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Contra Harmonic Mean Labelling of Graphs

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Abstract

A graph labelling is an assignment of integers to the vertices or edges or both subject to certain conditions. A Graph G(V, E) with p vertices and q edges is called a Contra Harmonic mean graph if it is possible to label all the vertices $x \in V$ with distinct labels f(x) from {1, 2, 3, 4, …., p} in such a way that each edge e = uv is labelled with $f(uv) = \left\lceil \frac{\{f(u)\}^2+\{f(v)\}^2}{f(u)+f(v)} \right\rceil$ or $f(uv) = \left\lfloor \frac{\{f(u)\}^2+\{f(v)\}^2}{f(u)+f(v)} \right\rfloor$ are distinct. In this paper the Contra Harmonic mean labelling of several standard graphs such as path, cycle, $C_m \cup P_n$, $C_m \cup C_n$, nK_3, $nK_3 \cup P_m$, $nK_3 \cup C_m$ etc. are explored.

Keywords: Graph Labelling, Contra harmonic mean graph.

1. Introduction

Mean Labelling of a graph G with p vertices and q edges is an injective function f from G to {0, 1, 2, 3, 4…q} such that when each edge uv is labelled with (f(u) + f(v))/2 , if f(u) + f(v) is even, and (f(u) + f(v) + 1)/2 if f(u) + f(v) is odd, then the resulting edge labels are distinct.[1] In this paper the Contra Harmonic mean labelling of some standard graphs are introduced.

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Let G = (V, E) with p vertices and q edges be a simple, finite and undirected graph. The graph G–e is obtained from G by deleting an edge e. The sum $G_1 + G_2$ of two graphs G_1 and G_2 has vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv \text{ where } u \in V(G_1) \text{ and } v \in V(G_2)\}$.

The union of two graphs G_1 and G_2 is a graph $G_1 \cup G_2$ with vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. The square G^2 of the graph G has $V(G^2)$ with u, v adjacent in G^2, whenever $d(u, v) \leq 2$ in the graph G. For detailed survey on graph labeling we refer to Gallian.[3] According to Beineke and Hegde[4] graph labeling serves as a frontier between number theory and structure of graphs. The definitions which are useful for the present investigation are given below.

Definition 1.1

A function f is called a mean labeling of a graph G = (V, E) if $f: V \to \{0, 1, 2, 3, \cdots, q\}$ is injective and the induced function $f^*: E \to \{1, 2, 3, \cdots, q\}$ defined as

$$f^*(uv) = \begin{cases} \dfrac{f(u) + f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \dfrac{f(u) + f(v) + 1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

is bijective. A graph which admits mean labeling is called a mean graph.

Definition 1.2

A Graph G with p vertices and q edges is called a Contra Harmonic mean graph if it possible to label the vertices $x \in V$ with distinct labels f(x) from $1, 2, 3, 4, \cdots, p$ in such a way that when each edge e=uv is labeled with $f^*(uv) = \left\lceil \frac{\{f(u)\}^2 + \{f(v)\}^2}{f(u) + f(v)} \right\rceil$ or $f^*(uv) = \left\lfloor \frac{\{f(u)\}^2 + \{f(v)\}^2}{f(u) + f(v)} \right\rfloor$ are distinct. In this case f is called Contra Harmonic Mean labeling of G.

2. Main Results

Theorem 2.1

Any Path P_n is Contra Harmonic mean graph.

Proof

Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of the path P_n of length n. Let us define $f: V \to \{1, 2, 3, \cdots, p\}$ by $f(v_i) = i,\ 1 \le i \le n$, such that the induced function $f^*: E(G) \to \{1, 2, 3, \cdots, q\}$ given by $f^*(uv) = \left\lfloor \frac{\{f(u)\}^2 + \{f(v)\}^2}{f(u) + f(v)} \right\rfloor$ for every $uv \in E(G)$ are all distinct. Hence path P_n is Contra Harmonic mean graph.

Illustration 2.2: Consider the path of length 5. The labelling is as shown in Figure -1.

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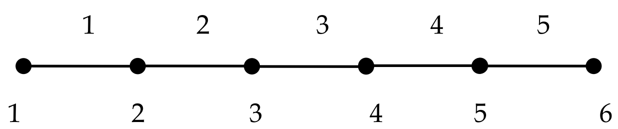


Fig. 1

Theorem 2.3

Any cycle C_n of length n $n \ge 3$, is a Contra Harmonic mean graph.

Proof

Let C_n be the cycle with vertices $v_1, v_2, v_3, \cdots, v_n$.

Define a function $f: V(C_n) \to \{1, 2, 3, \cdots, p\}$ by $f(v_i) = i,\ 1 \le i \le n$, here f is an increasing function on $V(C_n)$, so f^* is also an increasing function on $E(C_n) - \{v_n v_1\}$. For every $e_i, e_j \in E(C_n) - \{v_n v_1\}$, we assign the label $f^*(v_i v_j) = \left\lfloor \frac{\{f(v_i)\}^2 + \{f(v_j)\}^2}{f(v_i) + f(v_j)} \right\rfloor$ and $f^*(v_n v_1) = \left\lfloor \frac{\{f(v_i)\}^2 + \{f(v_j)\}^2}{f(v_i) + f(v_j)} \right\rfloor$. Hence $f^*(e_i) \ne f^*(e_j),\ i \ne j$. Therefore, f^* is injective and f is a Contra Harmonic labelling on C_n. So C_n is a Contra Harmonic mean graph.

Illustration 2.4: Consider the cycle of length 4 & 5. The labelling is as shown in Figure-2.

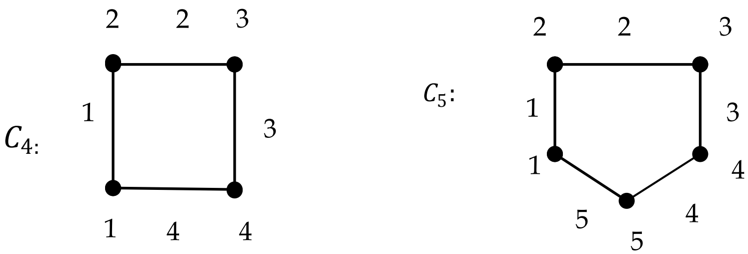


Fig. 2

Theorem 2.5

The Complete graph K_n is a Contra Harmonic mean graph for $n \leq 3$.

Proof

The Contra Harmonic mean labeling of the complete graph K_n for $n \leq 3$ is given in Figure - 3 below.

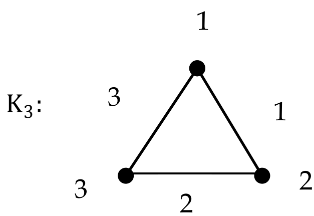


Fig. 3

Let K_n and $f: V(G) \rightarrow \{1, 2, 3, \cdots, n\}$, be a vertex function which induces a function f^* given by $f^*(v_i v_j) = \left\lceil \frac{\{f(v_i)\}^2 + \{f(v_j)\}^2}{f(v_i)+f(v_j)} \right\rceil$ for every $e = v_i v_j \in E(G) - \{v_n v_1\}$ and $f^*(v_n v_1) = \left\lfloor \frac{\{f(v_n)\}^2 + \{f(v_1)\}^2}{f(v_n)+f(v_1)} \right\rfloor$. Assume $n \geq 4$ and since the graph is complete we have two edges e_1 and e_2 such that $f^*(e_1 = (3,4)) = \left\lceil \frac{3^2+4^2}{3+4} \right\rceil = \lceil 3.555 \rceil = 4$ and $f^*(e_2 = (2,4)) = \left\lceil \frac{2^2+4^2}{2+4} \right\rceil =$

$\lceil 3.33 \rceil = 4$ are same. So we conclude that when $n \geq 4$, f^* is not injective. Therefore K_n for $n \geq 4$ is not a Contra harmonic mean graph.

Hence K_n is a Contra Harmonic mean graph for $n \leq 3$.

Theorem 2.6

The graph mK_2 , m copies of K2 is a Contra Harmonic mean graph.

Proof

mK_2 consists of m pair wise disjoint edges as shown in Figure - 4. Let

$V(mK_2) = \{a_1, b_1, a_{2,} b_2, \cdots a_m, b_m\}$, $E(mK_2) = \{(a_{i,} b_i): 1 \leq i \leq m\}$.

Define f: $V(mK_2) \rightarrow \{1, 2, 3, \cdots, p\}$ as follows: $f(a_i) = 2i - 1$, $1 \leq i \leq m$ and $f(b_i) = 2i, 1 \leq i \leq m$. So that $f^*(a_i, b_i) = \left\lfloor \frac{(2i-1)^2+(2i)^2}{(2i-1)+(2i)} \right\rfloor$, $1 \leq i \leq m$. Moreover, f is an increasing function on V(G) and so $f^*(a_i, b_i) \neq f^*(a_j, b_j)$, $i \neq j$ $1 \leq i, j \leq m$. Hence f^* is injective and f is a Contra harmonic mean graph.

Illustration 2.7: Consider a graph $6K_2$. The labelling is as shown in Figure - 4.

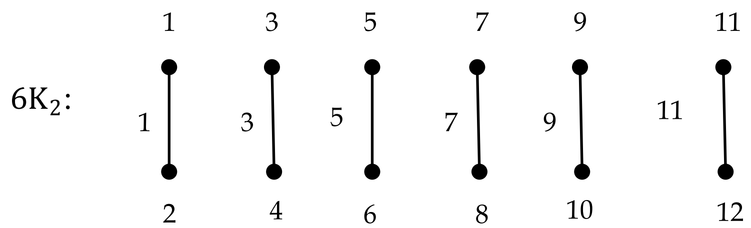


Fig. 4

Theorem 2.8

The graph mK_3 , m copies of K3 is Contra Harmonic mean graph.

Proof

The graph mK_3 which is m copies of triangles as shown in Figure – 5. Let the vertex set of mK_3 be $V = V_1 \cup V_2 \cup \cdots \cup V_m$, where $V_i = \{V_1^i, V_2^i, V_3^i\}$. We define

$f: V(mK_3) \to \{1, 2, 3, \cdots, p\}$ by $f(V_j^i) = 3(i-1) + j, 1 \le i \le m,\ 1 \le j \le 3$ is an injective function. So that the graph with vertices having labels 1, 2, 3 of first copy of K_3, for the edge joining the vertices 1 and 2 assign label 1, for the edge joining the vertices 2 and 3 assign label 2 and for the edge joining the vertices 1 and 3 assign label 3. Similarly second copy of K_3 the vertices are labeled by 4, 5, 6, for edge joining the vertices 4 and 5, 5 and 6, 4 and 6. Similarly, in the i^{th} copy of K_3 the vertices are labeled with $3i-2,\ 3i-1$ and $3i$. For the edge joining the vertices $3i-2, 3i-1$ assign the label

$f^*(3i-2, 3i-1) = \left\lfloor \frac{(3i-2)^2 + (3i-1)^2}{(3i-2)+(3i-1)} \right\rfloor,\ 1 \le i \le m$. For the edge joining the vertices $3i-1$ and $3i$ assign the label

$f^*(3i-1, 3i) = \left\lfloor \frac{(3i-1)^2 + (3i)^2}{(3i-1)+(3i)} \right\rfloor,\ 1 \le i \le m$ and for the edge joining the vertices $3i-2$ and $3i$ assign the label $f^*(3i-2, 3i) = \left\lceil \frac{(3i-2)^2 + (3i)^2}{(3i-2)+(3i)} \right\rceil$. Hence $f^*(e_i) \ne f^*(e_j),\ i \ne j$. Therefore f^* is injective and f is a Contra harmonic mean labeling on mK_3.Therefore mK_3 is a Contra harmonic mean graph.

Illustration 2.9: Consider a graph $3K_3$. The labeling is as shown in Figure - 5.

$3K_3$:

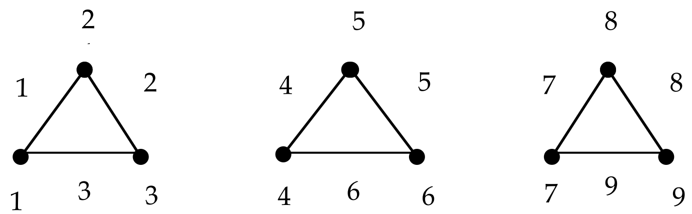


Fig. 5

3. Consequences

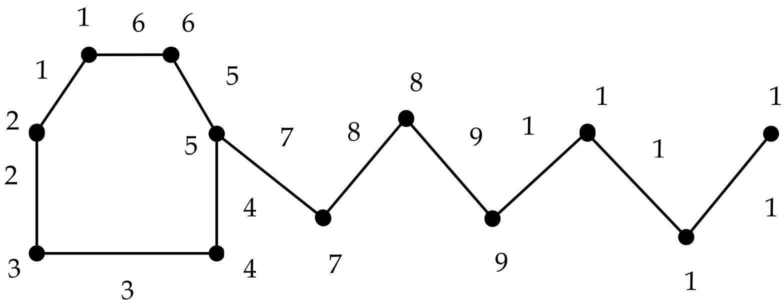
1. The graph $(nK_3 \cup P_m)$ consists of n disjoint triangles and a path of length m is a Contra Harmonic mean graph.

2. $C_m \cup C_n$ is a Contra Harmonic mean graph for $m \ge 3,\ n \ge 3$.

3. $nK_3 \cup C_m$ is a Contra Harmonic mean graph for $n \ge 1,\ m \ge 3$.

4. Dragons $C_n@P_m$ are contra Harmonic mean graphs.

$C_6@P_7$ Consists of C_6 a cycle of length 6 and a path of length 7. The labeling is as shown in figure.



5. The square graph of a path P_n^2 , Ladder L_n are not a Contra Harmonic mean graph.

4. Conclusion

Since Contra Harmonic Mean Labeling of graphs holds good only for a few types of graphs, the other areas in which this concept can be applied are explored. Also we are able to find the contra harmonic sequence which has graphical representation. It is planned to explore different graph properties in future work regarding to this concept.

5. References

[1] S. Somasundaram and R. Ponraj, "Mean Labeling of graphs," *National Academy science Letters*, 26, pp.210 - 213.

[2] F. Harary, Graph theory, Massachusetts: Addison Wesley, 1972.

[3] J. A. Gallian, "A dynamic survey of graph labeling," *The Electronic Journal of Combinatorics,"* 17, 2010.

[4] L. W. Beineke and S. M. Hegde, "Strongly Multiplicative graphs", *Disc Math.* 21, pp.63 – 75, 2001.

[5] S. Somasundaram and R. Ponraj,"On mean graphs of order<5", *J. Decision and Mathematical sciences.*

[6] D. B. West, Introduction to Graph Theory, 2nd Ed., Prentice Hall. 2000.