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Michael Knoblach

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Skill-Biased Technological Change, Endogenous Labor Supply, and the Skill Premium*

Michael Knoblach†
Technische Universität Dresden

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Abstract The evolution of the U.S. skill premium over the past century has been characterized by a U-shaped pattern. The previous literature has attributed this observation mainly to the existence of exogenous, unexpected technological shocks or changes in institutional factors. In contrast, this paper demonstrates that a U-shaped evolution of the skill premium can also be obtained using a simple two-sector growth model that comprises both variants of skill-biased technological change (SBTC): technological change (TC) that is favorable to high-skilled labor and capital-skill complementarity (CSC). Within this framework, we derive the conditions necessary to achieve a non-monotonic evolution of relative wages and analyze the dynamics of such a case. We show that in the short run for various parameter constellations an educational, a relative substitutability, and a factor intensity effect can induce a decrease in the skill premium despite moderate growth in the relative productivity of high-skilled labor. In the long run, as the difference in labor productivity increases, the skill premium also rises. To underpin our theoretical results, we conduct a comprehensive simulation study.

JEL classification: E24; J24; J31; O33; O41

Keywords: Skill-Augmenting Technological Change; Capital-Skill Complementarity; Skill Premium; Neoclassical Growth Model

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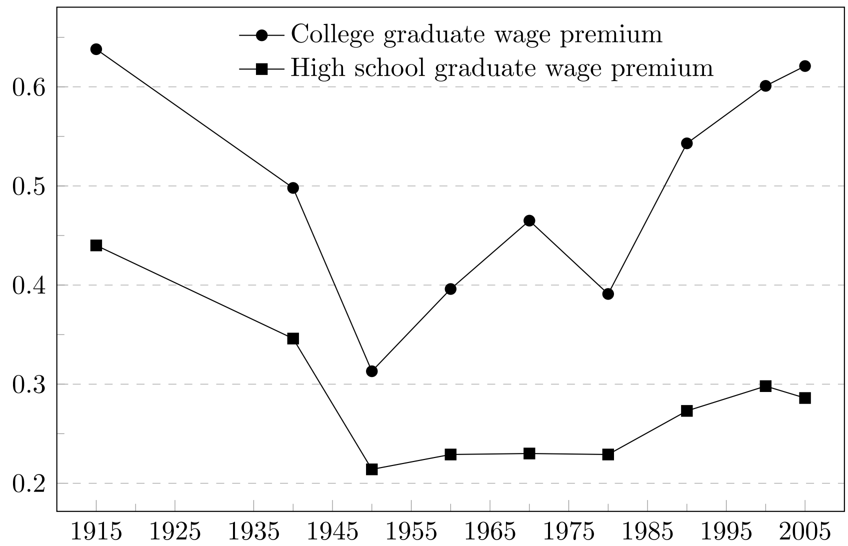
†Faculty of Business and Economics, Technische Universität Dresden, 01062 Dresden, Germany. E-mail: michael.knoblach@tu-dresden.de

1 Introduction

The pattern of movements in the U.S. skill premium, defined as the wage of high-skilled relative to low-skilled labor, varied substantially during the twentieth century. Figure 1 plots the development of the wage premium for college and high school graduates in the U.S. from 1915 through 2005.[1] It illustrates that the college and high school premium collapsed from 1915 to 1950 and fluctuated during the years 1950 to 1980. Thereafter, the wage differential between college and high school graduates expanded considerably starting in the 1980s and continued to increase, although at a slower rate, in the 1990s and early 2000s. By 2005, the college premium had almost come full circle to its 1915 level. Although less pronounced, a similar picture can be obtained for the development of the high school premium. While relatively constant from 1950 to 1980, the wage differential began to increase during the 1980s and 1990s. Consequently, the time series of the U.S. college and high school premium have been U-shaped over the past century. In addition to the U.S., a non-monotonic development of relative wages between high- and low-skilled labor has been documented for several developed countries, most prominently Sweden (Lindquist, 2005, Domeij and Ljungqvist, 2019), Japan (Lise et al., 2014), and Germany (Glitz and Wissmann, 2017).[2]

Figure 1: Evolution of the U.S. college and high school graduate wage premium



The time series of the U.S. skill premium are obtained from Goldin and Katz (2008). The plotted series are based on the log (college/high school) and (high school/eighth grade) wage differential series from 1915 to 2005.

To explain changes in the structure of relative wages, numerous potential approaches have been proposed, including low-skilled immigration, labor market institutions

[1]In addition, see Autor (2014) for the development of the college premium based on yearly data from 1963 to 2012.

[2]See Peracchi (2006) for an extensive survey of the empirical literature.

and the international fragmentation of production.[3] However, most of the recent empirical literature, e.g., McAdam and Willman (2018), suggests that skill-biased technological change (SBTC) has been the most important driver of the wage structure development.[4] In essence, SBTC can be interpreted as a shift in the production technology that raises the relative productivity of high-skilled labor and thus its relative wage rate (Violante, 2008). The literature distinguishes two distinct approaches of SBTC: technological change (TC), which is favorable to high-skilled labor, and capital-skill complementarity (CSC).[5] The first approach was popularized by Bound and Johnson (1992) and Katz and Murphy (1992). Under this explanation, an increase in the skill premium is induced by an increase in technological change that augments high-skilled more than low-skilled labor. In addition, this requires that the elasticity of substitution between the two types of labor must exceed unity. The second approach is capital-skill complementarity. CSC is present if an increased use of capital in production raises the wage of high-skilled labor more than that of low-skilled labor. Consequently, the skill premium increases. As shown by Griliches (1969), under this explanation, capital and high-skilled labor must be more complementary than capital and low-skilled labor.

However, in most preceding studies, both approaches of SBTC have generally been applied to explain the rapid increase in the U.S. college premium over the past four decades. In comparison, with respect to the U-shaped development of the skill premium, only a few studies have applied the framework of SBTC. Moreover, in most of these papers (e.g. Galor and Tsiddon, 1997, Caselli, 1999, Lloyd-Ellis, 1999, Jacobs and Nahuis, 2002, Guvenen and Kuruscu, 2012, Kishi, 2015), the non-monotonic behavior of the skill premium has mainly been explained through the adjustment process to exogenous, unexpected technological shocks. For instance, Caselli (1999) assumes the existence of learning costs for new technologies. He argues that the cyclical behavior of the skill premium can be explained by the interplay of skill-biased and de-skilling technological revolutions. Guvenen and Kuruscu (2012) present an overlapping-generations model in which individuals differ in their ability to accumulate human capital. The authors show that a U-shaped pattern of the skill premium emerges in response to a one-time permanent increase in the productivity of human capital. In addition to technological change, another strand of the literature explains the development of the skill premium through changes in institutional factors. For example, Galor and Moav (2000) present an overlapping-generations model with ability-biased technological progress where education requires a costly investment financed through a loan. The authors show that within this framework, a sufficient decline in the degree of credit market imperfections can explain the cyclical pattern of wage inequality over recent decades.

In contrast to this literature, the main aim of this paper is to demonstrate that a U-shaped development of the skill premium is attainable under both approaches

[3]For a detailed discussion of different explanations, see Bound and Johnson (1992), Katz and Autor (1999), Acemoglu (2003), Hornstein et al. (2005), Autor et al. (2008), Goldin and Katz (2008), Acemoglu and Autor (2011) and van Reenen (2011), among others.

[4]See Card and DiNardo (2002) for a critical review of the literature linking SBTC to wage inequality in the U.S. economy.

[5]See the appendix for a detailed description of both approaches.

of SBTC and can emerge in the absence of any exogenous shocks or changes in institutional factors. By extending a simplified approach developed by Steger (2008), we demonstrate that the non-monotony in in the development of relative wages can be derived as an equilibrium outcome driven by factor-augmenting technological change and the accumulation of capital. For this purpose, this paper develops a two-sector Solow growth model in which a final good is produced through a four-factor, two-level CES production function that combines two types of capital with low-skilled and high-skilled labor. To allow for SBTC, production is characterized by factor-augmenting technological progress and differences in the sectoral possibility to substitute capital and labor. The labor force consists of heterogeneous workers that differ with respect to their inherent abilities. Each worker can decide whether to spend a fraction of time on education and work as high-skilled labor or to enter the labor market directly as a low-skilled worker without education. In each period, the composition of labor is endogenously determined by the decisions of individual workers and solely depends on the skill premium and the distribution of inherent abilities. In this economic environment, we derive the conditions necessary to achieve a U-shaped pattern in the behavior of relative wages. Furthermore, by conducting comparative statics of changes in central model parameters, we test for the sensitivity of this result. We demonstrate that a U-shaped pattern of the skill premium can be achieved due to the interplay of three distinct effects: an educational, a relative substitutability, and a factor intensity effect. While the first effect relies on the opportunity cost of education, the second effect is driven by sectoral differences in the elasticity of substitution between capital and labor. Both effects increase the requirement of relative high-skilled augmenting technological change necessary to induce an increase in the skill premium. Consequently, for a moderate rise in relative labor productivity, the wage ratio will decrease in the short run. In the long run, as the difference in labor productivity increases, the skill premium also rises. The factor intensity effect is driven by sectoral differences in the relative usage of capital and labor in production. Depending on the set of parameter combinations, the effect can either strengthen or weaken the U-shaped pattern of the skill premium. Especially for a strong sectoral reallocation of capital, the skill premium might increase steadily over time. Although the model appears relatively simple, it cannot generally be solved analytically. Therefore, we conduct a comprehensive simulation study to present the transitional dynamics of the model. We show that with empirically plausible parameter choices, our neoclassical growth model is able to generate a U-shaped development of the skill premium under both approaches of SBTC.

Furthermore, the mechanisms derived in this paper are also related to those discussed in the literature on structural transformation. Notable contributions that examine technological explanations for the observed reallocation of resources across the broad sectors of agriculture, manufacturing and services include Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), and Alvarez-Cuadrado et al. (2017).[6] Specifically, Alvarez-Cuadrado et al. (2017) present a model where the process of

[6]For a review of the main forces driving structural change, see van Neuss (2019). Alvarez-Cuadrado et al. (2018) jointly examine the influence of technological aspects (sectoral differences in capital-labor substitutability, productivity, and capital intensity) together with non-homothetic preferences on changes in the labor share and structural transformation.

structural change is induced by cross-sectoral differences in the elasticity of substitution between capital and labor.[7] However, as the authors assume homogeneous labor, the wage rate is always equalized across sectors, and the model is therefore unable to study the development of the skill premium. To some extent, our model can be seen as an extension of Alvarez-Cuadrado et al. (2017), as we study the development of the skill premium under the assumption of heterogeneous labor.

The remainder of the paper is organized as follows. In the next section, our model is introduced. After a description of the static equilibrium in section 3.1, we analyze the dynamics of the model in section 3.2. Section 4 presents some numerical applications. We close with some concluding remarks in section 5.

2 A two-sector model of endogenous labor supply

In this section, we present our model, which introduces elements from the literature on endogenous labor supply (Cahuc and Michel, 1996, Cervellati and Sunde, 2005, Meckl, 2006) into a two-sector growth model in the spirit of Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), Steger (2008), and Alvarez-Cuadrado et al. (2017). We consider a closed economy operating under perfect competition in an infinite, continuous-time horizon. To simplify the analysis, we abstract from a household sector and solely focus on the production structure of the economy. Within the latter, in each period of time, a homogeneous final good is produced as a combination of two intermediates. One intermediate uses capital and low-skilled labor, while the other uses capital and high-skilled labor.

2.1 Labor supply

The economy is populated by a continuum of different workers who live for one period. Each period, a new generation is born such that the population is constant over time. To simplify the analysis, we normalize the mass of workers to unity. Workers are endowed with one unit of time and are heterogeneous with respect to their inherent abilities $a \in [0, 1]$. Abilities are distributed uniformly over the continuum of workers. In each period t, the composition of labor is endogenously determined by the decisions of individual workers.

Ability a characterizes each worker by the effort that is required of him or her to become a high-skilled worker. Thus, following Cahuc and Michel (1996), education is not assumed to raise individual human capital but to enable a worker to be engaged in the high-skilled sector. The costs of education depend negatively on the inherent abilities. A worker with ability a can choose to spend an exogenously given fraction of time $\eta = 1 - a$ on education. This enables him or her to inelastically supply $1 - \eta$ units of high-skilled labor once the education process is completed. The educational

[7] In a related approach, Wingender (2015) argues that structural transformation in developing countries is mainly driven by differences in the sectoral elasticity of substitution between high-skilled and low-skilled workers.

costs are completely borne by workers. Therefore, the wage income of the individual high-skilled worker is given by $(1-\eta)w_H$. Alternatively, a worker can decide against education and enter the labor market directly as a low-skilled worker. In that case, he or she inelastically supplies one unit of low-skilled labor and earns the wage rate w_U, irrespective of ability. Based on these assumptions, the individual worker chooses to become high-skilled if and only if his or her ability a is not smaller than a threshold value $\tilde{a}$ determined by (suppressing time subscripts for legibility)

$$\omega = (1-\tilde{\eta})^{-1} = (\tilde{a})^{-1}, \tag{1}$$

where $\omega \equiv \frac{w_H}{w_U}$ denotes the skill premium in the economy. As will be shown in the following, in each period t, there exists a unique, interior, threshold level of ability $0 < \tilde{a} < 1$.[8] Workers with an ability $a \geq \tilde{a}$ decide in favor of education to become high-skilled, while those with $a < \tilde{a}$ decide to remain low-skilled. The threshold level $\tilde{a}$ is inversely related to the relative wage rate ω. Any increase in ω induces labor employment shifts from the low-skilled to the high-skilled intermediate sector, as it increases the incentive to become educated. Finally, the aggregate supplies of low- and high-skilled labor, L_U and L_S, respectively, measured in effective working time and the aggregate time of high-skilled labor spent on education, E, are

$$\begin{aligned} L_U &= \int_0^{\tilde{a}} f(a)\mathrm{d}a = \tilde{a} \\ L_S &= \int_{\tilde{a}}^1 af(a)\,\mathrm{d}a = 0.5(1-\tilde{a}^2) \\ E &= \int_{\tilde{a}}^1 (1-a)f(a)\,\mathrm{d}a = 0.5(1+\tilde{a}^2) - \tilde{a} \end{aligned} \tag{2}$$

where $f(a)$ is the density function of low-skilled workers given the uniform distribution of abilities.

2.2 Technology

In each period t, a two-level, four-factor CES production system is applied, where a unique final good, Y, is produced by a representative firm as a combination of capital, K, high-skilled labor, L_S, and low-skilled labor, L_U. In the following analysis, we adopt the normalization procedure developed in de La Grandville (1989) and Klump and de La Grandville (2000). The basic idea behind normalization is to express production functions in terms of index numbers to ensure deep or dimensionless parameters (Cantore and Levine, 2012).[9] During our analysis, normalization is

[8] Note that as a consequence, aggregate labor supply measured in effective working time is necessarily smaller than unity. Furthermore, this ensures that the wage rate of each effective unit of high-skilled labor strictly exceeds that of low-skilled labor.

[9] For a comprehensive survey of the concept of normalization, see Klump et al. (2012).

essential, as it guarantees a meaningful and consistent comparison of model results relying on different values of production parameters. Hence, in the following, the subscript 0 indicates an arbitrarily chosen benchmark value used for normalization at the point $t = 0$. In the upper level of the production system, normalized final output, $\widetilde{Y} = Y/Y_0$, is produced based on the following CES technology

$$\widetilde{Y} = \left[\alpha_0 \widetilde{Y}_S^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0)\widetilde{Y}_U^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where σ refers to the elasticity of substitution between a normalized high-skilled and low-skilled intermediate good, $\widetilde{Y}_i = Y_i/Y_{i,0}$, $i \in \{S, U\}$. The production of intermediates is defined by two lower level CES production functions

$$\widetilde{Y}_S = \left[\beta_0 (A_{KS}\widetilde{K}_S)^{\frac{\sigma_S-1}{\sigma_S}} + (1-\beta_0)(A_S \widetilde{L}_S)^{\frac{\sigma_S-1}{\sigma_S}}\right]^{\frac{\sigma_S}{\sigma_S-1}} \tag{4}$$

and

$$\widetilde{Y}_U = \left[\gamma_0 (A_{KU}\widetilde{K}_U)^{\frac{\sigma_U-1}{\sigma_U}} + (1-\gamma_0)(A_U \widetilde{L}_U)^{\frac{\sigma_U-1}{\sigma_U}}\right]^{\frac{\sigma_U}{\sigma_U-1}} \tag{5}$$

where $\widetilde{K}_i = K_i/K_{i,0}$ and $\widetilde{L}_i = L_i/L_{i,0}$ depict normalized sectoral inputs of capital and labor, respectively. Production functions (3) to (5) are linearly homogeneous. At the point of normalization, $t = 0$, the distribution parameter $\alpha_0 \in (0, 1)$ represents the output share of the high-skilled intermediate in the final good production. Analogously, β_0 (γ_0) $\in (0, 1)$ depicts the factor share of capital in the production of the high-skilled (low-skilled) intermediate. Capital market clearing requires the aggregate capital stock, K, at each date to be fully allocated across intermediate sectors:

$$K = K_S + K_U \tag{6}$$

In the following, we assume capital to be fully mobile across intermediate sectors. Thus, the equalization of the interest rates, $r_S = r_U = r$, allows us to endogenously determine the allocation of capital between sectors. In contrast, labor differs with respect to its degree of education. Based on educational attainment, high- or low-skilled labor will only be employed in its respective intermediate sector. Within these sectors, the elasticity of substitution between capital and the skill-specific labor input is defined by σ_i, $i \in \{S, U\}$.[10] Based on the results of Krusell et al. (2000) and McAdam and Willman (2018), among others, we make the following assumption with respect to these elasticities:

[10]Note that for production functions (3) to (5), the Cobb-Douglas case occurs when $\sigma(\sigma_S)\{\sigma_U\} \to 1$, the Leontief case with perfect complements occurs when $\sigma(\sigma_S)\{\sigma_U\} \to 0$, and the von Neumann case with perfect substitutes occurs when $\sigma(\sigma_S)\{\sigma_U\} \to \infty$.

Assumption 1. Within the production of intermediates, capital and high-skilled labor are more complementary than are capital and low-skilled labor, that is,

$$\sigma_S < \sigma_U.$$

Furthermore, factor-specific technological progress for each input, K_S, K_U, L_S, and L_U, is assumed to be an exogenous result of research and development. It is defined according to the following exponential functions of time

$$A_{j,t} = e^{\lambda_j (t-t_0)} \tag{7}$$

where the coefficients A_{KS}, A_{KU}, A_S and A_U indicate the efficiency in production of the two types of capital, high-skilled and low-skilled labor, respectively.[11]

Assumption 2. Technological change augments high-skilled labor more than low-skilled labor, that is,

$$\lambda_S > \lambda_U.$$

Based on Assumption 2, we follow the majority of the empirical literature, as summarized in Hornstein et al. (2005) and Acemoglu and Autor (2011), and assume that technological change is unbalanced and favorable to high-skilled labor. Inserting equations (4) and (5) into (3), the two-level CES production function becomes

$$\widetilde{Y} = \begin{bmatrix} \alpha_0 \quad \left[\beta_0 (A_{KS}\widetilde{K}_S)^{\frac{\sigma_S-1}{\sigma_S}} + (1-\beta_0)(A_S\widetilde{L}_S)^{\frac{\sigma_S-1}{\sigma_S}}\right]^{\frac{\sigma_S}{\sigma_S-1}\frac{\sigma-1}{\sigma}} + \\ (1-\alpha_0) \quad \left[\gamma_0 (A_{KU}\widetilde{K}_U)^{\frac{\sigma_U-1}{\sigma_U}} + (1-\gamma_0)(A_U\widetilde{L}_U)^{\frac{\sigma_U-1}{\sigma_U}}\right]^{\frac{\sigma_U}{\sigma_U-1}\frac{\sigma-1}{\sigma}} \end{bmatrix}^{\frac{\sigma}{\sigma-1}}, \tag{8}$$

which imposes the following symmetry restrictions on the substitution elasticities.[12] In the case of (8), the elasticity of substitution between the high-skilled and low-skilled intermediate, σ, is restricted to be the same as between the pairs K_S and K_U, K_S and L_U, and L_S and L_U, together with L_S and K_U. Most of the empirical literature reports estimates of these elasticities to be greater than unity.[13] Therefore, in the following, we assume $\sigma, \sigma_S, \sigma_U \geq 1$. Moreover, to ensure strictly positive values for all inputs, we consider finite values for all substitution elasticities. Associated with production function (8), the skill premium

$$\omega = \frac{\alpha_0}{1-\alpha_0}\left(\frac{\widetilde{Y}_S}{\widetilde{Y}_U}\right)^{-\frac{1}{\sigma}} \frac{1-\beta_0}{1-\gamma_0} \frac{(\widetilde{Y}_S/L_S)^{\frac{1}{\sigma_S}}}{(\widetilde{Y}_U/L_U)^{\frac{1}{\sigma_U}}} \frac{(A_S/L_{S,0})^{\frac{\sigma_S-1}{\sigma_S}}}{(A_U/L_{U,0})^{\frac{\sigma_U-1}{\sigma_U}}} \tag{9}$$

[11] Thus, at the point of normalization, $t = 0$, we have $A_{j,0} = 1$.

[12] See Sato (1967) for the theoretical foundation of two-level CES production functions. A complete derivation of the normalized production system can be found in the appendix.

[13] See chapter 4 for a summary.

is given by the marginal value product of high-skilled labor relative to that of low-skilled labor.[14] Finally, along the lines of Solow (1956), we assume a constant rate of capital depreciation, $\delta \in (0,1)$, and a constant exogenous fraction of final output, $s \in (0,1)$, to be saved and invested every period.[15] The law of motion for the capital stock takes the form

$$\dot{K}_t = sY_t - \delta K_t \tag{10}$$

where a dot denotes the time derivative of a variable. The aggregate resource constraint of the economy requires consumption, C, and gross investment, I, to be equal to the output of the final good:

$$\dot{K}_t + \delta K_t + C_t = I_t + C_t = Y_t \tag{11}$$

After presenting the full model, in the following, we turn to the characterization of the static and dynamic equilibrium of the economy.

3 The static equilibrium and model dynamics

We normalize the price of the final good, p, to unity at all points in time. This leads to

$$p \equiv 1 = \left[\alpha_0^{\sigma} p_S^{1-\sigma} + (1-\alpha_0)^{\sigma} p_U^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

where the prices of the high-skilled and low-skilled intermediates, p_S and p_U, respectively, are obtained as

$$p_S = \alpha_0 \left[\frac{\widetilde{Y}}{\widetilde{Y}_S}\right]^{\frac{1}{\sigma}} \qquad \text{and} \qquad p_U = (1-\alpha_0)\left[\frac{\widetilde{Y}}{\widetilde{Y}_U}\right]^{\frac{1}{\sigma}}.$$

We can now turn to the definition of the competitive equilibrium of the economy.

Definition 1. The competitive equilibrium of the economy, given factor supply and technology sequences $\{K_t, L_t, A_{S,t}, A_{U,t}, A_{KS,t}, A_{KU,t}\}_{t=0}^{\infty}$, is defined as paths for factor and intermediate good prices $\{r_t, w_{S,t}, w_{U,t}, p_{S,t}, p_{U,t}\}_{t=0}^{\infty}$, factor allocations $\{K_{S,t}, K_{U,t}, L_{S,t}, L_{U,t}\}_{t=0}^{\infty}$ and output levels $\{Y_{S,t}, Y_{U,t}, Y_t\}_{t=0}^{\infty}$ such that the representative firm maximizes profits and all markets clear.

The competitive equilibrium can be solved by maximizing the profit of the representative firm, that is,

[14] A complete list of first-order conditions corresponding to (8) is presented in the appendix.

[15] Thus, following Barro (1974), we implicitly assume the existence of dynastic families with intergenerational transfers that behave as though they are single, infinitely lived individuals.

$$(12)\qquad \max_{\{L_{S,t},L_{U,t},K_{S,t},K_{U,t}\}_{t=0}^{\infty}} \int_{t=0}^{\infty} \left(Y_t - r_t(K_{S,t}+K_{U,t}) - w_{S,t}L_{S,t} - w_{U,t}L_{U,t}\right) dt$$

subject to (2), (6), (7), and the resource constraint (10) in addition to the initial conditions $K_0 > 0$, $L_0 > 0$, $A_{KS,0} > 0$, $A_{KU,0} > 0$, $A_{S,0} > 0$, and $A_{U,0} > 0$. Since there are no intertemporal elements in the firm's maximization problem, the model can be solved sequentially by maximizing the static profits of the representative firm at all points in time. Thanks to this property, we follow Acemoglu and Guerrieri (2008) and divide the solution of the maximization problem into two parts: *static* and *dynamic*. In the static part of the equilibrium characterization, presented in section 3.1, we take the state variables, $\{K, L, A_{KS}, A_{KU}, A_S, A_U\}$, at any point in time as given. We first determine the composition of labor into high-skilled and low-skilled as well as the allocation of capital between sectors. This is followed by a discussion of the comparative statics of the equilibrium allocation. In section 3.2, we characterize the dynamic behavior of the economy, which is given as a sequence of static problems as formalized above.

3.1 The static equilibrium

For the economy to be in equilibrium, all markets must clear simultaneously. As can immediately be seen from (12), under perfect competition, the maximization problem of the representative firm comprises two equilibrium conditions with respect to factor inputs.[16] First, at any point in time, free mobility of capital implies the equalization of the marginal value product of capital across intermediate sectors, that is, $\partial Y/\partial K_S = \partial Y/\partial K_U$. This results in the following capital mobility condition (CMC):

$$(\text{CMC})\qquad \frac{\alpha_0}{(1-\alpha_0)}\left(\frac{\widetilde{Y}_S}{\widetilde{Y}_U}\right)^{-\frac{1}{\sigma}} \frac{\beta_0}{\gamma_0}\frac{(\widetilde{Y}_S/K_S)^{\frac{1}{\sigma_S}}}{(\widetilde{Y}_U/K_U)^{\frac{1}{\sigma_U}}}\frac{(A_{KS}/K_{S,0})^{\frac{\sigma_S-1}{\sigma_S}}}{(A_{KU}/K_{U,0})^{\frac{\sigma_U-1}{\sigma_U}}} = 1$$

Second, at any point in time and for any prices, the allocation of labor between the high-skilled and low-skilled intermediate sector is captured by the following labor mobility condition (LMC). The LMC equalizes the wage income of the marginal worker, who is indifferent between becoming high-skilled or remaining low-skilled, i.e., $(1-\eta)\partial Y/\partial L_S = \partial Y/\partial L_U$:

$$(\text{LMC})\qquad (1-\eta)\frac{\alpha_0}{1-\alpha_0}\left(\frac{\widetilde{Y}_S}{\widetilde{Y}_U}\right)^{-\frac{1}{\sigma}} \frac{1-\beta_0}{1-\gamma_0}\frac{(\widetilde{Y}_S/L_S)^{\frac{1}{\sigma_S}}}{(\widetilde{Y}_U/L_U)^{\frac{1}{\sigma_U}}}\frac{(A_S/L_{S,0})^{\frac{\sigma_S-1}{\sigma_S}}}{(A_U/L_{U,0})^{\frac{\sigma_U-1}{\sigma_U}}} = 1$$

[16]Due to the concavity of production system (8) with respect to factor inputs, the associated first-order conditions are both necessary and sufficient for the characterization of the competitive equilibrium.

Both conditions, the CMC and the LMC, ensure the optimal allocation of capital and labor between the two intermediate sectors. They hold at any point in time and for any given set of state variables. We can now turn to a discussion of the comparative statics of the static equilibrium. Within our model, shifts in the skill premium can in general be decomposed into shifts induced by factor-augmenting technological change and capital accumulation. In the following, we investigate the influence of these sources on the determination of ω by considering both approaches of SBTC. For that reaseon, we distinguish two different cases with respect to the ordering of substitution elasticities: i) $\sigma > \sigma_U > \sigma_S$ and ii) $\sigma_U > \sigma > \sigma_S$. As we will see below, in the first special case, $\sigma > \sigma_U > \sigma_S$, factor-augmenting technological change is also factor-biased because it increases the relative marginal value product of the respective factor. As a result, under this constellation of the substitution elasticities, an increase in the skill premium can be explained by technological change that augments high-skilled labor more than low-skilled labor. The second case, $\sigma_U > \sigma > \sigma_S$, features capital-skill complementarity. That is, an increase in the aggregate stock of capital, K, unambiguously increases the relative marginal value product of high-skilled labor and thus the skill premium.

Special case 1: factor-biased technological change Consider in the following the ordering $\sigma > \sigma_U > \sigma_S$. Assume first that the aggregate capital stock, K, is constant, and focus solely on technological progress. Proposition 1 summarizes the quantitative impact of labor- and capital-augmenting technological change on the general equilibrium allocation of workers to the high-skilled intermediate sector, L_S, and thus the skill premium of the economy.[17]

PROPOSITION 1. Consider $\sigma > \sigma_U > \sigma_S$. Under this constellation of the substitution elasticities, in the competitive equilibrium,

$$\frac{\partial L_S}{\partial A_S} = -\frac{\partial \Sigma(\cdot)/\partial A_S}{\partial \Sigma(\cdot)/\partial L_S} > 0, \tag{13}$$

$$\frac{\partial L_S}{\partial A_U} = -\frac{\partial \Sigma(\cdot)/\partial A_U}{\partial \Sigma(\cdot)/\partial L_S} < 0, \tag{14}$$

$$\frac{\partial L_S}{\partial A_{KS}} = -\frac{\partial \Sigma(\cdot)/\partial A_{KS}}{\partial \Sigma(\cdot)/\partial L_S} > 0, \tag{15}$$

and

$$\frac{\partial L_S}{\partial A_{KU}} = -\frac{\partial \Sigma(\cdot)/\partial A_{KU}}{\partial \Sigma(\cdot)/\partial L_S} < 0, \tag{16}$$

where the function $\Sigma(\cdot) = \Sigma(A_S, A_U, A_{KS}, A_{KU}, K, L_S)$ and its partial elasticities with respect to A_S, A_U, A_{KS}, A_{KU}, and L_S are defined in the appendix.

[17]Note that, given by (1), $\omega = \frac{1}{a}$, where $a = \sqrt{1 - 2L_S}$. Thus, we have $\frac{\partial \omega}{\partial L_S} = (1 - 2L_S)^{-1.5} > 0$, or, alternatively, $\frac{\partial L_S}{\partial \omega} = \omega^{-3} > 0$.

Proof: See Appendix A.4. ■

Proposition 1 states that in the competitive equilibrium, assuming $\sigma > \sigma_U > \sigma_S$, the level of labor allocated to the high-skilled intermediate sector and thus the skill premium strictly increases with A_S and A_{KS} but decreases with A_U and A_{KU}.[18] The intuition for this result is as follows. Under $\sigma > \sigma_U > \sigma_S$, an improvement in the productivity of high-skilled workers, A_S, increases their relative marginal value product

$$\frac{\partial\left(\frac{\partial Y/\partial L_S}{\partial Y/\partial L_U}\right)}{\partial A_S} > 0$$

and thus raises the demand for high-skilled labor. Consequently, the skill premium increases. This increase, in turn, generates an incentive for additional workers to become educated and thus raises the supply of high-skilled labor. Therefore, following Acemoglu (2002), under $\sigma > \sigma_U > \sigma_S$, high-skilled augmenting technological change is also high-skilled biased. The effects of an increase in A_U, A_{KS} or A_{KU} appear similar. Moreover, the results in Proposition 1 can also be applied to K_S. In particular, for a constant aggregate capital stock, K, and the ordering $\sigma > \sigma_U > \sigma_S$, we have $\partial K_S/\partial A_S > 0$, $\partial K_S/\partial A_U < 0$, $\partial K_S/\partial A_{KS} > 0$, and $\partial K_S/\partial A_{KU} < 0$.

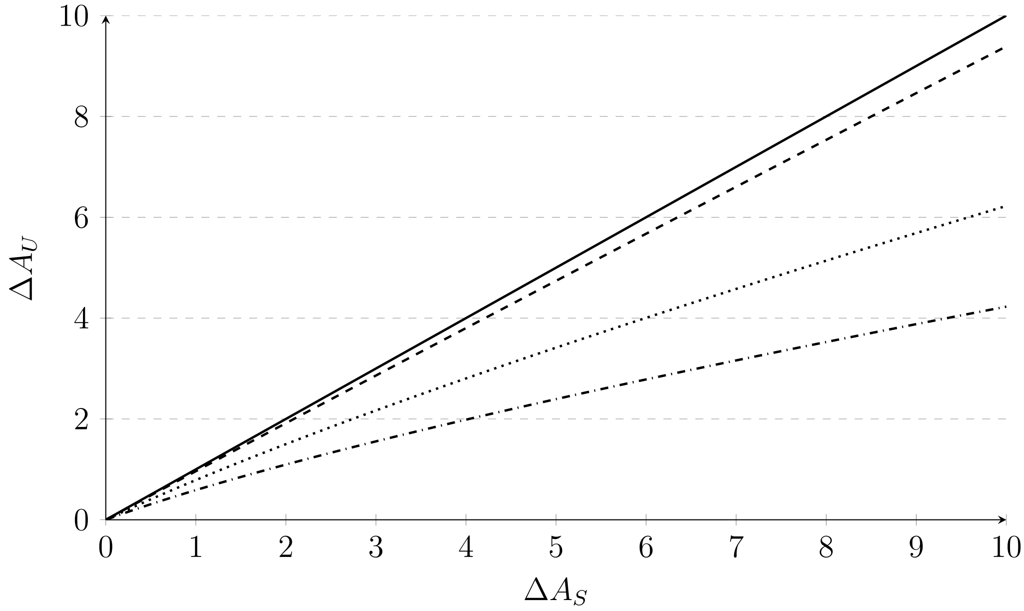
Following the results from Proposition 1, it becomes clear that for any given capital stock, in the long run, due to Assumption 2, ω and thus the level of high-skilled labor always increases as a result of labor-augmenting technological change. Thus, in the long run, our model results are consistent with the hypothesis put forward by Bound and Johnson (1992) and Katz and Murphy (1992). However, in the short run, the model also entails a feature that can account for the U-shaped development of the skill premium. A necessary condition to allow for this feature is that the relative influence of technological change augmenting low-skilled labor has to be more pronounced than that of high-skilled labor. Consequently, for a moderate increase in relative labor productivity, the skill premium will decrease in the short run.

Figure 2 shows the result of a numerical example to visualize the interplay between the two types of labor-augmenting technological change on the determination of the skill premium. The 45° solid line indicates equality of high- and low-skilled augmenting technological progress, that is, $\Delta A_S = \Delta A_U$. Based on the chosen model configuration, we first calculate the initial skill premium, ω_0, of the economy. As a second step, the dashed, dotted, and dot-dashed line illustrate, under different assumptions on the underlying production parameters, the relative increase in labor-augmenting technological change necessary to keep ω_0 constant. As seen from figure 2, under the assumptions made here, for any increase in A_U, constancy in relative wages requires that the increase in A_S has to be strictly higher than that of A_U. In other words, to induce an increase in the skill premium, high-skilled augmenting

[18] This result is true even in the limiting case of $\sigma_S = 1$, where production function (4) reduces to the Cobb-Douglas case, that is, $\widetilde{Y}_S = (A_{KS}\widetilde{K}_S)^{\beta_0}(A_S\widetilde{L}_S)^{1-\beta_0}$. Note that $\sigma_U = 1$ is ruled out due to Assumption 1.

technological change has to sufficiently exceed low-skilled augmenting technological change. Within our model, this result is driven by three distinct effects.

Figure 2: Static effects of labor-augmenting technological change on the skill premium



The scenario depicted in the figure above is based on production function (8), where the distribution parameters are $\alpha_0 = \gamma_0 = 0.5$, the initial aggregate capital stock is $K_0 = 2$, and the elasticity of substitution between intermediates is $\sigma = 3$. Furthermore, we have $\beta_0 = 0.5$, $\sigma_S = \sigma_U = 1.2$ (dashed line), $\beta_0 = 0.5$, $\sigma_S = 1.2$ and $\sigma_U = 1.7$ (dotted line), and $\beta_0 = 0.7$, $\sigma_S = \sigma_U = 1.2$ (dot-dashed line).

First, there is an *educational effect.* It is depicted by the difference between the solid and the dashed line. The rationale behind the effect is simple. Consider an increase in the productivity of low-skilled labor. As shown in Proposition 1, this increase in A_U shifts labor from the high-skilled to the low-skilled intermediate sector, as it causes a decrease in the skill premium ω. To induce an increase in L_S back to its initial level, high-skilled augmenting technological change has to compensate not only for the increase in A_U but also for the extent of educational costs. These costs occur because in our model, education is not free, as in, for instance, Alvarez-Cuadrado et al. (2017), but requires a costly investment of time. The strength of the educational effect depends on the initial allocation of labor to both intermediate sectors and the corresponding threshold ability $\tilde{a}$. However, in general, the effect implies that to ensure constancy in an initial skill premium, ΔA_S has to exceed ΔA_U.

Second, there is a *relative substitutability effect.* This effect relies on a fundamental property of the CES production function.[19] As demonstrated by de La Grandville

[19]Note that the effect is similar to the *factor rebalancing effect* developed in Alvarez-Cuadrado et al. (2017).

and Solow (2017), sectoral production described by CES production functions (4) and (5) is an increasing function of the sectoral elasticity of substitution. The rationale behind the relative substitutabiltiy effect is that when $\sigma_i, i \in \{S, U\}$ is high, diminishing marginal returns of capital and labor set in less pronounced than in a case where σ_i is low (Brown, 1966). Consequently, for any given set of state variables, $\{K, L, A_{KS}, A_{KU}, A_S, A_U\}$, and production parameters, the skill premium is an increasing (decreasing) function of σ_S (σ_U).[20] On the basis of this property and due to Assumption 1, that is $\sigma_S < \sigma_U$, an identical increase in labor productivity, i.e., $\Delta A_S = \Delta A_U$, induces a decrease in the skill premium. As a result, to keep the initial skill premium constant, the relative substitutability effect also requires $\Delta A_S > \Delta A_U$. In figure 2, the effect can approximately be measured by the difference between the dotted and the dashed lines.

Third, in addition to the educational and the relative substitutablity effect, the model comprises an additional effect. Referring to Acemoglu and Guerrieri (2008), we label it the *factor intensity effect* because it compares the relative usage of capital and labor in both intermediate sectors. To visualize the effect, we need to relax the assumption of a symmetric constellation of the production system with respect to the distribution parameters, i.e., $\alpha_0 = \beta_0 = \gamma_0 = 0.5$.[21] The intuition for the factor intensity effect is as follows. Consider an increase in β_0. As seen from the sectoral production function (4), this is tantamount to a decrease in the relative importance of high-skilled labor, L_S, in producing the high-skilled intermediate, Y_S. Consequently, the effect of high-skilled augmenting technological change on the skill premium decreases. It then follows that for a given increase in A_U, constancy in an initial skill premium, ω_0, now requires a larger increase in high-skilled augmenting technological change. In figure 2, the factor intensity effect can approximately be measured by the difference between the dot-dashed and dashed lines. Moreover, a similar effect appears for a decrease in γ_0. This increases the importance of low-skilled labor, L_U, in producing the low-skilled intermediate, Y_U, which in turn increases the relative influence of low-skilled augmenting technological progress on the skill premium.[22]

The discussion of the special case underlying figure 2 demonstrates that for a fixed stock of capital, a U-shaped development of relative wages can be achieved, as long as in the short run, relative labor-augmenting technological change $\Delta A_S/\Delta A_U$ stays below a certain threshold value. This threshold level is defined as the degree of high-

[20]This statement remains true as long as for the normalized capital-labor ratio in efficiency units $k_{i,t} = \frac{(A_{Ki,t}K_{i,t})/(A_{Ki,0}K_{i,0})}{(A_{i,t}L_{i,t})/(A_{i,0}L_{i,0})} < 1$, $i \in \{S, U\}$, holds. A formal proof based on a single CES production function can be found in Mallick (2012). In addition, see the numerical results provided by figure 3.

[21]At the point of normalization, $t = 0$, the distribution parameters are given by $\alpha_0 = \frac{p_{S,0}Y_{S,0}}{p_{S,0}Y_{S,0}+p_{U,0}Y_{U,0}}$, $\beta_0 = \frac{r_0 K_{S,0}}{r_0 K_{S,0}+w_{S,0}L_{S,0}}$, and $\gamma_0 = \frac{r_0 K_{U,0}}{r_0 K_{U,0}+w_{U,0}L_{U,0}}$, respectively, and thus solely rely on chosen baseline values. Although the normalization procedure developed by de La Grandville (1989) and Klump and de La Grandville (2000) allows these values to be arbitrarily chosen, the following discussion will show how variations in the distribution parameters influence the impact of factor-augmenting technological progress and the accumulation of capital on the skill premium. For a more detailed discussion on the calibration of CES production functions, see Temple (2012).

[22]Of course, the effects are reversed for a decrease in β_0 and an increase in γ_0.

skilled relative to low-skilled augmenting technological progress required to ensure constancy of a given initial skill premium ω_0. Based on Proposition 1 and solely focusing on labor-augmenting technological progress, for a constant level of high-skilled labor, $\bar{L}_S$, the threshold value can be calculated by the total differential of L_S with respect to A_S and A_U:

$$\begin{aligned} dL_S &= \frac{\partial L_S}{\partial A_S} dA_S + \frac{\partial L_S}{\partial A_U} dA_U \overset{!}{=} 0 \\ \Leftrightarrow \left. \frac{dA_S}{dA_U} \right|_{L_S=\bar{L}_S} &= -\frac{\partial \Sigma(\cdot)/\partial A_U}{\partial \Sigma(\cdot)/\partial A_S} \end{aligned} \tag{17}$$

We denote this threshold value by $\tilde{\vartheta}$. Consequently, for $\vartheta > \tilde{\vartheta}$ ($\vartheta < \tilde{\vartheta}$), relative labor-augmenting technological change leads to an increase (decrease) in the skill premium. The above equation is of particular importance because it enables us to test the sensitivity of the threshold value $\tilde{\vartheta}$ to changes in the central parameter of our model. This in turn allows for a quantitative evaluation of both the relative substitutabilty and the factor intensity effect. For that reason, we choose a certain configuration of production system (8) and calculate $\tilde{\vartheta}$ while systematically varying the individual parameter values $\alpha_0, \beta_0, \gamma_0$, and σ_U.

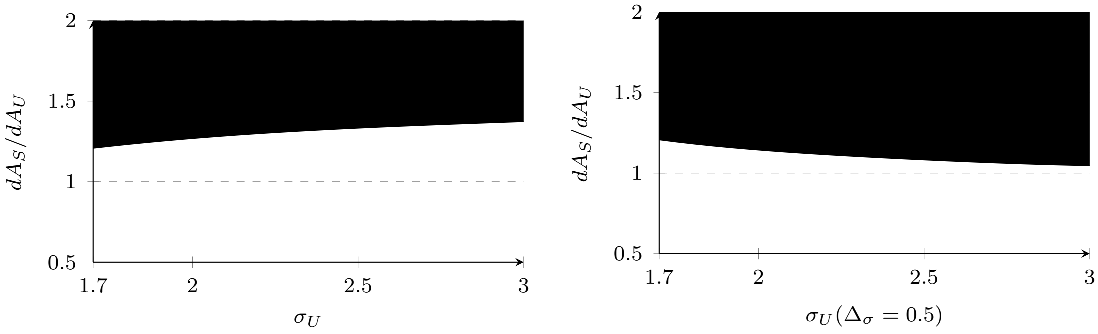
Figure 3 shows the results of this exercise. The black (white) area indicates the extent of relative labor-augmenting technological change that, given the particular parameter configuration, induces an increase (decrease) in the initial skill premium ω_0. Panel *a* visualizes the sensitivity of $\tilde{\vartheta}$ with respect to the substitution elasticities. As seen from the left figure, for a given value of σ_S, the strength of the relative substitutability effect strictly increases with σ_U, although at a substantially diminishing marginal rate. By contrast, as shown by the right figure, given a fixed difference between the two sectoral substitution elasticities, $\Delta_\sigma = \sigma_U - \sigma_S = const.$, the strength of the effect decreases with σ_U. This occurs because for large values of the substitution elasticities, the two sectors become increasingly similar. However, and most interesting, the relative substitutability effect disappears at a relatively fast rate as σ_U increases and almost completely vanishes as σ_U approaches towards $\sigma = 3$. Panel *b* illustrates the sensitivity of $\tilde{\vartheta}$ with respect to the distribution parameters. Based on the underlying model configuration, the threshold value monotonically increases in both α_0 and β_0 but decreases in γ_0. Interestingly, while the effect of changes in β_0 and γ_0 is relatively strong, $\tilde{\vartheta}$ does not change much in response to variation in α_0.[23] Especially for small values of α_0, the threshold value remains relatively constant. In addition, for very low values of β_0 or very high values of γ_0, an increase in ω_0 can also be achieved for net low-skilled augmenting technological change, that is, $0 < \lambda_S < \lambda_U$.[24] Overall, based on the chosen model configuration, figure 3 suggests that a change in sectoral distribution parameters appears to have a greater influ-

[23]However, note that the initial skill premium, ω_0, and thus L_S is increasing in α_0 and γ_0 but decreasing in β_0.

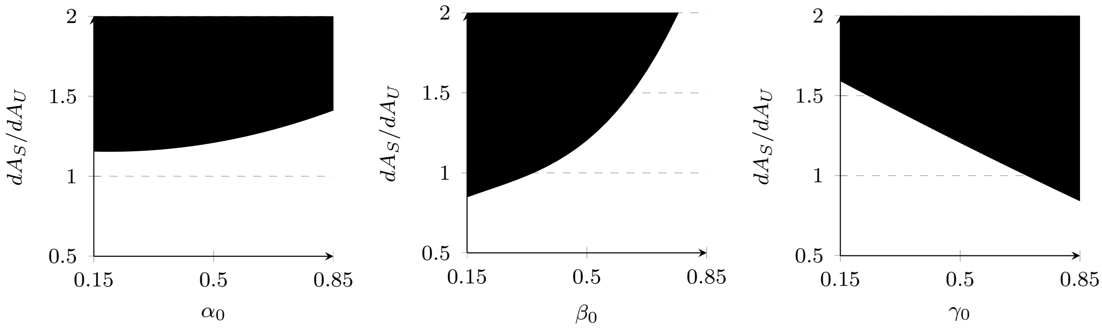
[24]Note that in our model, due to Assumption 1, that is, $\lambda_S > \lambda_U$, for this constellation of production function parameters, ω is monotonically increasing in labor-augmenting technological change.

ence on the development of the skill premium than a similar change in the value of sectoral substitution elasticities.

Figure 3: Sensitivity of the threshold value $\tilde{\vartheta}$ to the choice of model parameters



(a) Sensitivity with respect to substitution elasticities



(b) Sensitivity with respect to distribution parameters

The calculations in the figure above are based on a baseline configuration, where the distribution parameters are $\alpha_0 = \beta_0 = \gamma_0 = 0.5$, the initial aggregate capital stock is $K_0 = 2$, and the substitution elasticities are given by $\sigma_S = 1.2$, $\sigma_U = 1.7$, and $\sigma = 3$. For the sensitivity analysis, we choose $\alpha_0, \beta_0, \gamma_0 \in [0.15, 0.85]$, and $\sigma_U \in [1.7, 3]$. The borders of the distribution parameters are chosen to ensure the numerical tractability of the model.

After discussing the impact of factor-augmenting technological change, we next examine the effects of an increase in the aggregate capital stock, K, on the allocation of labor to the high-skilled intermediate sector. Contingent on the assumption of finite values of both sectoral substitution elasticities, the marginal products of capital are positive and downward sloping. From this, it follows that for a fixed level of productivity, an increase in K leads to a strictly positive increase in both K_S and K_U. This result holds for any ordering of substitution elasticities and remains true even in the extreme case of an aggregate von Neumann production technology, where $\sigma \to \infty$.[25] The following lemma characterizes the effect of capital accumulation on the composition of labor with respect to the first special case:

[25] However, note that in this case, with $\alpha_0 = 0.5, \beta_0 = \gamma_0$, $K_{S,0} = K_{U,0}$, and neglecting technological change, we have $\lim\limits_{K_S \to \infty} \frac{\partial Y}{\partial K_S} = \frac{\alpha_0 Y_0}{K_{S,0}} \beta_0^{\frac{\sigma_S}{\sigma_S - 1}} > 0$ and $\lim\limits_{K_U \to \infty} \frac{\partial Y}{\partial K_U} = \frac{(1-\alpha_0) Y_0}{K_{U,0}} \gamma_0^{\frac{\sigma_U}{\sigma_U - 1}} > 0$. Assumption 1, that is, $\sigma_S < \sigma_U$, leads to $\frac{\alpha_0 Y_0}{K_{S,0}} \beta_0^{\frac{\sigma_S}{\sigma_S - 1}} < \frac{(1-\alpha_0) Y_0}{K_{U,0}} \gamma_0^{\frac{\sigma_U}{\sigma_U - 1}}$. Therefore, in the limit,

Lemma 1. Under $\sigma > \sigma_U > \sigma_S$, in the competitive equilibrium, the level of high-skilled labor, L_S, is increasing in K_S but decreasing in K_U.

Proof: See Appendix A.5. ■

The result follows from the elasticities of the left-hand side of (LMC) and the (CMC) with respect to L_S, K_S, and K_U. Therefore, for the first special case, the influence of an increase in the aggregate capital stock, K, on the allocation of labor to the high-skilled intermediate sector, L_S, and the skill premium, ω, is generally ambiguous and dependent on parameters. We can now turn to a discussion of special case 2.

Special case 2: capital-skill complementarity In the second case, we consider the ordering $\sigma_U > \sigma > \sigma_S$. Under this constellation of the substitution elasticities, the impact of factor-augmenting technological change on the allocation of labor to the high-skilled intermediate sector and the skill premium are generally ambiguous and dependent on parameter values. This can immediately be seen from an inspection of the proof of Proposition 1 in Appendix A.4 while considering the ordering $\sigma_U > \sigma > \sigma_S$.

Turning to the accumulation of capital, equivalent to the first special case, an increase in the aggregate capital stock K leads to a positive increase in both K_S and K_U. Under $\sigma_U > \sigma > \sigma_S$, the impact of capital accumulation on the composition of labor across the two intermediate sectors is unambiguous. The following lemma establishes the existence of capital-skill complementarity.

Lemma 2. Under $\sigma_U > \sigma > \sigma_S$, in the competitive equilibrium, an increase in the aggregate capital stock, K, strictly raises the level of high-skilled labor, L_S, and the skill premium ω.

Proof: See Appendix A.5. ■

This result can be obtained analogously to that of Lemma 1. The intuition is as follows. Under $\sigma_U > \sigma > \sigma_S$, an increase in both types of capital, K_S and K_U, respectively, always induces an increase in the relative marginal value product of high-skilled labor. Formally, we have $\partial\left(\frac{\partial Y/\partial L_S}{\partial Y/\partial L_U}\right)/\partial K_S > 0$ and $\partial\left(\frac{\partial Y/\partial L_S}{\partial Y/\partial L_U}\right)/\partial K_U > 0$. Consequently, considering $\sigma_U > \sigma > \sigma_S$, our model supports the hypothesis advanced by Griliches (1969). That is, an increase in the aggregate capital stock *ceteris paribus* raises the demand for high-skilled labor and thus the relative wage rate. As a result, to obtain a U-shaped evolution of the skill premium under CSC, the transitional decline in ω has to be induced completely by factor-augmenting technological change.

After analyzing the effects of factor-augmenting technological change and capital accumulation for all three possible cases with respect to the constellation of substitution elasticities, we can now turn to the dynamic behavior of the model.

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the marginal product of K_S is equal to the asymptotical value of the marginal product of K_U, i.e., $\frac{\alpha_0\beta_0 Y_0}{K_{S,0}}\left(\frac{\widetilde{Y}_S}{\widetilde{K}_S}\right)^{\frac{1}{\sigma_S}} = \alpha_0\beta_0^{\frac{\sigma_U}{\sigma_U-1}}$. From this, it follows that as K goes to infinity, K_S is bounded from above while $\lim\limits_{K\to\infty} K_U = \infty$.

3.2 Dynamic behavior

In the previous section, we discussed the comparative statics of the static equilibrium for arbitrary, one-time changes in the technology levels, A_S, A_U, A_{KS}, A_{KU}, or the aggregate capital stock, K. In this section, we briefly discuss the dynamics of the central variables of our model before we turn to the numerical evaluation of the model. We first define

$$\begin{aligned}
&\frac{\dot{L}_i}{L_i} \equiv n_i, \quad \frac{\dot{K}_i}{K_i} \equiv z_i, \quad \frac{\dot{Y}_i}{Y_i} \equiv g_i, \quad \text{for} \quad i \in \{S, U\} \\
&\frac{\dot{Y}}{Y} \equiv g, \quad \frac{\dot{\omega}}{\omega} \equiv g_\omega,
\end{aligned}$$

where n_i, z_i and g_i denote the growth rates of labor, capital, and output in both intermediate sectors. The growth rate of aggregate production is denoted by g. Finally, g_ω refers to the growth rate of the skill premium. To decompose these growth rates into different components, we follow Krusell et al. (2000) and McAdam and Willman (2018). We first log-linearize (3), (4), (5), and (9) and differentiate the resulting equations with respect to time. From the result, and with some algebra, we obtain the following two propositions that characterize the dynamic behavior of our model in terms of the growth rates of output, Y, Y_S and Y_U, as well as of ω.

Proposition 2. In the competitive equilibrium, the dynamic behavior of the model is described by the following growth rates:

$$g = \alpha_0 \left(\frac{\widetilde{Y}_S}{\widetilde{Y}} \right)^{\frac{\sigma-1}{\sigma}} g_S + (1-\alpha_0) \left(\frac{\widetilde{Y}_U}{\widetilde{Y}} \right)^{\frac{\sigma-1}{\sigma}} g_U$$

where

$$g_S = \beta_0 \left(\frac{A_{K,S}\widetilde{K}_S}{\widetilde{Y}_S} \right)^{\frac{\sigma_S-1}{\sigma_S}} (z_S + \lambda_{KS}) + (1-\beta_0) \left(\frac{A_S\widetilde{L}_S}{\widetilde{Y}_S} \right)^{\frac{\sigma_S-1}{\sigma_S}} (n_S + \lambda_S)$$

and

$$g_U = \gamma_0 \left(\frac{A_{K,U}\widetilde{K}_U}{\widetilde{Y}_S} \right)^{\frac{\sigma_U-1}{\sigma_U}} (z_U + \lambda_{KU}) + (1-\gamma_0) \left(\frac{A_U\widetilde{L}_U}{\widetilde{Y}_U} \right)^{\frac{\sigma_U-1}{\sigma_U}} (n_U + \lambda_U)$$

where $z_i = z_i(A_S, A_U, A_{KS}, A_{KU}, K, L_S)$ and $n_i = n_i(A_S, A_U, A_{KS}, A_{KU}, K, L_S)$ with $i \in \{S, U\}$.

Not surprisingly, the growth rate of aggregate output, g, is a weighted average of the sectoral growth rates g_S and g_U. Due to our assumption of $\sigma > 1$, this implies that as $t \to \infty$, aggregate output will be determined by the asymptotically dominant sector. Formally, we have $g^* = max\{g_S^*, g_U^*\}$, where the asterisks denote asymptotic growth rates. The growth rates of intermediate production are in turn weighted averages of the sectoral growth rates of capital and labor, measured in efficiency units. Moreover, the following proposition decomposes the growth rate of the skill premium into three components.[26]

Proposition 3. At the point of normalization, $t = 0$, the growth rate of the skill premium, ω, can be decomposed into:

$$
\begin{aligned}
(18) \qquad g_\omega = & -\left[\tfrac{\sigma_S}{\sigma}(1-\beta_0)+\beta_0\right]n_S + \left[\tfrac{\sigma_U}{\sigma}(1-\gamma_0)+\gamma_0\right]n_U && : \text{RSS} \\
& + \left(\tfrac{\sigma-\sigma_S}{\sigma_S\sigma}\right)\beta_0 z_S - \left(\tfrac{\sigma-\sigma_U}{\sigma_U\sigma}\right)\gamma_0 z_U && : \text{CA} \\
& + \left(\tfrac{\sigma-\sigma_S}{\sigma_S\sigma}\right)\beta_0 \lambda_{KS} - \left(\tfrac{\sigma-\sigma_U}{\sigma_U\sigma}\right)\gamma_0 \lambda_{KU} && \\
& + \left(\tfrac{\sigma-\sigma_S}{\sigma_S\sigma}(1-\beta_0)+\tfrac{\sigma_S-1}{\sigma_S}\right)\lambda_S && \text{TC} \\
& - \left(\tfrac{\sigma-\sigma_U}{\sigma_U\sigma}(1-\gamma_0)+\tfrac{\sigma_U-1}{\sigma_U}\right)\lambda_U &&
\end{aligned}
$$

The first component, RSS, refers to the relative supply of high-skilled labor. The coefficients $\left[\frac{\sigma_S}{\sigma}(1-\beta_0)+\beta_0\right]$ and $\left[\frac{\sigma_U}{\sigma}(1-\gamma_0)+\gamma_0\right]$ are both strictly positive. Due to our assumption of a constant population, we further have $sgn(n_S) = -\,sgn(n_U)$. From this, it follows that for an increase in the reallocation of labor to the high-skilled (low-skilled) intermediate sector, the growth rate of the skill premium strictly decreases (increases) according to the RSS component. This result relies on the usual *substitution effect* (Acemoglu, 2002). According to this effect, for competitive markets, an increase in the supply of a factor depresses its marginal value product and thus the wage rate. The second component, CA, describes the effect of capital accumulation. In this channel, the condition for an increase in g_ω is given by the inequality $\frac{z_S}{z_U} > \frac{(\sigma-\sigma_S)\sigma_U}{(\sigma-\sigma_U)\sigma_S}\frac{\gamma_0}{\beta_0}$. From the previous discussion, we obtained $sgn\left(\frac{z_S}{z_U}\right) = 1$ for any increase in the aggregate capital stock, independent of the ordering of substitution elasticities. Furthermore, in special case 2, that is $\sigma_U > \sigma > \sigma_S$, we obtain $\frac{(\sigma-\sigma_S)\sigma_U}{(\sigma-\sigma_U)\sigma_S}\frac{\gamma_0}{\beta_0} < 0$. Consequently, similar to the results obtained from Lemma 2, under CSC, capital accumulation has a strictly positive impact on the growth rate of the skill premium. Finally, the third component, TC, refers to the impact of factor-augmenting technological change. As seen from (18), capital-augmenting technological progress has the same quantitative impact on the growth rate of the skill premium as capital accumulation. A positive contribution requires $\frac{\lambda_{KS}}{\lambda_{KU}} > \frac{(\sigma-\sigma_S)\sigma_U}{(\sigma-\sigma_U)\sigma_S}\frac{\gamma_0}{\beta_0}$. In addition, labor-augmenting technological progress has a positive influence on the TC channel, if $\lambda_S\left(\frac{\sigma-\sigma_S}{\sigma_S\sigma}(1-\beta_0)+\frac{\sigma_S-1}{\sigma_S}\right) > \lambda_U\left(\frac{\sigma-\sigma_U}{\sigma_U\sigma}(1-\gamma_0)+\frac{\sigma_U-1}{\sigma_U}\right)$.

Unfortunately, due to the complexity of the underlying production system, we cannot analytically solve for the endogenous growth rates z_S, z_U, n_S and n_U. Therefore, we resort to the following chapter, in which we conduct a comprehensive simulation analysis.

[26] However, note that in a general equilibrium framework, these components cannot be separated in a strict sense, as a reallocation of labor between intermediate sectors induces a reallocation of capital and *vice versa*.

4 Numerical examples

In this section, we provide several numerical examples of the above model to examine the development of the skill premium and some further key variables along the transitional path of a growing economy. To simulate the model, we choose parameter and benchmark values that reflect the results of empirical studies wherever possible. However, note that the following simulations should not be interpreted as a calibration but instead as a simple numerical illustration.

Our model economy is fully characterized by three substitution parameters, σ, σ_S, and σ_U, four growth parameters, λ_S, λ_U, λ_{KS}, and λ_{KU}, the savings rate, s, the depreciation rate, δ, and 10 benchmark values, Y_0, $Y_{S,0}$, $Y_{U,0}$, $K_{S,0}$, $K_{U,0}$, $L_{S,0}$, $L_{U,0}$, $\mu_0 = \left[\frac{p_U}{p_S}\right]_0$, $\mu_{S,0} = \left[\frac{\omega_S}{r}\right]_0$, and $\mu_{U,0} = \left[\frac{\omega_U}{r}\right]_0$. We choose parameter and benchmark values as follows. Estimates of substitution elasticities can be found in, among other works, Krusell et al. (2000), Duffy et al. (2004), Goldin and Katz (2008), and Polgreen and Silos (2008). While the results vary considerably, most of them exceed unity. Unfortunately, due to different specifications of the underlying production function, none of these estimates directly fit to our model. In comparison, McAdam and Willman (2018) estimate the parameters of the production function (8) for the U.S. economy over the period $1963 - 2008$. The authors obtain the substitution elasticities $\sigma = 2.951$, $\sigma_S = 1.234$, and $\sigma_U = 1.697$, which are suitable to our case of factor-biased technological change, when treating K_S as structure (or building) capital and K_U as equipment capital. A second specification assumes K_S to be equipment and K_U to be structure capital. In that case, the authors obtain $\sigma = 2.505$, $\sigma_S = 1.317$, and $\sigma_U = 7.327$, which are supportive of capital-skill complementarity. For reasons of comparability, in the following, we simulate our model based on the results provided by McAdam and Willman (2018).

With respect to technological change, Katz and Murphy (1992), Krusell et al. (2000), Autor et al. (2008), and Acemoglu and Autor (2011), among others, estimate the difference in the growth rates of labor productivity, i.e., $\lambda_S - \lambda_U$, based on U.S. data for different specifications and time periods. The results range from 0.006 to 0.033. Furthermore, McAdam and Willman (2018) provide estimates of the growth rates $\lambda_S, \lambda_U, \lambda_{KS}$, and λ_{KU} for U.S. data based on the production function (8). While λ_S is estimated at approximately 0.033, the corresponding value for low-skilled labor, λ_U, lies slightly below zero. For different specifications of the underlying production function, the difference between the two estimates narrows, while lower values of λ_S and higher values of λ_U are obtained. Furthermore, with respect to capital-augmenting technological change, the results are rather diverse, and the estimates lie between -0.06 and 0.05. Against this background, we exclude capital-augmenting technological progress, i.e., $\lambda_{KS} = \lambda_{KU} = 0$, and solely focus on labor-augmenting technological change. We set high-skilled augmenting technological change to be $\lambda_S = 0.035$, slightly exceeding that of low-skilled labor, $\lambda_U = 0.033$. The choice of these growth rates allows us to provide sensible examples. To set the parameters of the Solow growth model, we follow Miyagiwa and Papageorgiou (2007) and choose $\delta = 0.1$. Series of gross savings as a percentage of gross national income are available from the U.S. Bureau of Economic Analysis (BEA). Over the period 1947 to 2018, average annual savings lie slightly above 20%. Therefore, we set $s = 0.2$.

Finally, with respect to the distribution parameters, we distinguish two cases. In the following figures, the left panel shows transitional dynamics based on our symmetric benchmark case, where $\alpha_0 = \beta_0 = \gamma_0 = 0.5$. In that case, the results are purely driven by the educational and the relative substitutability effect. Additionally, in the right panel, we utilize the results provided by McAdam and Willman (2018) and set the distribution parameters to $\alpha_0 = 0.5, \beta_0 = 0.7$ and $\gamma_0 = 0.3$. As a consequence, the transitional dynamics presented in the right panel also account for the factor intensity effect. Based on these choices, table 1 summarizes the remaining benchmark values. An outline of its calculation is provided in Appendix A.6.

Table 1: Benchmark values

Benchmark values for $\alpha_0 = \beta_0 = \gamma_0 = 0.5$

$\sigma > \sigma_U > \sigma_S$		$\sigma_U > \sigma > \sigma_S$	
$L_{S,0} = 0.333$	$L_{U,0} = 0.577$	$L_{S,0} = 0.333$	$L_{U,0} = 0.577$
$K_{S,0} = 1.000$	$K_{U,0} = 1.000$	$K_{S,0} = 1.000$	$K_{U,0} = 1.000$
$\mu_{S,0} = 2.436$	$\mu_{U,0} = 1.382$	$\mu_{S,0} = 2.303$	$\mu_{U,0} = 1.078$

Benchmark values for $\alpha_0 = 0.5$, $\beta_0 = 0.7$, $\gamma_0 = 0.3$

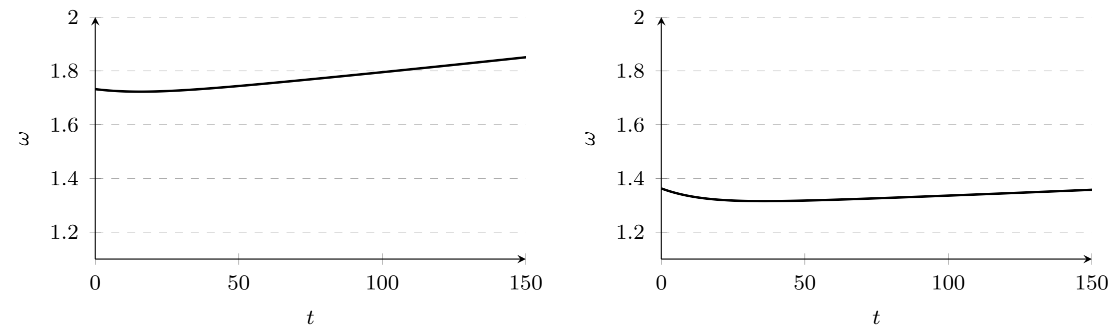
$\sigma > \sigma_U > \sigma_S$		$\sigma_U > \sigma > \sigma_S$	
$L_{S,0} = 0.231$	$L_{U,0} = 0.734$	$L_{S,0} = 0.231$	$L_{U,0} = 0.734$
$K_{S,0} = 1.400$	$K_{U,0} = 0.600$	$K_{S,0} = 1.400$	$K_{U,0} = 0.600$
$\mu_{S,0} = 1.847$	$\mu_{U,0} = 2.072$	$\mu_{S,0} = 1.685$	$\mu_{U,0} = 2.270$

Note: Across all specifications, we employed the benchmark values $Y_0 = Y_{S,0} = Y_{U,0} = \mu_0 = 1$, together with an initial aggregate capital stock of $K_0 = 2$. The remaining benchmark values provided in the table are calculated as outlined in Appendix A.6 and rounded to 3 digits.

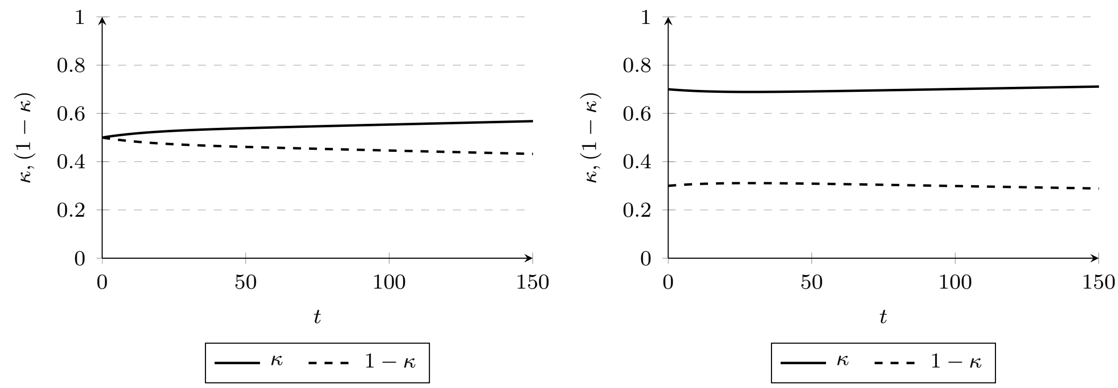
We can now turn to our numerical results. Figures 4 and 5 plot the time series of some key variables of our model, considering both special cases with respect to the constellation of substitution elasticities. Within each figure, chart a) illustrates the development of the skill premium, ω. Chart b) shows the share of capital allocated to the high-skilled intermediate sector, $\kappa = \frac{K_S}{K}$, and to the low-skilled intermediate sector, $(1 - \kappa) = \frac{K_U}{K}$. Finally, chart c) displays the development of the functional distribution of income between labor, $S_L = \frac{\omega_S L_S + \omega_U L_U}{Y}$, and capital, $S_K = \frac{r(K_S + K_U)}{Y}$. As a consequence of the assumption of constant returns to scale and purely competitive markets, the two factor shares sum to a value of unity by Euler's theorem.

Figure 4 illustrates the transitional dynamics for the case of factor-biased technological change, i.e., $\sigma > \sigma_U > \sigma_S$. As seen in the left panel, considering a symmetric constellation of distribution parameters, i.e., $\alpha_0 = \beta_0 = \gamma_0 = 0.5$, a U-shaped development of the skill premium can be achieved under the stated model parametrization. However, the magnitude of the pattern is very small. Relative wages decline by only 0.5% during the first 17 periods, followed by a monotonic increase afterwards. The share of capital allocated to the high-skilled intermediate sector, κ, increases

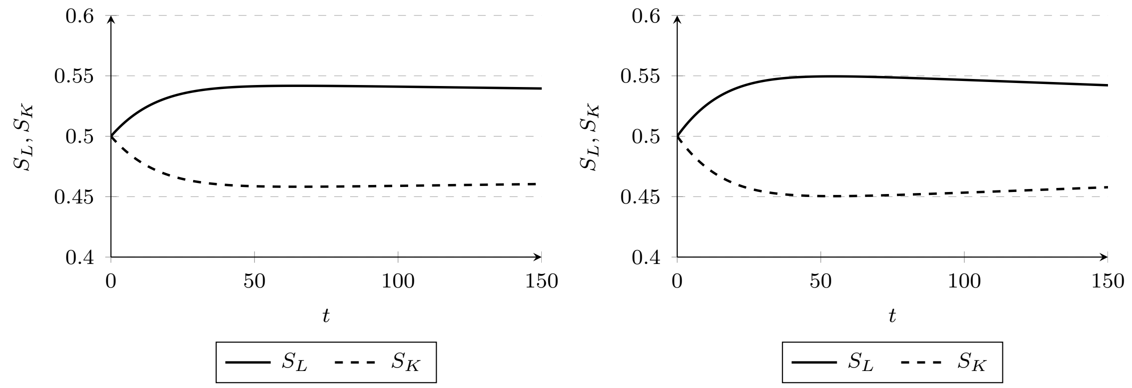
Figure 4: Transitional dynamics for the case of factor-biased technological change, i.e., $\sigma > \sigma_U > \sigma_S$



(a) skill premium



(b) share of capital



(c) functional distribution of income

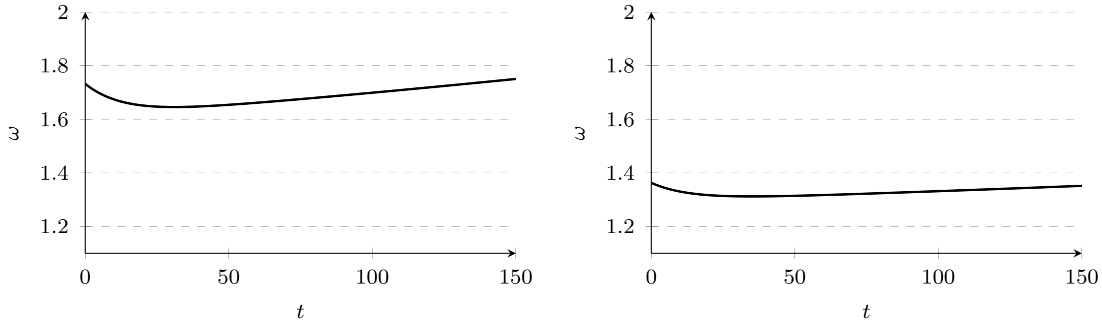
The illustrations above are constructed while assuming the following parameter values: $\sigma_S = 1.234$, $\sigma_U = 1.697, \sigma = 2.951, K_0 = 2, \lambda_S = 0.035, \lambda_U = 0.033, \lambda_{KS} = \lambda_{KU} = 0, \delta = 0.1$, and $s = 0.2$. The distribution parameters are $\alpha_0 = \beta_0 = \gamma_0 = 0.5$ (left panel) and $\alpha_0 = 0.5, \beta_0 = 0.7$, and $\gamma_0 = 0.3$ (right panel), respectively.

constantly from 0.50 to 0.57. With respect to the functional distribution of income, the labor share, S_L, increases during the first 66 periods and shows a slight decline afterwards. In comparison, the right panel of figure 4 shows results for the case of an asymmetric constellation of distribution parameters, i.e., $\alpha_0 = 0.5$, $\beta_0 = 0.7$ and $\gamma_0 = 0.3$. As can be deduced from equation (8), an increase (decrease) in β_0 (γ_0) decreases the relative importance of high-skilled labor in producing aggregate output Y. Consequently, as illustrated in chart 4*b*, the initial skill premium, ω_0, declines. Moreover, the chart also reveals the impact of the factor intensity effect from a dynamic perspective. From the discussion of figure 3, we have seen that an increase (decrease) in β_0 (γ_0) implies a rise in the threshold value $\tilde{\vartheta}$. This, in turn, should result in a longer and more pronounced transitional decrease in the skill premium. Such a pattern can be obtained from the time series underlying panel *b*. The skill premium decreases by approximately 3.5% and reaches an inflection point after 37 periods. Subsequently, relative wages begin to rise, but it takes more than 150 periods for ω to come full circle. Furthermore, as illustrated by the right panel of chart *c*, the share of capital allocated to the high-skilled intermediate sector, κ, rises considerably relative to the left panel. The reason for this behavior is that an increase (decrease) in β_0 (γ_0) also increases the relative importance of capital in producing the high-skilled intermediate good. As a result, a higher share of capital is allocated to the high-skilled intermediate sector. As seen in panel 4*b*, at the point of normalization, $t = 0$, κ jumps to a high value of approximately 0.70. Thereafter, the value remains roughly constant. Finally, the development of factor shares is approximately the same as in the left panel, although the subsequent decline in the labor share is more pronounced.

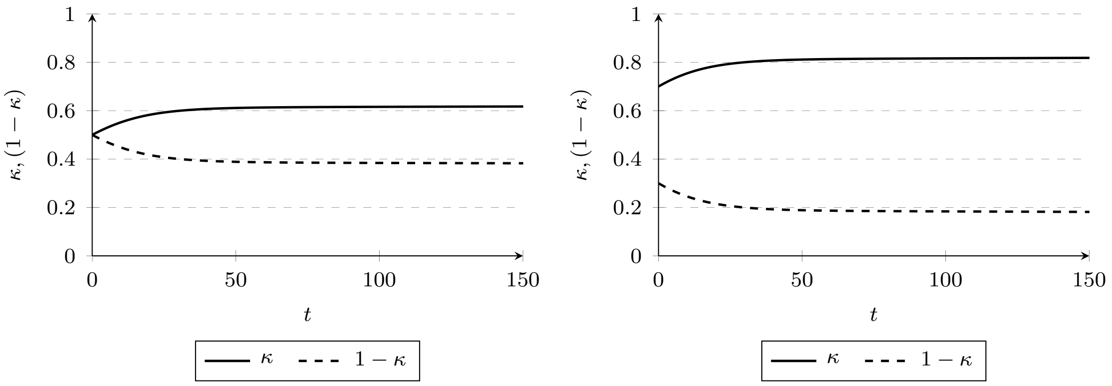
Figure 5 illustrates the transitional dynamics for the case of capital-skill complementarity, that is, $\sigma_U > \sigma > \sigma_S$. As seen in the left panel, the transitory decrease in ω is more pronounced than in the case of factor-biased technological change. During the first 32 periods, the skill premium declines by approximately 5.0%. This result is largely induced by the greater difference between the two sectoral substitution elasticities, σ_S and σ_U, i.e., a greater impact of the relative substitutability effect. Furthermore, under CSC, we obtain a higher share of capital allocated to the high-skilled intermediate sector. After a brief but considerable adjustment process, κ stabilizes at approximately 0.6 (left panel) and 0.8 (right panel). This relatively strong sectoral reallocation of capital during the initial periods also explains why in the right panel, the transitional decline in the skill premium is less pronounced than in the left panel. From Lemma 2, we observed that under CSC, the skill premium is strictly increasing in both K_S and K_U. However, conditioned by the inequality $\beta_0 > \gamma_0$, an increase in K_S leads to a stronger increase in ω than a similar increase in K_U. As a result, an increase in κ reduces the threshold value $\tilde{\vartheta}$ necessary to induce a rise in the skill premium and thus dampens the transitional decline in ω. Interestingly, the development of the functional distribution of income is almost identical under the two assumptions on distribution parameters. The labor share reaches a maximum of 59% after 56 periods and slightly declines afterwards.

Our numerical application thus yields the following conclusion. To achieve a U-shaped development of the skill premium, the ordering of substitution elasticities

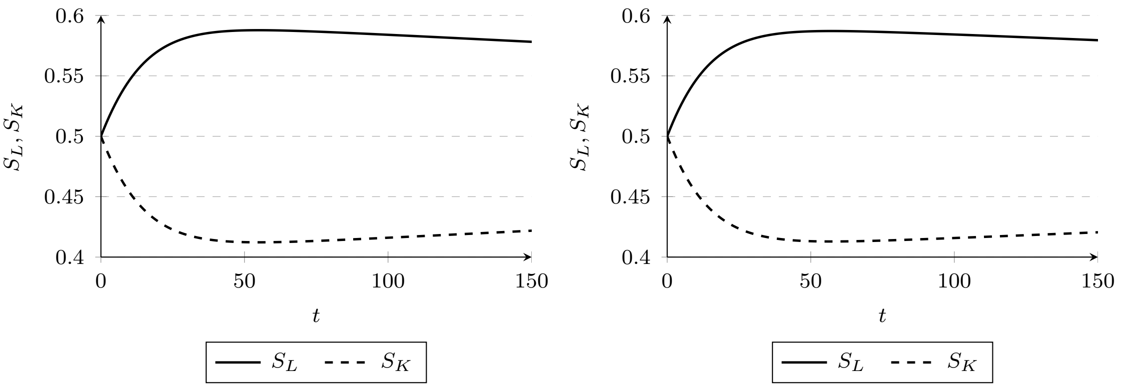
Figure 5: Transitional dynamics for the case of capital-skill complementarity, i.e., $\sigma_U > \sigma > \sigma_S$



(a) skill premium



(b) share of capital



(c) functional distribution of income

The illustrations above are constructed while assuming the following parameter values: $\sigma_S = 1.317$, $\sigma_U = 7.327, \sigma = 2.505, K_0 = 2, \lambda_S = 0.035, \lambda_U = 0.033, \lambda_{KS} = \lambda_{KU} = 0, \delta = 0.1$, and $s = 0.2$. The distribution parameters are $\alpha_0 = \beta_0 = \gamma_0 = 0.5$ (left panel) and $\alpha_0 = 0.5, \beta_0 = 0.7$, and $\gamma_0 = 0.3$ (right panel), respectively.

seems to be of minor importance. By contrast, in all cases we examined, relative wages first decline and then monotonically increase. The results thus demonstrate that a U-shaped development of ω is attainable under both approaches of SBTC and can emerge in the absence of any exogenous shocks or changes in institutional factors. As obtained from our symmetric benchmark cases, both the educational and the relative substitutability effect can induce a considerable transitional decline in ω. In comparison, we demonstrated that the factor intensity effect can both strengthen and weaken the U-shaped pattern of the skill premium. In particular, for a strong reallocation of capital between intermediate sectors, the transitional decline of ω can considerably be dampened. As this effect becomes strong, the skill premium may increase steadily over time. Furthermore, our model also generates time series of the functional distribution of income that are largely in line with observations for the U.S. economy. As has been documented by several authors, e.g., Karabarbounis and Neiman (2014) and Herrendorf et al. (forthcoming), the U.S. labor share of income remained roughly constant during the mid-twentieth century but has declined since the 1980s.[27] Our results obtained under CSC are broadly in accordance with that observation. However, based on the chosen parametrization, our model produces relatively slow dynamics with respect to ω. Generally, the interim decline in the skill premium is less pronounced than the dramatic decrease shown by the data.[28] Thus, in the following, we conduct a brief sensitivity analysis to better align our model results with the data.

Table 2: Sensitivity analysis

Sensitivity analysis for $\sigma > \sigma_U > \sigma_S$, asymmetric case

savings rate		*technological change*		*initial capital stock*	
$s = 0.15$	7.19 (45)	$\lambda_U = 0.025,\ \lambda_S = 0.023$	2.26 (34)	$K_0 = 3$	8.51 (47)
$s = 0.25$	0.34 (16)	$\lambda_U = 0.045,\ \lambda_S = 0.043$	4.56 (38)	$K_0 = 5$	13.17 (53)
$s = 0.30$	increase	$\lambda_U = 0.055,\ \lambda_S = 0.053$	5.56 (38)	$K_0 = 10$	17.39 (57)

Sensitivity analysis for $\sigma_U > \sigma > \sigma_S$, asymmetric case

savings rate		*technological change*		*initial capital stock*	
$s = 0.15$	6.91 (42)	$\lambda_U = 0.025,\ \lambda_S = 0.023$	2.55 (34)	$K_0 = 3$	7.98 (43)
$s = 0.25$	0.48 (18)	$\lambda_U = 0.045,\ \lambda_S = 0.043$	4.70 (36)	$K_0 = 5$	11.67 (48)
$s = 0.30$	increase	$\lambda_U = 0.055,\ \lambda_S = 0.053$	5.56 (36)	$K_0 = 10$	15.24 (52)

Table 2 reports the results under alternative calibrations of our model economy. We consider different values for the savings rate, s, labor-augmenting technological progress, λ_S and λ_U, and the initial capital stock, K_0. All other parameter values remain unchanged. For each alternative parametrization, the first value shows the maximum percentage decrease in the skill premium relative to ω_0, while the value

[27] From a level of roughly 64% in 1975, the U.S. labor share has constantly declined, reaching 59% in 2012.

[28] Based on the data provided by Goldin and Katz (2007), the U.S. college (high-school) wage gap declined by approximately 50.94% (51.36%) between 1915 and 1950. In the left panel of figure 5, the decrease is only 5.0%.

in brackets indicates the inflection point. For instance, in the asymmetric case of $\sigma > \sigma_U > \sigma_S$, for a savings rate of $s = 0.15$, ω decreases by approximately 7.19% over the course of 45 periods. In general, consistent with our expectations, a change in the savings rate can have a substantial impact on both the length and the strength of the decline in relative wages. Especially for high values of s, the skill premium increases monotonically. In comparison, the sensitivity results show that for higher values of labor-augmenting technological change, the U-shaped evolution of the skill premium is more pronounced. Finally, the appropriate choice of the initial capital stock, K_0, seems to be the most important aspect. As can be seen under both parameter configurations, the U-shaped pattern in the development of the skill premium is more pronounced the higher the initial capital stock is.

5 Conclusion

In this paper, we introduced a simple neoclassical growth model with endogenous labor supply to investigate the transitional dynamics of the wage ratio between high-skilled and low-skilled labor, referred to as the skill premium. The empirical literature has documented that for several developed countries, the behavior of the skill premium was U-shaped over the last century. We demonstrated that such a development of relative wages can be obtained using a simple two-sector growth model that comprises both variants of skill-biased technological change: technological change that is favorable to high-skilled labor and capital-skill complementarity. Our approach thus contrasts with most earlier studies that explain the U-shaped development of the skill premium through exogenous, unexpected technological shocks or changes in institutional factors. Within our framework, the transitional decline in the skill premium can be explained by the interplay of an educational, a relative substitutability, and a factor intensity effect. The first two effects unambiguously increase the requirement of relative high-skilled augmenting technological change necessary to induce an increase in the skill premium. Consequently, for a moderate rise in relative labor productivity, the wage ratio will decrease in the short run. We showed that such a transitional decline can be achieved for a broad range of parameter values. However, in all cases, in the long run, as relative labor productivity increases, the skill premium also rises. In contrast to the first two effects, the factor intensity effect can either increase or decrease the transitional decline in the skill premium. Especially for a production structure that is characterized by a high share of capital allocated to the high-skilled intermediate sector, the accumulation of capital can considerably dampen the transitional decline in ω. As a result, rapid capital accumulation might induce a strictly increasing wage ratio. Overall, however, our numerical evaluation illustrates that a U-shaped development of the skill premium can be achieved under both approaches of SBTC.

In future research, our model could be extended in several important directions. In the model, the formation of labor is stylized and abstracts from some important aspects of education and labor markets. For instance, we assumed the distribution of abilities to be uniform. This simplifies the analysis but prevents a deeper discussion of within-group income inequality for high-skilled labor. Over recent decades, several authors, e.g., Autor et al. (2008), have documented a gradual increase in wage dispersion within the group of high-skilled labor. A more detailed model could focus on this issue, for instance, by incorporating a versatile beta distribution of abilities. A second aspect concerns the educational process itself. In our model, the time spent on education is fixed for a certain ability level and thus independent of the stage of technological development. However, consistent with the empirical literature, the effort required to become a high-skilled worker should increase with the level of technological progress. While the effect may be negligible for low differences in labor productivity, it becomes increasingly important as technological change continues. Incorporating this aspect might help to explain the recent deceleration in the growth of the skill premium as observed for the U.S. economy.

References

Acemoglu, D. (2002). Directed Technical Change. *The Review of Economic Studies*, 69(4):781–809.

Acemoglu, D. (2003). Patterns of Skill Premia. *The Review of Economic Studies*, 70(2):199–230.

Acemoglu, D. and Autor, D. H. (2011). Skills, Tasks, and Technologies: Implications for Employment and Earnings. In Card, D. and Ashenfelter, O., editors, *Handbook of Labor Economics*, volume 4B, chapter 12, pages 1043–1171. Elsevier B.V.

Acemoglu, D. and Guerrieri, V. (2008). Capital Deepening and Nonbalanced Economic Growth. *Journal of Political Economy*, 116(3):467–498.

Alvarez-Cuadrado, F., Van Long, N., and Poschke, M. (2017). Capital-Labor Substitution, Structural Change, and Growth. *Theoretical Economics*, 12(3):1229–1266.

Alvarez-Cuadrado, F., Van Long, N., and Poschke, M. (2018). Capital-labor substitution, structural change and the labor income share. *Journal of Economic Dynamics and Control*, 87:206–231.

Arrow, K. J., Chenery, H. B., Minhas, B. S., and Solow, R. M. (1961). Capital-Labor Substitution and Economic Efficiency. *The Review of Economics and Statistics*, 43(3):225–250.

Autor, D. H. (2014). Skills, education, and the rise of earnings inequality among the “other 99 percent”. *Science*, 344(6186):843–851.

Autor, D. H., Katz, L. F., and Kearney, M. S. (2008). Trends in U.S. Wage Inequality: Revising the Revisionists. *The Review of Economics and Statistics*, 90(2):300–323.

Barro, R. J. (1974). Are Government Bonds Net Wealth? *Journal of Political Economy*, 82(6):1095–1117.

Bound, J. and Johnson, G. (1992). Changes in the Structure of Wages in the 1980‘s: An Evaluation of Alternative Explanations. *The American Economic Review*, 82(3):371–392.

Brown, M. (1966). *On the Theory and Measurement of Technological Change*. Cambridge University Press.

Cahuc, P. and Michel, P. (1996). Minimum wage unemployment and growth. *European Economic Review*, 40(7):1463–1482.

Cantore, C. and Levine, P. (2012). Getting normalization right: Dealing with ‘dimensional constants’ in macroeconomics. *Journal of Economic Dynamics and Control*, 36(12):1931–1949.

Card, D. and DiNardo, E. J. (2002). Skill-Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles. *Journal of Labor Economics*, 20(4):733–783.

Caselli, F. (1999). Technological Revolutions. *The American Economic Review*, 89(1):78–102.

Cervellati, M. and Sunde, U. (2005). Human Capital Formation, Life Expectancy, and the Process of Development. *The American Economic Review*, 95(5):1653–1672.

Cummins, J. G. and Violante, G. L. (2002). Investment-Specific Technical Change in the United States (1947-2000): Measurement and Macroeconomic Consequences. *Review of Economic Dynamcis*, 5(2):243–284.

David, P. A. and van de Klundert, T. (1965). Biased Efficiency Growth and Capital-Labor Substitution in the U.S., 1899-1960. *The American Economic Review*, 55(3):357–394.

de La Grandville, O. (1989). In Quest of the Slutsky Diamond. *The American Economic Review*, 79(3):468–481.

de La Grandville, O. (2017). *Economic Growth: A Unified Approach*. Cambridge University Press, second edition.

de La Grandville, O. and Solow, R. M. (2017). Capital-Labour Substitution and Economic Growth. In de La Grandville, O., editor, *Economic Growth - A Unified Approach*, pages 92–113. Cambridge University Press, Cambridge.

Domeij, D. and Ljungqvist, L. (2019). Public Sector Employment and the Skill Premium: Sweden versus the United States 1970-2012. *The Scandinavian Journal of Economics*, 121(1):3–31.

Duffy, J., Papageorgiou, C., and Perez Sebastian, F. (2004). Capital-skill Complementarity? Evidence from a Panel of Countries. *The Review of Economics and Statistics*, 86(1):327–344.

Galor, O. and Moav, O. (2000). Ability-Biased Technologiacl Transition, Wage Inequality, and Economic Growth. *The Quarterly Journal of Economics*, 115(2):469–497.

Galor, O. and Tsiddon, D. (1997). Technological Progress, Mobility, and Economic Growth. *The American Economic Review*, 87(3):363–382.

Glitz, A. and Wissmann, D. (2017). Skill Premiums and the Supply of Young Workers in Germany. *CESifo Working Paper No. 6576.*

Goldin, C. and Katz, L. F. (2007). The Race Between Education and Technology: The Evolution of U.S. Educational Wage Differentials, 1890 to 2005. *NBER Working Paper No. 12984.*

Goldin, C. D. and Katz, L. F. (2008). *The Race Between Education and Technology*. Harvard University Press, Cambridge, MA.

Greenwood, J., Hercowitz, Z., and Krusell, P. (1997). Long-Run Implications of Investment-Specific Technological Change. *The American Economic Review*, 87(3):342–362.

Griliches, Z. (1969). Capital-Skill Complementarity. *The Review of Economics and Statistics*, 51(4):465–468.

Guvenen, F. and Kuruscu, B. (2012). Understanding the Evolution of the US Wage Distribution: A Theoretical Analysis. *Journal of the European Economic Association*, 10(3):482–517.

Herrendorf, B., Rogerson, R., and Valentinyi, A. (forthcoming). Balanced Growth and the Kaldor Facts. *Review of the Federal Reserve Bank of St. Louis*.

Hornstein, A., Krusell, P., and Violante, G. L. (2005). The Effects of Technical Change on Labor Market Inequalities. In Aghion, P. and Durlauf, S. N., editors, *Handbook of Economic Growth*, volume 1B, chapter 20, pages 1275–1370. Elsevier B.V., Amsterdam.

Hulten, C. R. (1992). Growth Accounting When Technical Change is Embodied in Capital. *The American Economic Review*, 82(4):964–980.

Jacobs, B. and Nahuis, R. (2002). A general purpose technology explains the Solow paradox and wage inequality. *Economics Letters*, 74(2):243–250.

Karabarbounis, L. and Neiman, B. (2014). The Global Decline in the Labor Share. *The Quarterly Journal of Economics*, 129(1):61–103.

Katz, L. F. and Autor, D. H. (1999). Changes in the Wage Structure and Earnings Inequality. In Card, D. and Ashenfelter, O., editors, *Handbook of Labor Economics*, volume 3A, pages 1463–1555. Elsevier B.V., Amsterdam.

Katz, L. F. and Murphy, K. (1992). Changes in Relative Wages, 1963-1987: Supply and Demand Factors. *The Quarterly Journal of Economics*, 107(1):35–78.

Kishi, K. (2015). Dynamic analysis of wage inequality and creative destruction. *Journal of Economics*, 115(1):1–23.

Klump, R. and de La Grandville, O. (2000). Economic Growth and the Elasticity of Substitution: Two Theorems and Some Suggestions. *The American Economic Review*, 90(1):282–291.

Klump, R., McAdam, P., and Willman, A. (2012). The Normalized CES Production Function: Theory and Empirics. *Journal of Economic Surveys*, 26(5):769–799.

Klump, R. and Preissler, H. (2000). CES Production Functions and Economic Growth. *Scandinavian Journal of Economics*, 102(1):41–56.

Krusell, P., Ohanian, L. E., Rios-Rull, J.-V., and Violante, G. L. (2000). Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis. *Econometrica*, 68(5):1029–1053.

Lindquist, M. J. (2005). Capital-Skill Complementarity and Inequality in Sweden. *The Scandinavian Journal of Economics*, 107(4):711–735.

Lise, J., Sudo, N., Suzuki, M., Yamada, K., and Yamada, T. (2014). Wage, income and consumption inequality in Japan, 1981-2008: From boom to lost decades. *Review of Economic Dynamics*, 17(4):582–612.

Lloyd-Ellis, H. (1999). Endogenous Technological Change and Wage Inequality. *The American Economic Review*, 89(1):47–77.

Mallick, D. (2012). The role of the elasticity of substitution in economic growth: A cross-country investigation. *Labour Economics*, 19(5):682–694.

McAdam, P. and Willman, A. (2018). Unraveling the Skill Premium. *Macroeconomic Dynamics*, 22(1):33–62.

Meckl, J. (2006). Does European Unemployment Prop Up American Wages? National Labor Markets and Global Trade: Comment. *The American Economic Review*, 96(5):1924–1930.

Miyagiwa, K. and Papageorgiou, C. (2007). Endogenous aggregate elasticity of substitution. *Journal of Economic Dynamics and Control*, 31(9):2899–2919.

Ngai, L. R. and Pissarides, C. A. (2007). Structural Change in a Multisector Model of Growth. *The American Economic Review*, 97(1):429–443.

Papageorgiou, C. and Chmelarova, V. (2005). Nonlinearities in Capital-Skill Complementarity. *Journal of Economic Growth*, 10(1):55–86.

Peracchi, F. (2006). Educational Wage Premia and the Distribution of Earnings: An International Perspective. In Hanushek, E. A. and Welch, F., editors, *Handbook of the Economics of Education*, volume 1, chapter 5, pages 189–254. Elsevier B.V.

Polgreen, L. and Silos, P. (2008). Capital-skill Complementarity and Inequality: A Sensitivity Analysis. *Review of Economic Dynamics*, 11(2):302–313.

Sato, K. (1967). A Two-Level Constant-Elasticity-of-Substitution Production Function. *Review of Economic Studies*, 34(2):201–218.

Solow, R. M. (1956). A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics*, 70(1):65–94.

Steger, T. M. (2008). Comment on "Looking back, looking ahead: Biased technological change, substitution and the wage gap". *Journal of Macroeconomics*, 30(2):714–717.

Temple, J. (2012). The calibration of CES production functions. *Journal of Macroeconomics*, 34(2):294–303.

van Neuss, L. (2019). The Drivers of Structural Change. *Journal of Economic Surveys*, 33(1):309–349.

van Reenen, J. (2011). Wage inequality, technology and trade: 21st century evidence. *Labour Economics*, 18(6):730–741.

Violante, G. L. (2008). Skill-Biased Technical Change. In Durlauf, S. N. and Blume, L. E., editors, *The New Palgrave Dictionary of Economics*, volume 2. Palgrave Macmillan, London, UK.

Wingender, A. M. (2015). Skill complementarity and the dual economy. *European Economic Review*, 74:269–285.

A Appendices

A.1 Two approaches of SBTC

The first approach of SBTC was popularized by Bound and Johnson (1992) and Katz and Murphy (1992). For illustrative purposes, suppose that high-skilled, L_S, and low-skilled labor, L_U, with factor-specific productivities, A_S and A_U, build a compound labor input, L, according to the following constant elasticity of substitution (CES) production function:

$$L = \left[(A_S L_S)^{\frac{\epsilon-1}{\epsilon}} + (A_U L_U)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \tag{19}$$

where ϵ is the elasticity of substitution between the two labor inputs. Based on (19), with competitive labor markets, the log of relative wages (ω_S/ω_U) is a function of relative productivity (A_S/A_U) and relative labor supply (L_S/L_U):

$$\log \left(\frac{\omega_S}{\omega_U} \right) = \frac{\epsilon - 1}{\epsilon} \log \left(\frac{A_S}{A_U} \right) - \frac{1}{\epsilon} \log \left(\frac{L_S}{L_U} \right). \tag{20}$$

As can immediately be seen from (20), if $\epsilon > 1$, that is, high- and low-skilled labor are (imperfect) substitutes, an increase in relative labor productivity will lead to an increase in the skill premium. Given observable time series of relative wages and relative factor supplies together with an estimate of ϵ, relative labor-augmenting technological change can be estimated residually from (20). Following Bound and Johnson (1992) and Katz and Murphy (1992), this inherently unobservable trend component has been the main explanation for the rapid increase in the U.S. skill premium starting in the 1980s. However, despite its simplicity, a main drawback of the first approach is that it does not explicitly account for the role of capital accumulation.[29] This gap was subsequently closed by the second approach.

The concept of capital-skill complementarity was initially formalized by Griliches (1969) and further applied by Krusell et al. (2000), Duffy et al. (2004), and Papageorgiou and Chmelarova (2005), among others. Capital-skill complementarity is present if the relative marginal product of high-skilled labor, and thus the wage premium, increases with capital accumulation. The most simple model of CSC applies the following two-level, three-factor CES production technology

$$Y = \left[\alpha \left(\beta K^{\theta} + (1-\beta) L_S^{\theta} \right)^{\frac{\psi}{\theta}} + (1-\alpha) L_U^{\psi} \right]^{\frac{1}{\psi}} \tag{21}$$

[29] As shown by Hulten (1992), Greenwood et al. (1997), and Cummins and Violante (2002), the substantial decline in the quality-adjusted price of equipment capital, since at least the 1960s, induced a considerable expansion of such capital in production.

with capital, K, high- and low-skilled labor, two distribution parameters, α and β, and two constant substitution parameters, θ and ψ. Based on (21), relative wages can be derived as

$$\frac{\omega_S}{\omega_U} = \frac{\alpha\left(\beta K^\theta + (1-\beta)L_S^\theta\right)^{\frac{\psi}{\theta}-1}(1-\beta)L_S^{\theta-1}}{(1-\alpha)L_U^{\psi-1}}. \tag{22}$$

Differentiation of the right-hand side of (22) with respect to K yields

$$\frac{\partial\left(\frac{\omega_S}{\omega_U}\right)}{\partial K} = \left(\frac{\psi-\theta}{\theta}\right)\frac{\theta\beta K^{\theta-1}}{\beta K^\theta + (1-\beta)L_S^\theta}\frac{\omega_S}{\omega_U}. \tag{23}$$

Equation (23) reveals that CSC requires $\psi > \theta$. That is, based on a production structure as formalized by (21), capital and high-skilled labor must be more complementary than capital and low-skilled labor.

A.2 Explicit normalization of the production system

In this appendix, we derive the explicitly normalized, two-stage CES production function (8), as applied in this paper.[30] The derivation can start with the primal Arrow et al. (1961) specification of the CES production function

$$Y = C\left[aK^{\frac{\sigma-1}{\sigma}} + (1-a)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{24}$$

where σ is the elasticity of substitution between capital and labor, C denotes a Hicks-neutral "efficiency" parameter and $0 < a < 1$ refers to a "distribution" parameter.[31] For a meaningful specification of the latter two parameters, de La Grandville (1989) and Klump and de La Grandville (2000) introduced the idea of "normalizing" CES production functions. Based on a given set of baseline values, K_0, L_0, $\mu_0 = \left[\frac{F_L}{F_K}\right]_0 = \left[\frac{w}{r}\right]_0$ and Y_0, production function (24) can be transformed as follows:

[30]See Klump and Preissler (2000) and Klump et al. (2012) for further details.

[31]A detailed derivation of the CES function applying the primal approach developed in Arrow et al. (1961) can be found in de La Grandville (2017, p. 85 - 88).

$$
\begin{aligned}
(25) \qquad Y_0 &= C\left[aK_0^{\frac{\sigma-1}{\sigma}} + (1-a)L_0^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
\Rightarrow \mu_0 &= \left[\frac{F_L}{F_K}\right]_0 = \frac{1-a}{a}\left(\frac{K_0}{L_0}\right)^{\frac{1}{\sigma}} \\
\Leftrightarrow a(\sigma) &= \frac{K_0^{1/\sigma}}{K_0^{1/\sigma} + L_0^{1/\sigma}\mu_0} \\
\Rightarrow C(\sigma) &= Y_0\left[\frac{K_0^{1/\sigma} + L_0^{1/\sigma}\mu_0}{K_0 + L_0\mu_0}\right]^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

This procedure delivers an explicit relationship between the elasticity of substitution, σ, and both C and a. Inserting $a(\sigma)$ and $C(\sigma)$ into (24) provides, after some rearranging, a normalized CES production function

$$
(26) \qquad Y = Y_0\left[\pi_0\left(\frac{K}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)\left(\frac{L}{L_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}
$$

similar to (3). The parameter $\pi_0 = \frac{r_0K_0}{r_0K_0+w_0L_0}$ denotes the capital share in total income at the point of normalization $t = 0$. At that point, (26) reduces to $Y = Y_0$ and is thus independent of σ. Additionally, a CES production function with factor-augmenting technological change has been introduced by David and van de Klundert (1965). It can be written as

$$
(27) \qquad Y = \left[(E_{K,t}K)^{\frac{\sigma-1}{\sigma}} + (E_{L,t}L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}
$$

where $E_{j,t}, j \in (K, L)$ represent the levels of efficiency of the two input factors. The functional form for the growth rates of the two efficiency levels is assumed to be

$$
(28) \qquad E_{j,t} = E_{j,o}e^{\lambda_j(t-t_0)}
$$

where λ_j denotes growth in technical progress and t represents time. To ensure that at the common baseline point, $t = 0$, the factor shares are not distorted by the level of factor efficiencies but are just equal to the distribution parameter π_0 and $1 - \pi_0$, it follows that

$$
\begin{aligned}
(29) \qquad E_{K,0} &= \frac{Y_0}{K_0}\left(\frac{1}{\pi_0}\right)^{\frac{\sigma}{1-\sigma}} \\
E_{L,0} &= \frac{Y_0}{L_0}\left(\frac{1}{1-\pi_0}\right)^{\frac{\sigma}{1-\sigma}}.
\end{aligned}
$$

Inserting (28) and the normalized values (29) into (27) leads to a normalized CES function that can be written as

$$Y = Y_0 \left[\pi_0 \left(\frac{K_t}{K_0} e^{\lambda_K (t-t_0)} \right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0) \left(\frac{L_t}{L_0} e^{\lambda_L (t-t_0)} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{30}$$

which is equivalent to (4) and (5), respectively. Again, as a test of consistent normalization, for $t = 0$, (30) reduces to $Y = Y_0$.

A.3 First-order conditions of the production system

Normalizing the price of the final good, p, to unity at all points in time leads to

$$p \equiv 1 = \left[\alpha_0^\sigma p_S^{1-\sigma} + (1-\alpha_0)^\sigma p_U^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where p_S refers to the price of the high-skill intermediate and p_U denotes the price of the low-skill intermediate. Based on the chosen normalization of the price index, the CES production function (8) implies the following four first-order conditions for profit maximization:

$$r_S = \alpha_0 \beta_0 Y_0 \widetilde{Y}^{\frac{1}{\sigma}} \widetilde{Y}_S^{\left(\frac{1}{\sigma_S} - \frac{1}{\sigma}\right)} \left(\frac{A_{KS}}{K_{S,0}} \right)^{\frac{\sigma_S - 1}{\sigma_S}} K_S^{-\frac{1}{\sigma_S}} \tag{31}$$

$$r_U = (1-\alpha_0) \gamma_0 Y_0 \widetilde{Y}^{\frac{1}{\sigma}} \widetilde{Y}_U^{\left(\frac{1}{\sigma_U} - \frac{1}{\sigma}\right)} \left(\frac{A_{KU}}{K_{U,0}} \right)^{\frac{\sigma_U - 1}{\sigma_U}} K_U^{-\frac{1}{\sigma_U}} \tag{32}$$

$$\omega_S = \alpha_0 (1-\beta_0) Y_0 \widetilde{Y}^{\frac{1}{\sigma}} \widetilde{Y}_S^{\left(\frac{1}{\sigma_S} - \frac{1}{\sigma}\right)} \left(\frac{A_S}{L_{S,0}} \right)^{\frac{\sigma_S - 1}{\sigma_S}} L_S^{-\frac{1}{\sigma_S}} \tag{33}$$

$$\omega_U = (1-\alpha_0)(1-\gamma_0) Y_0 \widetilde{Y}^{\frac{1}{\sigma}} \widetilde{Y}_U^{\left(\frac{1}{\sigma_U} - \frac{1}{\sigma}\right)} \left(\frac{A_U}{L_{U,0}} \right)^{\frac{\sigma_U - 1}{\sigma_U}} L_U^{-\frac{1}{\sigma_U}} \tag{34}$$

where $\widetilde{Y}, \widetilde{Y}_S$, and $\widetilde{Y}_U$ are given by equations (8), (4), and (5) in the text.

A.4 Proof of Proposition 1

In the following, we consider the ordering $\sigma > \sigma_U > \sigma_S$. We start by rearranging CMC and define

$$(35) \qquad P(A_S, A_U, A_{KS}, A_{KU}, K, L_S, K_S) \equiv \frac{\alpha_0}{(1-\alpha_0)}\left(\frac{\widetilde{Y}_S}{\widetilde{Y}_U}\right)^{-\frac{1}{\sigma}}\frac{\beta_0}{\gamma_0}\frac{(\widetilde{Y}_S/K_S)^{\frac{1}{\sigma_S}}}{(\widetilde{Y}_U/(K-K_S))^{\frac{1}{\sigma_U}}}\frac{(A_{KS}/K_{S,0})^{\frac{\sigma_S-1}{\sigma_S}}}{(A_{KU}/K_{U,0})^{\frac{\sigma_U-1}{\sigma_U}}} - 1 = 0$$

where $\widetilde{Y}_S$ and $\widetilde{Y}_U$ depend on both K_S and L_S. Equation (35) yields the implicit function $K_S = g(A_S, A_U, A_{KS}, A_{KU}, K, L_S)$, with

$$\frac{\partial P(\cdot)}{\partial K_S} = -\frac{1}{\sigma}\frac{\alpha_0}{(1-\alpha_0)}\frac{\partial\widetilde{Y}_S/\partial K_S}{\partial\widetilde{Y}_U/\partial K_U}\left\{\left(\frac{\partial\widetilde{Y}_S}{\partial K_S}\right)\widetilde{Y}_U^{\frac{1}{\sigma}}\widetilde{Y}_S^{-\frac{1+\sigma}{\sigma}} + \left(\frac{\partial\widetilde{Y}_U}{\partial K_S}\right)\widetilde{Y}_S^{-\frac{1}{\sigma}}\widetilde{Y}_U^{\frac{1-\sigma}{\sigma}}\right\} + \frac{\alpha_0}{(1-\alpha_0)}\left(\frac{\widetilde{Y}_S}{\widetilde{Y}_U}\right)^{-\frac{1}{\sigma}}\left\{\frac{\partial^2\widetilde{Y}_S/\partial K_S^2}{\partial\widetilde{Y}_U/\partial K_U} - \frac{\partial\widetilde{Y}_S/\partial K_S}{(\partial\widetilde{Y}_U/\partial K_U)^2}\frac{\partial^2\widetilde{Y}_U}{\partial K_U\partial K_S}\right\} < 0,$$

$$\frac{\partial P(\cdot)}{\partial A_S} = X\left(\frac{1}{\sigma_S} - \frac{1}{\sigma}\right)\alpha_0\widetilde{Y}_U^{\frac{1}{\sigma}-\frac{1}{\sigma_U}}\widetilde{Y}_S^{\frac{2\sigma-\sigma_S}{\sigma_S\sigma}-1}\widetilde{L}_S^{\frac{\sigma_S-1}{\sigma_S}}A_S^{-\frac{1}{\sigma_S}} > 0,$$

$$\frac{\partial P(\cdot)}{\partial A_U} = X\left(\frac{1}{\sigma} - \frac{1}{\sigma_U}\right)\beta_0\widetilde{Y}_U^{\frac{1-\sigma}{\sigma}}\widetilde{Y}_S^{\frac{1}{\sigma_S}-\frac{1}{\sigma}}\widetilde{L}_U^{\frac{\sigma_U-1}{\sigma_U}}A_U^{-\frac{1}{\sigma_U}} < 0,$$

$$\frac{\partial P(\cdot)}{\partial A_{KS}} = X\widetilde{Y}_U^{\frac{1}{\sigma}-\frac{1}{\sigma_U}}\left\{\left(\frac{\sigma_S-1}{\sigma_S}\right)\widetilde{Y}_S^{\frac{1}{\sigma_S}-\frac{1}{\sigma}}A_{KS}^{-1} + \left(\frac{1}{\sigma_S}-\frac{1}{\sigma}\right)\beta_0\widetilde{Y}_S^{\frac{2\sigma-\sigma_S}{\sigma_S\sigma}-1}A_{KS}^{-\frac{1}{\sigma_S}}\widetilde{K}_S^{\frac{\sigma_S-1}{\sigma_S}}\right\} > 0,$$

$$\frac{\partial P(\cdot)}{\partial A_{KU}} = X\widetilde{Y}_S^{\frac{1}{\sigma_S}-\frac{1}{\sigma}}\left\{\left(\frac{1-\sigma_U}{\sigma_U}\right)\widetilde{Y}_U^{\frac{1}{\sigma}-\frac{1}{\sigma_U}}A_{KU}^{-1} + \left(\frac{1}{\sigma}-\frac{1}{\sigma_U}\right)\gamma_0\widetilde{Y}_U^{\frac{1-\sigma}{\sigma}}A_{KU}^{-\frac{1}{\sigma_S}}\widetilde{K}_U^{\frac{\sigma_U-1}{\sigma_U}}\right\} < 0,$$

and

$$\frac{\partial P(\cdot)}{\partial L_S} = X\left\{\left(\frac{1}{\sigma_S}-\frac{1}{\sigma}\right)(1-\beta_0)\widetilde{Y}_U^{\frac{1}{\sigma}-\frac{1}{\sigma_U}}\widetilde{Y}_S^{\frac{2\sigma-\sigma_S}{\sigma_S\sigma}-1}\left(\frac{A_S}{L_{S,0}}\right)^{\frac{\sigma_S-1}{\sigma_S}}L_S^{-\frac{1}{\sigma_S}} + \left(\frac{1}{\sigma_U}-\frac{1}{\sigma}\right)(1-\gamma_0)\widetilde{Y}_U^{\frac{1-\sigma}{\sigma}}\widetilde{Y}_S^{\frac{1}{\sigma_S}-\frac{1}{\sigma}}\left(\frac{A_U}{L_{U,0}}\right)^{\frac{\sigma_U-1}{\sigma_U}}(1-2L_S)^{-\frac{1+\sigma_U}{2\sigma_U}}\right\} > 0,$$

where $X = \frac{\alpha_0}{(1-\alpha_0)}\frac{\beta_0}{\gamma_0}\frac{K_U^{1/\sigma_U}}{K_S^{1/\sigma_S}}\left(\frac{A_{KS}}{K_{S,0}}\right)^{\frac{\sigma_S-1}{\sigma_S}}\left(\frac{A_{KU}}{K_{U,0}}\right)^{\frac{1-\sigma_U}{\sigma_U}} > 0$. From this, it follows that $\frac{\partial g(\cdot)}{\partial L_S} = -\frac{\partial P(\cdot)/\partial L_S}{\partial P(\cdot)/\partial K_S} > 0$, $\frac{\partial g(\cdot)}{\partial A_S} = -\frac{\partial P(\cdot)/\partial A_S}{\partial P(\cdot)/\partial K_S} > 0$, $\frac{\partial g(\cdot)}{\partial A_U} = -\frac{\partial P(\cdot)/\partial A_U}{\partial P(\cdot)/\partial K_S} < 0$, $\frac{\partial g(\cdot)}{\partial A_{KS}} = -\frac{\partial P(\cdot)/\partial A_{KS}}{\partial P(\cdot)/\partial K_S} > 0$, and $\frac{\partial g(\cdot)}{\partial A_{KU}} = -\frac{\partial P(\cdot)/\partial A_{KU}}{\partial P(\cdot)/\partial K_S} < 0$. Next, we arrange (LMC) and define

(36) $$\Sigma(A_S, A_U, A_{KS}, A_{KU}, K, g(A_S, A_U, A_{KS}, A_{KU}, K, L_S)) \equiv$$
$$\Sigma(A_S, A_U, A_{KS}, A_{KU}, K, L_S) \equiv$$
$$(1-\eta)\frac{\alpha_0}{1-\alpha_0}\left(\frac{\widetilde{Y}_S}{\widetilde{Y}_U}\right)^{-\frac{1}{\sigma}}\frac{1-\beta_0}{1-\gamma_0}\frac{(\widetilde{Y}_S/L_S)^{\frac{1}{\sigma_S}}}{(\widetilde{Y}_U/L_U)^{\frac{1}{\sigma_U}}}\frac{(A_S/L_{S,0})^{\frac{\sigma_S-1}{\sigma_S}}}{(A_U/L_{U,0})^{\frac{\sigma_U-1}{\sigma_U}}} - 1 = 0$$

which yields L_S as an implicit function of A_S, A_U, A_{KS}, A_{KU}, and K, which we denote as $L_S = f(A_S, A_U, A_{KS}, A_{KU}, K)$. Based on (1) and (2), we have $1-\eta = L_U = (1-2L_S)^{0.5}$. From that, we can derive

$$\begin{aligned}\frac{\partial\Sigma(\cdot)}{\partial L_S} =&(1-2L_S)^{0.5}\frac{\alpha_0}{(1-\alpha_0)}\Bigg[\frac{1}{\sigma}\frac{\partial\widetilde{Y}_S/\partial L_S}{\partial\widetilde{Y}_U/\partial L_U}\left\{\left(\frac{\partial\widetilde{Y}_U}{\partial L_S}\right)\widetilde{Y}_S^{-\frac{1}{\sigma}}\widetilde{Y}_U^{\frac{1-\sigma}{\sigma}} - \left(\frac{\partial\widetilde{Y}_S}{\partial L_S}\right)\widetilde{Y}_U^{\frac{1}{\sigma}}\widetilde{Y}_S^{-\frac{1+\sigma}{\sigma}}\right\}\\
&+\left(\frac{\widetilde{Y}_S}{\widetilde{Y}_U}\right)^{-\frac{1}{\sigma}}\left\{\frac{\partial^2\widetilde{Y}_S/\partial L_S^2}{\partial\widetilde{Y}_U/\partial L_U} - \frac{\partial\widetilde{Y}_S/\partial L_S}{(\partial\widetilde{Y}_U/\partial L_U)^2}\frac{\partial^2\widetilde{Y}_U}{\partial L_U\partial L_S}\right\}\\
&-(1-2L_S)^{-1}\left(\frac{\widetilde{Y}_S}{\widetilde{Y}_U}\right)^{-\frac{1}{\sigma}}\frac{\partial\widetilde{Y}_S/\partial L_S}{\partial\widetilde{Y}_U/\partial L_U} + \frac{1-\beta_0}{1-\gamma_0}\frac{L_U^{1/\sigma_U}}{L_S^{1/\sigma_S}}\frac{(A_S/L_{S,0})^{\frac{\sigma_S-1}{\sigma_S}}}{(A_U/L_{U,0})^{\frac{\sigma_U-1}{\sigma_U}}}\times\\
&\left\{\left(\frac{1}{\sigma_S}-\frac{1}{\sigma}\right)\beta_0\widetilde{Y}_U^{\frac{1}{\sigma}-\frac{1}{\sigma_U}}\widetilde{Y}_S^{\frac{2\sigma-\sigma_S}{\sigma_S\sigma}-1}\left(\frac{A_{KS}}{K_{S,0}}\right)^{\frac{\sigma_S-1}{\sigma_S}}g(\cdot)^{-\frac{1}{\sigma_S}}\frac{\partial g(\cdot)}{\partial L_S}\right.\\
&\left.+\left(\frac{1}{\sigma_U}-\frac{1}{\sigma}\right)\gamma_0\widetilde{Y}_U^{\frac{1-\sigma}{\sigma}}\widetilde{Y}_S^{\frac{1}{\sigma_S}-\frac{1}{\sigma}}\left(\frac{A_{KU}}{K_{U,0}}\right)^{\frac{\sigma_U-1}{\sigma_U}}(K-g(\cdot))^{-\frac{1}{\sigma_U}}\frac{\partial g(\cdot)}{\partial L_S}\right\}\Bigg] < 0,\end{aligned}$$

where the latter two terms are positive due to the complementarity between capital and labor in production. However, as an increase in K_S is induced by an increase in L_S, the whole expression becomes negative. Furthermore, we obtain

$$\begin{aligned}\frac{\partial\Sigma(\cdot)}{\partial A_S} =&W\Bigg[\left(\frac{\sigma_S-1}{\sigma_S}\right)\frac{\widetilde{Y}_S^{\frac{1}{\sigma_S}-\frac{1}{\sigma}}}{A_S\widetilde{Y}_U^{\frac{1}{\sigma_U}-\frac{1}{\sigma}}} + \left(\frac{1}{\sigma_S}-\frac{1}{\sigma}\right)\widetilde{Y}_S^{\frac{2\sigma-\sigma_S}{\sigma_S\sigma}-1}\widetilde{Y}_U^{\frac{1}{\sigma_U}-\frac{1}{\sigma}}\times\\
&\left\{(1-\beta_0)\widetilde{L}_S^{\frac{\sigma_S-1}{\sigma_S}}A_S^{-\frac{1}{\sigma_S}} + \beta_0\left(\frac{A_{KS}}{K_{S,0}}\right)^{\frac{\sigma_S-1}{\sigma_S}}K_S^{-\frac{1}{\sigma_S}}\frac{\partial g(\cdot)}{\partial A_S}\right\}\\
&+\left(\frac{1}{\sigma_U}-\frac{1}{\sigma}\right)\gamma_0\left(\frac{A_{KU}}{K_{U,0}}\right)^{\frac{\sigma_U-1}{\sigma_U}}\widetilde{Y}_S^{\frac{1}{\sigma_S}-\frac{1}{\sigma}}\widetilde{Y}_U^{\frac{1-\sigma}{\sigma}}(K-K_S)^{-\frac{1}{\sigma_U}}\frac{\partial g(\cdot)}{\partial A_S}\Bigg] > 0,\end{aligned}$$

$$\begin{aligned}\frac{\partial\Sigma(\cdot)}{\partial A_U} =&W\Bigg[\left(\frac{1-\sigma_U}{\sigma_U}\right)\frac{\widetilde{Y}_S^{\frac{1}{\sigma_S}-\frac{1}{\sigma}}}{A_U\widetilde{Y}_U^{\frac{1}{\sigma_U}-\frac{1}{\sigma}}}+\left(\frac{1}{\sigma_U}-\frac{1}{\sigma}\right)\widetilde{Y}_S^{\frac{1}{\sigma_S}-\frac{1}{\sigma}}\widetilde{Y}_U^{\frac{1-\sigma}{\sigma}}\times\\ &\left\{\gamma_0\left(\frac{A_{KU}}{K_{U,0}}\right)^{\frac{\sigma_U-1}{\sigma_U}}(K-K_S)^{-\frac{1}{\sigma_U}}\frac{\partial g(\cdot)}{\partial A_U}-(1-\gamma_0)A_U^{-\frac{1}{\sigma_U}}\widetilde{L}_U^{\frac{\sigma_U-1}{\sigma_U}}\right\}\\ &+\left(\frac{1}{\sigma_S}-\frac{1}{\sigma}\right)\beta_0\left(\frac{A_{KS}}{K_{S,0}}\right)^{\frac{\sigma_S-1}{\sigma_S}}\widetilde{Y}_S^{\frac{2\sigma-\sigma_S}{\sigma_S\sigma}-1}\widetilde{Y}_U^{\frac{1}{\sigma_U}-\frac{1}{\sigma}}K_S^{-\frac{1}{\sigma_S}}\frac{\partial g(\cdot)}{\partial A_U}\Bigg]<0,\end{aligned}$$

$$\begin{aligned}\frac{\partial\Sigma(\cdot)}{\partial A_{KS}} =&W\Bigg[\left(\frac{1}{\sigma_S}-\frac{1}{\sigma}\right)\beta_0\widetilde{Y}_U^{\frac{1}{\sigma}-\frac{1}{\sigma_U}}\widetilde{Y}_S^{\frac{2\sigma-\sigma_S}{\sigma_S\sigma}-1}\widetilde{K}_S^{\frac{\sigma_S-1}{\sigma_S}}\left\{A_{KS}^{-\frac{1}{\sigma_S}}+A_{KS}^{\frac{\sigma_S-1}{\sigma_S}}K_S^{-1}\frac{\partial g(\cdot)}{\partial A_{KS}}\right\}\\ &+\left(\frac{1}{\sigma_U}-\frac{1}{\sigma}\right)\gamma_0\left(\frac{A_{KU}}{K_{U,0}}\right)^{\frac{\sigma_U-1}{\sigma_U}}\widetilde{Y}_S^{\frac{1}{\sigma_S}-\frac{1}{\sigma}}\widetilde{Y}_U^{\frac{1-\sigma}{\sigma}}(K-K_S)^{-\frac{1}{\sigma_U}}\frac{\partial g(\cdot)}{\partial A_{KS}}\Bigg]>0,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial\Sigma(\cdot)}{\partial A_{KU}} =&W\Bigg[\left(\frac{1}{\sigma_U}-\frac{1}{\sigma}\right)\gamma_0\widetilde{Y}_S^{\frac{1}{\sigma_S}-\frac{1}{\sigma}}\widetilde{Y}_U^{\frac{1-\sigma}{\sigma}}\widetilde{K_U}^{\frac{\sigma_U-1}{\sigma_U}}\left\{A_{KU}^{\frac{\sigma_U-1}{\sigma_U}}(K-K_S)^{-1}\frac{\partial g(\cdot)}{\partial A_{KU}}-A_{KU}^{-\frac{1}{\sigma_S}}\right\}\\ &+\left(\frac{1}{\sigma_S}-\frac{1}{\sigma}\right)\beta_0\left(\frac{A_{KS}}{K_{S,0}}\right)^{\frac{\sigma_S-1}{\sigma_S}}\widetilde{Y}_U^{\frac{1}{\sigma}-\frac{1}{\sigma_U}}\widetilde{Y}_S^{\frac{2\sigma-\sigma_S}{\sigma_S\sigma}-1}K_S^{-\frac{1}{\sigma_S}}\frac{\partial g(\cdot)}{\partial A_{KU}}\Bigg]<0,\end{aligned}$$

where $W=(1-2L_S)^{0.5}\frac{\alpha_0}{(1-\alpha_0)}\frac{1-\beta_0}{1-\gamma_0}\left(\frac{A_S}{L_{S,0}}\right)^{\frac{\sigma_S-1}{\sigma_S}}\left(\frac{A_{LU}}{L_{U,0}}\right)^{\frac{1-\sigma_U}{\sigma_U}}L_S^{-\frac{1}{\sigma_S}}L_U^{\frac{1}{\sigma_U}}>0$. Finally, we receive $\frac{dL_S}{dA_S}=-\frac{\partial\Sigma(\cdot)/\partial A_S}{\partial\Sigma(\cdot)/\partial L_S}>0$, $\frac{dL_S}{dA_U}=-\frac{\partial\Sigma(\cdot)/\partial A_U}{\partial\Sigma(\cdot)/\partial L_S}<0$, $\frac{dL_S}{dA_{KS}}=-\frac{\partial\Sigma(\cdot)/\partial A_{KS}}{\partial\Sigma(\cdot)/\partial L_S}>0$, and $\frac{dL_S}{dA_{KU}}=-\frac{\partial\Sigma(\cdot)/\partial A_{KU}}{\partial\Sigma(\cdot)/\partial L_S}<0$.

A.5 Proof of Lemmas 1 and 2

We initially show that the left-hand side of (LMC) is strictly decreasing in L_S. The CES production function (4) yields $\frac{\partial Y_S}{\partial L_S}=(1-\beta_0)Y_{S,0}\left(\frac{\widetilde{Y}_S}{L_S}\right)^{\frac{1}{\sigma_S}}\left(\frac{A_S}{L_{S,0}}\right)^{\frac{\sigma_S-1}{\sigma_S}}>0$ and $\frac{\partial^2 Y_S}{\partial L_S^2}=(1-\beta_0)\sigma_S^{-1}(A_S\widetilde{L}_S)^{\frac{\sigma_S-1}{\sigma_S}}\widetilde{Y}_S^{\frac{1-\sigma_S}{\sigma_S}}[(1+\frac{\beta_0}{1-\beta_0}\left(\frac{A_{KS}\widetilde{K}_S}{A_S\widetilde{L}_S}\right)^{\frac{\sigma_S-1}{\sigma_S}})^{-1}-1]<0$. Analogously, as $\partial L_U/\partial L_S<0$, production function (5) yields $\partial Y_U/\partial L_S<0$ and $\partial^2Y_U/\partial L_S^2>0$. Finally, $L_S=0.5-0.5a^2$ leads to $(1-\eta)=a=\sqrt{1-2L_S}$. Partial derivation of a with respect to L_S yields $\partial a/\partial L_S=-(1-2L_S)^{-0.5}<0$. Combining these effects results in $\partial LMC/\partial L_S<0$, which is the usual *substitution effect* (Acemoglu, 2002). Furthermore, we denote the numerator in (LMC) as

$$\Psi=(1-\eta)\alpha_0\widetilde{Y}_S^{-\frac{1}{\sigma}}(1-\beta_0)(\widetilde{Y}_S/L_S)^{\frac{1}{\sigma_S}}(A_S/L_{S,0})^{\frac{\sigma_S-1}{\sigma_S}} \tag{37}$$

where Y_S is defined by (4). Analogously, we denote the denominator in (LMC) as

$$(38) \qquad \xi = (1-\alpha_0)\widetilde{Y}_U^{-\frac{1}{\sigma}}(1-\gamma_0)(\widetilde{Y}_U/L_U)^{\frac{1}{\sigma_U}}(A_U/L_{U,0})^{\frac{\sigma_U-1}{\sigma_U}}$$

where Y_U is defined by (5). Partial derivation of (37) with respect to K_S yields

$$(39) \qquad \frac{\partial\Psi}{\partial K_S} = (1-\eta)\alpha_0(1-\beta_0)\beta_0(L_S K_S)^{-\frac{1}{\sigma_S}}\left(\frac{A_{KS}A_S}{K_{S,0}L_{L,0}}\right)^{\frac{\sigma_S-1}{\sigma_S}} \times \left(\frac{1}{\sigma_S}-\frac{1}{\sigma}\right)\left[\beta_0(A_{KS}\widetilde{K}_S)^{\frac{\sigma_S-1}{\sigma_S}} + (1-\beta_0)(A_S\widetilde{L}_S)^{\frac{\sigma_S-1}{\sigma_S}}\right]^{\frac{\sigma_S}{\sigma_S-1}\left(\frac{1}{\sigma_S}-\frac{1}{\sigma}\right)-1}$$

and thus $sgn\left(\frac{\partial\Psi}{\partial K_S}\right) = sgn(\sigma-\sigma_S)$. Analogously, partial derivation of (38) with respect to K_U yields

$$(40) \qquad \frac{\partial\xi}{\partial K_U} = (1-\alpha_0)(1-\gamma_0)\gamma_0(L_U K_U)^{-\frac{1}{\sigma_U}}\left(\frac{A_{KU}A_U}{K_{U,0}L_{U,0}}\right)^{\frac{\sigma_U-1}{\sigma_U}} \times \left(\frac{1}{\sigma_U}-\frac{1}{\sigma}\right)\left[\gamma_0(A_{KU}\widetilde{K}_U)^{\frac{\sigma_U-1}{\sigma_U}} + (1-\gamma_0)(A_U\widetilde{L}_U)^{\frac{\sigma_U-1}{\sigma_U}}\right]^{\frac{\sigma_U}{\sigma_U-1}\left(\frac{1}{\sigma_U}-\frac{1}{\sigma}\right)-1}$$

and thus $sgn\left(\frac{\partial\xi}{\partial K_U}\right) = sgn(\sigma-\sigma_U)$. Analogously, (CMC) is strictly decreasing in K_S. The production function (4) yields $\frac{\partial Y_S}{\partial K_S} = \beta_0 Y_{S,0}\left(\frac{\widetilde{Y}_S}{K_S}\right)^{\frac{1}{\sigma_S}}\left(\frac{A_{KS}}{K_{S,0}}\right)^{\frac{\sigma_S-1}{\sigma_S}} > 0$ and $\frac{\partial^2 Y_S}{\partial K_S^2} = \beta_0\sigma_S^{-1}(A_{KS}\widetilde{K}_S)^{\frac{\sigma_S-1}{\sigma_S}}\widetilde{Y}_S^{\frac{1-\sigma_S}{\sigma_S}}[(1+\frac{1-\beta_0}{\beta_0}\left(\frac{A_S\widetilde{L}_S}{A_{KS}\widetilde{K}_S}\right)^{\frac{\sigma_S-1}{\sigma_S}})^{-1}-1] < 0$. Furthermore, as $\partial K_U/\partial K_S = -1$, production function (5) yields $\partial Y_U/\partial K_S = -(\partial Y_U/\partial K_U) < 0$ and $\partial^2 Y_U/\partial K_S^2 > 0$. Combining these partial effects results in $\partial CMC/\partial K_S < 0$, which is again the usual *substitution effect.* Next, we denote the numerator in the (CMC) as

$$(41) \qquad \Omega = \alpha_0\widetilde{Y}_S^{-\frac{1}{\sigma}}\beta_0(\widetilde{Y}_S/K_S)^{\frac{1}{\sigma_S}}(A_{KS}/K_{S,0})^{\frac{\sigma_S-1}{\sigma_S}}$$

where Y_S is defined by (4). Partial derivation of (41) with respect to L_S yields:

$$(42) \qquad \frac{\partial\Omega}{\partial L_S} = \alpha_0\beta_0(1-\beta_0)(L_S K_S)^{-\frac{1}{\sigma_S}}\left(\frac{A_S A_{KS}}{K_{S,0}L_{S,0}}\right)^{\frac{\sigma_S-1}{\sigma_S}} \times \left(\frac{1}{\sigma_S}-\frac{1}{\sigma}\right)\left[\beta_0(A_{KS}\widetilde{K}_S)^{\frac{\sigma_S-1}{\sigma_S}} + (1-\beta_0)(A_S\widetilde{L}_S)^{\frac{\sigma_S-1}{\sigma_S}}\right]^{\frac{\sigma_S}{\sigma_S-1}\left(\frac{1}{\sigma_S}-\frac{1}{\sigma}\right)-1}$$

and thus $sgn\left(\frac{\partial\Omega}{\partial L_S}\right) = sgn(\sigma - \sigma_S)$. Analogously, we denote the denominator in (CMC) as

$$\Theta = (1-\alpha_0)\widetilde{Y}_U^{-\frac{1}{\sigma}}\gamma_0(\widetilde{Y}_U/K_U)^{\frac{1}{\sigma_U}}(A_{KU}/K_{U,0})^{\frac{\sigma_U-1}{\sigma_U}} \tag{43}$$

where Y_U is defined by (5). Accordingly,

$$\begin{aligned}\frac{\partial\Theta}{\partial L_U} =& (1-\alpha_0)\gamma_0(1-\gamma_0)(L_U K_U)^{-\frac{1}{\sigma_U}}\left(\frac{A_U A_{KU}}{A_{U,0}K_{U,0}}\right)^{\frac{\sigma_U-1}{\sigma_U}} \times \\ &\left(\frac{1}{\sigma_U}-\frac{1}{\sigma}\right)\left[\gamma_0(A_{KU}\widetilde{K}_U)^{\frac{\sigma_U-1}{\sigma_U}} + (1-\gamma_0)(A_U\widetilde{L}_U)^{\frac{\sigma_U-1}{\sigma_U}}\right]^{\frac{\sigma_U}{\sigma_U-1}\left(\frac{1}{\sigma_U}-\frac{1}{\sigma}\right)-1}\end{aligned} \tag{44}$$

with $sgn\left(\frac{\partial\Theta}{\partial L_U}\right) = sgn(\sigma - \sigma_U)$ where $\partial L_U/\partial L_S < 0$.

A.6 Choice and construction of benchmark values

In this appendix, we outline the choice and construction of benchmark values applied for the numerical evaluation of our model in chapter 4. We start by setting $Y_0 = 1$. In appendix A.2, we saw that the distribution parameter α_0 can also be written as

$$\alpha_0 = \frac{Y_{S,0}^{\frac{1}{\sigma}}}{Y_{S,0}^{\frac{1}{\sigma}} + Y_{U,0}^{\frac{1}{\sigma}}\mu_0} \tag{45}$$

We set $Y_{S,0} = Y_{U,0} = \mu_0 = 1$ to obtain $\alpha_0 = 0.5$. Furthermore, at the point of normalization, $t = 0$, we have $L_S = L_{S,0}, L_U = L_{U,0}, K_S = K_{S,0}$, and $K_U = K_{U,0}$. From that, (LMC) simplifies to

$$L_{U,0} = \left(L_{S,0}\frac{1-\alpha_0}{\alpha_0}\frac{1-\gamma_0}{1-\beta_0}\right)^{0.5} \tag{46}$$

We can substitute the benchmark value of high-skilled labor, $L_{S,0} = 0.5 - 0.5L_{U,0}^2$, and rearrange (46) to obtain

$$L_{U,0} = \left(1 + \frac{2\alpha_0(1-\beta_0)}{(1-\alpha_0)(1-\gamma_0)}\right)^{-0.5} \tag{47}$$

For given values of α_0, β_0, and γ_0, the benchmark allocation of labor, $L_{S,0}$ and $L_{U,0}$, can unequivocally be obtained from (47). Next, at the point of normalization, (CMC) reduces to

$$K_{U,0} = K_{S,0}\frac{1-\alpha_0}{\alpha_0}\frac{\gamma_0}{\beta_0} \tag{48}$$

where capital market clearing requires $K_{S,0} = K_0 - K_{U,0}$. As before, for given values of α_0, β_0, and γ_0, together with an assumption on the initial aggregate capital stock K_0, the benchmark values for $K_{S,0}$ and $K_{U,0}$ can be obtained from (48). Finally, based on the above calculations and chosen values for σ_S and σ_U, the marginal rates of technical substitution $\mu_{S,0}$ and $\mu_{U,0}$ within the two intermediate sectors are given by

$$\beta_0 = \frac{K_{S,0}^{\frac{1}{\sigma_S}}}{K_{S,0}^{\frac{1}{\sigma_S}} + L_{S,0}^{\frac{1}{\sigma_S}}\mu_{S,0}}$$
$$\Leftrightarrow \mu_{S,0} = \frac{(1-\beta_0)K_{S,0}^{\frac{1}{\sigma_S}}}{\beta_0 L_{S,0}^{\frac{1}{\sigma_S}}} \tag{49}$$

and

$$\gamma_0 = \frac{K_{U,0}^{\frac{1}{\sigma_U}}}{K_{U,0}^{\frac{1}{\sigma_U}} + L_{U,0}^{\frac{1}{\sigma_U}}\mu_{U,0}}$$
$$\Leftrightarrow \mu_{U,0} = \frac{(1-\gamma_0)K_{U,0}^{\frac{1}{\sigma_U}}}{\gamma_0 L_{U,0}^{\frac{1}{\sigma_U}}} \tag{50}$$

This completes the choice and the construction of the benchmark values Y_0, $Y_{S,0}$, $Y_{U,0}$, $K_{S,0}$, $K_{U,0}$, $L_{S,0}$, $L_{U,0}$, $\mu_0 = \left[\frac{p_U}{p_S}\right]_0$, $\mu_{S,0} = \left[\frac{\omega_S}{r}\right]_0$, and $\mu_{U,0} = \left[\frac{\omega_U}{r}\right]_0$.