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THE MILLIKAN OIL DROP EXPERIMENT: A SIMULATION SUITABLE FOR CLASSROOM USE

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ABSTRACT

Due to advancements in computing techniques it has become possible to extend the accessibility of physics experiments across the physics curriculum by means of computational simulations. The widespread availability of computers in modern classrooms provides virtual access to hands-on physics, chemistry, and biology experiments, among others. Here, specifically, we consider Robert Millikan's famous oil drop experiment. This experiment requires equipment that can be dangerous and expensive. A more practical approach is achieved via a computer simulation, a useful and a universally available alternative. The goal is to encourage scientific thinking, literacy, and innovation while promoting a free network of academic tools. Here we present a simulation that allows the user to carry out Millikan's ingenious experiment by measuring the velocities of oil drops as they are influenced by an electric field. This is repeated until enough data is produced to deduce the charge of the electron. The simulation provides a clean and simple user interface, allowing for realistic interactions within a computational environment. For best results, a basic understanding of the theory and experimental procedure is valuable. We use Easy Java Simulations as the programming environment to carry out the simulation presented here.

Keywords: Robert Millikan, oil drop experiment, equations of motion, Easy Java Simulations

INTRODUCTION

The renowned oil drop experiment, performed by Robert Millikan in 1909, was designed precisely to investigate the total electric charge on a single drop of oil in order to ascertain the fundamental charge of the electron (Millikan 1911) as discussed in many modern physics courses (Thornton et al. 2006). Millikan was so successful in his endeavor that he came within one percent of the currently accepted value of the electron charge and was subsequently able to determine the mass of the electron. Today the experiment is carried out in a similar fashion as in Millikan's time and a modern apparatus, such as that made by Pasco (2015), is sometimes found in a university's physics laboratory. A great deal of appreciation for Millikan's ingenuity comes with performing the experiment, however, the apparatus is expensive and can be dangerous if not used properly. The purpose of this simulation is to make the experimental experience available to nearly everyone. Although there are many free simulations on the web, none give the user a full appreciation of the experiment. The simulation produced here offers a full demonstration

which provides the experience and insight very similar to what one would achieve from performing the actual oil drop experiment.

THE MASS OF AN OIL DROP

When the oil drops are initially introduced into the chamber, they appear as a cloud. After a short time, typically within a minute, the drops spread out due to some falling more quickly than others as well as movement in horizontal directions known as horizontal drift. Once the user selects a drop to observe, the latter movement requires the user to maintain focus on the drop using the microscope's focusing adjustment, as the drop may move closer to or farther away from the microscope. This movement is excluded from the simulation, so focusing is not needed here.

When falling, the drop is subjected to the following three forces: the gravitational force due to Earth F_g, a drag force due to resistance to the motion from the surrounding air F_d, and a buoyant force due to the density of the surrounding air F_b. These forces are directed in the $\hat{z}$-direction (up, down) as shown in Fig. 1. Throughout this paper, for simplicity we will use a positive sign for the up direction and a negative sign for the down direction.
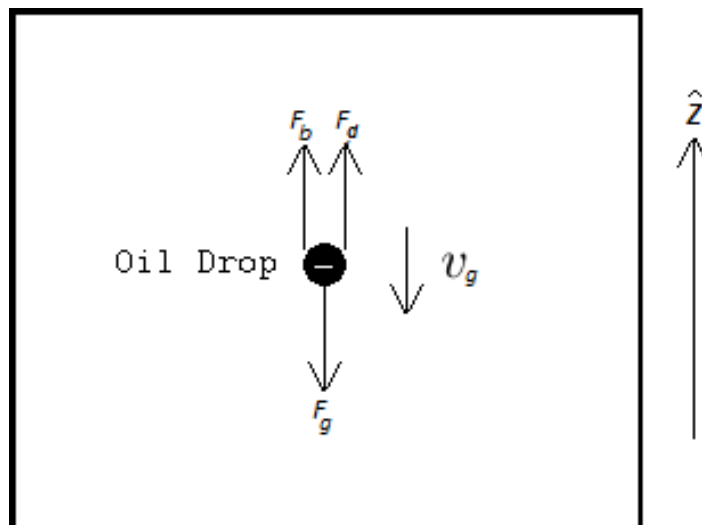

Figure 1. Three forces act on the drop with velocity v_g.

The gravitational force on the drop is

$$F_g = -mg,$$

where m is the mass of the drop and g is the acceleration due to gravity. The negative sign indicates that the force is directed downward.

The drag force given by Stoke's law is

$$F_d = -kv = 6\pi a \eta v_g,$$

where $k = 6\pi a \eta$, with a the drop radius and η the fluid viscosity. This basic form of Stoke's law holds for spherical, homogeneous objects under laminar flow (an explanation of Stoke's law and the theory of laminar flow is given by Crane [Crane Company 1988]),

which is characteristic of the oil drop when falling (or when forced up or down by the electric field). The negative sign indicates that the force is directed opposite to the direction of the velocity v, and therefore is canceled out on the right side of the equation because v_g is negative when the drop is falling. The dynamic viscosity η is affected by atmospheric pressure, which is directly related to temperature, as shown in Appendix 1. When the radius a of the object is comparable to that of the mean free path of air molecules, as is the case of an atomized oil drop, the viscosity η must be multiplied by a correction factor (explained further in *The Electron* [Millikan 1917]; it should also be mentioned that this variable is the only factor that contributes to any significant sources of error in the calculation of the charge) giving a new 'effective' viscosity η_{eff}:

$$\eta_{eff} = \eta \left(1 + \frac{b}{pa}\right)^{-1}.$$

The atmospheric pressure p is recorded at the time of the experiment, b is a constant, and a is again the radius of the oil drop. The new drag force is then

$$F_d = 6\pi a \eta_{eff} v_g.$$

Regarding the buoyant force, we use Archimedes' principle; that is, when an object is submersed in a fluid, there is a resulting upward buoyant force on the object that is equal to the weight of the fluid displaced by the object. The buoyant force is equal to the weight of the air displaced:

$$F_b = m_{air} g$$

Here, m_{air} is the mass of the air displaced by the drop, so the radius of this displacement is the same as the radius of the drop. With this, we arrive at the net force on the drop:

$$\begin{aligned} F_{net} &= F_d + F_b + F_g \\ &= k_{eff} v_g + m_{air} g - mg \\ &= (m_{air} - m)g + 6\pi a \eta_{eff} v_g \\ &= m' g + 6\pi a \eta_{eff} v_g \end{aligned} \qquad (1)$$

where $m' \equiv m - m_{air}$, $m_{air} = \rho_{air} \frac{4}{3}\pi a^3$, and $m = \rho \frac{4}{3}\pi a^3$, with ρ the density of oil and ρ_{air} the air density.

If measurements of the oil drop's velocity were made while the drop was at terminal velocity, the net force would be reduced to zero during all measurements – allowing for the determination of the drop's radius. We can check this by solving the differential equation of motion of the oil drop. From Eq. (1) we have:

$$\vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt} = -m'g - k\vec{v}$$

where we substitute the velocity v_g for the vector quantity $\vec{v}$. For convenience, we let $\gamma = \frac{k}{m}$ and $m'' = 1 - \frac{m_{air}}{m}$.

We then have

$$\frac{d\vec{v}}{dt} = -m''g - \gamma\vec{v} = -\gamma\left(\frac{m''g}{\gamma} + \vec{v}\right)$$

$$\frac{d\vec{v}}{\left(\frac{m''g}{\gamma} + \vec{v}\right)} = -\gamma\,dt$$

Solving the differential equation gives

$$\vec{v} = ce^{-\gamma t} - \frac{m''g}{\gamma},$$

where c is a constant of integration. If we let $\vec{v} = 0$ for $t = 0$, and solve for c, the equation for velocity becomes

$$\vec{v} = \frac{m''g}{\gamma}(1 - e^{-\gamma t})$$

Using experimentally gained values for the mass of an average oil drop and using the values in Appendix 2 to find k, we can create a velocity versus time graph for an average oil drop:
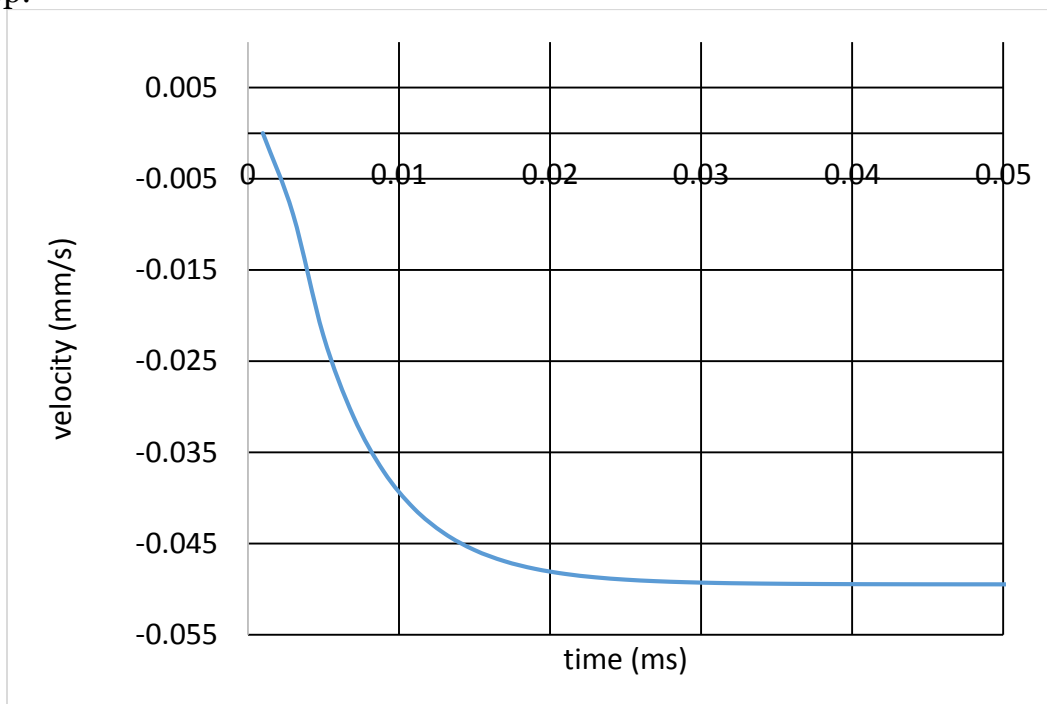

Figure 2. Velocity versus time graph showing how quickly an oil drop reaches terminal velocity.

It is easily seen from the graph above that the oil drops will reach terminal velocity within hundredths of a millisecond. To this end, the left side of Eq. (1) is made zero and replacing m' with $\frac{4}{3}\pi a^3 \rho'$ gives

$$0 = -\frac{4}{3}\pi a^3 \rho'g + 6\pi a\eta_{eff}v_g,$$

where $\rho' \equiv \rho - \rho_{air}$. Using the previous definition for η_{eff} and solving for a gives

$$a = \sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta v_g}{2\rho'g}} - \frac{b}{2p}.$$

It is then possible to calculate the mass of the oil drop:

$$m = \frac{4}{3}\pi a^3 \rho = \frac{4}{3}\pi \left(\sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta v_g}{2\rho'g}} - \frac{b}{2p} \right)^3 \rho. \qquad (2)$$

THE CHARGE ON THE OIL DROP

Applying a voltage to the parallel plates produces an electric field; charged objects experience a force equal to the product of the field strength and the charge held by the object:

$$F_e = qE = q\frac{V}{d}. \qquad (3)$$

Here, q is the charge held by the object, V is the electric field strength, and d is the distance between the plates that produce that electric field. The electric field exerts a force on the charge that, depending on its direction, can either favor or oppose the gravitational force. Thus depending on the electric field direction the experimenter can cause the drop to move so as to keep it in view. Measurements are then made of the velocity v_E of the drop when the field is on. If the electric force is greater than and opposite to the gravitational force, as shown in Figure 2, the drop will rise, and v_E will be positive. If the electric force is in the same direction as the gravitational force, the necessary signs would be changed.

With the field present, as mentioned above, the net force on the oil drop becomes

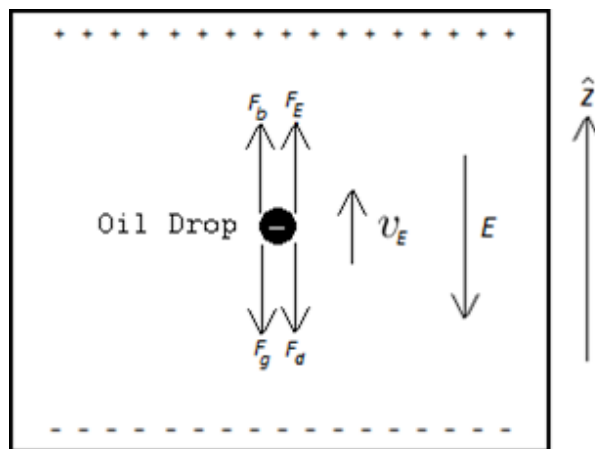$$F_{net} = qE - m'g - k_{eff}v_E. \qquad (4)$$

Figure 3. The four forces acting on the oil drop under the influence of an electric field E giving the drop a velocity v_E.

The electric force on the oil drop is constant causing the drop to reach terminal velocity very quickly, just as in the case with no electric field as mentioned above. The net force is again zero, and we solve Eq. (4) for q:

$$q = \frac{m'g + k_{eff}v_E}{E}. \tag{5}$$

Earlier the constant k was defined in terms of the drop's radius. We choose now to write it in terms of the drop's mass by going back to the $E = 0$ condition; that is, going back to the second line of Eq. (1) and setting the net force to zero gives:

$$0 = kv_g + m_{air}g - mg$$
$$= kv_g - m'g.$$

Solving for k gives:

$$k = \frac{m'g}{v_g}. \tag{6}$$

Finally, inserting the definition for k from above into Eq. (5) gives

$$q = \frac{m'g(v_g + v_E)}{Ev_g}, \tag{7}$$

which is ultimately used to calculate the net charge of the oil drop.

All that is required to determine the charge on any given oil drop then is a measurement of the velocities v_g and v_E. The values of the electric field and all constants used are shown in Appendix 2. Millikan's calculation of the electron's charge was within an outstanding one percent of the presently accepted value of the electric constant, falling short only because the experimentally-gained value used for the air viscosity was slightly low. Millikan made many improvements to his oil drop apparatus to eliminate any errors in measurement and answered any criticisms. Some of these include the fact that the size of drops affects their density; charges on the drops cause the drop to respond differently to air resistance; and charged drops interact with each other, causing variations in velocity and horizontal drift. In this way, having an accepted value for the electron's charge, Millikan was also able to calculate the mass of the electron as well as other fundamental constants that previously relied on it, such as Avogadro's number. More information can be found in Millikan's original publication (Millikan 1911).

EQUATIONS OF MOTION

Applying Newton's 2nd law to the case of an oil drop moving under the action of gravity, an electric force, and subjected to a drag force gives

$$F_x = ma_x = -kv_x$$
$$F_z = ma_z = m_{air}g - mg - kv_z + qE$$

Solving each equation for the acceleration gives

$$a_x = -\frac{k}{m}v_x \tag{8}$$

$$a_z = \frac{m_{air}}{m}g - g - \frac{k}{m}v_z + \frac{q}{m}E = \left(\frac{m_{air}}{m} - 1\right)g - \frac{k}{m}v_z + \frac{q}{m}E \tag{9}$$

Since $\vec{a} = \frac{d\vec{v}}{dt}$, we have

$$\frac{dv_x}{dt} = -\frac{k}{m}v_x$$

$$\frac{dv_z}{dt} = \left(\frac{m_{air}}{m} - 1\right)g - \frac{k}{m}v_z + \frac{q}{m}E$$

By convention, $\gamma = \frac{k}{m}$ and $\beta = \left(\frac{m_{air}}{m} - 1\right)$. Solving these separable differential equations for the velocities then gives

$$v_x = v_{x0}e^{-\gamma t} \qquad (10)$$

$$v_z = \left[v_{z0} + \frac{\beta g}{\gamma} - \frac{qE}{\gamma m}\right]e^{-\gamma t} - \frac{\beta g}{\gamma} + \frac{qE}{\gamma m} \qquad (11)$$

Since $\vec{v} = \frac{d\vec{r}}{dt}$, we have again two separable ODEs which upon solving gives

$$x = x_0 + \frac{v_{x0}}{\gamma}(1 - e^{-\gamma t}) \qquad (12)$$

$$z = z_0 + \frac{1}{\gamma}\left(v_{x0} + \frac{\beta g}{\gamma} - \frac{qE}{\gamma m}\right)(1 - e^{-\gamma t}) + \left(\frac{qE}{\gamma m} - \frac{\beta g}{\gamma}\right)t \qquad (13)$$

SIMULATION AND RESULTS

We make use of the Easy Java Simulations (EJS) software and develop a simulation of the Millikan oil drop experiment (Esquembre 2015). The programming environment allows us to focus more on the physics involved rather than on a deep knowledge of programming. One powerful tool is EJS's intrinsic ODE solver that allows the programmer to make use of the forces needed to provide the acceleration associated with the motion of the object. In the case of the oil drop, for example, it is only necessary to input equations (7) and (8). The motion of the drop is then solved numerically by the software's ODE solver.

In the initial programming, an array of drops with random radii is created. The radii are then used to calculate the mass, effective viscosity, and drag coefficient of each drop. The motion of the falling drops is calculated by the ODE solver. At this stage the acceleration of the ith drop is

$$a_{z\,i} = -k_i\frac{v_{z\,i}}{m_i} - g\left(1 - \frac{m_{air\,i}}{m_i}\right) + q_i\frac{E}{m_i},$$

with the last term being zero when the voltage is zero. When the voltage is negative, the electric field points down; if the net charge on the oil drop is negative – as it would be if it had excess electrons – the last term of the equation is positive. If it exceeds the total of the first two terms, the drop will rise, otherwise it will continue to fall with the new acceleration.

A snapshot of the simulation is shown in Appendix 3. When the simulation is begun, the viewing screen is populated with several oil drops. The user can select which drop to measure by simply left-clicking it, after which all other drops are grayed-out. The graphic user interface contains four main buttons. The first two allow for recording the velocity of the oil drop during without the electric field and again while being influenced

by the electric field. When the third button is depressed, the charge on the drop is calculated. To perform this, Eq. (6) is employed. The result is displayed in a table on the side of the GUI. The fourth button is the ionization button, which changes the charge on the drop (similar to the x-ray gun lever in the actual experiment). This randomly changes the charge on the drop. The velocities are recorded again followed by the calculation of the new charge. This process is repeated until several charges are listed in the table. It is then left to the user to determine the charge of the electron by finding the least common multiple of all the charges. The GUI contains other buttons, including the 'Spray Oil' button, which starts the simulation, a reset button, and a 'Polarity' button, which reverses the direction of the electric field. The field strength can be adjusted by the 'Voltage' slider. A final 'Save File' button allows for the charge values to be saved in a file which can be read using the MATLAB script of Appendix 4 which when run groups the common charges, finds the average of each group, and displays the separation between these groups. With a large enough sample size, the separation between the groups will be indicative of the elementary charge. The sample run shown in the Appendix 5 figure shows the smallest charge is $q = 1.602$ in units of 10^{-19} coulombs.

CONCLUSIONS

We have developed a simulation of the famous Millikan oil drop experiment. The simulation provides a broader accessibility of the experience and knowledge to undergraduate students as well as high schools. We have explored the laws of motion, point charges, and electric fields, and explained how Millikan used these ideas in his Nobel prize-winning experiment. We have presented the details of the experiment to serve students and teachers alike. We have developed a Java code that will be made accessible in the future (Hasbun et al. 2015) and which runs using the EJS program as well as a MATLAB program (see Appendix 4) with which to analyze the data (see sample output in Appendix 5). The MATLAB file can also be run on the free redistributable software GNU Octave, Eaton (2016).

ACKNOWLEDGEMENTS

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http://www.westga.edu/~jhasbun/osp/osp.htm (item 27) or directly here:
http://www.westga.edu/~jhasbun/osp/Millikan_Oil_Drop_Experiment.htm

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Appendix 1: Viscosity Graph

Viscosity of Dry Air as a Function of Temperature

$$y = 0.004774x + 1.7284$$

Nsm^{-2} x 10^{-5}

°C
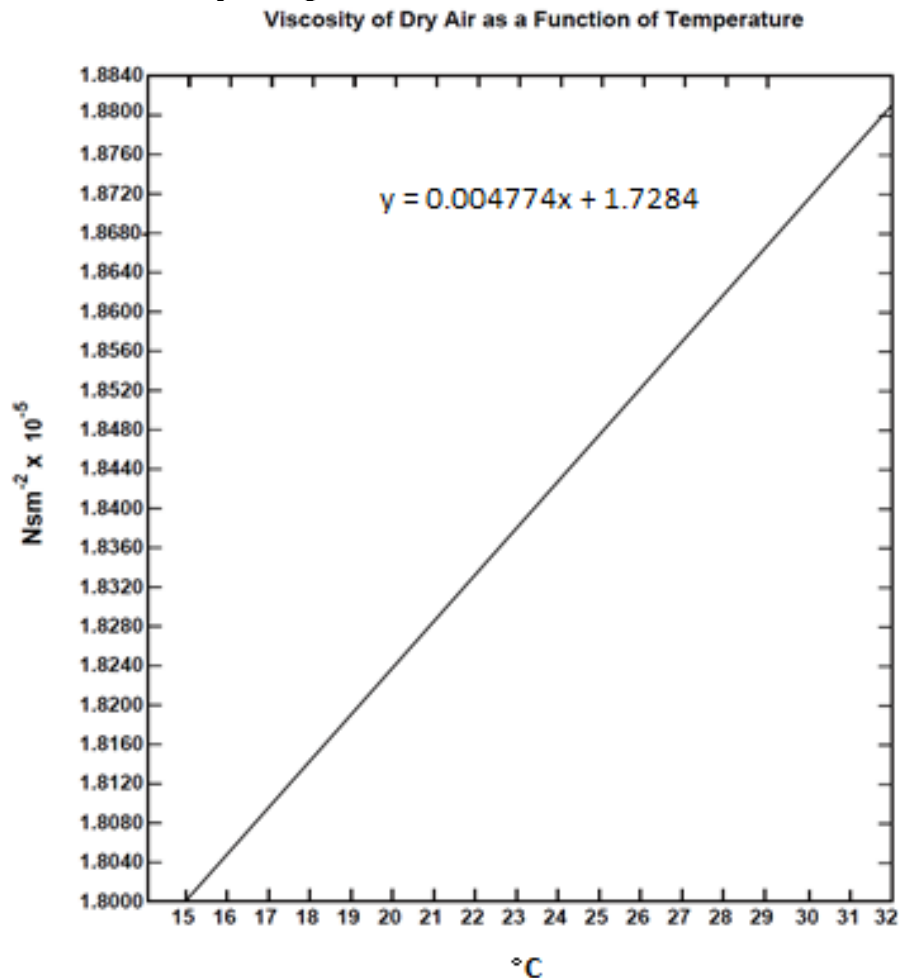
Substituting the ambient temperature in Celcius for x in the viscosity equation above will give the viscosity of the air in the chamber in units of Nsm^{-2} x 10^{-5}.

Appendix 2: Constant Values Used in the Simulation[2]

$d = 7.67 \times 10^{-3}\ m$

$\rho = 886\ kg \cdot m^{-3}$

$\rho_{air} = 1.204\ kg \cdot m^{-3}$

$g = 9.807\ m \cdot s^{-2}$

$\eta = 1.842 \times 10^{-5}\ N \cdot s \cdot m^{-2}$

$b = 8.20 \times 10^{-3}\ Pa \cdot m$

$p = 1.01 \times 10^5\ Pa$

Appendix 3: Snapshot of the simulation after calculating a few charges.
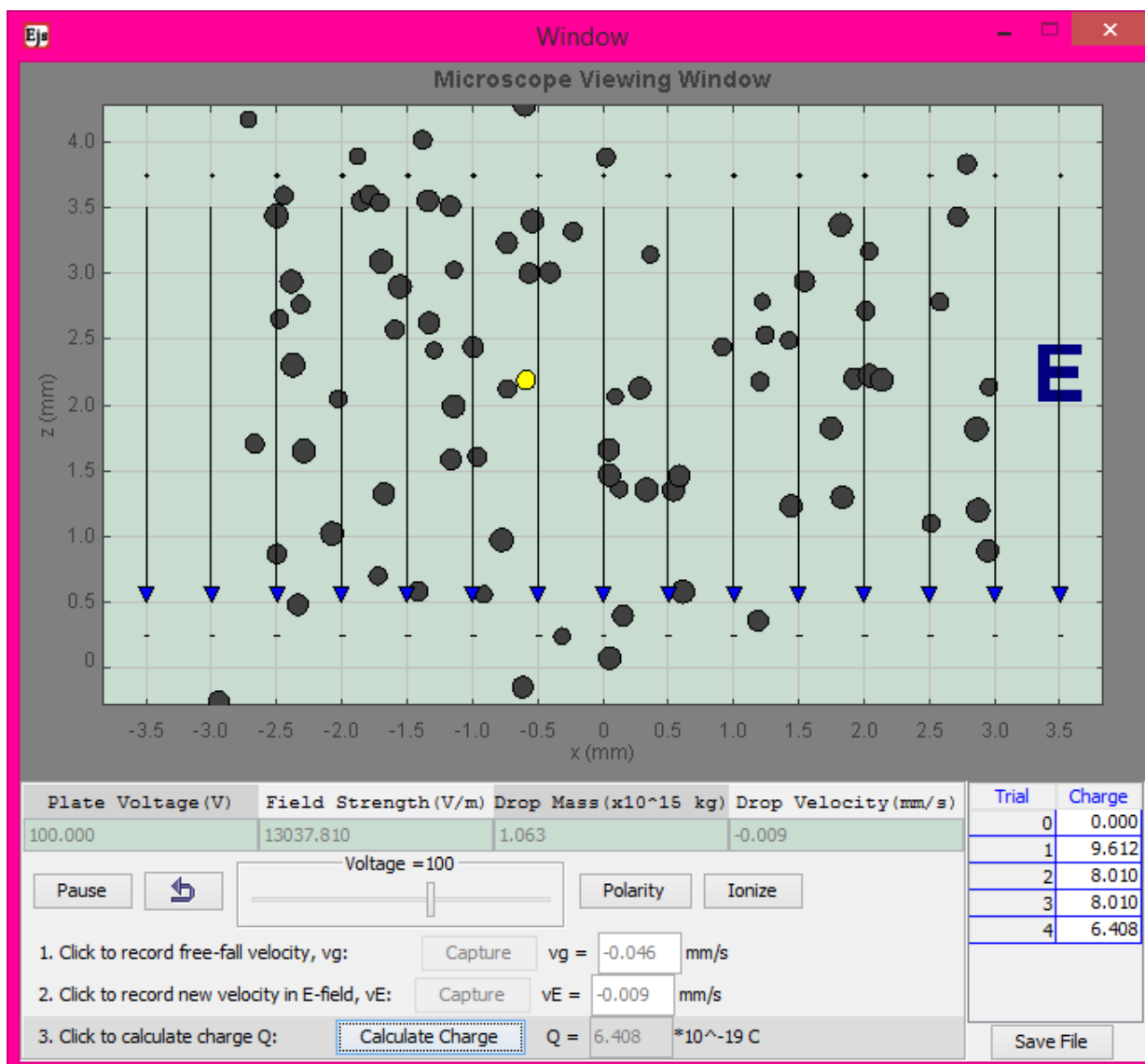
Appendix 4: MATLAB program
```
% Millikan_Oil_Drop_Experiment_Analysis.m
% A matlab file to analyze data from the EJS based
% Millikan_Oil_Drop_Experiment.xml program file.
% Written by Ben Hogan & Javier Hasbun, PhD (12/2015)

% This program takes in charge values obtained from the Oil-drop EJS
% program (ATTENTION: The charge file created from EJS must be in the same
% path as this .m file). The values are put into a list, categorized into
% charge groups (charges of values that fall within a certain variance from
% each other), and then plotted. The individual charge groups make possible
% finding the mean charge value for that group, which is also plotted on
% the same graph as a line. Finally, the program calculates the distance
% between all neighboring mean charge lines, giving the user a clear visual
% that the charges are multiples of one value - the charge of the electron.

% Clear any preexisting variables/conditions
clear all
clc
clear

% Y-Axis
% Get numerical data from charge file by prompting user to select the data
% file
datafile = uigetfile('*.txt','Select the file to run');
FullData = importdata(datafile);
q=abs(FullData.data);
% Set number of entries to I for future parameterization
I = length(q);

% X-Axis
i = 1:1:I;   % prepare i-matrix for I measurements

% Plot the charges
scatter(i,sort(q),'fill')

% Length of x axis based on sample number
xl = I + 3;

% Group common charges and find the mean and var of each group
Commons =
ordinal(q,{'1','2','3','4','5','6','7','8','9','10','11','12','13','14','15','16','17','18','19','20','21','22','2
3','24','25','26','27','28','29','30'},...
        [],[0.5, 2, 3.5, 5, 6.5, 8, 9.5, 11, 12.5, 14, 15.5, 17, 18.5, 20, 21.5, 23, 24.5, 26,
27.5, 29, 30.5, 32, 33.5, 35, 36.5, 38, 39.5, 41, 42.5, 44, 45.5]);
[xbar,s2,grp] = grpstats(q,Commons,{'mean','var','gname'});
```

```
% Remove from xbar any mean value that is not non-zero/finite
for i=1:length(xbar)
   if isnan(xbar(i)) == 1
      xbar(i) = 0;
   end
end
 xbar(xbar==0) = [];

% Plot mean charge of each group as reference lines
for i=1:length(xbar)
   line([0,xl-xl/10],[xbar(i),xbar(i)])
   text(xl-xl/11,xbar(i),num2str(xbar(i)))
end

% Calculate and show separation between neighboring charge groups
chavg(1) = 0;
chdif(1) = xbar(1);
line([0.5*xl,0.5*xl],[0,xbar(1)],'LineWidth',2)
text(0.5*xl+0.25,xbar(1)/2,num2str(chdif(1)))
for i=2:length(xbar)
   j = i-1;
   k = 0.5*xl+i/length(xbar);
   line([k,k],[xbar(j),xbar(i)],'LineWidth',2)
   chavg(i) = (xbar(i)+xbar(j))/2;
   chdif(i) = xbar(i)-xbar(j);
   text(k+0.25,(chavg(i)),num2str(chdif(i)))
end

% Template
grid on
axis([0 xl 0 max(q)+2])
title('Measured Charge Values of Oil-drop','FontSize',15)
xlabel('measurement','FontSize',16)
ylabel('charge q','FontSize',16)
a = line([0,I+1],[xbar(1),xbar(1)]);
legend(a,'Average Charge Group Value','Location','NorthWest')
```

Appendix 5: Snapshot of the MATLAB program output.