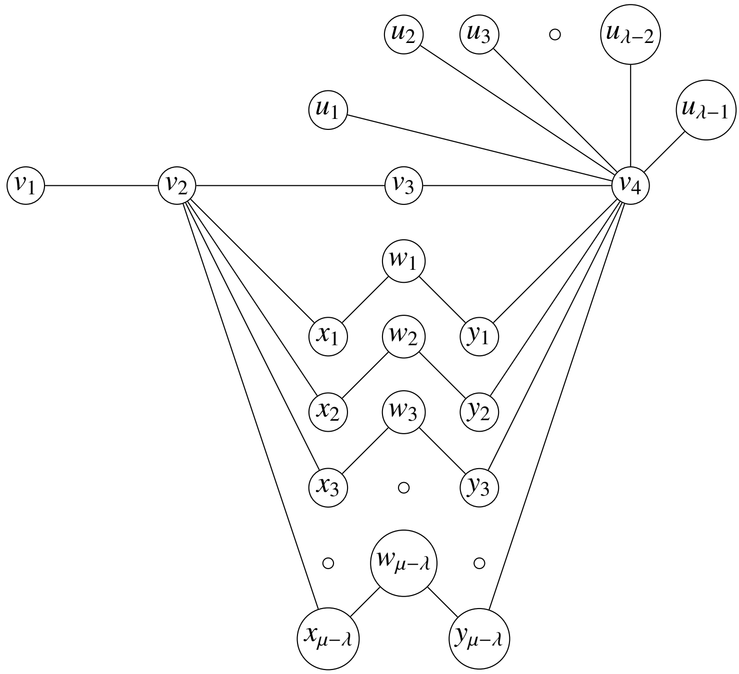


**Figure 5:** A graph $G$ with $m_\alpha(G) = \lambda$, $g(G) = \mu$ and $\beta(G) = \mu + 1$

Consider $\mu - \lambda$ copies of a path graph $P_i$ $(i = 1, 2, 3 \ldots \mu - \lambda)$ of length 2 where $P_1 : [x_1, w_1, y_1]$, $P_2 : [x_2, w_2, y_2]$, $\ldots$, and

$$P_{\mu-\lambda} : [x_{\mu-\lambda}, w_{\mu-\lambda}, y_{\mu-\lambda}].$$

Also choose another path graph on four vertices $P : [v_1, v_2, v_3, v_4]$. We obtain a new graph $G$ by joining each $x_i$ in $P_i$ to $v_2$ in $P$ and joining each $y_i$ in $P_i$ to $v_4$ in $P$. Finally adding a new set of $\lambda - 1$ non-adjacent vertices $\{u_1, u_2, u_3, \ldots u_{\lambda-1}\}$ to end vertex $v_4$ of $P$. The new graph $G$ is given in Figure5.

Since $M = \{v_1, u_1, u_2, \ldots u_{\lambda-1}\}$ is a set of monophonic vertices of $G$, $m(G) = 1 + \lambda - 1 = \lambda$. Also, all the elements of $M$ are independent but not maximal. Therefore $M$ is the minimum independent monophonic set of $G$ so that the variable $m_\alpha(G) = \lambda$.

If we add the set of vertices $N = \{v_3, w_1, w_2, \ldots w_{\mu-\lambda}\}$ to $M$, then $S = M \cup N$ becomes the maximal independent set of vertices in $G$ so that independence number of $G$ is

$$\beta(G) = |S| = \lambda + 1 + \mu - \lambda = \mu + 1.$$

Finally we have to show that geodetic number $g(G) = \mu$. All the vertices of $S$ except the vertex $v_3$ are under an extreme category or end vertices. So that $S - \{v_3\}$ is the minimum geodetic set of $G$. Hence we obtain a minimum geodetic set $S - \{v_3\}$ of the graph $G$ so that geodetic number $g(G) = |S| - 1 = \mu$. □

## 5. Conclusion

This work can be extended to find the number of non-adjacent vertices in an edge monophonic sets, connected monophonic sets and upper monophonic sets etc in the connected graphs.

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