TECHNISCHE UNIVERSITÄT DRESDEN
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Makespan Minimization in Re-entrant Permutation Flow Shops

Dissertation

to achieve the academic degree
Doctor rerum politicarum (Dr. rer. pol.)

by
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Date of submission: 01.02.2017
Date of defense: 29.08.2017

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List of Abbreviations

ABC	Artificial bee colony
ACO	Ant colony optimization
B&B	Branch and bound
B&C	Branch and cut
BN	Best neighbor
B&P	Branch and price
CDS	Algorithm of Campbell, Dudek and Smith
EA	Evolutionary algorithms
EDD	Earliest due date
FI	First improvement
FL	Fuzzy logic
GA	Genetic algorithm
GRASP	Greedy randomized search procedure
LB	Limited buffer capacity
LPT	Longest processing time first
LTPT	Longest total processing time first
LTPTJ	Longest total processing time job first
LTPTL	Longest total processing time level first
MA	Memetic algorithm
MIP	Mixed integer programming
MO	Missing operations
NEH	Algorithm of Nawaz, Enscore and Ham
NEHJ	NEH job
NEHL	NEH level
NNC	Non-negativity constraints
PN	Petri nets
PR	Priority rules
PSO	Particle swarm optimization
RFS	Re-entrant flow shop
RJS	Re-entrant job shop

RPFS	Re-entrant permutation flow shop
SA	Simulated annealing
SI	Swarm intelligence
SIRO	Service in random order
SIROJ	Service in random order job
SIROL	Service in random order level
SPT	Shortest processing time first
ST	Setup times
STPT	Shortest total processing time first
STPTJ	Shortest total processing time job first
STPTL	Shortest total processing time level first
TA	Threshold accepting
TP	Throughput
TS	Tabu search
VNS	Variable neighborhood search
WIP	Work-in-process

List of Symbols

A	A sufficiently large number
a	Neighborhood hierarchy index
b	Block index
B	Number of blocks in a permutation
BS	Label indicating values concerning model Y with basic sequence
batch	Batch processing
C_{max}^{BS}	Makespan obtained by model Y with basic sequence
C_i	Completion time of job i
$C_{max}^{I}(\pi_t)$	Income makespan value of a permutation
C_{max}	Makespan
$C_{max}(\pi)$	Makespan of permutation π
$C_{max}^{O}(\pi_t)$	Outcome makespan value of a permutation
C_{max}^{PC}	Makespan obtained by the model of Pan / Chen (2003)
$C_{max}^{Q,\,consistent}$	Makespan with Q consistent sublots per job
$C_{max}^{Q,\,consecutive}$	Makespan with Q consistent consecutive sublots
$C_{max}^{Q,\,consecutive\,equal}$	Makespan with Q consecutive equal sublots per job
$C_{max}^{Q,\,equal}$	Makespan with Q equal sublots per job
C_{max}^{Q}	Makespan obtained by the model with Q consistent sublots per job
$C_{max}^{Q,\,resize}$	Makespan with resizing of Q sublots per job
$C_{max}^{Q,\,resize\,consecutive}$	Makespan with resizing of Q consecutive sublots per job
C_{max}^{X}	Makespan obtained by model X
C_{max}^{Y0}	Makespan obtained by model Y with missing operations
C_{max}^{Y}	Makespan obtained by model Y
ΔC_{max}^{*}	Makespan reduction obtained by a metaheuristic with a changed neighborhood setting
$\Delta C_{max}^{3\|1}$	Makespan reduction with $Q = 3$ instead of $Q = 1$ sublots per job
ΔC_{max}^{best}	Makespan deviation between a certain solution and the best of the generated solutions
ΔC_{max}^{init}	Makespan deviation between the initial solution and the solution obtained by an improvement method

ΔC_{max}^{LS}	Makespan reduction of using lot streaming without predetermined number of sublots per job
ΔC_{max}	Makespan deviation
ΔC_{max}^{MIP}	Makespan deviation between the MIP solution and the solution obtained by a metaheuristic
ΔC_{max}^{Q}	Makespan deviation between any tested lot streaming model and a model with simple consistent sublots with Q sublots per job
$\Delta C_{max}^{Q\|Q-1}$	Makespan deviation if there is one sublot more per job
$chains$	Chain precedence constraints
ct	Computation time
ct^{BS}	Computation time with model Y with basic sequence
ct^{PC}	Computation time with the model of Pan / Chen (2003)
ct^{X}	Computation time with model X
ct^{Y}	Computation time with model Y
ct^{Y0}	Computation time with model Y with missing operations
Δct	Computation time difference
D	Number of parts in a job / lot size of a job
D^i	Number of parts in job i / lot size of job i
d_i	Due date of job i
D_i	Absolute deviation between a job i's completion and its due date d_i
E_i	Earliness of job i
F	Indicating a flow shop if $\alpha_1 = F$
$\overline{F}$	Total flow time
F^{asp}	Function value of an aspiration function in a tabu search
F_I	Income value of an aspiration function in a tabu search
$f_i\left(C_i\right)$	Sum objective function based on the on completions time of jobs
$F_I\left(C_{max}\left(\pi_t\right)\right)$	Income makespan value of a permutation π_t
$f_{max}\left(C_i\right)$	Bottleneck objective function based on completions time of jobs
F_O	Outcome value of an aspiration function in a tabu search
$F_O\left(C_{max}\left(\pi_t\right)\right)$	Outcome makespan value of a permutation π_t
$\overline{F}_\omega$	Total weighted flow time
G	Indicating a general shop if $\alpha_1 = G$
g, g'	Operation indices
$\overline{G}$	Graph representing precedence constraints
G^Q	Gap value for problems with a maximum of Q sublots per job
h_{kj}	Starting time of the jth member of a permutation of jobs on machine k

i, i', i''	Job indices
$\overline{I}$	Total idle time of all machines
Inc_1	Set of test instances 1
Inc_2	Set of test instances 2
Inc_3	Set of test instances 3
Inc_4	Set of test instances 4
Inc_5	Set of test instances 5
$intree$	Intree precedence constraints
It	Number of CPLEX iterations
It^{BS}	Number of CPLEX iterations with model Y with basic sequence
It^{PC}	Number of CPLEX iterations with the model of Pan/Chen (2003)
It^{Q}	Number of CPLEX iterations with a model with Q sublots per job
It^{X}	Number of CPLEX iterations with model X
It^{Y}	Number of CPLEX iterations with model Y
It^{Y0}	Number of CPLEX iterations with model Y with missing operations
ΔIt	Difference of the numbers of iterations
J	Number of positions in a permutation
J	Indicating a job shop if $\alpha_1 = J$
j, j', j'', j'''	Sequence position indices
j_i	Sequence of position of job i
j_{il}	Sequence of position of job i' level l
j_{iql}	Sequence of position of job i' sublot q in level l
k, k', k''	Machine indices
K, $\overline{K}$	Parameters used to calculate a new temperature
l, l'	Level indices
L	Number of levels per job
L_i	Lateness of job i
m	Number of machines
MRT_k	Machine ready time of machine k
n	Number of jobs
$\mathcal{N}$	Neighborhood, denoted with "N" in figures
$N^{available}$	Set of available jobs
$\mathcal{N}_t^a$	Neighborhood t in neighborhood hierarchy a
N^{ready}	Set of ready jobs
$\mathcal{N}_t$	Neighborhood t
$\mathcal{N}(\pi)$	Neighborhood of permutation π
O	Indicating an open shop if $\alpha_1 = O$

o_i	Number of operations of job i
$outtree$	Outtree precedence constraints
Par	Parallel machines
P	Identical parallel machines
$\overline{P}$	Probability
p	Processing time
PC	Label indicating values concerning model PAN / CHEN (2003)
p_g^i	Processing time of the gth operation of job i
p_{lk}^i	In Chapters 3 and 4:
	Processing time of job i in level l on machine k
p_{lk}^i	In Chapter 5:
	Processing time of one unit of job i in level l on machine k
p_{lk}	Processing time of level l on machine k in an one-job problem
$PMPM$	Multipurpose machines with identical processing speed
$pmtn$	Job preemption
$prec$	Precedence constraints
q, q', q''	Sublot indices
Q	In Chapters 2: Denoting uniform parallel
	machines in the α_1 field of the 3-field classification scheme
Q	In Chapters 5: Maximum number of sublots per job
Q_{max}	Limit for incrementing the number of sublots per job level
$QMPM$	Multipurpose machines with uniform processing speed
R	Unrelated parallel machines
$\overline{r}$	Uniformly distributed random number, $0 \leq \overline{r} \leq 1$
$recrc$	Re-entrant material flow
res	Resource constraints
r_i	Release date of job i
$R^{iqq'l}$	Binary variable, takes the value 1 if sublot q of job i starts
	into level $l+1$ before sublot q' of the same job finished level l
RT_i	Ready time of job i
RT_{il}	Ready time of job i's level l
S_i	Squared deviation between completion time and due date of job i
s_g^i	Starting time of the gth operation of job i
s_{lk}^i	Starting time of job i in level l on machine k
s_{lk}^{iq}	Starting time of job i's sublot q in level l on machine k
s_{lk}	Starting time of level l on machine k
t, t'	Iteration indices and subiterition indices

	(Neighborhood indices for the variable neighborhood search)
T_i	Tardiness of job i
t_{max}	Maximum number of iterations of an heuristic
T^{min}	Minimum temperature in a simulated annealing algorithm
TPT_i	Total processing time of job i
TPT_{il}	Total processing time of level l of job i
T^t	Temperature in iteration t in a simulated annealing algorithm
t'^t_{max}	Maximum number of iteration on the tth temperature level
u_b	Last member of block b
U_i	Unit penalty of job i
VNSa BN	Variable neighborhood search with neighborhood hierarchy a and best neighbor local search
VNSa BN*	Variable neighborhood search with neighborhood hierarchy a and best neighbor local search and Nowicki across block moves
VNSa FI	Variable neighborhood search with neighborhood hierarchy a and first improvement local search
VNSa FI*	Variable neighborhood search with neighborhood hierarchy a and first improvement local search and Nowicki across block moves
W	Matrix containing all parameters w^i_g
w_{ig}	Integer parameter indicating on which machine the gth operation of job i is performed
X	Indicating a mixed shop if $\alpha_1 = X$
X	Label indicating values concerning model X
$\overline{X}$	Remainder of parts after dividing the lot size of the job by Q
x, x'	Specific solutions of optimization problems
x_{ilj}	Binary variable, takes the value 1 if level l of job i is scheduled on position j
X^{iq}	Size of job i's sublot q
X^{iq}_l	Size of job i's sublot q in level l
X^q	Size of sublot q
Y	Label indicating values concerning model Y
Y0	Label indicating model Y with missing operations
$y_{ii'}$	Binary variable, takes the value 1 if level l of job i is scheduled before the same level of job i'
$y_{ii'k}$	Binary variable, takes the value 1 if job i is scheduled before job i' on machine k
$y^{il}_{i'l'}$	Binary variable, takes the value 1

	if level l of job i is scheduled before level l' of job i'
$y_{iql}^{i'q'l'}$	Binary variable, takes the value 1 if sublot q of job i in level l is scheduled before sublot q' of job i' in level l'
α	Machine characteristics
β	Job characteristics
γ	Objective characteristics
π, π'	Permutations of job levels
π_{best}	Best known permutation of job levels
π_{init}^{best}	Best initial permutation of job levels
π_{BN}	Permutation of job levels obtained by a best neighbor search
π_{init}	Initial permutation of job levels
π_{meta}	Permutation of job levels obtained by a metaheuristic
π_{meta^*}	Permutation of job levels obtained by a metaheuristic with changed neighborhood setting
π_{meta}^{LS}	Permutation of job level sublots obtained by a metaheuristic with a framework dependent number of sublots per job
π_{meta}^{Q}	Permutation of job level sublots obtained by a metaheuristic with Q sublots per job
π_{MIP}^{Q}	Permutation of job level sublots obtained by a mixed integer model with Q sublots per job
π_t	Permutation of job levels in iteration t
ω_i	Weight of job i in the objective value
$\circ$	No value for a problem characteristic

1 Introduction

1.1 Motivation

Flow shops have received much research interest since 1954 when Johnson[1] first addressed the problem. The fundamental characteristic of a flow shop is that the sequence of working stations to visit is the same for each production job. Common objectives for defining the sequences of operations for all machines are to increase the machine usage and to minimize the throughput times of production jobs in the manufacturing system. Both objectives are competing.[2] There is a multitude of extensions for the flow shop problem, e.g. Setup times (ST) or sequence-dependent setup times, batch processing, waiting time restrictions and lot streaming. An extension that receives great attention, especially in the semiconductor industry, is a re-entrant material flow. This means that the jobs need to visit at least one working station multiple times. The rising complexity of production environments and material flows leads to a growing importance of re-entrant characteristics in scheduling problems.

The literature review of Danping / Lee (2011) on re-entrant scheduling problems of for the period between 1994 and 2009 contains 61 journal articles.[3] After conducting search queries for the time between 2010 and 2015, 94 journal articles on re-entrant scheduling problems were found, indicating the emerging relevance of the topic. The fields of applications are numerous. Specifically, Re-entrant flow shop (RFS) scheduling problems occur in practical applications, such as the manufacturing of semiconductors and electronic devices, airplane engines, and petrochemical production.[4] In integrated circuit manufacturing, a particular integrated circuit may return several times to the photo-lithographic process in order to place several layers of patterns on the wafer.[5] In a painting shop, parts may move back and forth between the painting and baking departments for successive coats of paint.[6] Further occurrences of Re-entrant permutation

[1] See Johnson (1954): *Optimal two- and three-stage production schedules*, pp. 61–68.
[2] See Gutenberg (1983): *Produktion*, p. 216.
[3] See Danping / Lee (2011): *Review of research for re-entrant scheduling*, p. 2222.
[4] See Hekmatfar / Fatemi Ghomi / Karimi (2011): *Reentrant flow shops with setup times*, p. 4530.
[5] See Kang / Lee (2007): *Make-to-order scheduling in foundry semiconductor fabrication*, p. 616.
[6] See Emmons / Vairaktarakis (2013): *Flow shop scheduling*, p. 271.

flow shop (RPFS) are found in the automotive industry[7], weapon production[8], mold and die processes[9], and the textile industry[10].

The computational complexity of RPFS problems requires heuristic solution approaches for large problem sizes. The problem provides interesting structural properties for the application of a Variable neighborhood search (VNS) because of the repeated processing of jobs on several machines. In addition, the effects of lot streaming have not been investigated in connection with re-entrant characteristics for permutation flow shops until now, despite the massive savings in makespan provided by applying lot streaming in regular flow shop and job shop problems. Hence, the different characteristics of job sublots and their impact on the makespan of a schedule are examined in this thesis and the heuristic solution methods are adjusted to manage the problem's extension.

1.2 Structure and Methodology

The aim of this work is to examine re-entrant permutation flow shops regarding makespan minimization.

An overview of the current literature is provided, which has been selected based on a search methodology described in Section 3.2. Within the literature review, the occurrence of certain problem characteristics is quantified based on the 3-field classification scheme for machine scheduling problems of Graham et al. (1979).[11] An overview is also given of the different methods of modeling and solving scheduling optimization problems that are applied to re-entrant permutation flow shop problems. These methods can be divided into exact methods, including Mixed integer programming (MIP) models, that are used in connection with commercial solver software, such as CPLEX or Gurobi by applying Branch and bound (B&B) or Branch and cut (B&C) algorithms. The second group of solution methods contains heuristics. Within this group of solution methods constructive heuristics can be differentiated from metaheuristics. Constructive heuristics also include Priority rules (PR), which can be used to provide initial solutions for metaheuristic improvement methods, such as Tabu search (TS), Simulated annealing (SA) and variable neighborhood search.

Two modeling approaches are tested regarding their computational performance in different problem sizes. The approaches differ in the kind of binary variables used to

[7] See Chong / Jingshan (2010): *Approximate Analysis of Reentrant Lines*, p. 708 and See Liu / Li / Chiang (2010): *Re-entrant lines with unreliable machines and finite buffers*, p. 1151.

[8] See Chen et al. (2012): *Flexible job shop scheduling*, p. 10016.

[9] See Gomes / Barbosa-Póvoa / Novais (2013): *Reactive scheduling*, pp. 5120–5121.

[10] See Topaloglu / Kilincli (2010): *Shifting bottleneck heuristic for reentrant job shops*, p. 790.

[11] See Graham et al. (1979): *Optimization and approximation in sequencing and scheduling*, pp. 288–290.

represent the permutation. The models in Chapter 5 are based on the formulation that performed best in the computational experiments in Section 4.4.1. Furthermore, the necessity of including Missing operations (MO) in re-entrant permutation flow shops is constituted. The structure of the permutation is justified based on the makespan values achieved. The impact of both the structure of the permutation and the appropriate management of missing operations on the makespan is examined with different MIP models.

Various heuristics are developed due to the complexity of scheduling problems. The solutions obtained by heuristics are compared to each other and to the results delivered by applying solver software to the suggested MIP models. The first group of the proposed heuristics contains priority rules, mainly used for the initialization of the later used metaheuristic improvement methods. The tested metaheuristics are tabu search, simulated annealing, variable neighborhood search and simple local search approaches such as Best neighbor (BN) and First improvement (FI). As suggested by a variable neighborhood search, several mechanisms for modifying a solution are developed for the problem and implemented within each improvement method. The computational experiments compare the solution methods regarding solution quality and computation time. All experiments are performed on a 64-bit Windows 10 system with a 2.5 GHz Intel i7-4710HQ quad core processor and 16 GB RAM. IBM CPLEX 12.4 is used as the MIP solver and the examined heuristics are coded in C++.

The models are extended to include lot streaming, which allows different modes of sublots regarding size and processing sequence in re-entrant permutation flow shops. The different constraints determining the characteristics of sublots are tested in different MIP models. Additionally, the heuristic solution approaches are adjusted and tested for the RPFS with lot streaming and the preferred form of sublots.

The examined research questions in this thesis are:

Q1: What is the state of research for re-entrant permutation flow shops?

Q2: How can missing operations and mixed levels be formulated in a mathematical model and what are the effects on the optimal makespan of a schedule? What problem sizes can be solved optimally?

Q3: How does the application of problem-specific constructive heuristics and adjusted metaheuristics affect the solution quality and computational performance in re-entrant permutation flow shop problems?

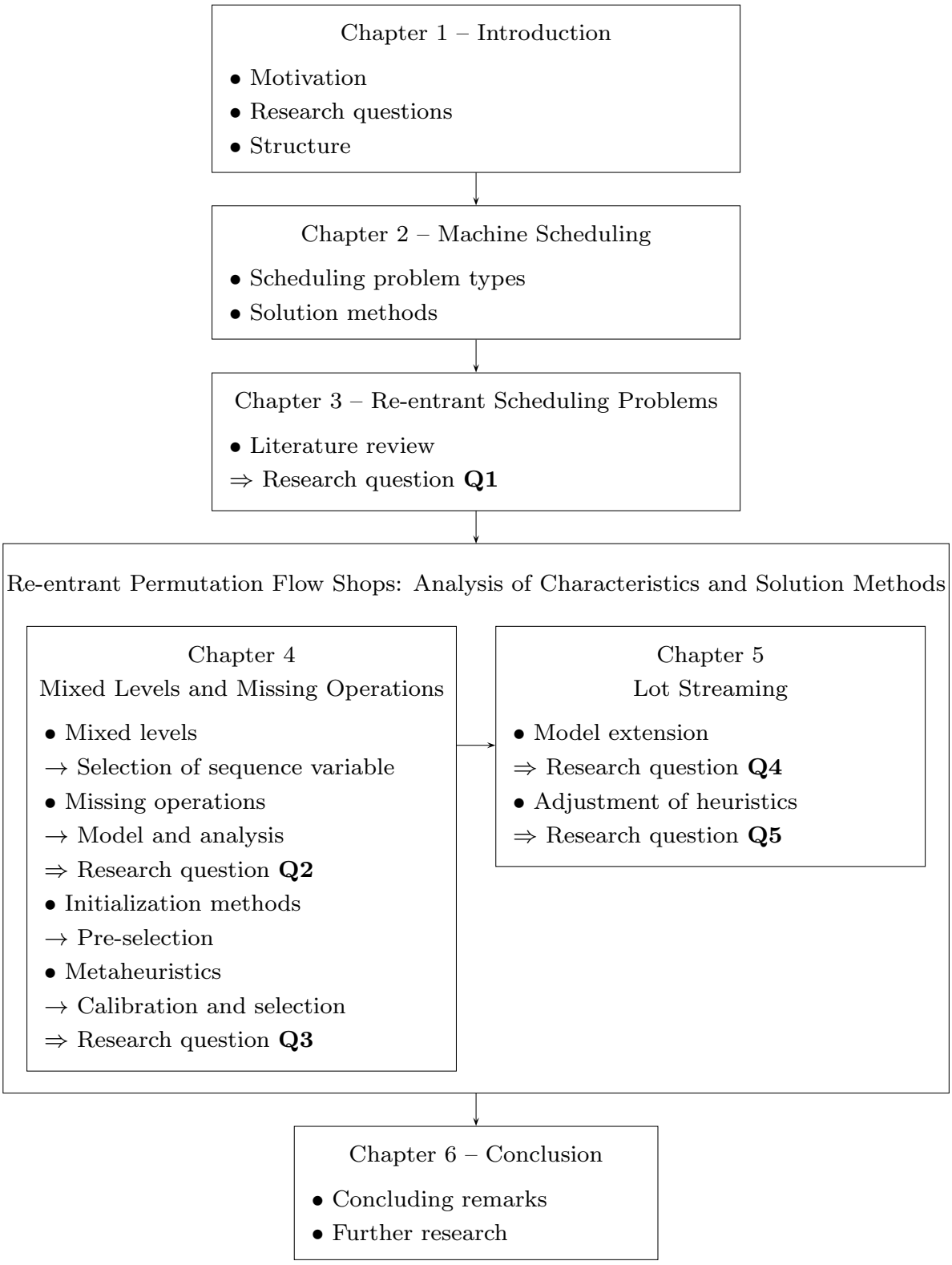
Q4: What is the impact of different forms of lot streaming on the makespan?

Q5: What numbers of sublots per job dependent on the problem size are suitable for

metaheuristics?

The structure of the thesis is shown in Figure 1.1. Chapter 1 discusses the motivation for the work and explains the methodology. Chapter 2 describes the fundamentals in scheduling and introduces expressions used in the following chapters. Chapter 3 contains a survey on literature concerning the examined problem and answers research question **Q1**. A model and heuristics for solving the re-entrant permutation flow shop problem with missing operations are proposed and examined in Chapter 4 answering research questions **Q2** and **Q3**. The research questions **Q4** and **Q5** are answered in Chapter 5, followed by the conclusion in Chapter 6.

Figure 1.1: Structure of the thesis



2 Machine Scheduling

This chapter provides insight into the problem classification and solution methods for machine scheduling problems.

2.1 Introduction

Scheduling generally is a decision making process, which assigns limited resources to tasks in the course of time to achieve predefined objectives.[1] This assignment decision is a combinatorial search problem under parameters describing the problem and its specific characteristics. The combinatorial search problem becomes an optimization problem, if an objective function is added.[2]

Pinedo (2002) listed five different cases for scheduling in manufacturing: i) project scheduling, ii) machine scheduling, iii) scheduling of flexible assembly systems, iv) economic lot scheduling and v) scheduling in supply chains.[3] This work focuses on machine scheduling. The machines of a production environment are the limited resources in this problem class.[4] The tasks that need to be assigned to machines are the operations to finish production jobs. Hence the set of tasks can be divided into n subsets, each containing all necessary operations to finish a single job. Sequencing the operations of all jobs is the combinatorial search problem in machine scheduling.

This is one of the scheduling problems in manufacturing companies. Błażewicz et al. (1996) described the instance of machine scheduling problems with three main parameter sets, which are the set of tasks that need to be performed, the processors and the set of additional resources necessary for production process. Machine scheduling problems, considering an objective function, are optimization problems.[5] The set of processors includes all machines, which are operating the tasks. In the following the processors are called machines.

Graham et al. (1979) proposed the 3-field classification scheme for machine schedul-

[1] See Pinedo (2002): *Scheduling: theory, algorithms, and systems*, p. 1.
[2] See Błażewicz et al. (1996): *Scheduling Computer and Manufacturing Processes*, p. 11.
[3] See Pinedo (2005): *Planning and Scheduling in Manufacturing and Services*, p. 14.
[4] See Pinedo (2002): *Scheduling: theory, algorithms, and systems*, p. 1.
[5] See Błażewicz et al. (1996): *Scheduling Computer and Manufacturing Processes*, p. 57.

ing problems $\alpha|\beta|\gamma$.[6] The scheme considers the fields machine environment (α), job characteristics (β) and optimality criteria (γ) that can be represented by an objective function. This categorization was extended by Brucker (1995) and Błażewicz et al. (1996),[7] and is further described in Section 2.2.

2.2 Classification of Machine Scheduling Problems

This section describes the extended 3-field categorization scheme of Brucker (1995) and Błażewicz et al. (1996), explains the main expressions and problem types in machine scheduling, and categorizes the examined problem of re-entrant permutation flow shops.

Machine Environment

The machine environment is described by the α-field and its two subcategories α_1 and α_2. α_1 describes which machines are able to process which jobs and the directions of possible material flows. α_2 indicates the number of processors.

α_1 can take the symbols $\circ$, P, Q, R, $PMPM$, $QMPM$, G, X, O, J, and F. The status of the machine environment in which each job consists of only one operation and needs to be processed on a dedicated machine is represented by $\alpha = \circ$. P, Q, R indicate parallel machines. Each job needs to be processed just once on any of the machines to be finished. P represents an environment with identical parallel machines, which means that the processing times of a job are the same on each machine. Q indicates uniform parallel machines. The processing speed differs between machines, but the relation of processing speeds is independent from the jobs. Unrelated parallel machines (R) are also characterized by different processing times for each job's operation, but those time variations depend on the assigned job.

$PMPM, QMPM$ stand for multi-purpose environments. That means the machines can perform different processing functions depending on the tools, they are equipped with. The two forms vary in the two kinds of processing speed: identical ($PMPM$) and uniform processing speed ($QMPM$).

A general shop (G) is a multi-operation model with dedicated machines. A certain set of processing steps is defined for each job that should be operated by the machines. It contains the three sub-forms open shop (O), job shop (J), flow shop (F) and mixed shop (X). There are no precedence constraints on the jobs' operations in open shops. The set of operations needs to be processed, but it does not matter in which sequence.[8] The

[6] Graham et al. (1979): *Optimization and approximation in sequencing and scheduling*, pp. 288–291.
[7] See Brucker (1995): *Scheduling algorithms*, pp. 2–7 and Błażewicz et al. (1996): *Scheduling Computer and Manufacturing Processes*, pp. 68–69.
[8] See Gonzalez / Sahni (1976): *Open Shop Scheduling to Minimize Finish Time*, p. 665.

operations of each job need to follow a certain sequence in a job shop. The predecessors and successors of each operation are defined and may differ from job to job. Also in flow shops, there is a defined order of operations, which is identical for each job. So, the sequence of the machines to be processed on is the same for each job. The problem is called permutation flow shop if the job sequence is the same on all machines. A mixed shop environment is a combination of open and job shop. An additional class of flow shop problems are hybrid or flexible flow shop problems. In this class of problems multiple parallel machine resources are available for at least one processing step.[9] The second characterization parameter α_2 can be a positive integer number or $\circ$. α_2 indicates the number of machines, i.e. the system contains two machines if $\alpha_2 = 2$. For $\alpha_2 = \circ$, the number of machines is variable.

For the problem covered in this work, the α-parameters are: $\alpha_1 = F$ and $\alpha_2 = m$. The problem considered is a permutation flow shop with a number of dedicated machines m.

Job Characteristics

The job characteristics can be divided into six categories; hence there are $\beta_1, \ldots, \beta_9$. There are two options for β_1. Job preemption, $\beta_1 = pmtn$, allows interruptions and the later resumption of any operation that needs to be processed. If interruptions are not allowed, then β_1 is not mentioned in the problem description or is represented by $\circ$.

β_2 determines whether any additional resources are necessary to process the jobs in addition to the machines. Such resources can be renewable, non-renewable and doubly constrained. Renewable resources are only available at certain points of time, but the amount of usage is not limited. Non-renewable resources are limited in quantity but not connected to availability times. Doubly constrained resources are limited in availability time and quantity. The requirement of additional resources is indicated with $\beta_2 = res$, otherwise $\beta_2 = \circ$.

The precedence constraints between the jobs are represented by β_3. To explain these constraints, a graph $\overline{G}$ is used, which is acyclic and directed. The set of nodes in $\overline{G}$ represents the jobs or the jobs' operations, and the set of arcs $\overline{A}$ illustrates the precedence relations between the jobs. If there is an arc $i \rightarrow i'$, then job i needs to be finished before job i' is allowed to begin. The parameter $\beta_3 = prec$ indicates precedence constraints between jobs in general. If there is at most one successor for each job, then $\beta_3 = intree$. Hence, only the roots of the tree have an outdegree of zero, and all other nodes have an outdegree of one. For $\beta_3 = outtree$, all jobs have at most one direct predecessor

[9] See Linn / Zhang (1999): *Hybrid flow shop scheduling: a survey*, p. 57 and Sriskandarajah / Sethi (1989): *Scheduling algorithms for flexible flowshops*, p. 143.

that they need to wait for. In that case, the roots of $\overline{G}$ have no predecessor, and all other nodes have only one predecessor. For the form $\beta_3 = chains$ there is a maximum of one predecessor and one successor for each job. Another form of β_3 is series-parallel. Series parallel graphs include vertices, which can have more than one successor and predecessor. The classification scheme is extended at this point by adding $\beta_3 = reentr$ for jobs that enter the production environment multiple times. $\beta_3 = \circ$ indicates no precedence constraints between jobs.

If there is a release date r_i given for any job i, β_4 will be r_i. The fifth job characteristic β_5 is related to the processing times. The jobs have a unit processing requirement if β_5 is equal to 1. A β_5 of $p \geq 0$ indicates possible missing operations.

Due dates d_i for the jobs are represented in $\beta_6 = d_i$.

β_7 states the maximum number of operations necessary to finish a job in a job shop. $\beta_7 = \circ$ indicates no limits to the number of operations. When $\beta_7 = (o_i \leq m)$, the number of operations for each job $i = 1, \ldots, n$ is not allowed to exceed m.

β_8 indicates whether job waiting times are permitted or not. $\beta_8 = \circ$ declares waiting times as permitted, and $\beta_8 = no{-}wait$ indicates, that waiting times are not allowed. This means that the processing of a job needs to start at a machine $k+1$ immediately after it has been finished on machine k. Błażewicz et al. (1996) proposed a statement on the buffer capacities of the machine environment within parameter β_8.[10] The buffers have an unlimited capacity in the case of $\beta_8 = \circ$. However, waiting times of jobs can also occur in the case of Limited buffer capacity (LB) between machines. No buffer is necessary, if $\beta_8 = no - wait$. Grouping jobs into batches is indicated by $\beta_9 = batch$. A batch of jobs is processed on a machine without being interrupted by the processing of jobs of another batch. Brucker (1995) indicates batch criteria with β_6[11], Błażewicz et al. (1996) do not indicate batch characteristics.

Preemptions are forbidden, and no resource constraints are required. Therefore, β_1 and β_2 are omitted. The re-entrant material flow is indicated by $\beta_3 = recrc$. Driessel / Mönch (2012a) and Eskandari / Hosseinzadeh (2014) used the same notation to indicate job re-entrants.[12] Alternatively the scheduling of jobs in a re-entrant permutation flow shop can be characterized with $\beta_3 = chains$ if the permutation consists of the (re-)entry levels of the jobs, since a level $l+1$ can just begin after level l is finished. The occurrence of missing operations and the maximum processing time in the test instances leads to a β_5 of$0 \leq p \leq 99$. Ready times and deadlines are not considered, leading to $\beta_4 = \beta_6 = \circ$. Also, β_7, β_8 and β_9 are omitted, since the problem is not a job shop and

[10] See Błażewicz et al. (1996): *Scheduling Computer and Manufacturing Processes*, p. 69.
[11] See Brucker (1995): *Scheduling algorithms*, pp. 6–7.
[12] See Driessel / Mönch (2012a): *Integrated scheduling and material-handling*, p. 5968 and Eskandari / Hosseinzadeh (2014, p. 3).

the material buffers between the machines are not limited.

Optimality Criteria

The optimality criteria of scheduling problems are their objective functions. Two main types of objective functions are categorized by GRAHAM et al. (1979)[13], which are called bottleneck and sum objectives by BRUCKER (1995).[14] Both types use cost functions, $f_{max}(C)$ and $f_i(C_i)$, which for each job are based on the completion times C_i of the n jobs. The completion time is the point of time when a job is finished. The scheduling problem is to find a valid solution that minimizes the cost function.

Bottleneck objectives are generally formulated with a cost function, which uses the maximum cost value among all jobs as the objective value:

$$f_{max}(C) := \max_{i=1,\ldots,n} f_i(C_i). \tag{2.1}$$

The sum objectives use the total cost value over all jobs as objective value:

$$\sum f_i(C_i) := \sum_{i=1}^{n} f_i(C_i). \tag{2.2}$$

In the 3-field categorization scheme, γ can take the values f_{max} and $\sum f_i$. Common concrete objective values that are minimized are makespan $C_{max} = \max_{i=1,\ldots,n} C_i$, the total flow time $\sum_{i=1}^{n} C_i$ denoted with $\overline{F}$, weighted total flow time $\overline{F}_\omega$ as $\sum_{i=1}^{n} \omega_i C_i$. The makespan is the time between the start of the first operation of the schedule and the end of the last operation. It is equal to the maximum flow time. Flow time is measured per job and is also called completion time for jobs with release dates equal to 0. Flow time begins with the release time of a job and ends for each job with the end of the last operation of the job. Other common objectives include:

- Total lateness: $\sum_{i=1}^{n} L_i$ with $L_i := C_i - d_i$
- Total earliness: $\sum_{i=1}^{n} E_i$ with $E_i := max\{0, d_i - C_i\}$
- Total tardiness: $\sum_{i=1}^{n} T_i$ with $T_i := max\{0, C_i - d_i\}$
- Total absolute deviation: $\sum_{i=1}^{n} D_i$ with $D_i := |C_i - d_i|$
- Total squared deviation: $\sum_{i=1}^{n} S_i$ with $S_i := (C_i - d_i)^2$
- Total unit penalty: $\sum_{i=1}^{n} U_i$ with $U_i := 0$ if $C_i \leq d_i$ and $U_i := 1$ if $C_i > d_i$

[13] See GRAHAM et al. (1979): *Optimization and approximation in sequencing and scheduling*, p. 290.
[14] See BRUCKER (1995): *Scheduling algorithms*, p. 6.

The minimization of the total unit penalty is equal to minimizing the number of tardy jobs. All of these objective functions can also be formulated as weighted sum objectives. In addition, it is also possible to use the variables L_i, E_i, T_i, D_i, S_i in bottleneck (weighted bottleneck) formulations. This means that the maximum values of lateness, earliness, tardiness and absolute or squared deviation can be minimized. Linear combinations of the different objective functions are also possible optimization targets. In consideration of the objective functions and constraints, a schedule is described as active, if the operations cannot be scheduled earlier without violating any constraint. Semi-active schedules do not allow an operation to be processed earlier without changing the processing sequence or obtaining an invalid solution. An open γ field means, that a feasible solution should be generated, without respect to any objective value.

The γ-parameter to classify the examined problem is $\gamma = C_{max}$.

2.3 Solution Approaches

Solution methods for machine scheduling problems, as for other optimization problems, can be divided into exact methods and heuristics. Heuristic solution approaches are used to generate valid solutions with good objective values, since exact methods like branch and bound, branch and cut and Branch and price (B&P) are not always appropriate to solve the problem in an acceptable time. This section will give an overview of the heuristics used for scheduling. Common methods for obtaining valid schedules for flow shops are priority rules[15], also called dispatching rules.[16] Examples of these rules are Shortest processing time first (SPT), Longest processing time first (LPT) and Earliest due date (EDD). Random schedules are created with Service in random order (SIRO) rules. As the considered problem within this thesis is a permutation flow shop problem, the described priority rules are global rules. Furthermore, the Algorithm of Nawaz, Enscore and Ham (NEH) and the Algorithm of Campbell, Dudek and Smith (CDS) are other explained constructive heuristics. Metaheuristic solution approaches are performed either on a single solution generated by constructive methods or on multiple solutions generated by one or multiple constructive methods. The group of metaheuristics can be divided into two main groups:[17]

- Trajectory methods,
- Population based methods.

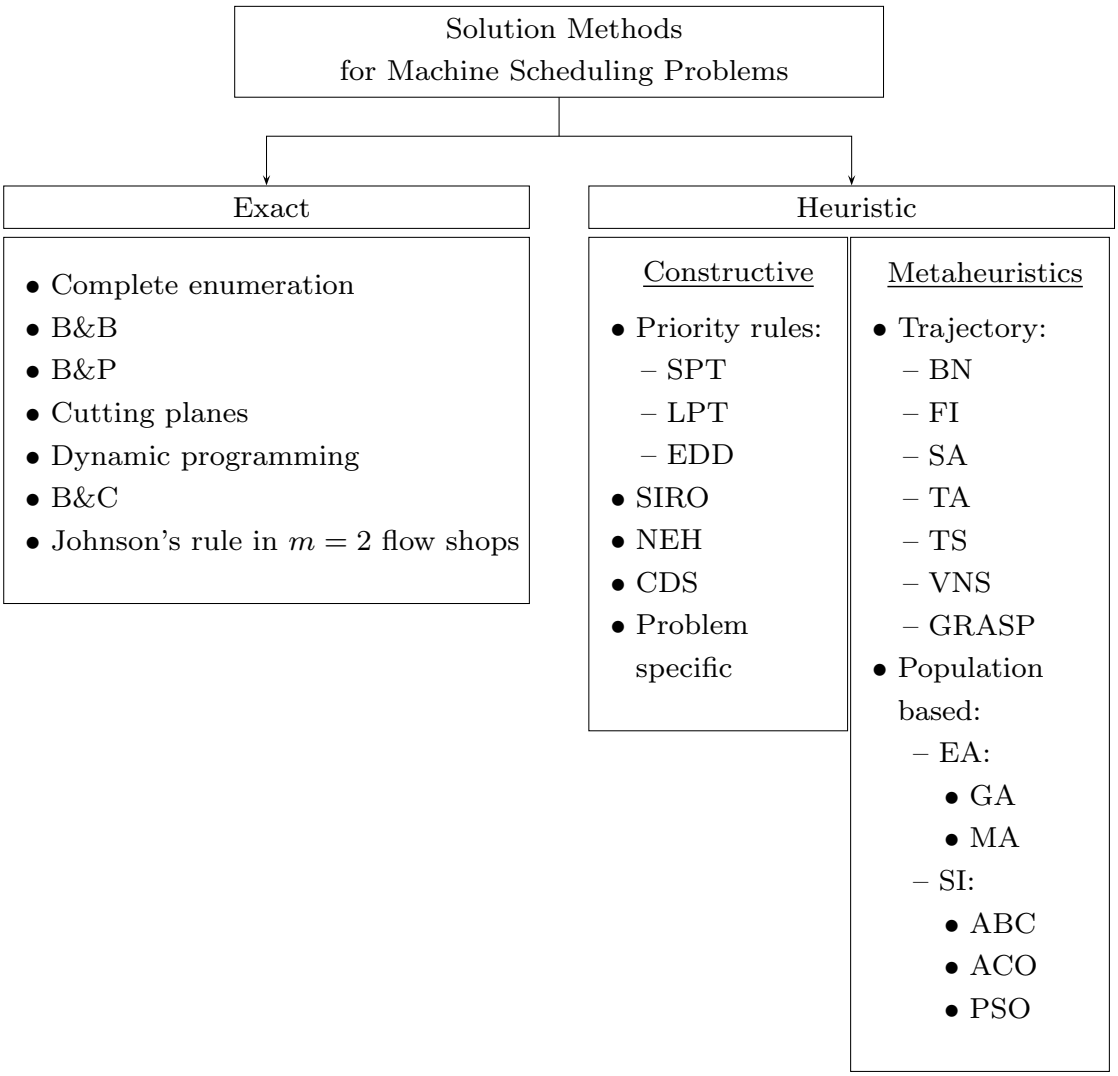
[15] See Hunsucker / Shah (1994): *Analysis of priority rules in a constrained flow shop*, p. 105.
[16] See Ruiz / Maroto (2005): *Review and evaluation of permutation flowshop heuristics*, p. 486.
[17] See Blum / Roli (2003): *Metaheuristics in combinatorial optimization*, pp. 272–292.

Trajectory methods are performed on a single solution and apply changes to this single solution successively. Simple trajectory methods are first improvement and best neighbor, while simulated annealing, tabu search, variable neighborhood search, Greedy randomized search procedure (GRASP) and Threshold accepting (TA) are more sophisticated approaches. Population based algorithms work with multiple solutions in each iteration by applying changing evaluation patterns and combination schemes to multiple solutions. They can be divided into Evolutionary algorithms (EA) (e.g. Genetic algorithm (GA) and Memetic algorithm (MA)) and Swarm intelligence (SI) algorithms (e.g. Artificial bee colony (ABC), Ant colony optimization (ACO) and Particle swarm optimization (PSO)).

This work focuses on trajectory methods.

The explained classification of some solution methods is summarized in Figure 2.1.

Figure 2.1: Classification of solution methods



The terms "move", "neighbor" and "neighborhood" are briefly explained here because they are used to describe different solution methods in this thesis. The changes applied to modify solutions are called moves. Two basic move strategies for permutation flow shops are to swap the sequence positions of different permutation members (swap moves)[18] or to place an item at another position in the permutation (insertion moves)[19]. There are different names for these move strategies: swap moves are also called exchange moves[20], interchange moves[21], E-moves[22] or S-moves[23]. Synonym names for insertion moves are shift moves[24] and I-moves[25]. The preferred terms in this thesis are swap and insertion moves. The new solution obtained by moving is called neighbor. The set of all (valid) neighbors that can be obtained by a specified move or set of moves is called a neighborhood. A class of moves for makespan minimization problems that is based on the critical path of a solution are block moves. All the moves considered for a re-entrant permutation flow shop are explained in detail in Section 4.6.

Problem modeling and exact methods

This section gives an overview of the exact methods for solving machine scheduling problems, specifically the methods used in the IBM ILOG CPLEX optimization studio 12.4 to solve optimization problems formulated as a mathematical programming model.

Machine scheduling problems are often formulated as MIP models[26], which are mathematical programming models that involve integer variables. CPLEX uses linear programming relaxations for its branch and cut algorithm. Linear programming relaxations allow an MIP's integer variables to be continuous. Heuristics are used to repair invalid continuous solutions into valid integer solutions.[27]

There are three main groups of exact methods for solving combinatorial optimization problems:

- Search tree methods / enumeration tree: explicit enumeration, implicit enumeration (branch and bound, branch and price, dynamic programming)
- Cutting planes methods

[18] See Grabowski / Pempera (2005): *Local search algorithms for no-wait flow-shop problem*, p. 2199.
[19] See Ogbu / Smith (1990): *Simulated annealing for the n/m/C max flowshop problem*, p. 246.
[20] See Chen / Pan / Wu (2007): *Reentrant flow-shops and hybrid tabu search*, p. 357.
[21] See Osman / Potts (1989): *Simulated annealing for permutation flow-shop scheduling*, p. 552.
[22] See Nowicki / Smutnicki (1996): *Fast taboo search for the job shop problem*, p. 162.
[23] See Grabowski / Pempera (2005): *Local search algorithms for no-wait flow-shop problem*, p. 2199.
[24] See Osman / Potts (1989): *Simulated annealing for permutation flow-shop scheduling*, p. 552.
[25] See Nowicki / Smutnicki (1996): *Fast taboo search for the job shop problem*, p. 162.
[26] See Table 3.7 in Section 3.2.
[27] See IBM (2011): *CPLEX user's manual*, pp. 215–216.

- Hybrid methods (branch and cut).

The explicit enumeration evaluates every possible solution. The best valid solution found is the global optimum. Evaluating all possible solutions requires long computation times for large NP-hard problems. Therefore, more structured enumerations can be used to reduce the computational effort. Branch and bound is a structured implicit enumerative approach to solve combinatorial optimization problems.
Implicit enumeration approaches are branch and bound, branch and price and dynamic programming.[28] Branch and bound algorithms divide optimization problems into subproblems. Based on the solutions of the subproblems, the lower / upper bounds of the global solutions are calculated by solving either relaxations or specified bounds on the specific problem. A common relaxation for combinatorial problems is the linear programming relaxation[29]. A global minimum (maximum) is found if the lower (upper) bound is equal to or higher (lower) than the objective value of the solution found. A lower bound gives an approximation of the best possible objective value in a minimization if the solution of the subproblem is extended to the complete problem. An upper bound is the estimation of the best possible objective value in a maximization. A further search for a better solution is not necessary if the gap between the bound and incumbent solution is closed. In the worst case, all combinatorial possibilities need to be evaluated. In all other cases, at least one solution is evaluated implicitly. Ignall / Schrage (1965) introduced a branch and bound procedure for two and three machine flow shop problems in order to minimize the makespan or total completion time.[30]
Branch and price is a method of using column generation to generate new branches during a branch and bound algorithm but is not explained in this thesis since it is not part of the solver software CPLEX used.[31] Further information on branch and price can be found in Barnhart et al. (1998).[32]
Dynamic programming works by using recursion relations between solutions of different problem sizes. The recursion used determines the type of dynamic programming. There are forward and backward dynamic programming. Forward dynamic programming builds a sequence from the beginning, whereas backward dynamic programming starts to build a sequence at the rear end. Pinedo (2005) shows an example of both approaches for the minimization of objective functions similar to the total flow time for a single machine scheduling problem without preemption.[33]
The cutting plane approach tries to find additional constraints to an integer optimization

[28] See Błażewicz et al. (2007): *Handbook on Scheduling: From Theory to Applications*, p. 33.
[29] See Domschke / Scholl / Voss (1997): *Produktionsplanung*, p. 42.
[30] See Ignall / Schrage (1965): *Branch and bound technique to flow-shop scheduling*, pp. 401–406.
[31] See IBM (2011): *CPLEX user's manual*, p. 215.
[32] See Barnhart et al. (1998): *Column generation for solving huge integer programs*, pp. 316–329.
[33] See Pinedo (2005): *Planning and Scheduling in Manufacturing and Services*, pp. 397–399.

problem in order to cut off non-integer solutions.[34] An early cutting plane approach was introduced by Gomory (1958). It applies the simplex algorithm to a linear MIP and generates additional inequalities if the solution obtained is not an integer. Additional inequalities are not allowed to affect the validity of the unknown integer optimal solution, but exclude the current optimal continuous solution.[35]
The combination of cutting planes and branch and bound is called branch and cut.[36]
Johnson's rule provides optimal solutions for the makespan minimization in two-machine permutation flow shop problems in every case. The job with the shortest processing time of all jobs is determined. If the corresponding operation is performed on the first machine, then the job is assigned to the first free position of the job sequence; otherwise the job is placed on the last non-occupied position. This job's processing times are then removed from the list and the shortest processing time is searched again. The process repeats until all jobs are assigned to sequence positions.[37]

Constructive heuristics

Priority rules, also called dispatching rules, are a group of constructive heuristics. This group of solution methods can be divided based on their influence on the schedule. Local priority rules determine which operation should be selected next for a single machine. Global priority rules determine a sequence of jobs for machines at once.[38] This subsection introduces some priority rules. The focus is on global priority rules since the job sequence in permutation flow shops does not differ between machines.
A global dispatching is the Shortest total processing time first (STPT) rule, which is derived from the local SPT rule. The local rule says that if several jobs are available to be processed on a machine, the job with the lowest processing time is preferred. The global STPT rule determines a job sequence by sequencing the jobs in a non-increasing order of each job's total processing time. The rule is suggested to obtain a low total completion time of jobs.
Longest total processing time first (LTPT) is another global rule. It operates in the opposite way to the STPT rule because it sequences the jobs in a non-decreasing order of their total processing time on all machines. The local equivalent is the LPT rule.
The SIRO rule can be used to check whether other priority rules have a relevant influence on objective values. It puts operations (local version) or jobs (global version) in a random sequence.

[34] See Korte / Vygen (2012): *Kombinatorische Optimierung: Theorie und Algorithmen*, p. 129.
[35] See Gomory (1958): *Outline of an algorithm for integer solutions to linear programs*, p. 275.
[36] See Korte / Vygen (2012): *Kombinatorische Optimierung: Theorie und Algorithmen*, p. 624.
[37] See Johnson (1954): *Optimal two- and three-stage production schedules*, pp. 61–64.
[38] See Pinedo (2002): *Scheduling: theory, algorithms, and systems*, p. 336.

A priority that is useful to achieve relatively low values of total tardiness is the EDD rule. It sorts the jobs in a non-increasing order of their due date / delivery date. The local version decides to perform the operation of the available job with the lowest due date on a specific machine. Another priority rule based on due dates is the slack time remaining rule. The remaining slack time of a job is calculated by subtracting the current time, when the machine is empty, from the due dates of the single available jobs for the machine. The job with the lowest slack time value is chosen to be processed on the machine.
The first come first serve rule schedules the job amongst the available jobs on a machine that becomes available first. The first come first serve rule is automatically a global rule in all permutation flow shops, which provide job release dates different from zero, since the job sequence of the first machine is the same for all other machines.

The CDS method divides the $m > 3$ machine flow shop into $m - 1$ subproblems and applies the rule of Johnson to the problems. In every kth subproblem, with $k = 1, \ldots, m - 1$, the sum of the processing times of the machines $k' = 1, \ldots, k$ represents the processing times of the surrogate machine 1. The processing times of the second surrogate machine are represented by the sum of processing times of the machine $k'' = m + 1 - k, \ldots, m$. The makespan values for $m - 1$ solutions are calculated and the best solution selected.[39]

The NEH heuristic sequences jobs in a non-increasing order of the total processing times of each job. The first job is selected and scheduled on the m machines. Each job that is added to the sequence is inserted in all possible sequence positions and the best configuration is chosen, then the next job is selected.[40]

Another constructive heuristic for the permutation flow shop is an insertion heuristic proposed by Widmer / Hertz (1989) to obtain an initial solution for a tabu search.[41]

Eight different constructive heuristics are used to initialize the meta heuristics in Chapters 4 and 5. These eight heuristics are based on the LTPT rule, STPT rule, NEH algorithm and SIRO rule. The LTPT and STPT rule are selected because they are common and simple. The NEH algorithm is tested to examine the influence of a more sophisticated constructive heuristic on the initial solution. The solutions of the six constructive heuristics based on the STPT rule, the LTPT rule and the NEH method are compared to two randomly generated solutions per problem instance.

[39] See Campbell / Dudek / Smith (1970): *The n job, m machine sequencing problem*, p. 631.
[40] See Nawaz / Enscore / Ham (1983): *Heuristic for m-machine, n-job flow-shops*, pp. 92–94.
[41] See Widmer / Hertz (1989): *A new heuristic method for flow shops*, pp. 187–188.

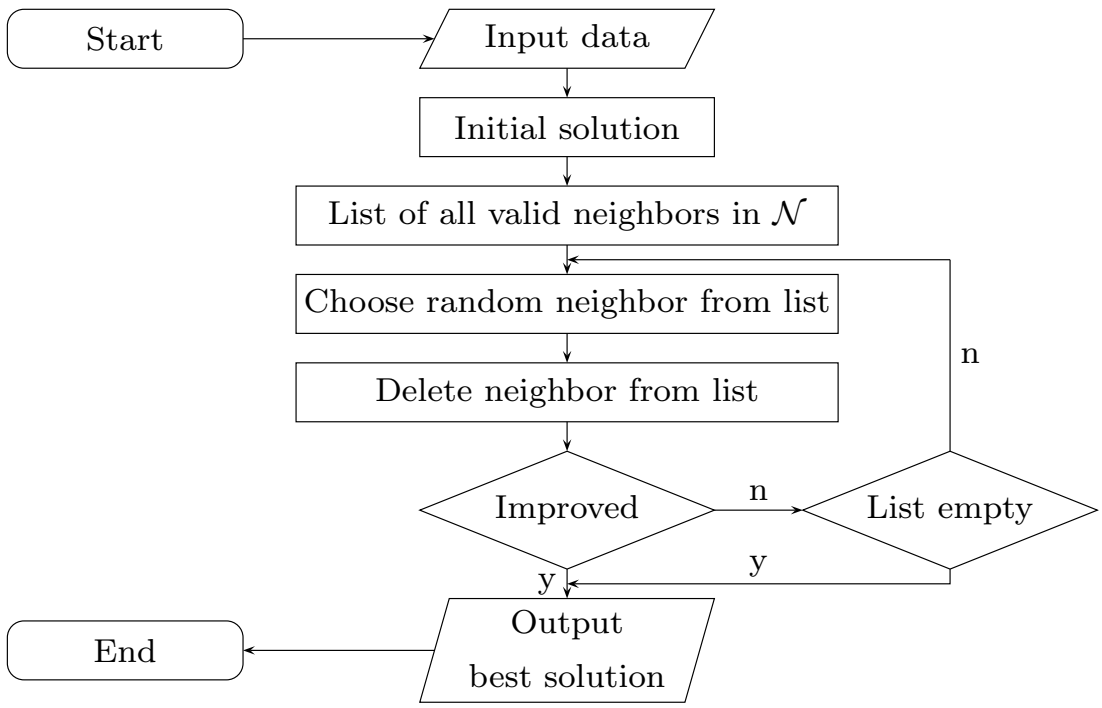
Metaheuristics

Basic improvement methods like FI and BN are explained within this section as they are part of the tested methods SA, TS and VNS, which are explained here in general. Additionally the GRASP and TA are briefly explained because they are mentioned in the literature review in section 3.2 The general elements of some population based algorithms are also described to show the difference with trajectory methods.

First Improvement

The first improvement method is a fast local search. A random valid neighbor of a given solution is selected, and the objective value calculated. The solution is accepted, and the method terminates if an improvement of the initial objective value is achieved or the objective values of all valid neighbors in the selected neighborhood are calculated and no improvement has been found. The general procedure is shown in Figure 2.2.

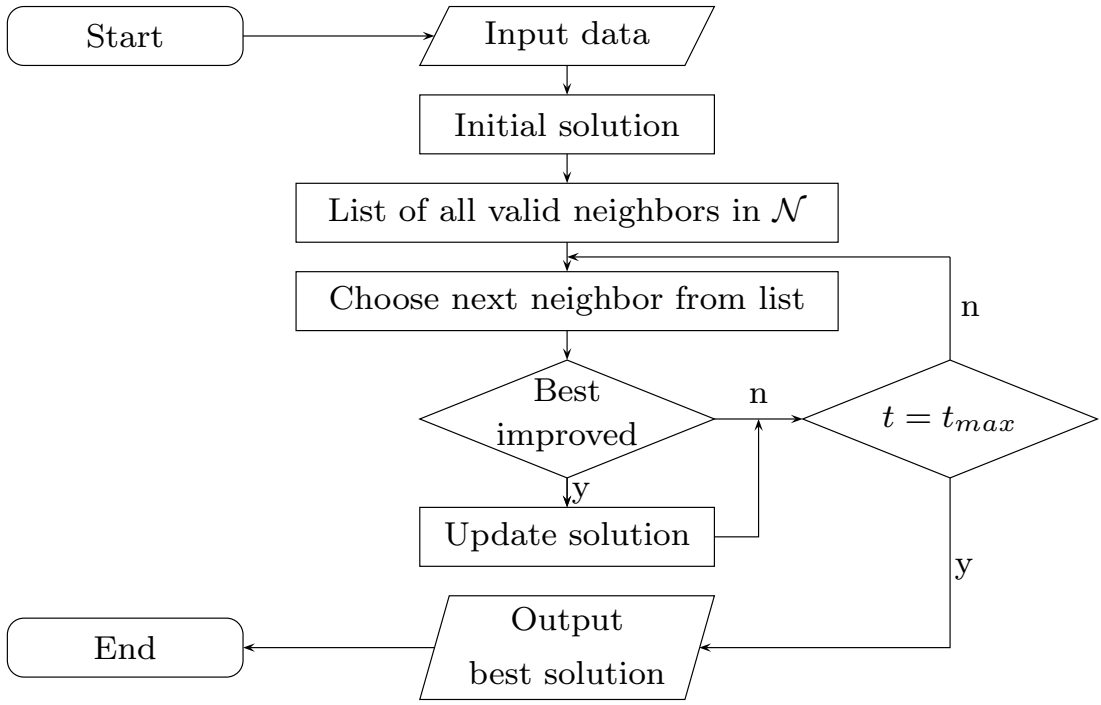
Figure 2.2: First improvement



Best Neighbor

The best neighbor local search calculates the objective values of all valid neighbors within a given neighborhood and chooses the solution with the best objective value if it improves the initial solution. A list with all valid moves is created and the list's items respectively the resulting solutions are successively evaluated as shown in Figure 2.3.

Figure 2.3: Best neighbor



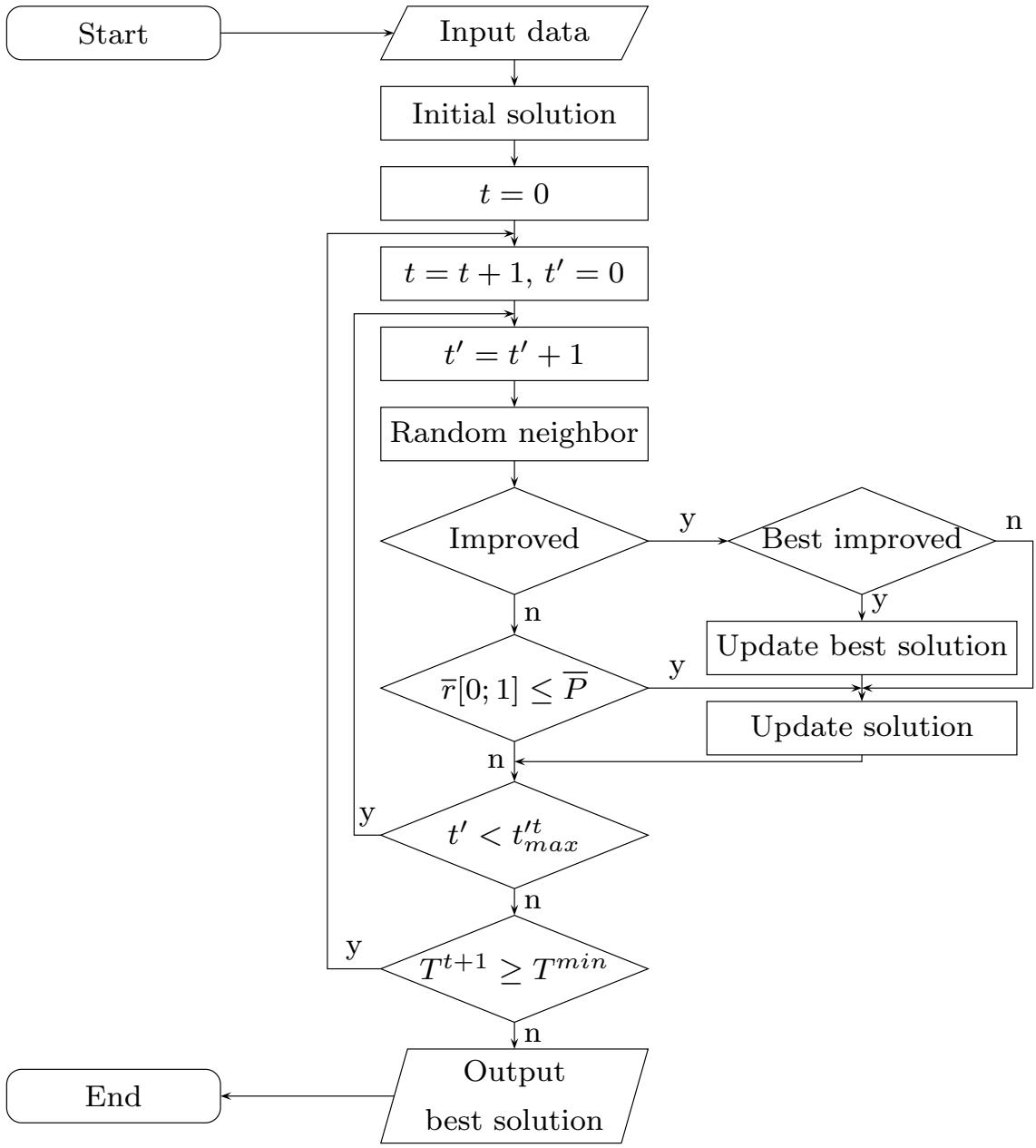
Simulated Annealing

Simulated annealing is a metaheuristic improvement method first mentioned by Kirkpatrick / Gelatt / Vecchi (1983) to solve traveling salesman problems.[42] Černỳ (1985) developed the method independently from Kirkpatrick / Gelatt / Vecchi (1983) and also applied it to the traveling salesman problem.[43] The method is based on calculating the energetic state or thermodynamic equilibrium of a fluid or solid with a given temperature. The energetic state is based on probabilistic behavior. A high energy state can be reached by a material with a certain probability depending on the material temperature. This behavior is applied to finding the solution in optimization problems. A neighbor of an incumbent solution is selected and evaluated regarding the objective value. The neighbor is accepted as a new incumbent solution if its objective value is better than the objective value of the incumbent solution. If the neighbor delivers a worse result, it can also be accepted with the probability $\overline{P}$, i.e. if a uniformly distributed random number $\overline{r}$ is lower or equal to $\overline{P}$. Parameter t counts the temperature states, i.e. the main iterations of the algorithm. $\overline{P}$ depends on the current temperature value T^t. The initial temperature is given by T^1. It is not allowed to fall below a temperature

[42] See Kirkpatrick / Gelatt / Vecchi (1983): *Optimization by Simulated Annealing*, pp. 671-680.
[43] See Błażewicz / Kobler (2002): *Properties of precedence graphs for scheduling problems*, p. 41, Černỳ (1985): *Thermodynamical approach to the traveling salesman problem*, pp. 41-51 and Ogbu / Smith (1990): *Simulated annealing for the n/m/C max flowshop problem*, p. 244.

T^{min}. A solution is denoted with x and a newly generated neighboring solution is called x'. $f(x)$ is the objective value of solution x. t' counts the iterations per temperature level t. Figure 2.4 gives an overview of the procedure.

Figure 2.4: Simulated annealing



Osman / Potts (1989) determined the initial temperature in a simulated annealing for a permutation flow shop dependent on the total processing time of all jobs, the number of jobs n as well as the number of machines m. The temperature follows a geometric annealing scheme. The lowest possible temperature is set equal to 1. The number of iterations with the same temperature was estimated by Osman / Potts (1989) to be the

maximum of either 2000 or $3300\,ln\,n + 7500\,ln\,m - 18250$, if $n \leq 100$ and $m \leq 20$. For higher values of the number of jobs and machines, the number of iterations needs to be less than $7500(ln\,m + ln\,n)$ and less than $15.000ln(m+n)$. In one version, the neighbors are selected via forward insertion moves. That means a job at position j of the permutation will be placed at a position $j' > j$. The job changes to any position $j' < j-1$ if a backward insertion move is applied. Also swap moves are tested. The initial solution is created by the NEH algorithm. The insertion moves deliver better results than the swap moves.[44] Ogbu / Smith (1990) initialized an SA with randomly generated solutions. A candidate is created by applying the last improvement scheme to a neighborhood that combines insertion and swap moves. A neighborhood consisting solely of insertion moves delivered better results than that consisting of swap moves. The neighborhood is searched in random sequence to find improvements. The latest found improvement of the incumbent solution is selected after the last neighbor is evaluated. The initial temperature is calculated during the first iteration and is based on the mean improvement of the makespan over all possible candidates.[45] The SA algorithms of Osman / Potts (1989) and Ogbu / Smith (1990) are compared in Ogbu / Smith (1991). The SA of Osman / Potts (1989) obtained better results than the version of Ogbu / Smith (1990) but requires longer computation time.[46] Zegordi / Itoh / Enkawa (1995) used a restricted swap neighborhood in their SA approach. The initial temperature was also calculated based on objective changes within the first iteration. The objective within this problem is the makespan minimization.[47]

Threshold Accepting

Threshold accepting is similar to simulated annealing but with the difference of an equal probability of accepting solutions that do not lead to improvement of the incumbent objective value.[48]

Tabu Search

Tabu search was initially proposed by Glover (1986).[49] Tabu search creates a list of candidates within a specified neighborhood of the incumbent solution. One of the candidates is selected to replace the current solution. The chosen candidate has the best objective value amongst all the created candidates and the objective value of the

[44] See Osman / Potts (1989): *Simulated annealing for permutation flow-shop scheduling*, pp. 553–556.
[45] See Ogbu / Smith (1990): *Simulated annealing for the n/m/C max flowshop problem*, pp. 244–252.
[46] See Ogbu / Smith (1991): *Simulated annealing for the permutation flowshop problem*, pp. 64–66.
[47] See Zegordi / Itoh / Enkawa (1995): *Minimizing makespan for flow shop scheduling*, pp. 517–523.
[48] See Dueck / Scheuer (1990): *Threshold accepting*, pp. 161–164.
[49] See Glover (1986): *Integer programming and links to artificial intelligence*, pp. 541–546.

new solution does not necessarily need to be better than the current best solution. To prevent falling back to previous solutions, certain moves are forbidden and noted in a tabu list. If a candidate includes a forbidden change, it must not be selected. Either a candidate is selected randomly via first improvement or the best candidate is selected. A forbidden candidate can be accepted depending on an aspiration function.[50,51] The function allows the acceptance of forbidden moves, if a certain improvement is achieved. This will prevent the algorithm from rejecting relatively large improvements. The tabu tenure, which means the number of iterations a move is forbidden, may be constant or may depend on the number of the current iteration of the algorithm. A restriction of the neighborhood in a tabu search was proposed by Adenso-Díaz (1992) for the minimization of weighted tardiness in a proportionate permutation flow shop.[52] The initial solution is created by prioritizing jobs based on their influence on machine idle time. A swap neighborhood is limited to moves within a certain range, which is defined as the positions in the permutation between possible swap partners. The limits on the range are changed depending on the current iteration.[53] Armentano / Ronconi (1999) also applied a tabu search to minimize total tardiness in permutation flow shops. The initial solution is created by using a modified due date rule. The selected neighborhood is an insertion neighborhood. The aspiration function allows a forbidden move to be made, if the best solution found will be improved by the move. The tabu tenure is variable and is adjusted every 20 iterations. The tabu search terminates after a certain computation time depending on the problem size. The results are compared to the ones achieved with the NEH algorithm and show improvements.[54] Ben-Daya / Al-Fawzan (1998) compared a tabu search approach for minimizing the makespan in a permutation flow shop with a simulated annealing from Ogbu / Smith (1990). In this tabu search approach, the neighborhood consisted of three parts. The first part contains all the solutions that can be obtained by pairwise swaps of jobs. The second part of the neighborhood can be accessed by performing insertion moves. The third part considers inserting multiple jobs in a certain position of the permutation. A significant improvement of results could not be registered, if a variable tabu list size is used. Thus, the size of the tabu list and the tabu tenure are fixed in the suggested tabu search. The aspiration criterion is the same as that in the tabu search of Armentano / Ronconi

[50] See Glover (1986): *Integer programming and links to artificial intelligence*, p. 543.
[51] Nowicki / Smutnicki (1996): *Fast taboo search for the job shop problem*, p. 166.
[52] According to Ow (1985): *Focused scheduling in proportionate flowshops*, p. 852, the processing times of jobs on a machine are similar to the processing times of other jobs on the same machine, i.e. there are machines with a tendency to be visited for a relatively short processing time by all jobs, and machines with relatively long processing times in a proportionate flow shop.
[53] See Adenso-Díaz (1992): *Restricted neighborhood for the flowshop problem*, pp. 28–30.
[54] See Armentano / Ronconi (1999): *Total tardiness minimization in flowshops*, pp. 224–225.

(1999).[55]

A tabu search approach, especially for re-entrant flow shop and permutation flow shop problems, was proposed by Chen / Pan / Wu (2007) and Chen / Pan / Wu (2008). The proposed algorithms are hybrid tabu search algorithms, which is a traditional tabu search combined with another heuristic. The objective of both algorithms is to reduce the makespan. Within the first tabu search, the block criteria of Nowicki / Smutnicki (1996) were applied to identify promising moves. Although Nowicki / Smutnicki (1996) used insertion moves of operations to another block, Chen / Pan / Wu (2007) used swap moves within the same block.[56] The method is hybridized by applying constructive heuristics to random parts of the permutation of operations if the tabu search cannot find improvements within a given number of iterations. The second hybrid TS is based on the approach of Chen / Pan / Wu (2007) but uses NEH to create an initial solution and to hybridize the method.[57]

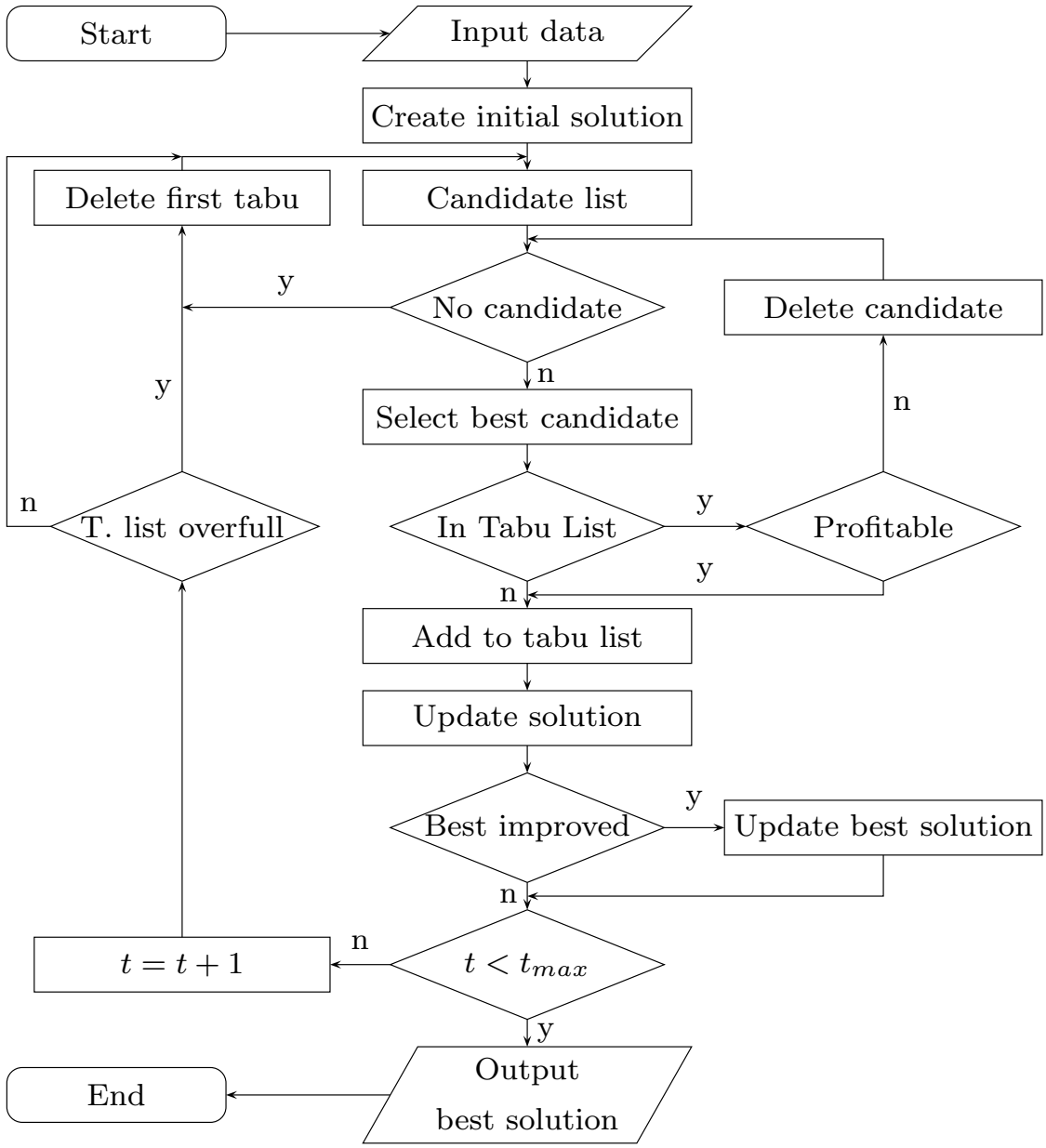
Grabowski / Pempera (2001) and Grabowski / Wodecki (2004) proposed a tabu search for reducing the makespan in a permutation flow shop using block criteria based on the critical path of the solution. The suggested promising moves are insertion moves that place a job out of its block into a neighboring block. The neighborhood size is reduced even further than by Nowicki / Smutnicki (1996).[58]

Furthermore, Solimanpur / Vrat / Shankar (2004) used the block criteria to identify promising neighborhoods for the reduction of the makespan in permutation flow shops. This approach is an extension of the tabu search of Nowicki / Smutnicki (1996) and also uses insertion moves to positions in another block. The algorithm is initialized with a solution obtained by applying the NEH algorithm.[59]

The general tabu search procedure with its elements is shown in Figure 2.5, where t counts the algorithm's iterations.

[55] See Ben-Daya / Al-Fawzan (1998): *A tabu search for the flow shop scheduling problem*, pp. 90–92.
[56] See Chen / Pan / Wu (2007): *Reentrant flow-shops and hybrid tabu search*, p. 356.
[57] See Chen / Pan / Wu (2008): *Hybrid tabu search for re-entrant flow-shops*, pp. 1925–1927.
[58] See Grabowski / Pempera (2001): *New block properties for the permutation flow shop problem*, pp. 210–220 and Grabowski / Wodecki (2004): *A very fast tabu search for the permutation flow shop problem*, pp. 1891–1909.
[59] See Solimanpur / Vrat / Shankar (2004): *A neuro-tabu search for flow shops*, pp. 2151–2164.

Figure 2.5: Tabu search

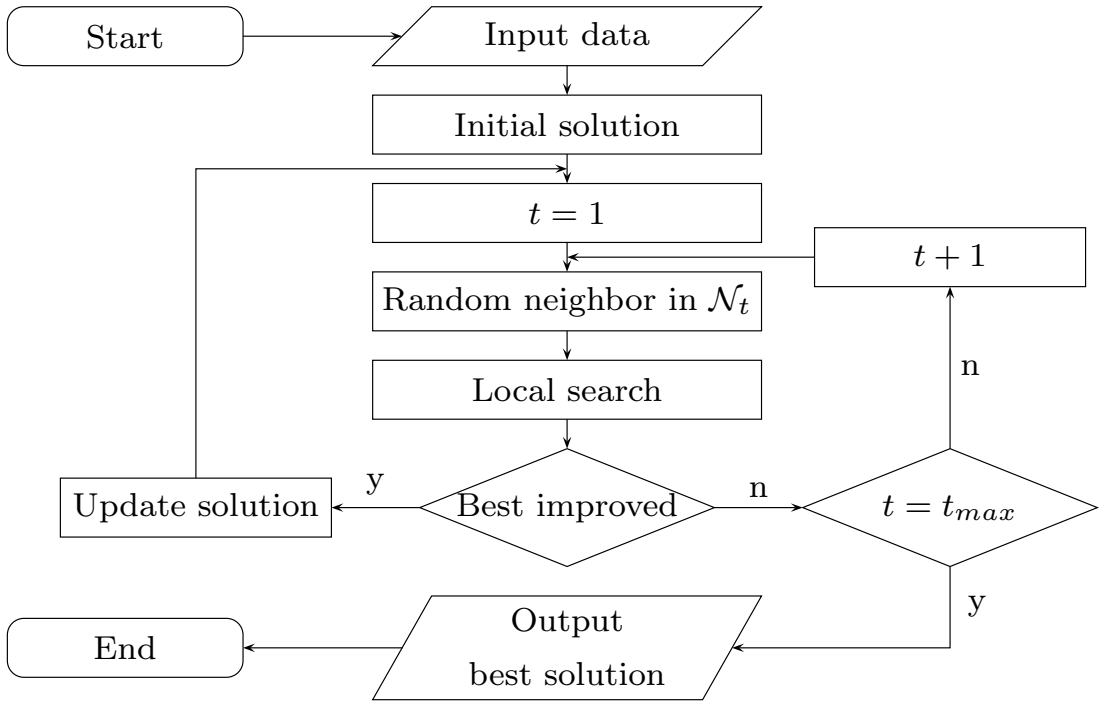


Variable Neighborhood Search

Mladenović / Hansen (1997) introduced the VNS for improving initial solutions. The problem considered was a traveling salesman problem. A variable neighborhood search defines different ways to change an existing solution. These different modification options are the variable neighborhoods. The procedure includes a local search method, e.g. best neighbor or first improvement, and combines it with random moves within different neighborhoods. The neighborhood hierarchy is defined by the sequence in which the neighborhoods are changed. Beginning with neighborhood $\mathcal{N}_1$, the algorithm then

switches to a neighborhood $\mathcal{N}_{t+1}$ if no improvement of the best solution is found in $\mathcal{N}_t$. A random neighborhood move is applied to the current best solution, after changing the neighborhood. This happens to escape local optima. If an improvement to the current best solution is found, the neighborhood is set back to $\mathcal{N}_1$.[60] The method was applied to a location problem in 1997.[61]

Figure 2.6: Variable neighborhood search



GRASP

GRASP is a metaheuristic solution approach, which requires the construction of an initial solution based on randomly choosing a construction step from a list of preselected construction steps. The pre-selection of construction steps per iteration is done based on the solution quality of the partial solutions, which would be obtained by performing the construction steps. The approach further applies a local search mechanism to the initial solution to achieve improvement of the objective values.[62] In permutation flow shops, the partial solution during the construction may be a permutation of a subset of jobs, which are selected based on their influence on the makespan of the partial solution. The construction steps can be adding additional jobs step-by-step to the permutation.[63]

[60] See Mladenović / Hansen (1997): *Variable neighborhood search*, pp. 1097–1100.
[61] See Hansen / Mladenović (1997): *Variable neighborhood search for the p-median*, pp. 207–226.
[62] See Feo / Resende (1995): *Greedy randomized adaptive search procedures*, pp. 109–113.
[63] See Prabhaharan / Khan / Rakesh (2006): *Grasp in flow shops*, pp. 1026–1028.

Genetic Algorithm

Genetic algorithms and also memetic or hybrid genetic algorithms are based on the genetic operations performed by chromosomes. The initial point of the procedure is a parent generation, where each parent represents a particular solution of an optimization problem. These solutions do not necessarily need to be a valid solution to the problem. A parent is "genetically" coded as a chromosome. Different operations can be applied to the chromosomes to achieve a generation of children, i.e. a new solution of the problem. Each child is evaluated by a fitness function that measures the performance of the solution regarding a given objective. So, new solutions can be accepted or rejected depending on their performance. The options to modify a parent generation are mutation and cross-over. Cross-over operations combine existing solutions, i.e. different individuals to generate new solutions (children generation), while mutations modify an existing solution in selected characteristics.[64] Transferring a solution unchanged to the next iteration (next generation) is called reproduction.[65]

Ant Colony Optimization

Ant colony optimization generates multiple solutions step by step. A process of generating a solution is performed by a so-called artificial ant. The ant adds a component to a partial solution in each iteration. The selection of an additional solution component is based on probability functions. The values of the probability functions depend on a component's position in the new solution and its appearance in other promising solutions at a proximal position. Promising solutions are identified by comparing their objective values to other existing solutions. Multiple ants can operate in parallel, generating multiple solutions. For permutation flow shops, this means that jobs are subsequently added to one or more sequences.[66] Another artificial swarm intelligence algorithm is the ABC algorithm, which is similar to ACO.[67]

Particle Swarm Optimization

The PSO generates multiple solutions in each iteration, and the solutions are called particles. The particles are evaluated regarding an objective function. The solutions not delivering the so far best result are adjusted to change the solution structure to obtain a partial higher similarity to the best solution found at that time. The degree of adjustment is called velocity in most PSO approaches. The different velocities in

[64] See Pinedo (2005): *Planning and Scheduling in Manufacturing and Services*, pp. 431–432.
[65] See Błażewicz et al. (2007): *Handbook on Scheduling: From Theory to Applications*, p. 49.
[66] See Rajendran / Ziegler (2004): *Ant-colony algorithms for permutation flowshops*, pp. 428–438.
[67] See Karaboga / Akay (2009): *A comparative study of artificial bee colony algorithm*, pp. 113–114.

each iteration depend on the differences in objective values between the solutions, the previous velocity and random influences.[68]

Method selection

This thesis compares different configurations of the SA, TS and VNS, including first improvement and best neighbor as integrated local search. The different configurations of SA and the TS are mainly restricted to the neighborhoods included for choosing moves. The VNS is chosen, because it can use different altering neighborhood structures that exploit the problem's specifications. The other methods are implemented since they are common in scheduling and deliver results to evaluate the solution quality of the VNS. Genetic algorithms are not considered in this thesis because they have been examined in several publications.[69]

[68] See Eberhart / Kennedy (1995): *An optimizer using particle swarm theory*, pp. 39–45.
[69] See section 3.2 for an overview of literature.

3 Re-entrant Scheduling Problems

This chapter describes the characteristics of re-entrant permutation flow shops and their consideration in current literature. Section 3.3 describes a MIP model that is frequently used as the basis for extended model formulations in re-entrant scheduling problems.

3.1 Problem Assumptions and Classification

The considered problem is classified with: $\alpha_1 = F$, $\alpha_2 = m$, $\beta_1 = \circ$, $\beta_2 = \circ$, $\beta_3 = chains$ or $\beta_3 = recrc$, $\beta_4 = \circ$ for the first levels of each job, the ready times of the following levels depend on the schedule, $\beta_5 = \circ$ (for the test instances $\beta_5 = 0 \leq p \leq 99$ is chosen), $\beta_6 = \circ$, $\beta_7 = \circ$, $\beta_8 = \circ$. The objective is $\gamma = C_{max}$.

Within the 3-field classification scheme, the considered problem is denoted as $F, m|recrc, 0 \leq p \leq 99|C_{max}$ or $F, m|chains, 0 \leq p \leq 99|C_{max}$.

The general assumptions on flow shop problems are given in the following, since the examined problem is a flow shop problem. These assumptions are also summarized in Gupta / Stafford Jr (2006).[1]

The requirements and characteristics of the machine environment are described in detail:

- There is only one machine available for each operation.
- All machines are available at the time 0.
- All machines operate independently.
- A machine cannot process more than one job at a time.
- No breakdowns or other causes of machine unavailability are assumed.

The assumptions regarding the jobs are:

- No release dates different from 0.
- Due dates cannot be changed.

[1] See Gupta / Stafford Jr (2006): *Flowshop scheduling research after five decades*, pp. 701–703.

- The jobs are independent of each other.
- Each operation necessary to finish a job is dedicated to a single machine.
- Setup and transportation time are included within deterministic, finite processing times, if not stated differently.
- Each job is not allowed to visit a machine multiple times.
- Waiting times are allowed.
- Operations are not postponed if all requirements to perform the operation are fulfilled.
- Jobs are not allowed to be split into several sub-jobs.
- Jobs need to be finished.
- Operations are not allowed to be interrupted, so preemption is not allowed.
- Jobs cannot be on more than one machine at a time.
- There are no limits to buffer areas between machines.
- The machine environment is used exclusively by the jobs that need to be scheduled within the considered period of time.
- The job sequence for each machine is the same (permutation flow shop).

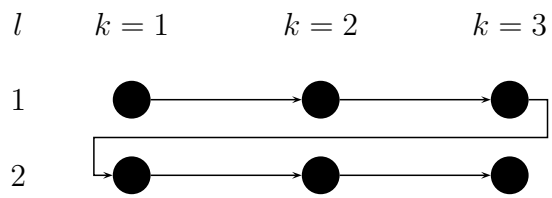
The characteristic feature of a re-entrant flow shop is that jobs are processed more than once on some machines due to repetitive processes, also known as levels[2], layers[3], loops[4] or cycles[5]. After one level is finished, the job re-enters the manufacturing system to be repeatedly processed and to accomplish the remaining levels. The jobs are not finished until every level is completed. In general, a re-entrant flow shop differs from a simple flow shop by the requirement that one or more jobs may need to be processed repeatedly at one or more stations. The re-entrant may be determined initially by the process flow or it may be caused by quality reasons. As described in Section 2.2, the problem considered is different from the general flow shop assumptions of Gupta / Stafford Jr (2006) based on several points:

[2] See Lee et al. (2011): *A genetic algorithm for bi-objective flow shops with re-entrants*, p. 1106.
[3] See Chen / Chao-Hsien Pan (2006): *Models for the re-entrant shop scheduling*, p. 578.
[4] See Rifai / Nguyen / Dawal (2016): *Large neighborhood search*, p. 43.
[5] See Choi et al. (2009): *Minimizing makespan under the maximum allowable due dates*, p. 965.

- The members of a permutation are not independent of each other, since the structure is a $\beta_3 = chains$.
- Jobs visit machines multiple times, since it is a re-entrant scheduling problem.
- Skipping of machines is allowed.
- Jobs are allowed to be split into several sublots in Chapter 5.

These assumptions are based on the problem characteristics in Arisha / Young / El Baradie (2002).[6] An example of a machine sequence for jobs in a re-entrant flow is shown for a three-machine flow shop with two levels per job in Figure 3.1.

Figure 3.1: Example of a machine sequence with re-entrant



Emmons / Vairaktarakis (2013) identified five classes of re-entrant flow shop problems:[7]

- Cyclic re-entrants: All jobs are processed several times on every machine. Therefore, the jobs move through the production system in levels, where each level starts at the first machine and ends at machine m.
- Chain re-entrants: Each job needs to return to the first machine to be finished after it has been processed on all m machines.
- Hub re-entrants: A central machine needs to be attended by each job before moving to the next machine.
- V-re-entrants: The jobs start in the first machine and move to machine m step-by-step. After being processed on machine m, the job moves successively back to the first machine by attending all other machines in the reverse sequence.
- (1-2-1) re-entrants: The production system of this type consists of two machines. Each job is operated once on the second machine and twice on the first one.

[6] See Arisha / Young / El Baradie (2002): *Flow shop scheduling*, p. 544.
[7] See Emmons / Vairaktarakis (2013): *Flow shop scheduling*, pp. 270–271.

Wang / Sethi / van de Velde (1997) showed that (1-2-1)-re-entrant flow shop problems considering a makespan minimization, which belong to the simplest class of re-entrants, are already NP-hard. The makespan minimization in other classes of re-entrant flow shop problems is an NP-hard problem as well. This causes high computation times for solving the model exactly. However, the model proposed in this thesis has the ability to represent every form of re-entrant listed above.[8]

3.2 Literature Review

Graves et al. (1983) first proposed a scheduling problem called the re-entrant flow shop problem and described an integrated circuit production in the early 1980s, which consists of 69 working stations performing about 185 operations on each job. Some facilities need to operate a single job up to eight times until it is completely finished.[9] Uzsoy / Lee / Martin-Vega (1992) and Uzsoy / Lee / Martin-Vega (1994) provided reviews of the production planning in the semiconductor industry.[10] These reviews also cover publications on re-entrant scheduling problems since they were first mentioned in Graves et al. (1983). Later, Danping / Lee (2011) gave a survey on publications between 1994 and 2009 regarding re-entrant scheduling problems.[11]

In regard to queuing theory, Kumar (1993) introduced the term "re-entrant line" and described this line as a closed shop scheduling problem, as mentioned by Graves (1981), with the additional re-entrant product flow.[12] Re-entrant line problems such as closed shop problems require the calculation of lot or batch sizes and inventory positioning between different work stations beside the sequencing decisions.[13] Re-entrant line scheduling problems are cyclic scheduling problems according to Chu / Chu / Desprez (2010). The problem focuses on determining job releases into a production system as well as queuing theoretical aspects like finite buffer capacities and buffer sizing. The performance measures for re-entrant lines are Work-in-process (WIP), Throughput (TP), production rate, tardiness and cycle time.[14]

The task in re-entrant flow shop and job shop scheduling problems is to sequence jobs on the machines of the production system. The main objectives are the makespan, flow time, completion time, throughput time, and due date related targets such as the tardiness or earliness of jobs.

[8] See Wang / Sethi / van de Velde (1997): *Minimizing makespan in reentrant shops*, pp. 703–704.
[9] See Graves et al. (1983): *Scheduling of re-entrant flow shops*, pp. 197–207.
[10] See Uzsoy / Lee / Martin-Vega (1992): *Semiconductor industry part I*, pp. 47–60 and Uzsoy / Lee / Martin-Vega (1994): *Semiconductor industry part II*, pp. 44–55.
[11] See Danping / Lee (2011): *Review of research for re-entrant scheduling*, pp. 2221–2242.
[12] See Kumar (1993): *Re-entrant lines*, p. 106.
[13] See Graves (1981): *A review of production scheduling*, p. 655.
[14] See Chu / Chu / Desprez (2010): *Series production in a basic re-entrant shop*, p. 257.

Danping / Lee (2011) gave an overview of re-entrant scheduling between 1994 and 2009. Hence, this section will cover the literature published from 2010 until 2015, distinguishing between re-entrant shop problems and re-entrant lines.

3.2.1 Search Methodology

The years reviewed are 2010 until June 2015. Several data-bases and search engines have been used to search the title, keywords and the abstract of articles for the following phrases:

- "scheduling",
- "flow shop" and "flowshop",
- "job shop" and "jobshop",
- "reentrant" and "re-entrant".

The databases and search engines used to get an overview on recent literature are:

- Google Scholar
- Web of Science
- ScienceDirect
- Ebscohost
- Taylor and Francis
- IEEE Xplore

The detailed query logic for the search engines and databases is presented in Appendix A.

The numbers of articles published between 2010 and June 2015 are given in Table 3.1. The numbers in parentheses for Google Scholar are values that include inproceedings, considering that Google Scholar had no filter to select journal articles only.

Table 3.1: Number of articles per year

Source	Hits	2010	2011	2012	2013	2014	2015
Google Scholar	53 (78)	10 (15)	15 (18)	7 (13)	8 (15)	12 (15)	2 (2)
Web of Science	68	8	16	14	12	12	6
ScienceDirect	22	5	7	6	1	2	1
Ebscohost	23	4	6	4	4	2	3
IEEE Xplore	7	2	1	1	3	0	0
Taylor and Francis	3	1	0	1	0	1	0
Total (no redundancies)	94	16	20	21	16	15	6

Table 3.2: Number of articles per journal

Journal	Hits
International Journal of Production Research	20
International Journal of Advanced Manufacturing Technology	7
IEEE Transactions on Semiconductor Manufacturing	5
Computers & Industrial Engineering	5
IEEE Transactions on Automation Science & Engineering	4
Applied Mechanics and Materials	3 (0)
European Journal of Operational Research	3
Expert Systems with Applications	3
Applied Soft Computing	2
CIRP Annals Manufacturing Research	2
Computer Integrated Manufacturing Systems	2 (0)
Advanced Materials Research	2
Journal of the Operational Research Society	1
Others	35
Total	94

More than 5000 articles containing the term "scheduling" in the title, abstract or as a keyword were found in Science Direct. 59 articles with the term "reentrant" and 58 with "re-entrant" were found.

Contrary to the review by Danping / Lee (2011), the search does not include the term "cyclic", due to the definition given on cyclic scheduling in Hanen (1994).[15] The search query found 105 articles, which were mainly published in the International Journal of Production Research, The International Journal of Advanced Manufacturing Technology,

[15] See Hanen (1994): *The recurrent job-shop*, p. 83.

Computers & Industrial Engineering, IEEE Transactions on Semiconductor Manufacturing, European Journal of Operational Research and Expert Systems with Applications.

The articles Cao / Peng / Wu (2010) and Qiao et al. (2010) in Computer Integrated Manufacturing Research are written in Chinese. The articles by Bareduan / Gani (2014) Pan / Ye / Zhou (2011) and Ye et al. (2014) are not relevant; therefore, no articles in Applied Mechanics and Materials are relevant. The article by Danping / Lee (2011) is included in the number of publications found but is not classified in the review table 3.7 because it is a literature review itself.

The publications found are described in Sections 3.2.2, 3.2.3 and 3.2.4. The literature reviewed in this section includes conference articles if they are closely related to the research published in journal articles.

3.2.2 Re-entrant Flow Shops

Chu / Chu / Desprez (2010) examined the optimality criteria for the minimization of the makespan and total flow time in a (1-2-1) re-entrant flow shop problem. Therefore the number of machines is limited to $m = 2$. The jobs are processed on two machines once and return to the first machine to be finished. Chu / Chu / Desprez (2010) tackled the problem of minimizing the makespan by applying a rule to avoid idle time on the first machine, since it is identified as the bottleneck machine of the (1-2-1) RFS. The first operation of a job is called the A-operation, the last operation the C-operation, and both are performed on machine $k = 1$. The rule of Chu / Chu / Desprez (2010) is to not allow C-operations on machine $k = 1$ before all A-operations are performed. For the minimization of total flow time, the C-operation of a job is performed before the A-operation of its direct successor.[16]

Lee et al. (2011) performed a bi-objective optimization in a stochastic RFS. Therefore a MIP model is suggested to minimize the total weighted tardiness of jobs and the makespan of the schedule. The solution is generated in two phases. The first phase is to generate a job sequence to minimize tardiness. The total weighted tardiness obtained is added as a constraint to the model for the second phase, which minimizes the makespan for the given tardiness value. Also a genetic algorithm is proposed to obtain near optimal job sequences.[17]

Yan / Wang (2012) proposed a scheduling framework for a dynamic RFS. It consists of two parts: the first one is to select a scheduling rule from a pool of given rules, while the second part is to apply it in real time. The problem is modeled as a series of discrete events. These events are the ends of each performed operation. The schedule

[16] See Chu / Chu / Desprez (2010): *Series production in a basic re-entrant shop*, pp. 258–259.
[17] See Lee et al. (2011): *A genetic algorithm for bi-objective flow shops with re-entrants*, pp. 1105–1113.

is recalculated after an operation is finished. The aim is to minimize the tardiness and earliness of jobs. Both values are connected with weights in a joint objective function.[18]

Eskandari / Hosseinzadeh (2014) proposed a MIP for a rework-related hybrid RFS with parallel-unrelated machines. The formulation is operation-based, since the problem considered is not a permutation flow shop. Additionally, a variable neighborhood search is suggested for this hybrid re-entrant permutation flow shop with sequence-dependent setup times. The re-entry feature of the problem appears through a rework probability and is limited to a maximum of one rework operation per job. The initial solutions for the VNS are created via dispatching rules. The integrated local search algorithm is a first improvement method. The stochastic nature of the rework problem is simulated with a Monte Carlo simulation module. The VNS is tested in two different versions: a VNS, where the jobs are scheduled at the machines with the shortest queue, and a VNS that assigns the jobs to machines with the shortest processing times. Both approaches have been compared with several dispatching rules that have been outperformed.[19]

Dugardin / Amodeo / Yalaoui (2010) considered a hybrid re-entrant flow shop problem with the objectives makespan and total tardiness. The proposed solution method is a so-called strength Pareto[20] evolutionary algorithm, which is a special variation of a genetic algorithm.[21] This method is used to handle multiple objectives.[22] The approach is improved by using Fuzzy logic (FL).[23] The membership values of solutions in previous iterations to fuzzy sets based on the obtained objective values and the degree of how much a solution changed during an iteration determine the crossover and mutation probabilities in the current iteration. In a later article, Dugardin / Amodeo / Yalaoui (2011) compared a multi-objective ant colony system algorithm and fuzzy ant colony system, showing that the implementation of a fuzzy logic controller also improves the ant colony algorithm.[24] For another multi-objective (bottleneck utilization and makespan) hybrid RFS with stochastic characteristics regarding rework and machine repairing, Dugardin / Yalaoui / Amodeo (2010) proposed a dominance relationship genetic algorithm. It outperformed a strength Pareto evolutionary algorithm and a variation of a genetic algorithm as a specific evolutionary algorithm.[25]

[18] See Yan / Wang (2012): *Minimising earliness and tardiness of a re-entrant line*, pp. 499–515.
[19] See Eskandari / Hosseinzadeh (2014): *Variable neighbourhood search for flow-shops*, pp. 1–10.
[20] Pareto optimality means that there is no possibility to improve one objective value without deteriorating another objective See Censor (1977): *Pareto optimality in multiobjective problems*, p. 43.
[21] See Dugardin / Amodeo / Yalaoui (2010): *FLC-archive*, pp. 324–327.
[22] See Zitzler / Laumanns / Thiele (2001): *Strength Pareto Evolutionary Algorithm 2*, p. 2.
[23] The fuzzy set theory was introduced by Zadeh (1965). It assigns objects gradually to sets or classes by determining a membership value. The membership value is between, at minimum (not assigned a considered set), 0 and 1 at maximum (only assigned to the considered set).
[24] See Dugardin / Amodeo / Yalaoui (2011): *Fuzzy Lorenz Ant Colony System*, pp. 1–6.
[25] See Dugardin / Yalaoui / Amodeo (2010): *Multi-objective method for reentrant hybrid flow shops*, pp. 22–31.

Yalaoui et al. (2010) also applied a fuzzy logic controller on a hybrid re-entrant flow shop problem in the framework of particle swarm optimization. Their publication focuses on the minimization of total tardiness. The solutions obtained by particle swarm optimization with a fuzzy logic controller are better than the solution without a fuzzy logic controller.[26]

Chamnanlor et al. (2014) examined a re-entrant permutation flow shop problem with missing operations and due windows in the context of hard disk drive manufacturing with job families based on the results of Chamnanlor et al. (2012). The job families differ in the machine sequence that they need to follow. The due windows are not allowed to be infringed. The missing operations are necessary to model the possibility to re-enter the production system on a machine different from $k = 1$ and to end a level on machine different from $k = m$. The makespan is the considered objective. A hybrid genetic algorithm with fuzzy logic is applied to the problem.[27]

For a two-stage flexible re-entrant flow shop problem, a hybrid genetic algorithm and another modified genetic algorithm, called a random key genetic algorithm, have been compared to modified versions of the SPT rule, LPT rule and NEH algorithm by Hekmatfar / Fatemi Ghomi / Karimi (2011). The hybrid genetic algorithm obtained the best results for makespan minimization. Additionally, an MIP model is suggested to represent the problem.[28]

The same problem was also examined by Huang / Yu / Kuo (2014) and extended with due dates for the jobs that need to be processed. Three algorithms are assessed for the problem: an ant colonization algorithm, a particle swarm optimization and a tuned particle swarm optimization approach. The latter outperformed the first two in regards to the minimization of the total weighted earliness and tardiness.[29]

Also, Cho et al. (2011) examined the hybrid RFS with minimizing the makespan and total tardiness as target functions in a multi-objective optimization. To solve this problem, a genetic algorithm with a Pareto objective function is used. The algorithm is compared to another variation of the genetic algorithm. The Pareto approach reaches better objective values, but needs longer computation times.[30]

Pareto criteria for multi-objective optimization are also used by Ying / Lin / Wan (2014). A greedy algorithm with Pareto criteria is suggested to minimize the makespan and total tardiness of jobs in a hybrid re-entrant flow shop. Initial solutions are created

[26] See Yalaoui et al. (2010): *Particle swarm optimization for a hybrid Reentrant Flow Shops*, pp. 1–6.
[27] See Chamnanlor et al. (2014): *Re-entrant flow shops with time windows*, pp. 2612–2629.
[28] See Hekmatfar / Fatemi Ghomi / Karimi (2011): *Reentrant flow shops with setup times*, pp. 4530–4539.
[29] See Huang / Yu / Kuo (2014): *Reentrant multiprocessor flow shop with due windows*, pp. 1263–1276.
[30] See Cho et al. (2011): *Bi-objective scheduling for reentrant hybrid flow shop*, pp. 529–541.

randomly.[31]

The problem of scheduling the production of stacked chips in semiconductor manufacturing has been identified as a re-entrant flow shop problem under stochastic influences by Han / Choi (2010), who tackled the problem with a framework based on Petri nets (PN) [32] to maximize the system throughput.[33]

In Lin / Lee / Wu (2011) and Lin / Lee / Wu (2012), a genetic algorithm with an integrated analytic hierarchy process[34] was applied on a hybrid re-entrant flow shop. The analytic hierarchy process is used to identify promising parents for the genetic operations in order to optimize multiple criteria. These criteria are the total cost, total tardiness, the number of tardy jobs and makespan. The proposed combination outperformed the pure genetic algorithm. It was also compared to a first in first out dispatching rule, which attempts to schedule the re-entering job as soon as possible in the job sequence.[35] The objective in Lin / Lee / Wu (2012) is total weighted tardiness.[36]

The sequencing of moves of a robot arm in semiconductor test facilities was examined by Sangsawang et al. (2015). The associated problem is modeled as hybrid no-wait RFS with identical parallel machines. The re-entries occur on the robot arm since the parts need to be repeatedly handled by the robot arm to be moved to different places of the facility, which makes it a hub re-entrant problem. The schedules are created with a hybrid genetic algorithm and a hybrid particle swarm optimization with makespan as the objective.[37]

A Lagrangian decomposition algorithm was used by Kaihara / Kurose / Fujii (2012) to solve a real-world semiconductor hybrid / flexible RFS problem with due dates and identical parallel machines to minimize tardiness. The algorithm proposed is based on the relaxation of the machine capacity constraint of a suggested MIP model.[38]

Choi / Kim / Lee (2011) suggested real-time dispatching rules for a five stage hybrid

[31] See Ying / Lin / Wan (2014): *Bi-objective reentrant hybrid flowshop scheduling*, pp. 5735–5747.

[32] Petri nets are graphs that represent events and conditions. Conditions are represented by a set of nodes, which are called places. Events are represented by a set of nodes, which are called transitions. The sets of nodes differ in their graphical representation. The status of a system is described by the tokens at each place. Arcs connect places and transitions and describe possible changes in the modeled system, i.e. a change in the number of tokens at specific places. For further information, see Murata (1989): *Petri nets: Properties, analysis and applications*.

[33] See Han / Choi (2010): *A GSPN-based approach to stacked chips scheduling problem*, pp. 4–12.

[34] The analytical hierarchy process is a structured way to evaluate alternative options in a decision making process. The weights of evaluation criteria are determined by pairwise comparisons of the criteria. The evaluations of the alternatives' characteristics are also based on pairwise comparisons of the alternatives. The total decision values are obtained over the eigenvectors of the evaluation matrix and the criteria weight matrix. For more information see e.g. Saaty (1990): *The analytic hierarchy process*.

[35] See Lin / Lee / Wu (2011): *Integrated GA and AHP for re-entrant flow shops*, pp. 496–500.

[36] See Lin / Lee / Wu (2012): *Analytical hierarchy process and genetic algorithm*, pp. 1813–1824.

[37] See Sangsawang et al. (2015): *Reentrant flexible flow shop with blocking*, pp. 2395–2410.

[38] See Kaihara / Kurose / Fujii (2012): *Actual-scale semiconductor manufacturing*, pp. 467–470.

re-entrant flow shop in thin film transistor display manufacturing. The scheduling is dynamic due to possible machine break downs. The considered objectives are flow time and tardiness related values.[39]

The minimization of total tardiness is the objective in a (1-2-1) hybrid re-entrant flow shop problem in the semiconductor industry investigated by Kim et al. (2011). The first stage consists of multiple material handling machines. These machines load chips into holding boards, which proceed to the machines in the second stage. These machines are workstations to test the chips. After being tested the chips need to be unloaded by stage one again. Several priority rules are suggested to create promising schedules.[40]

Lee / Lin (2010) minimized the makespan and total tardiness for a re-entrant permutation no-wait flow shop without any hybrid characteristics. Within the objective function, the tardiness of each job is multiplied with a special weight. A hybrid genetic algorithm is used to solve the problem. Specific job weights are calculated according to the number of re-entries of a job. The suggested methods are tested in a simulation environment.[41] The results have also been published in Lee et al. (2011).[42]

A multi-level genetic algorithm for makespan minimization in a resource constraint re-entrant flow shop was proposed by Lin / Lee (2012) and Lin / Lee / Ho (2013). The limited resources in these cases can be human resources.[43,44]

Qian et al. (2013b) proposed a so-called differential evolution algorithm for re-entrant permutation flow shop problems in order to minimize the makespan. The results of the suggested method show significant improvements compared to a hybrid genetic algorithm developed by Chen / Pan / Lin (2008).[45,46]

A special case for a four-machine RFS was examined in Bareduan / Hasan (2010) and Bareduan / Hasan (2012). Each job is operated twice on machines $k = 3$ and $k = 4$. Therefore, a heuristic to minimize the makespan based on the identification of bottleneck machines is developed. The identified bottlenecks in that system are machines $k = 1$ and $k = 4$. The method produces improved results compared with the NEH heuristic.[47]

Jeong / Kim (2014) developed a branch and bound algorithm for minimizing total tardiness in a two-machine re-entrant flow shop with sequence-dependent setup times for

[39] See Choi / Kim / Lee (2011): *Real-time scheduling for reentrant hybrid flow shops*, pp. 3514–3521.
[40] See Kim et al. (2011): *Minimizing tardiness in a semiconductor manufacturing system*, pp. 14–26.
[41] See Lee / Lin (2010): *Bi-objective flow shops with re-entrant jobs*, pp. 1240–1245.
[42] See Lee et al. (2011): *A genetic algorithm for bi-objective flow shops with re-entrants*, pp. 1105–1113.
[43] See Lin / Lee (2012): *Resource-Constrained Re-Entrant Flow Shop Scheduling Problem*, pp. 653–657.
[44] See Lin / Lee / Ho (2013): *Resource-constrained re-entrant scheduling*, pp. 1282–1290.
[45] See Qian et al. (2013b): *Reentrant permutation flow-shops with different job reentrants*, pp. 22–27.
[46] See Chen / Pan / Lin (2008): *A hybrid genetic algorithm for re-entrant flow-shops*, pp. 572–572.
[47] See Bareduan / Hasan (2010): *Internet-Based Collaborative Manufacturing*, pp. 91–97 and See Bareduan / Hasan (2012): *Re-Entrant Flow Shop With Dominant Machines*, pp. 81–93.

the second machine. The branch and bound procedure is compared to some dispatching rules and modified versions of the NEH algorithm and to an algorithm suggested in Framinan / Leisten (2003), which is designed to minimize the total flow time in permutation flow shops.[48] The branch and bound procedure could find optimal solutions for problems with ten jobs and one re-entry for each job.[49]

The makespan minimization in a re-entrant permutation flow shop problem with mixed levels was examined in Li et al. (2013). A population-based algorithm combined with a first improvement local search was used to solve the problem. The results are compared to the solutions of a rebuilt genetic algorithm of Chen / Pan / Lin (2008) in computational experiments. The suggested combination of a population-based solution scheme and the simple local search obtained better results for the tested instances.[50]

Kaihara et al. (2010) examined the scheduling of jobs and maintenance operations in an RFS. The objective in the suggested model and proposed Lagrangian relaxation method is the total tardiness of jobs and maintenance operations. The approach was tested in a simulation study and resulted in near optimal solutions.[51]

A memetic algorithm as a form of hybrid genetic algorithm is suggested for a re-entrant permutation flow shop with separated job levels in Xu et al. (2014). It is a combination of the operations cross-over and mutation of a genetic algorithm and the integration of a local search phase. The initial population is generated randomly, while one individual is generated by an NEH procedure. The results are compared to MIP solutions and are near optimal for small instances.[52]

3.2.3 Re-entrant Job Shops

Re-entrant job shop (RJS) scheduling problems do not require the jobs to be processed with the same machine sequence. Hence, a minimum of one machine needs to be visited more than once by one or more jobs, since the problem is re-entrant. Hybrid or flexible job shop problems have multiple parallel machines available on at least one production stage, which is similar to hybrid / flexible flow shops.

Elmi et al. (2011) proposed an MIP for re-entrant job shops with manufacturing cells to minimize the makespan. Furthermore, a simulated annealing algorithm is suggested for the problem.[53]

An ant colony optimization algorithm for the re-entrant job shop with batch processing

[48] See Framinan / Leisten (2003): *Flowtime minimisation in permutation flow shops*, pp. 311–317.
[49] See Jeong / Kim (2014): *Two-machine re-entrant flowshop with setup times*, pp. 72–80.
[50] See Li et al. (2013): *Population-Based Learning Algorithm*, pp. 1636–1641.
[51] See Kaihara et al. (2010): *Proactive maintenance scheduling in a re-entrant flow shop*, pp. 453–456.
[52] See Xu et al. (2014): *A memetic algorithm for the re-entrant permutation flowshop*, pp. 277–283.
[53] See Elmi et al. (2011): *A simulated annealing algorithm for the job shop cell scheduling*, pp. 171–178.

machines was proposed by Guo et al. (2012). The algorithm and underlying MIP aim for a minimization of the makespan.[54]

Jung / Lee (2012) suggested a Petri net based solution approach for a hub re-entrant job shop with finite buffer capacities between the machines in order to minimize the makespan in a robotic cell. The robot arm in a robotic manufacturing cell is the re-entrant machine, which the jobs need to use multiple times.[55]

Topaloglu / Kilincli (2010) tested a shifting bottleneck heuristic[56] for a re-entrant job shop in the textile industry. The target is to minimize the makespan, where the shifting bottleneck heuristic is proposed for a regular job shop. The solution quality and computational performance are compared to an MIP solved by LINGO 8.0.[57]

Bard et al. (2013) proposed an MIP for a flexible re-entrant job shop with precedence constraints, setup times and due dates. The machines are assumed to be multi-tools, which means that different machines are able to perform the same operation in the flexible re-entrant job shop. The operation does not necessarily take the same amount of time to be completed on different machines. The considered objectives are the makespan, job waiting times and tardiness. Also, a GRASP is tested.[58] An enhancement of the approach is suggested by Bard et al. (2015).[59]

Assembly processes have strong precedence constraints since the job relations are an intree. An MIP for a flexible re-entrant job shop in assembly is proposed by Gomes / Barbosa-Póvoa / Novais (2013). The objective is to minimize total waiting times and the earliness and tardiness of jobs. Also, different reactive scheduling strategies are compared in a predictive way for the case of new jobs entering the production system. These strategies include partial changes, complete rescheduling and applying no changes to the schedule.[60]

A flexible re-entrant job shop with setup times for identical parallel machines and job due dates is considered by Chen et al. (2012). Makespan, machine idle time and tardiness are optimized with a genetic algorithm.[61]

Driessel / Mönch (2012a) decomposed a hybrid RJS problem for automated material handling systems in semiconductor manufacturing in job-related and transport subproblems. The job-related subproblems are re-entrant job shops with precedence

[54] See Guo et al. (2012): *Decomposition-based classified ant colony optimization*, pp. 141–151.
[55] See Jung / Lee (2012): *Model for Cluster Tool Scheduling Problems*, pp. 186–199.
[56] The shifting bottleneck heuristic was proposed by See Adams et al. (1988): *The Shifting Bottleneck Procedure for Job Shop Scheduling*, pp. 392–397 for job shop problems. The method sequences the jobs on each machine separately.
[57] See Topaloglu / Kilincli (2010): *Shifting bottleneck heuristic for reentrant job shops*, pp. 785–792.
[58] See Bard et al. (2013): *Scheduling at assembly and test facilities*, pp. 7047–7070.
[59] See Bard et al. (2015): *Optimisation and simulation for assembly and test operations*, pp. 2617–2632.
[60] See Gomes / Barbosa-Póvoa / Novais (2013): *Reactive scheduling*, pp. 5120–5141.
[61] See Chen et al. (2012): *Flexible job shop scheduling*, pp. 10016–10021.

constraints, release dates, setup times, due dates and batch processing. The transport related subproblems are transformed into a scheduling problem with identical parallel machines and setup times. The objective is to minimize the tardiness of jobs. A shifting bottleneck heuristic is used to solve the job-related problems, while a VNS is used to solve the transport related subproblems.[62] The neighborhoods used in the VNS are based on swap and insertion moves of operations on either a single machine or all possible machines. These neighborhoods were examined for a non re-entrant job shop in Driessel / Mönch (2011).[63] Shifting bottleneck heuristics are also applied to complex job shops including re-entrant features by Driessel et al. (2010) and Driessel / Mönch (2012b). Complex job shops include sequence-dependent setup times, parallel machines, batch processing, machine breakdowns and re-entrant product flows. For testing the solution approach, a simulation environment designed by INTEL, called INTEL minifab, was used. The optimization objective was the weighted tardiness.[64,65]

A hybrid re-entrant job shop scheduling problem with job release dates and batch processing was investigated by Jampani / Mason (2010). The objective function of the proposed MIP is the total weighted completion time. A column generation heuristic based on the MIP is developed, due to the problem complexity. The objective gap to the global optimal solution of the problem is between 2 and 13 % for the test instances.[66]

Johnson's rule is the basis for a heuristic developed by Xie / Tang / Li (2011) to schedule jobs in a hub re-entrant job shop and a hybrid hub re-entrant shop job. The objective is makespan minimization. The re-entrant machine in the underlying practical case is a crane in a packing process in the steel industry. The proposed heuristic in the worst case is 20 % weaker than the optimal solution.[67]

Chen / Wang (2013) examined an RJS problem in the wafer manufacturing industry. They considered uncertain re-entry numbers and processing times. A fuzzy logic was applied to a dispatching rule to minimize the average flow time, flow time standard deviation, maximum lateness and the number of tardy jobs. This method is considered to lead to a disadvantageous schedule if the estimation of processing times and re-entries is not accurate enough.[68]

Dispatching rules are suggested by Chiang / Fu (2012) to minimize mean tardiness and maximum tardiness or to maximize the on-time delivery rate in a flexible RJS.[69]

[62] See Driessel / Mönch (2012a): *Integrated scheduling and material-handling*, pp. 5966–5985.
[63] See Driessel / Mönch (2011): *Jobs with precedence constraints and ready times*, pp. 336–340.
[64] See Driessel et al. (2010): *A parallel shifting bottleneck heuristic for complex job shops*, pp. 81–86.
[65] See Driessel / Mönch (2012a): *Integrated scheduling and material-handling*, pp. 413–418.
[66] See Jampani / Mason (2010): *A column generation heuristic for complex job shop*, pp. 108–118.
[67] See Xie / Tang / Li (2011): *Hub reentrant job shop*, pp. 743–753.
[68] See Chen / Wang (2013): *A Fuzzy Rule for Multiobjective Job Dispatching*, pp. 1–18.
[69] See Chiang / Fu (2012): *Rule-based scheduling in wafer fabrication*, pp. 2820–2835.

This approach was extended by Chiang (2013).[70]

A disjunctive graph model for an RJS with sequence-dependent setup times was developed by Dehghanian / Homayouni (2013). The applied solution method is a genetic algorithm with an integrated fuzzy logic controller. The new method outperformed the simple genetic algorithm with, on average, 5 % lower makespan values.[71]

Li / Linhao / Yunfeng (2012) developed three algorithms for makespan minimization in a real re-entrant job shop. These include a genetic algorithm, memetic-climbing algorithm and memetic simulated annealing algorithm. Memetic algorithms are genetic algorithms combined with a trajectory method. The integration of local search algorithms lead to better objective values.[72]

Beside the application on re-entrant flow shop problems, Qian et al. (2013a) also used a differential evolution algorithm for re-entrant job shops, which they combined with a problem-dependent local search for a multi-objective minimizing of total machine idle time and maximum tardiness.[73]

Fattahi et al. (2010) proposed a mixed integer programming model for a bi-objective re-entrant flexible job shop problem. The first objective is to minimize the working load of the bottleneck machines by minimizing their idle time; the second one concerns minimizing the WIP by reducing the total flow time of jobs.[74]

Yugma et al. (2012) developed an MIP formulation for a wafer fabrication scheduling problem, which includes re-entrant characteristics. They proposed a priority-rule based insertion method for job scheduling.[75]

3.2.4 Re-entrant Line Problems

The focus of a re-entrant line problem is on the increase of throughput rate, with low and smooth WIP over the course of time. A main research point is the identification of bottleneck machines and performance measurement. In some cases, a predefined order of job release is given, but the jobs need to be assigned to parallel machines and the activities of material handling devices, e.g. robot arms, need to be scheduled.[76]

Choi / Kim (2012) proposed a method to estimate the throughput in a re-entrant line problem achieved by a greedy scheduling method if new jobs enter the system randomly. The material flow is assumed to be re-entrant with finite buffer capacity between the

[70] See Chiang (2013): *Enhancing rule-based scheduling by evolutionary algorithms*, pp. 524–535.
[71] See Dehghanian / Homayouni (2013): *Re-entrant job shops with setup times*, pp. 1–5.
[72] See Li / Linhao / Yunfeng (2012): *Release control*, pp. 977–990.
[73] See Qian et al. (2013a): *Multi-objective reentrant job-shop scheduling problem*, pp. 485–489.
[74] See Fattahi et al. (2010): *A hybrid algorithm for re-entrant manufacturing systems*, pp. 268–278.
[75] See Yugma et al. (2012): *A batching and scheduling algorithm*, pp. 2118–2132.
[76] See Wikborg / Lee (2013): *Scheduling single-armed cluster tools*, p. 700.

stages of the production system. Each stage consists of parallel identical machines.[77]

The determination of release dates of jobs in a re-entrant line with finite buffer capacities was investigated by Dong / He (2012). Partial differential equations were used to control the WIP and throughput.[78]

To reach a smooth WIP level over the course of time in a semiconductor wafer fabrication system, Hu et al. (2010) proposed a genetic algorithm. Avoiding idle times on bottleneck machines was used to achieve a target level of WIP. The problem is described for machines with stochastic break-downs. Side objectives are the minimization of tardy jobs as well as jobs being finished too early.[79]

Jia / Jiang / Li (2013) considered identical parallel batch processing machines in a re-entrant line. The set of jobs is segmented into job families with release and due dates. The waiting times of jobs are not allowed to exceed a predetermined value. A genetic algorithm and priority rules based on a Petri net formulation are applied to the problem to minimize total weighted tardiness. The solution method is tested in a simulation study. Further performance indices are flow time and WIP level.[80] Also in Jia / Jiang / Li (2015), a genetic algorithm was developed for a re-entrant line problem to sequence job batches. The objective was also to minimize total weighted tardiness.[81]

A re-entrant line problem with stochastic processing times and machine break-downs in the semiconductor industry with a given target flow time and throughput rate was examined by Kim / Cox / Mabin (2010). A simulation study was used to determine how much WIP and buffer capacity is necessary to reach certain target values.[82]

Liu / Li / Chiang (2010) considered a re-entrant line in the automotive industry with exponentially distributed machine break downs. A method for estimating the production rate is suggested. A dispatching rule for job assignment is applied which prefers jobs that have spent the longest time in the production system.[83] In a later publication, methods were developed to identify bottlenecks and to approximate the throughput for a system with machine break-downs, which are assumed to be exponentially distributed. The average deviation from a target throughput obtained in 100 simulations is about 2 %. The bottleneck machine is identified by comparing scenarios with additional capacity on different machines. The configuration with the highest improvement of throughput indicates the bottleneck machine.[84]

[77] See Choi / Kim (2012): *Capacitated re-entrant line scheduling problem*, pp. 2353–2362.
[78] See Dong / He (2012): *A new continuous model for re-entrant manufacturing systems*, pp. 659–668.
[79] See Hu et al. (2010): *A decomposition based algorithm for scheduling*, pp. 2066–2070.
[80] See Jia / Jiang / Li (2013): *Real-time dispatching on parallel batch machines*, pp. 4570–4584.
[81] See Jia / Jiang / Li (2015): *Re-entrant batch-processing machines*, pp. 4570–4584.
[82] See Kim / Cox / Mabin (2010): *Protective inventory in a re-entrant line*, pp. 4153–4178.
[83] See Liu / Li / Chiang (2010): *Re-entrant lines with unreliable machines and finite buffers*, pp. 1151–1159.
[84] See Liu / Li / Chiang (2012): *Performance approximation and bottleneck identification*, pp. 977–990.

Petri net-based dispatching rules were used by Shin (2015) to increase the quality of work in a re-entrant line problem with a single stage production system consisting of multiple identical parallel machines. The performance measures are the process capability index, tardiness and cycle time. The process capability index shows the degree to which it is possible to achieve a specified output level within a limited time.[85]

A re-entrant line problem with limited resources was considered by Liu et al. (2010). The impact of dispatching rules on machine utilization and throughput in an example system is examined in a simulation.[86]

Chong / Jingshan (2010) tested an approach to measure the performance in re-entrant lines. The performance index is the system throughput. Also the estimation of different WIP was examined.[87] The estimation of flow time in re-entrant lines to give information on due date assignments was examined by Tai / Pearn / Lee (2012).[88] Another publication on performance estimation was prepared by Starkov et al. (2013), who consider a re-entrant line scheduling problem with infinite buffer capacity. The influence of the buffer inventory on how the production output follows changes in demand was examined, but sequencing decisions were not examined.[89]

A re-entrant line in a layer coating process was examined by Wu et al. (2011). A robot arm handles the parts in a cluster tool with multiple workstations that perform coating operations on wafers. The coating process places integrated circuits on the wafer. The workstations and the robot are visited multiple times by the jobs. A Petri net model represents the work-flow. The problem concerns the work station where the wafer needs to be placed in order to have short movements for the robot. The target is to obtain a schedule with the minimal makespan of a cyclic schedule.[90]

Yan / Hassoun / Meerkov (2012) analyzed two dispatching rules for loading a bottleneck work-center with down times. The jobs are repeatedly processed at the work-center without visiting other workstations between the operations. The work-center has different buffers for different levels of re-entries. The first dispatching rule prefers jobs from lower levels, while the other one prefers jobs on higher levels. The rule preferring the lower levels yielded better results regarding WIP and the production rate.[91]

The decision about the size of limited buffers and individual release dates of jobs in a re-entrant line was considered by Yang / Hsieh / Cheng (2011). By applying a drum

[85] See Shin (2015): *Dispatching in re-entrant production lines*, pp. 249–259.
[86] See Liu et al. (2010): *Dynamic reentrant scheduling simulation*, pp. 2418-2422.
[87] See Chong / Jingshan (2010): *Approximate Analysis of Reentrant Lines*, pp. 708–715.
[88] See Tai / Pearn / Lee (2012): *Cycle time estimation*, pp. 581–592.
[89] See Starkov et al. (2013): *Performance analysis of re-entrant manufacturing*, pp. 1563–1586.
[90] See Wu et al. (2011): *Petri Net-Based Scheduling*, pp. 42–55.
[91] See Yan / Hassoun / Meerkov (2012): *Equilibria, stability, and transients in re-entrant lines*, pp. 211–229.

buffer rope scheduling method[92] to the problem, the WIP, buffer occupation and flow time are reduced in connection with an increase of the production rate for a thin-film transistor liquid-crystal display manufacturing plant.[93] A simplified drum buffer rope approach to influence the performance of a so-called stochastic re-entrant flow shop is suggested by Chang / Huang (2014). Despite referring to a flow shop in the title of the publication, the problem is more related to re-entrant lines than to a classical re-entrant flow shop. The performance measures to evaluate the simplified drum buffer rope are due date related values, the average queue length of the capacity constraint resource, WIP, throughput and flow time. These measures are influenced by assigning due dates and release dates to jobs. The job sequence on every machine is determined by dispatching rules. The simplified drum buffer rope control is tested in simulation with uniformly distributed processing times and random machine breakdowns. The main result is an increased utilization of the capacity constraint resource compared to previous methods.[94]

A machine learning approach for a single stage re-entrant line problem with parallel machines in semiconductor test facilities is suggested by Zhang et al. (2011). The jobs are processed in batches and require setup times. The objective is to minimize the total weighted unsatisfied demand. The heuristic was able to reduce the objective values by an average of 65 % in experiments compared to the industrial method.[95]

The determination of job release dates is the focus of Qiao / Wu (2013). Therefore, a mechanism to identify the system bottleneck in a re-entrant line manufacturing system is proposed. The throughput of the system bottleneck and, therefore, of the whole system is increased by determining the job releases into the different parts of the manufacturing system.[96] In addition, Hu et al. (2013) examined the bottleneck identification in re-entrant lines. Machine break downs are a special characteristic of the considered line. They aim to protect the bottlenecks from being empty by smoothing the capacity usage in non-bottleneck machines in order to increase the system throughput.[97]

Choi (2015) tested a mechanism to avoid deadlocks in a re-entrant line clustertool with limited buffer capacity. The aim of the deadlock analysis and avoidance policy is to increase system throughput.[98]

[92] Drum buffer rope is a control policy that determines a series of operations by utilizing the resource with the tightest constraints. Such a resource or machine should not be empty, since its processing time is considered the most valuable. The schedule of this machine is determined first and is called drum. The materials required for this operation are provided by the operation of the schedule on previous working stations. Their schedule is derived from the drum. This mechanism is called rope. For more information, see Schragenheim / Ronen (1990).

[93] See Yang / Hsieh / Cheng (2011): *Lean-pull strategy in a re-entrant manufacturing*, pp. 1511–1529.

[94] See Chang / Huang (2014): *SDBR in a random reentrant flow shop environment*, pp. 1808–1826.

[95] See Zhang et al. (2011): *Semiconductor final test scheduling*, pp. 446–458.

[96] See Qiao / Wu (2013): *Layered Drum-Buffer-Rope-Based*, pp. 178–187.

[97] See Hu et al. (2013): *Multiple bottlenecks in wafer fabrication*, pp. 111–120.

[98] See Choi (2015): *Banker's algorithm*, pp. 2605–2616.

Optimality criteria for moves of a robot arm regarding total flow time in a cyclic schedule for robotic cells were estimated by FOUMANI / JENAB (2012). A special characteristic of the studied problem is the ability of the robot arm to exchange parts with a machine without a loadlock being involved.[99]

YAN et al. (2013) aimed for smooth capacity usage, setup avoidance and a postponed expiration of additional manufacturing equipment by applying a branch and cut approach for a re-entrant lines problem in wafer manufacturing with litho machines.[100]

Comprehensive review

A comprehensive overview of the articles is provided in Table 3.7. The publications included are journal articles only. The classification of the problem has a similar structure to the 3-field classification scheme as presented in Section 2.2. Parallel machines are denoted with Par. Parallel identical, uniform and unrelated machines are not distinguished. The last character in the column "Machines" tells the number of machines in non-flexible / non-hybrid problems or the number of stages in the case of flexible shops. Precedence constraints on the jobs are abbreviated with *prec* in the following Table 3.3. No distinction is made between *intree*, *outtree* and series parallel precedence constraints. All articles in Table 3.7 concern re-entrant scheduling problems.
The additional problem characteristics are abbreviated by the symbols shown in Tables 3.3, 3.4, 3.5 and 3.6.
Table 3.3 shows the symbols used to describe the machine characteristics.

Table 3.3: List of machine characteristics

Par	Parallel machines
F	Flow shop
J	Job shop
m	Arbitrary number of machines

The symbols addressing job properties are shown in Table 3.4.

[99] See FOUMANI / JENAB (2012): *Cycle time analysis in reentrant robotic cells*, pp. 6372–6387.
[100] See YAN et al. (2013): *Litho Machine Scheduling*, pp. 928–937.

Table 3.4: List of job characteristics

prec	Precedence constraints between the jobs
r_i	Release dates
batch	Batch processing
ST	Setup times
res	Resource constraints
MO	Missing operations
LB	Limited buffer capacity

The third group of characteristics contains the objectives. An overview on the symbols used for this group is given by Table 3.5.

Table 3.5: List of objectives

C_{max}	Makespan
C_i	Total completion / cycle / flow time
WIP	Work-in-process
$\overline{I}$	Total idle time / machine utilization
TP	Through put
E_i / T_i	Earliness / tardiness related objectives

The literature review in Table 3.7 uses the abbreviations of solution methods provided by Table 3.6.

Table 3.6: List of solution methods

MIP	Mixed integer programming
PN	Petri nets
EA	Evolutionary algorithms
SA	Simulated annealing
PR	Dispatching rules
VNS	Variable neighborhood search
SI	Swarm intelligence algorithms
FL	Fuzzy logic

Table 3.7: Literature Review 2010–2015

		Job Characteristics							Objectives							Solution Method								
Author	Machines	*prec*	r_i	*batch*	ST	*res*	MO	LB	C_{max}	C_i	WIP	$\overline{I}$	TP	E_i / T_i	other	MIP	PN	EA	SA	PR	VNS	SI	FL	other
Bard et al. (2013)	$Par/J/m$				x		x		x			x	x		x	x								x
Bard et al. (2015)	$Par/J/m$				x		x		x			x	x		x	x								x
Chamnanlor et al. (2014)	F/m						x		x									x					x	
Chang / Huang (2014)	m		x	x											x					x				x
Chen et al. (2012)	$Par/J/m$				x				x			x		x				x						
Cho et al. (2011)	$Par/F/m$						x		x					x		x		x						
Choi / Kim / Lee (2011)	$Par/F/m$							x		x			x	x	x					x				x
Choi / Kim (2012)	Par/m							x					x											x
Choi (2015)	Par/m							x					x											x
Chong / Jingshan (2010)	m							x					x							x				
Chu / Chu / Desprez (2010)	$F/2$					x			x	x										x				
Dong / He (2012)	Par/m							x			x		x				x							x
Driessel / Mönch (2012a)	$Par/J/m$	x	x	x	x									x						x	x			x
Dugardin / Yalaoui / Amodeo (2010)	$Par/F/2$									x		x						x						
Elmi et al. (2011)	J/m								x							x			x					
Eskandari / Hosseinzadeh (2014)	$Par/F/m$		x	x	x		x		x							x				x	x			
Foumani / Jenab (2012)	2									x														

Author	Machines	Job Characteristics							Objectives							Solution Method								
		prec	r_i	*batch*	ST	*res*	MO	LB	C_{max}	C_i	WIP	$\overline{I}$	TP	E_i / T_i	other	MIP	PN	EA	SA	PR	VNS	SI	FL	other
Gomes / Barbosa-Póvoa / Novais (2013)	$Par/J/m$	x									x			x		x								
Guo et al. (2012)	J/m	x							x							x						x		x
Han / Choi (2010)	$Par/F/6$	x						x					x			x	x							
Hekmatfar / Fatemi Ghomi / Karimi (2011)	$Par/F/2$				x				x									x						
Hu et al. (2013)	m							x		x	x		x	x						x				
Huang / Yu / Kuo (2014)	$Par/F/2$													x		x						x		
Jampani / Mason (2010)	$Par/J/m$		x	x						x						x								x
Jia / Jiang / Li (2013)	$Par/3$		x	x				x						x			x	x		x				
Jia / Jiang / Li (2015)	$Par/4$						x							x		x				x				
Jung / Lee (2012)	J/m							x	x							x	x							
Kaihara et al. (2010)	F/m													x		x								x
Kaihara / Kurose / Fujii (2012)	F/m						x							x		x								x
Kim / Cox / Mabin (2010)	m									x			x											
Kim et al. (2011)	$Par/F/2$													x						x				
Lee et al. (2011)	F/m								x					x		x		x						
Li et al. (2013)	F/m								x															x
Lin / Lee / Wu (2012)	F/m													x		x		x						x
Liu et al. (2010)	Par/m		x	x	x	x						x	x											
Liu / Li / Chiang (2010)	m							x					x											

Author	Machines	Job Characteristics							Objectives							Solution Method								
		prec	r_i	*batch*	ST	*res*	MO	LB	C_{max}	C_i	WIP	$\overline{I}$	TP	E_i / T_i	other	MIP	PN	EA	SA	PR	VNS	SI	FL	other
Liu / Li / Chiang (2012)	m							x					x											
Qiao / Wu (2013)	Par/m							x	x		x	x								x				x
Sangsawang et al. (2015)	$Par/F/2$								x									x				x		
Shin (2015)	$Par/1$		x	x	x					x				x	x					x				
Starkov et al. (2013)	m																							
Tai / Pearn / Lee (2012)	m									x														
Topaloglu / Kilincli (2010)	J/m								x											x				x
Wikborg / Lee (2013)	Par/m	x							x			x			x									
Wu et al. (2011)	m							x	x								x							
Xie / Tang / Li (2011)	$Par/J/3$								x															x
Xu et al. (2014)	F/m								x									x						
Yan / Wang (2012)	F/m													x						x			x	x
Yan / Hassoun / Meerkov (2012)	m										x		x		x					x				
Yan et al. (2013)	$Par/1$				x	x									x	x								x
Yang / Hsieh / Cheng (2011)	Par/m							x	x		x		x							x				
Ying / Lin / Wan (2014)	$Par/F/m$								x					x										x
Zhang et al. (2011)	$Par/1$	x			x	x							x		x									x
Total		6	7	7	10	4	7	14	22	9	6	7	16	21	9	17	5	10	1	16	2	3	2	21

Problem categories and applications

A strong motivation for investigating solution approaches and the modeling of re-entrant scheduling problems comes from the semiconductor industry. Of 33 articles related to real semiconductor factories or simulation tools, 4 articles deal with the production and repair of electric components and machines as well as electronic devices, and 2 publications provide a background to the automotive industry. Specially discussed in this content are hub re-entrants, which occur in the cluster tools of wafer fabrication facilities. A robot serves several processing stations at once. It picks wafers, puts them into a processing station and passes them to another processing station after the operation is completed. The wafers, considered as manufacturing jobs, return to the robot before they can be processed at another station/machine. The objectives of the considered problems are the makespan, flow time, earliness/tardiness and work in process related in equal proportions. Maximizing the utilization of a bottleneck machine is used to increase the throughput rate. Another field of research in re-entrant scheduling problems is the scheduling of rework, with 5 publications in the observed period. Rework cases are mainly examined in flow shop environments. The re-entry for repair operations of a job needs to be scheduled. The objective functions considered are the makespan, total flow time, total idle time, and total earliness / tardiness. 19 articles consider flow shop problems, 11 publications deal with job shops, and 23 articles are related to re-entrant line problems. In 32 articles, parallel machines are considered. The ability to split jobs into sublots and sequence the sublots, i.e. lot streaming, is not discussed in the reviewed articles. Minimization of the makespan is the most frequently pursued objective, with 22 appearances, followed by due-date related objectives with 21 appearances. Re-entrant line scheduling problems are often investigated in connection with limited buffer capacity between workstations (14 times). Setup times are required for the operations to finish the production lots in 10 publications. The work on re-entrant flow shops considers either scheduling single operations or sequencing complete jobs. The sequencing of single job levels has received little attention, although it represents a detailed permutation in re-entrant permutation flow shops. Only 4 journal articles (Chamnanlor et al. (2014), Cho et al. (2011), Kaihara / Kurose / Fujii (2012) and Eskandari / Hosseinzadeh (2014)) consider skipping of machines in re-entrant flow shops. The suggested models and algorithms schedule single operations but do not necessarily deliver a permutation schedule. Specifically, the use of binary sequence variables for every single operation instead of complete jobs or job levels in MIP results in high computational effort.

Solution methods

The nine major methods applied to the re-entrant scheduling problem in the reviewed articles are listed in Table 3.7 under "Solution Method". Other methods with just one appearance are summarized in the column "other". The most common exact method in the context of optimizing one or more objective values is mathematical programming. 17 MIP models have been developed for problems with different characteristics, operating to sequence either complete jobs or single operations. For re-entrant scheduling problems, dispatching rules are the most common heuristic approach to create schedules. Shortest processing time, longest processing, earliest due date, individual designed dispatching rules, etc. are applied in 16 articles. In 8 of the 16 articles, the rules are used to create initial solutions for improvement methods. Most of the effort is put into developing evolutionary algorithms (in 10 articles) and swarm intelligence algorithms (3 articles). A promising approach is the variable neighborhood search, because it is able to integrate other solution methods. Driessel / Mönch (2012a) and Eskandari / Hosseinzadeh (2014) used the approach either to solve the subproblem for a total tardiness minimization or for makespan minimization.[101] Tabu search, as another common solution method, is not applied in any of the 53 articles between 2010 and June 2015. The author of this thesis proposed an iterated local search for the considered problem in Hinze / Sackmann (2016).[102]

The literature review shows that, regarding problem characteristics, a permutation of single job levels has not been examined deeply. The makespan is an eligible criterion to compare solution approaches. Due to the structure of re-entrant permutation flow shops and the possibility to schedule all levels of a job or only a single job level, the variable neighborhood search seems to be a promising approach. Furthermore, it has not received much attention in re-entrant scheduling problems until now.

3.3 A Mathematical Formulation for Re-entrant Permutation Flow Shops

Three MIP formulations for the re-entrant permutation flow shop have been suggested by Pan / Chen (2003). These formulations are based on the classic flow shop and job shop MIP models of Wagner (1959), Manne (1960) and Wilson (1989).[103] These

[101] See Driessel / Mönch (2012a): *Integrated scheduling and material-handling*, pp. 5968–5977 and Eskandari / Hosseinzadeh (2014): *Variable neighbourhood search for flow-shops*, pp. 5–7.

[102] See Hinze / Sackmann (2016): *An Iterated Local Search for a Re-entrant Flow Shop*, pp. 221–226.

[103] See Wagner (1959): *An integer programming model for machine scheduling*, pp. 137–138, Manne (1960): *On the job-shop scheduling problem*, pp. 220–221 and Wilson (1989): *Alternative formulations of a flow-shop scheduling problem*, pp. 396–397.

re-entrant scheduling models are used many times in research.[104] The proposed models determine a permutation of jobs for the flow shop[105], but permutations in re-entrant permutation flow shop problems actually consist of job levels. These levels represent the different (re)-entries of the same job into the production environment. PAN / CHEN (2004) proposed a model for sequencing single operations in a re-entrant flow shop, which does not guarantee the same permutation of job levels on every machine.[106] In CHEN / CHAO-HSIEN PAN (2006), several integer programming models are suggested for re-entrant non-permutation flow shops.[107] An example of a permutation of three jobs in a regular flow shop is shown in Figure 3.2. The symbol j_i indicates the sequence position of job i. The total number of different permutations in a regular permutation flow shop is $n!$.

Figure 3.2: Example of a permutation of three jobs in a regular flow shop

j_1	j_2	j_3

Contrarily, a permutation of three jobs in three levels in an RPFS is shown in Figure 3.3. The number of sequence positions increased from n to $n \cdot L$. The sequence position of job i's level l is represented by j_{il}.

Figure 3.3: Example of a permutation of three jobs in a re-entrant flow shop

j_{11}	j_{21}	j_{31}	j_{12}	j_{22}	j_{32}	j_{13}	j_{23}	j_{33}

To give a short introduction to modeling RPFS, the approach of PAN / CHEN (2003) based on the model of MANNE (1960) is described in detail below. The variable used to determine a sequence of jobs and their loops through the production system is $y_{ii'} \in \{0; 1\}$. It equals 1, if job i precedes job i' in every loop $l = 1, \ldots, L$, otherwise it equals 0. The indexing of the sequence variables in connection with the restrictions (3.4) leads to a schedule with separated job levels. The permutation begins with all levels $l = 1$ of the n jobs. Then, the levels $l = 2$ follow, and so on. It is not allowed that a job's level $l + 1$ precedes another job's level l. The total number of different permutations in a re-entrant permutation flow shop with separated levels equals the number of permutations

[104] See CHEN / PAN / WU (2008): *Hybrid tabu search for re-entrant flow-shops*, pp. 223–224 and LEE et al. (2011): *A genetic algorithm for bi-objective flow shops with re-entrants*, pp. 1107–1108.
[105] See PAN / CHEN (2003): *Minimizing makespan in re-entrant permutation flow-shops*, pp. 643–647.
[106] See PAN / CHEN (2004): *Schedule-generation procedures for the reentrant shops*, p. 316.
[107] See CHEN / CHAO-HSIEN PAN (2006): *Models for the re-entrant shop scheduling*, pp. 584–588.

in a regular flow shop. The objective is to minimize the makespan:

$$\min C_{max}. \tag{3.1}$$

The jobs $i = 1, \ldots, n$ are operated by the machines $k = 1, \ldots, m$ as in a regular flow shop. Since the jobs are processed multiple times on each machine, it is necessary to introduce a level index $l = 1, \ldots, L$. A level represents a loop of the job within the production system. The inequalities (3.2) describe a job level passing from machine to machine. A level l of a job i starts on machine $k = 1$ and ends on machine $k = m$. The passing from one machine k to a machine $k + 1$ is only possible after the operation on machine k is finished. The end of the operation of job level i, l on machine k is calculated by $s^i_{lk} + p^i_{lk}$, thus the starting time of the next operation of the same level has to be greater than or equal to the end time of the current operation.

$$\begin{gathered} s^i_{lk} + p^i_{lk} \leq s^i_{l,k+1} \\ \forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m-1. \end{gathered} \tag{3.2}$$

The re-entry of a job i, i.e. the change from one level l to the next level $l+1$ is controlled by the constraints (3.3). The last operation of a job on level l is performed on machine m. The end time of this operation is given by $s^i_{lm} + p^i_{lm}$. The re-entry of a job on its next level, $l + 1$, on the first machine must be greater than or equal to the end time of the current level l.

$$\begin{gathered} s^i_{lm} + p^i_{lm} \leq s^i_{l+1,1} \\ \forall i = 1, \ldots, n; l = 1, \ldots, L-1. \end{gathered} \tag{3.3}$$

The relation between different levels l and $l + 1$ of different jobs i, i' is regulated by the inequalities (3.4). Without these restrictions, two succeeding levels of different jobs can be assigned to machine k at the same time. A level $l + 1$ of any job i' is allowed to start on a machine k if all levels l of all jobs $i = 1, \ldots, n$ have already been processed on that machine.

$$\begin{gathered} s^i_{lk} + p^i_{lk} \leq s^{i'}_{l+1,k} \\ \forall i, i' = 1, \ldots, n; l = 1, \ldots, L-1; k = 1, \ldots, m. \end{gathered} \tag{3.4}$$

The relation between different jobs on the same level l is regulated by the restrictions (3.5) and (3.6). Variable $y_{ii'}$ equals 1 if job i precedes job i', and the difference between the starting of job i' on machine k and the starting time of i on the same machine is

greater than or equal to the processing time p^i_{lk}. This is ensured by inequalities (3.5). The constraints (3.6) take effect in the contrary case, since the equations (3.5) have no impact if the levels of job i' precede the levels with the same level index l of job i. If i does not precede i', the difference between s^i_{lk} and $s^{i'}_{lk}$ must be greater than or equal to $p^{i'}_{lk}$ since i is only allowed to start on machine k when i' is finished. The job index i is not allowed to equal i' as this would lead to a contradiction, where i must wait to start on machine k until i is finished.

$$A\left(1 - y_{ii'}\right) + \left(s^{i'}_{lk} - s^i_{lk}\right) \geq p^i_{lk}, \tag{3.5}$$

$$Ay_{ii'} + \left(s^i_{lk} - s^{i'}_{lk}\right) \geq p^{i'}_{lk} \tag{3.6}$$

$$\forall i = 1, \ldots, n; i' = 1, \ldots, n, \left(i' < i\right); l = 1, \ldots, L; k = 1, \ldots, m.$$

The original model of Pan / Chen (2003) considers the makespan as an objective. The last operation of any job needs to be performed on the last machine $k = m$ and during a job's last loop $l = L$ in the production system. Thus, makespan C_{max} needs to be greater than or equal to the end of the last operation of each job $i = 1, \ldots, n$.

$$s^i_{Lm} + p^i_{Lm} \leq C_{max} \tag{3.7}$$

$$\forall i = 1, \ldots, n.$$

An alternative objective function is total flow time. The constraints (3.8) measure the completion times of the jobs $i = 1, \ldots, n$. Completion is described as the end of the last operation of each job. The last operation in a regular re-entrant permutation flow shop is performed on machine $k = m$ and must be part of a job's level L.

$$s^i_{Lm} + p^i_{Lm} \leq C_i \tag{3.8}$$

$$\forall i = 1, \ldots, n.$$

The inequalities (3.9) are the Non-negativity constraints (NNC).

$$s^i_{lk} \geq 0 \tag{3.9}$$

$$\forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m.$$

The binary constraints are given by the restrictions (3.3).

$$y_{ii'} \in \{0; 1\} \, \forall i, i' = 1, \ldots, n, \left(i' < i\right).$$

Table 3.8 gives an overview of the number of constraints of the Pan / Chen (2003)

model. The rows C_{max} and C_i refer to the constraints necessary to measure the makespan and job completion times.

Table 3.8: Numbers of constraints of the Pan/Chen (2003) model

Constraints	Number
(3.2)	$n \cdot L \cdot (m-1)$
(3.3)	$n \cdot (L-1)$
(3.4)	$n^2 \cdot (L-1) \cdot m$
(3.6)	$\frac{1}{2}n \cdot (n-1) \cdot m \cdot L$
(3.7)	$\frac{1}{2}n \cdot (n-1) \cdot m \cdot L$
Total	$n^2 \cdot (2L-1) \cdot m - n$
C_{max}	n
C_i	n
NNC	$n \cdot L \cdot m$
Binary	$n \cdot (n-1)$

4 Re-entrant Permutation Flow Shop Problems with Mixed Levels and Missing Operations

4.1 Introduction

The re-entries of a job are considered levels. A job enters a new level, when it leaves the machine sequence $1, \ldots, m$ and returns to a machine to be processed again. The permutation of a re-entrant permutation flow shop is a sequence of job levels. A sequence of separated levels does not allow scheduling of single levels. In contrast to separated level schedules, mixed levels allow processing of a level $l + 1$ of a job i before a level l of a job i' $(i \neq i')$. Missing operations allow the jobs to skip machines, re-entries into the productions system on machines $k > 1$, and exits on machines $k < m$. Existing approaches do have some weaknesses in time calculations if missing operations occur, as is shown in subsection 4.4.3.

Sections 4.2 and 4.3 explain the mixed levels and missing operations in detail. Then section (4.4) compares two different approaches to model mixed level permutation schedules without using an operation index for the single processing steps of the jobs. The preferred approach to model mixed levels is adjusted to deal with missing operations correctly. Furthermore, the reduction of the makespan by applying mixed levels and dealing appropriately with missing operations is examined in computational experiments.

The models presented for a re-entrant permutation flow shop are based on the re-entrant flow shop formulations of Pan / Chen (2003).[1] They are originally derived from a job shop formulation by Manne (1960)[2] and a flow shop model proposed by Wilson (1989)[3]. The number of possible permutations for problems with separated levels is $n!$, which increases for a problem with mixed levels to $(n \cdot L)! / (L!)^n$.[4]

Sections 4.5, 4.6 and 4.7 examine heuristic solution methods for the re-entrant permu-

[1] See Pan / Chen (2003): *Minimizing makespan in re-entrant permutation flow-shops*, pp. 643–647.
[2] See Manne (1960): *On the job-shop scheduling problem*, pp. 219–221.
[3] See Wilson (1989): *Alternative formulations of a flow-shop scheduling problem*, pp. 395–397.
[4] See Li et al. (2013): *Population-Based Learning Algorithm*, p. 1637.

Table 4.1: Parameters and variables in the re-entrant permutation flow shop models

A	Sufficiently large number
C_{max}	Makespan
C_i	Completion / cycle / flow time of job i
i, i'	Job indices
j, j', j''	Sequence position indices
k, k'	Machine indices
L	Total number of levels per job
m	Total number of machines
n	Total number of jobs
p^i_{lk}	Processing time of job i on machine k in level l
s^i_{lk}	Starting time of job i on machine k in level l
x_{ilj}	Binary variable, takes the value 1 if level l of job i is on position j
$y^{il}_{i'l'}$	Binary variable, takes the value 1 if level l of job i is scheduled before level l' of job i'

tation flow shop with mixed levels and missing operations to obtain promising solutions for larger problems. The problem with separated levels is already NP-hard as a regular permutation flow shop, and the number of possible permutations increases for mixed level schedules.

The preferred re-entrant permutation flow shop model is also able to solve job shop problems. Section 4.8 compares the model with a traditional job shop model regarding the performance in solving job shop problems.

The symbols used for modeling a re-entrant permutation flow shop with mixed levels are summarized in Table 4.1.

4.2 Mixed Levels

An example that shows the effects of mixing levels consists of $n = 3$ jobs, which are processed twice ($L = 2$) on each of the $m = 2$ machines. The processing times are given by p^i_{lk}:

$$p^1_{lk} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}, \qquad p^2_{lk} = \begin{pmatrix} 4 & 4 \\ 3 & 1 \end{pmatrix}, \qquad p^3_{lk} = \begin{pmatrix} 1 & 1 \\ 4 & 3 \end{pmatrix}.$$

The permutation resulting from applying the model of Pan / Chen (2003) to the problem is shown in Table 4.2. First, all jobs start start on level $l = 1$, which is followed by a section of the same jobs in level $l = 2$.

Table 4.2: Optimal permutation of the example with separated levels

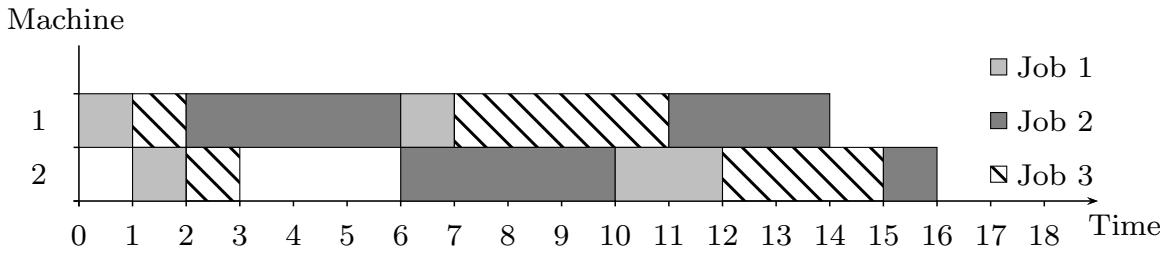
Position	1	2	3	4	5	6
Job	1	3	2	1	3	2
Level	1	1	1	2	2	2

The starting times of every operation are summarized by the values of s^i_{lk}.

$$s^1_{lk} = \begin{pmatrix} 0 & 1 \\ 6 & 10 \end{pmatrix}, \qquad s^2_{lk} = \begin{pmatrix} 2 & 6 \\ 11 & 15 \end{pmatrix}, \qquad s^3_{lk} = \begin{pmatrix} 1 & 2 \\ 7 & 12 \end{pmatrix}$$

The resulting Gantt chart in Figure 4.1 visualizes the permutation and the start and end times of every operation.

Figure 4.1: Solution to the example with separated levels



The makespan C_{max} in this solution of the problem is 16 time units.

Applying an approach that allows mixed levels leads to a different optimal solution from the one presented above. The permutation of the solution is presented in Table 4.3. The job levels on the permutation positions $j = 3$ and $j = 4$ switched their positions. The second level of job $i = 1$ starts after the level $l = 1$ of job $i = 3$ and before the level $l = 1$ of job $i = 2$.

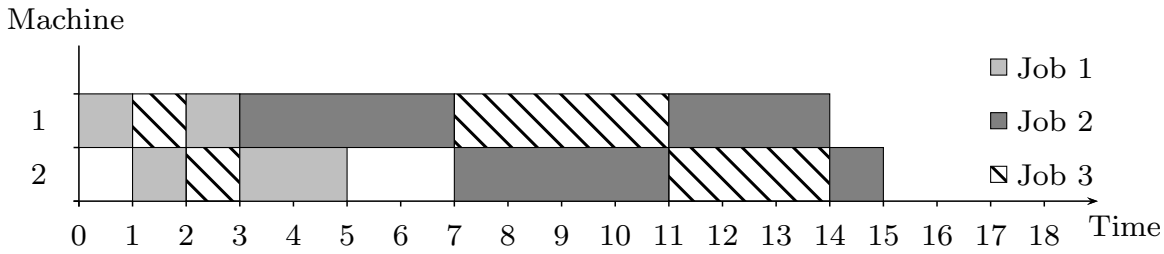
Table 4.3: Optimal permutation of the example with mixed levels

Position	1	2	3	4	5	6
Job	1	3	1	2	3	2
Level	1	1	2	1	2	2

Figure 4.2 shows the solution achieved using a model that allows mixed levels. The starting times of the single operations are given below by the values s^i_{lk}. The makespan of this solution is $C_{max} = 15$. So, the objective value is reduced using a mixed levels

model. The problem itself still is a permutation flow since the sequence of job levels is the same on each machine.

Figure 4.2: Solution to the example with mixed levels



The starting times for the jobs are provided by the values s^i_{lk}:

$$s^1_{lk} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}, \qquad s^2_{lk} = \begin{pmatrix} 3 & 7 \\ 11 & 14 \end{pmatrix}, \qquad s^3_{lk} = \begin{pmatrix} 1 & 2 \\ 7 & 11 \end{pmatrix}.$$

The possible reduction of objective values, while maintaining the structural benefits of a permutation flow shop, is the motivation to examine a mixed level re-entrant permutation flow shop.

Some examples of the numbers of possible sequences are presented in Table 4.4 to give an impression of the problem size.

Table 4.4: Number of possible permutations

n	L	Separated levels	Mixed levels
2	2	2	6
	3	2	20
	4	2	70
3	2	6	90
	3	6	1680
	4	6	34650
4	2	24	2520
	3	24	369600
	4	24	63063000
10	2	3628800	$2.38 \cdot 10^{15}$
	5	3628800	$4.91 \cdot 10^{43}$

Changes in the number of jobs n have a greater effect on the number of possible permutations than changes in the number of levels L. For instance there are 20 possible

permutations for a problem with $n = 2$ jobs and $L = 3$ levels facing 90 permutations for $n = 3$ and $L = 2$.

4.3 Missing Operations

Missing operations can occur in three different ways in re-entrant flow shops:

1. Jobs re-enter the production system on a machine $k > 1$,
2. A level of processing is finished on a machine $k < m$,
3. The job skips one or more machines in a row on a specific level of processing.

The three cases are shown in Figures 4.3, 4.4 and 4.5 for a system with $m = 3$ machines, which is passed by jobs in two levels ($L = 2$):

Figure 4.3: Example of missing operations case 1

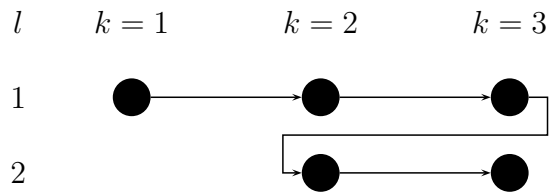


Figure 4.4: Example of missing operations case 2

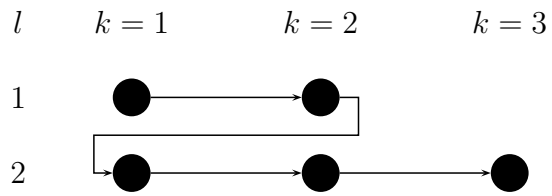
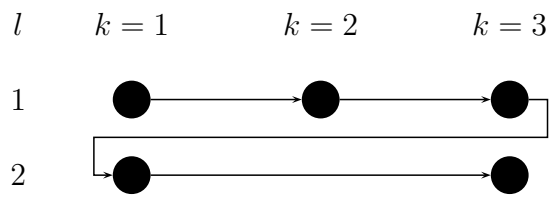


Figure 4.5: Example of missing operations case 3



To explain the impact of dealing appropriately with missing operations the following example is considered. There are two jobs, $i = 1$ and $i = 2$, that need to be scheduled on $m = 3$ machines. Both jobs run $L = 2$ times through the production system. During the second level machine, $k = 1$ is skipped, so the jobs re-enter the system on machine $k = 2$. The processing times p^1_{21} and p^2_{21} are set to 0 to represent the missing operation.

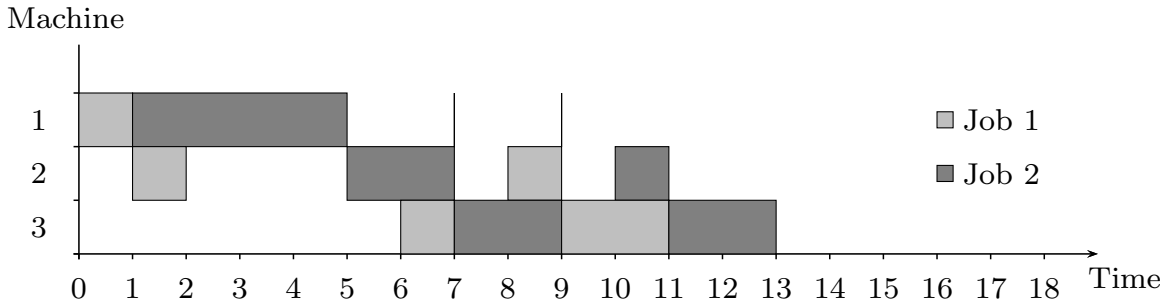
$$p^1_{lk} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}, \qquad p^2_{lk} = \begin{pmatrix} 4 & 2 & 2 \\ 0 & 1 & 2 \end{pmatrix}.$$

The result of a makespan minimization, if missing operations are not considered as a model property, is shown in Figure 4.6. The starting times of the operations are given by the values s^i_{lk}. The minimum makespan in such a model would be $C_{max} = 13$.

$$s^1_{lk} = \begin{pmatrix} 0 & 1 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \qquad s^2_{lk} = \begin{pmatrix} 1 & 5 & 7 \\ 9 & 10 & 11 \end{pmatrix}.$$

It is necessary for the jobs' operations in level $l = 2$ on $k = 2$ to wait to be processed on machine $k = 1$ in the same levels, although the processing time is zero. The missing operations are performed by machine $k = 1$ after seven and nine time units, which delays the schedule compared to the optimal solution.

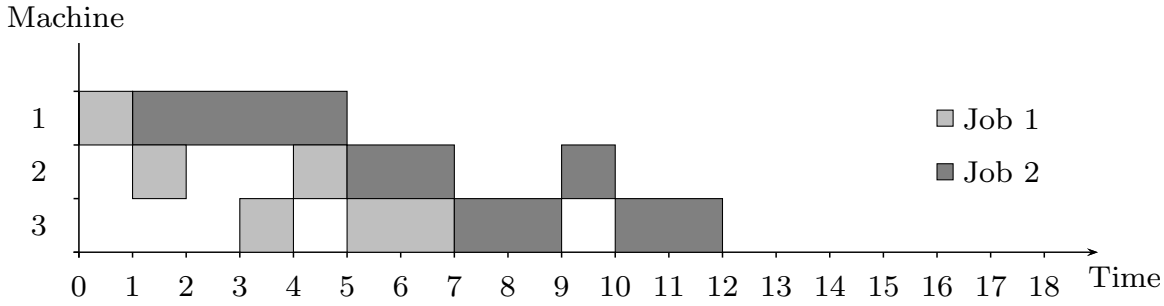
Figure 4.6: Solution to the example if missing operations are not properly managed



The best possible solution results in a makespan of $C_{max} = 12$ if a job does not need to wait for an operation with a processing time of zero.

$$s^1_{lk} = \begin{pmatrix} 0 & 1 & 3 \\ 0 & 4 & 5 \end{pmatrix}, \qquad s^2_{lk} = \begin{pmatrix} 1 & 5 & 7 \\ 1 & 9 & 10 \end{pmatrix}.$$

Figure 4.7: Solution to the example if missing operations are appropriately managed



Models with the possibility of scheduling particular job levels and that do not require a free machine for missing operations are able to solve the different forms of re-entrant flow shop problems mentioned by Emmons / Vairaktarakis (2013) and described in Section 3.1. These problems are cyclic re-entrants, chain re-entrants, hub re-entrants, V re-entrants and (1-2-1) re-entrants. The following examples show how to model these forms with zero processing times.

Cyclic re-entrants:
A job is processed L times on every machine.

$$p_{lk} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Chain re-entrants:
A job needs a finishing operation on machine $k = 1$.

$$p_{lk} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Hub re-entrants:
The jobs return to a central machine before they are passed to the next machine.

$$p_{lk} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

V re-entrants:
The jobs pass from machine $k = 1$ step by step to machine $k = m$ and in the reverse

order to machine $k = 1$.

$$p_{lk} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

(1-2-1) re-entrants:
The jobs are processed twice on machine $k = 1$ and once on machine $k = 2$ in a two-machine system.

$$p_{lk} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

4.4 Mathematical Models

This section introduces two different approaches for modeling a re-entrant permutation flow shop with mixed levels. The models differ in the kind of sequence variable that is used. The first model is called model Y, while the second model is model X. Both formulations are evaluated regarding their computational performance. Additionally, a problem property called basic job sequence is examined regarding its influence on the computation time and solution quality. One configuration of model Y in connection with makespan and total flow time minimization was also published by Hinze et al. (2013).[5]

4.4.1 Comparison of Sequence Variables

Model Y

Model Y is tested regarding makespan minimization. The objective function is given by equation (4.1).

$$\min C_{max}. \tag{4.1}$$

The time restrictions (4.2) and (4.4) are explained by describing the simplified non-missing operation compatible restrictions (4.3) and (4.5).

$$p^i_{lk'} \left(s^i_{lk'} + p^i_{lk'} \right) p^i_{lk} \leq p^i_{lr} s^i_{lk} p^i_{lk} \tag{4.2}$$
$$\forall i = 1, \ldots, n; l = 1, \ldots, L; k = 2, \ldots, m; k' = 1, \ldots, m-1, (k' < k).$$

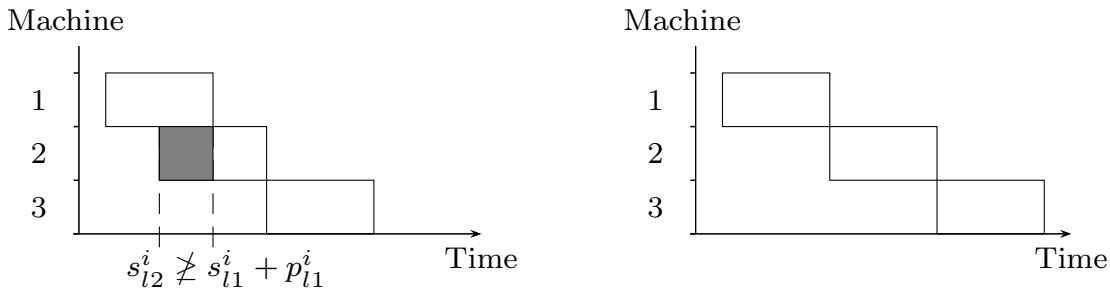
[5] See Hinze et al. (2013): *A contribution to the reentrant flow-shop scheduling problem*, pp. 718–723.

The inequalities (4.2) describe how a job i proceeds from machine 1 to machine m within one level l. The inequalities (4.3) illustrate the restriction set of (4.2).

$$s^i_{lk} + p^i_{lk} \leq s^i_{l,k+1} \tag{4.3}$$
$$\forall i = 1, \ldots, n; l = 1, \ldots, L; k = 2, \ldots, m; k' = 1, \ldots, m-1, (k' < k).$$

Figure 4.8 shows an example of how the constraints (4.2) can be violated by a job i on its lth level in a machine environment with three machines. The left side of the figure is an invalid schedule. The operation on the second machine starts before the operation on the first machine is finished. The right part of the figure shows a possible valid schedule for the considered job level.

Figure 4.8: Starting times for job i in level l: invalid on the left and valid on the right



The job's starting time on a machine k has to be greater than or equal to the end time of all prior operations $1, \ldots, k'$ and $k' < k$. The end time is computed by adding the processing time $p^i_{lk'}$ to the starting time $s^i_{lk'}$ of the operation. The multiplication of the processing times of the job's operation on machine k' and k triggers the constraint. The constraint does not take effect if any of these processing times equals 0. A zero operation time stands for a missing operation. Both sides of the inequality equal zero if one of the two considered operations is missing. There is no need to set any requirements to the operation of the job on a machine k if the operation does not take place. On the other hand, the starting time of the operation on machine k cannot be determined on the end time of the operation on machine k' if job i is not operated on machine k' in level l. The end time of the last operation with $p^i_{lk'} > 0$ before the job is processed on machine k is checked by running the index k' from 1 to $k-1$. The processing times in previous job levels are checked in the inequalities (4.4) if there are no operations with $p^i_{lk'} > 0$ in the

current level before s^i_{lk}.

$$p^i_{lk'}\left(s^i_{lk'} + p^i_{lk'}\right) p^i_{l+1,k} \leq p^i_{lk'} s^i_{l+1,k} p^i_{l+1,k} \tag{4.4}$$
$$\forall i = 1, \ldots, n; l = 1, \ldots, L-1; k, k' = 1, \ldots, m.$$

The constraints (4.4) regulate the level transition from l to $l+1$ for all jobs. The simple form of this constraint is presented in the inequalities (4.5).

$$s^i_{lk'} + p^i_{lk'} \leq s^i_{l+1,k} \tag{4.5}$$
$$\forall i = 1, \ldots, n; l = 1, \ldots, L-1; k, k' = 1, \ldots, m.$$

Job i is not allowed to start on any machine k in level $l+1$ before all of its processing steps on the machines $k = 1, \ldots, m$ in level l are completed. Again, the multiplication of the processing times $p^i_{lk'}$ and $p^i_{l+1,k}$ in the constraints (4.4) triggers the restrictions.

Figures 4.9 and 4.10 illustrate the functionality of the constraints (4.5) on level transition. The operation of level l on machine $k = 3$ and level $l+1$ on machine $k = 1$ overlap in the schedule shown in Figure 4.9, which leads to invalid starting times.

Figure 4.9: Invalid starting times for a job i changing from level l to $l+1$

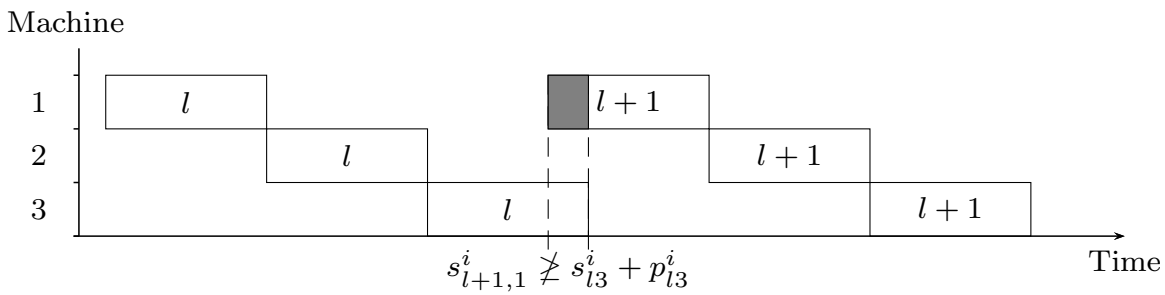
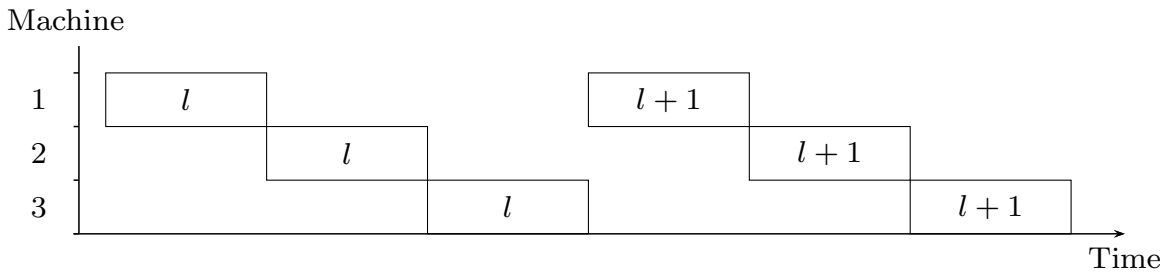


Figure 4.10: Valid starting times for a job i changing from level l to $l+1$



The inequality sets (4.6) and (4.7) ensure that only one job at a time can be processed

on a machine k. Therefore, the variables $y^{il}_{i'l'}$ describe the job sequence. $y^{il}_{i'l'}$ equals 1 if level l of job i precedes level l' of i' on all machines; otherwise, the variable 0. The difference between the starting times of the two jobs i and i' in their levels l and l' on a machine k should be greater than or equal to the processing time of the preceding job. The term $A(1-y^{il}_{i'l'})$ in the constraint set (4.6) equals 0 if level l of job i precedes level l' of i', because of $y^{il}_{i'l'} = 1$. In this case, level l' of job i has to wait at least p^i_{lk} time units after the start of job i's level l on machine k to start on the same machine. $A(1-y^{il}_{i'l'})$ is equal to A if level l of job i succeeds level l' of i'. Then, the left side of the constraint is greater than p^i_{lk}, since A is a large enough number. In the case of $y^{il}_{i'l'} = 0$, level l' of job i' precedes level l of job i. The inequalities (4.7) include the term $Ay^{il}_{i'l'}$, which becomes zero if $y^{il}_{i'l'} = 0$. Then, job i's level l needs to wait at least for job i''s level l' to finish on machine k. The difference between both starting times s^i_{lk} and $s^{i'}_{l'k}$ should be greater than or equal to the processing time, $p^{i'}_{l'k}$, of the job level that is scheduled on an earlier sequence position.

$$A(1 - y^{il}_{i'l'}) + (s^{i'}_{l'k} - s^i_{lk}) \geq p^i_{lk} \tag{4.6}$$

$$Ay^{il}_{i'l'} + (s^i_{lk} - s^{i'}_{l'k}) \geq p^{i'}_{l'k} \tag{4.7}$$

$$\forall i, i' = 1, \ldots, n; (i' < i); l, l' = 1, \ldots, L; k = 1, \ldots, m.$$

Figure 4.11 shows an invalid assignment of operations if level l of job i precedes job i''s level l', i.e. $y^{il}_{i'l'} = 1$. The operation of $i'l'$ on machine $k = 2$ is not allowed to start $(s^{i'}_{l'2})$ before the operation of il on this machine is finished $(s^i_{l2} + p^i_{l2})$. The valid schedule for the same precedence relation of the two job levels is shown in Figure 4.12.

Figure 4.11: Invalid starting times if $y^{il}_{i'l'} = 1$

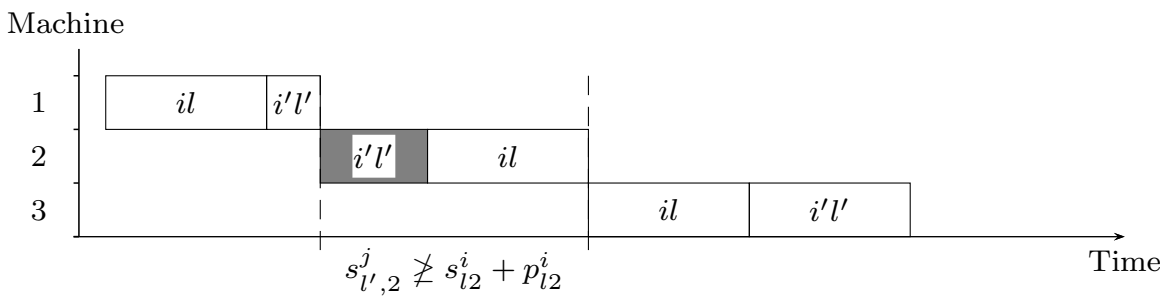
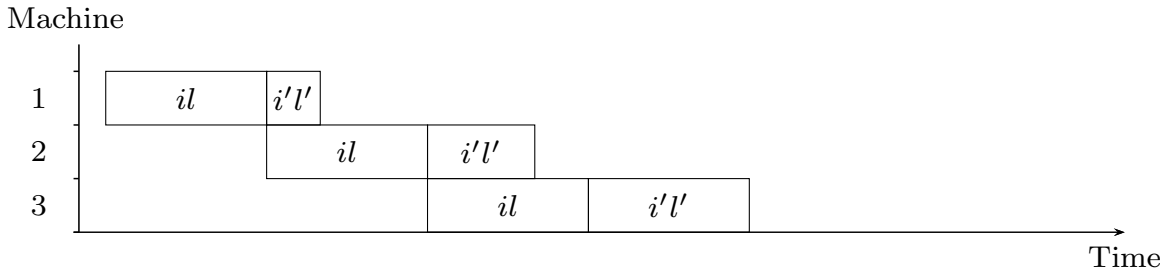
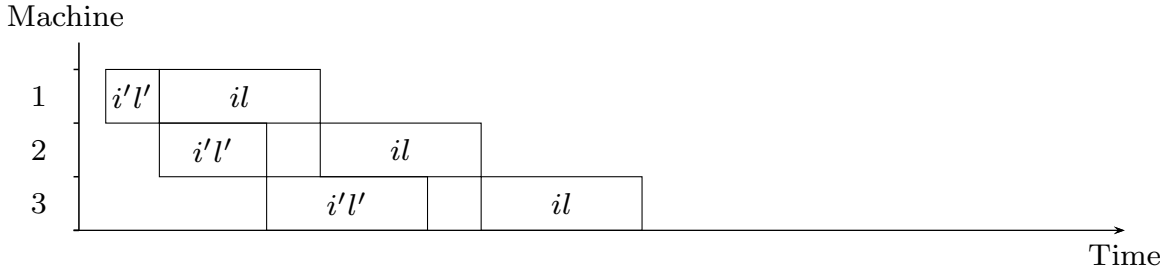


Figure 4.12: Valid starting times if $y_{i'l'}^{il} = 1$



The valid Gantt chart for the case of job i's level l not preceding level l' of job i', i.e. $y_{il}^{i'l'} = 0$, is presented in Figure 4.13. All operations of il on the machines $k = 1, 2, 3$ are necessary to start after the processing of $i'l'$ is finished.

Figure 4.13: Valid starting times if $y_{i'l'}^{il} = 0$.



The constraints (4.8) determine the makespan. The end times of all operations need to be checked due to the fact that some jobs may not have their last processing time greater than zero in the last level or on the last machine.

$$s_{lk}^{i} + p_{lk}^{i} \leq C_{max} \tag{4.8}$$
$$\forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m.$$

The starting times and makespan need to be greater than or equal to 0.

$$C_{max} \geq 0 \tag{4.9}$$
$$s_{lk}^{i} \geq 0 \tag{4.10}$$
$$\forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m.$$

$y^{il}_{i'l'}$ is a binary variable as previously described.

$$y^{il}_{i'l'} \in \{0;1\} \tag{4.11}$$
$$\forall\, i = 1,\ldots,n; i' = 1,\ldots,n-1, (j < i); l, l' = 1,\ldots,L.$$

Table 4.5: Number of constraints of model Y

Constraints	Without missing operations	With missing operations
(4.2)	$n \cdot L \cdot (m-1)$	$\frac{1}{2}n \cdot L \cdot m \cdot (m-1)$
(4.4)	$n \cdot (L-1)$	$n \cdot (L-1)\, m^2$
(4.6)	$\frac{1}{2}n \cdot (n-1) \cdot m \cdot L \cdot (L+1)$	$\frac{1}{2}n \cdot (n-1) \cdot m \cdot L \cdot (L+1)$
(4.7)	$\frac{1}{2}n \cdot (n-1) \cdot m \cdot L \cdot (L+1)$	$\frac{1}{2}n \cdot (n-1) \cdot m \cdot L \cdot (L+1)$
Total	$L\,(L \cdot m\,(n-1) \cdot n + m \cdot n^2) - n$	$m \cdot n \cdot \Big(m \cdot \Big(\frac{3}{2}L - 1\Big)$ $+L \cdot \Big(n - \frac{3}{2} + L \cdot (n-1)\Big)\Big)$
C_{max}	n	$n \cdot L \cdot m$
C_i	n	$n \cdot L \cdot m$
NNC	$n \cdot L \cdot m$	$n \cdot L \cdot m$
Binary	$n \cdot L \cdot (n \cdot L - 1)$	$n \cdot L \cdot (n \cdot L - 1)$
Basic Sequence	$\frac{1}{2}n \cdot (n-1) \cdot L$	$\frac{1}{2}n \cdot (n-1) \cdot L$

Model X

This model contains variables different from the ones used in model Y. The sequence assignment is done by the variables x_{ilj}. The variable equals 1 if job i on level l is assigned to position j and is 0 otherwise. The starting times for the jth job level on the kth machine are represented by the variable h_{kj} $\forall k = 1,\ldots,m; j = 1,\ldots,n \cdot L$. The model X is capable of generating an optimal sequence with mixed levels, but not for problems with missing operations. Nevertheless, its computational performance is compared to the model Y's concerning instances without missing operations in order to find the superior sequence variable for extended formulations.

The objective is to minimize the makespan ((4.12)).

$$\min C_{max}. \tag{4.12}$$

Each sequence position j has to be assigned to exactly one job level i, l, so the sum of

binary sequence variables over all jobs and levels must be 1 in the equations (4.13).

$$\sum_{i=1}^{n}\sum_{l=1}^{L} x_{ilj} = 1 \tag{4.13}$$
$$\forall j = 1, \ldots, n \cdot L;$$

Further, each job level must be assigned to exactly one sequence position, so the sum of sequence variables over all positions $j = 1, \ldots, n \cdot L$ needs to be 1 for every job level in the constraints (4.14).

$$\sum_{j=1}^{n \cdot L} x_{ilj} = 1 \tag{4.14}$$
$$\forall i = 1, \ldots, n; l = 1, \ldots, L;$$

A level $l + 1$ of a job i needs be scheduled after the lth level of the same job. This is ensured with the inequalities (4.15). The left side of the equation is 1 if level $l + 1$ is scheduled on one of the positions $j = 1, \ldots, j'$, and level l on a later position $j = j' + 1, \ldots, n \cdot L$. This situation is not allowed.

$$\sum_{j=1}^{j'} x_{i,l+1,j} - \sum_{j=1}^{j'} x_{ilj} \leq 0 \tag{4.15}$$
$$\forall i = 1, \ldots, n; l = 1, \ldots, L - 1; j' = 1, \ldots, n \cdot L;$$

The constraints (4.16) are equivalent to the restrictions (4.24) in model Y. They maintain a basic job sequence but permit a mixed level sequence.

$$\sum_{j=1}^{j''} x_{i',l+1,j} - \sum_{j=j'+1}^{n \cdot L} x_{ilj} - \left(\sum_{j=1}^{j'} x_{i'lj} + \sum_{j=1}^{j''} x_{i,l+1,j} \right) \leq 0; \tag{4.16}$$
$$\forall i, i' = 1, \ldots, n (i < i'); j', j'' = 1, \ldots, n \cdot L, (j' < j''); l = 1, \ldots, L - 1.$$

The inequalities (4.17) forbid a machine k to process two jobs at the same time and ensure the correct order of job levels processed on the machine. A job level on position $j + 1$, is allowed to start at machine k when the operation of the job that is on the previous sequence position j, is finished on machine k.

$$h_{k,j+1} \geq h_{kj} + \sum_{i=1}^{n}\sum_{l=1}^{L} x_{ilj} \cdot p_{lk}^{i} \tag{4.17}$$
$$\forall i = 1, \ldots, n; j = 1, \ldots, n \cdot L - 1; l = 1, \ldots, L; k = 1, \ldots, m.$$

A job level il on position j needs to be processed completely on machine k before it is allowed to move to machine $k+1$ ((4.18)).

$$h_{k+1,j} \geq h_{kj} + \sum_{i=1}^{n} \sum_{l=1}^{L} x_{ilj} \cdot p_{lk}^{i} \tag{4.18}$$
$$\forall i = 1, \ldots, n; j = 1, \ldots, n \cdot L; k = 1, \ldots, m-1.$$

The level transition is regulated by the constraints (4.19). Level $l+1$ of job i is on position j' if $\sum_{j=1}^{j'} x_{i,l+1,j} - \sum_{j=j'}^{n \cdot L} x_{i,l+1,j} = 2$, and level $l+1$ of the same job i is on position j'' if $\sum_{j=1}^{j''} x_{i,l+1,j} - \sum_{j=j''}^{n \cdot L} x_{i,l+1,j} = 2$. The constraints (4.19) take effect if these requirements are met. Then the processing of the j''th job level on machine m must be finished before the j'th job level is allowed to start on machine $k = 1$.

$$A \cdot \left(2 - \sum_{j=1}^{j'} x_{i,l+1,j} - \sum_{j=j'}^{n \cdot L} x_{i,l+1,j}\right) + A \cdot \left(2 - \sum_{j=1}^{j''} x_{ilj} - \sum_{j=j''}^{n \cdot L} x_{ilj}\right) + (h_{1j'} - h_{mj''}) \geq p_{lk}^{m} \tag{4.19}$$
$$\forall i = 1, \ldots, n; j', j'' = 1, \ldots, n \cdot L; l = 1, \ldots, L-1.$$

The makespan is calculated by the inequalities:

$$h_{m,n \cdot L} + \sum_{i=1}^{n} x_{iL,n \cdot L} p_{Lm}^{i} \leq C_{max} \tag{4.20}$$
$$\forall i = 1, \ldots, n.$$

Table 4.6 gives an overview of the number of constraints of model X.

Table 4.6: Number of constraints of model X

Constraints	Number
(4.13)	$n \cdot L$
(4.14)	$n \cdot L$
(4.15)	$n^2 \cdot L \cdot (L-1)$
(4.17)	$n \cdot (n \cdot L - 1) \cdot L \cdot m$
(4.18)	$n^2 \cdot L \cdot (m-1)$
(4.19)	$n^3 \cdot L^2 (L-1)$
Total	$n^2 \cdot L^2 \cdot (n \cdot L + m - n + 1) + n \cdot L \cdot (n \cdot m - 2n - m + 2)$
C_{max}	n
C_i	$n^2 \cdot L$
NNC	$n \cdot L \cdot m$
Binary	$n^2 \cdot L^2$
Basic Sequence ((4.16))	$\frac{1}{4} n^2 \cdot (n-1) \cdot (n \cdot L - 1) \cdot L \cdot (L-1)$

Alternate Objective Function

The total flow time is the sum of the completion times of every job $i = 1, \ldots, n$. The objective function is given by equation (4.21).

$$\min \sum_{i=1}^{n} C_i \tag{4.21}$$

The single completion times of the jobs are measured by the constraints (4.22). The end time $s_{lk}^i + p_{lk}^i$ of every operation that is performed on the machines $k = 1, \ldots, m$ during the levels $l = 1, \ldots, L$ of a job i, needs to be lower than or equal to the completion time C_i of the job.

$$s_{lk}^i + p_{lk}^i \leq C_i \tag{4.22}$$
$$\forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m.$$

Alternatively, the completion time can be measured by the constraints (4.23).

$$h_{m,j} + \sum_{i=1}^{n} x_{iLj} p_{Lm}^i \leq C_i \tag{4.23}$$
$$\forall i = 1, \ldots, n; j = 1, \ldots, n \cdot L.$$

Comparing models with two different sequence variables

The processing times of the test instances are uniformly distributed random numbers 1 and 99. The number are generated by the suggested method of TAILLARD (1993).[6] This method of processing time generation is used in all the tests in Chapter 4. Missing operations are not considered for comparing model Y and model X. The test will indicate, what formulation is more suitable for RPFS problems. Ten test instances are solved for each of the 64 problem sizes considered. The computation time is limited to 1 hour. The number of jobs n varies between two and five jobs, as does the number of levels L. The values for the numbers of machines m are two, five, six and ten.[7]

Explanations of the symbols used in the evaluation tables are listed in Table 4.7. The average relative gap values and makespan deviations ΔC_{max} are the measures for solution quality. The average relative deviation of the performed iterations, ΔIt, and the average relative deviation of the computation time, Δct, are the measurements for computing performance.

Table 4.7: List of symbols in the model comparison

ΔC_{max}	$\Delta C_{max} = \left(C^{\mathrm{X}}_{max}/C^{\mathrm{Y}}_{max}\right) - 1$
ct	Computation time
Δct	$\Delta ct = \left(ct^{\mathrm{X}}/ct^{\mathrm{Y}}\right) - 1$
Gap	Gap between lower bound and best found solution either after solving the problem or a computation time of 1 hour
It	Number of iterations until a problem instance is solved or the computation time limit is reached
ΔIt	$\Delta It = \left(It^{\mathrm{X}}/It^{\mathrm{Y}}\right) - 1$
L	Number of levels per job
m	Number of machines
n	Number of jobs
X	Model with x_{ilj} as sequence variable
Y	Model with $y^{i'l'}_{il}$ as sequence variable

The positive values in Figure 4.14 indicate better average results of model Y after a computation time of 1 hour. The instances that are solved to optimality result in the same makespan values for both models. The deviations increase with an increasing problem size.

[6] See TAILLARD (1993): *Benchmarks for basic scheduling problems*, pp. 279–280.
[7] See Table B.2 in Appendix B (p. 204) for detailed information on the problem sizes.

Figure 4.14: Average makespan deviations between model X and Y

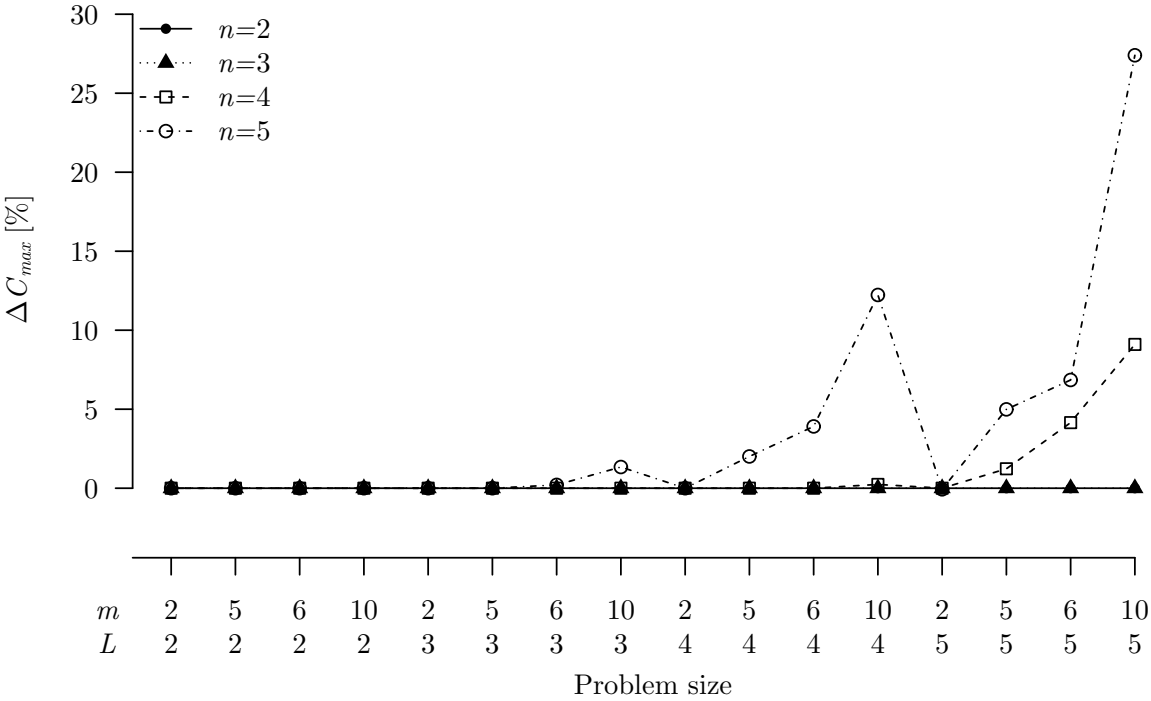


Table 4.8 compares the behavior of both models depending on the number of machines. Model Y is not able to close the gap between the lower bound and best-found solution for $m = 2$ and $m = 5$ machines. The gap to the optimal solution of these instances is small enough that the deviation of makespan, ΔC_{max}, between the models Y and X is near zero, despite the fact that model X finds the optimal solution for each of $m = 2$ and $m = 5$ instances. The positive deviation of $\Delta ct = 66.3$ % occurs due to the single instances with relatively high computation times of model X, e.g. an $n = 3,\ L = 5,\ m = 2$ instance with a computation time of 1 second for model Y and 7 seconds for model X, leading to a relative deviation of 600 % for a single instance.

Model Y is superior for instances with a higher number of machines ($m = 6$ and $m = 10$). It uses less computation time and iterations. The solution process for solving the $m = 10$ problems with model X requires 9679 more iterations than solving it with model Y. Most of the high machine number instances are not solved by the X model within 1 hour, leading to a gap of 3 and 7 %. Also, the deviation of makespan increases with the increasing number of machines.

Table 4.8: Influence of the number of machines on the models X and Y

m	Gap Y [%]	Gap X [%]	ΔC_{max} [%]	ΔIt [%]	Δct [%]
2	1.68	0.00	0.00	309.68	66.30
5	0.07	1.67	0.47	1,251,671.01	13,832.35
6	0.00	3.07	0.94	242,996.54	15,928.13
10	0.00	7.00	3.14	967,914.71	33,488.72

The influence of the number of levels L on the performance of both models X and Y is provided in Table 4.9. The highest gaps for both models occur for instances with three levels. Small problems with just two levels are solved optimally by both models. The gaps of model X are higher than those of model Y for three, four and five levels. Hence, the deviation of makespan is positive, meaning the makespan obtained by model X is, on average, higher than the best makespan values of model Y. The computational effort of model X increases with increasing values of L.
The required computation time of model X is an average of 121 times higher than if model Y is used for instances with five levels. Additionally 8310 times more iterations are performed for model X.

Table 4.9: Influence of the number of levels per job on the models X and Y

L	Gap Y [%]	Gap X [%]	ΔC_{max} [%]	ΔIt [%]	Δct [%]
2	0.00	0.00	0.00	6411.36	170.42
3	1.17	5.70	2.45	306,189.14	9048.95
4	0.38	3.23	1.15	535,994.05	21,130.43
5	0.78	4.79	1.92	831,014.25	12,057.65

The largest mean deviations of makespan are measured for problems that require five jobs to be scheduled, as shown in Table 4.10. Neither model X nor model Y are able to prove the solution they found after a computation time of 1 hour to be optimal, as indicated by the gap values of both models. Despite that fact, the X model provides, on average, a 0.92 % worse solution for $n = 4$ instances and 3.66 % worse makespan for instances with five jobs on average. The computational effort in computing the solutions is higher in all four problem classes for model X, as shown in columns $\Delta Iter$ and Δct.

Table 4.10: Influence of the number of jobs on the models X and Y

n	Gap Y [%]	Gap X [%]	ΔC_{max} [%]	ΔIt [%]	Δct [%]
2	0.00	0.00	0.00	384.49	21.56
3	0.00	0.00	0.00	1,132,804.34	1940.63
4	0.00	3.42	0.92	1,012,697.84	37,792.37
5	1.74	8.69	3.66	252,030.28	14,464.58

Figure 4.15 shows the mean absolute computation time for all problem classes with $n = 2$ jobs. The average computation times are between 1 and 2 seconds for all $n = 2$ problem sizes. Differences of more than 0.5 seconds occur only for problems with $L = 5$ levels. Model Y requires less time to solve these problem instances. However, both models can find optimal solutions for the instances with two jobs in a short time.

Figure 4.15: Average computation times of models X and Y for $n = 2$

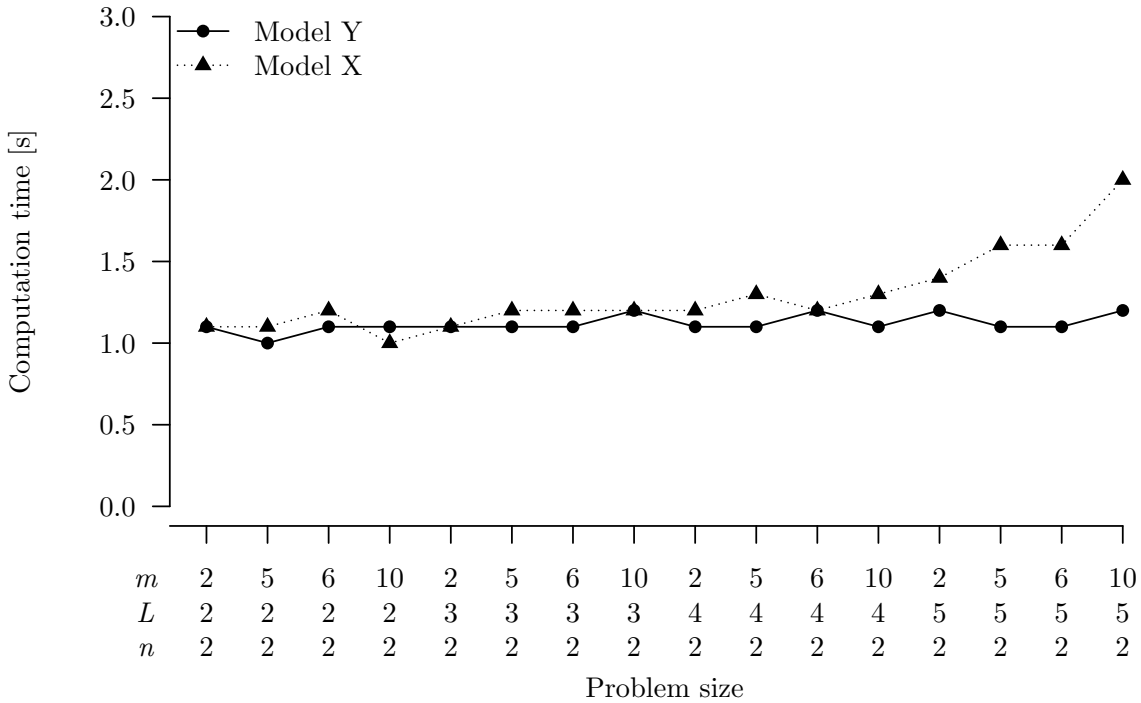
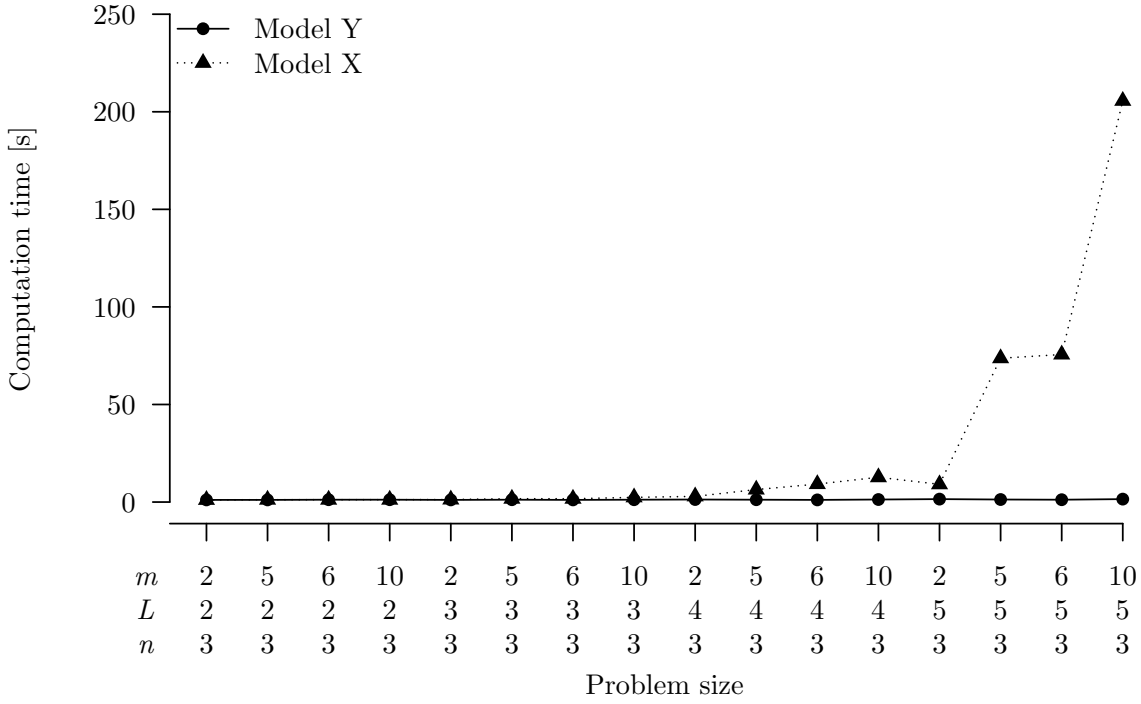


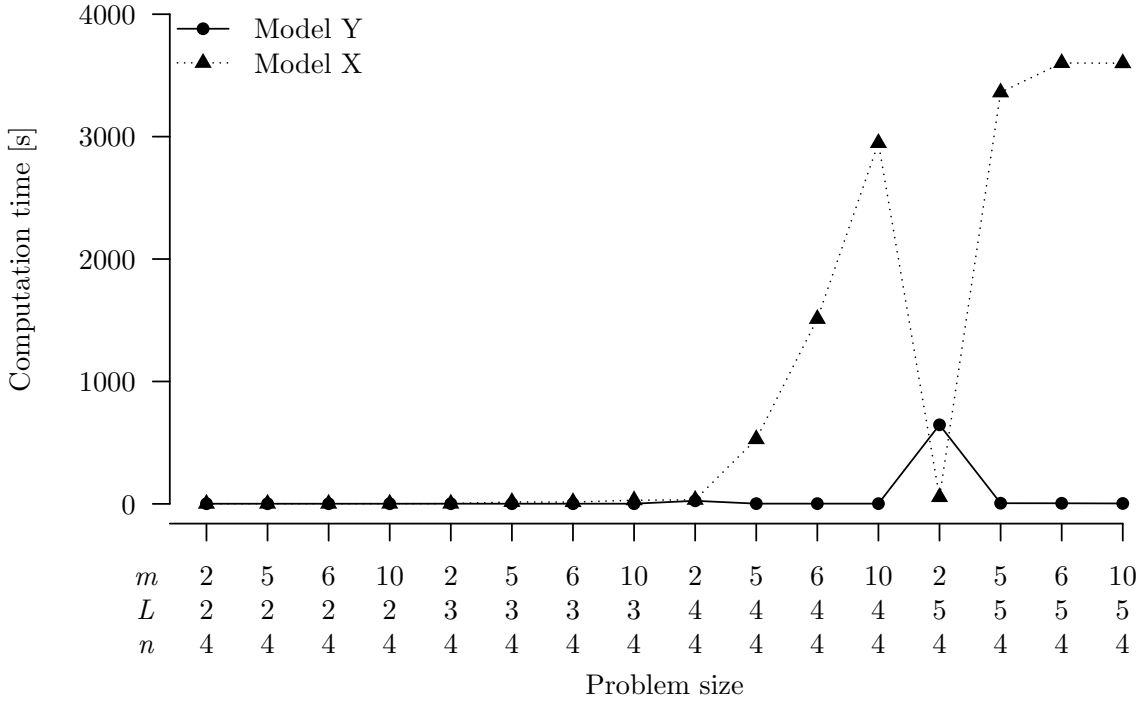
Figure 4.16 shows the mean absolute computation time for all problem classes with $n = 2$ jobs. The mean computation times are between 1 and around 200 seconds for all $n = 3$ problem classes. Only problem sizes with five levels per job exceed mean computation times of 1 minute for model X. The mean computation times of model Y for these problem classes remain about 1 second.

Figure 4.16: Average computation times of models X and Y for $n = 3$



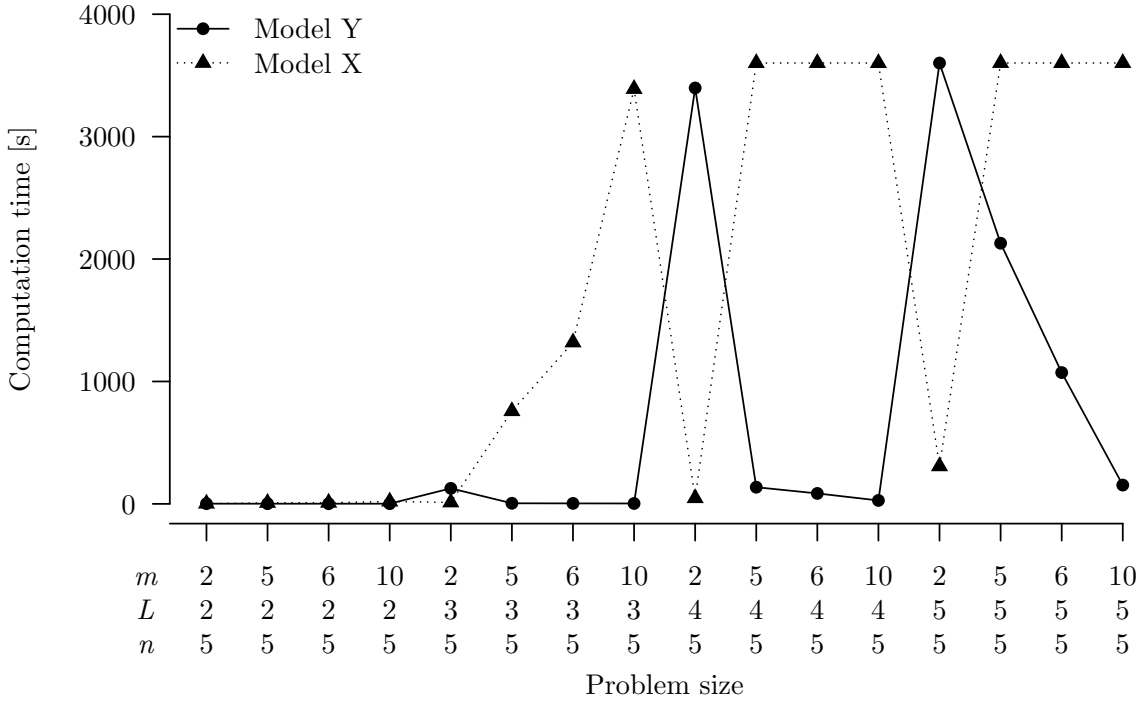
A visualization of the mean computing times for solving problems with four jobs is provided in Figure 4.17. While the computation times are relatively high for the X model, the Y model solves the instances in a relatively short time period.

Figure 4.17: Average computation times of models X and Y for $n = 4$



The effect of having a high computation time for a low number of machines, if model Y is used, can also be seen in Figure 4.18. The computation time decreases for higher machine numbers. In contrast to this behavior of model Y, model X solves problems with $m = 2$ machines in a short time frame.

Figure 4.18: Average computation times of models X and Y for $n = 5$



The number of jobs has the greatest influence on the computation time, before the number of levels, as seen by comparing Figures 4.15, 4.16, 4.17 and 4.18.

The computational superiority of the Y model for higher numbers of jobs, levels and machines compared to the X model leads to the decision to use the Y model in the further considerations.

Influence of Mixed Levels

In this section the Pan / Chen (2003) model, which features separated job levels instead of mixed levels, is compared to the mixed level solutions obtained with the Y model. The test instances are the same as in section 4.4.1.[8] The evaluation examines the models' behavior regarding machine, level and job numbers.

The symbols used for evaluation of the effect of mixed levels are shown in Table 4.11.

[8] See Table B.2 in Appendix B (p. 204) for detailed information on the problem sizes.

Table 4.11: List of symbols for the evaluation of mixed and separated levels

ΔC_{max}	$\Delta C_{max} = \left(C_{max}^{\mathrm{PC}}/C_{max}^{Y}\right) - 1$
ct	Computation time
Δct	$\Delta ct = \left(ct^{\mathrm{PC}}/ct^{Y}\right) - 1$
It	Number of iterations until a problem instance is solved or the computation time limit is reached
ΔIt	$\Delta It = \left(It^{\mathrm{PC}}/It^{Y}\right) - 1$
Gap	Gap between lower bound and best found solution either after solving the problem or a computation time of 1 hour
L	Number of levels per job
m	Number of machines
n	Number of jobs
PC	Model of Pan / Chen (2003)
Y	Model Y

The biggest differences in the mean computation time and mean number of iterations occur for two machine instances as shown in Table 4.12. For these instances, the number of iterations is on average 75 % lower for the model with separated levels than for the mixed level model and the mean computation time is 35 % lower. The positive value for the mean relative deviation of the number of iterations used for the five machine instances is due to an instance with three jobs and five levels, which does not require an iteration by the mixed level model, but is 137 by the model of Pan / Chen (2003). The makespan values of the mixed level model are exceeded by 1.97 % by the model with separated levels.

Table 4.12: Influence of the number of machines on the models Y and PC

m	Gap Y [%]	Gap PC [%]	ΔC_{max} [%]	ΔIt [%]	Δct [%]
2	1.68	0.00	1.97	−75.44	−35.05
5	0.07	0.00	0.99	167.44	−23.53
6	0.00	0.00	0.98	−41.47	−22.03
10	0.00	0.00	0.61	−26.87	−25.70

Table 4.13 shows the performance values depending on the number of levels. Model Y achieves, on average, lower makespan values for each number of levels, $L = 2, \ldots, 5$.

Table 4.13: Influence of the number of levels per job on the models Y and PC

L	Gap Y [%]	Gap PC [%]	ΔC_{max} [%]	ΔIt [%]	Δct [%]
2	0.00	0.00	0.65	−29.63	−2.60
3	1.17	0.00	1.28	21.99	−46.48
4	0.38	0.00	1.37	−53.81	−36.60
5	0.78	0.00	1.31	6.49	−27.40

The mean reduction of the makespan of the Y model increases with the increasing number of jobs, as shown in Table 4.14.

Table 4.14: Influence of the number of jobs on the models Y and PC

n	Gap Y [%]	Gap PC [%]	ΔC_{max} [%]	ΔIt [%]	Δct [%]
2	0.00	0.00	0.77	−24.24	0.00
3	0.00	0.00	0.99	63.02	−3.75
4	0.00	0.00	1.39	71.54	−31.88
5	1.74	0.00	1.39	−86.54	−70.53

The model with mixed levels clearly outperforms the model with separated levels regarding solution quality, but leads to longer computation times.

4.4.2 Basic Job Sequence

The equations (4.24) declare that the sequence of jobs should be the same in all levels. If job i precedes a job i' on a level l', then all of its levels $l = 1, \ldots, L$ precede the corresponding level l of job i'. This keeps a basic job sequence.

$$y_{il}^{i'l} = y_{iL}^{i'L} \qquad (4.24)$$
$$\forall i, i' = 1, \ldots, n, (i' < i); l = 1, \ldots, L.$$

The test problems are identical to the experiments in Section 4.4.1.[9]

The symbols used for comparison are described in Table 4.15.

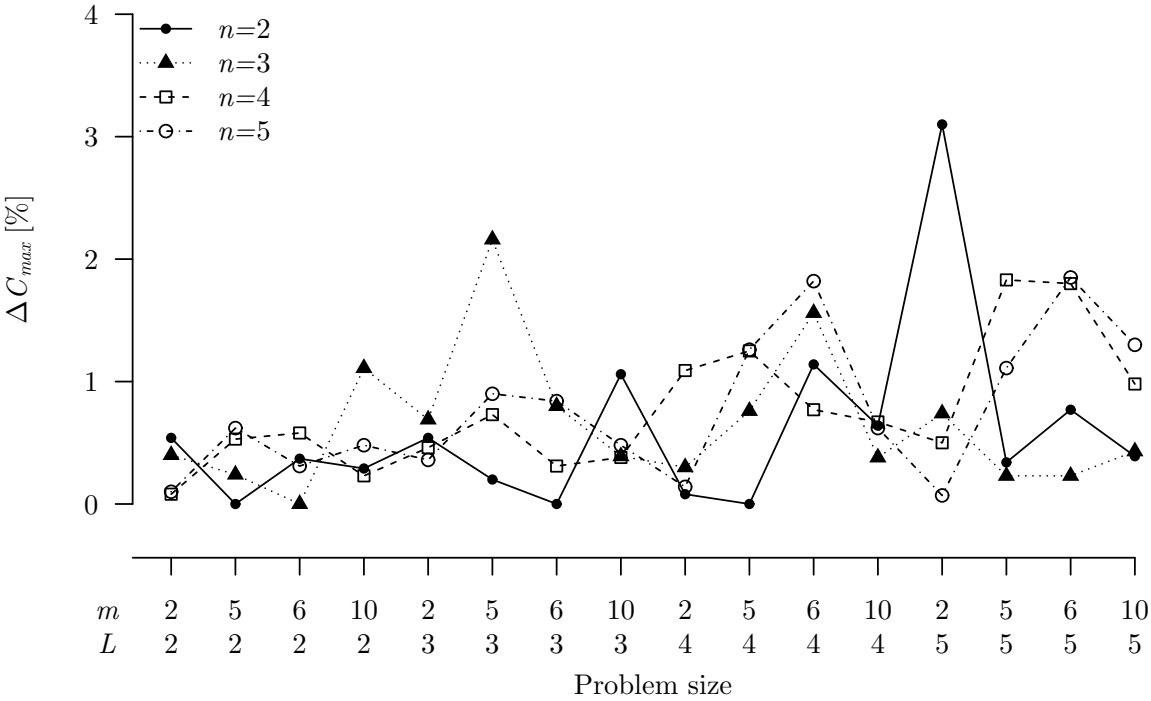
[9] See Table B.2 in Appendix B (p. 204) for detailed information on the problem sizes.

Table 4.15: List of symbols in evaluation tables concerning basic sequence

BS	Model Y with basic sequence
ΔC_{max}	$\Delta C_{max} = \left(C_{max}^{\mathrm{BS}}/C_{max}^{\mathrm{Y}}\right) - 1$
ct	Computation time
Δct	$\Delta ct = \left(ct^{\mathrm{BS}}/ct^{\mathrm{Y}}\right) - 1$
Gap	Gap between lower bound and best found solution either after solving the problem or a computation time of 1 hour
It	Number of iterations until a problem instance is solved or the computation time limit is reached
ΔIt	$\Delta It = \left(It^{\mathrm{BS}}/It^{\mathrm{Y}}\right) - 1$
L	Number of levels per job
m	Number of machines
n	Number of jobs
Y	Model Y

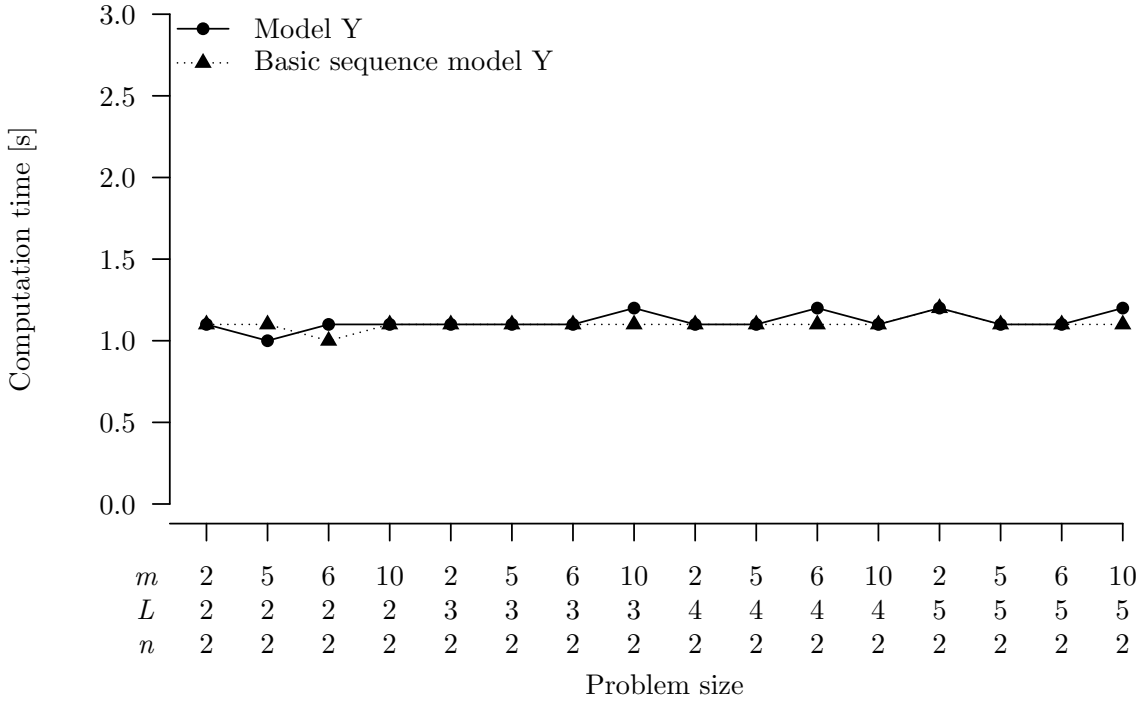
Figure 4.19 shows the average deviation of makespan values between the model without prescribing a basic sequence and the model with a basic sequence. The deviations are based on makespan values obtained using the model without a basic sequence. No average deviations of the makespan are observed for the problem sizes $n = 2, L = 2, m = 5$, $n = 2, L = 3, m = 6$, $n = 2, L = 4, m = 5$ as well as for $n = 3, L = 2, m = 6$. The mean makespan deviations increase if the number of levels increases.

Figure 4.19: Average makespan deviations between model Y and model BS



The impact of a basic level sequence on the computation time required to solve the problems optimally is shown in Figure 4.20 for $n = 2$ job problems and in Figure 4.21 for $n = 5$ job problems. The mean computation times for $n = 2$ instances are around 1 second for every problem class and both models.

Figure 4.20: Average computation times of models Y and BS for $n = 2$



The average computation times for problems with five jobs in Figure 4.21 show some differences between the models if the number of levels per job $L \geq 4$. Then, the model with a basic level sequence needs less time compared to the model without a basic sequence constraint. For $n = 2, L = 5, m = 2$ both models require a high mean computation time close to 1 hour.

Figure 4.21: Average computation times of models Y and BS for $n = 5$

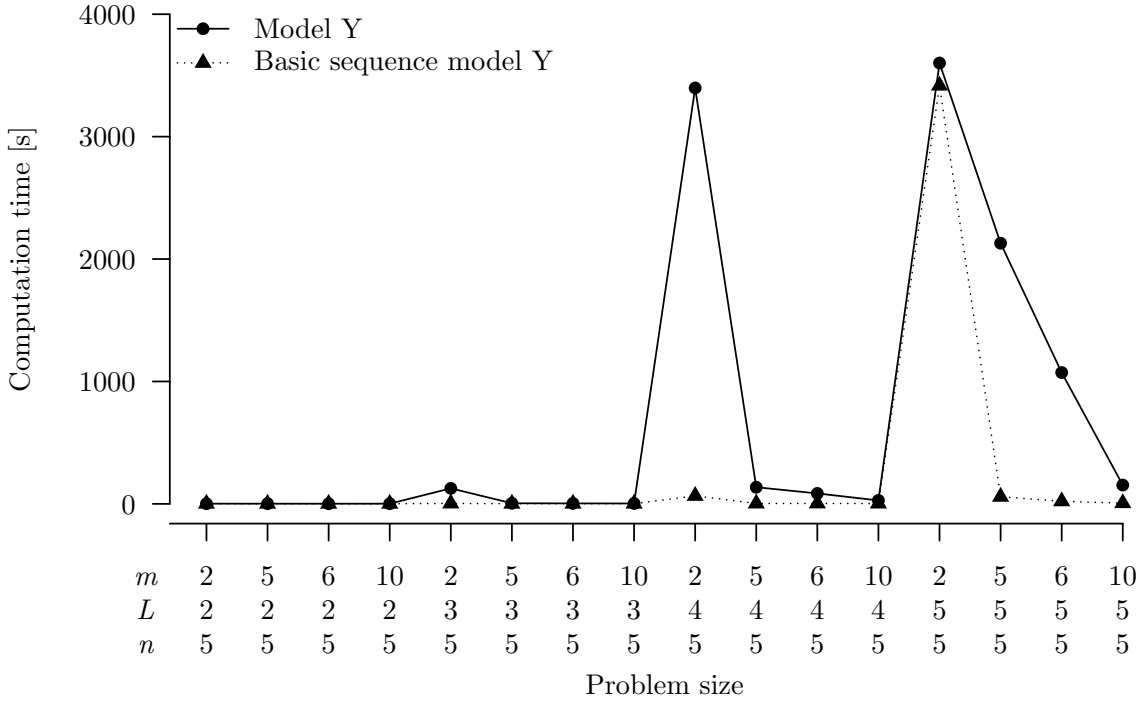


Table 4.16 shows the difference in solution quality and computational performance of model Y with and without basic sequence constraints depending on the number of jobs. The solution quality decreases with the increasing number of jobs since the average makespan deviation is also increasing. Also, the gap values for the model with a basic sequence are smaller than for those of the model without this constraint for every single number of jobs tested.

Table 4.16: Influence of the number of jobs on model Y with basic sequence

n	Gap Y [%]	Gap BS [%]	ΔC_{max} [%]	ΔIt [%]	Δct [%]
2	0.00	0.00	0.59	−6.09	2.19
3	0.00	0.00	0.65	97.07	3.75
4	0.00	0.00	0.76	89.42	−23.28
5	1.74	0.33	0.77	−59.57	−57.52

Looking at Table 4.17, it can be observed that higher numbers of re-entries lead to weaker results of the model with a basic sequence.

Table 4.17: Influence of the number of levels per job on model Y with basic sequence

L	Gap Y [%]	Gap BS [%]	ΔC_{max} [%]	ΔIt [%]	Δct [%]
2	0.00	0.00	0.37	−20.72	1.67
3	1.17	0.22	0.77	47.28	−37.34
4	0.38	0.00	0.78	−41.86	−30.41
5	0.78	0.19	0.87	44.51	−16.19

The influence of the number of machines is shown in Table 4.18. Mean gap values greater than zero appear for $m = 2$ and $m = 5$ machines. Both models need more computational effort if the number of machines is $m = 2$.

Table 4.18: Influence of the number of machines on model Y with basic sequence

m	Gap Y [%]	Gap BS [%]	ΔC_{max} [%]	ΔIt [%]	Δct [%]
2	1.68	0.33	0.57	−38.75	−22.71
5	0.07	0.00	0.76	214.96	−18.09
6	0.00	0.00	0.82	−34.61	−17.51
10	0.00	0.00	0.61	−21.01	−16.91

A basic sequence between the job levels is not considered in the following experiments in this thesis since it does not lead to much lower computation times for larger problems if a time limit of 1 hour is applied, and the results are up to 3.10 % weaker than those without a basic sequence.

4.4.3 Influence of Missing Operations

The different problem sizes and processing time generation procedure are identical to the test instances in Section 4.4.1, 4.4.1 and 4.4.2.[10]

In contrast to the previous tests, missing operations or incomplete levels are considered. Table 4.19 gives an overview of the number of generated later entries / earlier exits per jobs depending on the number of levels L per job. There are four different problem sets. The jobs in the problem set "Complete" need to be processed on all machines in every level. The sets "Inc_1", "Inc_2", "Inc_3" and "Inc_4" allow earlier exits and later level re-entries. The label "Inc" stands for incomplete levels because not all machines are visited.[11]

[10] See Table B.2 in Appendix B (p. 204) for an overview on the different problem sizes.
[11] See Table B.1 in Appendix B (p. 203) for an overview of missing operations.

Table 4.19: Number of later entries / earlier exits

Instance set	Levels per job							Chapter / Section
	2	3	4	5	10	20	40	
Complete	0	0	0	0	0	0	0	4.4.1, 4.4.2, 4.4.1
Inc_1	1	1	1	1	1	2	4	4.4.3, 4.5.3, 4.6.4, 4.7.5, 4.7.6, 5.2, 5.4
Inc_2	-	-	2	2	4	7	12	4.4.3, 5.2
Inc_3	-	-	-	3	5	9	20	4.4.3, 4.5.3, 4.6.4, 4.7.5, 4.7.6, 5.4
Inc_4	-	-	-	-	9	18	36	4.5.3, 4.6.4, 4.7.5, 4.7.6

The model that omits the scheduling of missing operations is labeled with Y0. The model requiring times to be assigned to missing operations is labeled with Y. Tables 4.21, 4.22 and 4.23 compare the influence of missing operations, depending on the number of machines, levels and jobs on the computational performance and solution quality. The formulas used to calculate the values are contained in the overview of the evaluation symbols used (Table 4.20).

Table 4.20: List of symbols in evaluation tables concerning missing operations

ΔC_{max}	$\Delta C_{max} = \left(C_{max}^{\mathrm{Y}}/C_{max}^{\mathrm{Y0}}\right) - 1$
ct	Computation time,
Δct	$\Delta ct = \left(ct^{\mathrm{Y}}/ct^{\mathrm{Y0}}\right) - 1$
Gap	Gap between lower bound and best found solution either after solving the problem or a computation time of 1 hour
It	Number of iterations until a problem instance is solved or the computation time limit is reached
ΔIt	$\Delta It = \left(It^{\mathrm{Y}}/It^{\mathrm{Y0}}\right) - 1$
L	Number of levels per job
m	Number of machines
n	Number of jobs
Y	Model without appropriate dealing with missing operations
Y0	Model with appropriate dealing with missing operations

Table 4.21 shows the influence of dealing appropriately with missing operations depending on the number of machines in the flow shops. Gap values for both models as well as the mean makespan deviations decrease with an increase in the number of machines. The relative difference in required or performed iterations is ascending if m rises. The difference in computation time is not affected by m in the Inc_1 instance set but is affected in the sets with higher numbers of incomplete levels, i.e. in sets with a

higher number of missing operations. Negative values of Δct indicate a lower average computation time of the simple model Y compared to the model Y0.

Table 4.21: The influence of missing operations depending on the number of machines

Inc	m	Gap Y0 [%]	Gap Y [%]	ΔC_{max} [%]	ΔIt [%]	Δct [%]
1	2	1.98	1.80	0.75	6.96	44.30
	5	0.12	0.08	3.08	86.43	47.67
	6	0.02	0.00	2.30	346.66	46.85
	10	0.00	0.00	1.31	596.01	49.26
2	2	4.43	4.26	1.23	12.81	−0.26
	5	0.46	0.55	4.62	−1.36	59.89
	6	0.06	0.03	4.87	403.11	171.65
	10	0.00	0.00	3.79	344.42	207.22
3	2	5.53	5.14	2.27	48.11	−22.77
	5	1.29	0.79	5.41	261.90	83.70
	6	0.46	0.34	6.56	1156.62	157.85
	10	0.00	0.12	4.54	156.48	119.51

A higher number of levels leads to higher gap values for both models, with the exception of $L = 3$ instances in test set Inc_1. In most cases, the solution quality of the Y0 model compared to the Y model increases with an increasing number of job levels.

Table 4.22: The influence of missing operations depending on the number of levels

Inc	L	Gap Y0 [%]	Gap Y [%]	ΔC_{max} [%]	ΔIt [%]	Δct [%]
1	2	0.00	0.00	1.49	19.64	17.51
	3	1.41	1.25	2.22	119.62	44.29
	4	0.65	0.57	2.29	913.94	84.10
	5	0.84	0.75	2.06	30.91	40.40
2	4	0.63	0.61	3.63	78.35	145.04
	5	1.48	1.45	3.60	243.03	62.95
3	5	1.82	1.60	4.69	405.78	84.57

If the number of jobs increases and the problems are still solved optimally in time, i.e. for most of the $n = 4$ problems, then the computational performance of the Y0 model increases compared to the simple Y model.

Table 4.23: The influence of missing operations depending on the number of jobs

Inc	n	Gap Y0 [%]	Gap Y [%]	ΔC_{max} [%]	ΔIt [%]	Δct [%]
1	2	0.00	0.00	1.26	−0.01	16.56
	3	0.00	0.00	2.32	7.26	45.31
	4	0.00	0.00	1.91	851.89	93.48
	5	2.11	1.87	1.94	176.67	32.52
2	2	0.00	0.00	2.92	−8.24	10.63
	3	0.00	0.00	3.52	−1.24	13.24
	4	0.71	0.53	4.41	638.05	109.68
	5	4.19	4.27	3.65	129.51	301.04
3	2	0.00	0.00	3.45	−7.62	5.00
	3	0.00	0.00	4.36	771.11	−36.21
	4	1.21	0.85	5.51	623.96	232.92
	5	5.93	5.40	5.59	241.55	137.71

The gap values increase with the increasing number of incomplete levels in all three evaluation tables. As expected, the advantage of dealing appropriately with missing operations increases with a higher number of incomplete levels, which is reflected in the values ΔC_{max}. In most cases, the Y0 model requires fewer iterations and less computation time compared to the simple Y model, which is indicated by the positive values of ΔIt and Δct.

The use of mixed levels leads to makespan reductions of up to 2 % compared with the separated level models for the tested small instances. Appropriate handling of missing operations leads to further reductions. Since only small problem sizes have been tested until now, the following section contains heuristic solution approaches to solve larger instances.

4.5 Initialization Methods

Constructive heuristics create a solution from scratch. They are necessary to initiate improvement methods like tabu search, simulated annealing and variable neighborhood search. This section explains several constructive heuristics for initializing improvement methods.

4.5.1 Constructive Heuristics for Separated Levels

Priority rules and simple constructive heuristics that lead to initial schedules with separated levels are presented in this subsection.

Longest Total Processing Time Jobs First

The Longest total processing time job first (LTPTJ) rule uses each job's sum of processing times. The first level of the job with the maximum sum of processing times, TPT_i, of all jobs is put on sequence position one. The first level of the job with the second highest sum of processing times follows on the second position and so on. The levels $l = 2, \ldots, L$ follow the same job sequence after all levels $l = 1$ have been assigned to their positions. The TPT_i of the scheduled job is set to 0, before the next job is selected. Assigning the sequence positions in this way results in a separated level schedule.

The procedure is implemented as shown in algorithm 4.1.

Algorithm 4.1 Longest total processing time jobs first rule

Data: n, L, m, TPT_i and $p^i_{lk}\, \forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m$
(1) *Empty Solution:* Create a sequence of $n \cdot L$ empty slots.
Assign job levels to sequence positions:
for $t = 1, \ldots, n$ **do**
  (2) Choose job i with $TPT_i = \max_{i'=1,\ldots,n} TPT_{i'}$
  for $l = 1, \ldots, L$ **do**
    (3) Assign level l of job i to position $t + (l-1) \cdot n$.
  end for
  (4) $TPT_i \leftarrow 0$
end for
(5) Calculate all starting times s^i_{lk} and C_{max}.

Shortest Total Processing Time Jobs First

The Shortest total processing time job first (STPTJ) rule also uses the sum of processing times of each job as a criterion to sequence the job levels. These values are given by the total processing times of jobs TPT_i. The job with the lowest TPT_i value is scheduled next. Its first level is assigned to position i and the following levels $l > 1$ to the positions $i + (l-1) \cdot n$. Afterwards, the job's TPT_i is set to a sufficiently large number A, and the next job is selected.

This priority rule is set out in algorithm 4.2.

Algorithm 4.2 Shortest total processing time jobs first rule

Data: n, L, m, TPT_i, A and $p^i_{lk}\,\forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m$
(1) *Empty Solution:* Create a sequence of $n \cdot L$ empty slots.
Assign job levels to sequence positions:
for $t = 1, \ldots, n$ **do**
  (2) Choose job i with $TPT_i = \min_{i'=1,\ldots,n} TPT_{i'}$
  for $l = 1, \ldots, L$ **do**
    (3) Assign level l of job i to position $t + (l-1) \cdot n$.
  end for
  (4) $TPT_i \leftarrow A$
end for
(5) Calculate all starting times s^i_{lk} and C_{max}.

NEH Jobs Algorithm

The one adoption of the NEH algorithm[12] for the RPFS presented in this section is called the NEH job (NEHJ) algorithm. There are two criteria for sequencing the jobs in this heuristic. The job that needs to be scheduled in iteration t is selected by its total processing. The jobs are selected in non-ascending order of their total processing time, i.e. the first job to be assigned is the one with the highest total processing time. The last job for which a sequence position is searched is the job with the lowest total processing time. So, the number of main iterations that the algorithm performs is equal to the number of jobs n. The number of the current iteration is identical to the number of already selected jobs. Within each of the iterations $t = 1, \ldots, n$ the selected job is inserted into each possible position current sequence, resulting in $t' = 1, \ldots, t$ subiterations for each main iteration with the resulting permutations π'. The levels of the jobs are added in a way that leads to a separated level schedule. The objective value for each of the resulting solutions is calculated. The best current solution will be accepted (π_{best}), and the algorithm will operate on this permutation (π) during the next iteration. The algorithm's implementation is simplified in algorithm 4.3.

[12] See Nawaz / Enscore / Ham (1983): *Heuristic for m-machine, n-job flow-shops*, pp. 91-95.

Algorithm 4.3 NEH job algorithm

Data: n, L, m, TPT_i, A and $p^i_{lk}\,\forall i = 1,\ldots,n; l = 1,\ldots,L; k = 1,\ldots,m$
Assign job levels to sequence positions:
for $t = 1,\ldots,n$ **do**
  (1) Choose job i with $TPT_i = \max_{i'=1,\ldots,n} TPT_{i'}$
  (2) $C_{max}(\pi_{best}) \leftarrow A$
  for $t' = 1,\ldots,t$ **do**
    (3) $\pi' \leftarrow \pi$
    for $l = 1,\ldots,L$ **do**
      (4) Insert level l of job i to position $t' + (l-1)\cdot n$ to update π'.
    end for
    (5) Calculate all starting times s^i_{lk} and $C_{max}(\pi')$.
    if $C_{max}(\pi') < C_{max}(\pi_{best})$ **then**
      (6) $\pi_{best} \leftarrow \pi'$
    end if
  end for
  (7) $\pi \leftarrow \pi_{best}$
  (8) $TPT_i = 0$
end for

Service in Random Order Job

Service in random order job (SIROJ) generates a job sequence randomly. A job is chosen by generating a uniformly distributed number and the job sequence is repeated for each level l. The pseudo code of the procedure is shown in 4.4. This method is one of the constructive methods for a schedule to show the impact of structured schedule generation procedures like priority rules or the NEH algorithm.

Algorithm 4.4 Service in random order job

Data: n, L, m and $p^i_{lk}\,\forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m$
(1) *Empty Solution:* Create a sequence of $n \cdot L$ empty slots.
Assign job levels to sequence positions:
for $t = 1, \ldots, n$ **do**
  (2) Choose a random remaining job i.
  for $l = 1, \ldots, L$ **do**
    (3) Assign level l of job $'$ to position $t + (l - 1) \cdot n$.
  end for
end for
(5) Calculate all starting times s^i_{lk}.

4.5.2 Constructive Heuristics for Mixed Levels

Dispatching rules and simple constructive heuristics that lead to initial schedules with mixed levels are described in this subsection.

Longest Total Processing Time Levels First

Similar to the method using the sum of all processing times of a job as criteria, the sums of processing times of each level are used as a criteria in the Longest total processing time level first (LTPTL) rule. These methods avoid assignments of a complete partition of a permutation with all job levels of the same level number l. A job level l is allowed to be assigned to a sequence position if the predecessor level $l - 1$ of the same job has a lower sequence position and has completed its last operation. In total, two criteria are used to generate a schedule. The first criterion is the total processing time of a job level and the second criterion is the ready time of a job level. Only levels whose predecessor levels are already scheduled are allowed to be added to the permutation. These levels are part of the set $N^{available}$. The job levels are not part of $N^{available}$ but are part of a second set N^{ready} if their ready time, RT_{il}, is smaller than or equal to the time when the machine k for their first operation is free after processing the preceding level, i.e. if $RT_{il} \leq MRT_k$. The comparison of total processing times is done on all levels that are in N^{ready}. When multiple job levels are ready, the level with the longest total processing time is chosen. If N^{ready} is empty, then the job level with the lowest ready time of all levels in $N^{available}$ is added to the permutation.

Algorithm 4.5 Longest and shortest total processing time level first rule

Data: n, L, m, TPT_{il}, A and $p^i_{lk}\,\forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m$
(1) Add all levels $l = 1$ to the ready job levels N^{ready}.
Assign job levels to sequence positions:
for $t = 1, \ldots, n \cdot L$ **do**
  if $N^{ready} \neq \emptyset$ **then**
    (2a) Choose job level il with $TPT_{il} = \max_{i' \in N^{ready}} TPT_{i'}$ (STPTL: $TPT_{il} = \min_{i' \in N^{ready}} TPT_{i'}$) and remove it from N^{ready}
  else
    (2b) Choose job level il with $RT_{il} = \min_{i' \in N^{available}} RT_{i'}$ and remove it from $N^{available}$
  end if
  (3) Add job level il at position t of existing job level sequence.
  (4) Calculate all starting times s^i_{lk} and all machine ready times MRT_k.
  if $l < L$ **then**
    (5) Calculate the level ready time $RT_{i,l+1}$.
    if $RT_{i,l+1} \leq MRT_k$ (with k being the first machine for level $l+1$ of job i with $p^i_{l+1,k} > 0$) **then**
      (6a) Add $i, l+1$ to the ready levels N^{ready}
    else
      (6b) Add $i, l+1$ to the available levels $N^{available}$
    end if
  end if
end for

Shortest Total Processing Time Levels First

The Shortest total processing time level first (STPTL) rule prefers the job level with the minimum sum of processing times among the levels that are ready to be processed. If no level is ready, the job level that becomes ready at the earliest point of time is chosen. The difference between algorithms LTPTL and STPTL is shown in step (2a) of algorithm 4.5. In contrast to the LTPTL rule, the STPTL rule chooses the ready job level with the lowest total processing time.

NEH Level Algorithm

The NEH level (NEHL) heuristic schedules the job levels separately. The job levels with the highest sum of processing times among all the ready levels is chosen to be scheduled next. The level with the earliest ready time is chosen next if there are no ready job levels.

A detailed overview of the procedure as it is implemented is given in algorithm 4.6. The chosen job level is inserted in each valid position, $j = t' > j_{i,l-1}$, in the permutation. The resulting solutions are evaluated regarding makespan. The best insertion position of all valid possibilities is saved and used as the basic permutation for the next iteration of the algorithm.

Algorithm 4.6 NEH level algorithm

Data: n, L, m, TPT_{il}, A and $p^i_{lk}\,\forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m$
(1) Add all levels $l = 1$ to the ready job levels N^{ready}.
Assign job levels to sequence positions:
for $t = 1, \ldots, n \cdot L$ **do**
  if $N^{ready} \neq \emptyset$ **then**
    (2a) Choose job level il with $TPT_{il} = \max_{i' \in N^{ready}} TPT_{i'}$ and remove from N^{ready}
  else
    (2b) Choose job level il with $RT_{il} = \min_{i' \in N^{available}} RT_{i'}$ and remove from $N^{available}$
  end if
  (3) $C_{max}(\pi_{best}) \leftarrow A$
  for $t' = j_{i,l-1} + 1, \ldots, t$ with $t_{i0} = 1$ **do**
    (4) $\pi' \leftarrow \pi$
    (5) Insert level il at position $j = t'$ to update π'.
    (6) Calculate all starting times s^i_{lk} and $C_{max}(\pi')$.
    if $C_{max}(\pi') < C_{max}(\pi_{best})$ **then**
      (7) $\pi_{best} \leftarrow \pi'$
    end if
  end for
  (8) $\pi \leftarrow \pi_{best}$.
  (9) Calculate all machine ready times MRT_k.
  if $l < L$ **then**
    (10) Calculate the level ready time $RT_{i,l+1}$.
    if $RT_{i,l+1} \leq MRT_k$ (with k being the first machine for level $l+1$ of job i with $p^i_{l+1,k} > 0$) **then**
      (11a) Add $i, l+1$ to the ready levels N^{ready}
    else
      (11b) Add $i, l+1$ to the available levels $N^{available}$
    end if
  end if
end for

Service in Random Order Level

The previously described constructive heuristics are compared to a randomly generated sequence of job levels generated by the Service in random order level (SIROL) method. SIROL assigns the single job levels successively to the sequence positions $t = 1, \ldots, n \cdot L$. Only valid assignments are allowed, which means that a level $l + 1$ can not be assigned earlier than the corresponding level l of the same job, i.e. the levels added to the sequence need to be in set $N^{available}$. The probability of choosing a job level from $N^{available}$ is the same for all job levels in $N^{available}$. Machine ready times and level ready times are not considered in this method. Nevertheless, the generated schedules are all valid, since only levels whose predecessors are scheduled, are allowed to be added to the permutation. Algorithm 4.7 shows the single steps of the procedures.

Algorithm 4.7 Service in random order level

Data: n, L, m and $p^i_{lk}\ \forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m$
(1) Add all levels $l = 1$ to the ready job levels $N^{available}$.
Assign job levels to sequence positions:
for $t = 1, \ldots, n \cdot L$ **do**
  (2) Choose a random job level il from $N^{available}$ and remove it from $N^{available}$.
  (3) Add job level il at position t of job level sequence.
  if $l < L$ **then**
    (4) Add level $l + 1$ of job i to $N^{available}$.
  end if
end for
(5) Calculate all starting times s^i_{lk}.

4.5.3 Computational Experiments

The initialization methods are tested for small, medium and large problem sizes. The parameters of the problem size are $n \in \{2, 3, 4, 5\}$, $L \in \{2, 3, 4, 5\}$, $m \in \{2, 5, 6, 10\}$ for small problems, $n \in \{10, 20, 30, 40, 50\}$, $L \in \{5, 10\}$, $m \in \{10, 20, 30, 40, 50\}$ for medium problems and $n \in \{50, 100\}$, $L \in \{20, 40\}$, $m \in \{50, 100\}$ for large problems.[13] Three sets of test instances (Inc_1, Inc_2 and Inc_4) are generated. The number of missing operations depends on the test set with the lowest number of missing operations in Inc_1 and the highest in Inc_4.[14] Ten instances are tested for each problem size in the sets Inc_1, Inc_2 and Inc_4. The processing times of non-missing operations are uniformly distributed random numbers between 1 and 99.

[13] See Table B.2 in Appendix B (p. 204) for detailed information on the problem sizes.
[14] See Table B.1 in Appendix B (p. 203) for an overview of missing operations.

The results regarding the mean relative deviation from the best obtained makespan are shown in Table 4.24. The deviation to the best makespan achieved for a problem instance is calculated by:

$$\Delta C_{max}^{best} = \frac{C_{max}\left(\pi_{init}\right)}{C_{max}\left(\pi_{init}^{best}\right)} - 1.$$

$C_{max}\left(\pi_{init}\right)$ is the makespan obtained with a certain opening procedure and $C_{max}\left(\pi_{init}^{best}\right)$ is the lowest makespan value among the results of all tested opening procedures. The different opening procedures are numbered:

1 LTPTJ,
2 STPTJ,
3 NEHJ,
4 SIROJ,
5 LTPTL,
6 STPTL,
7 SIROL,
8 NEHL.

Table 4.24: Comparison of makespan of the constructive heuristics

Inc	Problem Size	No. Inst.	Average makespan deviation ΔC_{max}^{best} [%] 1	2	3	4	5	6	7	8
1	Small	640	7.66	6.57	5.15	7.83	8.89	3.61	4.32	52.11
	Medium	500	2.84	2.85	2.23	2.80	3.10	1.49	16.80	236.73
	Large	80	1.45	1.41	0.96	1.47	0.71	0.54	36.86	483.55
	Total 1	1220	5.28	4.71	3.68	5.35	5.98	2.54	11.57	156.06
3	Small	160	10.40	9.94	7.85	9.80	7.62	3.60	5.31	55.95
	Medium	500	7.06	7.10	6.28	7.20	3.91	1.73	13.48	214.86
	Large	80	10.36	10.43	9.95	10.43	1.06	0.58	33.86	462.05
	Total 3	740	8.14	8.08	7.02	8.11	4.40	2.01	13.92	207.22
4	Medium	250	12.94	13.12	12.06	13.14	4.43	1.80	12.24	207.56
	Large	80	19.79	19.84	19.31	19.66	2.35	0.66	28.49	438.87
	Total 4	330	14.60	14.75	13.82	14.72	3.92	1.52	16.18	263.64

The STPTL rule delivers the best results on average for the makespan in each of the sets of test instances Inc_1, Inc_3 and Inc_5. The deviation compared to the best

makespan values achieved by the opening procedures is on average the lowest for the STPTL rule. The second best value for the Inc_1 instances is delivered by the NEHJ method. It is the best constructive heuristics to generate a schedule with separated levels. For Inc_3 and Inc_5, the method is the third best opening procedure. The second best for these cases is the LTPTL rule. The weakest approaches in these tests are the NEHL rule and the SIROL rule. The rules LTPTJ, STPTJ and SIROJ perform roughly on the same level of solution quality. The makespan values obtained with these rules are better on average than the values of NEHL and SIROL, but weaker than the values achieved by applying the procedures LTPTL, STPTL and NEHJ.

The relative frequencies of obtaining the best makespan among all tested opening procedures are shown in Table 4.25. The numeration of the initialization methods is the same as those given in Table 4.24.

Table 4.25: Best makespan frequencies of the constructive heuristics

Inc	Problem Size	No. Inst.	Frequency of obtaining the lowest makespan [%]							
			1	2	3	4	5	6	7	8
1	Small	640	20.16	22.19	29.22	17.19	14.22	42.19	42.50	1.56
	Medium	500	12.60	6.20	18.80	9.80	11.20	37.00	5.60	0.00
	Large	80	10.00	1.25	25.00	2.50	18.75	42.50	0.00	0.00
	Total 1	1220	16.39	14.26	24.67	13.20	13.28	40.08	24.59	0.82
3	Small	160	8.75	11.88	19.38	10.63	17.50	45.00	35.00	0.00
	Medium	500	4.80	1.80	7.60	3.60	21.60	48.60	14.20	0.00
	Large	80	0.00	0.00	0.00	0.00	32.50	67.50	0.00	0.00
	Total 3	740	5.14	3.78	9.32	4.73	21.89	49.86	17.16	0.00
4	Medium	250	0.40	0.00	2.40	1.20	25.60	58.40	12.00	0.00
	Large	80	0.00	0.00	0.00	0.00	22.50	77.50	0.00	0.00
	Total 4	330	0.30	0.00	1.82	0.91	24.85	63.03	9.09	0.00

The relative solution quality of the STPTL rule, as the best performing constructive method in this test, increases with increasing problem size and increasing numbers of missing operations.

The STPTL rule is the constructive method with the best makespan values on average and the NEHJ procedure is the best method that generates a separated level schedule. Both of these methods are then chosen as initialization methods for the metaheuristics. Additionally, the two random schedule generation procedures, SIROJ and SIROL, are also used for initialization to investigate the influence of the initialization methods on the solution quality of the examined metaheuristics.

4.6 Neighborhood Structures in Re-entrant Permutation Flow Shops

Two main neighborhoods are differentiated within this section: the swap neighborhood and insertion neighborhood. These neighborhoods are examined in different stages:

1. Job stage,
2. Job level stage.

The number of neighbors in the lowest hierarchy stage, which represents changes in particular members of the permutation, can be limited with so-called block neighborhoods. Grabowski (1982) first mentioned the block structure in flow shops given by the critical path.[15] The block neighborhoods introduced by Nowicki/Smutnicki (1996) are also based on the critical path of a solution, which defines the makespan of a permutation.[16]

The levels $l < L$ of each job i are not allowed to be processed before their succeeding levels $l+1$. The levels $l > 1$ are not allowed to be scheduled before their preceding levels $l-1$.

4.6.1 Swap Moves

Swap moves imply that the sequence positions of two job levels are exchanged. A swap between a level l of a job i and a level l' of a job i' is valid if the following limits to the sequence position are not exceeded:

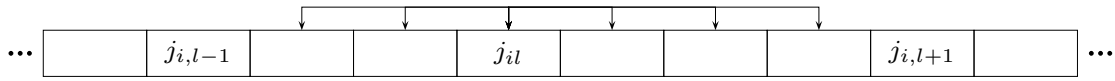
- $j_{il} < j_{i',l'+1}$ if $l' < L$,
- $j_{il} > j_{i',l'-1}$ if $l' > 1$,
- $j_{i'l'} < j_{i,l+1}$ if $l < L$,
- $j_{i'l'} > j_{i,l-1}$ if $l > 1$.

j_{il} is the sequence position of the first swap partner (the position of job i's level l). $j_{i'l'}$ is the position of the second swap partner. Swaps between levels of the same job are not possible, because they would violate the move limits. The sequence positions of all other job levels stay the same.

Figure 4.22 illustrates the limits of a swap move of a single job level at position j_{il} in a given permutation.

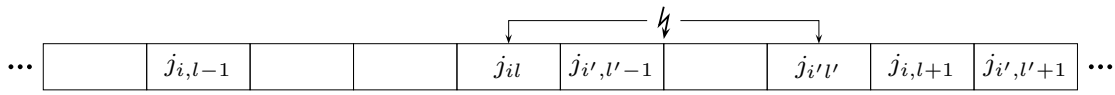
[15] See Grabowski (1982): *Solving the Flow—Shop Problem*, pp. 57–58.
[16] See Nowicki/Smutnicki (1996): *Fast taboo search for the job shop problem*, pp. 161–165.

Figure 4.22: Illustration of the swap move limits of a level



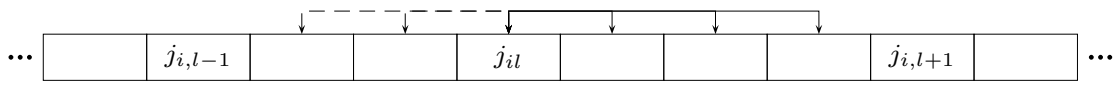
Considering the second swap partner $j_{i'l'}$ a possible swap may become invalid because of the additional move limits of level $i'l'$. An example is given by Figure 4.23.

Figure 4.23: Example of an invalid level swap



Not all swap moves need to be evaluated to find the best neighbor, as Figure 4.24 shows. The dashed lines indicate irrelevant swap moves.

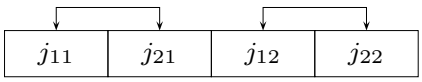
Figure 4.24: Illustration of relevant and irrelevant level swap moves



The moves to the left can be omitted in the evaluation of neighbors since they are equal to the moves to the right side by the corresponding swap partners between $j_{i,l-1}$ and j_{il}. Either all moves to the right or all moves to the left need to be evaluated. This leads to a number of $\frac{1}{2}n \cdot L\,(n \cdot L - 1)$ possible swap moves if the move limits of single levels are not considered. The original permutation on which the moves are performed determines the specific number of possible swap moves, as seen in the example with two jobs and $L = 2$ levels in Figures 4.25 and 4.26.

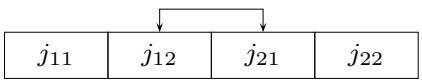
Two different swap moves can be applied to the permutation given in Figure 4.25.

Figure 4.25: Example of the number of possible level swap moves (I)



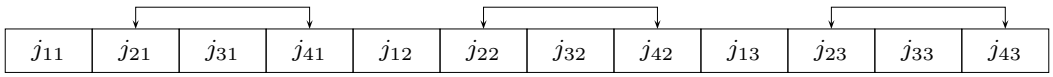
It is possible to apply only one swap move on the permutation shown in Figure 4.26.

Figure 4.26: Example of the number of possible level swap moves (II)



A swap on the job stage needs all levels $l = 1, \ldots, L$ of two jobs i and i' to be exchanged. Move limits of levels do not need to be considered, as long as both jobs have the same number of levels. An example is given in Figure 4.27. $i = 2$ and $i' = 4$ are the swapped jobs.

Figure 4.27: Example of a job swap



4.6.2 Insertion Moves

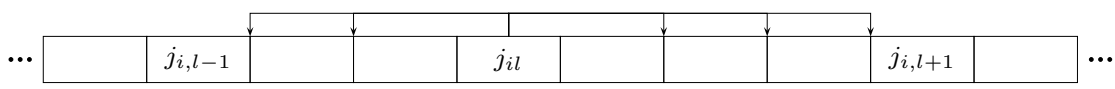
A job level at position j is picked to be placed at a new sequence position j'. The conditions on j' are:

$$j_{i,l-1} < j' \quad \forall i = 1, \ldots, n,\ l = 2, \ldots, L; \tag{4.25}$$

$$j' < j_{i,l+1} \quad \forall i = 1, \ldots, n,\ l = 1, \ldots, L-1. \tag{4.26}$$

$(n \cdot L - 1)^2$ different insertion moves can be applied to a permutation of the length $n \cdot L$ if move limits are not considered.[17] Therefore insertion neighborhoods are larger than swap neighborhoods if $n > 2$ and $L > 1$ or $n = 2$ and $L > 2$; otherwise, the size of the neighborhoods is equal. The position of the job levels $j'' = 1, \ldots, j' - 1$ does not change. The job levels on the positions $j''' = j' + 1, \ldots, j - 1$ are respectively shifted to $j''' + 1$. The remaining levels stay on the same positions. Figure 4.28 shows the limits of insertion moves. The earliest position for the job level il is right behind the position of level $i, l - 1$. The latest position is the one before the level $i, l + 1$.

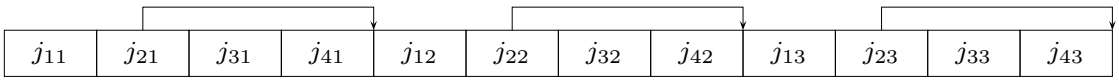
Figure 4.28: Illustration of the insertion move limits and valid moves of a level



A job insertion move is depicted in Figure 4.29. Two jobs $i = 2$ and $i' = 4$ are selected for the move. It it necessary to define whether the levels of job i should be inserted in the position of job i', or vice versa, before the move is performed. Job $i = 2$ is inserted in the positions of $i' = 4$ in the given example.

[17] See Taillard (1990): *Heuristic methods for the flow shop sequencing problem*, p. 61.

Figure 4.29: Example of a job insertion move



The resulting permutation is shown in Figure 4.30. The levels of job $i = 2$ are now on the previous sequence positions of the corresponding levels of job $i' = 4$.

Figure 4.30: Resulting permutation after the job insertion move

j_{11}	j_{31}	j_{41}	j_{21}	j_{12}	j_{32}	j_{42}	j_{22}	j_{13}	j_{33}	j_{43}	j_{23}

4.6.3 Block Neighborhoods

Block neighborhoods refer to sections in a critical path of a schedule. The blocks are the sequence positions between two different job levels that follow each other on a critical path. The critical path of operations determines the makespan of a schedule. Members of the same block are processed consecutively on the same machine according to Nowicki / Smutnicki (1996).[18]

The procedure to identify the critical path is shown in algorithm 4.8.

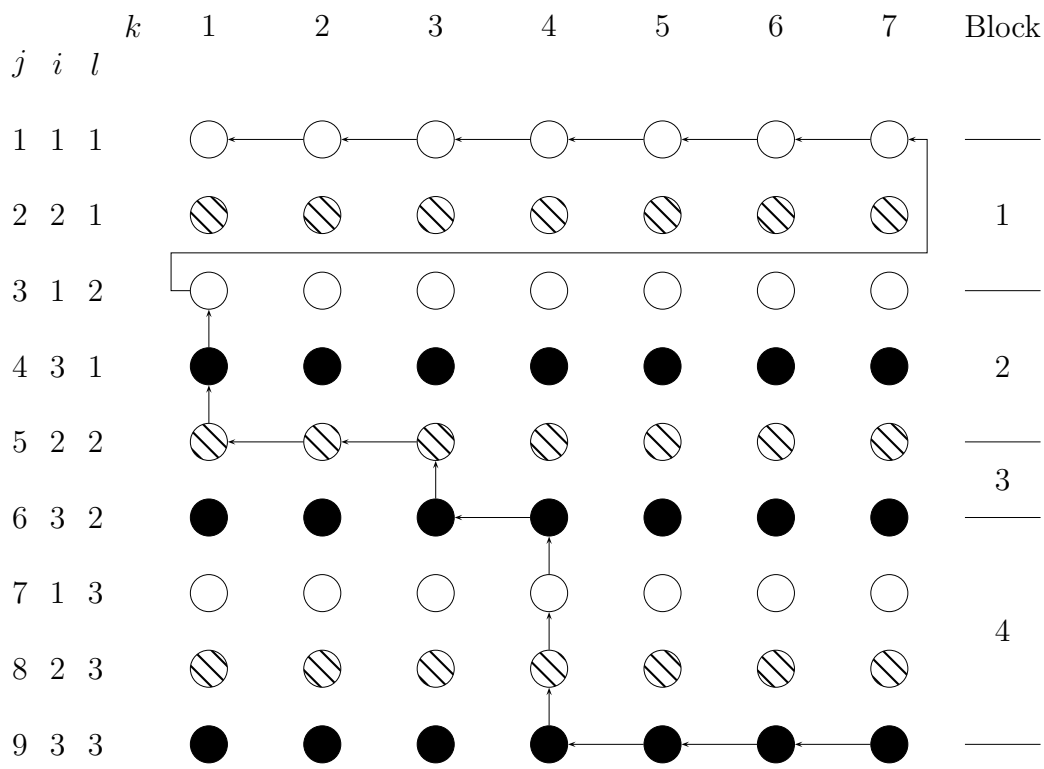
The identification starts with the last operation of the schedule, which is finished at the highest value $s^i_{lk} + p^i_{lk}$. The sequence position of the corresponding job level is added to the critical path. The following steps are repeated until the starting time of an operation equals 0, i.e. the first operation of the schedule is reached. For the latest identified operation on the critical path, it is determined whether there is an operation on any k' of the preceding machines k for the same job level il, which is finished right before the operation on k starts ($s^i_{lk'} + p^i_{lk'} = s^i_{lk}$). k is updated to k' as long as such a machine is found. If none of the preceding job levels match this requirement, then the last operation of the preceding level $l - 1$ of the same job i is added to the critical path. The last operation added is the first operation of the schedule with a starting time equal to 0 and a processing time greater than zero.

[18] See Nowicki / Smutnicki (1996): *Fast taboo search for the job shop problem*, p. 161.

Algorithm 4.8 Identifying a critical path in a re-entrant flow shop

(1) Find the job level, which is finished last (highest value $s^i_{lk} + p^i_{lk}$).
(2) Add its sequence position j to the critical path.
while $s^i_{lk} \neq 0$ **or** $p^i_{lk} = 0$ **do**
  if There is a machine $k' < k$ with $s^i_{lk'} + p^i_{lk'} = s^i_{lk}$ **then**
    (3) $k \leftarrow k'$.
  else
    if There is a job level $i'l'$ at sequence position j' with $s^{i'}_{l'k} + p^{i'}_{l'k} = s^i_{lk}$ **then**
      (4a) Add the sequence position j' of this job level to the critical path.
      (4b) $j \leftarrow j'$.
    else
      (4c) Add the sequence position j' of the job level $i, l-1$ to the critical path.
      (4d) $k \leftarrow k'$, where k' is the machine with the last operation in $i, l-1$.
      (4e) $j \leftarrow j'$.
    end if
  end if
end while
(5) Add the sequence position j with $s^i_{lk} = 0$ and $p^i_{lk} > 0$ to the critical path.
(6) Reverse the created list.

Figure 4.31: Example of identifying the critical path of operations



The members of the permutation are the job levels. Some of the job levels define the

border of the blocks based on the critical path. They are labeled with $u_b = 0, \ldots, B$. The critical path in the example shown in Figure 4.31 contains the job levels at the sequence positions $j = (1, 3, 5, 6, 9)$. The limits of critical blocks are identified as the positions $u_b = (1, 3, 5, 6, 9)\ \forall b = 0, \ldots, B$, where B is the number of blocks on the critical path. Between each pair of adjacent block limits, the operations defining the critical path are performed on the same machine.

There are two types of moves connected to the block properties. The first type is called intra block moves and the second type is called across block moves.[19] There are different definitions of intra and across block moves in related literature.[20] The definitions are explained in the following section for swaps and insertion moves. Equivalent moves yielding to the same permutation are included only once in every neighborhood.

Intra block moves

The figures in this subsection show all possible insertion moves following the particular block definition. No swap moves are omitted in the figures, despite the fact that some swap moves are not necessary to obtain all possible solutions in a swap neighborhood, as explained in subsection 4.6.1. The position of a job i's level l is labeled with j_{il} in the following.

Intra block swaps exclude moves to the block border positions u_{b-1} and u_b in the definition of Nowicki / Smutnicki (1996), as shown in Figures 4.32 and 4.34.[21] On the other hand, the Figures 4.33 and 4.35 show the Chen / Pan / Wu (2007) definition of intra block moves, which includes swaps to the block borders u_{b-1} and u_b.[22]

Figure 4.32: Example of a Nowicki intra block swap if j_{il} is not a block border

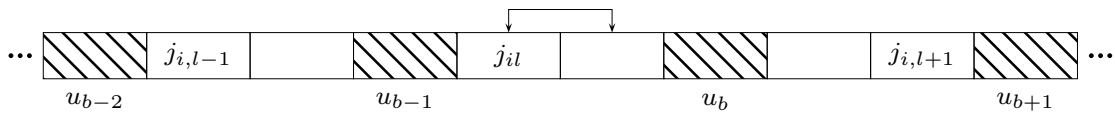
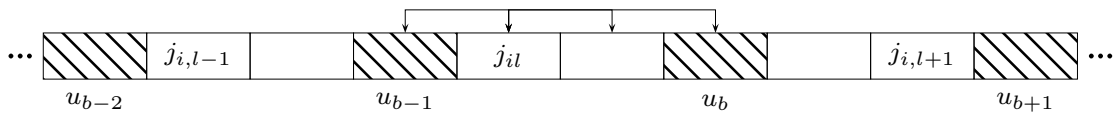


Figure 4.33: Example of a Chen intra block swap if j_{il} is not a block border



[19] See Shen / Buscher (2012): *Serial batching in job shops*, pp. 17–18.
[20] See Nowicki / Smutnicki (1996): *Fast taboo search for the job shop problem*, p. 164 and Chen / Pan / Wu (2007): *Reentrant flow-shops and hybrid tabu search*, p. 355.
[21] See Nowicki / Smutnicki (1996): *Fast taboo search for the job shop problem*, p. 164.
[22] See Chen / Pan / Wu (2007): *Reentrant flow-shops and hybrid tabu search*, p. 356.

A job level that is a block border u_b is part of the two blocks, $b-1$ and b. The possible intra block swaps for the members of the permutation are shown in Figures 4.34 and 4.35. The Nowicki intra block swaps again exclude moves to the block borders u_{b-1} and u_{b+1}, but these moves are included in the Chen intra block swaps.

Figure 4.34: Example of a Nowicki intra block swap if j_{il} is a block border

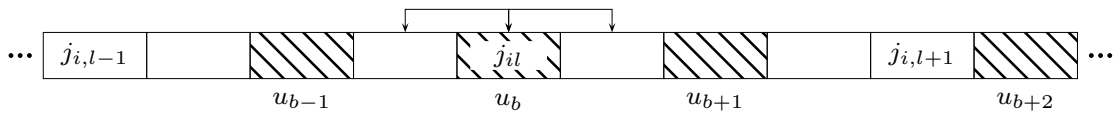
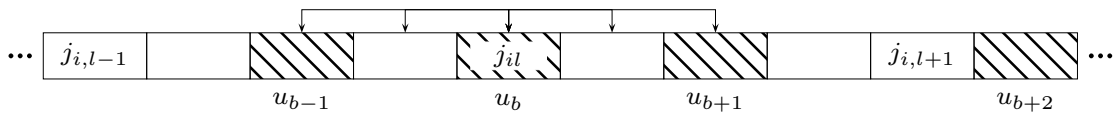


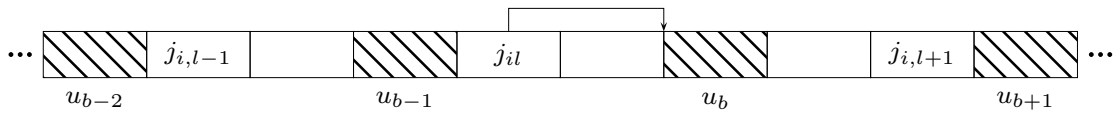
Figure 4.35: Example of a Chen intra block swap if j_{il} is a block border



The possible intra block insertion moves are illustrated in Figures 4.36 and 4.37, where the permutation member is not a block border.

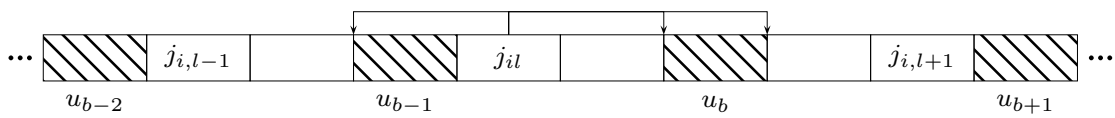
A Nowicki intra block neighborhood allows only moves between the two block borders u_{b-1} and u_b if the member to move is not on a block border position u_b, as can be seen in Figure 4.36.[23]

Figure 4.36: Example of a Nowicki intra block insertion if j_{il} is not a block border



The Chen intra block insertion shown in Figure 4.37, allows a job level to be moved to the position before the lower block border u_{b-1} and behind the the upper block border u_b, as well as on all positions between u_{b-1} and u_b.

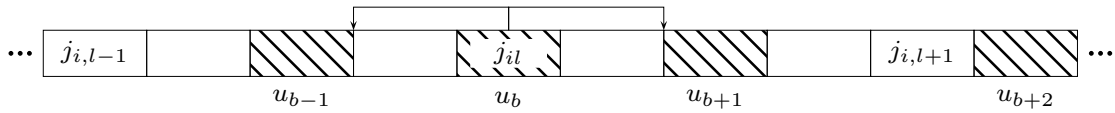
Figure 4.37: Example of a Chen intra block insertion if j_{il} is not part of the critical path



[23] See Nowicki / Smutnicki (1996): *Fast taboo search for the job shop problem*, p. 164.

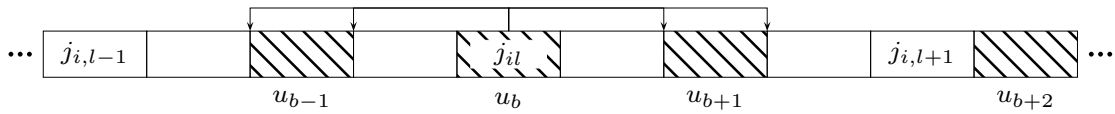
The intra block moves if the job level to move is a block border are displayed in Figures 4.38 and 4.39.
Nowicki moves do not allow changes in the positions of the lower and upper block borders u_{b-1} and u_{b+1}. In this case, insertion moves are only allowed between the mentioned block borders.

Figure 4.38: Example of a Nowicki intra block insertion if j_{il} is a block border



Chen intra block insertion moves allow the sequence positions of the block borders u_{b-1} and u_{b+1} to be change. Therefore, a job level that is a block border u_b itself is allowed to be inserted directly before u_{b-1} as well as directly behind u_{b+1}, as shown in Figure 4.39.

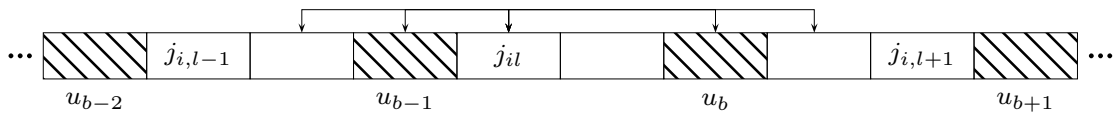
Figure 4.39: Example of a Chen intra block insertion if j_{il} is a block border



Across block moves

Figures 4.40, 4.41, 4.42 and 4.43 show the possible swap move of a permutation member j_{il} out of its block b. The possible Nowicki across block moves include swaps with the sequence positions u_{b-1} and u_b, defining the limits of the bth block, because these positions are also part of the neighboring blocks $b-1$ and $b+1$.

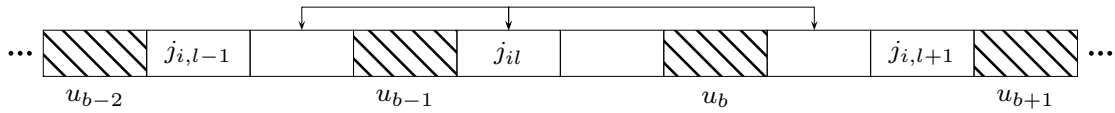
Figure 4.40: Example of a Nowicki across block swap if j_{il} is not a block border



The example in Figure 4.41 illustrates that Chen / Pan / Wu (2007) exclude moves to u_{b-1} and u_b from across block moves, since they are classified as intra block moves.[24] This means that for swap moves, the job levels on the positions u_{b-1} and u_b are not allowed to be swap partners of j_{il} (Figure 4.41).

[24] See Chen / Pan / Wu (2007): *Reentrant flow-shops and hybrid tabu search*, p. 356.

Figure 4.41: Example of a Chen across block swap if j_{il} is not a block border



Swaps to the adjacent block borders u_{b-1} and u_{b+1} are seen as across block moves by Nowicki / Smutnicki (1996) if the permutation member at the j_{il}th position is also a block border u_b, which will be swapped with a member at another position.[25]

To evaluate all across block swaps in the Nowicki definition of across block moves, not all moves to the left can be omitted. The move to the left border of the block still needs to be evaluated if the job level is not a block limit itself.

Figure 4.42: Example of a Nowicki across block swap if j_{il} is a block border

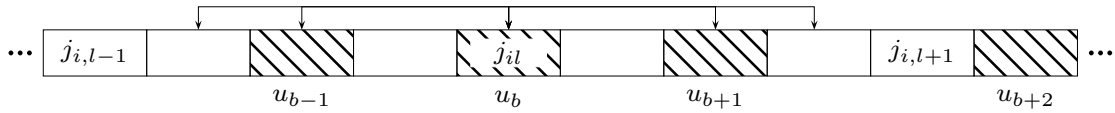
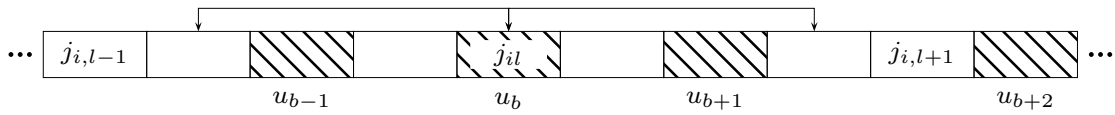


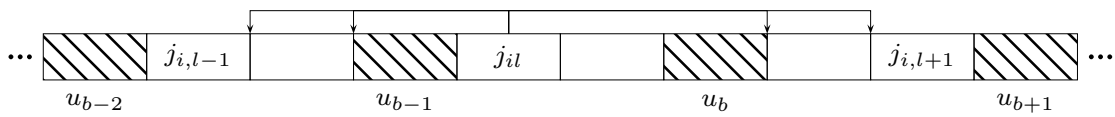
Figure 4.43 shows that swap partners of j_{il}, if $j_{il} = u_b$, need to be before u_{b-1} and behind u_{b+1} in the sequence of job levels.

Figure 4.43: Example of a Chen across block swap if j_{il} is a block border



The possible across block insertion moves are displayed in Figures 4.44, 4.45, 4.46 and 4.47. Nowicki across block insertion moves allow a job level to be inserted right before u_{b-1} and directly behind u_b.

Figure 4.44: Example of a Nowicki across block insertion if j_{il} is not a block border

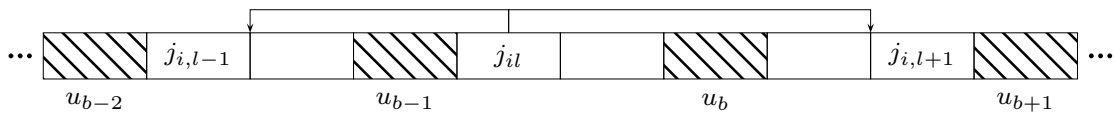


Chen across block moves do not allow a level to overtake the positions of the border job levels u_{b-1} and u_b of its block, i.e. the insertion position needs to be lower than or

[25] See Nowicki / Smutnicki (1996): *Fast taboo search for the job shop problem*, p. 164.

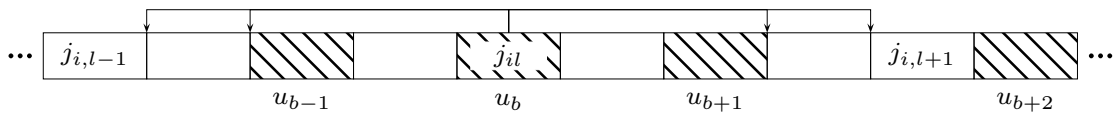
equal to $u_{b-1} - 1$, or greater than or equal to $u_b + 1$. An example is provided in Figure 4.45.

Figure 4.45: Example of a Chen across block insertion if j_{il} is not a block border



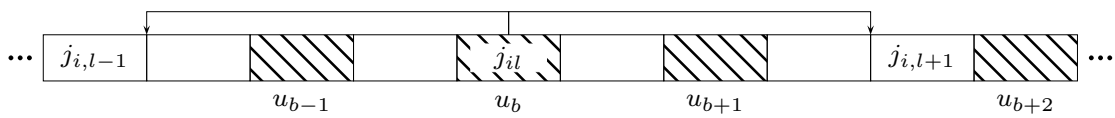
Nowicki across block insertion moves of a job level il are possible to the positions of u_{b-1} and u_{b+1} if il is also a block border u_b.

Figure 4.46: Example of a Nowicki across block insertion if j_{il} is a block border



Chen across block insertion moves in such a case do not allow job level il to be inserted into the positions u_{b-1} and u_{b+1}, as shown in Figure 4.47.

Figure 4.47: Example of a Chen across block insertion if j_{il} is a block border



Chen / Pan / Wu (2007) used swap moves just for members of the same block (intra block swap) for a re-entrant flow shop scheduling problem.[26] Dell'Amico / Trubian (1993) proposed to use only moves inside a block. According to Dell'Amico / Trubian (1993), across block swaps cannot improve the makespan. Within this definition, the moves to the block limits are intra block moves.[27] Nowicki / Smutnicki (1996) used insertion moves to positions that are not in the same block as the job level to be inserted.[28] According to Nowicki / Smutnicki (1996), intra block insertion moves cannot improve the makespan. Inserting at the position of the block limits is considered an across block move.

[26] See Chen / Pan / Wu (2007): *Reentrant flow-shops and hybrid tabu search*, p. 356.
[27] See Dell'Amico / Trubian (1993): *Applying tabu search to the job-shop scheduling problem*, p. 243.
[28] See Nowicki / Smutnicki (1996): *Fast taboo search for the job shop problem*, p. 164.

4.6.4 Computational Experiments

The following enumeration is used in Tables 4.26 and 4.27 to evaluate the neighborhoods presented in Section 4.6:

1 Level swap without considering blocks,
2 Chen intra block level swap,
3 Nowicki intra block level swap,
4 Chen across block level swap,
5 Nowicki across block level swap,
6 Level insertion without considering blocks,
7 Chen intra block level insertion,
8 Nowicki intra block level insertion,
9 Chen across block level insertion,
10 Nowicki across block level insertion,
J1 Job swap,
J2 Job insertion.

The neighborhoods are tested by applying the best neighbor algorithm to the SIROJ and SIROL solutions on the test instance sets Inc_1, Inc_3 and Inc_4. The test results in Table 4.26 show the results for small test problems with two to five jobs and levels per job as well as $m \in \{2, 5, 6, 10\}$ machines. The number of jobs and machines for large test instances can take the values 10, 20, 30, 40 and 50, and the number of levels per job is either 5 or 10.[29] Ten instances are generated for each combination within the small and large problems of the sets Inc_1, Inc_3 and Inc_4. Test set Inc_3 includes only problem sizes with $L \geq 5$, and set Inc_4 requires the number of levels L per job to be greater than or equal to 10.[30] The processing times greater than 0 are uniformly distributed random numbers between 1 and 99. The relative makespan improvement of the initial solution by applying the best neighbor algorithm is calculated by:

$$\Delta C_{max}^{init} = 1 - \frac{C_{max}(\pi_{BN})}{C_{max}(\pi_{init})}.$$

$C_{max}(\pi_{BN})$ is the makespan value of the permutation, π_{BN}, after applying the best neighbor algorithm. $C_{max}(\pi_{init})$ is the makespan of the initial solution. The average values of ΔC_{max}^{init} are displayed in Tables 4.26 and 4.27.

The mean relative makespan reduction by applying level swap moves is, in most cases, lower than the improvement values of the corresponding insertion moves for small test

[29] See Table B.2 in Appendix B (p. 204) for detailed information on the problem sizes.
[30] See Table B.1 in Appendix B (p. 203) for an overview of missing operations.

problems, which is shown in Table 4.26. Swap moves only deliver better average results for the level moves without block criteria (in test set Inc_1) and the Nowicki across block moves (with SIROL initialization in Inc_1 and SIROJ initialization in Inc_3). Job insertion moves also deliver better results on average than job swaps. In Chen block neighborhoods, intra block moves lead to higher mean average makespan improvements than across block moves and vice versa for the Nowicki block definition. The Nowicki across block moves deliver better results than the Chen intra block moves for small problems.

Table 4.26: Mean makespan deviations ΔC_{max}^{init} [%] of best neighbors for small problems

$\mathcal{N}$	Inc_1		Inc_3	
	SIROJ	SIROL	SIROJ	SIROL
1	3.08	13.39	3.89	9.05
2	2.73	10.76	3.28	7.82
3	1.01	1.20	1.31	1.58
4	1.29	9.00	2.22	5.34
5	3.08	13.39	3.89	9.05
6	3.26	13.26	4.09	10.29
7	2.85	11.08	3.37	8.76
8	1.38	1.85	1.56	2.31
9	1.65	11.16	2.60	8.92
10	3.11	13.25	3.87	10.26
J1	5.09	4.89	3.57	3.41
J2	5.28	16.15	3.84	16.20

The results for large test instances are shown in Table 4.27. The relation between Chen and Nowicki block moves is similar to the results for small test instances. Also, here Nowicki across block moves are preferred over the other block criteria moves. The highest rates of improvement are achieved by job moves. Job swaps are most effective if the initial solution is based on separated levels. Job insertion moves perform better on initial random solutions with mixed levels.

Table 4.27: Mean makespan deviations ΔC_{max}^{init} [%] of best neighbors for large problems

$\mathcal{N}$	Inc_1		Inc_3		Inc_4	
	SIROJ	SIROL	SIROJ	SIROL	SIROJ	SIROL
1	0.85	5.04	1.16	5.60	1.04	4.20
2	0.53	2.41	0.64	2.67	0.54	2.14
3	0.30	0.35	0.39	0.48	0.34	0.43
4	0.80	5.04	1.14	5.59	1.03	4.20
5	0.85	5.04	1.16	5.60	1.04	4.20
6	0.78	3.21	0.99	3.74	0.81	2.90
7	0.55	2.40	0.66	2.63	0.56	2.12
8	0.33	0.43	0.44	0.59	0.38	0.51
9	0.74	3.21	0.97	3.74	0.81	2.90
10	0.78	3.21	0.98	3.74	0.81	2.90
J1	1.63	1.42	1.80	1.59	1.59	1.40
J2	1.40	6.64	1.54	6.88	1.36	6.27

The computation times are given for the largest tested problem size, 50 jobs, 10 levels and 50 machines, in Table 4.28. The differences in computation time between the different neighborhoods are highest for this problem size. The computation times are compared for swaps and insertion neighborhoods, not considering block criteria, Nowicki across block moves and the two job neighborhoods. Chen block moves and Nowicki intra block moves are not considered because they deliver weak average makespan results. The computation times with the best neighbor algorithm are higher for insertion neighborhoods than for the corresponding swap neighborhoods. A higher number of missing operations leads to lower computation times for single level moves. The application of the Nowicki block criteria leads to a reduction in the computation time for insertion moves. The effect for swap moves is low and even increased computation times occur for some cases.

Table 4.28: Average computation times [s] of best neighbor algorithms

$\mathcal{N}$	Inc_1		Inc_3		Inc_4	
	SIROJ	SIROL	SIROJ	SIROL	SIROJ	SIROL
1	54.50	24.10	39.50	24.90	34.00	21.10
5	41.30	32.60	41.40	27.10	39.80	28.00
6	248.70	183.80	249.70	183.00	135.60	177.60
10	91.90	130.20	91.20	93.00	81.50	91.70
J1	1.80	1.50	1.60	1.60	1.40	1.50
J2	32.00	40.80	31.60	40.80	31.60	37.10

The makespan reductions led to the decision to omit the Chen block neighborhoods. The neighborhoods examined for the configuration of the VNS in section 4.7 are swaps and insertions for complete jobs and single levels without block criteria, since they achieve the highest values of makespan improvement. The effects of using Nowicki across block moves instead of level swaps and insertions without block criteria in the preferred neighborhood hierarchies of Section 4.7 are examined in subsection 4.7.6.

4.7 Improvement Methods

Improvement methods use an initial solution to search for better solutions in predefined neighborhoods. The metaheuristic improvement methods examined are variable neighborhood search, simulated annealing and tabu search.

4.7.1 Simple Local Search Algorithms

Two common local search algorithms are the first improvement and the best neighbor algorithms.[31] Both are often integrated in metaheuristic solution methods like GRASP[32], particle swarm optimization[33] and simulated annealing[34] for different flow shop problems. Both are trajectory methods, applying neighborhood moves to an incumbent solution. The first improvement algorithm evaluates neighbors in a predefined neighborhood $\mathcal{N}$ in a random order. The first solution found, which improves the initial solution, is accepted as the new solution. The procedure is shown in algorithm 4.9. The initial solution is kept, if no improvement can be found. The improvement criterion for the RPFS is the

[31] See Widmer / Hertz (1989): *A new heuristic method for flow shops*, p. 190.
[32] See Ruiz / Stützle (2007): *Iterated greedy algorithm*, p. 2037.
[33] See Tseng / Liao (2008): *Particle swarm optimization for lot-streaming*, p. 3105.
[34] See Naderi / Zandieh / Roshanaei (2009): *Scheduling hybrid flowshops with sequence dependent setup times*, p. 1189.

makespan of a schedule. The first improvement is considered to shorten the computation time in local search phases of different algorithms.[35]

Recalling the method introduced in Chapter 2, the implementation of the first improvement algorithm is given by algorithm 4.9.

Algorithm 4.9 First improvement

Data: n, L, m and $p^i_{lk} \forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m$.
(1) *Initial solution:* Result of a constructive heuristic π_{init}.
(2) *Update solution:* $\pi_{best} \leftarrow \pi_{init}$.
(3) Identify a list of all t_{max} valid moves in neighborhood $\mathcal{N}(\pi_{init})$.
(4) $t \leftarrow 1$.
while $t \leq t_{max}$ **do**
  5) *Update solution:* $\pi_t \leftarrow \pi_{init}$.
  (6) Perform a random move from the list of moves.
  if $C_{max}(\pi_t) < C_{max}(\pi_{best})$ **then**
    (7) $\pi_{best} \leftarrow \pi_t$ **and** $t \leftarrow t_{max}$.
  end if
  (8) Delete the move from the list of valid moves.
  (9) $t \leftarrow t + 1$.
end while

The best neighbor algorithm, shown in algorithm 4.10, evaluates all valid moves in a predefined neighborhood $\mathcal{N}$. The solution with the lowest makespan is accepted if it improves the initial solution.

[35] See Ishibuchi / Yoshida / Murata (2003): *Memetic algorithms for permutation flowshops*, p. 205.

Algorithm 4.10 Best neighbor

Data: n, L, m and $p^i_{lk}\,\forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m$
(1) *Initial solution:* Result of a constructive heuristic π_{init}.
(2) *Update solution:* $\pi_{best} \leftarrow \pi_{init}$.
(3) Identify a list of all t_{max} valid moves in neighborhood $\mathcal{N}(\pi_{init})$.
(4) $t \leftarrow 1$
while $t \leq t_{max}$ **do**
  (5) *Update solution:* $\pi_t \leftarrow \pi_{init}$
  (6) Perform move t in the list of moves
  if $C_{max}(\pi_t) < C_{max}(\pi_{best})$ **then**
    (7) $\pi_{best} \leftarrow \pi_t$
  end if
  (8) $t \leftarrow t + 1$
end while

Both local search mechanisms are tested in different calibrations of a VNS in Section 4.7.2.

4.7.2 Variable Neighborhood Search

The VNS applied to the RPFS is based on different neighborhood setups. The available neighborhoods are:

- Swaps of job levels without block criteria,
- Insertion moves of single job levels without block criteria,
- Nowicki across block swaps of single job levels,
- Nowicki across block insertion moves of single job levels,
- Swaps of complete jobs,
- Insertion moves of complete jobs.

The block criteria do not apply to complete jobs, since they are multiply represented by their levels within the permutation.

The various neighborhoods are used in different hierarchies. The criteria for selecting the neighborhoods are:

1. Job moves or level moves first,
2. Swap moves or insertion moves first,

3. Level moves without considering block criteria or Nowicki across block moves.

The first two characteristics of neighborhood hierarchies are examined in subsection 4.7.5. The third point is investigated in subsection 4.7.6.

The VNS was described first by MLADENOVIĆ / HANSEN (1997).[36] and basically consists of two alternating phases. One phase is called shaking and is used to escape local optima in order to find another, better, local optimum or the global optimum. The other phase applies a local search method to identify local optima. A basic overview of the method is shown in Algorithm 4.11.

Algorithm 4.11 Variable neighborhood search

Data: n, L, m and $p_{lk}^{i} \forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m$
(1) *Initial solution:* Result of a constructive heuristic π_{init}.
(2) *Update solution:* $\pi_{best} \leftarrow \pi_{init}$
(3) $t \leftarrow 1$
while $t \leq t_{max}$ **do**
  (4) *Shaking:*
  Select random neighbor $\pi \in \mathcal{N}_t(\pi_{best})$
  (5) *Local Search:*
  First improvement algorithm or best neighbor algorithm in $\mathcal{N}_t(\pi)$ to obtain the local search solution π'.
  if $C_{max}(\pi') < C_{max}(\pi_{best})$ **then**
    (6a) $\pi_{best} \leftarrow \pi'$ **and** $t \leftarrow 1$
  else
    (6b) $t \leftarrow t + 1$
  end if
end while

Initial Solution

The initial solution for the VNS is generated by one of the suggested constructive heuristics, i.e. NEHJ, SIROJ, STPTL or SIROL.

Shaking

The shaking phase provides the possibility to leave local optima in order to obtain an even better solution than the current best in an additional local search phase. The shaking applies if no improvement is found within a selected neighborhood. The next neighborhood is selected, and a random valid move within this neighborhood is made.

[36] See MLADENOVIĆ / HANSEN (1997): *Variable neighborhood search*, pp. 1097–1098.

The different $a = 1, \ldots, 8$ neighborhood hierarchies $\mathcal{N}_t^a$ are presented in Table 4.29. "J-swap" (neighborhood $\mathcal{N} = J1$) and "J-insert" ($\mathcal{N} = J2$) are neighborhood definitions that require moves of all levels of the selected jobs. "L-swap" (neighborhood $\mathcal{N} = 1$) and "L-insert" ($\mathcal{N} = 6$) are single level move neighborhoods. Block criteria are not considered for the level moves for the tests in subsection 4.7.5.

Table 4.29: Examined neighborhood hierarchies

t	$\mathcal{N}_t^1$	$\mathcal{N}_t^2$	$\mathcal{N}_t^3$	$\mathcal{N}_t^4$	$\mathcal{N}_t^5$	$\mathcal{N}_t^6$	$\mathcal{N}_t^7$	$\mathcal{N}_t^8$
1	J-swap	J-insert	J-swap	J-insert	L-insert	L-swap	L-insert	L-swap
2	J-insert	J-swap	L-swap	L-insert	L-swap	J-swap	J-insert	J-swap
3	L-swap	L-insert	J-insert	J-swap	J-insert	L-insert	L-swap	L-insert
4	L-insert	L-swap	L-insert	L-swap	J-swap	J-insert	J-swap	J-insert

Hansen / Mladenović (1997) preferred to start in neighborhoods that are defined by relatively small changes in the current solution.[37] It is disputable whether insertion or swap moves imply larger changes in a given solution. Insertion moves change the sequence position of a minimum of two job levels in the permutation, but may also affect more than one sequence position since they can shift several members of the permutation to the right or to the left. Swap moves of two permutation members, on the other hand, are limited to two changes in the permutation position. Job moves define neighborhoods that yield relatively large changes in the solution structure, since multiple job levels are inserted or swapped. The issue of dividing a permutation into blocks and the resulting neighborhoods of moves within a block or to another block are not directly linked to a larger or smaller neighborhood. There can only be a small number of valid moves due to tight technological move limits of a permutation member.

Integrated Local Search Algorithm

The shaking is followed by a local search phase. Two different methods are tested within this local search phase:

- First improvement,
- Best neighbor.

One of the local search methods is applied on the shaking solution. There is no change in the shaking solution if no improvement is found. In this case the VNS changes to the

[37] See Hansen / Mladenović (1997): *Variable neighborhood search for the p-median*, p. 211.

next neighborhood $\mathcal{N}_{t+1}$ and repeats the shaking on the best solution, π_{best}, found at that point.

Iterations and Termination

The algorithm terminates if the t_{max}th neighborhood is reached and no better solution compared to the current best is found.

4.7.3 Tabu Search

The tabu search looks for the best neighbor in each iteration. Neighbors that are part of the tabu list are not accepted if they do not lead to large improvements in the objective value. The tabu list stores forbidden moves to prevent the algorithm from becoming stuck in local optima. The size of such tabu lists and the tabu tenure can be fixed or variable. The calibration of the implemented tabu search is based on Nowicki / Smutnicki (1996).[38] The initial solutions are created with one of the constructive heuristics NEHJ, SIROJ, STPTL or SIROL.

The maximum number of tabu search iterations is limited to $t_{max} = 5000$ for every problem. The calibration of the tabu search, shown in Figure 4.12, is explained in the following.

[38] See Nowicki / Smutnicki (1996): *Fast taboo search for the job shop problem*, pp. 161–167.

Algorithm 4.12 Tabu search

Data: n, L, m, A and $p_{lk}^{i}\,\forall i = 1, \ldots, n; l = 1, \ldots, L; k = 1, \ldots, m$.
(1) *Initial solution:* Result of a constructive heuristic π_{init}.
(2) *Update solution:* $\pi_{best} \leftarrow \pi_{init}$, $\pi \leftarrow \pi_{init}$.
(3) $t \leftarrow 1$.
while $t \leq t_{max}$ **do**
  (4) *Create candidate list:* All moves to the permutations $\pi' \in \mathcal{N}(\pi)$ **and** evaluate the candidates.
  (5) *Select Candidate:* Choose the candidate π_t with the minimum $C_{max}(\pi_t)$ among all candidates.
  if Candidate is a tabu **and** Aspiration function value $F^{asp} \leq C_{max}(\pi_t)$. **then**
    if Tabu list size > 0 **then**
      (6a) $C_{max}(\pi_t) \leftarrow A$ **and** go back to (5).
    else
      (6b) Remove tabu list row 1 **and** restore candidate list and the C_{max} values.
    end if
  else
    (7) $\pi \leftarrow \pi_t$ **and** $t \leftarrow t + 1$ **and** Add move to tabu list.
    if $C_{max}(\pi) < C_{max}(\pi_{best})$ **then**
      (8) $\pi_{best} \leftarrow \pi$.
    end if
    if Tabu list size > 8 **then**
      (9) Remove tabu list row 1.
    end if
  end if
end while

Tabu list

The tabu list size is chosen to be static with eight rows. Nowicki/Smutnicki (1996) used a static tabu list of the size eight.[39] This leads to a tabu tenure for each row in the tabu list of eight iterations. The tabu moves are recorded by the following scheme:

For insertion moves, the two items registered within the tabu list are:

1. The job or job level that has been moved,
2. The job or job level that took its position within the permutation.

[39] See Nowicki/Smutnicki (1996): *Fast taboo search for the job shop problem*, p. 170.

An insertion move is tabu if it changes the sequence position of any of the recorded pairs of job levels or jobs in the tabu list. The entry for job swaps is the two participating jobs. The pair of participating job levels is added to the tabu list in case of a level swap neighborhood. A move is tabu if there is a pair of jobs or job levels that change their sequence position by applying the move and are identical to a pair of jobs or job levels within the tabu list. NOWICKI/SMUTNICKI (1996) used Nowicki across block level insertion moves (neighborhood $\mathcal{N} = 10$). These moves and job insertion moves ($\mathcal{N} = J2$) are the neighborhoods tested in subsection 4.7.5.

Aspiration function

Nevertheless, a tabu move can be accepted if a certain aspiration criterion is fulfilled. The aspiration function within this implementation of tabu search compares the makespan, $C_{max}(\pi_t)$, of the tabu move in iteration t and the tabu permutation, with so-called income and outcome values. Considering the tabu solution π_t of an iteration t, the income makespan value is defined as $C^I_{max}(\pi_t) = C_{max}(\pi_{t-1})$, i.e. the makespan value of the previous iteration. The outcome is defined as $C^O_{max}(\pi_t) = C_{max}(\pi_{t+1})$, i.e. the makespan value of the next iteration. Two values $F_I(C_{max}(\pi_t))$ and $F_O(C_{max}(\pi_t))$ are necessary to decide whether or not a tabu move should be accepted via the aspiration function. To determine the income value F_I and the outcome value F_O, the income and outcome values of the relevant iterations resulting in a permutation with an objective value of $C_{max}(\pi_{t'}) = C_{max}(\pi_t)$ are compared. The maximum values of these incomes $C^I_{max}(\pi_{t'})\ \forall t' = t' = 2, \ldots, t$ and outcomes $C^O_{max}(\pi_t)\ \forall t' = 1, \ldots, t-1$ are the values $F_I(C_{max}(\pi_t))$ and $F_O(C_{max}(\pi_t))$. The aspiration function chooses the minimum of value $F_I(C_{max}(\pi_t))$ and $F_O(C_{max}(\pi_t))$ by equation (4.31) and compares it to the objective value $C_{max}(\pi_t)$ of the tabu permutation. If the makespan value obtained by the tabu move is lower than the aspiration value, then the tabu move is accepted and the iteration counter t is increased by one, otherwise the next candidate will be chosen.

A summary of the calculations for the aspiration function value F^{asp} is given by the following equations (4.27)-(4.31).

$$C^I_{max}(\pi_{t'}) = C_{max}(\pi_{t'-1}), \tag{4.27}$$

$$F_I(C_{max}(\pi_t)) = \min_{t'=2,\ldots,t} \left\{ C^I_{max}(\pi_{t'}) \right\}, \tag{4.28}$$

$$C^O_{max}(\pi_{t'}) = C_{max}(\pi_{t'+1}), \tag{4.29}$$

$$F_O(C_{max}(\pi_t)) = \min_{t'=1,\ldots,t-1} \left\{ C^O_{max}(\pi_{t'}) \right\} \tag{4.30}$$

$$F^{asp} = \min\{F_I; F_O\}. \tag{4.31}$$

The implemented algorithm for finding the aspiration value F is shown with the pseudo code 4.13.

Algorithm 4.13 Finding the value of the aspiration function

Data: current iteration number t, $C_{max}(\pi_t)\,\forall t' = 1, \ldots, t$, A.
(1) $F_I \leftarrow A$, $F_O \leftarrow A$
(2) $t' \leq 1$
while $t' \leq t$ **do**
  if $C_{max}(\pi_{t'}) = C_{max}(\pi_t)$ **then**
    if $t' > 1$ **and** $C_{max}(\pi_{t'-1}) < F_I$ **then**
      (3a) $F_I \leftarrow C_{max}(\pi_{t'-1})$
    end if
    if $t' < t$ **and** $C_{max}(\pi_{t'+1}) < F_O$ **then**
      (3b) $F_O \leftarrow C_{max}(\pi_{t'+1})$
    end if
  end if
  (4) $t' \leftarrow t' + 1$
end while
(5) $F^{asp} \leftarrow \min\{F_I; F_O\}$

If a makespan value is lower than the aspiration value, then the tabu move is accepted and the next iteration begins.

4.7.4 Simulated Annealing

Simulated annealing uses another scheme to avoid being stuck in a local optimum. Worse objective values than in previous iterations are accepted with a certain probability. This probability declines with the number of iterations. The chosen implementation of simulated annealing is based on Osman / Potts (1989)[40] and summarized in algorithm 4.14. The initialization is done with one of the suggested constructive methods used also for the VNS and the TS. The temperature in each iteration is T^t.

[40] See Osman / Potts (1989): *Simulated annealing for permutation flow-shop scheduling*, pp. 551–557.

Algorithm 4.14 Simulated annealing

Data: n, L, m, T^1, $\overline{K}$ and $p^i_{lk}\,\forall i = 1,\ldots,n; l = 1,\ldots,L; k = 1,\ldots,m$.
(1) *Initial solution:* Result of a constructive heuristic π_{init}.
(2) *Update solution:* $\pi_{best} \leftarrow \pi_{init}$
while $T^t \geq T^{min}$ **do**
  (3) *Create random neighbor:* $\pi' \in \mathcal{N}(\pi)$.
  if $C_{max}(\pi') < C_{max}(\pi)$ **then**
    (4a) $\pi \leftarrow \pi'$.
    if $C_{max}(\pi') < C_{max}(\pi_{best})$ **then**
      (4b) $\pi_{best} \leftarrow \pi'$
    end if
  else
    (4c) Create random number $0 \leq \overline{r} \leq 1$.
    if $\overline{r} \leq \overline{P} = e^{-\frac{C_{max}(\pi)-C_{max}(\pi')}{T^t}}$ **then**
      (4d) $\pi \leftarrow \pi'$.
    end if
  end if
  (5) $T^{t+1} \leftarrow T^t / \left(1 + \overline{K} \cdot T^t\right)$.
end while

Annealing scheme

The initial temperature is calculated by $T^1 = \sum_{i=1}^{n}\sum_{l=1}^{L}\sum_{k=1}^{m} p^i_{lk} / (5 \cdot n \cdot L \cdot m)$. The algorithm terminates when $T^t < T_{min} = 1$. The number of iterations per temperature state is one. The annealing scheme by Osman/Potts (1989) is based on the number of temperature states, which depends on the initial temperature. The annealing function for the re-entrant permutation flow shop in this thesis is based on the calculation of annealing parameters of Osman/Potts (1989), but its parameters K and $\overline{K}$ depend on the neighborhood type. The first annealing parameter for level move neighborhoods is calculated by:

$$K = \max\{3300 \log(n \cdot L - L + 1) + 7500 \log m - 18250; 2000\}.$$

The calculation of the parameter is slightly different for job move neighborhoods:

$$K = \max\{3300 \log n + 7500 \log m - 18250; 2000\}.$$

The parameter K for job moves is calculated with $\log n$ because there are n different positions in a job permutation and every job is allowed to change to every position in every possible permutation. A level permutation consists of $n \cdot L$ positions, but not every level is allowed to change its position arbitrarily. This limitation of moves grows with a higher number of levels per job L. Therefore, L is subtracted from $n \cdot L$ in $\log(n \cdot L - L + 1)$ if the neighborhood is a level move neighborhood. The second annealing parameter is defined by:

$$\overline{K} = \frac{T^1 - T^{min}}{(K-1) \cdot T^{init} \cdot T_{min}}$$

The annealing scheme for each iteration t is $T_{t+1} = T / \left(1 + \overline{K} \cdot T_t\right)$.

Probabilistic function

The probabilistic function generates a probability $\overline{P}$, which is the probability of accepting a worse value of the makespan compared to the previous solution:

$$\overline{P} = e^{-\frac{C_{max}(\pi) - C_{max}(\pi')}{T}}.$$

Additionally, a uniformly distributed continuous random number $0 \leq \overline{r} \leq 1$ is generated. For $\overline{r} \leq \overline{P}$, the new solution is accepted and the objective value updated. The probabilistic function is suggested by Osman / Potts (1989), Ogbu / Smith (1990) and Ogbu / Smith (1991) and for permutation flow shop problems.[41]

Neighborhoods

The neighborhoods used to generate new solutions for the experiments in subsection 4.7.5 are the level insertion ($\mathcal{N} = J2$) and job insertion neighborhood ($\mathcal{N} = 6$), because Osman / Potts (1989) recommended insertion neighborhoods. Block criteria are not considered.

4.7.5 Computational Experiments

This subsection compares different configurations of the variable neighborhood search, tabu search and simulated annealing for the minimization of the makespan in re-entrant permutation flow shops. The aim of the computational experiments is to find an appropriate configuration for the VNS and to find out if it is superior to the TS and SA.

[41] See Osman / Potts (1989): *Simulated annealing for permutation flow-shop scheduling*, p. 553, Ogbu / Smith (1990): *Simulated annealing for the n/m/C max flowshop problem*, p. 64 and Ogbu / Smith (1991): *Simulated annealing for the permutation flowshop problem*, p. 244.

The VNS configurations are denominated by "VNSa" and "BN" for a best neighbor local search or "FI" for a first improvement local search. a stands for the number, suggested in subsection 4.7.2, of the neighborhood hierarchies for the VNS. The tabu search is tested with a job insertion ($\mathcal{N} = J2$) and a Nowicki across block level insertion neighborhood ($\mathcal{N} = 10$). The simulated annealing approach uses a simple level insertion ($\mathcal{N} = 6$) and a job insertion neighborhood ($\mathcal{N} = J2$). All metaheuristics are tested with four different initialization methods (NEHJ, SIROJ, STPTL and SIROL).

Test instances

The parameters of the problem size are $n \in \{2, 3, 4, 5\}$, $L \in \{2, 3, 4, 5\}$, and $m \in \{2, 5, 6, 10\}$ for small problems and $n \in \{20, 40\}$, $L \in \{5, 10\}$, and $m \in \{20, 40\}$ for large problems.[42] The processing times are uniformly distributed random numbers between 1 and 99. Ten test instances are generated for each problem size. Missing operations are generated about the Inc_1 and Inc_3 schemes.[43]

Results

Table 4.30 shows the average makespan deviations between all solutions calculated by the metaheuristics and the MIP solutions for the small test instances. The aim of this evaluation is to determine a method to use for small problem sizes, by taking into account the deviation to the MIP solution. The MIP solutions are obtained with CPLEX 12.4 with a computation time limit of 1 hour. The makespan deviation is defined as:

$$\Delta C_{max}^{MIP} = \frac{C_{max}\left(\pi_{meta}\right)}{C_{max}\left(\pi_{MIP}\right)} - 1.$$

$C_{max}\left(\pi_{MIP}\right)$ stands for the makespan value of a MIP solution. $C_{max}\left(\pi_{meta}\right)$ is the makespan with the permutation π_{meta} calculated by a metaheuristic. The VNS4 BN delivers the best results for the small instances of test set Inc_1 independently from the initialization method used, and for the Inc_3 instances if the initial solution is created with the SIROL rule. The VNS3 BN has three times the lowest average ΔC_{max}^{MIP} for small Inc_3 instances. These results imply the use of either the VNS3 BN or VNS4 BN for small problem sizes. The computation times are negligible for these problem sizes. The initialization with the STPTL rule delivers, on average, the best results for the VNS3 BN and VNS4 BN. There is a tendency of higher deviations to the MIP solutions in the Inc_3 set than in the Inc_1 set. This means that a higher number of missing operations makes it harder for the heuristic solution approaches to find a near optimal or optimal

[42] See Table B.2 in Appendix B (p. 204) for detailed information on the problem sizes.
[43] See Table B.1 in Appendix B (p. 203) for an overview of missing operations.

solution.

Table 4.30: Average makespan deviation ΔC_{max}^{MIP} [%] for small problems

	Inc_1				Inc_3			
	NEHJ	SIROJ	STPTL	SIROL	NEHJ	SIROJ	STPTL	SIROL
VNS1 BN	2.02	1.90	1.63	2.88	4.34	4.87	3.23	4.95
VNS2 BN	2.05	1.93	1.66	2.89	4.85	4.84	3.38	6.37
VNS3 BN	2.11	1.91	1.69	2.76	4.09	4.15	2.63	4.92
VNS4 BN	1.84	1.79	1.61	2.67	4.32	4.51	3.08	4.44
VNS5 BN	2.08	2.21	1.77	3.08	4.84	4.32	3.22	5.48
VNS6 BN	2.03	2.15	1.77	3.32	4.29	4.38	2.98	6.25
VNS7 BN	1.97	2.24	1.72	2.69	5.02	4.70	2.94	4.80
VNS8 BN	1.98	2.46	1.75	3.27	4.72	4.29	3.23	5.98
VNS1 FI	3.27	3.31	2.66	10.10	7.54	8.01	4.08	19.30
VNS2 FI	2.55	2.90	2.20	3.45	5.93	6.34	4.44	7.73
VNS3 FI	3.18	3.42	2.54	8.47	8.70	8.37	4.60	12.73
VNS4 FI	3.43	3.60	2.58	5.46	8.21	8.45	4.53	11.28
VNS5 FI	3.32	4.09	2.64	7.56	6.67	6.41	4.84	11.49
VNS6 FI	3.34	3.61	2.65	6.64	7.14	8.19	4.76	10.84
VNS7 FI	3.90	4.34	2.94	7.73	7.35	7.16	4.72	11.60
VNS8 FI	3.88	4.32	2.89	7.40	7.01	7.25	4.69	11.29
TS $\mathcal{N} = J2$	4.63	4.98	3.18	29.53	10.35	10.62	5.79	36.03
TS $\mathcal{N} = 10$	5.70	7.60	4.32	35.27	9.30	10.54	5.85	47.04
SA $\mathcal{N} = J2$	6.63	7.69	4.97	35.01	11.61	12.02	7.21	37.52
SA $\mathcal{N} = 6$	8.10	10.64	6.42	46.09	12.85	14.92	8.33	58.47
Best	1.84	1.79	1.61	2.67	4.09	4.15	2.63	4.44

The values of mean makespan reduction for large problem sizes are considered (Table 4.31) to get a pre-selection on the constructive methods used for initialization of the metaheuristics. The table shows the average deviation of a solution obtained with a certain method from the best solution over all methods. The values are calculated by:

$$\Delta C_{max}^{best} = \frac{C_{max}\left(\pi_{meta}\right)}{C_{max}\left(\pi_{best}\right)} - 1.$$

The lowest obtained makespan value for a problem instance is denoted with $C_{max}\left(\pi_{best}\right)$. The mean deviations from the best achieved makespan, which are obtained with the STPTL rule, an opening procedure for the metaheuristics, are the lowest for all metaheuristics in the test instance sets Inc_1 and Inc_3. The weakest results for every meta-

heuristic are obtained with the SIROL rule.

The values for the Inc_3 instances are slightly higher than the makespan deviations in the Inc_1 instances.

Table 4.31: Average makespan deviation ΔC_{max}^{best} [%] for large problems

	Inc_1				Inc_3			
	NEHJ	SIROJ	STPTL	SIROL	NEHJ	SIROJ	STPTL	SIROL
VNS1 BN	2.29	2.38	1.29	37.69	5.24	6.02	1.47	32.27
VNS2 BN	2.58	2.71	1.84	25.56	5.29	5.93	1.62	26.26
VNS3 BN	2.28	2.16	1.69	46.03	5.51	5.54	1.73	43.78
VNS4 BN	2.48	2.45	1.78	25.16	5.80	5.35	1.98	24.82
VNS5 BN	2.58	2.73	2.57	50.30	5.22	5.10	2.47	44.27
VNS6 BN	2.87	2.60	2.29	48.26	5.39	5.95	2.25	43.87
VNS7 BN	2.84	2.33	2.14	39.86	5.79	5.12	2.17	38.97
VNS8 BN	2.63	2.60	1.88	50.33	5.29	4.75	2.03	47.68
VNS1 FI	3.30	3.34	2.57	183.62	7.73	7.41	3.13	164.08
VNS2 FI	2.70	3.23	1.87	27.17	6.57	6.82	2.25	27.80
VNS3 FI	3.06	3.06	2.15	166.62	7.65	7.44	2.59	156.29
VNS4 FI	3.17	3.26	2.35	30.57	7.44	7.48	2.46	26.76
VNS5 FI	6.00	6.13	5.69	135.77	9.69	10.26	5.96	115.10
VNS6 FI	4.14	4.08	3.53	44.72	7.84	8.31	3.97	43.88
VNS7 FI	5.27	5.83	4.75	125.86	10.06	9.87	5.29	111.29
VNS8 FI	6.50	6.64	6.01	86.73	10.55	10.62	5.85	73.32
TS $\mathcal{N} = J2$	6.05	6.28	5.07	259.68	11.20	11.79	5.33	234.45
TS $\mathcal{N} = 10$	6.62	6.92	5.92	272.20	11.80	12.30	6.02	245.44
SA $\mathcal{N} = J2$	3.86	4.28	3.07	122.42	8.89	9.36	3.74	117.31
SA $\mathcal{N} = 6$	7.40	7.69	6.63	279.25	12.66	13.14	6.87	255.64
Best	2.28	2.16	1.29	25.16	5.24	4.75	1.47	24.82

The main area of application for heuristics contains large problem sizes; therefore, the results for the large problem sizes are examined in the following. The average makespan reductions are used to compare the different metaheuristic improvement methods and their different configurations. The relative makespan reductions of the metaheuristic solutions compared to the initial solutions are:

$$\Delta C_{max}^{init} = 1 - \frac{C_{max}\left(\pi_{meta}\right)}{C_{max}\left(\pi_{init}\right)}.$$

The mean values of the large instances are shown in Table 4.32.[44] The VNS with neighborhood hierarchy $a = 1$ in combination with a best neighbor local search delivers the highest mean improvements in three cases: two times when the the initial solution is provided by the STPTL rule (for Inc_1 and Inc_3 instances), and once for a initial solution with the NEHJ method for the Inc_3 instances. The other mean makespan reduction values of the VNS1 BN are not far from the best achieved, except for the initialization with the SIROJ rule in test set Inc_3. All metaheuristics deliver the worst results when they are initialized with the SIROL rule. If only VNS configurations with the first improvement local search are considered, then neighborhood hierarchy $a = 2$, i.e. VNS2 FI, is the most promising approach for large instances. The solutions achieved with best neighbor local search are better than those obtained with first improvement. Tabu search and simulated annealing deliver weak results compared to the VNS1 BN and VNS2 FI. The tested job neighborhoods are superior to the level moves for tabu search and simulated annealing.

[44] The average makespan reductions, ΔC_{max}^{init}, for small problems are provided by Table C.1 in Appendix C (p. 206).

Table 4.32: Average makespan reductions ΔC_{max}^{init} [%] for large problems

	Inc_1				Inc_3			
	NEHJ	SIROJ	STPTL	SIROL	NEHJ	SIROJ	STPTL	SIROL
VNS1 BN	4.79	4.96	5.05	63.43	6.61	6.38	5.13	62.04
VNS2 BN	4.52	4.66	4.54	66.51	6.53	6.45	4.98	63.58
VNS3 BN	4.80	5.16	4.69	61.30	6.35	6.77	4.90	58.94
VNS4 BN	4.61	4.87	4.60	66.52	6.08	6.94	4.65	64.11
VNS5 BN	4.51	4.62	3.85	60.15	6.57	7.12	4.21	58.76
VNS6 BN	4.23	4.74	4.13	60.72	6.42	6.39	4.41	58.96
VNS7 BN	4.27	4.98	4.26	62.93	6.07	7.09	4.47	60.32
VNS8 BN	4.47	4.75	4.51	60.19	6.53	7.42	4.62	57.88
VNS1 FI	3.86	4.07	3.88	25.59	4.46	5.18	3.60	25.88
VNS2 FI	4.41	4.18	4.52	66.12	5.43	5.69	4.40	63.12
VNS3 FI	4.08	4.35	4.27	30.04	4.53	5.18	4.09	28.29
VNS4 FI	3.99	4.15	4.06	65.10	4.68	5.12	4.21	63.42
VNS5 FI	1.36	1.50	0.99	38.26	2.69	2.65	0.97	39.50
VNS6 FI	3.09	3.38	2.97	61.52	4.31	4.38	2.81	58.86
VNS7 FI	2.04	1.78	1.85	40.77	2.40	3.03	1.58	40.07
VNS8 FI	0.90	1.02	0.68	50.74	1.94	2.31	1.07	51.00
TS $\mathcal{N} = J2$	1.33	1.37	1.54	6.23	1.43	1.38	1.56	6.55
TS $\mathcal{N} = 10$	0.79	0.78	0.77	2.98	0.90	0.94	0.92	3.55
SA $\mathcal{N} = J2$	3.35	3.23	3.41	40.95	3.47	3.51	3.03	38.14
SA $\mathcal{N} = 6$	0.08	0.07	0.11	1.10	0.15	0.20	0.14	0.60
Best	4.80	5.16	5.05	66.52	6.61	7.42	5.13	64.11

Further information on the solution quality of the tested metaheuristics is provided in Figures 4.48 and 4.49. They compare how often a metaheuristic obtains the best solution among all the tested methods. Additionally, the frequency of obtaining the best solution is shown if only solutions based on an initial STPTL solution are compared. The best solution is most often achieved by the VNS1 BN method. The VNS2 FI delivers most frequently the best solutions among the procedures with short computation times.

Figure 4.48: Frequency of obtaining best solutions for large Inc_1 problems

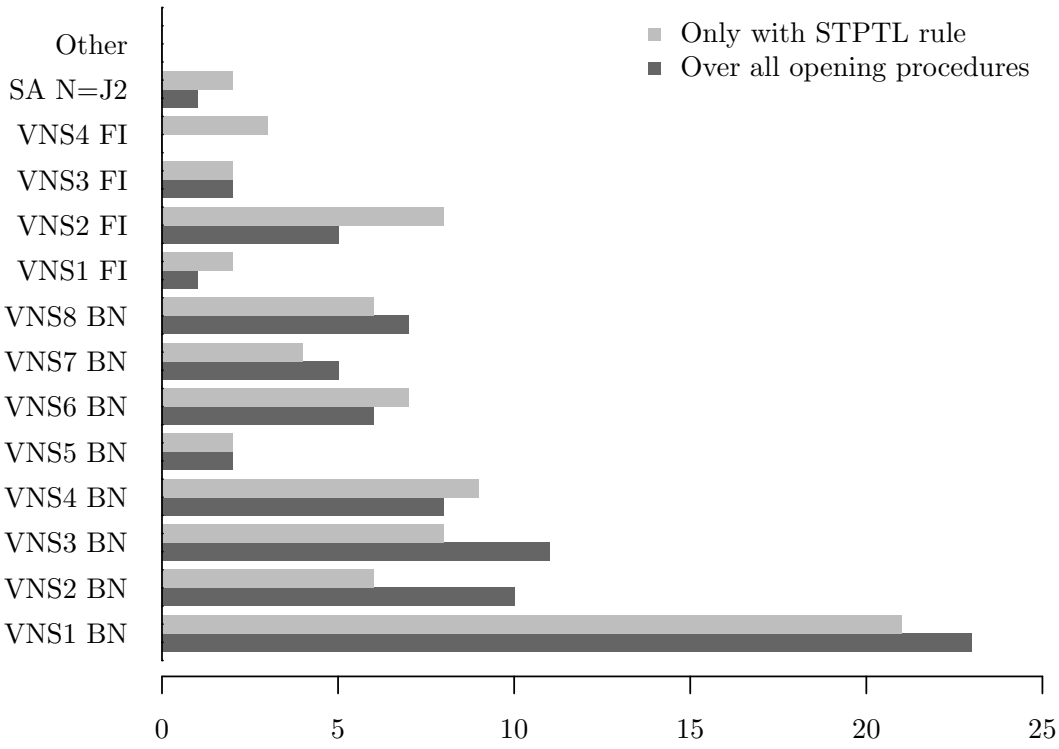
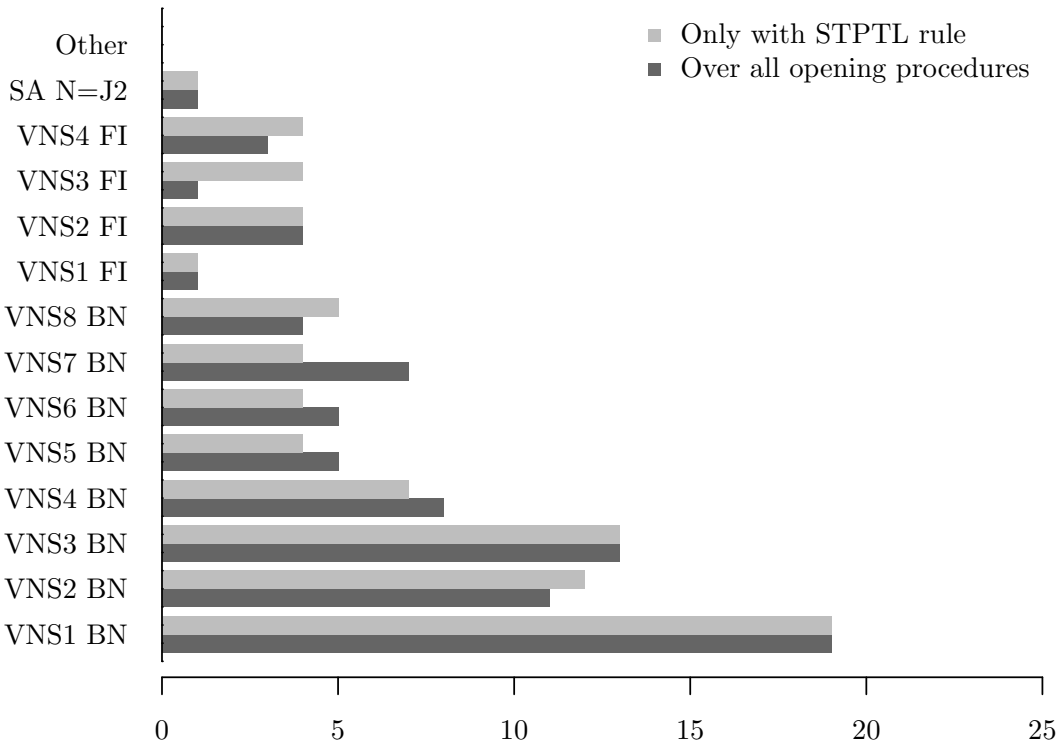


Figure 4.49: Frequency of obtaining best solutions for large Inc_3 problems



Figures 4.50 and 4.51 provide an overview of the computation times of the different improvement methods for Inc_1 instances. The computation times for problems with an increased number of missing operations, i.e. Inc_3 instances, can be seen in Appendix C. The value refers only to the time used to improve an initial solution and the computation times for initialization are not included. The shortest computation times are achieved with the first improvement VNS, tabu search in all configurations and the simulated annealing with level moves. The computation times are higher if the opening procedure is SIROL. This leads to values higher than 15000 seconds for the VNS5 BN and VNS7 BN.

Figure 4.50: Average computation times for large Inc_1 instances (I)

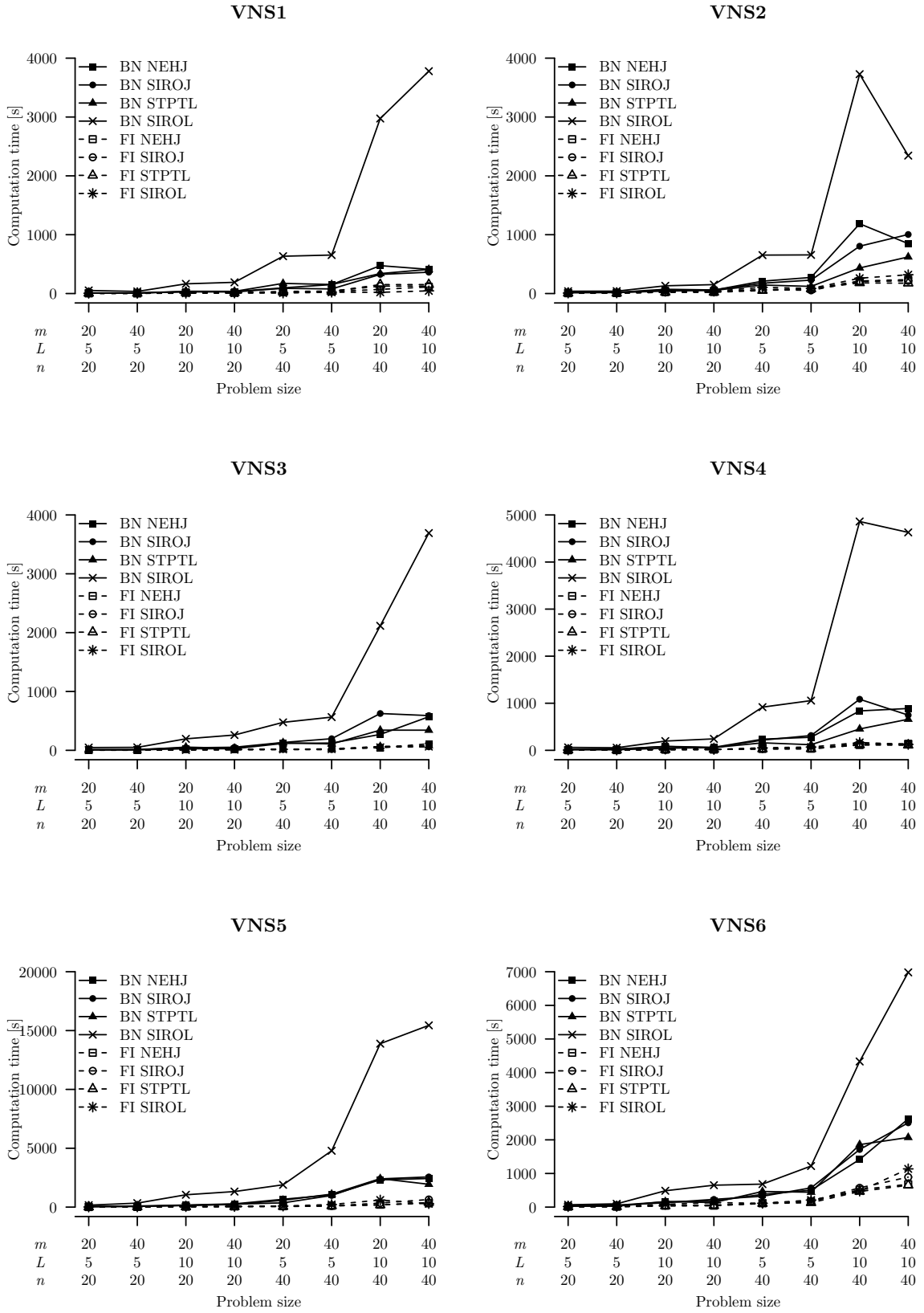
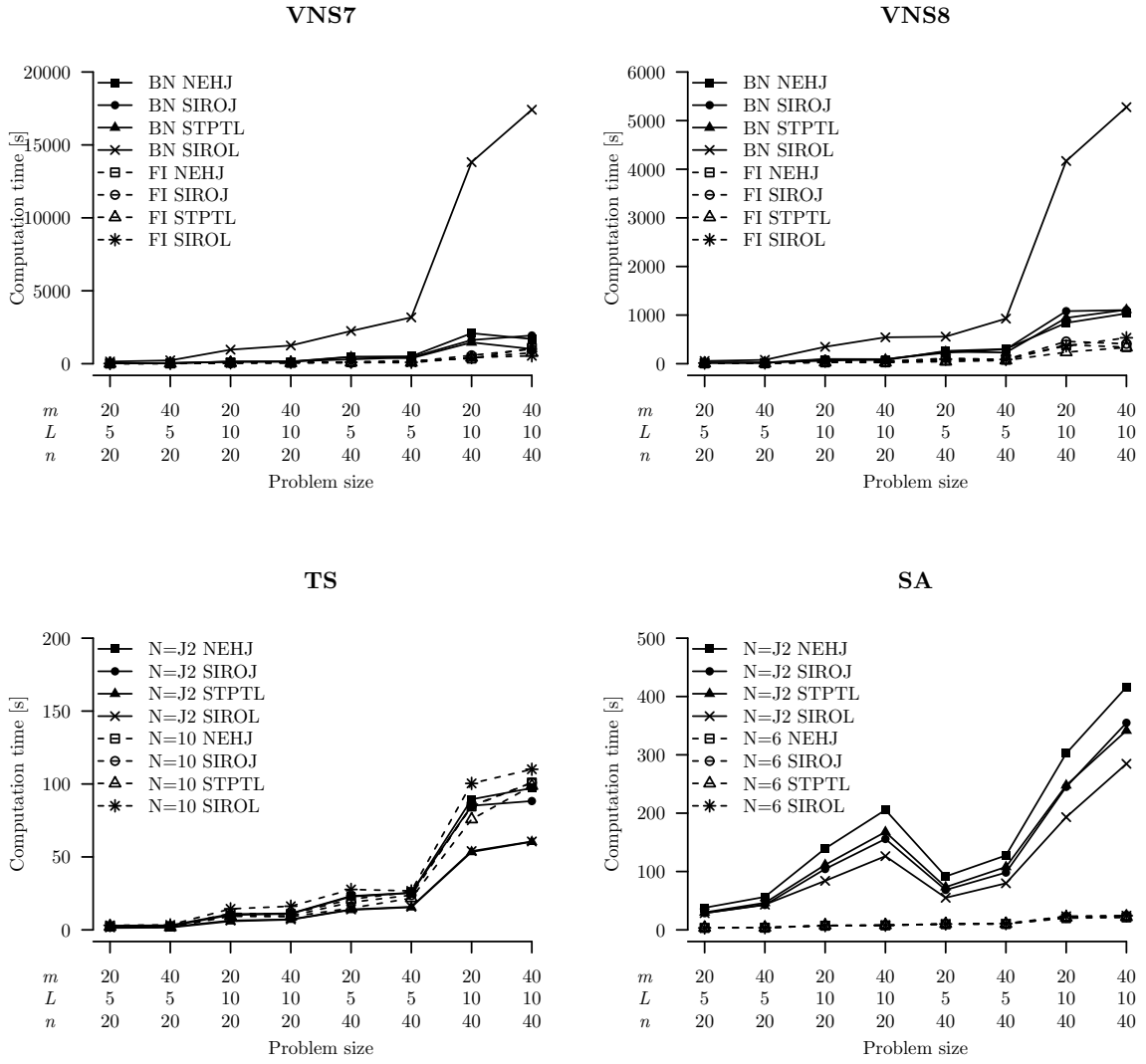


Figure 4.51: Average computation times for large Inc_1 instances (II)



The STPTL rule is suggested for further applications of the investigated metaheuristics for re-entrant permutation flow shops with mixed levels and missing operations, because of the results in this subsection. The VNS, especially its configurations VNS1 BN, is superior in solution quality to the investigated tabu search and simulated annealing configurations. The reason lies in the advanced exploitation of the solution space in re-entrant permutation flow shops by applying job and level moves. The VNS1 BN is further used in subsection 4.7.6 and Chapter 5 to minimize the makespan in re-entrant permutation flow shops. An advantage of an integrated first improvement local search for the VNS is its relatively short computation time. Hence, the VNS2 FI is also suggested for the makespan minimization in re-entrant permutation flow shops and is used in subsection 4.7.6 and Chapter 5.

Chen / Pan / Lin (2008), Chen / Pan / Wu (2008) and Qian et al. (2013b) ran tests with other heuristic solution methods on similar problem sizes for re-entrant permutation flow shop problems. These authors published improvement rates over their NEHJ solutions. Chen / Pan / Wu (2008) suggested a hybrid tabu search. The improvement rates of makespan, ΔC_{max}^{init}, over NEHJ solutions were between 2 and 4 % for test instances similar to the here tested large problems.[45] The hybrid genetic algorithm of Chen / Pan / Lin (2008) delivered improvement rates between 1 and 4.5 % for small problem sizes.[46] Qian et al. (2013b) tested two differential evolution algorithms and the hybrid genetic algorithm of Chen / Pan / Lin (2008) with medium and large problem instances. The average value of ΔC_{max}^{init} over the NEHJ solutions was 1.62 % for the hybrid genetic algorithm, 4.39 % for the first differential evolution algorithm and 5.32 % for the second differential evolution algorithm if only problem instances with $n \geq 20$ are considered.[47] The suggested variable neighborhood search configurations, VNS1 BN / VNS2 FI, deliver average improvement values of 5.50 / 5.10 % (Inc_1) and 7.50 / 6.03 % (Inc_3) for small instances and 4.79 / 4.41 % (Inc_1) and 6.61 % / 5.43 % (Inc_3) for large instances. The VNS configurations achieve obviously better makespan values. The tests by Qian et al. (2013a) were performed on a similar computer system to the one used for this thesis. The computation times of the differential equation algorithms for the large problems sizes were regularly several hours. The suggested variable neighborhood searches with only several hundred seconds are much faster than the approaches of Qian et al. (2013a). The VNS is also superior to the iterated local search, which only uses a level swap neighborhood, as proposed by Hinze / Sackmann (2016).[48]

4.7.6 Improvement of Metaheuristics

This subsection investigates the influence of exchanging the level moves without block criteria with the corresponding Nowicki across block swap and insertion moves for the VNS. The resulting VNS algorithms for the configurations suggested in subsection 4.7.5 are denoted with VNS1 BN* and VNS2 FI*. The tabu search with level moves uses across block swaps ($\mathcal{N} = 5$) instead of across block insertion moves, because of the results on large instances in the neighborhood experiments. The same neighborhood is also tested within the simulated annealing approach. Furthermore, neighborhoods in the tabu search and simulated annealing only using job moves are changed to job swaps ($\mathcal{N} = 11$) instead of job insertion moves. All metaheuristics are initialized with the STPTL rule. The target of the experiment is to find out how these changes in

[45] See Chen / Pan / Wu (2008): *Hybrid tabu search for re-entrant flow-shops*, pp. 1928–1929.
[46] See Chen / Pan / Lin (2008): *A hybrid genetic algorithm for re-entrant flow-shops*, p. 575.
[47] See Qian et al. (2013b): *Reentrant permutation flow-shops with different job reentrants*, p. 25.
[48] See Hinze / Sackmann (2016): *An Iterated Local Search for a Re-entrant Flow Shop*, pp. 221–226.

calibration affect the makespan values and computation times. The test instances are identical to those used in the previous subsection 4.7.5.[49] Table 4.33 shows the average reductions of makespan compared to the previous values from subsection 4.7.5. The makespan reduction ΔC^*_{max} is calculated with:

$$\Delta C^*_{max} = \frac{C_{max}\left(\pi_{meta^*}\right)}{C_{max}\left(\pi_{meta}\right)} - 1.$$

Applying a metaheuristic with the changed neighborhood setting results in the permutation π_{meta^*}. Negative values indicate an increase, i.e. worse makespan values, while positive values signal an improvement. The makespan improvements are higher for small problems than in the large problems for both tested VNS configurations. The investigated changes in a neighborhood structure lead to weaker results compared to the previous experiments for the variable neighborhood searches on large problems. Positive effects are observed for small problems, especially for the VNS2 with the first improvement local search. A reason for the negative effects in the variable neighborhood approaches is the restriction of moves during the shaking phase. This may prevent the algorithms leaving local optima. The neighborhood change for the level based tabu search leads to slight improvements in the large instances. The use of job swaps instead of insertions for the tabu search and simulated annealing does not lead to positive effects.

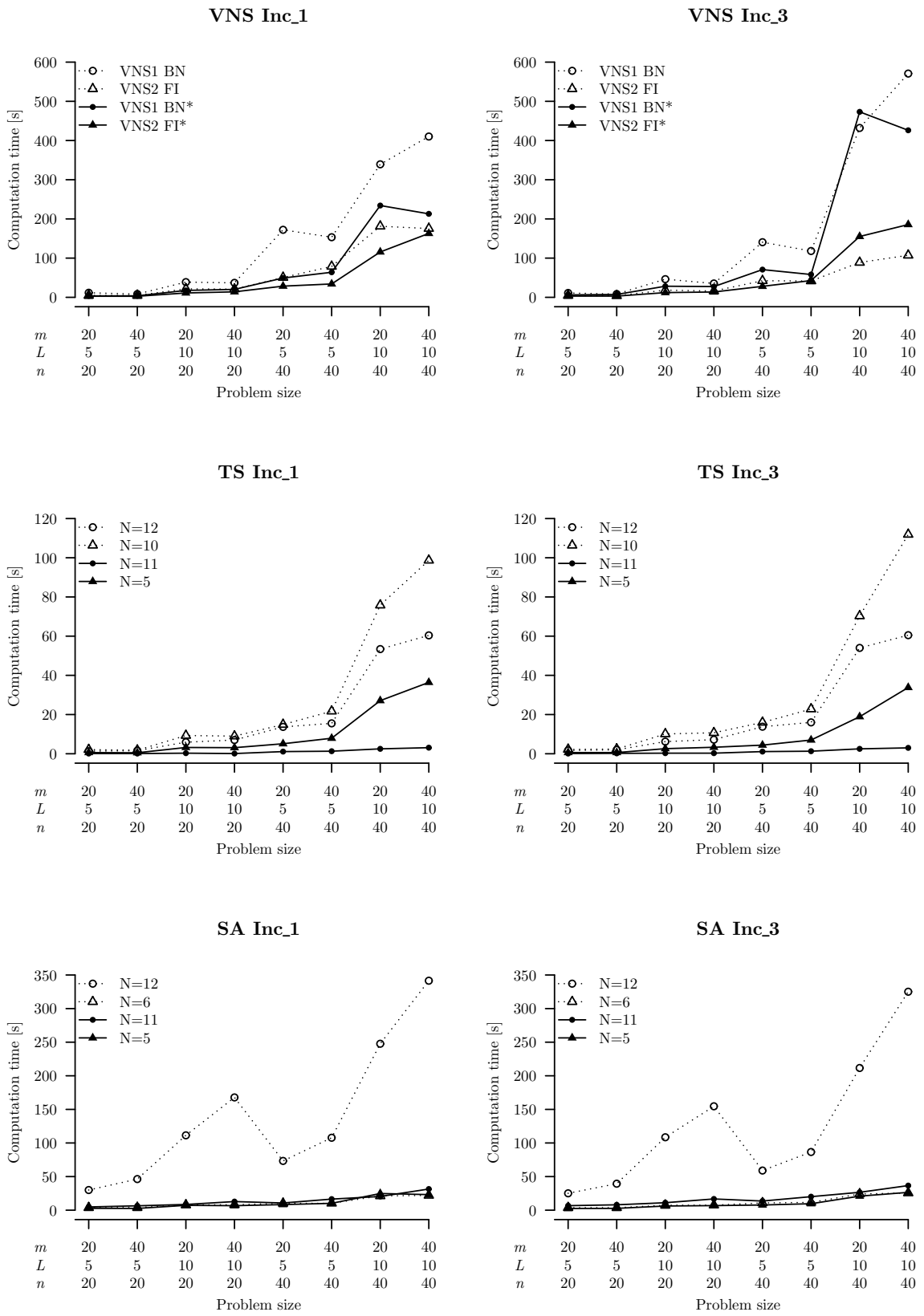
Table 4.33: Average makespan reductions ΔC^*_{max} [%] for large problems

	Inc_1			Inc_3		
	Small	Large	Total	Small	Large	Total
VNS1 BN*	−0.01	−0.43	−0.06	0.18	−0.28	0.03
VNS2 FI*	0.08	−0.25	0.04	0.07	−0.20	−0.02
TS $\mathcal{N} = J1$	−0.69	−0.06	−0.62	−0.27	0.00	−0.18
TS $\mathcal{N} = 5$	−0.38	0.02	−0.34	−0.23	0.02	−0.15
SA $\mathcal{N} = J1$	−0.52	−2.41	−0.73	−0.31	−1.94	−0.85
SA $\mathcal{N} = 5$	0.22	−0.04	0.19	0.14	−0.06	0.07

The effects of the different neighborhood calibrations on computation times are shown in Figure 4.52 for the large problem sizes. The changes lead to lower computation times for the VNS approaches in the Inc_1 test set but partly longer computing times in the test set with a higher number of missing operations, i.e. Inc_3. The changes in the calibration of the tabu search and simulated annealing lead to a decrease in computing time, especially for the job move neighborhoods.

[49] See Table B.2 in Appendix B (p. 204) and Table B.1 (p. 203) for detailed information on the problem instances.

Figure 4.52: Average computation times with changed neighborhood structures



Considering the decreasing solution quality for the VNS and the low impact on computation time, the examined changes of neighborhood structures are not recommended for re-entrant permutation flow shop problems without lot streaming.

4.8 Application on Job Shop Problems

The RPFS models are also able to solve job shop scheduling problems optimally. This section shows how missing operations can be used to represent a job shop problem. A common job shop model[50] and another formulation based on the model of Manne (1960)[51] are suggested. Afterwards, the computational performance of the job shop model is compared to the performance of the re-entrant permutation flow shop model with missing operations. The problem assumptions are:

1. All n jobs are available for processing at time zero.
2. There is a predetermined machine sequence for each job.
3. The machine sequence does not need to be identical for each job.
4. Each job may be processed on only one machine at a time, i.e., job splitting is not permitted.
5. There is only one of each type of machine available.
6. Only one job at a time can be processed on an individual machine.
7. Processing times of all n jobs on each of the m machines are predetermined.
8. Set-up times are not considered.
9. Unlimited in-process inventory is allowed between consecutive machines in the production system.
10. No preemption is allowed: once started, a task must be completed without interruption.

The job shop model from literature uses s_g^i as the starting time variable for the start of the gth operation of job i. p_g^i is the processing time of the gth operation of job i. The parameter w_g^i indicates on which machine the gth operation of job i is performed.

[50] See Domschke / Scholl / Voss (1997): *Produktionsplanung*, p. 403.
[51] See Manne (1960): *On the job-shop scheduling problem*, p. 220.

The objective in the considered job shop problem is to minimize the makespan (4.32).

$$\min C_{max} \tag{4.32}$$

The makespan is measured by the inequalities 4.33.

$$s_m^i + p_m^i \leq C_{max} \tag{4.33}$$
$$\forall i = 1, \ldots, n.$$

The inequalities 4.34 ensure the correct sequence of operations for every job i. The gth operation needs to be finished before operation $g+1$ is allowed to start.

$$s_g^i + p_g^i \leq s_{g+1}^i \tag{4.34}$$
$$\forall i = 1, \ldots, n; g = 1, \ldots, m-1.$$

The constraints 4.35 and 4.36 prevent the machines $k = 1, \ldots, m$ from being occupied by more than one job at a point of time. The constraints apply to the gth operation of job i and the g'th operation of job i', if they both need to be performed on machine $w_g^i = w_{g'}^{i'} = k$. The binary variable $y_{ii'k}$ equals 1 if job i is scheduled before job i' on machine k. Otherwise, it equals 0. The operation $i'g'$ is only allowed to start on machine k, after operation ig is finished, if $y_{ii'k} = 0$ (4.35).

$$A\left(1 - y_{ii'k}\right) + s_g^i + p_g^i \leq s_{g'}^{i'} \tag{4.35}$$
$$\forall i, i' = 1, \ldots, n; g, g' = 1, \ldots, m; k = 1, \ldots, m; if \left(w_g^i = w_{g'}^{i'} = k\right).$$

In the case of $y_{ii'k} = 0$, operation $i'g'$ needs to completed before operation ig is allowed to start (4.36).

$$Ay_{ii'k} + s_{g'}^{i'} + p_{g'}^{i'} \leq s_g^i \tag{4.36}$$
$$\forall i, i' = 1, \ldots, n; g, g' = 1, \ldots, m; k = 1, \ldots, m; if \left(w_g^i = w_{g'}^{i'} = k\right).$$

The non-negativity constraints (4.37) and binary constraints (4.37) apply to the starting times and sequence variables.

$$s_g^i \geq 0 \qquad \forall i = 1, \ldots, n; k = 1, \ldots, m. \tag{4.37}$$
$$y_{ii'k} \in \{0; 1\} \qquad \forall i, i' = 1, \ldots, n; k = 1, \ldots, m. \tag{4.38}$$

The following example considers three jobs that need to be processed on three machines. The machine sequences are given by W:

$$W = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{pmatrix}.$$

The processing times of the different operations are provided by the values of p_g^i:

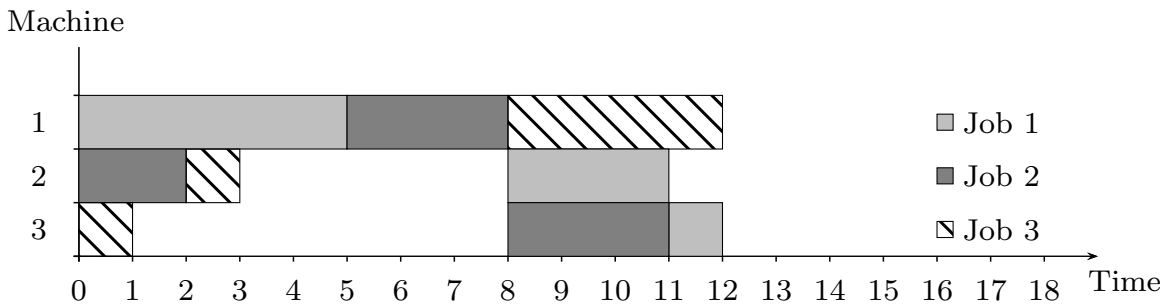
$$p_g^1 = \begin{pmatrix} 5 & 3 & 1 \end{pmatrix}, \qquad p_g^2 = \begin{pmatrix} 2 & 3 & 3 \end{pmatrix}, \qquad p_g^3 = \begin{pmatrix} 1 & 1 & 4 \end{pmatrix}$$

The number of levels in the corresponding RPFS problem with missing operations is $L = m = 3$. The processing time parameters are:

$$p_{lk}^1 = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad p_{lk}^2 = \begin{pmatrix} 0 & 2 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \qquad p_{lk}^3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 4 & 0 & 0 \end{pmatrix}$$

Both models lead to a result of $C_{max} = 12$. The solution is shown in Figure 4.53.

Figure 4.53: Solution to the job shop example



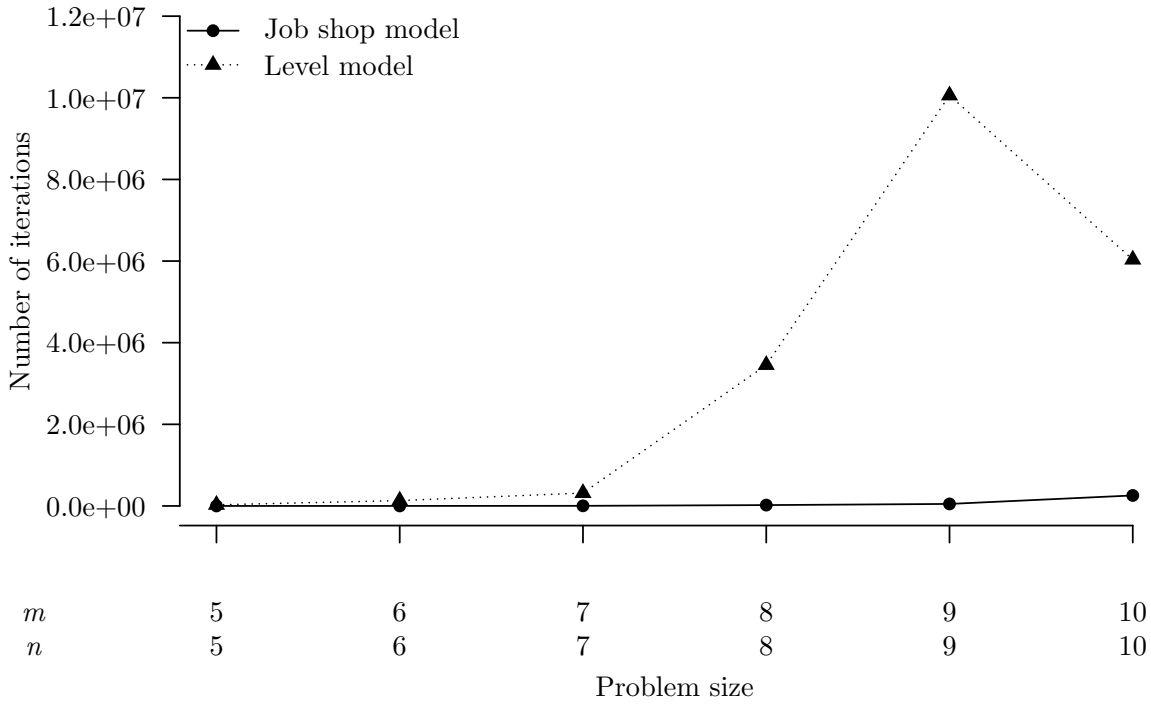
Both models are tested on several job shop problem instances. The number of jobs in the test instances is $n = 5, \ldots, 10$, which are processed on $m = 5, \ldots, 10$ machines with $n = m$ for all problem sizes. Ten different instances are generated and solved for each problem size. The processing times are uniformly distributed random numbers between 1 and 99.[52]

The results on computational performance are shown in Figures 4.54 and 4.55. The average number of iterations performed by the level model on the $n = 10, m = 10$ problems is lower than for the $n = 9, m = 9$ instances, because CPLEX runs out of time and the iterations are more time consuming. The number of iterations is generally

[52] See Table B.3 in Appendix B (p. 205) for detailed information on the problem sizes.

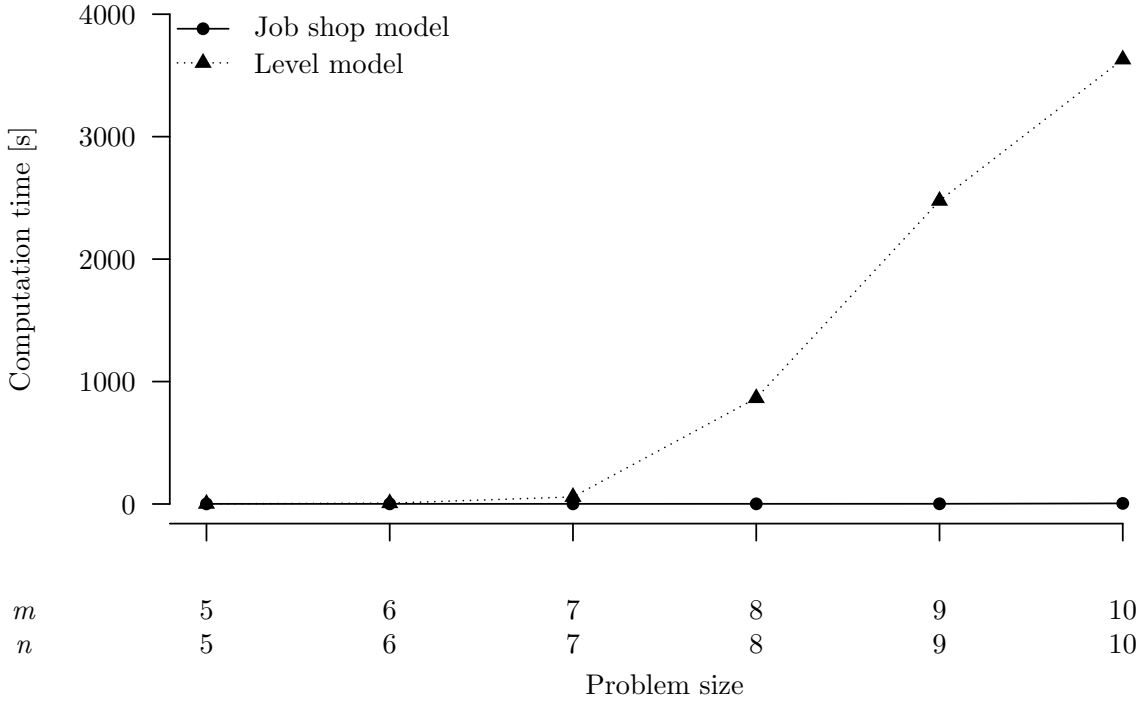
higher for the level model compared to the job shop model.

Figure 4.54: Average number of CPLEX iterations for job shops



The computation times of the level model are higher compared to the pure job shop model. The mean computation time of the level model reaches a limit of 1 hour for the problem size $n = 10, m = 10$, while the job shop needs just several seconds to optimally solve the problems.

Figure 4.55: Average computation times for job shops



The level model in this form and with this kind of input data does not seem to be a suitable alternative to solve job shop problems. A possible improvement is the reduction of levels by allowing multiple processing times greater than 0 within one job level. This gives the representation a more flow shop-like appeal and may lead to a better computational performance. Generally, the model is able to find the optimal solutions for job shop problems.

5 Re-entrant Permutation Flow Shop Problems with Mixed Levels, Missing Operations and Lot Streaming

The term "lot streaming" for splitting up a production job into several sublots was initially used for a job shop problem.[1] The basic condition for applying lot streaming is, that the jobs can be divided, for example, if a job consists of a discrete number of parts. The first part of this chapter investigates the impact of different sublot forms and processing requirements on the makespan and computational performance by using different MIP models to solve re-entrant lot streaming problems. A framework for metaheuristics is discussed in section 5.4 based on the findings from the MIP models.

5.1 Introduction

It is possible to split the operations to process a job if the job is assumed to be a lot consisting of multiple parts. Dividing the job into two sublots makes it possible to transfer the first sublot earlier to the next processing step.

An example of the ability to reduce the makespan is given for one job with the parameters p_{lk}. The job is processed in two levels and on two machines.

$$p_{lk} = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}.$$

The resulting starting times in a re-entrant flow shop without lot streaming are:

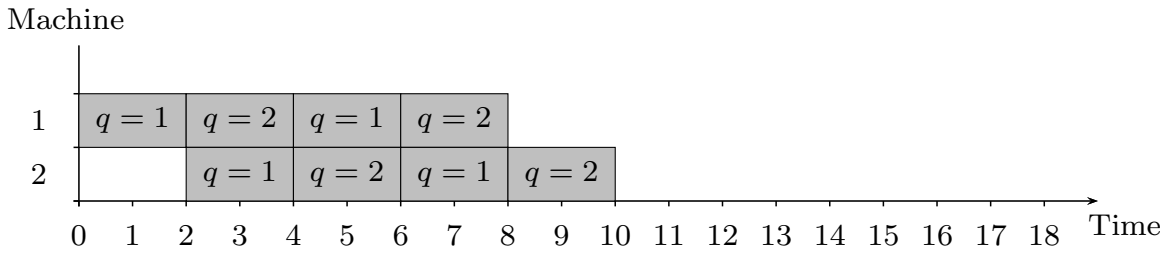
$$s_{lk} = \begin{pmatrix} 0 & 4 \\ 8 & 12 \end{pmatrix}.$$

The job is finished after 16 time units.

[1] Reiter (1966): *A system for managing job-shop production*, pp. 371–393.

Lot streaming makes it possible to schedule the sublots of a job. The models in this chapter are able to split each of the n jobs into Q sublots in order to reduce makespan. Figure 5.1 shows the processing of a job that needs to visit two machines twice if the job is split into two sublots. Assuming the job consists of $D = 100$ parts and is divisible into two sublots $q = 1$ and $q = 2$ each of the size $X^q = 50$, the following solution shown in Figure 5.1 is obtained.

Figure 5.1: Example of lot streaming



The time after the job is finished is reduced to 10 time units.

Literature Review on Lot Streaming

Lot streaming for scheduling problems was first proposed by Reiter (1966) for a job shop problem. The sublots are set to be equal in size for a job, but the number of sublots may differ from job to job to increase machine utilization and to reduce the flow times of jobs.[2]

The sublots of jobs can be characterized by the criteria of sublot size and processing. The sublot sizes are variable if the size of any sublot is allowed to differ between machines.[3] Sublots are called consistent if they are not allowed to vary from machine to machine[4]. Equal sublots mean that all sublots of a job are required to have the same size.[5] Sublots of the same job level can be scheduled either consecutively or non-consecutively in the permutation, characterizing the processing of sublots. It is called intermingling[6] or interleaving[7] of sublots, if the sublots belonging to the same job (in non re-entrant flow and job shops) or job levels (in the considered re-entrant problem) are not processed consecutively. Intermingling is considered to be possible in the investigated lot streaming problem in subsections 5.2.1 and 5.2.3. A consecutive processing of sublots is

[2] See Reiter (1966): *A system for managing job-shop production*, pp. 371–393.
[3] See Trietsch / Baker (1993): *Basic techniques for lot streaming*, pp. 1065–1066.
[4] See Potts / Baker (1989): *Flow shop scheduling with lot streaming*, p. 297.
[5] See Baker / Jia (1993): *Lot streaming procedures*, p. 562.
[6] See Feldmann / Biskup (2008): *Lot streaming intermingling*, pp. 198–199
[7] See Zhang / Jiang / Zhang (2008): *A Research on a Genetic Algorithm*, p. 1141 and Martin (2009): *Multi-family flowshop scheduling problem with lot streaming*, pp. 126–127

investigated separately in subsections 5.2.2 and 5.2.4. Subsection 5.2.5 also partly deals with a consecutive processing of sublots without sublot size consistency. Intermittent idling is when machine idle times are allowed between consecutive sublots of the same job (level).[8] Intermittent idling is allowed in all of the examined models.

Potts / Baker (1989) examined the effect of consistent and equal sublot sizes as a special case of consistent sublots for a one-job scheduling problem. Equal sublots deliver a maximum of 53 % worse results than simple consistent sublots for m machines in the single job case. The makespan is a maximum of 9 % higher for $m = 2$. Additionally, the multi-job case is examined using an example with two jobs. Within this example, different solution schemes are considered. The first scheme determines the optimal schedule of the two jobs and then divides the jobs into sublots, resulting in a consecutive sublot schedule. The second scheme uses lot streaming, leading to an optimal makespan and a schedule where the sublots are not processed consecutively. It is noted that if equal sublot sizes are used, there is no need to have no consecutive sublots, although the solution is worse than with the first solution scheme.[9]

The minimization of the makespan in a one-job scheduling problem with lot streaming was also considered by Trietsch / Baker (1993). Consistent and variable sublots, intermittent idling and whether the sublot size is allowed to be continuous or needs to be discrete are the different problem characteristics in this publication. Methods are suggested to solve the different problem variations optimally if the number of sublots is fixed and the amount of available transport equipment between the machines is limited.[10] The lot streaming solutions with sublots that are variable, consistent or equal and idle times being allowed or not were compared for a one-job scheduling problem with three machines in Baker / Jia (1993). The numbers of sublots per job are $Q \in \{2, 3, 5, 8, 10\}$. The makespan values obtained with consistent sublots are, on average, close to the makespan values of the solutions with variable sublots if more than two sublots are generated.[11]

Glass / Potts (1998) investigated the structural properties of makespan minimization in flow shop problems with consistent sublots and consecutive processing. They transferred a theorem on critical sublots of a three-stage production system proposed by Glass / Gupta / Potts (1994) to an m-stage flow shop.[12] It was stated that each sublot has at least has two critical operations if the sublot sizes are optimal. The property of dominant machines is introduced to find promising structural properties, including crit-

[8] See Trietsch / Baker (1993): *Basic techniques for lot streaming*, p. 1065.
[9] See Potts / Baker (1989): *Flow shop scheduling with lot streaming*, pp. 297–303.
[10] See Trietsch / Baker (1993): *Basic techniques for lot streaming*, pp. 1065–1076.
[11] See Baker / Jia (1993): *Lot streaming procedures*, pp. 561–566.
[12] See Glass / Gupta / Potts (1994): *Lot streaming in three-stage production processes*, pp. 381–382 and Glass / Potts (1998): *Structural properties of lot streaming in a flow shop*, pp. 633–634

ical block structure and sublot sizes of a solution with minimal makespan.[13] In a later work, the structural properties were used within a relaxation algorithm to find the optimal solutions for the flow shop problem with multiple jobs and consecutive sublots.[14]

Feldmann / Biskup (2008) considered a permutation flow shop with lot streaming. The problem was stated as a lot streaming problem with intermingling, which means that the sublots of a job do not need to form a consecutive sequence. The sublot sizes need to be consistent between different machines. Also a model with consecutive sublots was investigated. The model with consecutive sublots relies on a position-related variable, assigning sequence positions to single sublots. The model with intermingling uses a sequence-related binary variable to determine a permutation representing a predecessor-successor relation between sublots. Both formulations have been tested for the case of intermingling sublots with the LINGO solver software. The model using position-related binary variables to determine a permutation required that more branch and bound iterations occurred for every test instance to find the optimal solution regarding the makespan. For one of the test instances, the position-related formulation required 173.6 % more iterations.[15]

Makespan minimization in permutation flow shops was also investigated by Pan et al. (2010). The sublot sizes were determined to be equal and the setup times of the operations were sequence-dependent. The setup can be performed without the sublot present on the machine. The setup is only necessary for the first sublot of a job, since also consecutive sublots are assumed. The solution method applied to the problem is a heuristic called a discrete harmony search algorithm. The algorithm is compared to a hybrid genetic algorithm, an ant colony optimization algorithm, and two threshold accepting algorithm modifications, one with swap moves and the other with insertion moves. The discrete harmony search algorithm outperformed the other comparative solution methods.[16]

A combined objective function of weighted tardiness and weighted earliness was suggested byPan et al. (2011) to reduce the deviation of job completion time and job due date in a permutation flow shop with equal sublots and consecutive processing of same job sublots. An artificial bee colony algorithm was used to solve problem instances with and without intermittent idling. The number of sublots per job was chosen randomly between one and six sublots per job.[17]

Defersha (2011) proposed an MIP model for a hybrid flexible flow shop problem with multiple unrelated parallel machines at each stage and sequence-dependent setup

[13] See Glass / Potts (1998): *Structural properties of lot streaming in a flow shop*, pp. 624–639.
[14] See Glass / Possani (2011): *Lot streaming multiple jobs in a flow shop*, pp. 425–431.
[15] See Feldmann / Biskup (2008): *Lot streaming intermingling*, pp. 197–216.
[16] See Pan et al. (2010): *Discrete harmony search for lot-streaming flow shops*, pp. 1531–1536.
[17] See Pan et al. (2011): *Artificial bee colony algorithm for lot-streaming*, pp. 2455–2468.

times. The flexible property is referred to as the possibility for jobs to skip stages. This is formulated by using a precedence parameter defining the two succeeding stages for a job. The sublots do not need to be consistent or consecutive. The formulation is tested for multiple configurations of the maximal number of sublots per job. The reductions of the makespan with increasing sublot numbers are lower for higher sublot numbers. Generating more than five sublots per job is not appropriate for simple flow shops or for hybrid flexible flow shops.[18]

The minimization of the makespan in job shops with lot streaming was tackled with a combination of a tabu search and genetic algorithm by Buscher / Shen (2008). The tabu search sequenced the sublots, while a genetic algorithm was applied to change sublot sizes.[19] Also Buscher / Shen (2009) used a tabu search mechanism to sequence sublots in a job shop with lot streaming. The initially generated sublot sizes were generated randomly. The tabu search was applied repeatedly to the schedule, and every time the size of selected sublots was recalculated by the suggested procedure. Both the tabu search and the recalculation of sublot sizes, are based on the block structure suggested by Nowicki / Smutnicki (1996).[20] In Buscher / Shen (2011), a mixed integer lot streaming model for job shops with consistent sublots and setup times is proposed. The model is extended with no-wait and non-idling constraints. The objective is the makespan. The computational experiments show that the benefit of lot streaming decreases if the setup times are increased per operation. Additionally the impact of raising the number of sublots per job by one leads to a decreased benefit for higher numbers of sublots per job.[21]

To the authors knowledge, lot streaming has not been investigated for the re-entrant permutation flow shop problem, except by Hinze (2016).[22] The publication of Reiter (1966) considered re-entrant product flows in job shops without explicitly calling the repeated processing of jobs on one or more machines "re-entrant".[23] Queries in the databases mentioned in Section 3.2 did not deliver any relevant results regarding the search terms "re-entrant" ("reentrant") and "lot streaming". Also, the review on lot streaming by Chang / Chiu (2005), as well as the reviews on re-entrant scheduling, do not show any information regarding the application of lot streaming on re-entrant permutation flow shops.[24]

[18] See Defersha (2011): *Hybrid flexible flowshop lot streaming problem*, pp. 283–294.
[19] See Buscher / Shen (2008): *Lot Streaming in Job Shops*, pp. 425-431.
[20] See Buscher / Shen (2009): *Tabu search for the lot streaming problem in job shops*, pp. 385–399.
[21] See Buscher / Shen (2011): *An integer programming for lot streaming in a job shop*, pp. 1343–1348.
[22] See Hinze (2016): *Lot Streaming with Missing Operations*, pp. 152–155.
[23] See Reiter (1966): *A system for managing job-shop production*, p. 378.
[24] See Chang / Chiu (2005): *A comprehensive review of lot streaming*, pp. 1515–1536.

5.2 Mathematical Models

The number of sublots per job, Q, is predetermined in the models of this section. It is not considered as a solution variable. Contrary to the previous chapters, the symbol p^i_{lk} indicates the unit processing time, which refers to the processing time for one part of job i in level l on machine k.

5.2.1 Consistent Sublots

The equations (5.1) ensure that the set of sublots contains all units of the complete job. The sublots of the jobs are not variable, i.e. consistent during all operations. Therefore the sublot size X^{iq} has neither a level nor a machine index.

$$\sum_{q=1}^{Q} X^{iq} = D^i \tag{5.1}$$
$$\forall i = 1, \ldots, n.$$

The multiplication with the processing times in the constraint sets (5.2), (5.3) and (5.4) enables the ability to skip missing operations, equivalent to the model without lot streaming. Both sides of the constraints equal zero if one of the considered operations is missing. The constraints (5.2) regulate the machine sequence for each of the sublots $q = 1, \ldots, Q$ of a job i on level l. The sublots usually need to start on machine $k = 1$ and end the level on machine $k = m$. The operation on a machine k', with $k' > k$, can only start after the operations on all machines k are finished. The constraints are not relevant if an operation on either a machine k or k' is left out.

The sublots of a job i should be processed in a sequence starting with sublot $q = 1$ on each level $l = 1, \ldots, L$ and ending with sublot $q = Q$ in (5.3). The level transitions of a job's sublots are regulated in (5.4). A level $l + 1$ of a job's i sublot q is not allowed to start before its level l is finished.

$$p^i_{lk'} \cdot \left(s^{iq}_{lk} + p^i_{lk} \cdot X^{iq}\right) \cdot p^i_{lk} \leq p^i_{lk'} \cdot s^{iq}_{lk'} \cdot p^i_{lk} \tag{5.2}$$
$$\forall i = 1, \ldots, n; q = 1, \ldots, Q; l = 1, \ldots, L; k, k' = 1, \ldots, m; (k < k');$$
$$\left(s^{iq}_{lk} + p^i_{lk} \cdot X^{iq}\right) \cdot p^i_{lk} \leq s^{i,q+1}_{lk} \cdot p^i_{lk} \tag{5.3}$$
$$\forall i = 1, \ldots, n; q = 1, \ldots, Q-1; l = 1, \ldots, L; k = 1, \ldots, m;$$
$$p^i_{l+1,k'} \cdot \left(s^{iq}_{lk} + p^i_{lk} \cdot X^{iq}\right) \cdot p^i_{lk} \leq p^i_{l+1,k'} \cdot s^{iq}_{l+1,k'} \cdot p^i_{lk} \tag{5.4}$$
$$\forall i, = 1, \ldots, n; q = 1, \ldots, Q; l = 1, \ldots, L-1; k, k' = 1, \ldots, m.$$

The constraint sets (5.5) and (5.6) prevent jobs from being scheduled on a machine k

when there is already another job on k at the same time. The constraints (5.5) consider two sublots q and q' of two different jobs $i \neq i'$ or the same job $i = i'$ on the levels l and l'. If $y_{iql}^{i'q'l'} = 1$, i.e. iql precedes $i'q'l'$, then $i'q'l'$ needs to wait for iql to be finished before it can start on machine k. The inequalities (5.6) are applied for the case that $i'q'l'$ precedes iql, i.e. $y_{iql}^{i'q'l'} = 0$.

$$A \cdot \left(1 - y_{i'q'l'}^{iql}\right) + s_{i'q'}^{l'k} - s_{iq}^{lk} \geq p_{lk}^{i} \cdot X^{iq} \tag{5.5}$$
$$\forall i, i' = 1, \ldots, n; (i \geq i'); l, l' = 1, \ldots, L; k = 1, \ldots, m; q, q' = 1, \ldots, Q; if\ (i = i')\ \{q \neq q'\};$$
$$A \cdot y_{i'q'l'}^{iql} + s_{iq}^{lk} - s_{i'q'}^{l'k} \geq p_{l'k}^{i'} \cdot X^{i'q'} \tag{5.6}$$
$$\forall i, i' = 1, \ldots, n; (i \geq i'); l, l' = 1, \ldots, L; k = 1, \ldots, m; q, q' = 1, \ldots, Q; if\ (i = i')\ \{q \neq q'\}.$$

The job levels are kept in a basic sequence by the constraint set (5.7). Unless different levels can be mixed, the job sequence within each level is the same. If a sublot q of job i precedes a sublot q' of a job i' in level l, then it also precedes sublots q' of job i' on all other levels $l' \neq l$. These constraints are not included in the tested models, in order to identify the full impact of lot streaming on the objective values.

$$y_{i'q'l}^{iql} = y_{i'q'L}^{iqL} \tag{5.7}$$
$$\forall i, i' = 1, \ldots, n; (i > i'); l = 1, \ldots, L; q, q' = 1, \ldots, Q;\ if\ (i = i')\ \{q \neq q'\}.$$

The equations (5.8) ensure the unambiguousness of sequence variables. Given two sublots q and q' of two different jobs i and i' on any two levels l and l', one of these sublots needs to precede the other one, i.e. one of the variables $y_{i'q'l'}^{iql}$ and $y_{iql}^{i'q'l'}$ needs to be 1. Then the other variable must be 0.

$$y_{i'q'l'}^{iql} + y_{iql}^{i'q'l'} = 1 \tag{5.8}$$
$$\forall i, i' = 1, \ldots, n; i \geq i'; l, l' = 1, \ldots, L; q, q' = 1, \ldots, Q;\ if\ (i = i')\ \{q \neq q'\}.$$

The makespan of the schedule is calculated by (5.10). It is necessary to check the end of all operations to compute the makespan, since it is not known whether the operation on the last machine has a processing time equal to 0.

$$\min C_{max}; \tag{5.9}$$
$$s_{lk}^{iq} + p_{lk}^{i} \cdot X^{iq} \leq C_{max} \tag{5.10}$$
$$\forall i = 1, \ldots, n; q = 1, \ldots, Q; l = 1, \ldots, L; k = 1, \ldots, m.$$

An alternative objective function is the total flow time as the sum of completion times.

$$\min \sum_{i=1}^{n} C_i; \tag{5.11}$$

$$s_{lk}^{iq} + p_{lk}^{i} \cdot X^{iq} \leq C_i \tag{5.12}$$

$$\forall i = 1, \ldots, n; q = 1, \ldots, Q; l = 1, \ldots, L; k = 1, \ldots, m.$$

The equations (5.12) measure the completion time of each job, which is only necessary if the sum of completion times is the objective.

The constraints (5.13) are the binary constraints to the sequence variables. The non-negativity constraints apply to the sublot size (5.14), and (5.15) to the starting times of the sublots.

$$y_{i'q'l'}^{iql} \in \{0; 1\} \tag{5.13}$$

$$\forall i, i' = 1, \ldots, n; i \geq i'; l, l' = 1, \ldots, L; q, q' = 1, \ldots, Q; if\,(i = i' AND l = l')\, q \neq q'$$

$$s_{lk}^{iq} \geq 0 \tag{5.14}$$

$$\forall i = 1, \ldots, n; q = 1, \ldots, Q; l = 1, \ldots, L; k = 1, \ldots, m;$$

$$X^{iq} \geq 0 \tag{5.15}$$

$$\forall i = 1, \ldots, n; q = 1, \ldots, Q.$$

There are $n = 3$ jobs to schedule in an RPFS example with $m = 3$ machines. All jobs run $L = 2$ times through the production system. Missing operations are not considered in the example, as it should show only the effects of different forms of sublots. Each job consists of 100 parts:

$$D^1 = D^2 = D^3 = 100.$$

The unit processing times p_{lk}^{i} of each job are given with p_{lk}^{1}, p_{lk}^{2} and p_{lk}^{3}:

$$p_{lk}^{1} = \begin{pmatrix} 0.01 & 0.01 \\ 0.01 & 0.02 \end{pmatrix}, \qquad p_{lk}^{2} = \begin{pmatrix} 0.04 & 0.04 \\ 0.03 & 0.01 \end{pmatrix}, \qquad p_{lk}^{3} = \begin{pmatrix} 0.01 & 0.01 \\ 0.04 & 0.03 \end{pmatrix}$$

The maximum number of sublots per job is set to $Q = 2$.

The sublot sizes of the jobs are represented by the parameters X^{iq}. The first job is not divided into sublots, because the first sublot includes all of the job's parts.

$$X^{1q} = \begin{pmatrix} 100 & 0 \end{pmatrix}, \qquad X^{2q} = \begin{pmatrix} 80 & 20 \end{pmatrix}, \qquad X^{3q} = \begin{pmatrix} 80 & 20 \end{pmatrix}.$$

The optimal schedule, if the sublots are consistent but not equal and not consecutive,

leads to the permutation provided by Table 5.1. The values in parentheses are for sublots of size $X^{iq} = 0$.

Table 5.1: Optimal permutation of the example with consistent sublots

Position	1	2	3	4	5	6	7	8	9	10	11	12
Job	1	3	3	1	2	(1)	3	2	(1)	2	3	2
Level	1	1	1	2	1	(1)	2	1	(2)	2	2	2
Sublot	1	1	2	1	1	(2)	1	2	(2)	1	2	2

The starting times of all operations of the $i = 1, 2, 3$ job's sublots $q = 1, 2$ in the levels $l = 1, 2$ on the machines $k = 1, 2, 3$ are given by the values of s_{lk}^{iq}.

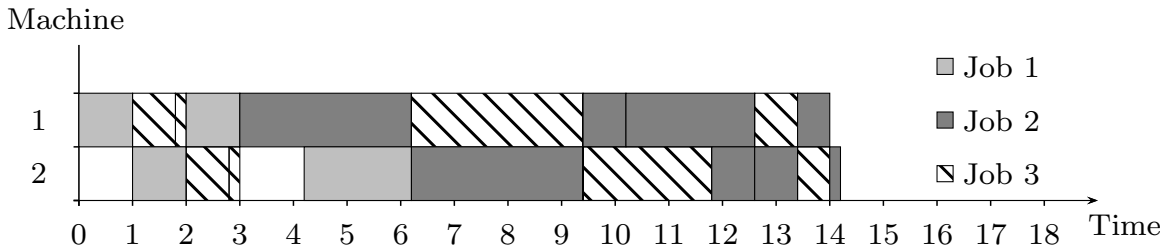
$$s_{lk}^{11} = \begin{pmatrix} 0 & 1 \\ 2 & 4.2 \end{pmatrix}, \qquad s_{lk}^{12} = \begin{pmatrix} 6.2 & 9.4 \\ 9.4 & 11.8 \end{pmatrix},$$

$$s_{lk}^{21} = \begin{pmatrix} 3 & 6.2 \\ 10.2 & 12.6 \end{pmatrix}, \qquad s_{lk}^{22} = \begin{pmatrix} 9.4 & 11.8 \\ 13.4 & 14 \end{pmatrix},$$

$$s_{lk}^{31} = \begin{pmatrix} 1 & 2 \\ 6.2 & 9.4 \end{pmatrix}, \qquad s_{lk}^{32} = \begin{pmatrix} 1.8 & 2.8 \\ 12.6 & 13.4 \end{pmatrix}.$$

The optimal schedule is shown in Figure 5.2. The resulting makespan is $C_{max} = 14.2$. The example is also presented in Section 4.2 without lot streaming. The makespan in the solution without lot streaming is $C_{max} = 16$.

Figure 5.2: Solution to the example with consistent sublots



Impact of Lot Streaming with Consistent Sublots

The test problem sizes are determined by the values of the number of jobs $n \in \{2, 3, 4\}$, the number of levels per job $L \in \{2, 3, 4\}$, the number of machines $m \in \{2, 5, 6, 10\}$ and

the number of sublots per job $Q \in \{2, 3, 4\}$.[25] Combining the different parameters results in 108 different problem sizes. The same ten instances as in the experiments of Chapter 5 are tested for each problem size regardless of the number of sublots per job. Missing operations are generated in ascending quantity for test instance sets Inc_1 and Inc_2. The instance set Inc_2 requires a minimum of $L = 4$ levels per job.[26] The processing times of the operations in the tested problem instances in Chapter 5 are identical to the tested problems in Chapter 4, i.e. uniformly distributed random numbers between 1 and 99 as suggested by Taillard (1993) for non-missing operations.[27] The unit processing times in this chapter are obtained by dividing the processing times generated for each job by the number of parts / units of the job. The lot sizes are required to be divisible by two and three for the tests with equal sublots in Sections 5.2.3 and 5.2.4. Additionally the divisibility by four is secured for additional tests in section 5.4.2. This allows job lot sizes of:

$$D \in \{60, 72, 84, 96, 108, 120\}$$

Figure 5.3 shows the mean relative values of makespan achieved by applying lot streaming with consistent sublots compared to the makespan values of the simple re-entrant permutation flow shop with missing operations for the Inc_1 instances. The relative makespan values are calculated with the formula:

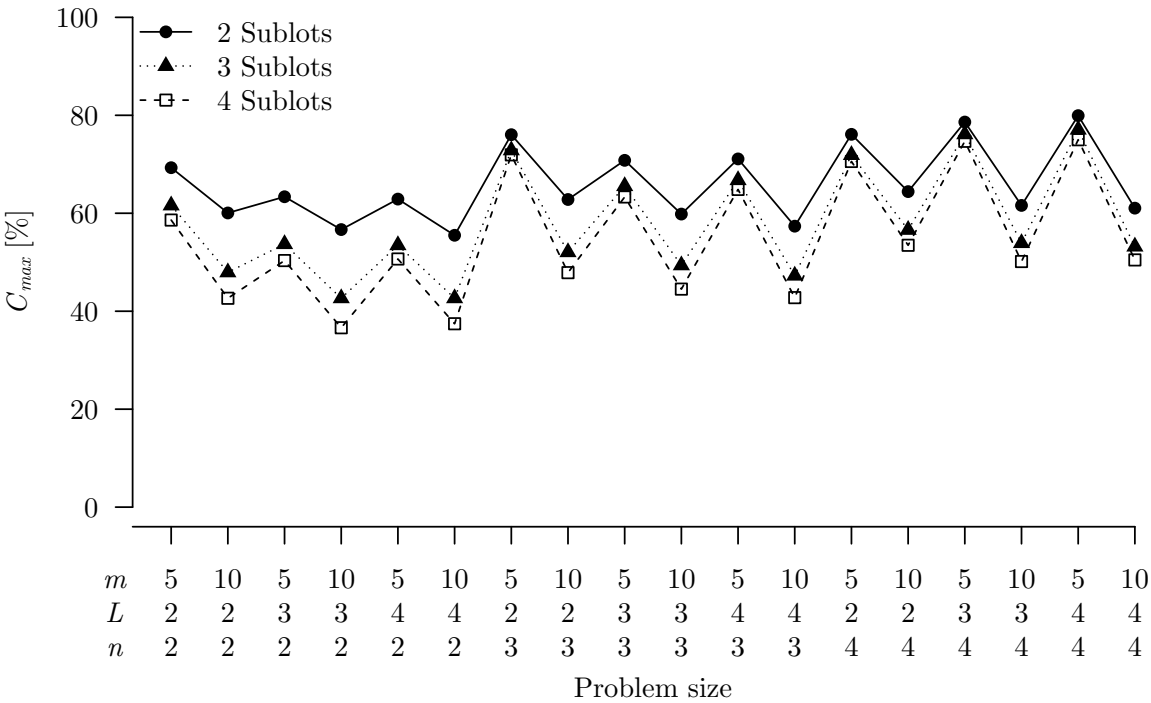
$$C_{max} = \frac{C^Q_{max}}{C^1_{max}}.$$

C^Q_{max} is the makespan value of the solution if the jobs are each split into $Q > 1$ sublots. The lot streaming problem with $Q = 1$ sublots, and the makespan values C^1_{max}, equal the problem without lot streaming. The values for splitting each job level into $Q = 2$ sublots are between 55.50 % ($n = 2$, $L = 4$, $m = 10$) and 79.93 % ($n = 4$, $L = 4$, $m = 5$) for every problem size. 55.50 % relative makespan means, that the average makespan computed for this problem size is 55.50 % of the makespan value if lot streaming is not applied. The reduction of the makespan between applying lot streaming with two sublots and no lot streaming is higher than the reduction from using three sublots instead of two and four sublots instead of three. The lowest values are obtained by splitting the job levels into $Q = 4$ sublots each. The mean values range from 37.44 % to 75.01 % compared to the mean makespan values of the model without lot streaming. The reduction of the makespan is generally higher for a larger number of machines, i.e. for problem sizes

[25] See Table B.3 in Appendix B (p. 205) for detailed information on the problem sizes.
[26] See Table B.1 in Appendix B (p. 203) for an overview of missing operations.
[27] See Taillard (1993): *Benchmarks for basic scheduling problems*, pp. 279–280.

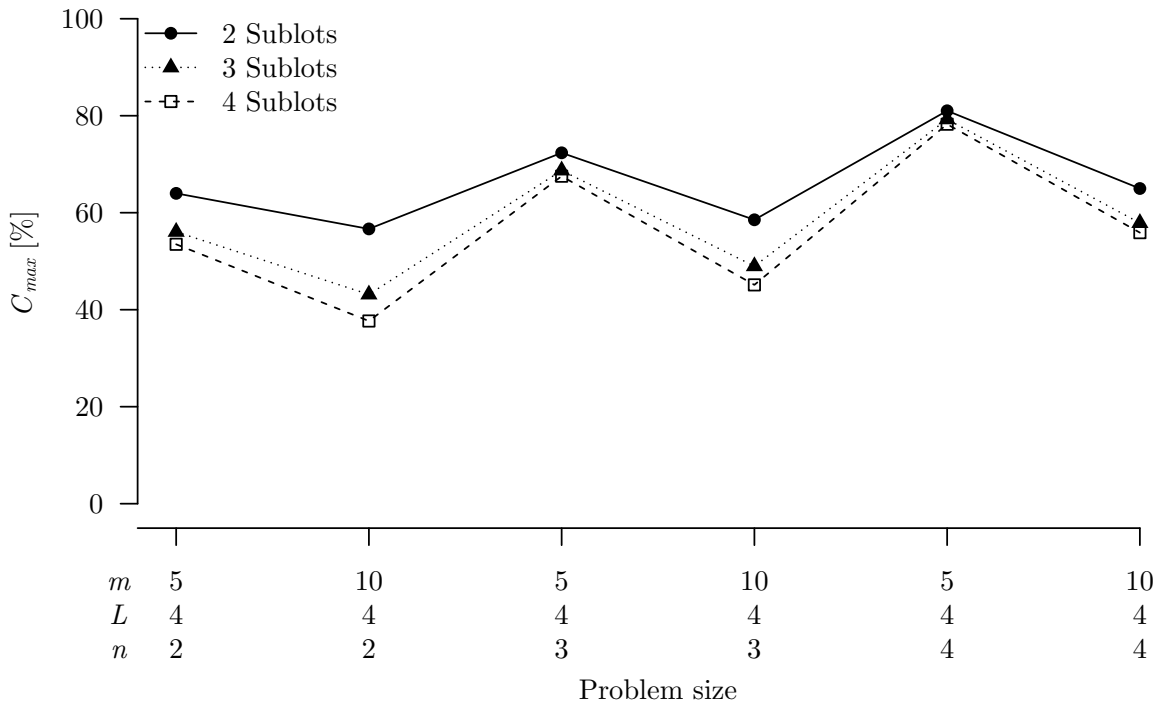
with $m = 10$. The advantage of using lot streaming increases with a higher number of machines that need to be visited by the job levels. The number of sequence positions to assign is $n \cdot L \cdot Q$.

Figure 5.3: Average makespan of consistent sublots compared to using no sublots (Inc_1)



The results of the mean relative makespan for the Inc_2 instances are shown in Figure 5.4. The lowest mean relative makespan is again achieved by splitting the job levels into four sublots. The average makespan for the instances of the problem size with ten machines, four levels and two jobs drops to 37.67 % of the makespan value without lot streaming. The lowest value for the three-sublot calculation appears for the same problem size with 43.14 % using two sublots per job, compared to the solution obtained if lot streaming is not allowed. The step from applying no lot streaming to lot streaming with two sublots is the largest step in makespan reduction compared to the step of increasing the number of sublots per job level from two to three and from three to four. Nevertheless, the mean relative makespan values for lot streaming with two sublots are higher than those for three and four sublots. The lowest value for the Inc_2 instances and $Q = 2$ is 56.66 % ($n = 2$, $L = 4$, $m = 10$).

Figure 5.4: Average makespan of consistent sublots compared to using no sublots (Inc_2)



The symbols used in the evaluation tables in this subsection (5.2.1) are G^Q, $\Delta C_{max}^{Q|Q-1}$ and It^Q. G^Q stands for the relative gap between the lower bound and best incumbent solution after 3600 seconds if lot streaming with Q consistent sublots per job is applied. It is 0 if the optimal solution is found. The makespan reduction by increasing the number of sublots by one is given by the symbol $\Delta C_{max}^{Q|Q-1}$ and calculated by:

$$\Delta C_{max}^{Q|Q-1} = \frac{C_{max}^{Q}}{C_{max}^{Q-1}} - 1.$$

It^Q denotes the number of CPLEX iterations for the lot streaming model with Q sublots per job.

Table 5.2 shows the average gaps for the the re-entrant permutation flow with missing operations and lot streaming for solutions with one, two, three and four sublots for each job depending on the number of machines. CPLEX comes closer to the optimal solution or closer to approving the solution found as optimal for problems with $m = 10$ machines compared to instances with $m = 5$ machines. Also the makespan reductions achieved by increasing the number of lots by one are higher for the ten machine instances. The makespan reductions are the highest for the step from one to two sublots per job. The weakest improvements of the makespan within a computation time of 1 hour are the ones for increasing Q from three to four. These facts apply for both test instance sets

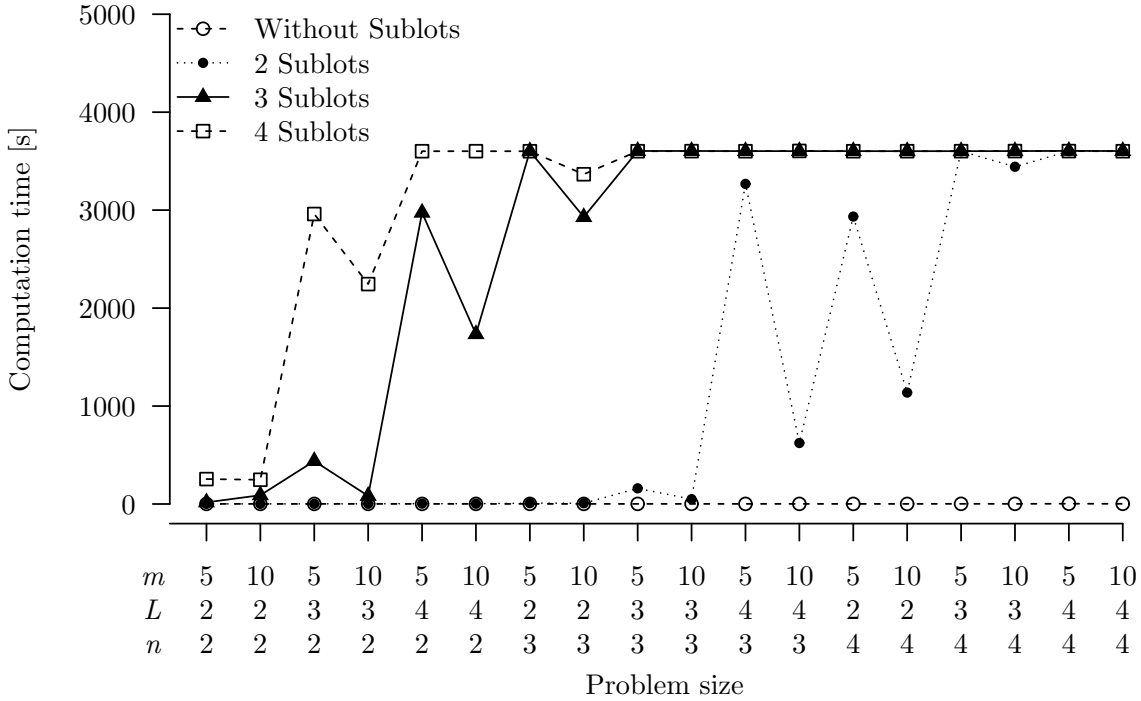
Inc_1 and Inc_2.

Table 5.2: Influence of m on makespan if the sublots are consistent

Inc	m	G^1 [%]	G^2 [%]	G^3 [%]	G^4 [%]	$\Delta C_{max}^{2\|1}$ [%]	$\Delta C_{max}^{3\|2}$ [%]	$\Delta C_{max}^{4\|3}$ [%]
1	5	0.00	5.86	21.25	31.14	−27.98	−7.97	−3.41
	10	0.00	1.66	12.12	21.14	−40.08	−17.57	−9.19
2	5	0.00	12.22	28.97	41.67	−27.55	−6.58	−2.56
	10	0.00	4.17	17.55	29.69	−39.94	−17.06	−8.04

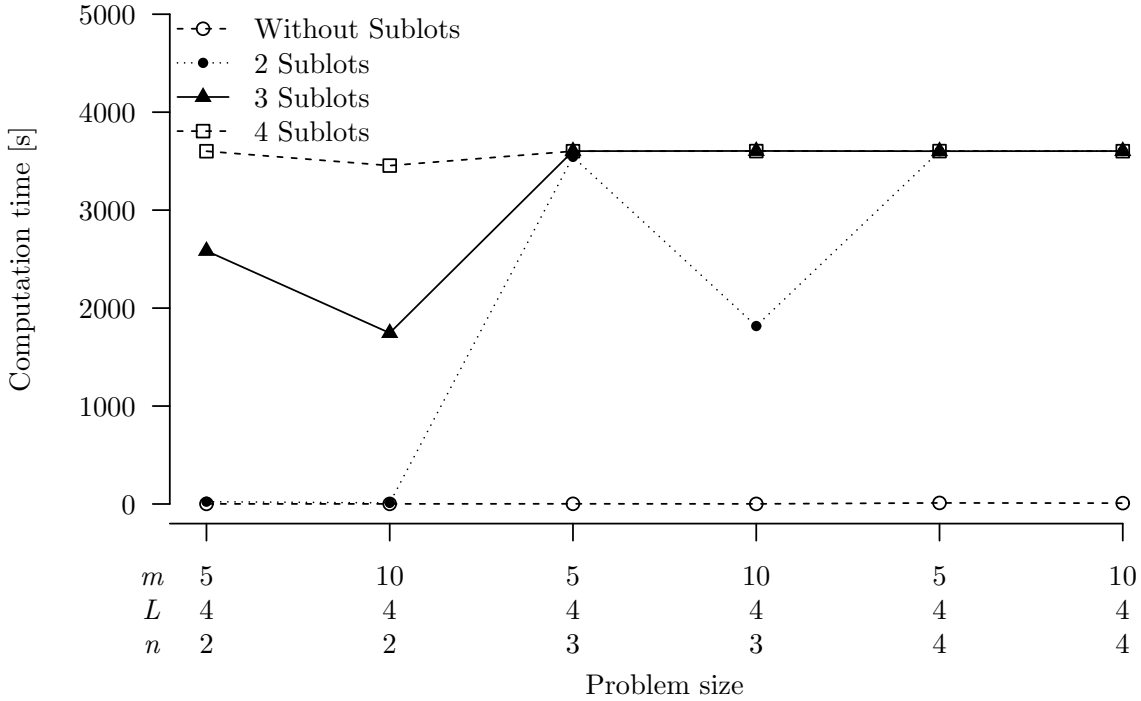
In Figure 5.3 and 5.4, a slight increase of the mean relative makespan values with an increasing number of jobs n can be seen. The reason for the effect can be explained by looking at Figures 5.5 and 5.6, which show the mean computation times for every problem size in the instance sets Inc_1 and Inc_2. An increase in the number of jobs leads to the largest increase of computation time compared to the influence of the number of levels. An increase in the number of machines leads to a lower computation time similar to the effect for the model without lot streaming. Solving the lot streaming problem with four consistent sublots prevents CPLEX from closing the gap between the incumbent solution and lower bound for most of the instances of the Inc_1 set. CPLEX either fails to find the optimal solution or fails to confirm the solution found as optimal. These problem sizes have an average computation time of 3600 seconds, which is the set limit of computing time. The calculations for three sublots and three jobs and more also lead to average computation time around 3600 seconds. The lot streaming problems with two sublots for each job are mostly solved optimally in less than 1 hour. The computation times without lot streaming, i.e. with just one sublot per job level, are around 1 second for the considered Inc_1 instances.

Figure 5.5: Average computation time with consistent sublots (Inc_1)



Increasing the number of missing operations, as done by switching to the Inc_2 instances, leads to higher computation times for the lot streaming problem with two sublots per job. On the other hand, the computation times for the three sublot problem drop slightly for the considered problems, as can be seen by comparing Figure 5.5 and 5.6. The computational effort for the problem without lot streaming does not change. The four-sublot problem is difficult to evaluate since the calculation times of the comparable problem sizes are around 3600 seconds in the cases of lower and higher numbers of missing operations.

Figure 5.6: Average computation time with consistent sublots (Inc_2)



Increasing the number of consistent sublots to values higher than two leads to greater computational effort, without reaching the same reduction of the makespan compared to the introduction of lot streaming.

The number of iterations also depends on the number of machines, as seen in Table 5.3. Problems with a higher number of machines need fewer iterations to be solved than comparable problems with a lower number of machines and the same number of jobs and levels per job. The calculated iterations for the problem without lot streaming and ten machines need the lowest number of CPLEX iterations with, on average, 284.28 iterations for the Inc_1 problem set and 3117.93 iterations for Inc_2 problems. The highest average number of iterations is 11923622.61 for the $Q = 4$ problems in Inc_1. For problems that are solved optimally or near optimally, i.e. mainly the calculation with $Q = 1$ and $Q = 2$, the number of iterations is higher if more missing operations are included within the job levels.

Table 5.3: Influence of m on CPLEX iterations if the sublots are consistent

Inc	m	It^1	It^2	It^3	It^4
1	5	3224.83	4,466,321.71	4,090,186.17	11,923,622.61
	10	284.28	1,015,293.40	1,980,717.80	5,534,364.94
2	5	8056.43	3,560,833.63	4,000,401.57	10,279,038.53
	10	3117.93	1,721,735.00	2,406,330.40	3,830,610.90

Table 5.4 shows the solution quality for different values of Q depending on the number of levels L per job. The Inc_2 test set includes only problems with $L = 4$ levels per job; therefore, a differentiation of the solution quality and number of CPLEX iterations based on the number of levels is not possible for these instances. The values of gaps after a maximum computation time of 1 hour increase with the increasing number of levels. The higher the number of levels, the more permutations are possible, which increases the computational effort necessary to solve the problem optimally. The values of makespan reductions for a given step from $Q - 1$ to Q are almost unaffected by the number of levels.

Table 5.4: Influence of L on makespan if the sublots are consistent

Inc	L	G^1 [%]	G^2 [%]	G^3 [%]	G^4 [%]	$\Delta C_{max}^{2\|1}$ [%]	$\Delta C_{max}^{3\|2}$ [%]	$\Delta C_{max}^{4\|3}$ [%]
1	2	0.00	0.67	11.48	19.45	−31.88	−11.78	−5.48
	3	0.00	3.80	16.93	25.62	−34.85	−13.46	−7.10
	4	0.00	6.82	21.65	33.35	−35.37	−13.07	−6.31
2	4	0.00	8.19	23.26	35.68	−33.74	−11.82	−5.30

The number of iterations distinguished by the number of levels and the number of sublots per job is shown in Table 5.5. The higher the number of levels, the higher the number of iterations performed by CPLEX. There are no large differences visible between the $L = 4$ instances with a lower or higher number of missing operations.

Table 5.5: Influence of L on CPLEX iterations if the sublots are consistent

Inc	L	It^1	It^2	It^3	It^4
1	2	178.80	2,282,915.23	3,973,201.18	10,873,712.45
	3	1829.28	2,625,889.20	2,661,204.70	8,827,528.18
	4	3255.58	3,313,618.23	2,471,950.07	6,485,740.70
2	4	5587.18	2,641,284.32	3,203,365.98	7,054,824.72

The following Tables 5.6 and 5.7 evaluate the solution quality using gap values and makespan reduction values as well as the computational effort (by the number of iterations) for different numbers of jobs.

Table 5.6: Influence of n on makespan if the sublots are consistent

Inc	n	G^1 [%]	G^2 [%]	G^3 [%]	G^4 [%]	$\Delta C_{max}^{2\|1}$ [%]	$\Delta C_{max}^{3\|2}$ [%]	$\Delta C_{max}^{4\|3}$ [%]
1	2	0.00	0.00	1.06	6.52	−38.70	−18.28	−9.03
	3	0.00	0.00	1.19	6.87	−38.48	−18.17	−9.01
	4	0.00	0.00	1.58	7.52	−38.47	−18.05	−8.93
2	2	0.00	0.00	3.42	14.99	−39.68	−18.13	−8.69
	3	0.00	5.32	25.90	39.69	−34.55	−10.75	−4.77
	4	0.00	19.26	40.46	52.36	−27.00	−6.57	−2.44

The four-job four-sublot problem instances required more than 3600 seconds of computation time on average for test sets Inc_1 and Inc_2. However, the number of iterations performed for Inc_1 instances is almost twice as high as for Inc_2 instances. Thus, the time for single CPLEX iterations increases with a higher number of missing operations.

Table 5.7: Influence of n on CPLEX iterations if the sublots are consistent

Inc	n	It^1	It^2	It^3	It^4
1	2	109.58	5927.38	919,157.70	7,637,007.68
	3	230.97	2,754,148.20	4,168,759.72	10,487,879.42
	4	4923.12	5,462,347.08	4,018,438.53	8,062,094.23
2	2	153.80	21,055.35	1,829,851.55	9,301,545.65
	3	379.70	3,055,880.65	3,713,227.00	6,871,212.80
	4	16,228.05	4,846,916.95	4,067,019.40	4,991,715.70

5.2.2 Consecutive Sublots

The equations (5.16) secure a consecutive processing of sublots that belong to the same job level. In the following, this is called consecutive sublots. If a sublot q' of a job level i', l' does not precede sublot q' of another job i's level l ($y_{i'q'l'}^{iq'l} = 0$), then all other sublots $q = 1, \ldots, Q$ precede sublot q' of job i' in level l' ($y_{iql}^{i'q'l'} = 1$).

$$y_{i'q'l'}^{iql} + y_{iq'l}^{i'q'l'} = 1 \tag{5.16}$$
$$\forall i, i' = 1, \ldots, n; i > i'; l, l' = 1, \ldots, L; q, q' = 1, \ldots, Q.$$

Considering the example from Section 5.2.1, the optimal makespan increases if consecutive processing is necessary. The new makespan is now $C_{max} = 14.25$. The variables of the solution are shown below.

Job one is split into two sublots, with 50 parts each. The sublots of job two are given by X^{2q}, with 75 parts in the first and 25 parts in the second sublot. The first sublot of job three is smaller than its second sublot.

$$X^{1q} = \begin{pmatrix} 50 & 50 \end{pmatrix}, \qquad X^{2q} = \begin{pmatrix} 75 & 25 \end{pmatrix}, \qquad X^{3q} = \begin{pmatrix} 29 & 71 \end{pmatrix}.$$

The resulting permutation is represented in Table 5.8. Job $i = 1$ starts the permutation with its first sublot in level $l = 1$. The last position in the sequence is occupied by the second sublot of job $i = 2$ in level $l = 2$.

Table 5.8: Optimal permutation of the example with consecutive consistent sublots

Position	1	2	3	4	5	6	7	8	9	10	11	12
Job	1	1	3	3	1	1	2	2	3	3	2	2
Level	1	1	1	1	2	2	1	1	2	2	2	2
Sublot	1	2	1	2	1	2	1	2	1	1	2	2

The associated starting times s_{lk}^{ij} are:

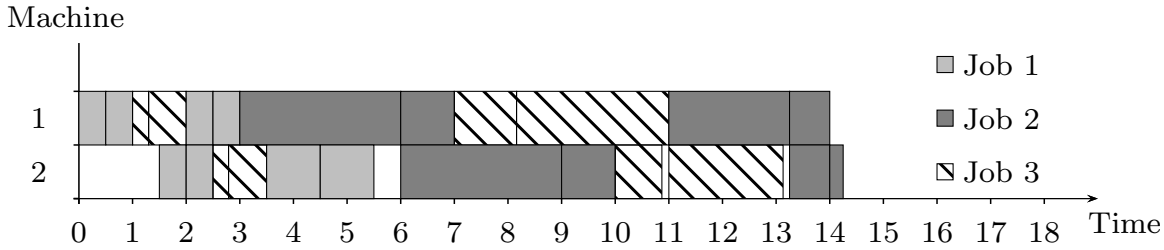
$$s_{lk}^{11} = \begin{pmatrix} 0 & 1.5 \\ 1.5 & 3.5 \end{pmatrix}, \qquad s_{lk}^{12} = \begin{pmatrix} 0.5 & 2 \\ 2.5 & 4.5 \end{pmatrix},$$

$$s_{lk}^{21} = \begin{pmatrix} 3 & 6 \\ 11 & 13.25 \end{pmatrix}, \qquad s_{lk}^{22} = \begin{pmatrix} 6 & 9 \\ 13.25 & 14 \end{pmatrix},$$

$$s_{lk}^{31} = \begin{pmatrix} 1 & 2 \\ 6.2 & 9.4 \end{pmatrix}, \qquad s_{lk}^{32} = \begin{pmatrix} 1.8 & 2.8 \\ 12.6 & 13.4 \end{pmatrix}.$$

The Gantt chart of the schedule is shown in Figure 5.7. The resulting makespan is $C_{max} = 14.25$, which is higher than the makespan without the consecutive sublot constraints (5.16).

Figure 5.7: Solution to the example with consecutive sublots



Influence of Consecutive Sublots

The test instances are identical to the instances tested in Section 5.2.1, except for a limitation of the number of sublots per job to $Q \leq 3$.[28]

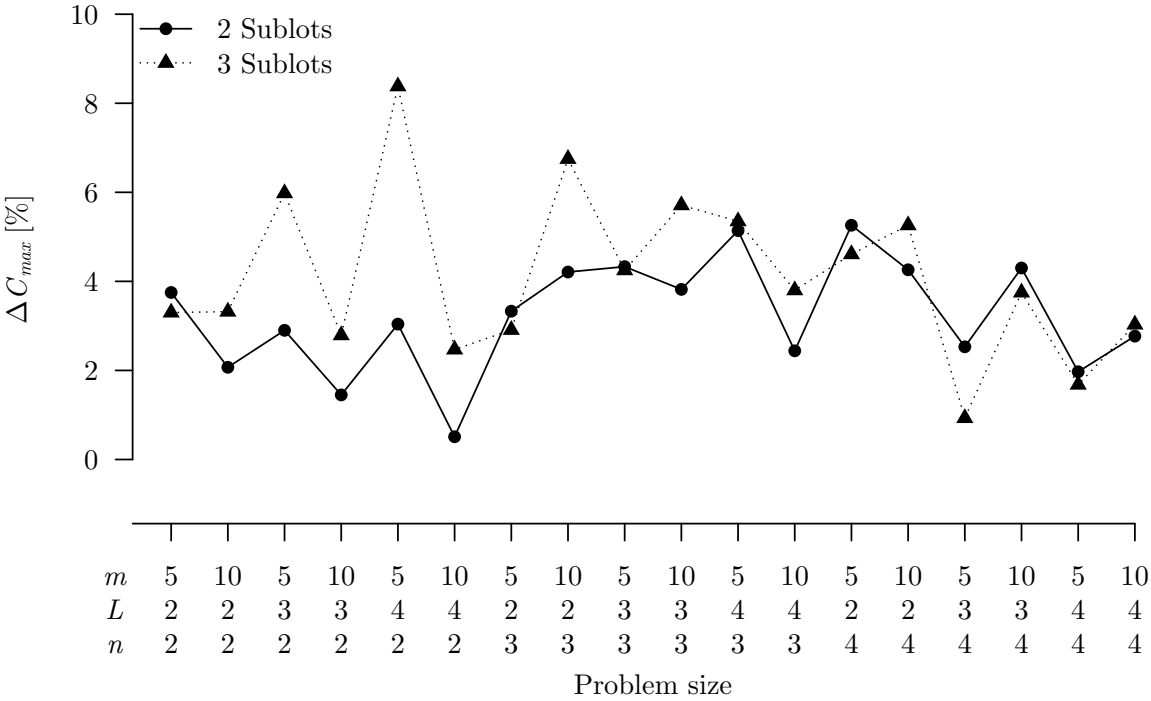
Figures 5.8 and 5.9 show the mean relative makespan deviation between the model with consecutive consistent sublots and the previously introduced model with consistent sublots. The values ΔC_{max}^{Q} in Table 5.9 are calculated by:

$$\Delta C_{max}^{Q} = \frac{C_{max}^{Q,\,consecutive}}{C_{max}^{Q,\,consistent}} - 1.$$

The mean relative makespan values are positive for all problem sizes of the Inc_1 problem set. This means that the makespan is higher on average in the consecutive sublot model than in the simple consistent case, even for the problem sizes that are not solved in time if the simple consistent sublots model is used. The lowest increase in makespan for the Inc_1 instances is 0.51 % for the $Q = 2, n = 2, L = 4, m = 10$ instances. The highest deviation is 8.38 % for the problem size $n = 2, L = 4, m = 5$ and $Q = 3$ sublots per job. The average makespan deviation of $Q = 3$ sublot lot streaming is higher for 11 of 18 problem sizes of Inc_1.

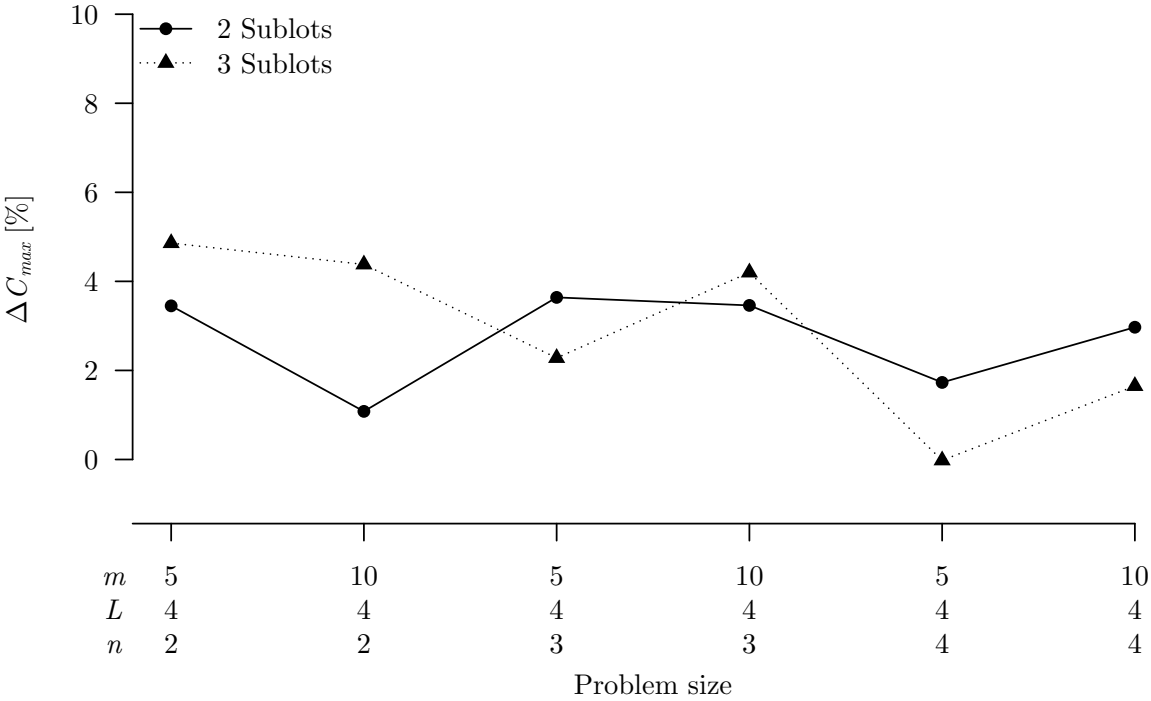
[28] See also the Tables B.3 and B.1 in Appendix B (p. 205 and p. 203).

Figure 5.8: Average makespan deviations with consecutive sublots (Inc_1)



The makespan deviations shown in Figure 5.9 represent the values for Inc_2 instances. The value for instances of the problem size $n = 4, L = 4, m = 10, Q = 3$ is -0.02 %, meaning that the makespan values achieved with the consecutive sublot model are on average better for this problem size, which is due to the limited computation time. The values for both two- and three-sublot problems in the Inc_2 set are lower for $m = 5$ problems than they are in the Inc_1 problem set. For $m = 10$ problems, the makespan deviations are on the same level as those for Inc_1 instances.

Figure 5.9: Average makespan deviations with consecutive sublots (Inc_2)



The average gap between the lower bound and incumbent solution after one hour of computation time is mostly closed for the problems with $Q = 2$ consecutive sublots per job, except for the $n = 4$ problem of Inc_2, as seen in Table 5.9. The gaps, G^Q, for $Q = 3$ consecutive sublots are higher than for $Q = 2$ problems and show an increase with the increasing number of jobs. The gap values are equal or lower for consecutive sublots than without consecutive sublots. The makespan deviation increases with the number of jobs, if $Q = 2$. It decreases with the number of jobs, if $Q = 3$, since the gaps for the formulation without consecutive sublots are relatively large for these instances compared to the formulation with consecutive sublots.

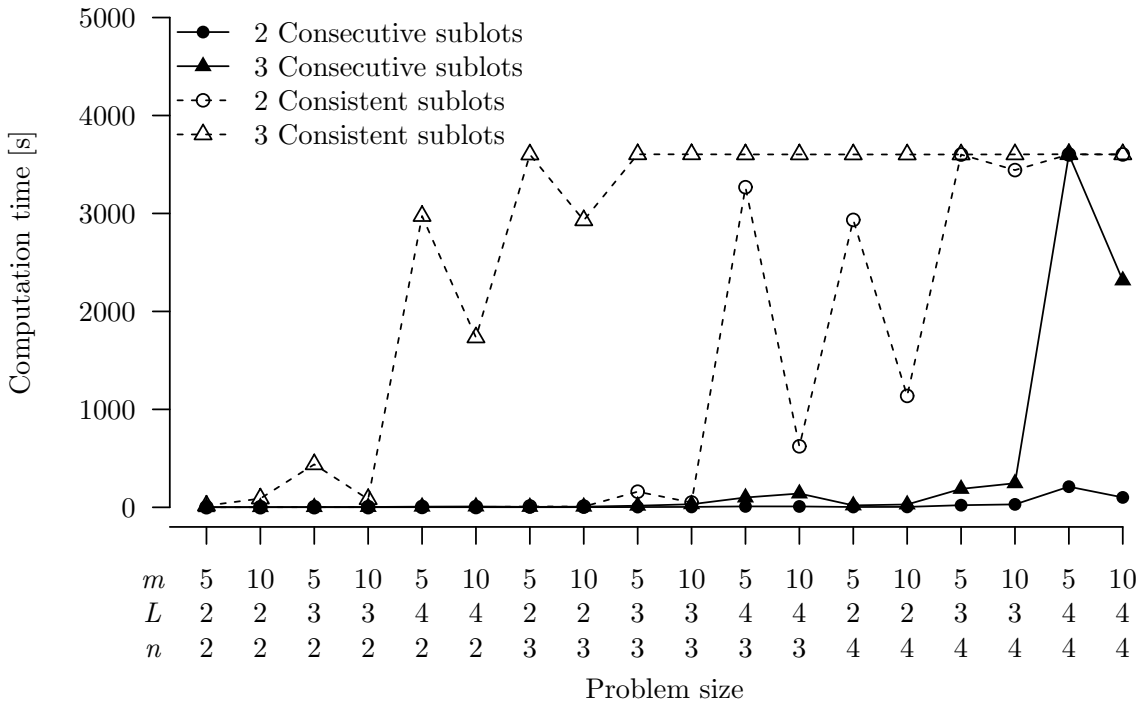
Table 5.9: Solution quality of consistent (I) and consecutive (II) sublots

Inc	n	G^2 I [%]	G^2 II [%]	G^3 I [%]	G^3 II [%]	ΔC^2_{max} [%]	ΔC^3_{max} [%]
1	2	0.00	0.00	1.06	0.00	2.28	4.38
	3	0.96	0.00	17.26	0.00	3.88	4.79
	4	10.32	0.00	31.73	1.41	3.52	3.21
2	2	0.00	0.00	3.42	0.00	2.26	4.62
	3	5.32	0.00	25.90	0.01	3.55	3.24
	4	19.26	0.12	40.46	3.76	2.35	0.82

The computation time of the models with consistent and consecutive sublots for Inc_1

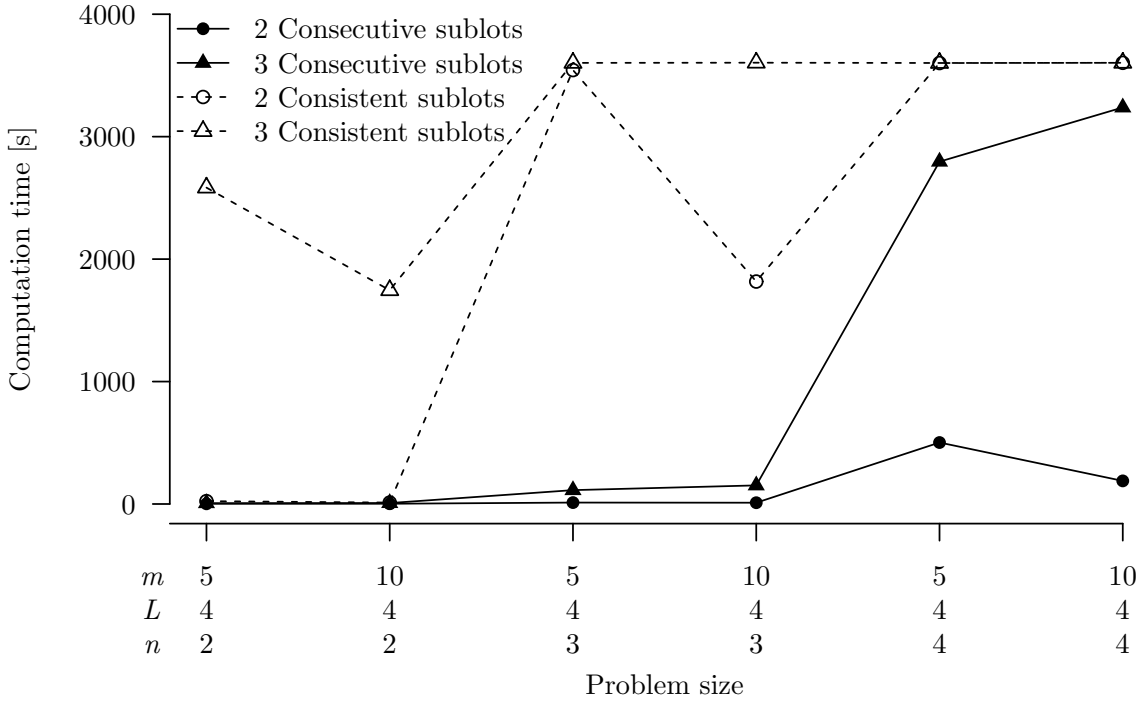
instances are shown in Figure 5.10. For most of the problem sizes, the average computation time of CPLEX for solving the problems with consecutive sublots is only a few seconds for the two- and three-sublot configuration. Except for the problem sizes with four jobs and four levels, the computation times in this case are much higher for $Q = 3$. The computation time reaches the limit of 3600 seconds in the case of $m = 5$ and is about 2300 seconds for $m = 3$, while the times for the same problems with the $Q = 2$ configuration are 210 and 100 seconds, respectively. Despite reaching the limit of computing time for one problem size, the computing times are generally lower for the consecutive sublot model than the computation times of the consistent sublot model, which does not prescribe a consecutive processing of sublots of the same job level.

Figure 5.10: Average computation times with consecutive sublots (Inc_1)



The same effect of lower computation times is also visible for Inc_2 instances in Figure 5.11. The computation times with the consecutive sublot formulation are lower than the computing times without consecutive sublots. There is no pattern visible that describes how the increased number of missing operations influences the computation time for the consecutive sublot model.

Figure 5.11: Average computation times with consecutive sublots (Inc_2)



The numbers of CPLEX iterations, It^Q, performed to solve the problems optimally or until a computation time of 1 hour is reached are shown in Table 5.10. The consecutive lot model requires more iterations than the consistent sublot model only for the $n = 4$ Inc_2 instances. In all other cases, the average numbers of iterations are lower.

Table 5.10: Influence of consecutive sublots on CPLEX iterations

Inc	n	Consistent		Consecutive	
		It^2	It^3	It^2	It^3
1	2	5927.38	919,157.70	268.08	5201.80
	3	2,754,148.20	4,168,759.72	11,629.53	188,970.62
	4	5,462,347.08	4,018,438.53	516,747.68	3,416,716.17
2	2	21,055.35	1,829,851.55	587.50	15,446.85
	3	3,055,880.65	3,713,227.00	50,285.45	616,849.80
	4	4,846,916.95	4,067,019.40	3,135,292.20	8,353,501.75

5.2.3 Equal Sublots

The intention of testing equally sized sublots is to reduce computational efforts without losing much of the solution quality. The size of the sublots is now solely predetermined by the number of sublots Q and the number of parts per job D^i. The results for equally sized

sublots are shown in Section 5.2.3. The model is similar to the simple consistent sublot model introduced in Section 5.2.1. However, the only difference is that the constraints (5.1) are replaced with the equations (5.17).

$$X^{iq} = \frac{D^i}{Q} \qquad (5.17)$$
$$\forall i = 1, \ldots, n; q = 1, \ldots, Q.$$

The solution to the problem above is given in Table 5.11.

Table 5.11: Optimal permutation of the example with equal sublots

Position	1	2	3	4	5	6	7	8	9	10	11	12
Job	3	2	1	3	1	2	1	3	2	1	3	2
Level	1	1	1	1	1	1	2	2	2	2	2	2
Sublot	1	1	1	2	2	2	1	1	1	2	2	2

The equal sublot constraints lead to a size of 50 parts per sublot.

$$X^{1q} = \begin{pmatrix} 50 & 50 \end{pmatrix}, \qquad X^{2q} = \begin{pmatrix} 50 & 50 \end{pmatrix}, \qquad X^{3q} = \begin{pmatrix} 50 & 50 \end{pmatrix}.$$

The starting times are given below with the values s_{lk}^{iq}. The starting time of the first sublot of job $i = 3$ in level $l = 1$ on machine $m = 1$ is equal to 0 since it is the first sublot in the sequence.

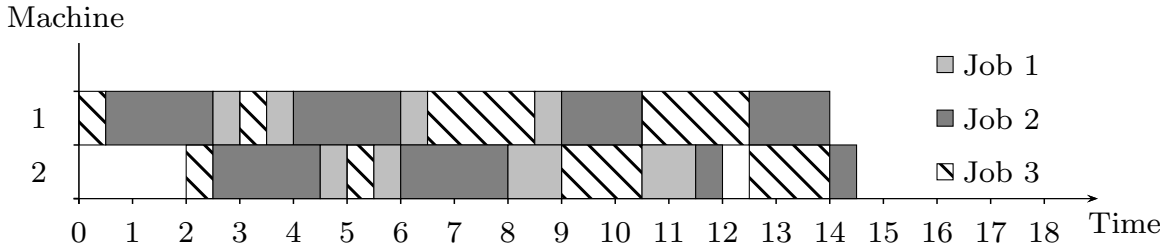
$$s_{lk}^{11} = \begin{pmatrix} 2.5 & 4.5 \\ 6 & 8 \end{pmatrix}, \qquad s_{lk}^{12} = \begin{pmatrix} 3.5 & 5.5 \\ 8.5 & 10.5 \end{pmatrix},$$

$$s_{lk}^{21} = \begin{pmatrix} 0.5 & 2.5 \\ 9 & 11.5 \end{pmatrix}, \qquad s_{lk}^{22} = \begin{pmatrix} 4 & 6 \\ 12.5 & 14 \end{pmatrix},$$

$$s_{lk}^{31} = \begin{pmatrix} 0 & 2 \\ 6.5 & 9 \end{pmatrix}, \qquad s_{lk}^{32} = \begin{pmatrix} 3 & 5 \\ 10.5 & 12.5 \end{pmatrix}.$$

The schedule with equally sized sublots for every job is shown in Figure 5.12. It leads to a makespan of $C_{max} = 14.5$. The value of the makespan is higher than that without the restriction of having equal sublots. In this example, it is even higher than the makespan of 14.25 achieved by the consecutive sublot model.

Figure 5.12: Solution to the example with equal sublots



Influence of Equal Sublots

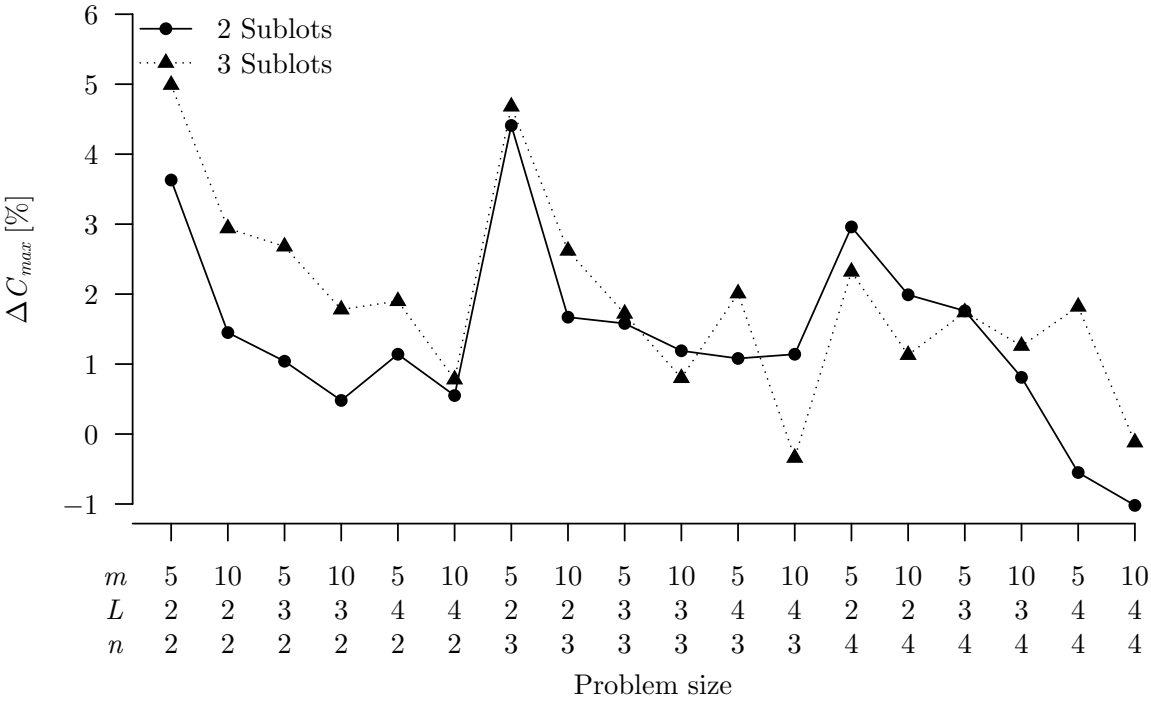
The test instances are identical to the instances tested in Sections 5.2.1 and 5.2.2. The number of sublots per job is limited to $Q \leq 3$.[29] The relative makespan deviations in this subsection are calculated by:

$$\Delta C_{max}^{Q} = \frac{C_{max}^{Q,\,equal}}{C_{max}^{Q,\,consistent}} - 1.$$

The mean relative makespan deviations for the Inc_1 instances depending on problem size, including the number of sublots, are presented in Figure 5.13. The deviations range from -1.02 % to 4.41 %. It is remarkable that the deviations for the problem sizes $n = 4, L = 4, m = 5$ and $n = 4, L = 4, m = 10$ for $Q = 2$ sublots and $n = 3, L = 4, m = 10$ for $Q = 3$ are all negative. This means that the equal sublot model delivers better results under the given maximum computation time than the less restricted, simple consistent sublot model.

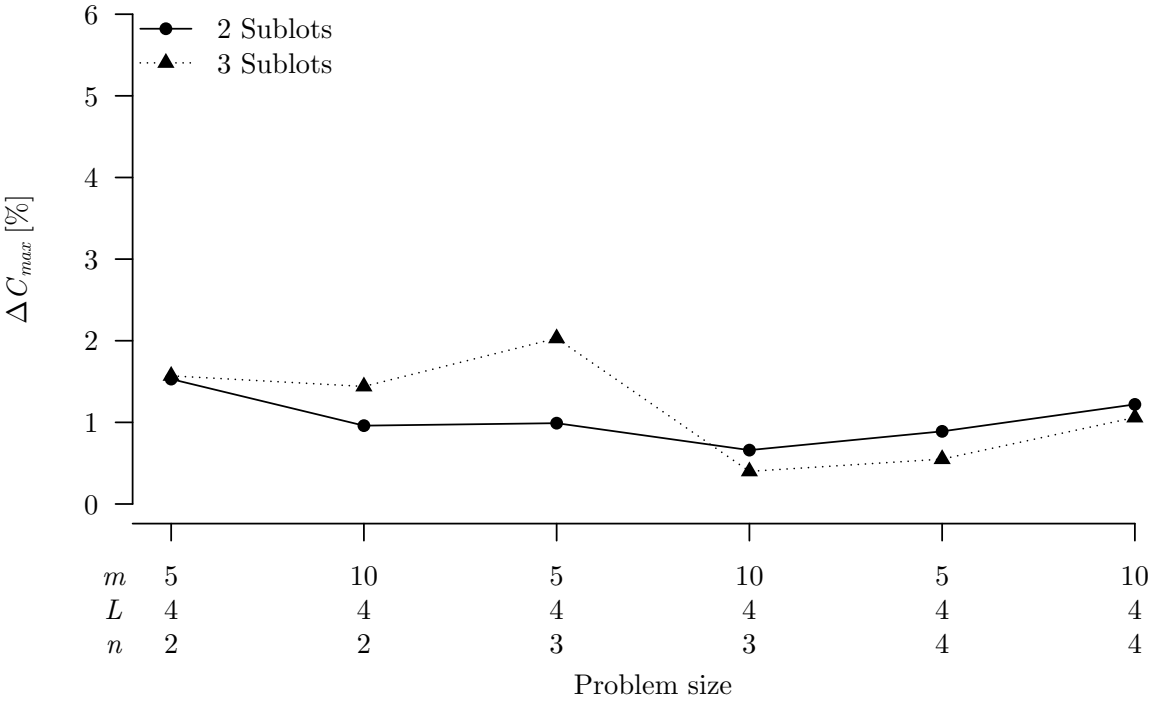
[29] See also the Tables B.3 and B.1 in Appendix B (p. 205 and p. 203).

Figure 5.13: Average makespan deviations with equal sublots (Inc_1)



The deviations of makespan shown for the Inc_2 instances in Figure 5.14 are all positive but on a relatively low range between 0.40 % and 2.03 % for the two- and three-sublot configurations.

Figure 5.14: Average makespan deviations with equal sublots (Inc_2)



Further details on the solution quality of the equal sublot model compared to the consistent sublot model are provided in Table 5.12. The gaps, G^Q, after the maximum computation time of 1 hour are all lower when the sublots are equally sized. Equal sublots seem to be suitable for a higher number of jobs since the makespan deviations of simple consistent sublots decrease if the number of jobs is increased. Five of the six mean relative makespan deviations, depending on the number of jobs, are lower for the Inc_2 instances. This means that a higher number of missing operations leads to a lower deviation between the models with consistent and equal consistent sublot sizes.

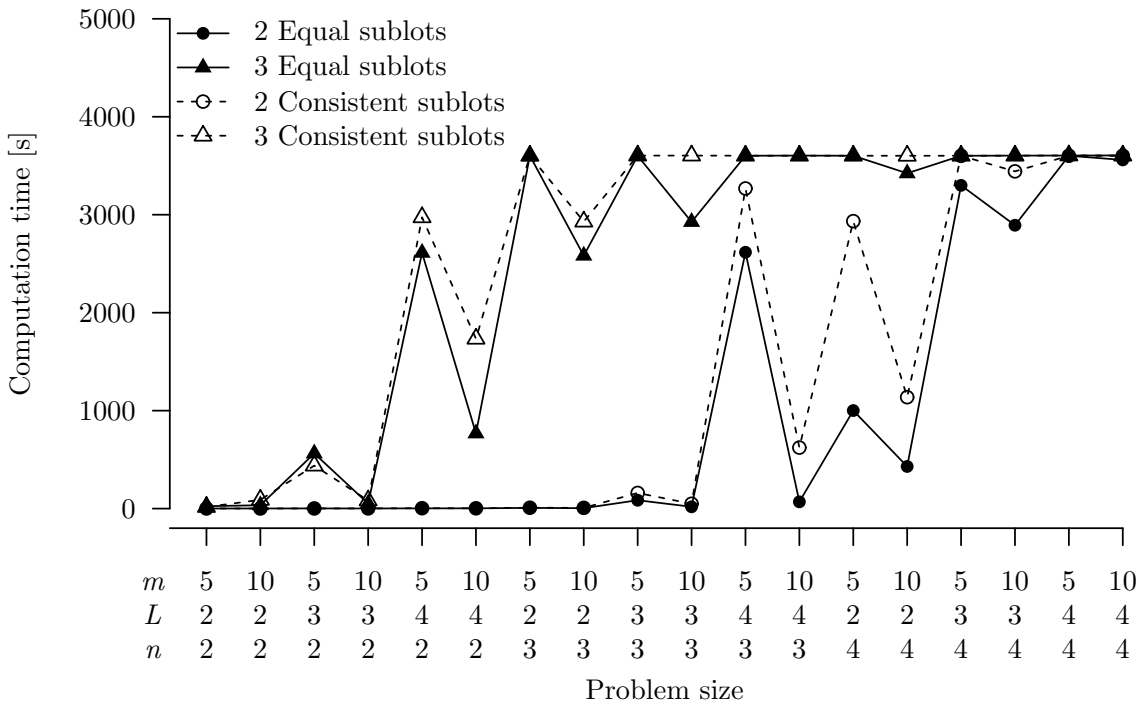
Table 5.12: Solution quality of consistent (I) and equal (II) sublots

Inc	n	G^2 I [%]	G^2 II [%]	G^3 I [%]	G^3 II [%]	ΔC^2_{max} [%]	ΔC^3_{max} [%]
1	2	0.00	0.00	1.06	0.77	1.38	2.51
	3	0.96	0.31	17.26	14.32	1.85	1.91
	4	10.32	6.98	31.73	28.13	0.99	1.36
2	2	0.00	0.00	3.42	2.36	1.25	1.50
	3	5.32	2.64	25.90	24.37	0.82	1.22
	4	19.26	18.10	40.46	38.54	1.06	0.81

The computation times of the equal sublot model are compared to the consistent sublot model in Figures 5.15 and 5.16. Both figures show that the average computation

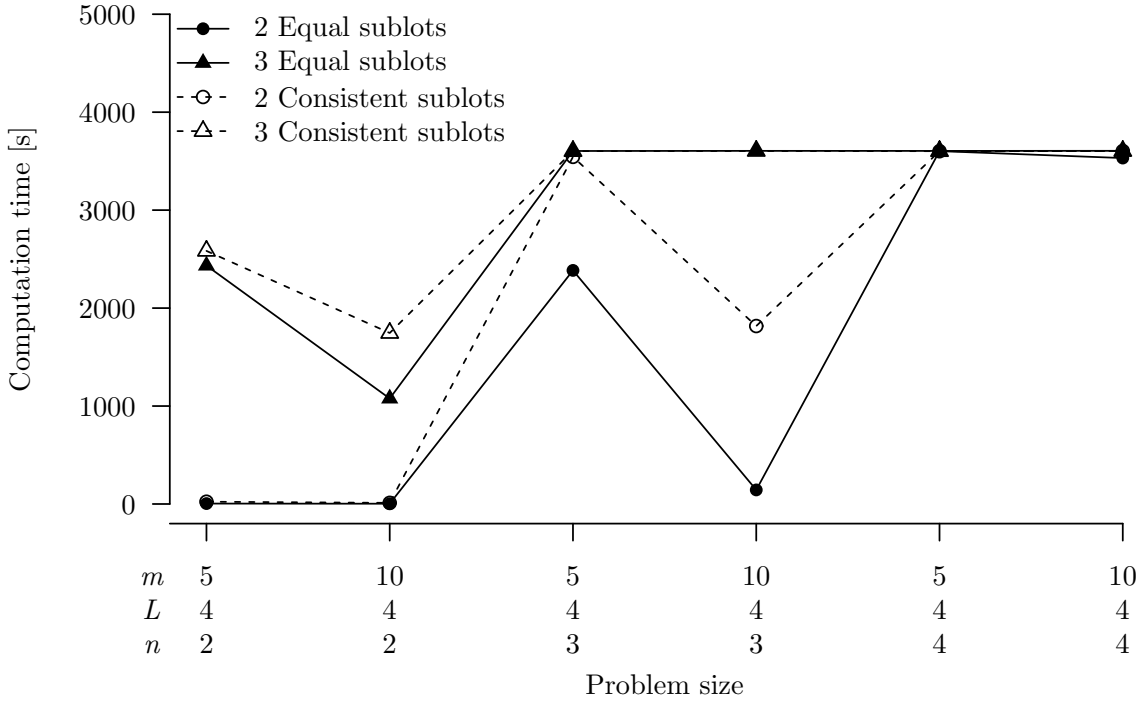
times for equal sublots is slightly below those of the consistent sublot model or that both models reach the limit of one hour. The single exception is the value for $n = 2, L = 3, m = 5, Q = 3$ in Figure 5.15 .

Figure 5.15: Average computation times with equal sublots (Inc_1)



The average computation times for the same problem sizes do not differ relevantly. Both show higher computation times for $m = 5$ problem sizes than for the corresponding $m = 10$ problems.

Figure 5.16: Average computation times with equal sublots (Inc_2)



The average numbers of iterations, It^Q, for the compared models are shown in Table 5.13. The instances with a number of jobs $n = 2$ are solved optimally for $Q = 2$. The mean number of iterations for these instances is seven times higher for the consistent sublot model than for the equal sublots. A similar relation can be observed for $n = 2$ instances with $Q = 3$ sublots, even if they are not all solved optimally by both models. The average number of iterations for $n = 4$, which are mostly not solved optimally by the consistent sublot model and the equal sublot model, are higher for the equal sublot model, indicating a shorter computation time for single iterations performed on this model.

Table 5.13: Influence of equal sublots on CPLEX iterations

Inc	n	Consistent		Equal	
		It^2	It^3	It^2	It^3
1	2	5927.38	919,157.70	779.17	182,664.53
	3	2,754,148.20	4,168,759.72	2,013,406.23	6,043,305.55
	4	5,462,347.08	4,018,438.53	7,092,066.87	4,480,098.72
2	2	21,055.35	1,829,851.55	6458.70	593,835.70
	3	3,055,880.65	3,713,227.00	4,817,891.50	2,302,744.60
	4	4,846,916.95	4,067,019.40	4,256,979.85	6,387,377.10

5.2.4 Consecutive Equal Sublots

A consecutive processing of sublots is achieved by the constraints (5.18).

$$y_{i'q'l'}^{iql} + y_{iq'l}^{i'q'l'} = 1 \tag{5.18}$$
$$\forall i, i' = 1, \ldots, n; i > i'; l, l' = 1, \ldots, L; q, q' = 1, \ldots, Q.$$

The jobs are split into Q equal sublots of size D^i/Q by the equations (5.19).

$$X^{iq} = \frac{D^i}{Q} \tag{5.19}$$
$$\forall i = 1, \ldots, n; q = 1, \ldots, Q.$$

The example from the previous sections is now solved with the consecutive and equal sublot constraints. The optimal permutation is shown in Table 5.14.

Table 5.14: Optimal permutation of the example with consecutive equal sublots

Position	1	2	3	4	5	6	7	8	9	10	11	12
Job	3	3	1	1	2	2	1	1	3	3	2	2
Level	1	1	1	1	1	1	2	1	2	2	2	2
Sublot	1	2	1	2	1	2	1	2	1	2	1	2

The sublot sizes are all 50 units per sublot.

$$X^{1q} = \begin{pmatrix} 50 & 50 \end{pmatrix}, \qquad X^{2q} = \begin{pmatrix} 50 & 50 \end{pmatrix}, \qquad X^{3q} = \begin{pmatrix} 50 & 50 \end{pmatrix}.$$

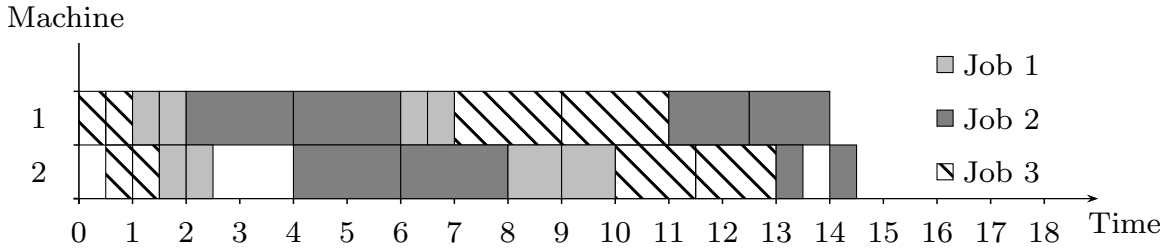
The starting times are given by the values s_{lk}^{iq} and visualized in Figure 5.17.

$$s_{lk}^{11} = \begin{pmatrix} 1 & 1.5 \\ 6 & 8 \end{pmatrix}, \qquad s_{lk}^{12} = \begin{pmatrix} 1.5 & 2 \\ 6.5 & 9 \end{pmatrix},$$

$$s_{lk}^{21} = \begin{pmatrix} 2 & 4 \\ 11 & 13 \end{pmatrix}, \qquad s_{lk}^{22} = \begin{pmatrix} 4 & 6 \\ 12.5 & 14 \end{pmatrix},$$

$$s_{lk}^{31} = \begin{pmatrix} 0 & 0.5 \\ 7 & 10 \end{pmatrix}, \qquad s_{lk}^{32} = \begin{pmatrix} 0.5 & 1 \\ 9 & 11.5 \end{pmatrix}.$$

Figure 5.17: Solution to the example with consecutive equal sublots



The resulting makespan is 14.5 time units, which is the worst value compared to simple consistent sublots, equal sublots and simple consistent consecutive sublots. This is reasonable, because the consecutive equal sublot model is the most restricted model among the mentioned formulations.

Influence of Consecutive Equal Sublots

The test instances are identical to the instances tested in Sections 5.2.1, 5.2.2 and 5.2.3. The number of sublots per job is limited to $Q \leq 3$.[30]

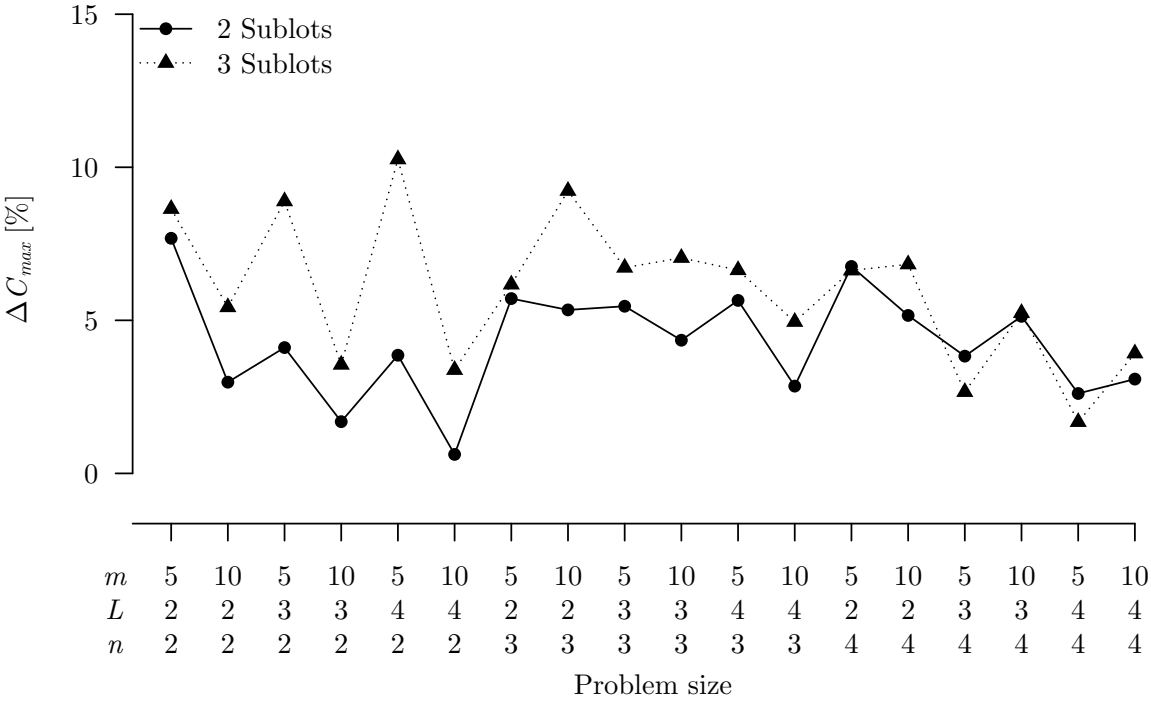
The relative makespan deviations in the tables and figures of this section are calculated with the formula:

$$\Delta C_{max}^{Q} = \frac{C_{max}^{Q,\,consecutive\,equal}}{C_{max}^{Q,\,consistent}} - 1.$$

Increases in the number of machines and levels lead to smaller mean relative makespan deviations which are visible in Figure 5.18. Increasing the number of jobs from $n = 2$ to $n = 3$ leads to higher deviations, i.e. to worse results for the consecutive equal sublot model compared to the consistent sublot model. The highest deviation is 10.26 % and occurs when each job level is split into three sublots. The value is 0.62 % for the Inc_1 instances and occurs for $Q = 2$. Therefore, the consecutive equal sublot model delivers results that are on average weaker than the consistent sublot model for all tested problem sizes in Inc_1.

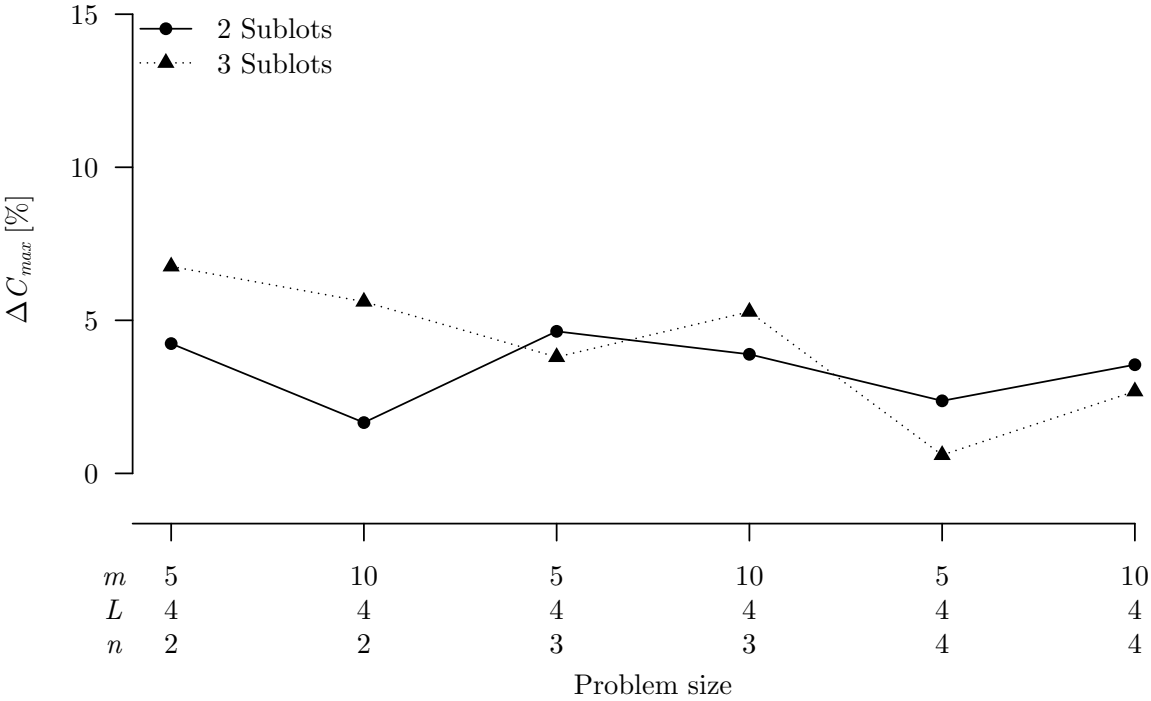
[30] See also the Tables B.3 and B.1 in Appendix B (p. 205 and p. 203).

Figure 5.18: Average makespan deviations with consecutive equal sublots (Inc_1)



The results shown in Figure 5.19 represent the makespan deviations for the Inc_2 instances. The range of deviations for this problem set is between 0.60 % and 6.76 %, which is similar to the same problem sizes in Inc_1.

Figure 5.19: Average makespan deviations with consecutive equal sublots (Inc_2)



The gap values, G^Q, of the consecutive equal sublot model shown in Table 5.15 are all lower than or equal to the gaps obtained with the consistent sublot model after a maximum computation time of 1 hour. Only three values are greater than zero, indicating that just a few of the $n = 4$ problems in Inc_1 and Inc_2 were not solved optimally or the incumbent was not approved as an optimal solution in time. The makespan deviations for consecutive equal sublots are on average higher than those for consecutive but not equal and equal but not consecutive sublots.

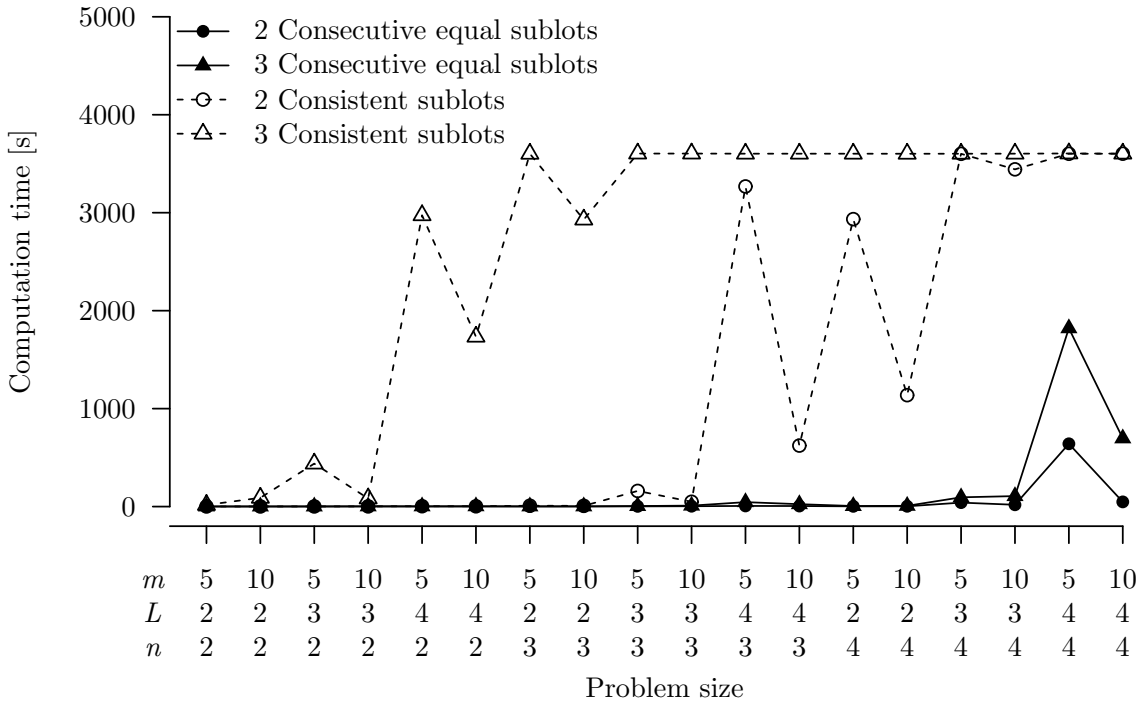
Table 5.15: Solution quality of consistent (I) and consecutive equal (II) sublots

Inc	n	G^2 I [%]	G^2 II [%]	G^3 I [%]	G^3 II [%]	ΔC^2_{max} [%]	ΔC^3_{max} [%]
1	2	0.00	0.00	1.06	0.00	3.49	6.69
	3	0.96	0.00	17.26	0.00	4.89	6.79
	4	10.32	0.00	31.73	0.05	4.43	4.50
2	2	0.00	0.00	3.42	0.00	2.95	6.18
	3	5.32	0.00	25.90	0.00	4.26	4.54
	4	19.26	0.19	40.46	0.94	2.96	1.64

Most of the average computation times of the model with consecutive equal sublots shown in Figure 5.20 are less than 10 seconds for the Inc_1 problem set. Only the values for the problem sizes with $n = 4$ and $L = 4$ are higher, between 18.50 and 640.90 seconds

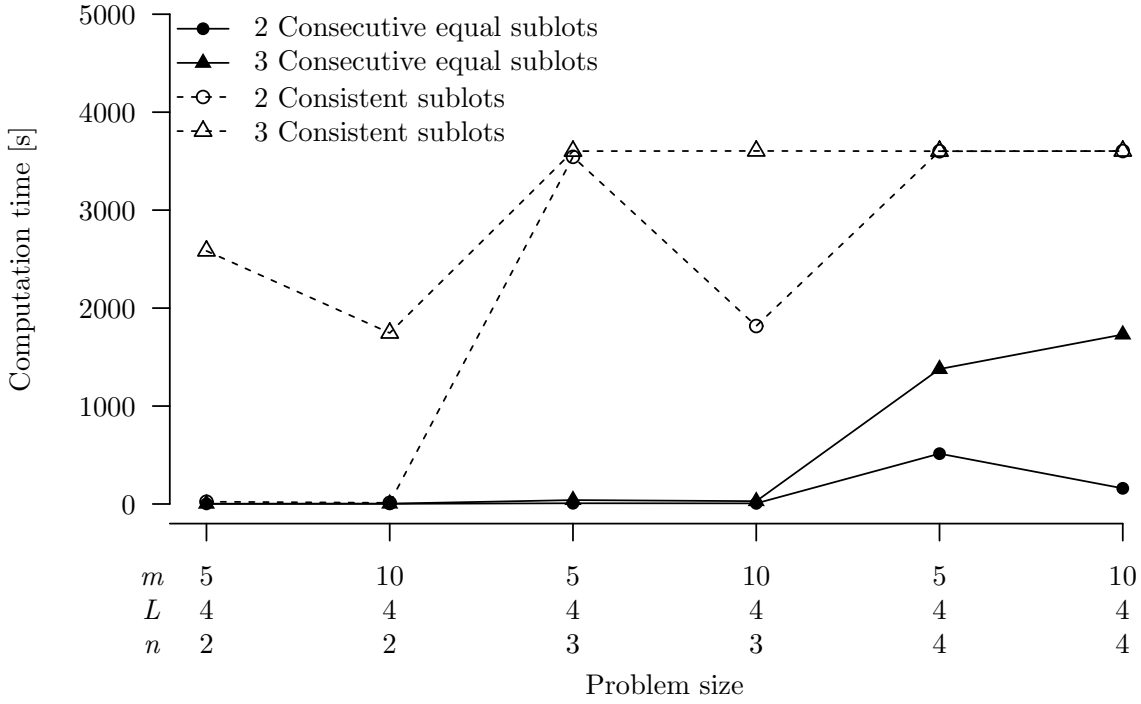
for $Q = 2$ sublots per job. For $Q = 3$ sublots, the mean computing times reach up to 1820.40 seconds.

Figure 5.20: Average computation times with consecutive equal sublots (Inc_1)



The corresponding computing times for Inc_2 presented in Figure 5.21 do not differ remarkably from the mean values in Figure 5.20.

Figure 5.21: Average computation times with consecutive equal sublots (Inc_2)



The average numbers of iterations, It^Q, depending on the number of jobs, are shown in Table 5.16. All the mean numbers of iterations performed on the problems of Inc_1 and Inc_2 with the consecutive equal sublot model are lower than those with the consistent sublot model. More iterations are necessary to solve a problem if the number of earlier exits from a level or of later re-entries is increased.

Table 5.16: Influence of consecutive equal sublots on CPLEX iterations

Inc	n	Consistent		Consecutive equal	
		It^2	It^3	It^2	It^3
1	2	5927.38	919,157.70	139.48	448.77
	3	2,754,148.20	4,168,759.72	1866.28	33,296.15
	4	5,462,347.08	4,018,438.53	243,235.55	1,538,305.07
2	2	21,055.35	1,829,851.55	217.45	1428.30
	3	3,055,880.65	3,713,227.00	12,754.10	136,345.25
	4	4,846,916.95	4,067,019.40	1,166,911.50	3,318,295.40

5.2.5 Resizing Sublots

This section proposes models that allow a resizing of sublots after a level is finished and the sublot re-enters the production system. The sublots are thus less strictly consistent.

Consistency remains within each level l, but not necessarily during re-entry.

To show the advantages of resizing, the following example is considered. $n = 2$ jobs with a number of parts equal to 100 each need to be processed in $L = 2$ levels on $m = 2$ machines.

$$D^1 = D^2 = 100.$$

The processing times per part are given below:

$$p^1_{lk} = \begin{pmatrix} 0.02 & 0.03 \\ 0.03 & 0.01 \end{pmatrix}, \qquad p^2_{lk} = \begin{pmatrix} 0.02 & 0.02 \\ 0.01 & 0.02 \end{pmatrix}.$$

The solution without resizing leads to a permutation shown in Table 5.17.

Table 5.17: Optimal permutation of the example with consistent sublots

Position	1	2	3	4	5	6	7	8
Job	2	1	2	2	1	1	2	1
Level	1	1	2	1	1	2	2	2
Sublot	1	1	1	2	2	1	2	2

The sublot sizes remain the same for all operations. The job $i = 1$ is divided into sublots of 39 and 61 parts. The second job, $i = 2$, is split into sublots with 28 and 72 parts.

$$X^{1q} = \begin{pmatrix} 39 & 61 \end{pmatrix}, \qquad X^{2q} = \begin{pmatrix} 28 & 72 \end{pmatrix}.$$

The starting times of each operation are given below. s^{11}_{lk} are the starting times of the first sublot of job $i = 1$, and s^{12}_{lk} are the starting times of the second sublot. The starting times s^{21}_{lk} and s^{22}_{lk} refer to job $i = 2$.

$$s^{11}_{lk} = \begin{pmatrix} 0 & 0.8 \\ 4.3 & 5.8 \end{pmatrix}, \qquad s^{12}_{lk} = \begin{pmatrix} 1.04 & 2.24 \\ 6.8 & 8.3 \end{pmatrix},$$

$$s^{21}_{lk} = \begin{pmatrix} 0.8 & 2 \\ 5.8 & 6.3 \end{pmatrix}, \qquad s^{22}_{lk} = \begin{pmatrix} 2.28 & 4.04 \\ 6.14 & 6.98 \end{pmatrix}.$$

The schedule results in a makespan of $C_{max} = 8.78$ for the processing of both jobs.

The permutation of sublots stays the same for the example concerning an enabled resizing of sublots (see Table 5.18).

Figure 5.22: Solution to the example without resizing of sublots

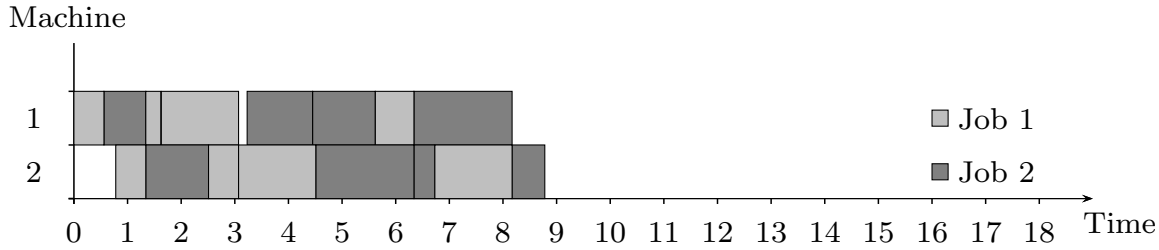


Table 5.18: Optimal permutation of the example with sublot resizing

Position	1	2	3	4	5	6	7	8
Job	2	1	2	2	1	1	2	1
Level	1	1	2	1	1	2	2	2
Sublot	1	1	1	2	2	1	2	2

The level index l is added to the sublot size variable X^{iq}, changing it to X_l^{iq}, to indicate different lot sizes for the sublots $q = 1, \ldots, Q$ of the jobs $i = 1, \ldots, n$ dependent on the levels $l = 1, \ldots, L$. The calculated sublot sizes with the possibility of resizing are:

$$X_l^{11} = \begin{pmatrix} 36 & 36 \end{pmatrix}, \qquad X_l^{12} = \begin{pmatrix} 64 & 64 \end{pmatrix},$$
$$X_l^{21} = \begin{pmatrix} 36 & 22 \end{pmatrix}, \qquad X_l^{22} = \begin{pmatrix} 64 & 78 \end{pmatrix}.$$

The corresponding starting times of the operations are:

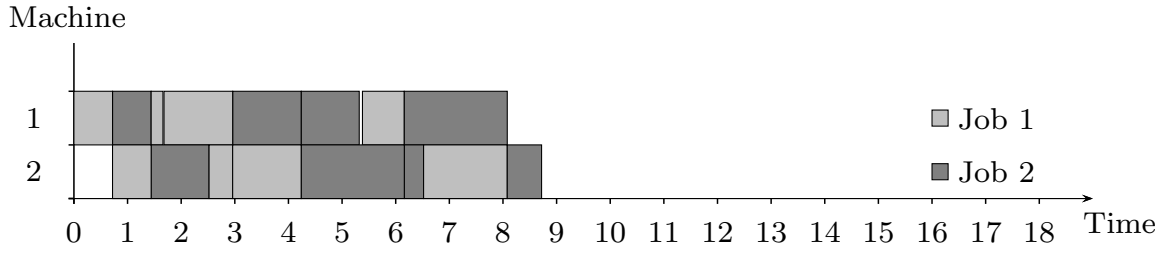
$$s_{lk}^{11} = \begin{pmatrix} 0.72 & 1.44 \\ 4.24 & 6.16 \end{pmatrix}, \qquad s_{lk}^{12} = \begin{pmatrix} 2.96 & 4.24 \\ 6.16 & 8.08 \end{pmatrix},$$

$$s_{lk}^{21} = \begin{pmatrix} 0 & 0.72 \\ 1.44 & 2.52 \end{pmatrix}, \qquad s_{lk}^{22} = \begin{pmatrix} 1.68 & 2.96 \\ 5.38 & 6.52 \end{pmatrix}.$$

The size of job $i = 2$'s sublot $j = 1$ decreases with the transition to level $l = 2$ from $X_1^{21} = 36$ to $X_2^{21} = 22$. The number of parts in the second sublot of job two increases from $X_1^{22} = 64$ to $X_2^{22} = 78$. After 1.44 time units, 36 parts are ready to be processed in level $l = 2$, since $s_{12}^{21} + p_{12}^{2} \cdot X_1^{21} = 1.44$. At this moment, the number of available parts decreases by 22, since $s_{21}^{21} = 1.44$ and $X_2^{21} = 22$. There are now 14 parts available to be added to the second sublot. The second sublot finishes level $l = 1$ after $s_{12}^{22} + p_{12}^{2} \cdot X_1^{22} = 2.96$ time units, making $X_1^{22} = 64$ parts available for forming the second

sublot in level $l = 2$. After 2.96 time units, 78 parts are waiting to be processed in level $l = 2$, forming sublot two with $X_2^{22} = 78$. The second sublot starts at a time $s_{21}^{22} \geq s_{12}^{22}$, resulting in a valid schedule regarding the starting times and numbers of ready parts.

Figure 5.23: Solution to the example with resizing of sublots



The possibility of resizing sublots reduces the makespan of this example to $C_{max} = 8.72$. The ΔC_{max} to the compared model without sublot resizing is 0.68% for this small example.

The following section introduces an MIP model to describe a valid schedule with a resizing option in order to minimize the makespan. The computational experiments give an overview of the effect of resizing for different problem sizes.

The models which feature sublot resizing before re-entering the production system do not consider equal lot sizes, since this would make the resizing obsolete. The resizing feature makes it necessary to modify the sublot size variables X^{iq} and add a level index l, resulting in a new variable X_l^{iq}. This variable distinguishes between the sizes of the lots in different levels. Also, a new binary variable $R^{iqq'l} \in \{0; 1\}$ is introduced to formulate the resizing. Two sublot indices are given by $q = 1, \ldots, Q$ and $q' = 1, \ldots, Q$ for every single job level. $R^{iqq'l}$ equals 1 if a sublot q of a job starts being processed at level $l + 1$ before sublot q' of the same job i has finished its level l; $R^{iqq'l}$ equals 0 otherwise.

The objective is the makespan:

$$\min C_{max}. \tag{5.20}$$

Every part of a job needs to be processed in the levels $l = 1, \ldots, L$, i.e. the sum of parts in the sublots per level needs to be equal to the parts contained by a job i. This is ensured by the constraints (5.21). The constraints need to be valid for each job and level since the size of a sublot is allowed to vary between levels.

$$\sum_{q=1}^{Q} X_l^{iq} = D^i \tag{5.21}$$

$$\forall i = 1, \ldots, n; l = 1, \ldots, L.$$

The correct order of operations of each single sublot is defined by the constraints (5.22). The equations (5.22) secure the correct machine order from machine 1 to m for each sublot of any job in every level.

$$p^i_{lk'} \cdot \left(s^{iq}_{lk} + p^i_{lk} \cdot X^{iq}_l\right) \cdot p^i_{lk} \leq p^i_{lk'} \cdot s^{iq}_{lk'} \cdot p^i_{lk} \tag{5.22}$$
$$\forall i = 1, \ldots, n; q = 1, \ldots, Q; l = 1, \ldots, L; k, k' = 1, \ldots, m; (k < k').$$

The constraints (5.23) ensure that a sublot q of a job i in level l is processed before sublot $q+1$ on every machine $k = 1, \ldots, m$.

$$\left(s^{iq}_{lk} + p^i_{lk} \cdot X^{iq}_l\right) \cdot p^i_{lk} \leq s^{i,q+1}_{lk} \cdot p^i_{lk} \tag{5.23}$$
$$\forall i = 1, \ldots, n; q = 1, \ldots, Q-1; l = 1, \ldots, L; k = 1, \ldots, m.$$

The equations (5.24) and (5.25) avoid the processing of two or more sublots on a machine at the same time. The cases for sublot $q = q'$ of the same job but in different levels are controlled by the constraints (5.26) and (5.27).

$$A \cdot \left(1 - y^{iql}_{i'q'l'}\right) + s^{l'k}_{i'q'} - s^{lk}_{iq} \geq p^i_{lk} \cdot X^{iql} \tag{5.24}$$
$$\forall i, i' = 1, \ldots, n; (i \geq i'); l, l' = 1, \ldots, L; k = 1, \ldots, m; q, q' = 1, \ldots, Q; if\ (i = i')\ \{q \neq q'\};$$
$$A \cdot y^{iql}_{i'q'l'} + s^{lk}_{iq} - s^{l'k}_{i'q'} \geq p^{i'}_{l'k} \cdot X^{i'q'l'} \tag{5.25}$$
$$\forall i, i' = 1, \ldots, n; (i \geq i'); l, l' = 1, \ldots, L; k = 1, \ldots, m; q, q' = 1, \ldots, Q; if\ (i = i')\ \{q \neq q'\}.$$

Equations (5.26) and (5.27) ensure that a sublot q of a job i is not processed simultaneously on a machine k in different levels l and l'.

$$A \cdot \left(1 - y^{iql}_{iql'}\right) + s^{l'k}_{iq} - s^{lk}_{iq} \geq p^i_{lk} \cdot X^{iql} \tag{5.26}$$
$$\forall i = 1, \ldots, n; l, l' = 1, \ldots, L\ (l \neq l'); k = 1, \ldots, m; q = 1, \ldots, Q;$$
$$A \cdot y^{iql}_{iql'} + s^{lk}_{iq} - s^{l'k}_{iq} \geq p^i_{l'k} \cdot X^{iql'} \tag{5.27}$$
$$\forall i = 1, \ldots, n; l, l' = 1, \ldots, L\ (l \neq l'); k = 1, \ldots, m; q = 1, \ldots, Q.$$

The resizing problem, i.e. providing a valid amount of parts for new sublot sizes, is depicted by the constraint sets (5.31), (5.28), (5.29) and (5.30). The equations (5.28) are time restrictions to the sublots $q > 1$ with respect to the ready times of parts forming new sublots. The constraints force the binary variable $R^{iqq'l}$ to be 1 if a certain sublot q' has finished a level and is available for providing parts for a new sublot q. The values of $R^{iqq'l}$ are regulated in the constraint set (5.28). If any starting time of a job i's sublot q on level $l+1$, $s^{iq}_{l+1,k}$, is lower than any of the times when an operation of a sublot q' of

the same job on the previous level l ends, $s_{lk}^{iq'} + X_{l+1}^{iq'} \cdot p_{lk}^{iq'}$, then $R^{iqq'l}$ equals 1.

$$p_{l+1,k'}^{i} \cdot \left(s_{lk}^{iq'} + p_{lk}^{i} \cdot X_{l}^{iq'}\right) \cdot p_{lk}^{i} \leq p_{l+1,k'}^{i} \cdot \left(A \cdot R^{iqq'l} + s_{l+1,k'}^{iq}\right) \cdot p_{lk}^{i} \tag{5.28}$$
$$\forall i = 1, \ldots, n; l = 1, \ldots, L-1; k, k' = 1, \ldots, m; q, q' = 1, \ldots, Q.$$

The limitation of starting times during level transition is not sufficient for the sublots $q > 1$ since the resizing of sublots can yield a situation where some parts of $q = 1$ are left over after starting $l + 1$. These parts can, for instance form a sublot $q = 2$, which starts into $l + 1$ before $q = 2$ is finished in l.

Another set of constraints (5.29) limits the level transition of all sublots $q = 1, \ldots, Q$ via the binary variables R^{iq1l}. All values of R^{iq1l} need to be equal to 0 in order to prevent the sublots from being processed in $l + 1$ before the first sublot has finished l.

$$R^{iq1l} = 0 \tag{5.29}$$
$$\forall i = 1, \ldots, n; l = 1, \ldots, L; q = 1, \ldots, Q.$$

The equations (5.30) secure a proper level transition of the last sublot of a job. The last sublot $q = Q$ of a job i is not allowed to start into level $l + 1$ before all sublots $q = 1, \ldots, Q$ have finished level l.

$$R^{iQql} = 0 \tag{5.30}$$
$$\forall i = 1, \ldots, n; l = 1, \ldots, L; q = 1, \ldots, Q.$$

The equations (5.31) determine the number of parts available to increase and decrease the number of parts of a sublot. Sublot q starts into level $l+1$ before sublot $q'+1$ finishes level l and after sublot q' finishes l, if $\sum_{q''=1}^{q'+1} R^{iqq''l} = 1$ and $\sum_{q''=1}^{q'} R^{iqq''l} = 0$. This means that the parts available for sublots $q'' = 1, \ldots, q$ in level $l + 1$ are not allowed to exceed the total number of parts provided by the sublots $q'' = 1, \ldots, q'$ in level l. But they are allowed to include fewer parts, as long as the constraints (5.21) hold.

$$\sum_{q''=1}^{q} X_{l+1}^{iq''} \leq \sum_{q''=1}^{q'} X_{l}^{iq''} + A \cdot \left(1 - \sum_{q''=1}^{q'+1} R^{iqq''l}\right) + A \cdot \sum_{q''=1}^{q'} R^{iqq''l} \tag{5.31}$$
$$\forall i = 1, \ldots, n; l = 1, \ldots, L-1; q = 1, \ldots, Q; q' = 1, \ldots, Q-1.$$

The makespan and completion times can be measured with the inequalities (5.32) and

(5.33).

$$s_{lk}^{iq} + p_{lk}^{i} \cdot X_{l}^{iq} \leq C_{max} \tag{5.32}$$
$$\forall i = 1, \ldots, n; q = 1, \ldots, Q; l = 1, \ldots, L; k = 1, \ldots, m;$$
$$s_{lk}^{iq} + p_{lk}^{i} \cdot X_{l}^{iq} \leq C_{i} \tag{5.33}$$
$$\forall i = 1, \ldots, n; q = 1, \ldots, Q; l = 1, \ldots, L; k = 1, \ldots, m.$$

The binary constraints on the variables $y_{i'q'l'}^{iql}$ and $R_{izz'l}$ are provided by equations (5.34) and (5.35).

$$y_{i'q'l'}^{iql} \in \{0; 1\} \tag{5.34}$$
$$\forall i, i' = 1, \ldots, n; i \geq i'; l, l' = 1, \ldots, L; q, q' = 1, \ldots, Q; if\ (i = i' AND l = l')\ q \neq q'$$
$$R_{iqq'l} \in \{0; 1\} \tag{5.35}$$
$$\forall i = 1, \ldots, n; q, q' = 1, \ldots, Q; l, l' = 1, \ldots, L.$$

The inequalities (5.36) and (5.37) are the non-negativity constraints for the starting times s_{lk}^{iq} and sublot sizes X_{l}^{iq}.

$$s_{lk}^{iq} \geq 0 \tag{5.36}$$
$$\forall i = 1, \ldots, n; q = 1, \ldots, Q; l = 1, \ldots, L; k = 1, \ldots, m;$$
$$X_{l}^{iq} \geq 0 \tag{5.37}$$
$$\forall i = 1, \ldots, n; q = 1, \ldots, Q; l = 1, \ldots, L.$$

Additional constraints ((5.38)) can be added to tighten the formulation regarding the unambiguousness of the sequence variables $y_{i'q'l'}^{iql}$.

$$y_{i'q'l'}^{iql} + y_{iql}^{i'q'l'} = 1 \tag{5.38}$$
$$\forall i, i' = 1, \ldots, n; i > i'; l, l' = 1, \ldots, L; q, q' = 1, \ldots, Q.$$

Possible additional restrictions regarding lot sizes and sequence

Equal sublot sizes are not reasonable in the context of resizing, because equally sized sublots do not allow differences between sublot sizes in the different levels. Nevertheless, it is possible to add constraints for consecutive sublots and a basic sublot sequence.

Consecutive sublots can be obtained by adding the constraints (5.39) to the formula-

tion.

$$y_{i'q'l'}^{iql} + y_{iq'l}^{i'q'l'} = 1 \tag{5.39}$$
$$\forall i, i' = 1, \ldots, n; i > i'; l, l' = 1, \ldots, L; q, q' = 1, \ldots, Q.$$

A possible basic sequence similar to the case of a re-entrant permutation flow shop without lot streaming can be achieved by adding the restrictions (5.40).

$$y_{i'q'l}^{iql} = y_{i'q'L}^{iqL} \tag{5.40}$$
$$\forall i, i' = 1, \ldots, n; i \geq i'; l = 1, \ldots, L; q, q' = 1, \ldots, Q; if\ (i = i')\ \{q \neq q'\}$$

Influence of Resizing Sublots

The tested problem sizes in Section 5.2.5 are described by the parameters $n \in \{2, 3\}$, $L \in \{2, 3\}$, $m \in \{2, 5, 6, 10\}$ and $Q \in \{2, 3\}$. Ten instances are generated for each problem size. These instances are the same as those in Sections 5.2.1 - 5.2.4 for identical problem size parameters.[31] The tested problems are part of the Inc_1 set.[32]

The resizing of sublots at a level transition is tested with and without consecutive processing of same level sublots. The makespan values are compared to the values obtained with the consistent sublot model. The consecutive processing of resized sublots is investigated to know whether the gain in solution space using sublot resizing still allows improvement of the consistent solution, even if the resizing is connected with a consecutive processing of sublots of the same job level. The average relative changes of makespan per problem size are displayed in Figure 5.24. The relative makespan deviations for the application of a sublot resizing option during level transition are calculated by:

$$\Delta C_{max}^{Q} = \frac{C_{max}^{Q,\,resize}}{C_{max}^{Q,\,consistent}} - 1,$$

and

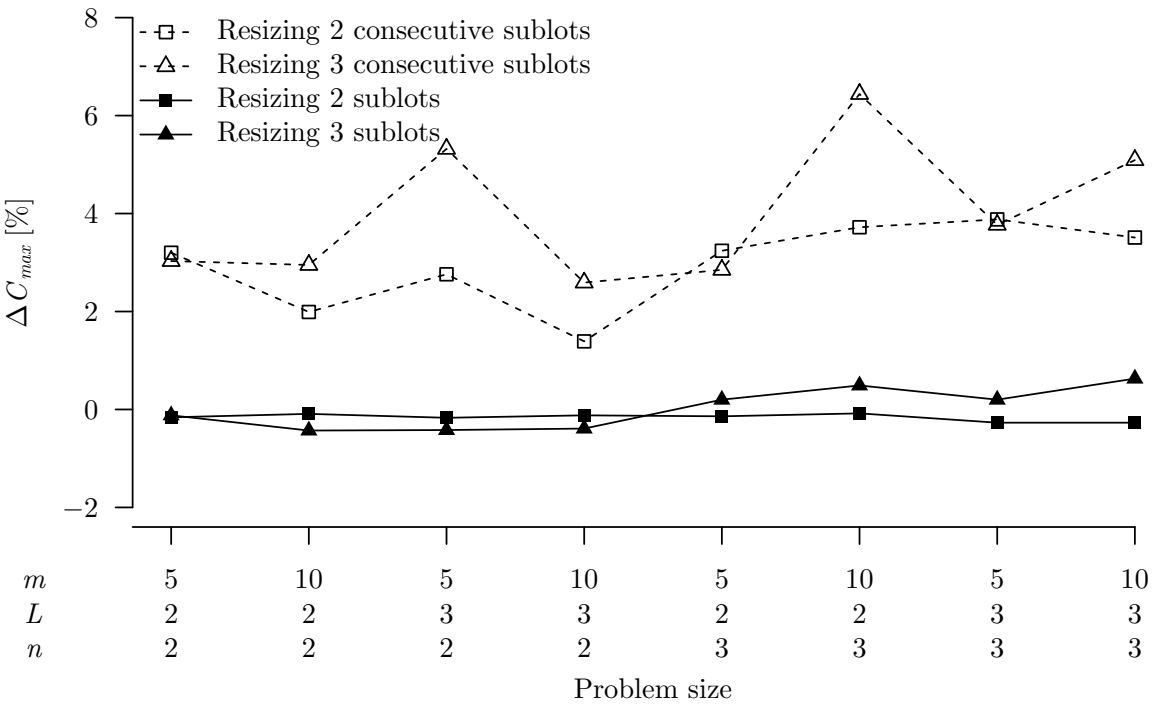
$$\Delta C_{max}^{Q} = \frac{C_{max}^{Q,\,resize\,consecutive}}{C_{max}^{Q,\,consistent}} - 1.$$

The resizing leads to slight improvements if the same level sublots do not need to be processed consecutively. The relative changes of makespan are slightly below 0 % if the problems are solved optimally, which is mainly the case for $Q = 2$ sublots per job. The mean relative changes in makespan are several times above 0 % for three sublots per job, because CPLEX runs out of time for these problem sizes. The consecutive processing

[31] See Table B.3 in Appendix B (p. 205) for detailed information on the problem sizes.
[32] See Table B.1 in Appendix B (p. 203) for an overview of missing operations.

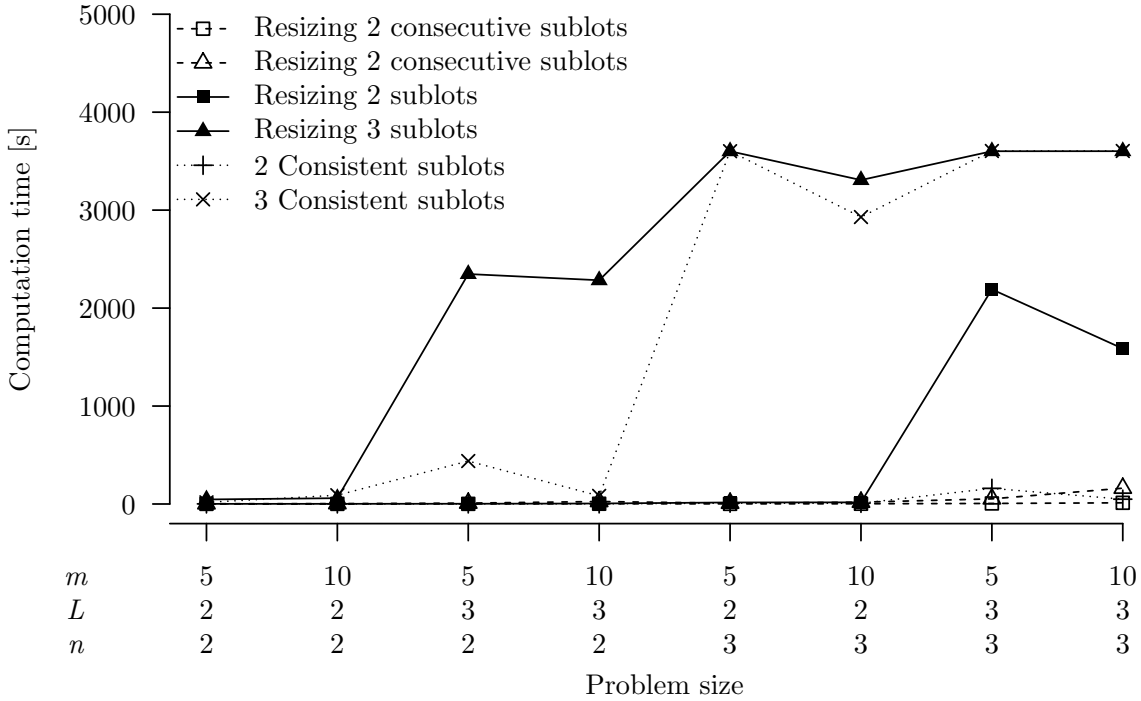
of the sublots still delivers weaker results than the consistent model if the sublots are allowed to be resized. The highest average makespan reduction of 0.43 % (- 0.43 % makespan deviation) during a computation time of 3600 seconds is achieved for $Q = 3$ sublots per job.

Figure 5.24: Average makespan deviations with resizing of sublots (Inc_1)



The computation times of the three different models, each with the two parameter configurations $Q = 2$ and $Q = 3$, are shown in Figure 5.25. A consecutive processing of sublots that are allowed to be resized, leads to low mean computation times under 170 seconds for the tested problem sizes. The resizing model without constraints for consecutive processing reaches the maximum computation time of 1 hour for $Q = 3$ sublots per job for most problem instances with $n = 3$ jobs. The possibility of resizing the sublots of a job during level transition leads to high computation times of more than 2000 seconds, even if just two jobs are split into three sublots. The resizing model requires computation times of only a few seconds for $n = L = 3$ instances on an average of around 2000 seconds to calculate the optimal solution if the number of sublots per job is $Q = 2$.

Figure 5.25: Average computation times with resizing of sublots (Inc_1)



5.2.6 Summary of Sublot Properties

This section gives an overview of the different lot streaming models for makespan minimization in re-entrant permutation flow shops. The consistent sublot model without consecutive processing delivers better results than the equal sublot models and the consecutive sublot models, but requires high computation times. One approach to balance the solution quality and computational requirements is to use equal sublots without a prescribed consecutive sublot processing. The makespan values are close to the results with a simple consistent model, but require less computation time. Although the consecutive processing of the sublots of the same job level reduces the computation time drastically, it also leads to a relatively low solution quality. A consecutive processing of sublots is not further pursued in Sections 5.3 and 5.4 due to the makespan values obtained and the fact that neighborhood moves of single sublots in the permutation are forbidden if consecutive processing is prescribed. The comparison between the consistent sublot model and the model with a resizing option during the level transition of sublots shows that the resizing option leads to much higher computational efforts, even for small problem sizes, than the consistent sublot model. The makespan reductions achieved by resizing the sublots is not high enough to compensate the increase in computational efforts. Therefore a resizing of sublots is not considered for the application of a variable neighborhood search for the lot streaming problem in Section 5.4.

Tables 5.19 and 5.20 summarize the number of constraints and variables for the different sublot configurations. Allowing a resizing of sublots results in a much higher number of constraints than without resizing.

Table 5.19: Number of constraints for lot streaming with consistent sublots

Constraints	Number
(5.2)	$\frac{1}{2}n \cdot L \cdot m \cdot (m-1) \cdot Q$
(5.3)	$n \cdot L \cdot m \cdot (Q-1)$
(5.4)	$n \cdot (L-1) \cdot m^2 \cdot Q$
(5.5)	$\frac{1}{2}n \cdot (n+1) \cdot L^2 \cdot m \cdot Q^2 - n \cdot L^2 \cdot m \cdot Q$
(5.6)	$\frac{1}{2}n \cdot (n+1) \cdot L^2 \cdot m \cdot Q^2 - n \cdot L^2 \cdot m \cdot Q$
Total	$n \cdot m \cdot \left(L \cdot Q \cdot \left((n+1) \cdot L \cdot Q - n \cdot L + \frac{1}{2}m + \frac{3}{2}\right) - L - Q\right)$
Consistent (5.1)	n
Equal (5.17)	$n \cdot Q$
Consecutive (5.16)	$\frac{1}{2}n \cdot (n-1) \cdot L^2 \cdot Q^2$
C_{max}	$n \cdot L \cdot m \cdot Q$
C_i	$n \cdot L \cdot m \cdot Q$
NNC	$n \cdot L \cdot m + n \cdot Q$
Binary	$n\,(n+1) \cdot L^2 \cdot Q^2 - n \cdot L \cdot Q$
Basic Sequence (5.7)	$\frac{1}{2}n \cdot (n+1) \cdot L \cdot Q^2 - n \cdot L \cdot Q$

Table 5.20: Number of constraints for lot streaming with sublot resizing

Constraints	Number
(5.22)	$n \cdot L \cdot \frac{1}{2}m \cdot (m-1) \cdot Q$
(5.23)	$n \cdot L \cdot m \cdot (Q-1)$
(5.24)	$\frac{1}{2}n \cdot (n+1) \cdot L^2 \cdot m \cdot Q^2 - n \cdot L^2 \cdot m \cdot Q$
(5.25)	$\frac{1}{2}n \cdot (n+1) \cdot L^2 \cdot m \cdot Q^2 - n \cdot L^2 \cdot m \cdot Q$
(5.26)	$n \cdot L \cdot (L-1) \cdot m \cdot Q$
(5.27)	$n \cdot L \cdot (L-1) \cdot m \cdot Q$
(5.28)	$n \cdot (L-1) \cdot m^2 \cdot Q^2$
(5.29)	$n \cdot L \cdot Q$
(5.30)	$n \cdot L \cdot Q$
(5.31)	$n \cdot (L-1) \cdot Q \cdot (Q-1)$
Total	$n \cdot L \cdot m \cdot Q \cdot \left(\frac{1}{2}m + (n+1) \cdot L \cdot Q + L + m \cdot Q - \frac{3}{2}\right)$
	$+n \cdot L \cdot m \cdot Q\,(Q+1)$
	$+n \cdot (-L \cdot m - m^2 \cdot Q^2 - Q \cdot (Q-1))$
Consistent level (5.21)	$n \cdot L$
Consecutive (5.39)	$\frac{1}{2}n \cdot (n-1) \cdot L^2 \cdot Q^2$
C_{max}	$n \cdot L \cdot m \cdot Q$
C_i	$n \cdot L \cdot m \cdot Q$
NNC	$n \cdot L \cdot m + n \cdot Q + C_{max}\,(1)$
Binary	$n\,(n+1) \cdot L^2 \cdot Q^2 - n \cdot L \cdot Q + n \cdot L \cdot Q^2$
Basic Sequence (5.40)	$\frac{1}{2}n \cdot (n+1) \cdot L \cdot Q^2 - n \cdot L \cdot Q$

5.3 Neighborhoods in Re-entrant Permutation Flow Shops with Lot Streaming

The move limits of the sublots in the MIP are:

- For sublots q the position of the same sublot q in the previous level, if $l \neq 1$,
- The position of the previous sublot $q-1$ of the same level l, if $q \neq 1$,
- The position of the next sublot $q+1$ of the same level l, if $q \neq Q$,
- The position of the same sublot q in the next level $l+1$, if $l \neq L$.

Since the number of possible permutations increases through the introduction of sublots to $(n \cdot L \cdot Q)!/\left((L \cdot Q)!\right)^n$, the moves are limited more strictly to reduce the computational effort for large test instances. The stricter move limits are:

- For sublots $q = 1$ the position of the last sublot $q = Q$ of the previous level, if $l \neq 1$,
- The position of the previous sublot $q - 1$ of the same level l, if $q \neq 1$,
- The position of the next sublot $q + 1$ of the same level l, if $q \neq Q$,
- The position of the first sublot $q = 1$ of the next level $l + 1$, if $l \neq L$.

The strict limits for any move of a sublot of a job i in level l in the sublot permutation are given by the levels $l - 1$ and $l + 1$ and the sublots $q - 1$ and $q + 1$. These limits do not consider the possibility that a sublot q can be processed before $q - 1$ of the same job level.

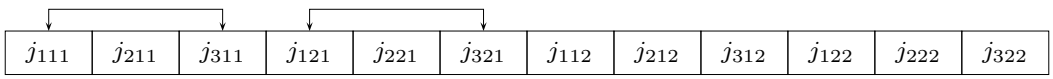
Technological restrictions to resizing the sublots after entering a new level are given by the number of available parts. There need to be enough parts available to reach the wanted sublot size. However, resizing is not considered in the heuristic solution approaches, since the experiments with the resizing models show that the effect on the makespan and total completion time are quite low considering the effort of selecting the new lot size.

Swap moves

Swap moves are also in the lot streaming problem, where the exchanges of sequence positions occur between two different sequence members. The sequence members are not the various job levels, but the single sublots of each job level. Without resizing of sublots, a sublot can start a new level after finishing the previous level. The earliest swap position is that of the same sublot in the preceding level. The latest swap position is the position of the same sublot in the next level.

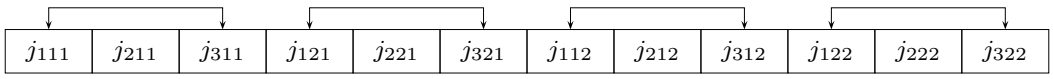
An example of a level swap is given in Figure 5.26. The sequence position of a sublot q of job i in level l is given by j_{iql}. All sublots of the job level $il = 11$ are exchanged with the sublots of $i'l' = 31$.

Figure 5.26: Example of a level swap of $i = 1, l = 1$ and $i' = 3, l' = 1$



A job swap is shown in Figure 5.27, where every sublot of job $i = 1$ swaps its sequence position with the corresponding sublot of job $i' = 3$.

Figure 5.27: Example of a job swap of $i = 1$ and $i' = 3$



Insertion moves

The insertion moves select a member of the sequence, i.e. a sublot of a job in a specific level, and place it into a new position. To be placed in a later position, the member is inserted between the former jth and $(j + 1)$th position. Since the members of the sequence between the old position j' and new position j move forward in the sequence, their sequence index is reduced by 1. If the member is inserted into a prior position, the sequence position of the members in between the old and new position will be increased by one because the number of the sublots is increased by 1.

Figures 5.28 and 5.29 give examples of level insertion and job insertion in a re-entrant flow shop with lot streaming, considering the example sublot permutation from the example for swap moves.

Figure 5.28: Example of a level insertion of $i = 1, l = 1$ to the positions of $i' = 3, l' = 1$

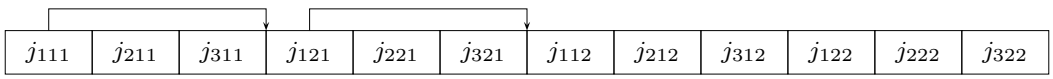
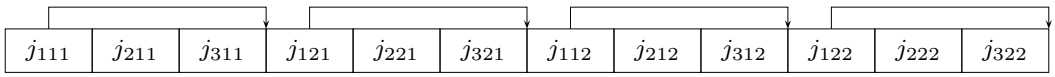


Figure 5.29: Example of a job insertion of $i = 1$ to the positions of $i' = 3$



5.4 A Variable Neighborhood Search for a Re-entrant Permutation Flow Shop with Lot Streaming

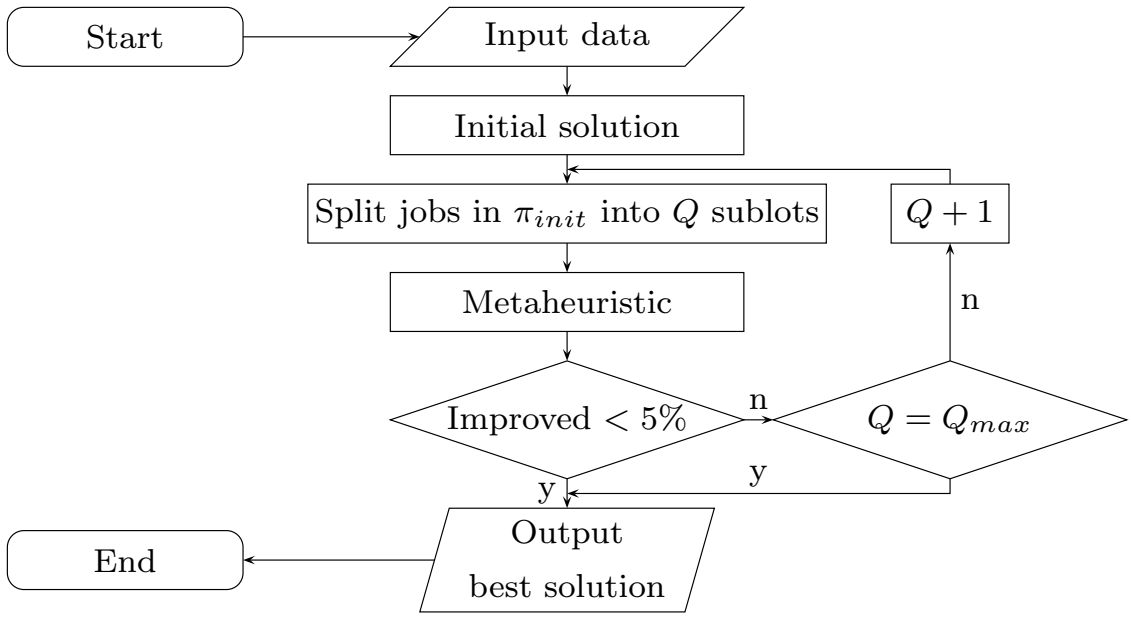
This section focuses on the adjustment and testing of variable neighborhood search configurations for the re-entrant permutation flow shop with missing operations and lot streaming. Other methods are not considered in this section, due to the relatively weak results in Chapter 4. The first part of the section introduces the tested calibration in a lot streaming framework, to define the sublot sizes for single job levels. The second

part presents the computational results to answer the research question concerning the number of sublots per job that should be chosen in the considered problem and the calibration of the VNS that obtains the best results.

5.4.1 Calibration

Consistent sublots are used for the heuristic solution methods in this section. The number of sublots may vary from iteration to iteration. Due to the results presented in Section 5.2.3, the sublots are considered to be equal but are not necessarily consecutive. Equal sublots do not require intense effort in calculating lot sizes and do not result in high losses of solution quality for makespan. The makespan values obtained by solving the model with equal sublots lead to even lower makespan values for some problems with four jobs to be scheduled with a maximum computation time of 1 hour.
The framework splits each job into $Q = 2$ sublots within the first main iteration. Then, the improvement method is applied to the permutation. The resulting objective value is compared to that of the previous iteration. The number of sublots Q is set to $Q + 1$ if the resulting makespan value, C^Q_{max}, of the iteration is $C^Q_{max} \leq 0.95C^{Q-1}_{max}$. Otherwise, the algorithm terminates. This lot streaming framework is chosen to determine the number of sublots per job that is useful in the context of re-entrant permutation flow shops and to quantify the improvement of metaheuristics for the problem with lot streaming. The VNS is suggested as an improvement method for this problem, since it showed the best results on the makespan with acceptable computational effort in Section 4.7.

Figure 5.30: Lot streaming framework with equal sublots



Initial solutions for an RPFS with missing operations and lot streaming

The initial solution for the re-entrant permutation flow shop with lot streaming can be generated by applying the suggested constructive heuristics of section 4.5. The selected method of initialization is the STPTL rule because of the achieved solution quality for the RPFS without lot streaming and the performance of the VNS operating on the initial solution generated by this procedure.
The mentioned constructive heuristic generates an initial sequence with mixed levels. The job levels in this solution are then split into Q equal consecutive sublots. If there is a remainder of parts of size $\overline{X} > 0$ after a job is divided into Q sublots, then the $\overline{X}$ parts are equally split between the $\overline{X}$ first sublots of the job. The improvement methods are then able to update the solutions to permutations with non-consecutive sublots, as presented by the neighborhood structures introduced in Section 5.3.

Neighborhood Structures for the RPFS with Lot Streaming

The neighborhood hierarchies 1 and 2 chosen in Section 4.7.5 are extended by two additional neighborhoods to sequence single sublots. The extended neighborhood hierarchies are shown in Table 5.21. S-swaps are swaps of two sublots in the sequence. An S-insert is an insertion move performed by a single sublot in the sequence.

Table 5.21: Neighborhood hierarchies with lot streaming

t	$\mathcal{N}_t^1$	$\mathcal{N}_t^2$
1	J-swap	J-insert
2	J-insert	J-swap
3	L-swap	L-insert
4	L-insert	L-swap
5	S-swap	S-insert
6	S-insert	S-swap

The Nowicki across block moves are also tested for the re-entrant permutation flow shop with lot streaming to investigate whether these improve their performance in the extended problem setting. The Nowicki across block moves can only be used by single sublots if lot streaming is applied.

Shaking

The shaking phase and the switch of neighborhoods follow the two preferred schemes in Section 4.7.5, which are extended to the neighborhood hierarchies shown in Table 5.21.

Integrated Local Search Algorithm

Again, both local search procedures, best neighbor and first improvement, are separately implemented within the local search phase.

5.4.2 Computational Experiments

This section examines what numbers of equally sized sublots per job are appropriate in a VNS framework for a re-entant permutation flow shop with lot streaming and how they affect the makespan and computation time. Furthermore, different calibrations of the VNS are evaluated regarding their performance in the considered problem.

Experimental Setup

The parameters of the problem size are $n \in \{2, 3, 4, 5\}$, $L \in \{2, 3, 4, 5\}$, $m \in \{2, 5, 6, 10\}$ for small problems and $n \in \{20, 40\}$, $L \in \{5, 10\}$, $m \in \{20, 40\}$ for large problems.[33] The processing times of complete operations are uniformly distributed random numbers between 1 and 99. Ten test instances are generated for each problem size. Missing operations are generated about the Inc_1 and Inc_3 schemes.[34]

Results

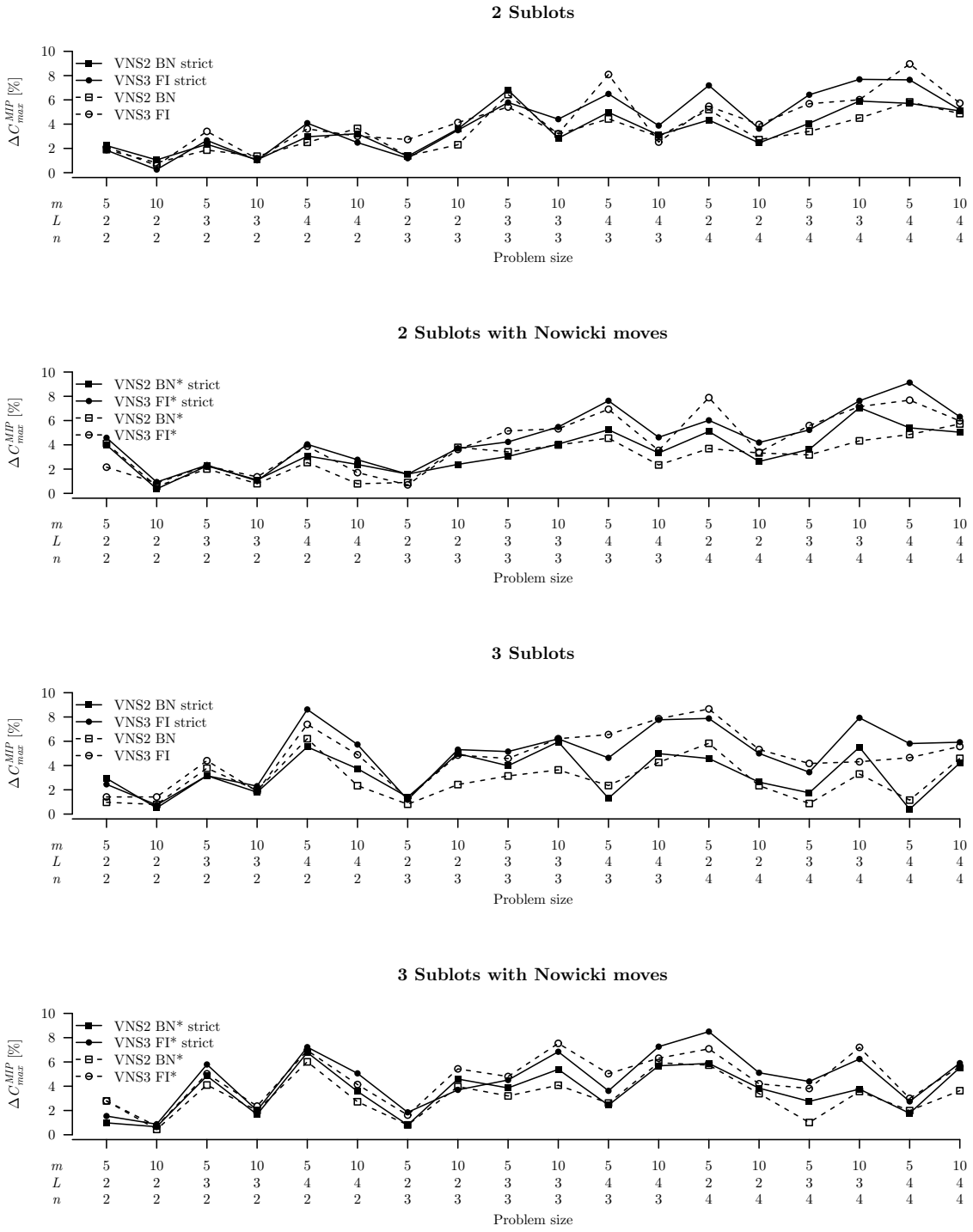
The makespan deviations, ΔC_{max}^{MIP}, of the VNS solutions from the MIP solutions with equally sized sublots are used to identify if strict limits should be applied to single sublot moves. The deviation is calculated via the equation:

$$\Delta C_{max}^{MIP} = \frac{C_{max}\left(\pi_{meta}^{Q}\right)}{C_{max}\left(\pi_{MIP}^{Q}\right)} - 1.$$

The permutation obtained by the metaheuristics if each job is split into Q equally sized sublots is denoted by π_{meta}^{Q}. The average values of ΔC_{max}^{MIP} per problem size are shown in Figure 5.31 for small problems. The deviations for strict and non-strict move limits of sublots are very close for every tested VNS configuration. Therefore, strict sublot move limits are used in all further tests with large instances to limit the computational effort.

[33] See Table B.3 in Appendix B (p. 205) for detailed information on the problem sizes.
[34] See Table B.1 in Appendix B (p. 203) for an overview of missing operations.

Figure 5.31: Average makespan deviation to MIP solutions (Inc_1)



The following evaluation tables and figures all provide the results for the different VNS approaches with strict sublot move limits.

Table 5.22 shows the mean makespan reductions of using lot streaming in different VNS configurations if the number of sublots per job depends on the makespan reduction

that is achieved by an increase of Q by one. The makespan reductions are denoted with ΔC_{max}^{LS} and calculated by:

$$\Delta C_{max}^{LS} = 1 - \frac{C_{max}\left(\pi_{meta}^{LS}\right)}{C_{max}\left(\pi_{meta}\right)}.$$

π_{meta}^{LS} is a permutation obtained with the tested methods for the re-entrant permutation flow shop problem with lot streaming. The number of sublots per job for each problem instance is determined by the framework illustrated in Figure 5.30. The average values of ΔC_{max}^{LS} are about 30 % for all approaches for small problems and for large instances the mean makespan reductions are about 20 %. The Inc_3 instances are only solved with the first improvement variations of the VNS due to the high computation times with an integrated best neighbor local search.

Table 5.22: Average makespan reductions ΔC_{max}^{LS} [%] with lot streaming

	Inc_1			Inc_3		
	Small	Large	Total	Small	Large	Total
VNS1 BN	30.93	21.35	29.87	-	-	-
VNS2 FI	30.17	20.68	29.11	27.92	19.66	25.16
VNS1 BN*	30.84	21.54	29.81	-	-	-
VNS2 FI*	30.33	20.83	29.27	30.83	23.17	28.27

The number of sublots per job reaches $Q = 7$ as shown in Table 5.23. The most frequently calculated sublots per job are $Q = 3$ and $Q = 4$.

Table 5.23: Number of sublots per job in the best solutions of small problems (Inc_1)

Q	VNS1 BN	VNS2 FI	VNS1 BN*	VNS2 FI*
2	64	73	67	70
3	210	222	204	208
4	202	197	208	201
5	109	100	98	107
6	41	32	49	43
7	13	15	12	9

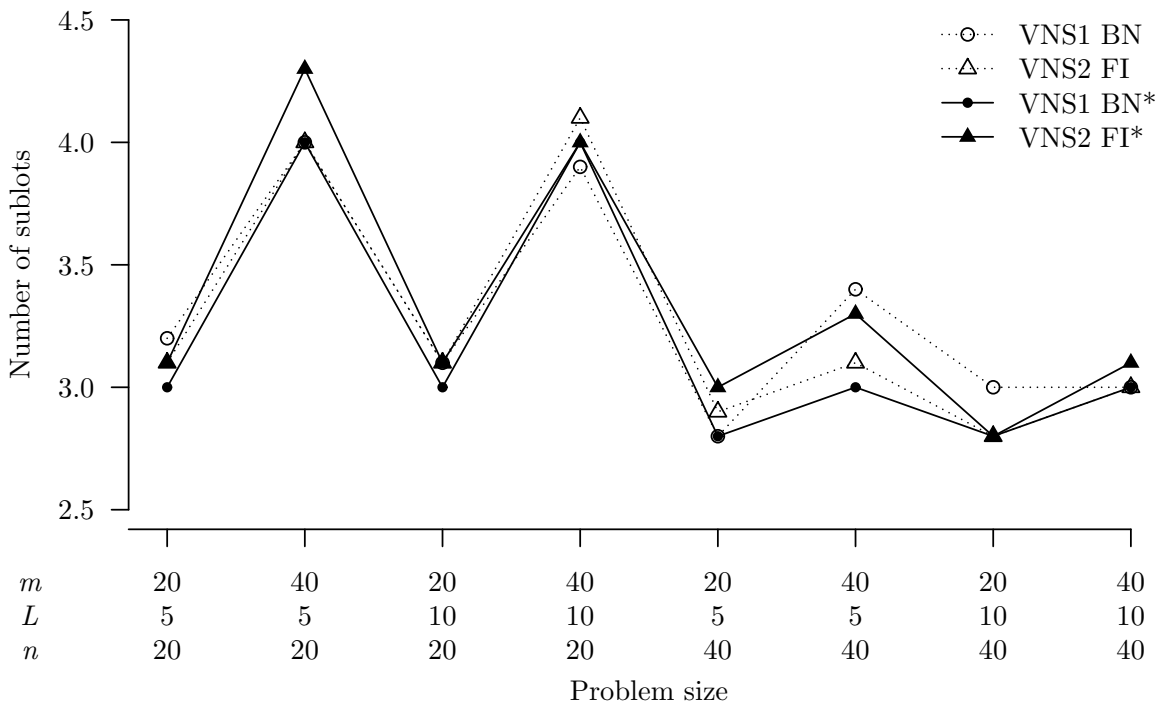
The number of sublots per job in the solutions obtained by metaheuristics in large instances are most frequently $Q = 3$ and $Q = 4$. Table 5.24 shows the frequencies of different numbers of sublots per job for large instances in the test sets Inc_1 and Inc_3. $Q = 5$ is the highest number of sublots per job calculated by the different VNS calibrations.

Table 5.24: Number of sublots per job in the best solution of large problems (Inc_1)

Q	VNS1 BN	VNS2 FI	VNS1 BN*	VNS2 FI*
2	2	3	5	2
3	53	55	54	52
4	24	20	21	23
5	1	2	0	3

Due to the results shown in Tables 5.23 and 5.24, it is suggested that the VNS starts with $Q = 3$ sublots per job and that the number of sublots per job should be limited to a maximum of $Q_{max} = 4$.

Figure 5.32: Average number of sublots per job in best solution (Inc_1)



The average improvement values, ΔC_{max}^{init}, shown in Figure 5.33[35] describe the relative reductions of makespan if a variable neighborhood search is applied to an STPTL solution, where jobs are each split into $Q = 3$ sublots. The following formula is used to calculate the single improvement values:

$$\Delta C_{max}^{init} = \frac{C_{max}\left(\pi_{meta}^{3}\right)}{C_{max}\left(\pi_{init}^{3}\right)} - 1.$$

[35] Figure 5.33 provides the values for the Inc_1 test set. The corresponding values for the Inc_3 are shown in Figure D.1 in Appendix D (p. 210).

High improvements of between 6 and 10 % are realized if the number of levels is $L = 5$. The mean makespan reductions for $L = 10$ instances are lower. The configurations with a best neighbor local search deliver better results than when first improvement is used as local search.

Figure 5.33: Average improvement of the $Q = 3$ STPTL solutions (Inc_1)

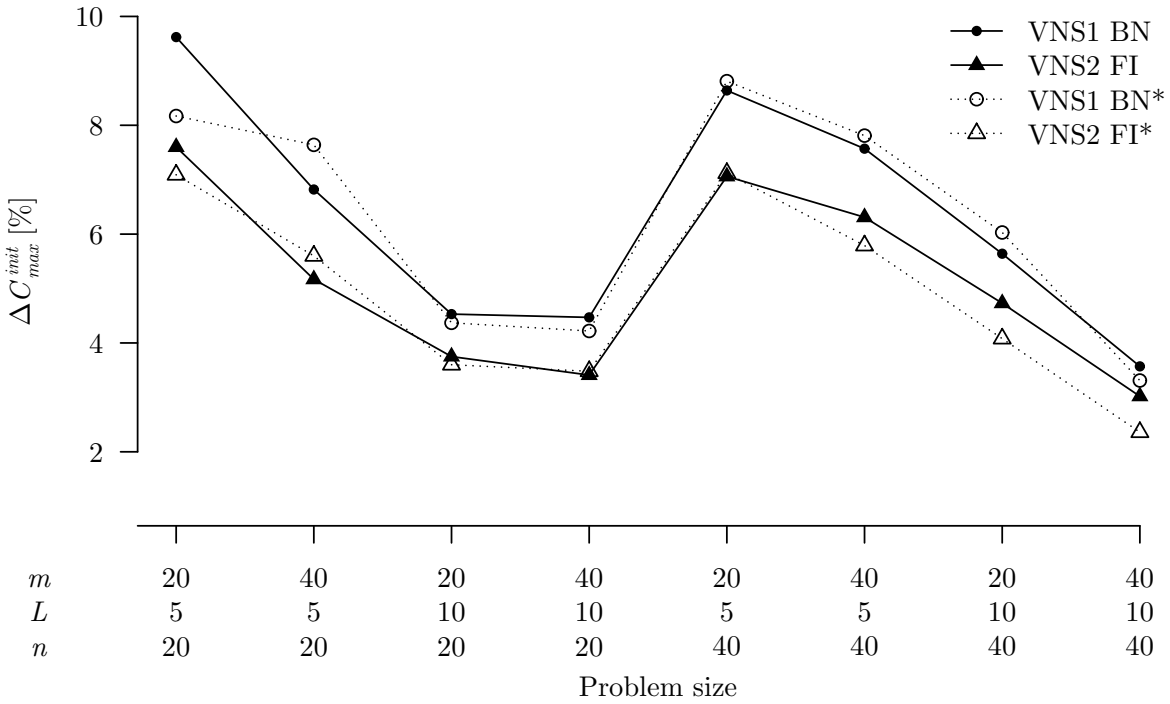
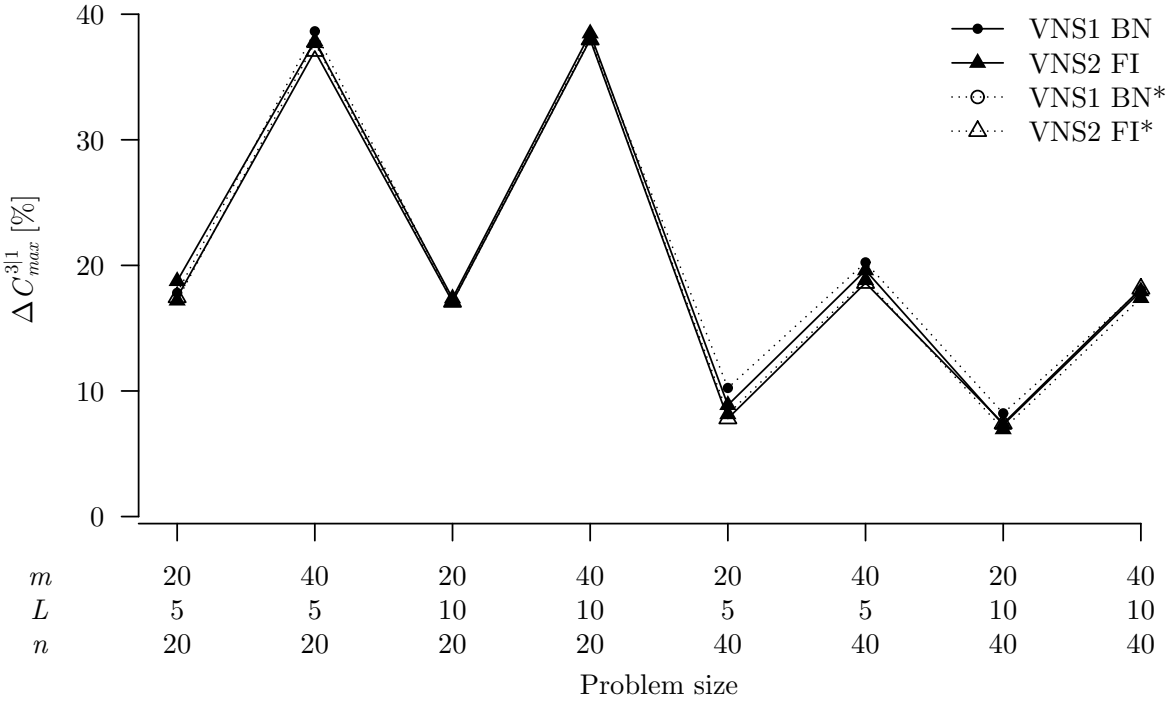


Figure 5.34 shows the average makespan reductions when lot streaming with $Q = 3$ sublots per job is applied to the Inc_1 test instances[36], as compared to the solutions obtained by the variable neighborhood searches without lot streaming. The makespan reductions are up to 40 % for $n = 20$ instances and up to 20 % for $n = 40$ instances. The values of reduction are calculated as follows:

$$\Delta C_{max}^{3|1} = 1 - \frac{C_{max}\left(\pi_{meta}^{3}\right)}{C_{max}\left(\pi_{meta}\right)}.$$

[36] The corresponding values for the Inc_3 are shown in Figure D.2 in Appendix D (p. 210).

Figure 5.34: Average makespan reductions compared to $Q = 1$ solutions (Inc_1)



The mean computation times for the tested problem sizes are shown in Table 5.25. The best neighbor variable neighborhood search based on the Nowicki block moves requires longer computation times on average for the largest instances ($n = 40$ and $L = 10$) than the equivalent VNS without block criteria. The VNS with a best neighbor local search is not tested for Inc_3 instances due to the relatively long computation times.

Table 5.25: Average computation times [s] with $Q = 3$ sublots per job

Set	n	L	m	VNS1 BN	VNS2 FI	VNS1 BN*	VNS2 FI*
Inc_1	20	5	20	500	119	128	82
			40	324	76	287	76
		10	20	941	398	526	270
			40	1180	284	613	346
	40	5	20	4320	650	2331	615
			40	4438	1350	3775	678
		10	20	9849	3978	11,163	1863
			40	7500	7276	9485	2696
Inc_3	20	5	20	-	147	-	68
			40	-	102	-	96
		10	20	-	611	-	362
			40	-	584	-	216
	40	5	20	-	844	-	489
			40	-	2154	-	987
		10	20	-	4326	-	2913
			40	-	3011	-	4014

The results of the test lead to the conclusion that the preferred algorithm configuration is the VNS2 FI. It works with acceptable computation times, even for large problems with about 1200 positions within a permutation, and delivers makespan reductions of initial solutions for large problem sizes which are only 1 to 2 % points lower than the reductions obtained with best neighbor variable neighborhood searches. Further, the VNS2 FI* with the Nowicki across block moves is a good configuration of the VNS. It delivers weaker results than the VNS2 FI but is up to around three times faster than the VNS2 FI.

6 Conclusion

This thesis considers a re-entrant permutation flow shop with missing operations. The permutation consists of job levels which represent the re-entries of jobs into a manufacturing system. The literature review showed the fields of application of the examined problem and addressed different problem characteristics and solution methods. An MIP model was suggested to solve small problem instances. Metaheuristic solution methods are developed for large problem instances. The main contributions of this work are provided by the answers to the research questions in the previous chapters and are summarized in the following.

Q1: What is the state of research for re-entrant permutation flow shops?

Current literature concerning re-entrant permutation flow shops only considers schedules with separated levels. MIP models only represent missing operations by using an operation index on the sequence variables. The literature review found applications of various metaheuristic solution approaches to the re-entrant scheduling problems. Evolutionary algorithms are a widely used group of heuristics for re-entrant scheduling problems. Beside this, also the tabu search approach and variable neighborhood search are applied on the problem in the literature. Simulated annealing has not been applied to a re-entrant permutation flow shop before. Nevertheless, the special neighborhood structures in re-entrant permutation flow shop problems have not been exploited extensively before. Additionally, no publication regarding lot streaming in re-entrant permutation flow shops is known to the author of this thesis, except for the author's earlier work Hinze (2016).

Q2: How can missing operations and mixed levels be formulated in a mathematical model and what are the effects on the optimal makespan of a schedule? What problem sizes can be solved optimally?

Two approaches were examined for modeling a re-entrant permutation flow shop with mixed levels without using a machine or operation index on the sequence variable. One model uses sequence variables which define predecessor-successor relations for each possible pair of job levels. This formulation is superior to the approach that assigns sequence

positions directly to single job levels. Furthermore, the influence of missing operations on the optimal makespan was examined. The preferred model was adjusted to deal with missing operations without using an operation index on the sequence variable. The proper management of missing operations leads to further makespan reductions. The models were able to optimally solve problem sizes with up to either four jobs and five levels or five jobs and four levels per job in an appropriate time.

Q3: How does the application of problem-specific constructive heuristics and adjusted metaheuristics affect the solution quality and computational performance in re-entrant permutation flow shop problems?

Eight different constructive heuristics were tested for the re-entrant permutation flow shop problem. The first set of heuristics generated schedules with separated levels, while the second set allowed mixed levels and considered the ready times of job levels. An STPTL priority rule delivered better results than the other methods. The best method for creating separated level schedules was the NEHJ algorithm. Both methods and two random schedule generation procedures were tested regarding their influence on the performance of various metaheuristics.

The suggested neighborhood structure for an application in metaheuristics contains swap and insertion moves of single job levels and all levels of a certain job. Furthermore, different definitions of block moves were examined. The block moves based on the definition of Chen/Pan/Wu (2007) delivered weaker results than the block moves based on the definition of Nowicki/Smutnicki (1996). Computation times were not cut down remarkably and the makespan values were not improved compared to the VNS configuration without block criteria.

Tabu search, simulated annealing and variable neighborhood search were examined regarding the makespan values achieved and the computation times required. Each metaheuristic was tested with different neighborhood settings based on the previous results of the tested neighborhoods. The variable neighborhood search obtained the best makespan values within a reasonable amount of computation time. Two different neighborhood hierarchies deliver the best results for the VNS with best neighbor and first improvement local search. The suggested neighborhood hierarchy for a VNS with a best neighbor local search uses swap neighborhoods before the corresponding insertion neighborhoods, and vice versa if a first improvement local search is integrated in the VNS. Block criteria did not lead to a better performance of the variable neighborhood search. The choice of the opening procedures affects the result of the VNS. The best results are obtained with the STPTL rule. Initializing with the SIROL rule leads to extensive computation times and weak makespan values. The VNS configurations delivered better average improvement values of NEHJ solutions than the suggested algorithms of Chen/

Pan / Lin (2008), Chen / Pan / Wu (2008) and Qian et al. (2013b) for similar problem sizes.

Q4: What is the impact of different forms of lot streaming on the makespan?

Applying lot streaming in re-entrant permutation flow shops led to large reductions of the makespan. Equally sized non-consecutively processed sublots were determined as a suitable form of sublots for a variable neighborhood search. Equal sublot sizes reduced the effort in determining sublot sizes but still obtained good results compared to not necessarily equal sublots. Schedules with a consecutive processing of sublots of the same job level were solved quickly by commercial solver software but lacked solution quality. A resizing of sublots during level transition did not lead to remarkable improvements of the makespan.

Q5: What numbers of sublots per job dependent on the problem size are suitable for metaheuristics?

Due to the weak results in the previous tests, tabu search and simulated annealing were not pursued further for the problem with lot streaming. The preferred configurations of the variable neighborhood search in the context of the problem without lot streaming were extended to neighborhood hierarchies with limited insertion and swap moves of single sublots. Splitting the jobs into three or four sublots was most appropriate for the application of the VNS to large problem instances. First improvement is the preferred integrated local search for the VNS due to the extended permutation if lot streaming is considered. The replacement of single sublot neighborhoods with across block neighborhoods for single sublots resulted in reduced computation times without a remarkable decrease in solution quality.

Future Research

Further points of interest in the research field of re-entrant permutation flow shops include in the following:

- Further neighborhood structures may lead to further improvements in solution quality, e.g. moving levels without move limits and re-assigning level numbers after invalid moves to repair the solution, or involving more than two levels in swap moves.
- Especially, more sophisticated opening procedures may lead to a better performance in computation time and solution quality. The final solutions of the trajectory metaheuristics depend strongly on the initial solution of the re-entrant

permutation flow shop and tend to terminate earlier if they are performed on a good initial solution.

- Different integrated local search methods in the framework of a variable neighborhood search could be used to profit from the short computation times of a first improvement search while maintaining the solution quality of a best neighbor search.
- It is necessary to investigate the influence of setup times and to solve the lot streaming problem in re-entrant permutation flow shops with setup times, either sequence-dependent or independent, to get a complete impression of the impact of lot streaming in re-entrant permutation flow shops.
- Lower bounds could be applied for the makespan to estimate the solution quality and to decrease the computation time of metaheuristics.
- The model performance on job shops could be increased by editing the number of necessary levels per job in a job shop representation.
- Different schemes to determine sublot sizes should be examined to get further makespan reductions for the lot streaming problem.

A Literature Review Search Methodology

This section shows the search methodology on different scientific search engines. The queries searched for literature between the years 2010 and 2015.

Google Scholar

The used url was:
`http://scholar.google.de/`

The search terms for Google Scholar were "scheduling" or "flow shop / flowshop" OR "job shop / jobshop" and "reentrant" or "re-entrant", since there was no keyword or abstract search available.

Web of Science or Web of Knowledge

The used url was:
`http://apps.webofknowledge.com/UA_GeneralSearch_input.do?product=UA`
`&search_mode=GeneralSearch&SID=X1RfCdK6zgXWmjlqEds&preferencesSaved=`

The dependencies between the search terms were:
TITLE: (Scheduling OR Flow Shop OR Job Shop OR Flowshop OR Jobshop) AND TITLE: (re-entrant OR reentrant) OR TITLE: (Scheduling OR Flow Shop OR Job Shop OR Flowshop OR Jobshop) AND TOPIC: (re-entrant OR reentrant) OR TOPIC: (Scheduling OR Flow Shop OR Job Shop OR Flowshop OR Jobshop) AND TITLE: (re-entrant OR reentrant) OR TOPIC: (Scheduling OR Flow Shop OR Job Shop OR Flowshop OR Jobshop) AND TOPIC: (re-entrant OR reentrant).

Ebsco Academic Search Complete

The used urls were:
`http://web.b.ebscohost.com/ehost/search/advanced?sid=440abe82-9a15-4fc7-a3d7-5b57ac3ae5cb\%40sessionmgr115&vid=0&hid=124`
and
`http://wwwdb.dbod.de:2105/ehost/search/advanced?sid=0242f8e0-6038-470e-87db-68f38ed704e2\%40sessionmgr4004&vid=0&hid=4104`

The dependencies between the search terms were:
(TI (Flowshop) OR TI (Flow-shop) OR TI (Flow Shop) OR TI (Jobshop) OR TI (Job-shop) OR TI (Job Shop) OR TI (Scheduling) OR SU (Flowshop) OR SU (Flow-shop) OR SU (Flow Shop) OR SU (Jobshop) OR SU (Job-shop) OR SU (Job Shop) OR SU (Scheduling) OR AB (Flowshop) OR AB (Flow-shop) OR AB (Flow Shop) OR AB (Job-shop) OR AB (Job-shop) OR AB (Job Shop) OR AB (Scheduling)) AND (TI (reentrant) OR TI (re-entrant) OR SU (reentrant) OR SU (re-entrant) OR AB (reentrant) OR AB (re-entrant)).

Only peer reviewed journals have been included within the search.

ScienceDirect

The used url was:
`http://www.sciencedirect.com/science/jrnlallbooks/a`

The dependencies between the search terms were:
(TITLE-ABSTR-KEY(scheduling) OR ("flow shop") OR ("flow-shop") OR (flowshop) OR ("job shop") OR ("job-shop") OR (jobshop)) AND (TITLE-ABSTR-KEY(("re-entrant") OR (reentrant))).

The search for journal articles in ScienceDirect was limited to the fields of computer science and mathematics.

IEEE xplore

The used url was:
`http://ieeexplore.ieee.org/Xplore/home.jsp`

The dependencies between the search terms were:
(((“Abstract”:QT.: ((Flowshop) OR (Flow Shop) OR (Jobshop) OR (Job Shop) OR (Scheduling))) OR “Document Title”:”: ((Flowshop) OR (Flow Shop) OR (Jobshop) OR (Job Shop) OR (Scheduling))) OR “Author Keywords”:”: ((Flowshop) OR (Flow Shop) OR (Jobshop) OR (Job Shop) OR (Scheduling))) AND ((((reentrant) OR (“re-entrant”))) OR “Document Title”:”: ((reentrant) OR (“re-entrant”)) OR “Author Keywords”:”: ((reentrant) OR (”re-entrant”))),
(((“Abstract”:Flowshop OR “Flow-Shop” OR “Job-Shop” OR Jobshop OR Scheduling) OR “Document Title”:Flowshop OR “Flow-Shop” OR “Job-Shop” OR Jobshop OR Scheduling) OR “Author Keywords”:Flowshop OR “Flow-Shop” OR “Job-Shop” OR Jobshop OR Scheduling),
(((“Abstract”:Flowshop OR “Flow-Shop” OR “Job-Shop” OR Jobshop OR Scheduling) OR “Document Title”:Flowshop OR “Flow-Shop” OR “Job-Shop” OR Jobshop OR Scheduling) OR “Author Index Terms”:Flowshop OR “Flow-Shop” OR “Job-Shop” OR Jobshop OR Scheduling).

The search querries were limited to “Journals & Magazines.”

Taylor and Francis

The used url was:
`http://www.tandfonline.com/`

The dependencies between the search terms were:
((scheduling) OR (“flow shop”) OR (“flow-shop”) OR (flowshop) OR (“job shop”) OR (“job-shop”) OR (jobshop)) AND (reentrant OR “re-entrant”).

B Test Instances

The problem size depends on the number of jobs n, number of levels per job L, and the number of machines m. The different settings for these three parameters are displayed in the Tables B.2 and B.3. A job leaving the production system after a machine $k < m$ to re-enter the system on a new level or to be finished is one possible characteristic of incomplete levels. Another possible characteristic can be an entrance to a new level on a machine $k > 1$. The numbers of earlier exits or later entries in a new job level are given by Table B.1.

Table B.1: Incomplete level scheme

Instance set	Levels per job							Chapter / Section
	2	3	4	5	10	20	40	
Complete	0	0	0	0	0	0	0	4.4.1, 4.4.2, 4.4.1
Inc_1	1	1	1	1	1	2	4	4.4.3, 4.5.3, 4.6.4, 4.7.5, 4.7.6, 5.2, 5.4
Inc_2	-	-	2	2	4	7	12	4.4.3, 5.2
Inc_3	-	-	-	3	5	9	20	4.4.3, 4.5.3, 4.6.4, 4.7.5, 4.7.6, 5.4
Inc_4	-	-	-	-	9	18	36	4.5.3, 4.6.4, 4.7.5, 4.7.6

The sizes of jobs in Chapter5 are required to be divisible by two and three for the tests with equal sublots in sections 5.2.3 and 5.2.4. Additionally the divisibility by four is secured for additional tests. So the lot sizes are allowed to take the values:

$$D \in \{60, 72, 84, 96, 108, 120\} .$$

All experiments are performed on a 64-bit Windows 10 system with a 2.5 GHz Intel i7-4710HQ quad core processor and 16 GB RAM. IBM CPLEX 12.4 is used as MIP solver and the examined heuristics are coded in C++.

Table B.2: Parameter settings of the test instances without lot streaming

Test	Size	Missing Operations	n	L	m
4.4.1 Sequence variable p. 72	-	Complete	2, 3, 4, 5	2, 3, 4, 5	2, 5, 6, 10
4.4.1 Mixed Levels p. 78	-	Complete	2, 3, 4, 5	2, 3, 4, 5	2, 5, 6, 10
4.4.2 Basic Sequence p. 80	-	Complete	2, 3, 4, 5	2, 3, 4, 5	2, 5, 6, 10
4.4.3 Missing Operations p. 85	-	Inc_1, Inc_2 ($L \geq 4$), Inc_3 ($L \geq 5$)	2, 3, 4, 5	2, 3, 4, 5	2, 5, 6, 10
4.5.3 Initialization p. 95	Small	Inc_1, Inc_3 ($L \geq 5$)	2, 3, 4, 5	2, 3, 4, 5	2, 5, 6, 10
4.5.3 Initialization p. 95	Medium	Inc_1, Inc_3 ($L \geq 5$), Inc_4 ($L \geq 10$)	10, 20, 30, 40, 50	5, 10	10, 20, 30, 40, 50
4.5.3 Initialization p. 95	Large	Inc_1, Inc_3 ($L \geq 5$), Inc_4 ($L \geq 10$)	50, 100	20, 40	50, 100
4.6.4 Neighborhoods p. 108	Small	Inc_1, Inc_3 ($L \geq 5$), Inc_4 ($L \geq 10$)	2, 3, 4, 5	2, 3, 4, 5	2, 5, 6, 10
4.6.4 Neighborhoods p. 108	Large	Inc_1, Inc_3 ($L \geq 5$), Inc_4 ($L \geq 10$)	10, 20, 30, 40, 50	5, 10	10, 20, 30, 40, 50
4.7.5 Meta heuristics p. 121	Small	Inc_1, Inc_3 ($L \geq 5$)	2, 3, 4, 5	2, 3, 4, 5	2, 5, 6, 10
4.7.5 Meta heuristics p. 121	Large	Inc_1, Inc_3 ($L \geq 5$)	20, 40	5, 10	20, 40
4.8 Job Shops p. 134	-	-	5, 6, 7, 8, 9, 10, 15	-	5, 6, 7, 8, 9, 10, 15

All experiments are performed on a 64-bit Windows 10 system with a 2.5 GHz Intel i7-4710HQ quad core processor and 16 GB RAM. IBM CPLEX 12.4 is used as MIP solver and the examined heuristics are coded in C++.

Table B.3: Parameter settings of the test instances with lot streaming

Test	Size	Missing Operations	n	L	m	Q
5.2.1 Consistent Sublots p. 144	-	Inc_1, Inc_2 ($L \geq 4$)	2, 3, 4, 5	2, 3, 4, 5	2, 5, 6, 10	2, 3, 4
5.2.2 Consecutive Sublots p. 155	-	Inc_1, Inc_2 ($L \geq 4$)	2, 3, 4, 5	2, 3, 4, 5	2, 5, 6, 10	2, 3
5.2.3 Equal Sublots p. 161	-	Inc_1, Inc_2 ($L \geq 4$)	2, 3, 4, 5	2, 3, 4, 5	2, 5, 6, 10	2, 3
5.2.4 Equal Consecutive Sublots p. 168	-	Inc_1, Inc_2 ($L \geq 4$)	2, 3, 4, 5	2, 3, 4, 5	2, 5, 6, 10	2, 3
5.2.5 Resizing Sublots p. 173	-	Inc_1	2, 3	2, 3	2, 5, 6, 10	2, 3
5.4.2 VNS p. 189	Small	Inc_1, Inc_3 ($L \geq 5$)	2, 3, 4, 5	2, 3, 4, 5	2, 5, 6, 10	-
5.4.2 VNS p. 189	Large	Inc_1, Inc_3 ($L \geq 5$)	20, 40	5, 10	20, 40	-

C Additional Computational Results of Metaheuristics

Table C.1: Average makespan deviations ΔC_{max}^{init} for small problems

	Inc_1				Inc_3			
	NEHJ	SIROJ	STPTL	SIROL	NEHJ	SIROJ	STPTL	SIROL
VNS1 BN	5.55	7.86	4.56	30.83	7.40	8.54	4.73	33.52
VNS2 BN	5.52	7.84	4.52	30.72	6.93	8.56	4.59	32.62
VNS3 BN	5.47	7.86	4.50	30.89	7.60	9.13	5.27	33.54
VNS4 BN	5.71	7.96	4.57	30.84	7.40	8.83	4.86	33.83
VNS5 BN	5.50	7.61	4.42	30.68	6.95	9.00	4.73	33.19
VNS6 BN	5.55	7.65	4.43	30.55	7.41	8.91	4.93	32.84
VNS7 BN	5.59	7.57	4.47	30.92	6.77	8.63	4.98	33.59
VNS8 BN	5.59	7.37	4.45	30.61	7.04	9.02	4.70	32.96
VNS1 FI	4.46	6.66	3.64	26.88	4.67	5.96	4.00	25.77
VNS2 FI	5.10	7.03	4.05	30.42	6.03	7.37	3.66	31.80
VNS3 FI	4.55	6.57	3.75	27.44	3.74	5.68	3.56	29.03
VNS4 FI	4.34	6.43	3.72	29.03	4.13	5.62	3.61	29.73
VNS5 FI	4.44	6.00	3.67	28.13	5.43	7.23	3.32	29.90
VNS6 FI	4.40	6.40	3.65	28.64	4.99	5.78	3.40	30.06
VNS7 FI	3.90	5.75	3.40	28.11	4.79	6.61	3.43	29.73
VNS8 FI	3.92	5.78	3.42	28.35	5.08	6.46	3.44	29.85
TS $\mathcal{N} = J2$	3.32	5.28	3.21	16.15	2.38	3.84	2.50	16.20
TS $\mathcal{N} = 10$	2.36	3.02	2.20	13.20	3.29	3.87	2.46	10.24
SA $\mathcal{N} = J2$	1.57	2.99	1.63	12.89	1.32	2.67	1.26	15.03
SA $\mathcal{N} = 6$	0.28	0.46	0.33	6.11	0.27	0.21	0.28	3.17
Best	5.71	7.96	4.57	30.92	7.60	9.13	5.27	33.84

Figure C.1: Average computation times for large Inc_3 instances (I)

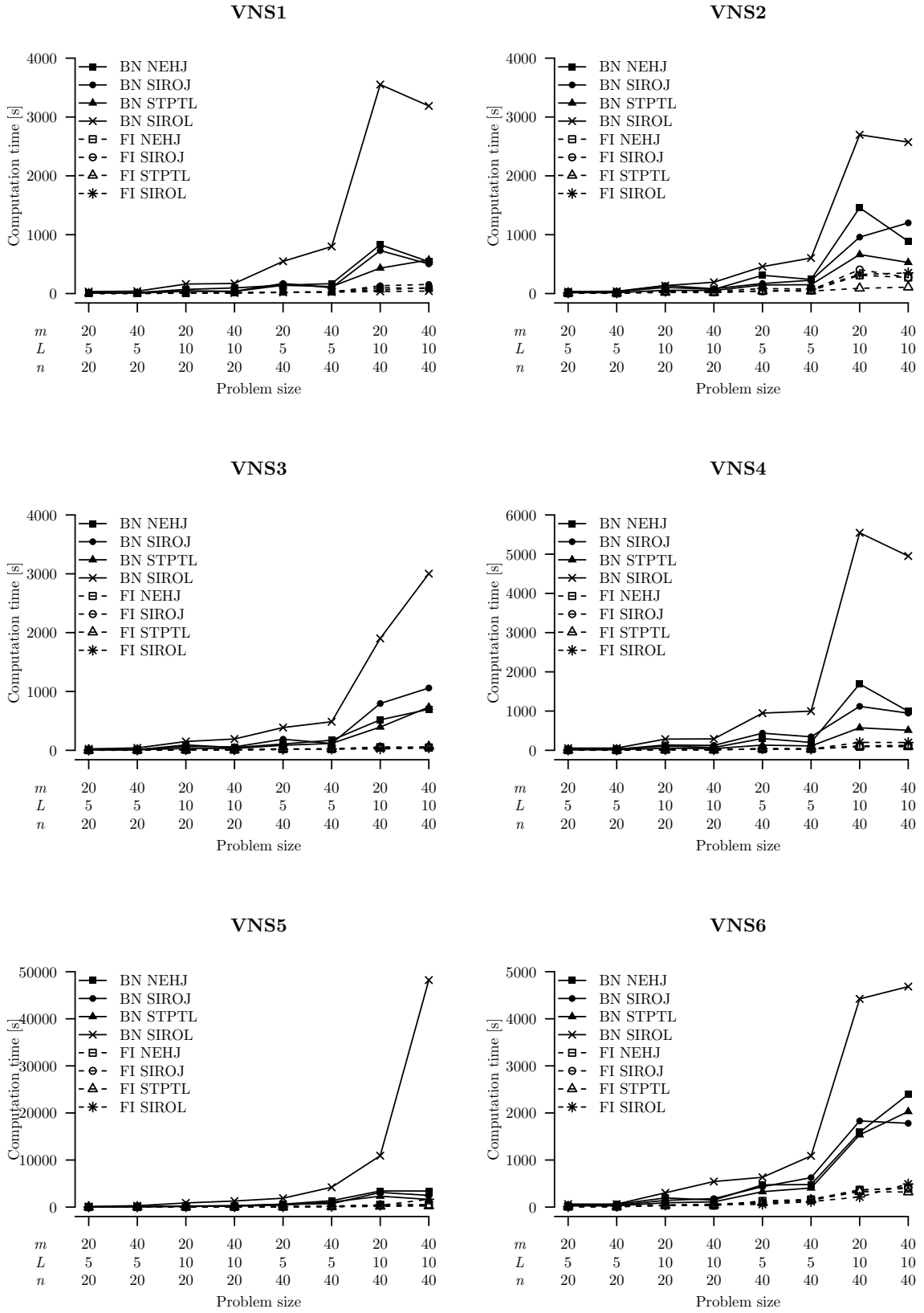
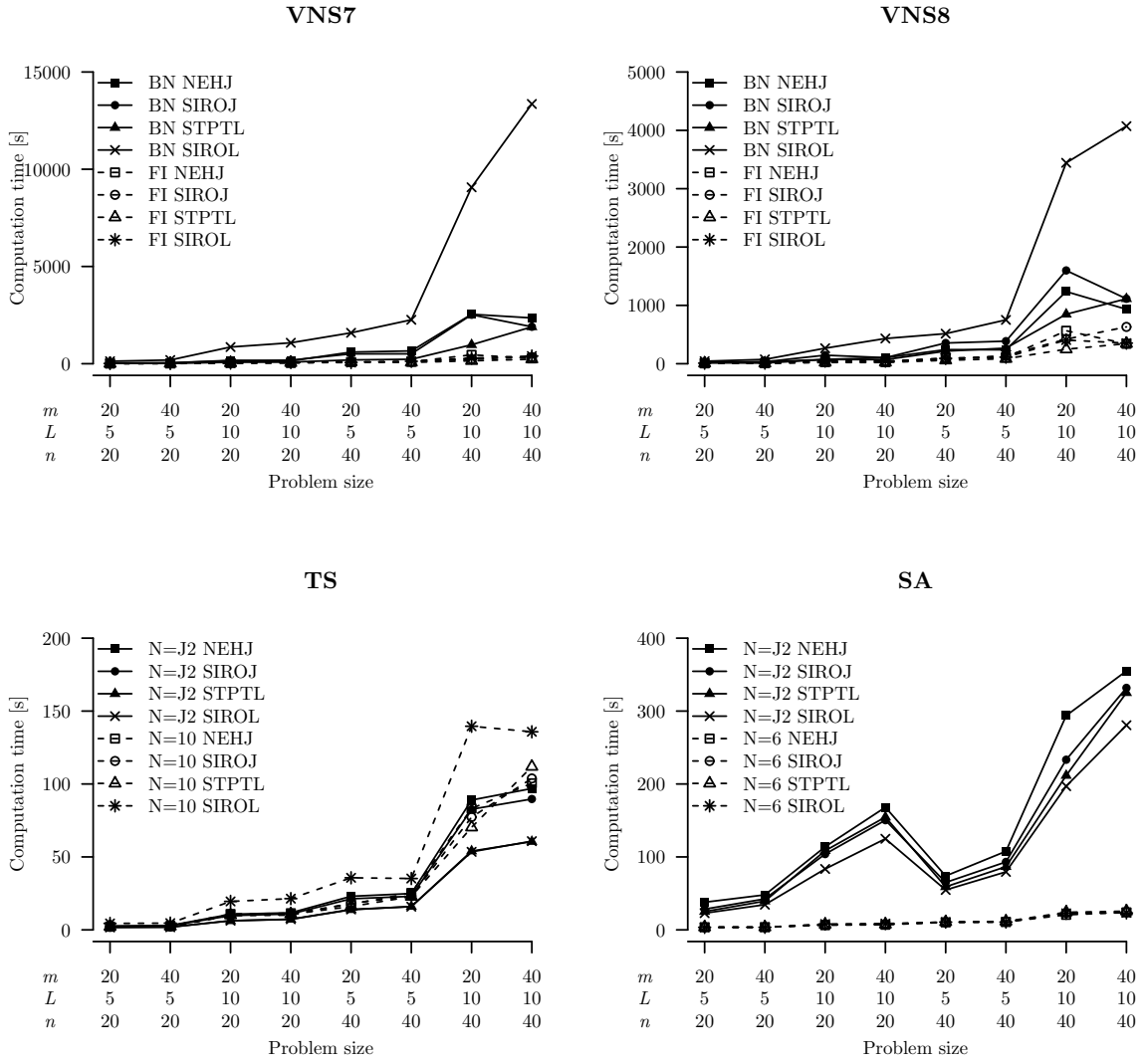


Figure C.2: Average computation times for large Inc_3 instances (II)



D Lot Streaming Results for Inc_3 Instances

Table D.1: Number of sublots per job in the best solutions of small problems (Inc_3)

Q	VNS2 FI	VNS2 FI*
2	17	18
3	62	66
4	57	46
5	17	22
6	7	6
7	0	2

Table D.2: Number of sublots per job in the best solutions of large problems (Inc_3)

Q	VNS2 FI	VNS2 FI*
2	8	7
3	49	50
4	16	20
5	6	3
6	1	0

Figure D.1: Average improvement of the $Q = 3$ STPTL solutions (Inc_3)

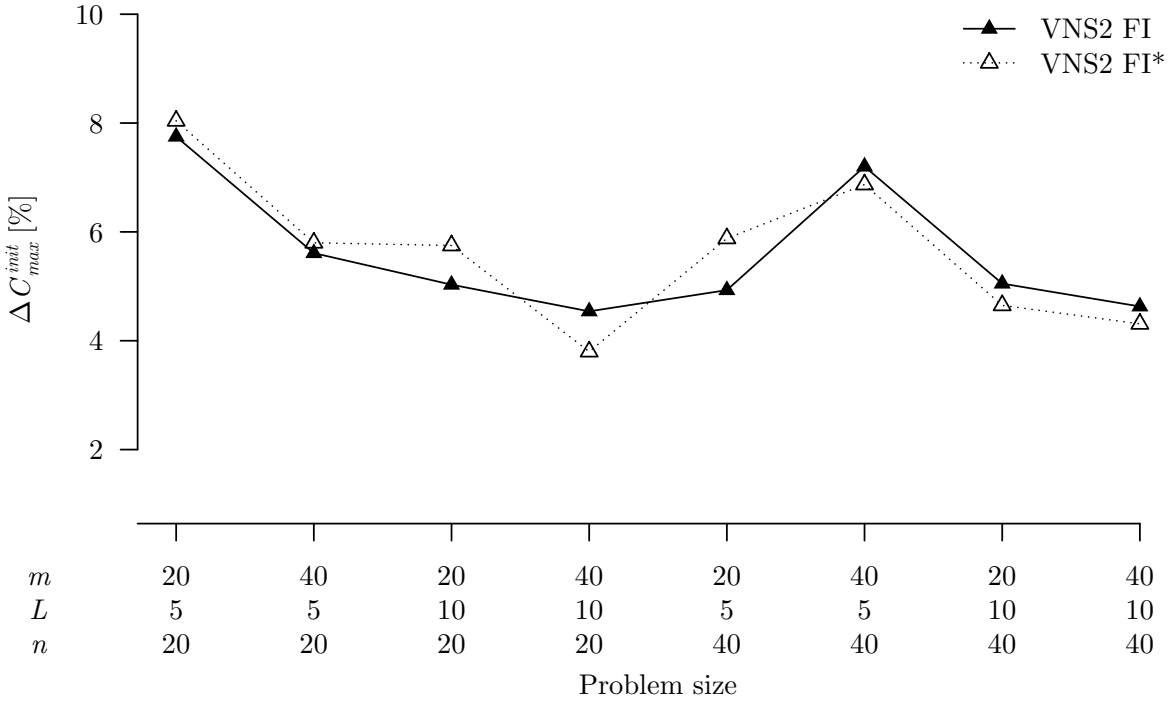
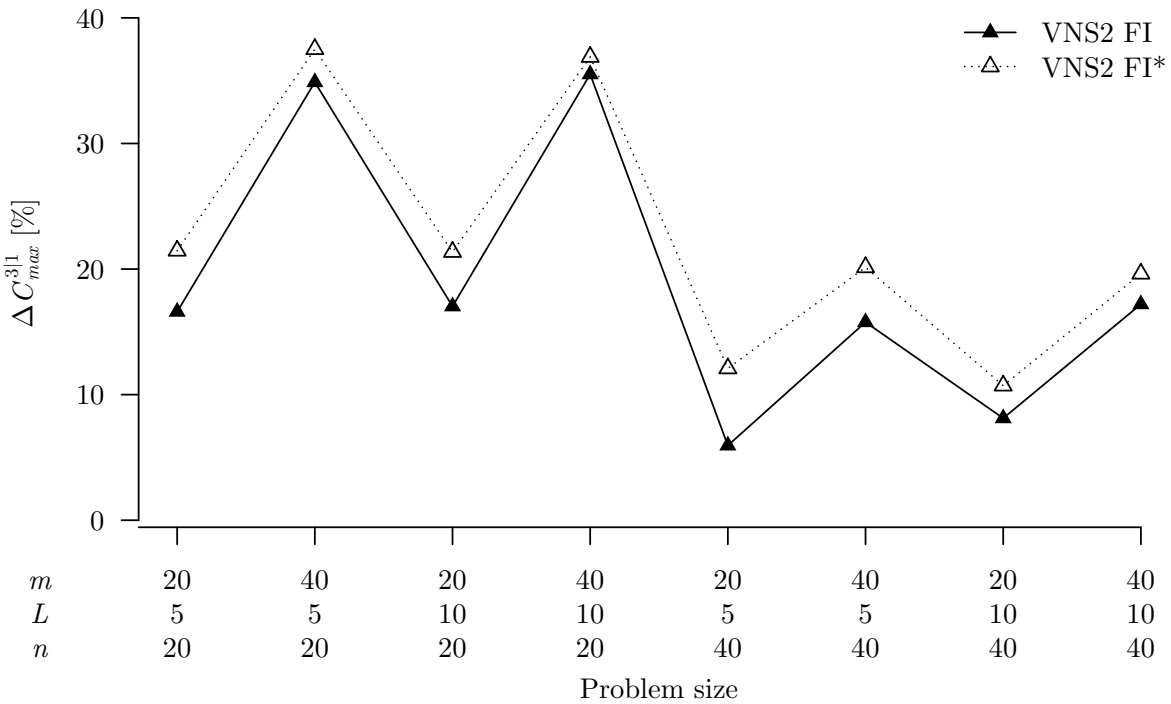


Figure D.2: Average makespan reductions compared to $Q = 1$ solutions (Inc_3)



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