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SIMPLE EXPERIMENTS AND MODELING OF INCANDESCENT LAMP SPECTRA

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ABSTRACT

The purpose of this work is to provide physics students and teachers with a simple experiment in modern physics, which utilizes modern spectroscopic methods and provides computational modeling of incandescent lamp spectra. Captured spectra are modeled with Planck's radiation distribution, so that a temperature can be extracted. Voltage across and current through the lamp are recorded at the time of spectra capture, and the power and temperature data are fit with the Stefan-Boltzmann law. This experiment is further expanded by investigating the lamp's resistance as a function of temperature. It is seen that typical incandescent lamps obtained at local retail stores are great examples of blackbody radiators, while the common energy efficient fluorescent lamps are not.

Key Words: Blackbody radiation, spectrometer, MATLAB, incandescent lamp, Planck's radiation law, spectrum

INTRODUCTION

In general, modern physics starts with phenomena and unsolved questions posed as far back as the year of 1895 (1). A fundamental issue was blackbody radiation which was treated classically, yet unsatisfactorily explained. In the experimental physics class, one can't easily find a simple experiment to study blackbody radiation while it is taught in the modern physics course, or to generate significant data that can be analyzed in a computational physics class. These simple experiments proposed here provide hands on experience in experimental physics, and the measurements of the blackbody spectra provide large amounts of data which can be used in a computational physics class. This paper covers these areas by focusing on blackbody radiation as the main topic.

The problem of determining filament temperatures has been seen before (2-4), but rather than focus on using the resistivity (2) or the irradiance over a range (3) or only at a few well defined wavelengths (4) to determine temperatures, we measure a wide wavelength range of the spectrum and use regression analysis to find the filament temperature. This process is unique in that it does not rely on a known resistance and temperature relationship (2-4); the temperature can be extrapolated without knowing any details of the incandescent lamp. This provides an avenue to investigate that relationship, rather than rely on it. Furthermore,

this process has only recently become feasible as an undergraduate experiment because of the low cost, modern spectrometers available today.

THEORY

Objects radiate electromagnetic waves in a distribution which depends on the object's temperature (5). This distribution is the Planck radiation law. The Planck radiation law is given by Eq. [1]:

$$S_\lambda = \frac{2\pi\alpha hc^2}{\lambda^5 (exp(\frac{hc}{\lambda kT})-1)}, \tag{1}$$

Where λ is wavelength (m), T is temperature (K), and α is a dimensionless scale factor known as the emissivity, that depends on the atomic structure, which can be a function of wavelength. For an ideal blackbody, $\alpha = 1$. the power spectral density, S_λ, as measured at some distance, r, from the source, falls off as an inverse square for a spherically symmetric emission source. Assuming our filament to be a spherical source of radius r, we write the spectral irradiance at some distance r' to be given by

$$S_\lambda(r') = \frac{2\pi\alpha hc^2}{\lambda^5 (exp(\frac{hc}{\lambda kT})-1)} \left(\frac{r}{r'}\right)^2, \tag{2}$$

Additionally, the Stefan-Boltzmann law is related to the Planck distribution by solving the following for P;

$$S_\lambda = \left(\frac{1}{A}\right)\frac{dP}{d\lambda}. \tag{3}$$

If we take S_λ to be given by Eq. (2), then solving Eq. (3) yields te Stefan-Boltzmann law,

$$P = \alpha\sigma T^4 \left(\frac{r}{r'}\right)^2, \tag{4}$$

where P is power, A is the effective surface area and σ is the Stefan-Boltzmann constant. We could estimate the total power radiated based on the distance from the source and the irradiance, $I = \int S_\lambda d\lambda$, but since we record the current i through and voltage V across the lamp at the time of spectra capture, we can assume that $P=iV$. We can then see how well the Stefan-Boltzmann law fits the collected data. Furthermore, according to Ohm's law, the resistance, R, is given by the ratio V/i, so we can show how the resistance varies with temperature, which reveals the non-Ohmic behavior of incandescent lamps.

EXPERIMENTAL APPROACH

To show that the incandescent lamp is described well by the Planck distribution, it is enough to record the lamp's spectrum at a single input power and perform regression. To gain more accurate results, a dark spectrum (lamp off) can be recorded and subtracted from the measurement. However, in order to investigate the Stefan-Boltzmann law, the circuit shown in Figure 1 should be constructed to carry out the experiment. The circuit uses a variable AC transformer,

yielding a sinusoidal output with 60 Hz frequency of variable amplitude. We performed this experiment with incandescent lamps (typical light bulbs) purchased at a local retail store of varying wattages (–7.5W, 15W, 40W, 60W, and 75W), and the lamps were successfully used. The voltage was varied between 15V and 130V for each lamp, each spectra recorded and temperature extrapolated.
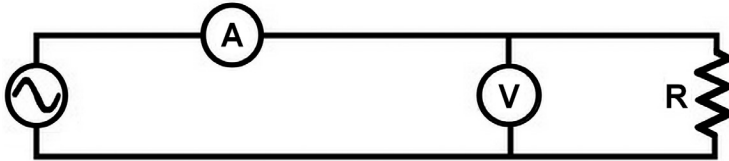
Figure 1. Schematic of the circuit required to measure voltage and current while capturing the spectra. The resistor, labeled R, represents the incandescent lamp. The voltage source (circled sine wave) should be a variable AC transformer, in order to regulate the output power to observe how the filament temperature varies. The circled A represents an ammeter, and the circled V represents a voltmeter.

After the circuit has been assembled, spectra can then be recorded along with the voltage and current. The process of logging voltage and current data can be done manually by using standard multimeters, or automated with voltage and current sensors in combination with some software, such as Logger Pro (6). After changing the voltage, the system should be allowed to stabilize. This can be seen by a change in the current during the experiment, as it will continue to rise or fall for a few moments after the voltage has been changed. We recommend waiting until the current stops fluctuating to capture the spectrum.

MODELING WITH MATLAB

The first goal is to import the data into MATLAB. If the data has been saved into a Microsoft Excel file, it can be imported into MATLAB through the method *xlsread('filename')*. Before we can perform the regression, we need to define a function which accepts the parameters and data, referenced here and in the appendix as *Sfunction*. Notice that the parameters to be found are stored in a single matrix, *O*.

$$Sfunction = @(O, \lambda) \frac{hc^2 O(1)}{\lambda^5 (\exp(\frac{hcO(2)}{\lambda k}) - 1)} \qquad (5)$$

In order to perform regression, the method *nlinfit(λ, S_λ, Sfunction, initialguess)* returns the best fitting values for the parameters of the model. Note that we have chosen $O(2)$ to be T^1. The following four arguments must be provided: λ, a matrix of λ values; S_λ, a matrix containing values of the spectral irradiance corresponding to λ; *Sfunction*, the function defined above in Eq. (5); and *initialguess*, a matrix containing an initial guess for each of the *O* parameters. Initial guesses for the temperature may be found by using the Stefan-Boltzmann law, solving for *T* yields

$$T_{initial} \approx \left(\frac{iV}{A\sigma}\right)^{\frac{1}{4}}, \qquad (6)$$

where A is an approximate value for the effective surface area of the filament. We successfully used the value $A = 3\times10^{-5}$ m^2. Initial guesses for the $O(1)$ parameter will most easily be found through trial and error. For our spectrometer roughly half a meter from the filament, we successfully used an initial guess of 3×10^{-12} for $O(1)$. MATLAB can also be used to produce plots of the results through the method *plot*(X,Y). See the appendix for a complete MATLAB script which imports a spectral data set, performs the regression, plots the data and model, and shows the determined temperature. Note that the excel data files used with the script have a two column, header-less format with λ values in the first column and S_λ values in the second.

RESULTS

Lamp wattage did not have a significant impact on our results. However, we did choose to only use clear lamps; the effects of frosted glass lamps are unknown to us. We present here our results for a 60W lamp.

Spectra were recorded at various voltages and currents, and fit with Planck's blackbody radiation distribution, as shown in Figure 2. The resulting models fit the spectra exceptionally well, with a lowest coefficient of determination (*i.e.* goodness of fit), $r^2 = 0.9509$. The goodness of fit can be calculated through the following equation,

$$r^2 = 1 - \frac{\sum(y_i - f_i)^2}{\sum(y_i - \overline{y})^2}, \qquad (7)$$

where y_i is a collected data point, f_i is the corresponding model point (in our case, both y_i and f_i have the same λ value), and $\overline{y}$ is the mean of y.
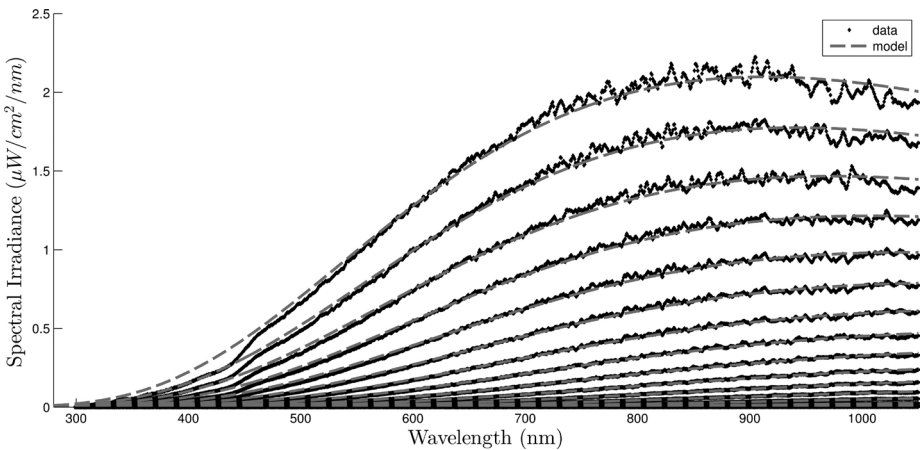

Figure 2. Spectra and models of a typical 120V/60W incandescent lamp at various voltages and currents. See Table I for corresponding values. Regression performed with Eq. (2) for the models.

Values for the regression parameters, voltages, currents, and temperatures corresponding to these spectra are given in Table I.

Table I: Values of voltage, current, power, resistance, temperature, $O(1)$ and goodness of fit for the spectra and models shown in Figure 2. Power is found via $P=iV$. Resistance is found via $R=V/i$. Temperature $(O(2)^{-1})$, and $O(1)$ values are found by regression.

| Voltage (V)	Current (A)	Power (W)	Resistance (Ω)	Temperature (K)	$O(1)^c \times 10^{12}$ (unitless)	r^2
14.7	0.174	2.56	84.48	1287	1.884	0.9509
20.4	0.200	4.08	102.00	1497	1.979	0.9961
26.1	0.225	5.87	116.00	1663	2.075	0.9981
32.5	0.250	8.13	130.00	1806	2.353	0.9988
39.3	0.275	10.81	142.91	1955	2.351	0.9983
46.3	0.300	13.89	154.33	2074	2.537	0.9985
53.8	0.325	17.49	165.54	2199	2.614	0.9984
61.8	0.350	21.63	176.57	2313	2.744	0.9986
70.6	0.375	26.48	188.27	2433	2.789	0.9985
79.3	0.400	31.72	198.25	2544	2.852	0.9986
89.0	0.425	37.83	209.41	2659	2.904	0.9984
99.0	0.450	44.55	220.00	2769	2.950	0.9980
109.3	0.475	51.92	230.11	2870	3.046	0.9979
120.4	0.500	60.20	240.80	2981	3.044	0.9967
131.4	0.525	68.99	250.29	3078	3.138	0.9965
143.3	0.550	78.82	260.55	3176	3.170	0.9953

To investigate the Stefan-Boltzmann law, we let $P = B\sigma T^4$, where B is our fitting parameter (see Eq. 4). B represents the product of the effective surface area, A, emissivity of the filament, α, and the distance factor, $(\frac{r}{r'})^2$. As shown in Figure 3, the Stefan-Boltzmann law provides excellent agreement for the relationship between P and T, with an $r^2 = 0.9997$.
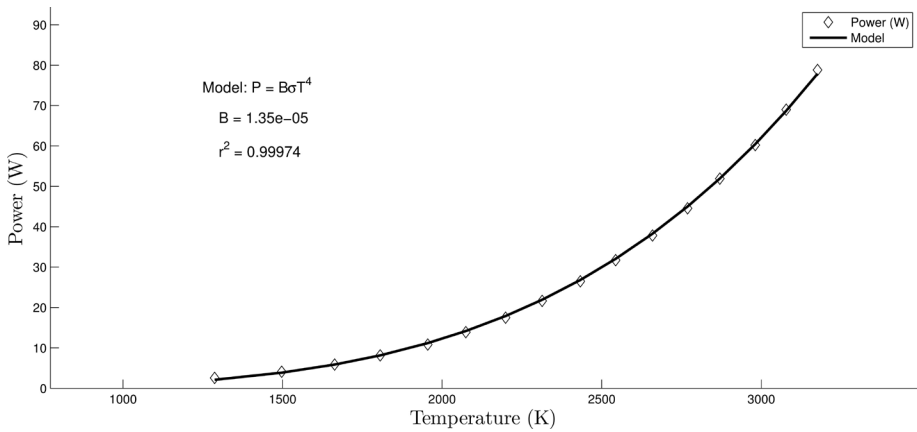
Figure 3. Power vs temperature fit with Stefan-Boltzmann law. We found $B = 1.35 \times 10^{-5}$ m^2, and r^2 = 0.9997.

Additionally, Ohm's law states that the resistance, R, is a constant that is independent of current. We can easily show that the incandescent lamp is a non-Ohmic conductor by looking at the relationship between V and i, or R and T, as shown in Figure 4. Because the filament temperature is proportional to some power of the current (through Stefan-Boltzmann law), it follows that for R to be independent of i (Ohmic), R must also be independent of T.
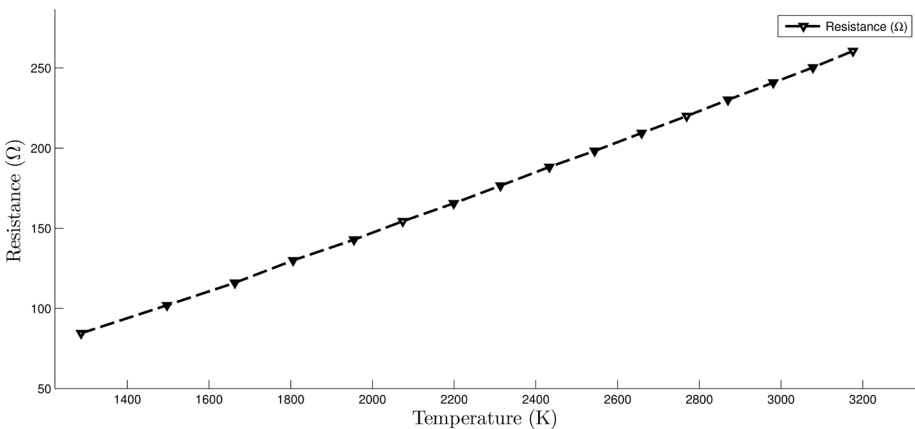
Figure 4. A plot of resistance vs temperature, revealing the non-Ohmic nature of incandescent lamps.

CONCLUSION

It can be seen that incandescent lamp spectra are described well by Planck's radiation distribution. Furthermore, these lamps were shown to obey the

Stefan-Boltzmann law. The processes involved with these experiments provide valuable experiences for students. Students who complete these experiments will have gained first-hand experience in applied modern physics, computational physics, and modern spectrometry. Building the circuit, using the spectrometer properly, recording the data and writing scripts to model the data develops many desirable skills for physics students. For the physics teacher, allowing students to observe the spectra of incandescent lamps and fluorescent lamps during these experiments reveals their fundamental differences; a subject for the modern physics lecture.

ACKNOWLEDGEMENTS

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APPENDIX
Example Script for MATLAB

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   @Author: Austin Kerlin, J.E. Hasbun, Ajith DeSilva %      %
%   University of West Georgia %                              %
%   Fall 2015 %                                              %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc; %these three lines clear the MATLAB IDE and give a fresh workspace
close all;
clear all;

c = 2.998e8;                  % speed of light in m/s
h = 6.62606957e-34;           % Planck's constant eV*s
k = 1.3806488e-23;            % Boltzmann's constant eV/K
sigma = 5.670373e-8;          % Stefan boltzman constant J/m^2sK^4
```

```
titleFontSize = 16;
axisFontSize = 18;

filename = '60W.xls';              % File name of data to import
Data = xlsread(filename);         % Imports data file
Offfilename = '60W_OFF.xls';
OffData = xlsread(Offfilename);

xdata=Data(:,1)./1e9; %converts xdata to meters (for regression)
ydata=Data(:,2)-OffData(:,2); %subtracts background signal from measurement
%Recorded data already in uW/cm^2/nm no need to convert

Amps=.50;                         %current
Voltage = 120.4;                  %voltage

initialguess=(3.1e-12, 1./( (Amps.*Voltage./(sigma*1.37e-5)).^(1/4) ));
%initial guess for regression

Sfunction = @(o,xdata) o(1).*c^2.*h./( xdata.^5.*...
 (exp(o(2).*h*c./(xdata.*k))-1));
%spectral density function declared as anonymous function

Xaxis = (100:1500)'./1e9;
%specifies x axis points for model

fit = nlinfit(xdata,ydata,Sfunction,initialguess);
% calls nonlinear regression function to evaluate fit
%coefficients

modelY = Sfunction(fit,Xaxis);
% evaluates the fitted function at the Xaxis points


temp = 1/fit(2);
% declares temp from fit coefficient 1/fit(2)

figure('Position',(100,50,1000,500));
plot(xdata.*1e9,ydata,'kd','MarkerSize',2)
%plots measurement, xdata converted to nm
hold on
plot(Xaxis.*1e9,modelY,'r-','LineWidth',2)
%plots model, Xaxis converted to nm

text(max(Xaxis.*1e9)*.3,max(ydata)*.9,('T = ',num2str(temp,'%4.0f'),'K')...
 ,'FontSize',12);
%adds text showing temperature to plot
```

```
%below title and axis labels in TeX
titleString = ('\bf{60W lamp spectra and model}');
title(titleString, 'FontSize',16,'Interpreter','latex');
TeXString1 = '\bf{Wavelength (nm)}';
TeXString2 = '\bf{Spectral Irradiance} ($\mu W/cm^2/nm$)';
xlabel(TeXString1,'FontSize',16,'Interpreter','latex');
ylabel(TeXString2,'FontSize',16,'Interpreter','latex');
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                                 %
%                        End program                              %
%                                                                 %
%                                                                 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```