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CHAOS-LOGISTIC MAP – TENT MAP – CORRESPONDING CELLULAR AUTOMATA

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ABSTRACT

We study the characteristic features of Tent Map and Logistic Map towards Choas. Cellular Automata is obtained for Tent Map and Logistic Map. The behavior of Tent Map and Logistic Map is reflected in the Cellular Automata generated.

Key words: *Chaos-Chaotic behavior, period, Cycle and Cellular Automata.*

Introduction

Remarkable progress has been made in exploring the Nonlinearity in Dynamical System [1-5]. Particularly Lyapunov exponent, correlation Dimension, Box Dimension and Fractal dimension etc., have been studied in detail [6]. Several investigations on ë were performed in the Logistic Map:

$$y_{t+1} = \Pi y_t (1-y_t)$$

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Lyapunov exponent, correlation dimension, fractal dimension were found by J.C. Sprott [6]. Grossberger (1984) and Tiffoli (1984) [7] discussed about Chaos through Cellular automata. In the present paper, we consider Tent Map and Logistic Map.

Tent Map

The binary Tent Map is defined by the Equation

$$S(n+1) = 2\,S(n) \qquad S(n) \in (0, 0.5)$$
$$\quad\quad = 2\,(1 - S(n)) \qquad S(n) \in (0.5, 1)$$

It is a one dimensional map. It is a chaotic map. It is a non linear map. This map explains chaotic non linear process. Choosing S(0) in [0, 1], S(n) is generated using the above equation. The map [S(n), S(n-1)] is shaped like a tent. Hence S (n) a non linear function of S(n-1). This tent map takes the unit interval [0,1] stretches it twice as long and folds it back onto itself. This type of folding and stretching is a characteristic of chaotic map. This makes the prediction difficult thus creating the illusion of randomness.

The Dynamics of this map is studied in Base - 2 arithmetic wherein we write all numbers and parameters as binary strings.

$$S(n) = \Sigma \epsilon_{\varphi}(n)/2^{i}$$
$$= 0.\ \epsilon_{1}(n)\epsilon_{2}(n)\ldots\ldots\ldots\ldots$$

Where $\epsilon_i(n) = 0$ or 1

Dynamics of Tent Map is also studied through Cellular Automata. In general if we write S(0) = a / b where a and b are integers. On iteration, S(n) can take on only one of b different possible values 0, 1/b,2/b,...(b-1)/b. Hence after almost b iterations, the initial condition is repeated and we have periodicity.

i.e If S(0) = 1/4+1/8+1/32+1/64 = 0.011011
S(1) = 0.11011
S(2) = 0.1011
S(3) = 0.011
S(4) = 0.11

Hence all rational initial conditions of the Tent Map yield periodic orbits. All these orbits are unstable. We infer that a small increase in b typically yields an entirely different orbit of longer period.

Similarly if we start with S(0) = 1/3 then the map hops into point S(1) = 2/3 whereas if S(0) = 2/9 yields 3 cycle made of the points 2/9, 4/9 & 8/9.

Whereas an irrational initial conditions on the other hand yield unstable

non periodic orbits since there is no periodicity in the binary string of an irrational number . Tent Map is sensitive with respect to initial condition.

Cellular Automation For Tent Map

For Tent Map,

$$S(n+1) = 2\,S(n), \qquad S(n) \in (0, 0.5)$$
$$ = 2\,(1 - S(n)), \qquad S(n) \in (0.5, 1)$$

Corresponding cellular Automaton is formulated as

$$\epsilon_i\,(n+1) = \epsilon_{i+1}(n) \qquad \text{if } \epsilon_1(n) = 0$$
$$ = 1 - \epsilon_{i+1}(n) \qquad \text{if } \epsilon_1(n) = 1$$

Complete Dynamics of Tent Map is observed through Cellular automata. We observe periodic orbits, non periodic orbits and quasi periodic orbits through Cellular Automata. Starting from any Tent Map one can form a Cellular automaton and vice versa. This tent map reduces to Logistic map by proper substitution. Since Logistic map is a particular case of Tent map, Tent map is also studied through Logistic map and vice versa.

Logistic map is of the form $y_{t+1} = r_i y_t (1 - y_t)$ where $y_t > 0$, $r_i > 0$, $0 \leq y_0 \leq 1$. Logistic Map is chaotic when $r_i > 3.57$. The dynamics of Logistic map $y_{t+1} = 4y_t(1 - y_t)$ can be studied through cellular automaton. y_{t+1} is generated for various initial values such as $y_0 = 1/8$, 1/10, 1/16, & 1/32. We obtain y_{t+1} for t = 0,1,2,3....,49.

We convert y_{t+1} value as a Binary equivalent (truncating beyond 6^{th} place) by doing so we observe cycles of various lengths (i.,e) 0.111111 is visible at various irregular intervals which implies Logistic map is Chaotic. The same type of behaviour is observed.

When y_0 = 1/10, 1/16, & 1/32. We study Cellular Automaton of Logistic Map for various initial values and the corresponding cellular automata are presented in the tables (1-4).

Conclusion

For different initial values we observe different orbits, which show that Logistic map is sensitive dependent on initial condition. By generating many values and finding Autocorrelation and Partial autocorrelation we observe interesting properties of this map. Cellular Automata can be generated to the solution of any Dynamical System. One can get the behaviour of any Dynamical System using corresponding Cellular Automata.

Henon Map, Lorenz equations, Rossler Maps etc., are studied through Cellular Automata. The concept of L-System and Cellular Automata are combined to study many more complicated Chaotic Systems. This kind of study is done for various base systems.

Outputs

Table 1. Cellular Automation for
Y(t+1)=4Yt(1-Yt)
Initial Value Y(0) = 1/8

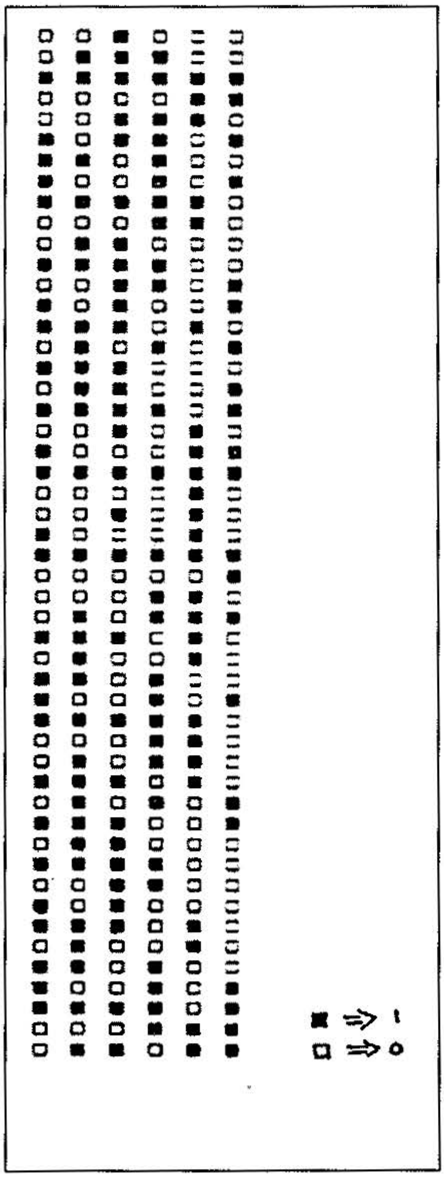


Table 2. Cellular Automation for
Y(t-1)=4Yt(1-Yt)
Initial Value Y(0) = 1/10

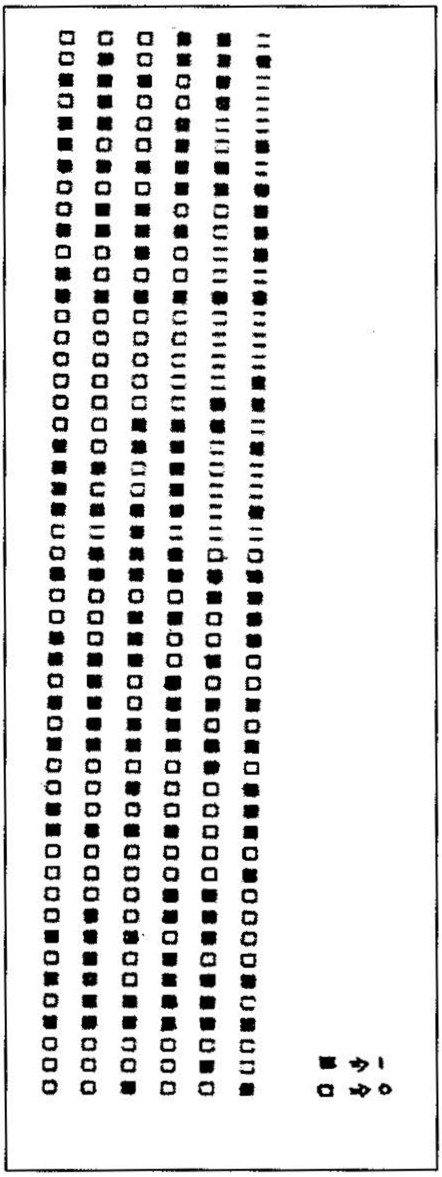


Table 3. Cellular Automation for
$Y(t+1)=4Yt(1-Yt)$
Initial Value $Y(0) = 1/16$

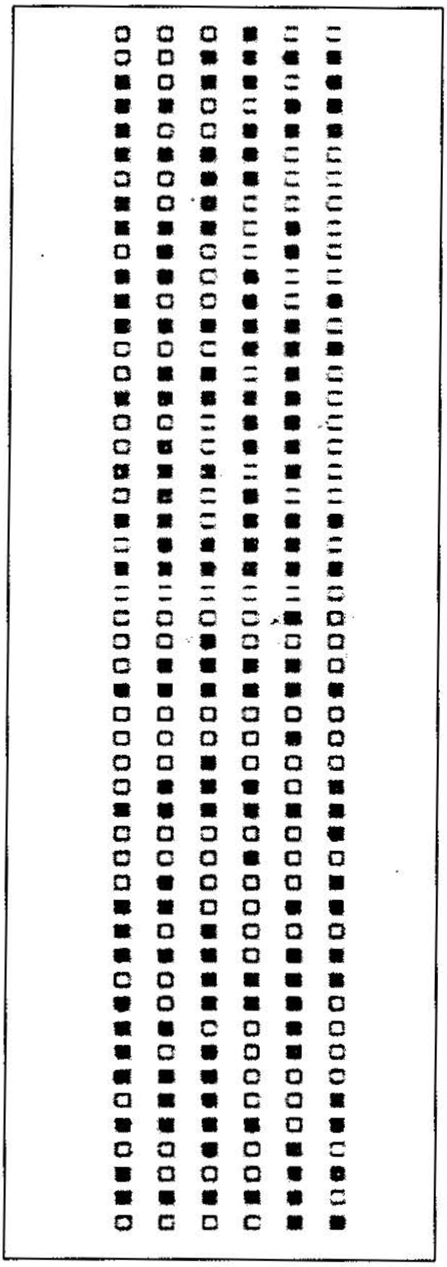
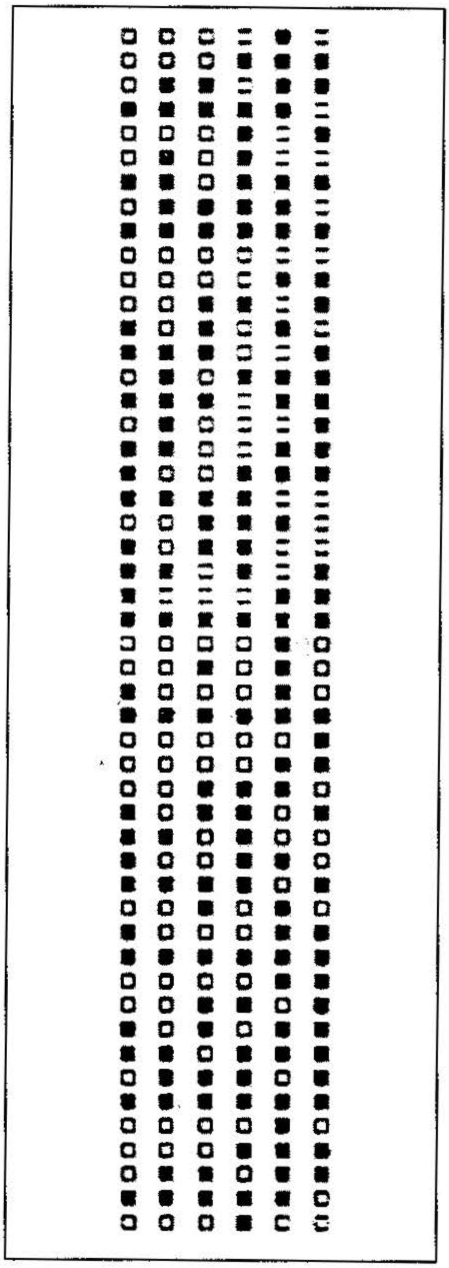


Table 4. Cellular Automation for
$Y(t-1)=4Yt(1-Yt)$
Initial Value $Y(0) = 1/32$



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