Waveform Advancements and Synchronization Techniques for Generalized Frequency Division Multiplexing

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Abstract

To enable a new level of connectivity among machines as well as between people and machines, future wireless applications will demand higher requirements on data rates, response time, and reliability from the communication system. This will lead to a different system design, comprising a wide range of deployment scenarios. One important aspect is the evolution of physical layer (PHY), specifically the waveform modulation. The novel generalized frequency division multiplexing (GFDM) technique is a prominent proposal for a flexible block filtered multicarrier modulation.

This thesis introduces an advanced GFDM concept that enables the emulation of other prominent waveform candidates in scenarios where they perform best. Hence, a unique modulation framework is presented that is capable of addressing a wide range of scenarios and to upgrade the PHY for 5G networks. In particular, for a subset of system parameters of the modulation framework, the problem of symbol time offset (STO) and carrier frequency offset (CFO) estimation is investigated and synchronization approaches, which can operate in burst and continuous transmissions, are designed.

The first part of this work presents the modulation principles of prominent 5G candidate waveforms and then focuses on the GFDM basic and advanced attributes. The GFDM concept is extended towards the use of OQAM, introducing the novel frequency-shift OQAM-GFDM, and a new low complexity model based on signal processing carried out in the time domain. A new prototype filter proposal highlights the benefits obtained in terms of a reduced out-of-band (OOB) radiation and more attractive hardware implementation cost. With proper parameterization of the advanced GFDM, the achieved gains are applicable to other filtered OFDM waveforms.

In the second part, a search approach for estimating STO and CFO in GFDM is evaluated. A self-interference metric is proposed to quantify the effective SNR penalty caused by the residual time and frequency misalignment or intrinsic inter-symbol interference (ISI) and inter-carrier interference (ICI) for arbitrary pulse shape design in GFDM. In particular, the ICI can be used as a non-data aided approach for frequency estimation. Then, GFDM training sequences, defined either as an isolated preamble or embedded as a midamble or pseudo-circular pre/post-amble, are designed. Simulations show better OOB emission and good estimation results, either comparable or superior, to state-of-the-art OFDM system in wireless channels.

Contents

1	Introduction			1	
	1.1	Motiva	ation and background	1	
	1.2	Outlin	.e	3	
2	Bas	Basics on Multicarrier Modulation			
	2.1	Overv	iew	5	
	2.2	OFDM	1	7	
	2.3	SC-FI	ЭЕ	12	
	2.4	FBMC	C and FTN	13	
	2.5	UFMC	3	15	
	2.6	GFDM	1	17	
	2.7	Summ	ary	23	
3	Adv	dvancements on GFDM 2			
	3.1	OQAN	<i>A</i> GFDM	25	
		3.1.1	QAM GFDM	25	
		3.1.2	TS-OQAM-GFDM	27	
		3.1.3	FS-OQAM-GFDM	30	
		3.1.4	A pulse shape design for FS-OQAM-GFDM	32	
		3.1.5	Performance analysis	35	
3.2 GFDM low-complexity signal processing		I low-complexity signal processing	38		
		3.2.1	Transmitter reformulation	38	
		3.2.2	Demodulator reformulation	40	
		3.2.3	Matrix representation and complexity analysis	42	
3.3		GFDM	<i>I</i> as a flexible framework	47	
	3.4	Summ	ary	52	

Contents

4	Coa	rse Sy	nchronization Strategy for GFDM		55	
	4.1	1 Problem formalism			55	
	4.2	Sensit	ivity to interference		58	
		4.2.1	Analysis of self-interference		59	
	4.3	4.3 Non-data aided search approaches			62	
		4.3.1	Sliding window for energy detection		62	
		4.3.2	STO and CFO estimation based on repetitive patterns $\ . \ .$.		64	
		4.3.3	CFO estimation based on ICI in GFDM		66	
	4.4	Summ	nary		69	
5	Fin	e Sync	chronization Strategy for GFDM		71	
	5.1	Data	aided search approaches used in OFDM		71	
	5.2	Traini	ing sequence designs for GFDM		73	
		5.2.1	Preamble synchronization		75	
		5.2.2	Midamble synchronization		83	
		5.2.3	Pseudo-circular pre/post-amble synchronization \ldots		88	
	5.3	Summ	nary		94	
6	Cor	Conclusions and Future Works				
A	\mathbf{Filt}	ter Functions 10				
В	GF	DM B	asic Experiments Using LabView	1	.03	
С	GF	GFDM PSD Experiments Using LabView/Mathscript				
D	Fun	damer	ntals of Estimation Theory	1	.09	
Ao	crony	/ms		1	13	
Li	st of	figure	S	1	18	
Li	st of	tables	3	1	19	
Pι	Publications and patents of the author				21	
Bi	Bibliography				35	

Chapter 1

Introduction

1.1 Motivation and background



Figure 1.1: 5G scenarios [G. 14].

The fifth generation (5G) physical layer (PHY) requires distinct levels of flexibility, reliability, efficiency, robustness, energy-awareness, and scalability [G. 14, OBB⁺14]. When considering new applications and services such as car-to-car and car-to-infrastructure communication (Car-2-x) and scenarios such as Internet of Things (IoT), Tactile Internet [Fet14], the requirements go far beyond increasing throughput [FA14]. To cite just a few examples: resilient communication for vehicle collision avoidance, loose synchronization for IoT devices; low latency for Tactile Internet, as well dynamic spectrum allocation with low out-of-band (OOB) emission, and cognitive radio (CR) techniques to allow opportunistic reuse of spectrum. An overview of what will be 5G is given in [Fet12, $ABC^{+}14$].

5G related topics have been intensively researched lately. Several collaborative research projects were carried out by industry and academia in Europe [Pir14], e.g., METIS[OBB+14] and 5GNOW [G. 14, WKtB+13b, WKW+14]. The envisioned 5G service architecture is reproduced in Fig. 1.1 [G. 14] and it is believed that the fundamental characteristics of the main multicarrier system used today, i.e., orthogonal frequency division multiplexing (OFDM), are no longer compliant with all depicted scenarios.

For instance, the high OOB emission of OFDM prevents its use in CR and in fine-grained use of fragmented spectrum. Accordingly, an OOB emission reduction by a factor of 100 times is one of the key performance indicators observed in 5GNOW [GW13]. Further, 5GNOW also targets for relaxing the oscillator accuracy requirements for IoT by a factor of 10 when compared to fourth generation (4G), which is beneficial in terms of price, settling time after sleep mode, and a more simple synchronization procedure. Notwithstanding, other scenarios, e.g., super-high-rate and low latency mobile broadband, should also be addressed in the new PHY conception. In summary, the diverse Quality of Service (QoS) requirements such as rate, spectral efficiency, reliability, delay, and energy, makes the task of developing a waveform modulation for the 5G PHY to be very ambitious. This goal is even more difficult considering that a careful performance comparison of these requirements has to be done based on a multitude of parameters and scenarios. Nonetheless, the flexibility of the waveform design has been a significant 5G research topic, in particular for the highly disputed spectrum of frequencies below 6 GHz, and this is a particular aspect being detailedly addressed in this thesis.

Given its low OOB emissions, filter bank multicarrier (FBMC) [BÖ3, BLRR⁺10] is rediscovered for CR and dynamic spectrum allocation. However, the long impulse response of the filters, typically leading to the overlapping of at least 4 data symbols, prohibits its use for applications with sporadic traffics and tight latency constraints. Waveform candidates might also use faster than Nyquist signaling (FTN) [Maz75]. Taking advantage of the Mazo limit, it is a promising solution for high data rate scenarios. But, the large complexity of the receiver is a drawback here and makes FTN unsuitable for IoT. The universally filtered multicarrier (UFMC) [VWS⁺13] is a recent proposal that can have shorter impulse response, once a group of subcarriers is filtered. One disadvantage is that UFMC does not hold compliance with the OFDM legacy in terms of cyclic prefix (CP) support. To date, there is no single waveform candidate that could accomplish the 5G PHY requirements, which motivates the search towards a common framework.

Likewise, it is also desirable that the corresponding signal processing for a new 5G waveform could be performed as close as possible to an ideally synchronized system, with minimum transmission length and overall delay. The waveform parameterization and proper estimation of symbol time offset (STO) and carrier frequency offset (CFO) have a key impact on the receiver performance and need to be addressed according to the specific properties of the employed waveform and the target application scenarios.

Given the diverse aspects of the 5G scenarios, the investigation of cyclic autocorrelation properties and also training sequences are of interest.

In this context, the generalized frequency division multiplexing (GFDM) [FKB09] provides a very flexible time-frequency structure that favors the accomplishment of a new 5G PHY. GFDM is based on a number of independent modulated blocks formed by subcarriers containing several circularly pulse shaped subsymbols. Besides compatibility with CP to allow frequency domain equalization (FDE) in multi-path channels, GFDM supports the use of guard subsymbols to produce smooth block boundaries and presents very low OOB radiation. GFDM can emulate OFDM and single-carrier with frequency domain equalization (SC-FDE) when a single sub-symbol and a rectangular pulse shape and single subcarrier with several subsymbols is respectively chosen. This backward compatibility allows GFDM to take advantage of existing synchronization algorithms.

1.2 Outline

This thesis focuses on finding advancements in classic GFDM [Mic15], enabling its use as a single flexible waveform that can be easily reconfigured to address a multitude of applications. It is assumed that after proper signal detection [Dat14] STO and CFO need to be estimated on the perspective of either a continuous or sporadic transmission. Analyses are carried out in single-input and single-output (SISO) wireless communication systems where echoes exhibit a typical exponential decaying gain behavior.

This thesis proposes the use of offset quadrature amplitude modulation (OQAM) features, not restricted to conventional time shifts. It explores the use of unitary transform to allow the use of OQAM in the frequency domain too, and exploits a new short pulse shape to achieve impressive performance results in terms of time-frequency localization (TFL).

This thesis proposes an implementation model for the modulation and demodulation of GFDM signals, considering a per subsymbol circular convolution in the frequency-domain, which can be performed as an element-wise multiplication in the time domain. This simple re-orientation of the data symbol processing allows for a considerable reduction in complexity. The proposed low complexity signal processing concepts are key tools for inner and outer receiver design. Particularly, they will be used in the thesis for supporting a pipeline inner receiver implementation of a synchronization block diagram for embedded training sequences.

This thesis proposes synchronization techniques for GFDM exploiting autocorrelation, correlation and other statistics properties methods. It is shown that the training sequence can be designed as an isolated preamble, embedded as a midamble, or as a pseudo circular pre/post-amble. A block diagram that is capable of performing a continuous search for the training information also constitutes an important contribution in this work. The achievable performance with the proposed schemes is compared with the results of a

state-of-the-art OFDM approaches and residual STO and CFO errors are modeled as an equivalent increased signal-to-noise ratio (SNR) in a flexible self-interference metric.

The remainder of this thesis is structured as follows:

- ▷ Chapter 2 briefly presents the multicarrier modulation principles of relevant waveforms as a background to the investigations carried out with GFDM. Moreover, GFDM waveform engineering and basic proof of concept implementation are described. These results have been published in [GNN⁺13, GNMF13, MMG⁺14].
- ▷ Chapter 3 focuses on the advancements on GFDM with respect to OQAM features and details an alternative way to perform the modem functionality with time domain signal processing. The chapter also discusses how other prominent waveform candidates can be emulated within the GFDM framework. These results have been published in [GMM⁺15a, GNN⁺13, GMM⁺15b, GMM⁺15c].
- ▷ In Chapter 4 the problem formalism of synchronization and the analysis of CFO and STO sensitivity in GFDM is presented. A non-data aided method for coarse estimation of frequency offset is presented. These results have been partially published in [GMMF14, KGMF14].
- ▷ Chapter 5 presents the training sequence design and synchronization strategy for preamble, midamble and pseudo-circular pre/post-amble. These results have been published in [GMMF14, GFF15, GF15].
- ▷ Finally, in Chapter 6, the thesis is concluded and future works are proposed.

Chapter 2

Basics on Multicarrier Modulation

This chapter presents an overview of multicarrier systems and corresponding accompanying synchronization schemes. The basic principles of orthogonal frequency division multiplexing (OFDM) and an analysis of its precoded version, single-carrier with frequency domain equalization (SC-FDE) [FABSE02], and filtered schemes, filter bank multicarrier (FBMC) [BÖ3], faster than Nyquist signaling (FTN) [Maz75], universally filtered multicarrier (UFMC) [VWS⁺13], and generalized frequency division multiplexing (GFDM) [MMG⁺14] are carried out. The analysis uses a common mathematical notation based on discrete time models.

2.1 Overview

Multicarrier systems were first used in the 1950s in military communication [MC58] and the concept of filtered multicarrier systems was presented in the middle of the 1960s [Cha66, Sal67], from where OFDM [Bin90] has emerged as the predominant waveform in communication systems.

The main idea of a multicarrier modulation system is that the data information is divided in parallel streams and transmitted over separate narrowband subcarriers, a simple frequency division multiplex (FDM) system (Fig. 2.1). This division enables combating inter-symbol interference (ISI) in a frequency selective channel (FSC) if the subcarriers span a fraction of the channel's coherence bandwidth, allowing a simpler equalization once the fading effects can be considered flat per subcarrier. Furthermore, the multicarrier modulation facilitates the management of spectrum resources and is less susceptible to interference caused by impulse noise [MADS13]. One restriction of this scheme is a high sensitivity to time variations of the fading channel during one symbol interval. Limitations also include requirement of strict linear power amplification with considerable output back-off due to a high peak-to-average power ratio (PAPR) [DW01].



Figure 2.1: Example of FDM transmissions. The subcarriers can be assigned in a way that they do not interfere with each other, or explored in a more efficient arrangement, which might lead to interference depending on the subcarrier spectrum properties.

At present, OFDM is a widely adopted solution mainly because of its efficient FDM arrangement and robust performance in multipath channels. OFDM can use a cyclic prefix (CP) to emulate a cyclic convolution with the channel and to perform frequency domain equalization (FDE) through an efficient fast Fourier transform (FFT) algorithm implementation. However, the application scenarios predicted for 5G networks present challenges that OFDM, or its variant SC-FDE, can only address in a limited way [G. 14], making it not the most promising waveform for the next generation networks. For instance, in OFDM, each single subcarrier is shaped using a rectangular window in the time domain, resulting in sinc-shaped subcarriers in the frequency domain, leading to a non-negligible out-of-band (OOB). The OOB leakage is especially prejudicial in fragmented spectrum applications or in multiple access schemes with loosened synchronization requirements.

There exists a trivial solution to reduce OOB in OFDM, which basically consists of applying a bandpass filter before transmission to achieve a desired spectral mask. Such solution has been successfully applied in systems such as terrestrial digital video broadcasting (DVB-T) and terrestrial integrated services digital broadcasting (ISDB-T), attaining a critical mask of -50 dB spectral emission at only 5% excess bandwidth [ITU14]. In these systems, the large number of subcarriers, in the order of thousands, and generous use of guard bands near the cut off frequencies, in the order of hundreds of subcarriers, prevents any significant degradation on the filtered modulation error ratio. Indeed, even when considering only hundreds of active subcarriers and a sharp emission masks criteria, the band pass approach is still efficient at a reasonable performance degradation price, as in the case of PAPR reduction techniques involving successive clipping and filtering [LCJ97].

Moreover, recently the band pass filter proposal has been re-introduced as a simple add on solution for the current Long Term Evolution (LTE) PHY [ZJC⁺15]. Although the proposal leads to inherent ISI, the approach including the modulation and coding scheme results in throughput performance gain larger than 30% in case of inter-subband interference and asynchronous transmission. An in-deep study of classic pulse shapers such as sinc pulses,root raised cosine (RRC) pulses, or classic digital filtering methods such as partial response signaling could reveal even higher gains, but they are not within the scope of what is presented next.

This thesis is particularly interested in how a flexible modulation can produce a desired spectrum mask without use of post-processing filters, but with direct parameterization of inherent pulse shapes and simple use of guard subsymbols and subcarriers. Nevertheless, observing the same considerations already presented, the same bandpass filter approach can still be applied over any possible waveform candidate for 5G physical layer (PHY) systems in future works. Alternative multicarrier schemes exist and are currently being evaluated as candidates for the PHY of future mobile communication systems, among them, FBMC, FTN, UFMC, and GFDM.

In the FBMC method, the subcarriers are individually pulse shaped to reduce the OOB emissions. The length of the transmit filter impulse response is usually long and good spectral efficiency is achieved in continuous transmission [Bel10]. This technique is able to deal with fragmented spectrum applications [DBCK14] but is limited in the context of low latency scenarios with high efficiency in short burst transmissions.

In the FTN method, which can be applied on top of the FBMC approach, an intentional overlapping of symbols is applied along the time axis [ARO13] or the frequency axis [KCRD09]. This property allows for a higher transmission rate at the cost of self-interference and a much more complex receiver design [LG03].

In the UFMC method, the filter impulse response length is reduced by filtering groups of subcarriers [VWS⁺13]. However, as in FBMC, UFMC does not consider the use of CP and straightforward FDE, leading to a lack of backward compatibility with OFDM and also making it more sensitive to small time misalignment.

In the GFDM method [MMG⁺14], modulation is performed with a given number of subcarriers and subsymbols in a block structure. The subcarriers are filtered with a prototype filter that is circularly shifted in the time and frequency domains, which can be designed to reduce the OOB emissions and, for instance, avoid severe interference in incumbent services or other users in the context of fragmented spectrum and dynamic spectrum allocation [MMG⁺14]. Moreover, OFDM and SC-FDE can be seen as special cases, retaining all main benefits, though, at the cost of additional implementation complexity to enable new degrees of freedom.

More details of the discussed waveforms will be presented next, based on discrete-time baseband models. Block transmission built on a matrix-vector framework will be established as the common notation for the further contributions to the GFDM concept in this thesis.

2.2 OFDM

On the transmitter side, the OFDM approach filters the information data with a rectangular impulse response, with length $T_{\rm S}$, upconverts it to the subcarrier frequency, and then sums all the subcarriers. This is equivalent to having frequency-shifted sinc-impulses as the transfer function for every subcarrier in the frequency domain (Fig. 2.2).



Figure 2.2: Rectangular pulse shaped subcarriers in the time domain and the corresponding sinc-impulse response in the frequency domain.

In Fig. 2.2, the highest peak in the middle of the sinc spectrums represents the sampling points for each subcarrier. The bandwidth between them, $B_{\rm S}$, is conditioned such that the neighboring subcarrier impulses have a null on the sampling point of another subcarrier. This is because of the fact that sine and cosine waves with frequencies that are multiples of $B_{\rm S} = \frac{1}{T_{\rm S}}$ are orthogonal on the time interval $T_{\rm S}$.

Observe in Fig. 2.2 that the second larger side lobe peak of the OFDM subcarrier has a normalized magnitude of near 0.22 (-13 dBc). Also, the remaining peaks present a slow decaying behavior. Consequently, there will be a considerable interference from adjacent subcarriers if the sampling occurs at off-peak positions. This is why a precise synchronization is needed. Considering digital domain implementation, it is convenient to describe the OFDM modulation using a discrete-time baseband representation. Considering a total number of K subcarriers, a sampling rate of $\frac{K}{T_{\rm S}}$ is assumed, i.e., a symbol length of K uniformly spaced samples. In practice, considering that the synthesis of the signal involves further operations of interpolation (upsampling followed by filtering to remove undesired spectral images) with hardware implementation through a digital-to-analog (D/A) conversion, the critical sampling condition is actually relaxed by restricting the number of active subcarrier, $K_{\rm on} < K$, nulling the data coefficients at the edges of the bandwidth.

The OFDM transmit signal can be represented as

$$x[n] = \sum_{k=0}^{K-1} g_k[n] d_k, \qquad (2.1)$$

where n is the discrete time sample index, d_k , with k = 0, ..., K - 1, is the data set of a single OFDM symbol, and $g_k[n] = g[n] \exp(j2\pi \frac{k}{K}n)$ represents the subcarriers with the rectangular pulse shaping function

$$g[n] = \begin{cases} \frac{1}{\sqrt{K}} & \text{for} \quad n = 0, \dots, K - 1\\ 0 & otherwise \end{cases}$$
(2.2)

normalized to unitary energy, $\sum_{n=0}^{K-1} g[n]^2 = 1$.

The OFDM modulation can also be compactly represented using a matrix notation given by

$$\mathbf{x} = \mathbf{W}_K^{\mathrm{H}} \mathbf{d},\tag{2.3}$$

where $\mathbf{W}_{K}^{\mathrm{H}} = (\mathbf{g}_{0} \dots \mathbf{g}_{K-1})$ contains the impulse responses of the OFDM subcarriers defined as column vectors, $\mathbf{g}_{k} = (g_{k}[n])^{\mathrm{T}}$, and $\mathbf{d} = (d_{k})$.

As $[\mathbf{W}_K]_{i,l} = \frac{1}{\sqrt{K}} \exp\left(-j2\pi \frac{il}{K}\right)$ is actually a unitary $K \times K$ discrete Fourier transform (DFT) matrix, the OFDM modulation can be efficiently implemented using FFT algorithms [HJB84, NP00].

An illustration of the obtained signal is shown in Fig. 2.3.



Figure 2.3: OFDM symbol in the time and frequency domains. Dotted lines show the rectangular pulse shape and the corresponding sinc-shaped subcarrier (normalized to 0 dB at the peak position). Total number of subcarriers is K = 64, number of active subcarriers is $K_{\rm on} = 24$.

To combat ISI caused by multipath transmission in a wireless channel, with channel length $N_{\rm ch}$, and to allow that consecutive OFDM symbols can be processed in a decoupled way, a guard interval filled with $N_{\rm CP} > N_{\rm ch}$ cyclic prefix samples, is added to the transmitted signal $\tilde{\mathbf{x}} = [x[(-N_{\rm CP}) \mod K] \dots x[0] \dots x[K-1]]$. The operator $(\cdot) \mod K$ denotes the remainder modulo K. Note, however, that the CP reduces the efficiency of the modulation system by a factor $\frac{K}{K+N_{\rm CP}}$.

Transmission through the wireless channel is modeled by $\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{w}}$, where $\tilde{\mathbf{y}}$ is the received counterpart of $\tilde{\mathbf{x}}$. Here, $\tilde{\mathbf{H}}$ denotes the channel matrix, which is a $K + N_{\rm CP} + N_{\rm ch} - 1$ by $K + N_{\rm CP}$ convolution matrix with band-diagonal structure [Hun72] based on a channel impulse response $\mathbf{h} = (h_0, \ldots, h_{N_{\rm ch}-1})^{\rm T}$, whereas $\tilde{\mathbf{w}} \sim \mathcal{CN} (\mathbf{0}, \sigma_w^2 \mathbf{I}_{K+N_{\rm CP}+N_{\rm ch}-1})$ denotes additive white Gaussian noise (AWGN).

At the receiver, after CP removal under the assumption of perfect time and frequency synchronization, and the channel being constant within the OFDM symbol duration, the wireless channel model can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w},\tag{2.4}$$

by replacing $\tilde{\mathbf{H}}$ with the K by K matrix \mathbf{H} , the corresponding circular convolution matrix.

The OFDM received symbol can be obtained by zero-forcing (ZF) demodulation, $\mathbf{z} = \mathbf{H}^{-1}\mathbf{y}$ [Bin90], if **h** is available at the receiver, and the estimated transmitted data can be obtained by estimating

$$\hat{\mathbf{d}} = \mathbf{W}_K \mathbf{z},\tag{2.5}$$

with a posterior hard decision detection performed by a slicer and demmaper operation. The slicer quantizes the estimated data to the nearest constellation point, while the demapper transfers the constellation point to a bit sequence.

Actually, the ZF equalization procedure can be performed without matrix inversion, given that **H** is a circulant matrix [Bin90] and can be diagonalized by a DFT operation, i.e., $\mathbf{H} = \mathbf{W}_{K}^{\mathrm{H}} \mathrm{diag} \left(\sqrt{K} \mathbf{W}_{K} \mathbf{h}' \right) \mathbf{W}_{K}$, with $\mathrm{diag}(\cdot)$ denoting a matrix with the vector (\cdot) on its diagonal and zeros elsewhere, and \mathbf{h}' is a properly zero padded version of \mathbf{h} .

Therefore, the equalization process can be combined with the demodulation as

$$\mathbf{\hat{d}} = \left[\operatorname{diag}\left(\sqrt{K}\mathbf{W}_{K}\mathbf{h}'\right)\right]^{-1}\mathbf{W}_{K}\mathbf{y},\tag{2.6}$$

which effectively requires the multiplication of a single coefficient for each data symbol due to the simple diagonal structure of the channel matrix. Note that because of the definition of the DFT matrix, $[\mathbf{W}_K]_{i,l} = \frac{1}{\sqrt{K}} \exp\left(-j2\pi \frac{il}{K}\right)$, the normalization factor \sqrt{K} is needed. The ZF equalization changes the noise variance per subcarrier proportional to the received power per subcarrier (depending on the frequency response), amplifying the additive white Gaussian noise (AWGN) term at subcarriers affected by deeper notches of the FSC.

OFDM synchronization is a very important aspect to be considered, alignment errors can destroy the orthogonality and result in severe performance degradation [Moo94]. This need of strict synchronization motivated the development of several methods for estimation of symbol time offset (STO) and carrier frequency offset (CFO). These methods can be basically categorized into non-data-aided approaches, e.g., using CP periodic structure [vdBSB97, LK00], and data-aided approaches, e.g., using null subcarriers [MTGB01], or special training symbols [SC97, MZB00]. The non-data-aided approach requires a large amount of symbols to present acceptable performance, whereas data-aided approaches can allow for faster timing and frequency synchronization, requiring a smaller number of received symbols at the cost of a reduced data rate.

The use of a data-aided sequence composed of two repeated OFDM symbols was investigated in [Moo94] to explore a strong autocorrelation property for CFO estimation, and shortened data symbols were suggested to address wider CFO estimation range. This idea was addressed by [SC97] with the construction of a single OFDM symbol constituted with two repeated parts, obtained by transmitting only even or odd subcarriers. Additional integration operations and use of repetition parts with different phase rotations was later proposed by [MZB00]. Today, these methods constitute the foundations of more recent proposals [AKE08, AZS11], and the overall principles are adaptable to other multicarrier systems in general.

2.3 SC-FDE

In the OFDM signal, the summation of a large amount of independent subcarriers results in a Gaussian probability density function (pdf). As a result, OFDM systems are known to have a high PAPR when compared to single-carrier systems. High PAPR decreases the signal-to-quantization noise ratio of the analog-to-digital (A/D) and D/A conversions and reduces the efficiency of the power-amplifier in the transmitter. This issue is especially critical in the uplink direction because of the limited battery power in a mobile terminal.

SC-FDE modulation principles can be seen as a variation of OFDM, which entails the benefits of CP and FDE, but transmits the information in a single carrier modulation to avoid the PAPR issues. The information data is precoded through a DFT operation, and the resulting output becomes the coefficients that modulates the subcarriers in the classic OFDM approach. Considering a selection of $K_{on} \leq K$ subcarriers to be modulated as a single carrier, the operation can be described as

$$\mathbf{x} = \mathbf{W}_{K}^{\mathrm{H}} \left(\mathbf{G}^{(K,K_{\mathrm{on}})} \mathbf{W}_{K_{\mathrm{on}}} \mathbf{d}_{K_{\mathrm{on}}} \right), \qquad (2.7)$$

where $\mathbf{d}_{K_{\text{on}}}$ is a column vector containing K_{on} time domain information symbols, $W_{K_{\text{on}}}$ is a K_{on} by K_{on} DFT operation, and $\mathbf{G}^{(K,K_{\text{on}})}$ is a K by K_{on} matrix that maps the precoded information to active subcarriers and attributes zeros to the remaining $K - K_{\text{on}}$ unused subcarriers.

The mapping operation can be defined either in localized or distributed mode. In localized mapping, the DFT outputs are mapped to a subset of consecutive subcarriers, e.g., $\mathbf{G}^{(K,K_{\text{on}})} = \left(\mathbf{I}_{K_{\text{on}}} \mid \mathbf{0}^{(K_{\text{on}},K-K_{\text{on}})} \right)^{\text{T}}$, where $\mathbf{I}_{K_{\text{on}}}$ is an identity matrix and $\mathbf{0}$ is a matrix with zero entries. While in distributed mapping, the DFT outputs of the input data are assigned to subcarriers over the entire bandwidth non-continuously. Nonetheless, the matrix $\mathbf{G}_{K,K_{\text{on}}}$ acts as filter mask in frequency domain and (2.7) actually performs an equivalent circular convolution operation in time domain, thus, enabling CP and FDE.

The SC-FDE demodulator is similar to OFDM, but with an additional reverse subcarrier selection and inverse discrete Fourier transform (IDFT) operation. This is, after synchronization and cyclic prefix removal, channel equalization is be performed in the frequency domain and the subset of subcarriers are converted back to time domain as

$$\hat{\mathbf{d}}_{K_{\text{on}}} = \mathbf{W}_{K_{\text{on}}}^{\text{H}} \left(\mathbf{G}^{(K,K_{\text{on}})} \right)^{\text{H}} \left[\text{diag} \left(\sqrt{K} \mathbf{W}_{K} \mathbf{h} \right) \right]^{-1} \mathbf{W}_{K} \mathbf{y}.$$
(2.8)

and a slicer is used to recover the data.

The reduced PAPR led Third Generation Partnership Project (3GPP) chose SC-FDE for the LTE uplink, resulting in a reduced energy consumption on the user terminal side. From the LTE base station perspective, the combination of the transmitted signals of several users equals single-carrier frequency division multiple access (SC-FDMA), a hybrid modulation scheme that combines individual low PAPR single-carrier systems with the multipath resistance and flexible subcarrier frequency allocation offered by orthogonal frequency division multiple access (OFDMA).

SC-FDE synchronization tasks and methods are very comparable to those developed for OFDM, for instance, when using an isolated preamble approach. The concept of using unique word (UW) prefixes instead of CP is also well investigated for SC-FDE [HWH03, CSBM06].

2.4 FBMC and FTN

FBMC modulation principles rely on a filter that spans over multiple symbols and is able to achieve an outstanding suppression of OOB emissions [FB11]. The filtering operation leads to self-interference between subcarriers and symbols, which is evaded by either reducing the efficiency using only even or odd subcarriers, or by enabling real-orthogonality [LAB95].

The real-orthogonality condition is achieved by using a half-symbol space delay between the in-phase and the quadrature components of quadrature amplitude modulation (QAM) symbols with half-Nyquist transmit and receive pulse shapes. This technique is known as offset quadrature amplitude modulation (OQAM).

The FBMC transmit signal using OQAM can be represented as

$$x[n] = \sum_{k=0}^{K-1} \left(j^k g[n] \Re\{d_k\} + j^{k+1} g\left[n - \frac{K}{2}\right] \Im\{d_k\} \right) \exp\left(j2\pi \frac{k}{K}n\right),$$
(2.9)

where $\Re\{d_k\}$ and $\Im\{d_k\}$ corresponds to the in-phase and quadrature components of the data set d_k , respectively. The j^k is a phase rotation among even and odd subcarriers that allows for a real-orthogonality condition when using the half-Nyquist prototype filter g[n] [LAB95]. The time-shift of $\frac{K}{2}$ implies that K must be even or that the sampling rate must be doubled $(n = ..., 0, \frac{1}{2}, 1, ...)$ when K is odd.

An illustration of a single FBMC symbol is shown in Fig. 2.4, using a near-perfect reconstruction (NPR) pulse [Bel10] (see Appendix A). The obtained impulse response in the time domain is 4.5 times larger than the OFDM case illustrated in Fig. 2.3. However, the spectrum is several orders of magnitude more localized, with a fast decay of -40 dBc OOB emission between subcarriers 12 and 13, and a signal strength less than -60 dBc at subcarriers 14 and 15.



Figure 2.4: FBMC symbol in the time and frequency domains. Dotted lines show a NPR pulse shape and the corresponding subcarrier spectrum. K = 64, $K_{on} = 24$. The signal generation and spectrum analysis is carried out in an asynchronous condition, highlighting the low OOB that can be achieved with FBMC.

FBMC systems do not consider the use of CP. To combat ISI introduced by the wireless channel, the symbol duration in FBMC is chosen to be much larger than the channel impulse response (CIR), allowing for a per subcarrier equalization. In bursty transmission of few symbols, the tails in FBMC dominate and decrease the modulation efficiency, on the contrary, the CP loss prevails in OFDM. In the frequency domain, as FBMC is much more localized, a given system can always use more subcariers than OFDM to achieve the same spectral mask, achieving more rate in the same bandwidth. So, in continuous transmission, the efficiency of FBMC is higher than OFDM with CP, once the influence of the tails introduced by the prototype filter can be neglected. With a maximum delay spread around 10% of the symbol duration, FBMC can compete with OFDM using CP for transmission over a multipath channel [LSL08].

FBMC synchronization methods can make use of an isolated preamble, or scattered pilots in long bursts transmissions [SIVR10]. By applying scattered pilots, the CFO, fractional time delay (FTD), and channel response can be estimated using an iterative interference cancellation approach [SIVR10].

The FTN signaling increases the spectral efficiency by imposing a denser time and frequency grid, intentionally creating interference among data symbols and subcarriers at the transmitter side. In view of the multicarrier techniques, the FTN concept can be achieved by packing the subcarriers at a closer distance, either increasing the overall number of transmitted data or by reducing the total bandwidth. This concept will be more specifically addressed in Section 3.3.

2.5 UFMC

The UFMC modulation applies filtering over a frequency band comprising several subcarriers. One of the design criteria of UFMC is to collect the advantages of filtered OFDM and FBMC while avoiding the respective disadvantages, thereby trading off the filtering functionality between the two techniques.

The UFMC transmit signal can be represented as

$$\tilde{x}[n] = \sum_{l=0}^{\mathcal{L}-1} g_l[n] * x_l[n], \qquad (2.10)$$

where l is a sub-band index, $g_l[n]$ is the *l*-th sub-band filter and $x_l[n]$ represents the *l*-th group of OFDM subcarriers modulated according to (2.1). An illustration of the concept is shown in Fig. 2.5 with 3 sub-bands using 8 subcarriers each.

UFMC filtering suppresses the spectral side-lobe levels and reduces the ICI between different sub-bands in case of CFO. Another advantageous effect of filtering per sub-band instead of per subcarrier is that the filter length may be significantly shorter than that of FBMC, e.g., in the order of the OFDM cyclic prefix, due to the larger sub-band bandwidth. The OOB emission is higher than the FBMC case, but enough to achieve -50 dBc at two sub-bands apart. Hence, the UFMC waveform is also a potential candidate for communication with short bursts, as they can appear in Internet of Things (IoT).



Figure 2.5: UFMC sub-band symbol in the time and frequency domains. In the top plot, dash-dotted line shows the Dolph-Chebyshev filter convolved with the rectangular pulse shape in the time domain and the corresponding side-lobe of -40 dBc applied to a sub-band in the frequency domain. In total, 3 sub-bands, with 8 subcarriers each, are used.

2.6 GFDM

GFDM [FKB09, MMG⁺14] is a block filtered multicarrier modulation scheme, in which multiple subsymbols can be transmitted per subcarrier in a block. GFDM applies circular pulse shaping of the individual subcarriers. GFDM can also resemble the well-known OFDM system, with the choice of a single subsymbol and a rectangular pulse shape, and the SC-FDE, using a single subcarrier with several subsymbols. The circularity principle allows GFDM to explore cyclic prefix (CP) and use FDE to handle multipath effects in FSC. GFDM can be designed to reduce the OOB emissions and, for instance, avoid severe interference in incumbent services or other users in the context of fragmented spectrum and dynamic spectrum allocation. In the following, GFDM basic principles will be detailed.

Consider a binary data vector source **b**, which is encoded to obtain \mathbf{b}_c . A mapper, e.g., QAM, maps the encoded bits to symbols from a 2^{μ} -valued complex constellation where μ is the modulation order. The resulting vector **d** denotes a data block that contains N elements, which can be decomposed into K subcarriers with M subsymbols each, according to $\mathbf{d} = \left(\mathbf{d}_0^{\mathrm{T}}, \ldots, \mathbf{d}_{M-1}^{\mathrm{T}}\right)^{\mathrm{T}}$ and $\mathbf{d}_m = (d_{0,m}, \ldots, d_{K-1,m})^{\mathrm{T}}$. The total number of symbols follows as N = KM. Therein, the individual element $d_{k,m}$ corresponds to the data transmitted on the k-th subcarrier and in the m-th subsymbol of a GFDM block.

The circular pulse shaping applied to each $d_{k,m}$ is given by

$$g_{k,m}[n] = g\left[(n - mK) \mod N\right] \cdot \exp\left(j2\pi \frac{k}{K}n\right), \qquad (2.11)$$

with n = 0, ..., N - 1 denoting the sampling index. Each $g_{k,m}[n]$ is a time and frequency shifted version of a prototype filter g[n], where the remainder modulo N operation makes $g_{k,m}[n]$ a circularly time shifted version of $g_{k,0}[n]$ and the complex exponential performs the shifting operation in the frequency domain.

The GFDM transmit signal is obtained by superposition of all modulated subsymbols

$$x[n] = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} g_{k,m}[n] d_{k,m}.$$
(2.12)

Collecting the filter samples in a vector $\mathbf{g}_{k,m} = (g_{k,m}[n])^{\mathrm{T}}$ allows to formulate (2.12) as

$$\mathbf{x} = \mathbf{A}\mathbf{d},\tag{2.13}$$

where $\mathbf{x} = (x[n])^{\mathrm{T}}$ and \mathbf{A} is a $KM \times KM$ transmitter matrix [GNN+13] with a structure according to

$$\mathbf{A} = \begin{pmatrix} \mathbf{g}_{0,0} & \dots & \mathbf{g}_{K-1,0} & \mathbf{g}_{0,1} & \dots & \mathbf{g}_{K-1,1} & \dots & \mathbf{g}_{0,M-1} & \dots & \mathbf{g}_{K-1,M-1} \end{pmatrix}.$$
(2.14)



The GFDM terminology is illustrated in Fig. 2.6.

Figure 2.6: GFDM terminology [GMM⁺15c].

At this point, \mathbf{x} contains the transmit samples that correspond to the GFDM data block \mathbf{d} .

In contrast to FBMC and UFMC, GFDM supports the use of a CP to provide robustness against interference between blocks at the cost of rate loss similar to OFDM. With the use of the CP, the convolution with the wireless FSC becomes circular, allowing for FDE.

As OOB is an important aspect to be addressed with GFDM, the use of windowed CP and guard symbols needs to be considered. These two approaches are detailed next.

A guard interval filled with a windowed cyclic prefix is a classic solution that reduces transmission efficiency, by transmitting redundant information. But to ease combating ISI, it can be shaped to reduce OOB [BLY13], e.g., pinching the block boundaries as follows

$$\tilde{x}[n] = p_{\mathrm{W}}[n]x[n], \qquad (2.15)$$

$$p_{\rm W}[n] = \begin{cases} p_{\rm UP}[n] & -N_{\rm W} - N_{\rm CP} \le n < -N_{\rm CP} \\ 1 & -N_{\rm CP} \le n < N - 1 \\ p_{\rm DW}[n] & N \le n < N + N_{\rm W} \\ 0 & \text{otherwise}, \end{cases}$$
(2.16)

where N = KM, $N_{\rm CP}$ is the CP length, which should be larger than the CIR, and $N_{\rm W}$ is the length expended with ramp up $p_{\rm UP}[n]$ and ramp down $p_{\rm DW}[n]$ smoothing functions. Several window functions can be used [NP00], where a commonly used one is the raised cosine window.

Inserting guard symbols (GS) is a particular solution that allows the GFDM block boundaries to naturally present a fade-in and fade-out behavior, being similar to the FBMC modulation in burst case. The natural smoothing transition is very effective when using an ISI-free transmission filter and CP with length ϑK samples, with $\vartheta \in \mathbb{N}$. Abrupt transitions at the block boundaries can be avoided by setting the 0-th and $(M - \vartheta)$ -th subsymbol to a fixed value, achieving a strongly attenuated OOB radiation.

The amount of useful time slots is termed $M_{\rm on}$, where $M_{\rm on} = M$ does not consider use of guard subsymbols, $M_{\rm on} = M - 1$ represents the case when a single guard symbol is used (Fig. 2.7), e.g., the first symbol null, $M_{\rm on} = M - 2$ considers the use of two guard subsymbols, e.g., when in combination with $N_{\rm CP} = \vartheta K$. Lower values of $M_{\rm on}$ can be used to emulate a FBMC burst case, as it will be described in Section 3.1.5.

In Fig. 2.7, it is possible to observe a configuration where four active subsymbols in GFDM are placed in a block occupying the length of 5 OFDM symbols. The filter tails of the subsymbols are cyclically confined within the duration of the GFDM block. The cyclic convolution approach contrasts with the FBMC example in Fig. 2.4. That is, one single FBMC symbol spreads along the duration of 4.5 OFDM symbols, while four FBMC symbols would spread along 7.5 OFDM symbols.

The use of one guard subsymbol, nulling the data at the first time slot, allows the GFDM signal to present smooth transitions at the block boundaries, achieving around -30 dBc between subcarriers 12 and 13, less than -40 dBc for subcarriers 14 and 15, and near to -70 dBc OOB emission at the extremes of the spectrum.

As GFDM presents a rich waveform engineering, two practical experiments have been prepared to explore the GFDM spectrum properties using windowing resources, guard subsymbols, and multiple filters (see Appendices B and C). More detailed results have been reported in [MMGF14, MMG⁺14].



Figure 2.7: GFDM symbol in the time and frequency domains. Dotted lines show a RC pulse shape, with roll-off $\alpha = 0.1$, cyclically shifted according to the subsymbol positions and the corresponding subcarrier spectrum when the first subsymbol is null. K = 64, $K_{\rm on} = 24$, M = 5, $M_{\rm on} = 4$. The signal generation and spectrum analysis is carried out in an asynchronous condition, highlighting the low OOB that can be achieved with GFDM.

The GFDM spectral efficiency using CP, windowing, and GS can be computed as

$$R_{\rm T} = \frac{KM_{\rm on}}{KM + N_{\rm CP} + 2N_{\rm W}},$$
(2.17)

showing that, from the N = KM transmitted symbols, only KM_{on} are used, and that the original block length of KM samples will be increased by $N_{CP} + 2N_W$ samples to provide protection against interblock interference and to reduce OOB by smoothing the block boundaries.

Without considering windowing, the higher the value of M, the smaller the reduction of the spectral efficiency due to the GS insertion. However, these guard subsymbols can also be used for inserting synchronization signals and pilots, as will be explored in Section 5.2.3. In case of high M, an increase of latency can be mitigated by proportionally reducing the subsymbol duration and enlarging the subcarrier bandwidth. The number of subcarriers can be reduced to keep the occupied bandwidth constant.

The overall GFDM transceiver equation can be written as $\mathbf{y} = \mathbf{H}\mathbf{A}\mathbf{d} + \mathbf{w}$. Introducing $\mathbf{z} = \mathbf{H}^{-1}\mathbf{H}\mathbf{A}\mathbf{d} + \mathbf{H}^{-1}\mathbf{w} = \mathbf{A}\mathbf{d} + \bar{\mathbf{w}}$ as the received signal after channel equalization, linear demodulation of the signal can be expressed as

$$\hat{\mathbf{d}} = \mathbf{B}\mathbf{z},\tag{2.18}$$

where **B** is a $KM \times KM$ receiver matrix and $\hat{\mathbf{d}}$ corresponds to the estimated received data before a slicer operation. This matrix can be defined as follows:

The matched filter (MF) receiver maximizes the signal-to-noise ratio (SNR) per subcarrier and is given by

$$\mathbf{B} = \mathbf{A}^{\mathrm{H}},\tag{2.19}$$

but with the effect of introducing self-interference when a non-orthogonal transmit pulse is applied, i.e., the scalar product $\langle g_{0,0}, g_{k,m} \rangle_{\mathbb{C}^N} \neq \delta_{0,k} \delta_{0,m}$ with Kronecker delta $\delta_{i,j}$.

The zero-forcing (ZF) receiver completely removes any self-interference with

$$\mathbf{B} = \mathbf{A}^{-1},\tag{2.20}$$

but at the cost of potentially enhancing the noise.

Over flat channels, the noise enhancement factor (NEF), ξ , determines the signal-to-noise ratio (SNR) reduction when using the ZF receiver. It is defined as

$$\xi = \sum_{i=0}^{N-1} |[\mathbf{B}]_{k,i}|^2 \ge 1,$$
(2.21)

which is equal for every k.

Also, there are cases in which **A** is ill-conditioned and thus the inverse does not exist [MMF14]. This behavior is illustrated in Fig. 2.8(a) where the reciprocal condition number [Ant05, p.316] of the transmission matrix **A** is shown when a RC filter with $\alpha = 0.5$ is used. Reciprocal condition numbers close to zero mean that the matrix is singular. For instance, the singularity of the system is observed for M even. This is also expressed in Fig. 2.8 (b), where the NEF is very high for M even.



Figure 2.8: (a) Reciprocal condition number (rcond) for GFDM matrix A depending on M and K. In this example, a RC prototype filter with $\alpha = 0.5$ is used. (b) Noise enhancement factor depending on the number of subsymbols, M (K = 15).

The minimum mean squared error (MMSE) receiver makes a compromise between self-interference and noise enhancement considering a jointly equalization, i.e., $\hat{\mathbf{d}} = \mathbf{B}\mathbf{y}$, where

$$\mathbf{B} = (\mathbf{R}_w + \mathbf{A}^{\mathrm{H}} \mathbf{H}^{\mathrm{H}} \mathbf{H} \mathbf{A})^{-1} \mathbf{A}^{\mathrm{H}} \mathbf{H}^{\mathrm{H}}, \qquad (2.22)$$

requires the knowledge of the covariance matrix of the noise, denoted as \mathbf{R}_w .

Finally, the received symbols $\hat{\mathbf{d}}$ are sliced and demapped to produce a sequence of bits $\hat{\mathbf{b}}_{c}$ at the receiver, which are then passed to a decoder to obtain $\hat{\mathbf{b}}$.

The subcarrier filtering can result in non-orthogonal subcarriers, and both ISI and inter-carrier interference (ICI) might arise. Nevertheless, there exist advanced receiver techniques that can eliminate the self-induced ICI and ISI interference, i.e., a MF or MMSE receiver with iterative interference cancellation [GNN⁺13, Mic15, MGZF15a, ZMM⁺15] can significantly improve the symbol error rate (SER) performance over wireless channels. The algorithms can combine maximum likelihood and successive interference cancellation detection techniques to exploit the inherent diversity of GFDM coming from self-interference, but the detailed analysis of such approaches are outside the scope of this thesis.

2.7 Summary

In this chapter, the basic principles of filtered multicarrier have been introduced, with an overview of the OFDM, SC-FDE, FBMC, FTN, UFMC, and GFDM modulation methods. Relevant synchronization literature was also briefly discussed.

This chapter introduces the common mathematical notation that is going to be used in remaining parts of the thesis. This notation relies on discrete-time baseband models and block transmission built on a matrix-vector framework.

The main contributions of this chapter have been published in [GNN⁺13, GNMF13, MMG⁺14], while [GMM⁺14, MMM⁺15, MMGF14, MGZF15a, MGZF15b, ZMM⁺15, AMGaG13] and [G. 14, WKtB⁺13b, WKTB⁺13a, WKW⁺14] can also be consulted for further details on the presented topics.

Details can be summarized as follows

- ▷ Multicarrier systems can combat multipath fading with enhanced immunity to inter-symbol interference, splitting the data into several narrow bandwidth subcarriers. OFDM modulation is a widely adopted solution.
- ▷ OFDM advantages include efficient implementation through FFT algorithms and robust performance in FSC using CP and FDE techniques. Disadvantages include non-negligible OOB emission (-13 dBc at first side lobe), high PAPR, and strict synchronization requirements.
- ▷ Precoding techniques, such as (SC-FDE), and more flexible filtered multicarrier systems (FBMC, UFMC, GFDM) aim to improve the OFDM drawbacks, targeting the requirements of future wireless systems.
- ▷ The vast literature for OFDM's STO and CFO estimation, exploring the block based structure with both non-data-aided and data-aided approaches, can be used as a basis to support the development of proper synchronization schemes for the new modulation approaches.
- ▷ FBMC achieves outstanding frequency localization, with great OOB reduction at neighbor subcarriers using long impulse responses. CP is not supported, requiring more complex multiple-tap equalizers.
- ▷ FTN principles can be explored to increase data rates or transmission efficiency. By reducing the subcarrier spacing and introducing intentional self-interference, data rates can be increased at the cost of more complex receivers.

- ▷ UFMC applies per sub-band filtering on the OFDM signal, achieving moderated OOB reduction with shorter filter responses. UFMC is not compatible with the legacy OFDM system using CP, it relies on zero padding guard interval to accommodate the filter tails.
- \triangleright GFDM divides a given time-frequency resource block into K subcarriers and M subsymbols and enables cyclic pulse shaping on a per subcarrier basis.
- \triangleright GFDM turns into OFDM when configured to use a single subsymbol, M = 1, with a rectangular pulse shape. SC-FDE is obtained when a single subcarrier is used, K = 1 and a rectangular pulse shape is defined in the frequency domain (resulting in a Dirichlet pulse in the time domain).
- ▷ GFDM is compatible with CP technique and can take advantage of techniques developed for OFDM, e.g., synchronization approaches. Low OOB radiation can be achieved by using windowing and/or guard subsymbols.
- ▷ Guard subsymbols can either receive null values (similar to a FBMC burst) or transmit known data (training sequences) along different blocks, which is helpful for synchronization.
- ▷ Experiments presented in the Appendices B and C are available in the public domain and can be used to demonstrate and verify the results presented in this thesis.
- ▷ GFDM can become non-orthogonal depending on the pulse shape design. The self-induced ICI and ISI can be mitigated by ZF and MMSE receivers, at the cost of noise enhancement, or by iterative non-linear receivers, at the cost of higher complexity.
- \triangleright For certain configurations, e.g. M even in combination with classic filters, the system performance is severely affected because of high NEF and the ill-conditioning of the GFDM modulation matrix **A**.

The advantages and contour solutions to the limitations of GFDM, as well as synchronization strategies, will be further discussed in the remaining chapters of this thesis.

Chapter 3

Advancements on GFDM

In this chapter GFDM advancements are presented to complement the basic principles introduced in chapter 2. The concept of OQAM is applied and a novel frequency-shift OQAM-GFDM configuration is discussed. A short prototype filter is proposed to achieve low OOB radiation in frequency-shift offset quadrature amplitude modulation (OQAM). A new low complexity model based on signal processing carried out in the time domain shows attractive hardware implementation costs. With proper parameterization, the advanced GFDM is shaped to represent other prominent filtered OFDM waveforms, serving as a common framework.

3.1 OQAM GFDM

This section presents a novel perspective to apply the OQAM scheme on generalized frequency division multiplexing (GFDM). The conventional time-shift OQAM is described for GFDM, and by introducing the general use of unitary transforms, an interesting counterpart, i.e., frequency-shift OQAM, is proposed. The conventional long prototype pulse with time-shift of one half subsymbol becomes a short prototype pulse with frequency-shift of one half subcarrier. The frequency-shift OQAM scheme offers advantages such as low out-of-band (OOB) emission and low implementation complexity. The concept can be applied to the broader scope of filtered OFDM without penalties in terms of performance in time variant frequency-selective channels.

3.1.1 QAM GFDM

In classic GFDM, using quadrature amplitude modulation (QAM) signaling, the transmission of well-localized half-Nyquist impulse responses, e.g., root raised cosine (RRC), assures inter-symbol interference (ISI)-free matched filter operation but can lead to undesirable inter-carrier interference (ICI). This aspect can be illustrated in the time domain in the form of an eye pattern [Pro01], as shown in Fig. 3.1.

The eye pattern is created with the superposition of multiple traces. Each trace describes a demodulated subcarrier using match filtering operation, that is, $\hat{y}_k[n] = g^*[-n]\exp\left(-j2\pi\frac{k}{K}n\right) \circledast y[n]$, where y[n] = x[n] is assumed. In this example, a real antipodal data is modulated using only the even subcarriers, i.e., $\Re\{d_{2k,m}\} = \{-1, 1\}$, $\Im\{d_{2k,m}\} = 0$, and $d_{2k+1,m} = 0$. To highlight the ICI, the roll-off of the half-Nyquist filter is set to $\alpha = 1$.



Figure 3.1: GFDM eye pattern obtained at the receiver side when a real antipodal data is modulated using only the even subcarriers. M = 8, K = 128, RRC, $\alpha = 1$. An ISI-free condition is achieved at every mK points for the even subcarriers, $\Re \{\hat{y}_{2k}[n]\}|_{n=mK}$. At the same sampling points, ICI is observed only at the the real components of the even subcarriers, that is, $\Re \{\hat{y}_{2k}[n]|_{n=mK}\} \neq 0$ and $\Im \{\hat{y}_{2k}[n]|_{n=mK}\} = 0$.

Fig. 3.1 shows that real data source (in-phase component) creates real ICI at the neighbor subcarriers at every mK sampling point, m = 0, ..., M - 1. In a corresponding

way, imaginary data source (quadrature component) would create imaginary ICI at the neighbor subcarriers at the same points. Note, however, that the eye pattern exhibit an ICI-free condition at half a subsymbol offset, $(m + \frac{K}{2})$, and this distinct behavior can be exploited to define a more favorable arrangement. In the following, the real and imaginary data will be shifted in time and phase rotated to achieve the real-orthogonality.

3.1.2 TS-OQAM-GFDM

In time-shifted OQAM-GFDM (TS-OQAM-GFDM), complex-valued data symbols, $d_{k,m} = d_{k,m}^{(i)} + jd_{k,m}^{(q)}$ are transmitted on K subcarriers, where the real and imaginary part are offset by half the symbol duration, i.e., $\frac{K}{2}$ samples. Each symbol is pulse shaped by a symmetric, real-valued, half-Nyquist prototype filter g[n]. A phase rotation of $\frac{\pi}{2}$ radians is applied among odd and even subcarriers, aligning them according to the ICI-free regions observed in Fig. 3.1.

Two sets of pulse shapes are defined to accomplish this task, one corresponding to the in-phase $(d_{k,m}^{(i)})$ and other corresponding to the quadrature $(d_{k,m}^{(q)})$ data input, leading to

$$g_{k,m}^{(i)}[n] = j^k g_{k,m}[n], \qquad (3.1)$$

$$g_{k,m}^{(\mathbf{q})}[n] = j^{k+1}g_{k,m+\frac{1}{2}}[n].$$
(3.2)

The in-phase pulse shapes, (3.1), correspond to the GFDM filter defined in (2.11) with an additional phased rotation, j^k . The quadrature pulse shapes, (3.1), shift the prototype pulse g[n] in time by half a subsymbol, and the phase rotation already includes one additional imaginary term, j^{k+1} , which is useful for a more compact notation.

The TS-OQAM-GFDM transmit signal is then given by [GMM⁺15a]

$$x[n] = \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} d_{k,m}^{(i)} g_{k,m}^{(i)}[n] + \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} d_{k,m}^{(q)} g_{k,m}^{(q)}[n].$$
(3.3)

Assuming a flat and noiseless channel, the corresponding demodulation using matched filtering operation can recover the data from the received signal, y[n] = x[n], as follows

$$\hat{d}_{k,m}^{(\cdot)} = \Re\left\{y[n] \circledast g_{k,m}^{(\cdot)*}[-n]\right\}|_{n=0},$$
(3.4)

where \circledast denotes circular convolution with period N, $\Re(\cdot)$ returns the real part. Here, $g_{k,m}^{(\cdot)*}[-n]$ denotes the matched filter version of either the in-phase $g_{k,m}^{(i)}$ or quadrature $g_{k,m}^{(q)}$ pulse shapes, returning respectively $\hat{d}_{k,m}^{(i)}$ and $\hat{d}_{k,m}^{(q)}$.

The real-orthogonality condition can be observed through the projection of any m'-th subsymbol and k'-th subcarrier onto all k, m in the resource grid, as illustrated in Fig. 3.2.



Figure 3.2: TS-OQAM-GFDM real-orthogonality illustration using the ambiguity function $\mathcal{X}(m,k)$, with the magnitude of the residual real (a) and imaginary (b) values along the neighbor subsymbols and subcarriers, for a GFDM transmitted real input using RRC prototype filter with roll-off=0.5.

Observe that the ambiguity function $\mathcal{X}(m,k) = g_{k,m}^{(\cdot)}[n] \circledast g_{k',m'}^{(\cdot)*}[-n]$ results in

$$\Re \left\{ g_{k,m}^{(i)}[n] \circledast g_{k',m'}^{(i)*}[-n] \right\} \Big|_{n=0} = \delta[k,m]\delta[k',m'], \tag{3.5}$$

$$\Re \left\{ g_{k,m}^{(\mathbf{q})}[n] \circledast g_{k',m'}^{(\mathbf{q})*}[-n] \right\} \Big|_{n=0} = \delta[k,m]\delta[k',m'], \tag{3.6}$$

$$\Re \left\{ g_{k,m}^{(i)}[n] \circledast g_{k',m'}^{(q)*}[-n] \right\} \Big|_{n=0} = 0,$$
(3.7)

$$\Re \left\{ g_{k,m}^{(\mathbf{q})}[n] \circledast g_{k',m'}^{(\mathbf{i})*}[-n] \right\} \Big|_{n=0} = 0,$$
(3.8)

and that the symmetric half-Nyquist band-limited filters [B03] assure zero ISI at any m'-th subsymbol and produce null imaginary interference at any k'-th neighboring subcarrier in Fig. 3.2. Hence, the phase rotation observed in (3.1) and (3.2) aligns the in-phase

and quadrature components into a null ICI arrangement. Interesting to observe that the eye diagram obtained in this condition reminds one of $\alpha = 0$ due the ICI influence. Actually, this pattern is kept constant for all $0 < \alpha \leq 1$, as the ICI and ISI influence scales accordingly, in other words, the smaller the α the less the influence of ICI and the higher the side lobe fluctuations of the prototype filter.



Figure 3.3: GFDM TS-OQAM-GFDM eye pattern obtained at the receiver side when a real antipodal data is modulated using all subcarriers. M = 8 (although only the first 4 subsymbols are exhibited), K = 128, RRC, $\alpha = 1$. A ISI-free condition is achieved at every mK and $(m + \frac{1}{2})K$ points, but the eye pattern reminds one of $\alpha = 0$ due the ICI influence out of the correct sampling points.

The TS-OQAM-GFDM matrix model allows for an even more convenient and compact notation, and can be written as

$$\mathbf{x} = \mathbf{A}^{(i)}\mathbf{d}^{(i)} + \mathbf{A}^{(q)}\mathbf{d}^{(q)}, \tag{3.9}$$

where the first/second term of (3.9) corresponds to the first/second double sum in (3.3) and the columns of the matrix $\mathbf{A}^{(i)}$ and $\mathbf{A}^{(q)}$ carry $\mathbf{g}_{k,m}^{(i)}$ and $\mathbf{g}_{k,m}^{(q)}$, respectively. Comparing (3.9) with (2.13), the OQAM approach can be seen as the combination of two independent GFDM systems, each one with real only input signals.

Using the matrix notation, the real-orthogonality condition can be abstracted in a more intuitive way. For instance, observing that the match filter operation can be performed with the Hermitian operation, i.e.

$$\left(g_{k,m}^{(\cdot)}[n] \circledast g_{k',m'}^{(\cdot)*}[-n]\right)\Big|_{n=0} = \left(\mathbf{g}_{k',m'}^{(\cdot)}\right)^{H} \mathbf{g}_{k,m}^{(\cdot)}, \tag{3.10}$$

the conditions given in (3.5) and (3.7) are equivalent to

$$\Re\left\{\left(\mathbf{A}^{(i)}\right)^{\mathrm{H}}\mathbf{A}^{(i)}\right\} = \Re\left\{\left(\mathbf{A}^{(q)}\right)^{\mathrm{H}}\mathbf{A}^{(q)}\right\} = \mathbf{I}_{N}, \qquad (3.11)$$

$$\Re\left\{\left(\mathbf{A}^{(i)}\right)^{\mathrm{H}}\mathbf{A}^{(q)}\right\} = \Re\left\{\left(\mathbf{A}^{(q)}\right)^{\mathrm{H}}\mathbf{A}^{(i)}\right\} = \mathbf{0}_{N}, \qquad (3.12)$$

where \mathbf{I}_N and $\mathbf{0}_N$ are N by N identity and null matrices, respectively.

3.1.3 FS-OQAM-GFDM

This subsection extends the use of OQAM by incorporating unitary transformations. In particular, the Fourier transform is an interesting corner case that allows to explore the duality between time and frequency domains to design the prototype filter, named frequency-shift OQAM-GFDM (FS-OQAM-GFDM) [GMM⁺15a]. In contrast to classic literature [LAB95, BÖ3], the impulse and frequency responses of the prototype filter are exchanged.

It will be shown that a half-Nyquist pulse, limited to the width of two subsymbols, requires a frequency-shift of one half subcarrier, instead of a time-shift of one half subsymbol. It turns out that the absence of a time-shift allows the FS-OQAM-GFDM to better use null subsymbols to achieve low OOB emission even with a very short impulse response.

A unitary transform, \mathbf{U}_N , is a key idea proposed in this thesis to expand the OQAM concept and can be straightforwardly applied to the proposed matrix model, as

$$\left(\mathbf{U}_{N}^{\mathrm{H}}\mathbf{A}^{(\cdot)}\right)^{\mathrm{H}}\mathbf{U}_{N}^{\mathrm{H}}\mathbf{A}^{(\cdot)} = \left(\mathbf{A}^{(\cdot)}\right)^{\mathrm{H}}\left(\mathbf{U}^{\mathrm{H}}\right)^{\mathrm{H}}\mathbf{U}^{\mathrm{H}}\mathbf{A}^{(\cdot)} = \left(\mathbf{A}^{(\cdot)}\right)^{\mathrm{H}}\mathbf{A}^{(\cdot)}.$$
(3.13)

As shown in (3.13), the influence of the unitary transformation applied to either the in-phase, $\mathbf{A}^{(i)}$, or quadrature modulation matrices, $\mathbf{A}^{(q)}$, can be canceled out of the match filter operation with a basic matrix manipulation, once $(\mathbf{U}^{H})^{H} \mathbf{U}^{H} = \mathbf{I}$. This observation shows that the orthogonality condition (3.11) is valid not only for the time domain grid, but with any unitary transform applied to the transmit filters.

Considering a discrete Fourier transform (DFT) matrix, $\mathbf{U}_N = \mathbf{W}_N$, $[\mathbf{W}_N]_{i,l} = \frac{\exp(-j2\pi \frac{il}{N})}{\sqrt{N}}$, and two new precoding matrices constructed from $\mathbf{A}^{(i)}$ and $\mathbf{A}^{(q)}$ as

$$\begin{aligned}
\tilde{\mathbf{A}}^{(i)} &= \mathbf{W}_N^{\mathrm{H}} \mathbf{A}^{(i)}, \\
\tilde{\mathbf{A}}^{(q)} &= \mathbf{W}_N^{\mathrm{H}} \mathbf{A}^{(q)},
\end{aligned}$$
(3.14)

a new class of offset QAM, termed FS-OQAM-GFDM, is introduced for the first time.

The matrix model for FS-OQAM-GFDM is then given by

$$\mathbf{x} = \tilde{\mathbf{A}}^{(i)} \mathbf{d}^{(i)} + \tilde{\mathbf{A}}^{(q)} \mathbf{d}^{(q)}$$
(3.15)

followed by the corresponding demodulation equation

$$\hat{\mathbf{d}} = \Re\{(\tilde{\mathbf{A}}^{(i)})^{\mathrm{H}}\mathbf{y}\} + j\Re\{(\tilde{\mathbf{A}}^{(q)})^{\mathrm{H}}\mathbf{y}\}.$$
(3.16)

The relation given in (3.14) effectively suggests that the signal modulation introduced into (3.15) is equivalent to applying the conventional OQAM modulation in frequency domain and then transforming the result back into the time domain.

The unitary transformation using matrix notation greatly simplifies the derivation and discussion on the new OQAM approach, but a deeper look using conventional notation can also be beneficial to visualize the changes applied to the pulse shape. More explicitly, the columns of $\tilde{\mathbf{A}}^{(i)}$ are inverse discrete Fourier transform (IDFT)s of the columns of $\mathbf{A}^{(i)}$,

$$\tilde{\mathbf{g}}_{k,m+c} = \mathbf{W}_N^{\mathrm{H}} \mathbf{g}_{k,m+c}, \qquad (3.17)$$

with $c \in \{0, \frac{1}{2}\}$ being used to describe both in-phase and quadrature filters sets. With the use of the auxiliary variable \tilde{n} , describing the samples in the transformed domain, and the IDFT operator, $U[\tilde{n}] = \mathcal{F}_N^{-1}\{u[n]\}|_{\tilde{n}} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} u[n] \exp(j2\pi \frac{n\tilde{n}}{N})$, the entries of $\tilde{\mathbf{g}}_{k,m+c}$ are therefore derived as

$$\tilde{g}_{k,m+c}[\tilde{n}] = \mathcal{F}_N^{-1} \{ g_{k,m+c}[n] \} |_{\tilde{n}}$$
(3.18)

$$= \mathcal{F}_N^{-1} \left\{ g \left[(n - (m + c)K) \mod N \right] \cdot \exp\left(j2\pi \frac{kM}{N}n\right) \right\} \right|_{\tilde{n}}$$
(3.19)

$$= G\left[(\tilde{n} - kM) \mod N \right] \cdot \exp\left(j2\pi \frac{(m+c)K}{N} \tilde{n}\right)$$
(3.20)

where $G[\tilde{n}]$ is the frequency representation of the prototype pulse shaping filter g[n] and it is symmetric. The time shift (m+c)K in $g_{k,m+c}[n]$ becomes the frequency offset while the subcarrier index k becomes the index for time shift in $\tilde{g}_{k,m+c}[\tilde{n}]$.

With this approach, half-Nyquist pulses, which have a localized rectangular-like shape with non-null roll-off, are now directly applied as the impulse response in the time domain with a sparse response expanding into up to two subsymbols width. Therefore, the necessary shift to obtain the real-orthogonality consists of one half subcarrier in the frequency domain. In this scheme, M would denote the number of subcarriers while Krepresents the number of subsymbols. Notice that the adjacent subsymbols will differ by a phase shift of $\frac{\pi}{2}$.

The TS-OQAM-GFDM and FS-OQAM-GFDM block diagram are depicted in (Fig. 3.4), (transmitter on the left side, receiver on the right side).


Figure 3.4: Block structure of the time-shift or frequency-shift OQAM-GFDM.

There is an apparent increased complexity of two modulators and two demodulators in OQAM-GFDM, compared to a single modulator and demodulator in QAM-GFDM. However, the implementation cost is in fact again reduced when considering that two real/imaginary modems can be equivalent to one complex in terms of memory and multiplications. Note also that M is not restricted to be odd [MMF14] (Fig. 2.8).

3.1.4 A pulse shape design for FS-OQAM-GFDM

Nyquist pulses consist of an impulse response that have periodic zero values at multiples of the symbol time period. In the frequency domain this condition turns into a vestigial symmetry [Ga10, p. 312-316]. This means that its spectrum exhibits odd symmetry around the cut-off frequency, which is half the symbol rate. Hence, Nyquist filters can be defined by imposing an odd symmetry to the frequency coefficients around the cut-off frequency. In transmission systems, the global Nyquist filter is generally split into two parts, a half-Nyquist filter in the transmitter and a half-Nyquist filter in the receiver. Then, the symmetry condition is satisfied by the squares of the frequency coefficients.

To design a proper Nyquist filter to be used in FS-OQAM-GFDM, the principles of odd symmetry need to be applied from the time perspective. A straightforward solution is to define a global raised cosine (RC) pulse in the time domain, as follows

$$g(t) = \begin{cases} 1, & \text{for } |t| \le (1-\alpha)\frac{T}{2} \\ \frac{1}{2} \left[1 + \cos\left(\pi \left(\frac{|t| - (1-\alpha)\frac{T}{2}}{\alpha T}\right)\right) \right], & \text{for } (1-\alpha)\frac{T}{2} < |t| \le (1+\alpha)\frac{T}{2} \\ 0, & \text{otherwise,} \end{cases}$$
(3.21)

and simply take the squared root of g(t) to obtain the half-Nyquist RRC pulse. For convenience, energy normalization is not applied in (3.21). The RC and RRC pulses defined in time are illustrated in Fig. 3.5(a), axis are normalized to unitary amplitude in time and 0 dB magnitude in frequency.

3.1 OQAM GFDM



Figure 3.5: Nyquist and half-Nyquist pulse design in the time and frequency domains. The half-Nyquist Meyer RRC pulse presents a more soft transition than the RRC pulse, resulting in a more localized OOB emission. T_{Sub} describes the subsymbol duration and roll-off is configured to $\alpha = 1$, which results in a time span of two subsymbols.

Although the global RC pulse is able to achieve a good balance on time and frequency span, it is not suitable for a match filter operation. That is, the concatenation of two RC filters does not result in vestigial symmetry. When considering RRC, the pulse exhibits abrupt transitions in time, leading to an undesired enhanced OOB.

As the half-Nyquist pulse has a major impact on the system design, an alternative solution to RRC defined in the time domain is required. The solution must assure that the derivative of the designed pulse is smooth in order to confine the frequency span as much as possible. The problem of defining smooth functions has been extensively investigated in the concept of wavelets $[D^+92]$, e.g., the Meyer Wavelet [Mey90]. A Wavelet is a wave-like

oscillation with an amplitude that begins at zero, increases, and then decreases back to zero. To assure this smooth transition, [Mey90] proposes the use of an auxiliary function to be used as an inner argument for sine and cosine waves. This is exactly the idea that has been used to construct the Meyer RC and Meyer RRC presented in Fig. 3.5(b).

The Meyer RRC pulse proposed in this thesis combines the RRC time domain pulse with the Meyer auxiliary function, as follows

$$g(t) = \begin{cases} 1, & \text{for } |t| \le (1-\alpha)\frac{T}{2} \\ \sqrt{\frac{1}{2} \left[1 + \cos\left(\pi v \left(\frac{|t| - (1-\alpha)\frac{T}{2}}{\alpha T}\right) \right) \right]}, & \text{for } (1-\alpha)\frac{T}{2} < |t| \le (1+\alpha)\frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$$
(3.22)

where the Meyer auxiliary function is

$$\mathbf{v}(t) = t^4 (35 - 84t + 70t^2 - 20t^3), \tag{3.23}$$

and α is the roll-off that defines the length of the ramp up and ramp down transitions.

When comparing Fig. 3.5 (a) and (b), it is possible to observe a smoother pulse transition in the Meyer RRC, resulting in an OOB of nearly -80dBc in comparison with the -50dBc achieved with the RRC pulse at a distance of ± 4 subcarriers.

To observe the difference between the FS-OQAM-GFDM and TS-OQAM-GFDM, highlighting the time and frequency shifts, a sequence of symbols modulated with the proposed Meyer RRC pulse and a near-perfect reconstruction (NPR) filter [Bel10] (see Appendix A), is compared in Fig. 3.6.



Figure 3.6: Burst sequences of 8 subsymbols, with first subsymbol being null, for (a) FS-OQAM-GFDM using Meyer RRC pulse and (b) TS-OQAM-GFDM using NPR.

The influence of α , already considering a FS-OQAM-GFDM complex subcarrier with expanded bandwidth, is illustrated in Fig. 3.7. For instance, with $\alpha = 0.5$, the time span of the pulse shape is reduced to only 1.5 subsymbol duration, while the OOB is still

reduced by more than 60dB at 6 subcarriers distance. This is an indication that even short busts can be obtained at the expense of more guard subcarriers to attain a given target OOB reduction. Note, however, that if the number of subcarriers is large, this reduction in relation to the overall bandwidth becomes more negligible. This aspect will be better explored in the next subsection.



Figure 3.7: Meyer RRC pulse with different roll-offs.

3.1.5 Performance analysis

One natural question regarding the FS-OQAM-GFDM is the OOB emission comparison with TS-OQAM-GFDM. To clarify this point, the time and frequency characterizations, normalized to the subsymbol time, $T_{\rm sub}$, and subcarrier bandwidth, $B_{\rm SC}$, of FS-OQAM-GFDM and TS-OQAM-GFDM pulses are compared in Fig. 3.8.

It can be observed that the FS-OQAM-GFDM is much shorter in length, with a maximum span of only 2 subsymbols against several in the TS-OQAM-GFDM case. The FS-OQAM-GFDM symbol structure can easily benefit from the use of null-subsymbols [MMG⁺14]. This is not straightforward in TS-OQAM-GFDM because the long impulse response and the pulse shape design can create signal discontinuities at the block boundaries.



Figure 3.8: FS-OQAM-GFDM using Meyer RRC pulse vs TS-OQAM-GFDM using RRC in time and frequency. The figure emphasizes the second subsymbol slot impulse response, the first subsymbol slot is left empty to allow smooth transition between blocks. Observe that the NPR pulse in TS-OQAM-GFDM still exhibits hard transitions at the block boundaries, leading to an increased OOB.

Although there might be other cyclic time offset arrangement that can reduce OOB for the considered TS-OQAM-GFDM pulse, it does not have equidistant zeros with interval of one half subsymbol. Hence, in this configuration, TS-OQAM-GFDM cannot outplay the spectrum achieved by FS-OQAM-GFDM, where the OOB radiation is several orders of magnitude lower. The TS-OQAM-GFDM can slightly outperform the FS-OQAM-GFDM only for frequencies surrounding the adjacent subcarriers and this advantage vanishes when a larger number of subcarriers is transmitted, as shown in Fig. 3.9.

When increasing the number of guard subsymbols at the block boundaries, the TS-OQAM-GFDM resembles the burst case of filter bank multicarrier (FBMC)/OQAM because the circular pulse shape is no longer observed. This simple consideration proves that the principles of frequency-shift OQAM presented in the context of GFDM also hold for more general filtered OFDM systems. Interesting to observe that the NPR still has a higher OOB than the Meyer RRC, this is because of the even smoother transition obtained with the latter one when using the span of two subsymbols.

In order to analyze the performance of the FS-OQAM-GFDM versus the TS-OQAM-GFDM, the symbol error ratio is evaluated in a time variant frequency

selective channel (FSC) considering perfect synchronization and that the channel impulse response (CIR) is known. A zero guard interval longer than the CIR is placed between two consecutive bursts to avoid interference. Cyclic convolution with the channel can be emulated by overlapping the tail samples over the initial samples of the GFDM block, allowing for simple zero-forcing (ZF) frequency domain equalization. Channel tap gains vary linearly in decibel scale from 0 to -10 dB. Each tap is multiplied by a Rayleigh distributed random variable with parameter $\sigma = 1/\sqrt{2}$. Fig. 3.9 shows that the obtained symbol error rate (SER) performance of both are equal, demonstrating that the proposed FS-OQAM-GFDM allows for very short burst transmission without penalties when compared with TS-OQAM-GFDM.



Figure 3.9: Simulated transmitted spectrum for time-shift with RRC and frequency-shift OQAM-GFDM using Meyer RRC. Burst sequences of 8 subsymbols are transmitted and 75 out of 128 subcarriers are active. The corresponding SER performance for both FS-OQAM-GFDM and TS-OQAM-GFDM is simulated in a FSC condition, showing there is no performance loss. For SER simulation, ideal synchronization and perfect knowledge of the channel are assumed. Channel tap gains vary linearly in decibel scale from 0 to -10 dB.

3.2 GFDM low-complexity signal processing

In this section, GFDM modulation and demodulation will be presented from the perspectives of time domain and frequency domain circular convolution. These models allow for a reduction of the implementation complexity based on DFT algorithms. In particular, the frequency domain convolution results in a time domain signal processing that is helpful for the synchronization algorithms presented in Chapter 5. In addition, a basic complexity analysis is carried out to compare these two signal processing chains.

3.2.1 Transmitter reformulation

The classical description of GFDM signal generation in (2.12) can also be expressed using the notation of circular convolution, \circledast , carried out in time with period N

$$x[n] = \sum_{k=0}^{K-1} \left(g[n] \exp\left(j2\pi \frac{k}{K}n\right) \right) \circledast d_k[n], \qquad (3.24)$$

where

$$d_k[n] = \sum_{m=0}^{M-1} d_{k,m} \delta[n - mK].$$
(3.25)

In (3.25), each $d_{k,m}$ data symbol multiplies a unit impulse function $\delta[n - mK]$, and can be seen as if stuffed with N - 1 zeros and time shifted by mK samples to the correct subsymbol position in the GFDM block. In (3.24), $d_k[n]$ is filtered by $g[n] = g[n \mod N]$ and shifted to the center frequency of the k-th subcarrier by the complex exponential $\exp(j2\pi \frac{k}{K}n)$. The transmit samples x[n], with sample index n = 0, ..., N - 1, are finally obtained through the superposition of all modulated subcarriers.

The naive interpretation of (3.24) is not suitable for straightforward implementation. In [MGK⁺12, GNN⁺13, GNMF13], the authors have investigated a low complexity frequency domain signal processing for (3.24) using the convolution theorem [Hun72]. With this perspective, the circular convolution at each subcarrier can be efficiently implemented as the IDFT of the product of the DFTs of the filter and the data.

Hence, (3.24) can be carried out as follows

$$x[n] = \frac{1}{\sqrt{N}} \mathcal{F}_N^{-1} \left\{ \sum_{k=0}^{K-1} \sqrt{N} \underbrace{\mathcal{F}_N\{g[n] \exp\left(j2\pi \frac{k}{K}n\right)\}}_{G[(f-kM) \mod N]} \underbrace{\sqrt{N} \mathcal{F}_N\{d_k[n]\}}_{\sqrt{M} \mathcal{F}_M\{d_{k,(\cdot)}\}|_f} \right\},$$
$$= \sqrt{M} \mathcal{F}_N^{-1} \left\{ \sum_{k=0}^{K-1} G[(f-kM) \mod N] \mathcal{F}_M\{d_{k,(\cdot)}\}|_f \right\},$$
(3.26)

where \mathcal{F}_N is the *N*-point DFT, $U[f] = \mathcal{F}_N\{u[n]\}|_f = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} u[n] \exp\left(-j2\pi \frac{nf}{N}\right)$, and $G[f] = \mathcal{F}_N\{g[n]\}.$

Notably, the multiplication by the complex exponentials, $g[n]\exp(j2\pi\frac{k}{K}n)$, turns out to be a simple shift in the frequency domain, $G[(f - kM) \mod N]$. The zero stuff operation in $d_k[n]$ results in a periodic spectrum, so $\mathcal{F}_N\{d_k[n]\}$ can be efficiently implemented as a K times concatenation of $\mathcal{F}_M\{d_{k,(\cdot)}\}|_f$, once $f = 0, \ldots, MK - 1$. Also, as the Fourier transform is a linear operation, the subcarriers summation can be performed in the frequency domain, prior to the IDFT operation.

The GFDM reformulated transmit signal is obtained by expressing the convolution in (2.12) explicitly as

$$x[n] = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} g[(n-mK) \mod N] \exp(j2\pi \frac{k}{K}n) d_{k,m}, \qquad (3.27)$$

and rearranging the subsymbol and subcarrier summations in a more convenient format

$$x[n] = \sum_{m=0}^{M-1} g[(n - mK) \mod N] \underbrace{\sum_{k=0}^{K-1} d_{k,m} \exp(j2\pi \frac{k}{K}n)}_{\sqrt{K}\mathcal{F}_{K}^{-1}\{d_{(\cdot),m}\}|_{n}}$$
$$= \sqrt{K} \sum_{m=0}^{M-1} g[(n - mK) \mod N] \mathcal{F}_{K}^{-1}\{d_{(\cdot),m}\}|_{n}, \qquad (3.28)$$

to reveal that a K-point IDFT operation is performed over the data.

The subcarrier summation in (3.28) is periodic, given that $\exp(j2\pi \frac{k}{K}n)$ is periodic at each K samples and $n = 0, \ldots, MK - 1$. Therefore, the operation over the data can be seen as an IDFT of length K concatenated M times in the time domain.

The subsymbol summation in (3.28) contains the time shifted versions of the prototype filter. The multiplication in the time domain expresses the convolution of the subcarrier filter with the data in the frequency domain for each subsymbol, i.e.,

$$X[f] = \sum_{m=0}^{M-1} G[f] \exp\left(j2\pi \frac{m}{M}f\right) \circledast \sum_{k=0}^{K-1} d_{k,m} \delta[f - kM].$$
(3.29)

In (3.29), $\sum_{k=0}^{K-1} d_{k,m} \delta[f - kM]$, f = 0, ..., N - 1, is now an impulse train in the frequency domain containing the K data symbols of a given *m*-th subsymbol. The time shift operation is performed in the frequency domain through the complex exponential multiplication, that is, $G[f]\exp(j2\pi\frac{m}{M}f)$.

The signal processing chains performed in frequency and in time domain are illustrated in Fig. 3.10.



Figure 3.10: GFDM terminology and corresponding signal processing chains that can be performed in the frequency domain, (3.26), or in the time domain, (3.28).

Comparing (3.26) with (3.28), the former requires to perform *M*-points DFTs and a *N*-point IDFT, and the latter requires the use of *K*-point IDFTs. Hence, (3.28) can be more straightforward implemented when *M* is odd (see Fig. 2.8), avoiding the need of fast Fourier transform (FFT) algorithms with lengths that are not a power of 2.

3.2.2 Demodulator reformulation

In a close analogy to the transmitter reformulation, a low-complexity demodulator is presented in the following. The necessary operations to receive a GFDM signal, i.e., down-conversion, pulse shaping and data downsampling, can also be described using DFT based algorithms.

Assuming a flat and noiseless channel, or simply y[n] = x[n], the transmitted data symbols can be recovered by using a set of receive filters, whose design can follow a matched filter (MF), ZF or minimum mean squared error (MMSE) criterion. For now, assume a prototype receive filter $\gamma[n]$ is used to demodulate the data transmitted with g[n]. In this case, the estimated data symbols are given by

$$\hat{d}_{k,m} = \left(\gamma^*[(-n) \mod N] \circledast y[n] \exp\left(-j2\pi \frac{k}{K}n\right)\right)|_{n=mK}.$$
(3.30)

Using the convolution theorem [Hun72] applied to (3.30), the receiver can also be rewritten as the IDFT of a product of DFTs, given by

$$\hat{d}_{k,m} = \sqrt{M} \mathcal{F}_N^{-1} \left\{ \underbrace{\mathcal{F}_N\{\gamma^*[(-n) \mod N]\}}_{\Gamma^*[f]} \underbrace{\mathcal{F}_N\{y[n]\exp\left(-j2\pi\frac{k}{K}n\right)\}}_{Y[(f+kM) \mod N]} \right\}_{mK}, \quad (3.31)$$

where $\mathcal{F}_N\{\gamma[n]\} = \Gamma[f]$ and $\mathcal{F}_N\{y[n]\} = Y[f]$, and \sqrt{M} is the corresponding normalization constant considering the mK decimation.

Complementary to upsampling operation (zero stuffing) in the transmitter side, a decimation operation is performed in (3.31) to recover the subsymbols at every mK samples. As an intuitive illustration, this is equivalent to sampling the filtered received signal at the open eye positions in Fig. 3.1. Data is then detected through a proper scaling and slicer operation, finding the closest point in the signal constellation to these sampled values.

In terms of implementation, the decimation can also be performed directly in the frequency domain. To do so, an aliasing operation $[OSB^+89, p.82-87]$ is performed. That is, the spectrum composed of N coefficient is K-fold accumulated, returning to the original M-point subcarrier bandwidth. With this approach, an IDFT of only M-point is required. The principle can also be explained according to the Poisson summation formula [BZ97]. The N-point DFT operation can actually be reduced to a $\frac{N}{K} = M$ point DFT once

$$\mathcal{F}_{N}\{u[n]\}|_{kM} = \mathcal{F}_{\frac{N}{K}}\left\{\frac{1}{\sqrt{K}}\sum_{k'=0}^{K-1}u[n+k'M]\right\}\Big|_{m}.$$
(3.32)

Processing the GFDM signal in the frequency domain considerably reduces the implementation complexity, once equalization and demodulation can be naturally combined. Also, when sparse filters are used in the frequency domain, the amount of coefficients for $\Gamma[f]$ can be reduced. For example, this leads to the low-complexity demodulator implementation described in [GNN⁺13] for the MF $\gamma[n] = g[n]$, which was later extended to more general filter types in [MMG⁺14, MMF14].

Nevertheless, processing the GFDM in the frequency domain requires strict block alignment, which means that it cannot be considered to support the synchronization operation. For that, a receiver that can operate without dependence on block alignment is of interest, as will be presented in the following. The GFDM reformulated demodulation is obtained by directly expressing the convolution and sampling in (3.30) as a time domain multiplication given by

$$\hat{d}_{k,m} = \sum_{n=0}^{N-1} \gamma^* [(n - mK) \mod N] y[n] \exp\left(-j2\pi \frac{kM}{KM}n\right)$$

$$= \sqrt{K} \mathcal{F}_N \{\gamma^* [(n - mK) \mod N] y[n]\}|_{kM},$$
(3.33)

where the N-point DFT is only evaluated at every kM-th sample. Again, according to (3.32), this operation can actually be reduced to a K-point DFT, i.e.,

$$\hat{d}_{k,m} = \sqrt{K} \mathcal{F}_K \left\{ \sum_{m'=0}^{M-1} u_m [n+m'K] \right\} \bigg|_k, \qquad (3.34)$$

with

$$u_m[n] = \gamma^*[(n - mK) \mod N]y[n].$$
 (3.35)

In an intuitive point of view, (3.33) can be seen a correlation receiver. That is, the received signal is first multiplied by the conjugated version of the transmitted pulse shape and integrated over the symbol period for each subcarrier (*K*-point DFT operation). These operations, integrate a series of values, use the result, and restart integrating, are also refereed in as integrate-and-dump [Ga10, Chapter 4]. What is not so intuitive is the *K*-fold summation performed in the time domain. This can be explained as a decimation carried out in the frequency domain, where each processed subcarrier is sampled at its center frequency, reverting the zero stuff operation described by the frequency domain convolution model in (3.29).

Other observation is that Eq. (3.33) describes a sampled short-time Fourier transform (STFT) [MMF14]. For this reason, (3.33) employs the same prototype filter for all subcarriers. Equalization can be performed before demodulation, or using a modified version of (3.33) with a different filter for each subcarrier, which would increase substantially the complexity. Alternatively, the presented scheme can be used as an initial step to access an early version of the channel state information by coarsely demodulating training data. Also, the proposed scheme facilitates the implementation of odd number of subsymbols [MMF14] avoiding the use of odd length *M*-point DFT and IDFT required in [GNN⁺13].

3.2.3 Matrix representation and complexity analysis

The data symbols $d_{k,m}$ can be arranged in a two-dimensional structure given by

$$\mathbf{D} = \left(\mathbf{d}_0 \ \mathbf{d}_1 \ \cdots \ \mathbf{d}_{M-1}\right). \tag{3.36}$$

where the columns of \mathbf{D} represent the data symbols transmitted in the *m*-th subsymbol,

$$\mathbf{d}_m = \begin{pmatrix} d_{0,m} & d_{1,m} & \dots & d_{K-1,m} \end{pmatrix}^{\mathrm{T}}, \qquad (3.37)$$

and the rows of \mathbf{D} represent the data symbols transmitted in the k-th subcarrier,

$$\mathbf{d}_{k} = \begin{pmatrix} d_{k,0} & d_{k,1} & \dots & d_{k,M-1} \end{pmatrix}^{\mathrm{T}}.$$
 (3.38)

The time domain circular convolution of the modulation process can be expressed in the frequency domain as $[\text{GNN}^+13]$

$$\mathbf{x} = \sqrt{M} \mathbf{W}_N^{\mathrm{H}} \sum_{k=0}^{K-1} \mathbf{P}^{(k)} \mathbf{G} \mathbf{R}^{(K,M)} \mathbf{W}_M \mathbf{d}_k.$$
(3.39)

In a nutshell, the function of each matrix in (3.39) is illustrated in Fig. 3.11.



Figure 3.11: GFDM matrix modulator, (3.39), with frequency domain processing

For each subcarrier, the data vector is taken to the frequency domain by the *M*-point DFT matrix \mathbf{W}_M . The corresponding time domain up sampling operation is realized in the frequency domain by duplicating the transformed data symbol vectors *K* times, using the repetition matrix $\mathbf{R}^{(K,M)} = \mathbf{1}_{K,1} \otimes \mathbf{I}_M$, where $\mathbf{1}_{i,j}$ is a $i \times j$ matrix of ones, i.e.,

$$\mathbf{R}^{(K,M)} = \left(\underbrace{\mathbf{I}_M \quad \mathbf{I}_M \quad \dots \quad \mathbf{I}_M}_{K} \right)^{\mathrm{T}}.$$
 (3.40)

Each subcarrier is then filtered with $\mathbf{G} = \text{diag}(\mathbf{W}_N \mathbf{g})$, where $\text{diag}(\bullet)$ returns a matrix that contains the argument vector on its diagonal and zeros otherwise and \mathbf{g} is the vector containing the transmit filter impulse response, i.e.,

$$\mathbf{G} = \begin{pmatrix} G[0] & 0 & 0 & 0 \\ 0 & G[1] & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & G[N-1] \end{pmatrix}.$$
 (3.41)

An up-conversion of the k-th subcarrier to its respective subcarrier frequency is performed by the shift matrix

$$\mathbf{P}^{(k)} = \mathbf{\Psi}\left(\mathbf{p}^{(k)}\right) \otimes \mathbf{I}_M,\tag{3.42}$$

where $\Psi(\bullet)$ returns the circulant matrix based on the input vector and $\mathbf{p}^{(k)}$ is a column vector of length K where the k-th element is 1 and all others are zero. The K subcarriers

are summed and transformed back to the time domain with \mathbf{W}_N^H to compose the GFDM signal.

On the demodulator side, the recovered data symbols for the k-th subcarrier are given by

$$\hat{\mathbf{d}}_{k} = \sqrt{M} \mathbf{W}_{M}^{\mathrm{H}} \left(\mathbf{R}^{(K,M)} \right)^{\mathrm{T}} \mathbf{\Gamma} \left(\mathbf{P}^{(k)} \right)^{\mathrm{T}} \mathbf{W}_{N} \mathbf{y}, \qquad (3.43)$$

where \mathbf{y} is the equalized vector at the input of the demodulator, $\mathbf{\Gamma} = \text{diag}(\mathbf{W}_N \boldsymbol{\gamma})$ with $\boldsymbol{\gamma}$ being the demodulation filter impulse response, e.g., MF, ZF or MMSE filters.

By transposing (Hermitian of real only values) the matrices, $(\mathbf{P}^{(k)})^{\mathrm{T}}$ performs a downconversion operation, shifting the spectrum of the *k*-th subcarrier to baseband. Then Γ filters the signal and $(\mathbf{R}^{(K,M)})^{\mathrm{T}}$ apply a *K*-fold summation to equivalently decimate the signal in time. These operations are conveniently summarized in Fig. 3.12.



Figure 3.12: GFDM matrix demodulator, (3.43), with frequency domain processing

The modulation and demodulation processes can be simplified just by changing the processing order of the data symbols, as derived in (3.28).

The GFDM transmitter reformulated matrix model is given by

$$\mathbf{x} = \sqrt{K} \sum_{m=0}^{M-1} \mathbf{P}^{(m)} \operatorname{diag}(\boldsymbol{g}) \mathbf{R}^{(M,K)} \mathbf{W}_{K}^{H} \mathbf{d}_{m}.$$
(3.44)

The function of each matrix in (3.39) is illustrated in Fig. 3.13.



Figure 3.13: GFDM matrix modulator, (5.18), with time domain processing

In fact, (5.18) can be seen as a polyphase filter structure [NBS92, Bel10]. A polyphase filter is composed of a filter bank which splits an input signal into a given number of equidistant N sub-bands. These sub-bands are subsampled by a factor of N, so they are critically sampled. In GFDM, through the \mathbf{d}_m arrangement, these sub-bands correspond directly to the subcarriers allocation. But the difference in (5.18) is that cyclic time-shifts are used and frequency domain convolution is performed in the time domain as element-wise vector multiplication, as shown in (3.29) and Fig. 3.10.

With this approach, the first step illustrated in Fig. 3.13 is to obtain a time domain version of \mathbf{d}_m by multiplying it with an inverse DFT (IDFT) matrix $\mathbf{W}_K^{\mathrm{H}}$. M times upsampling in the frequency domain is performed in time by duplicating the transformed data symbols with a repetition matrix $\mathbf{R}^{(M,K)}$. Each subsymbol is then pulse-shaped with \boldsymbol{g} and $\mathbf{P}^{(m)}$ shifts the *m*-th subsymbol to its respective position in time. The GFDM signal is obtained by summing all the pulse-shaped subsymbols, with no need of the IDFT domain conversion in (3.39).

The GFDM receiver reformulated matrix model is also simplified by using the circular-convolution in the frequency domain, leading to

$$\hat{\mathbf{d}}_{m} = \sqrt{K} \mathbf{W}_{K} \left(\mathbf{R}^{(M,K)} \right)^{\mathrm{T}} \operatorname{diag}(\boldsymbol{\gamma}) \left(\mathbf{P}^{(m)} \right)^{\mathrm{T}} \mathbf{y}.$$
(3.45)

The demodulation operations in time domain are conveniently summarized in Fig. 3.14.



Figure 3.14: GFDM matrix demodulator, (3.45), with time domain processing

The matrix formulation facilitates the complexity analysis of the different GFDM transceiver models presented in Table 3.1, which evaluates the number of complex valued multiplications and the use of DFT algorithms. Addition, Hermitian, and repetition operations, as well as other memory resources necessary to store the sampled signals, are neglected in this analysis. Their implementation costs are small compared to the complexity of a multiplication operation.

As the baseline for the complexity evaluation, the naive GFDM matrix model and the orthogonal frequency division multiplexing (OFDM) transceiver are included. It is assumed that N = KM complex data symbols are transmitted. While the GFDM naive model requires a N^2 complexity, the OFDM complexity is considered to be $C_{\text{DFT},N} = N \log N$, once is based on a split-radix FFT algorithm for power-of-two N[BM67]. The complexity of the GFDM models, obtained from the perspectives of convolution in time and frequency, are described using the same criterion used for OFDM.

Type	Matrix Model	Complexity
OFDM	$\mathbf{x} = \mathbf{W}_N^{ ext{H}} \mathbf{d}, \mathbf{\hat{d}} = \mathbf{W}_N \mathbf{y}$	$C_{\rm OFDM} = C_{\rm DFT,N} = N \log_2(N)$
GFDM	$\mathbf{x} = \mathbf{A}\mathbf{d}$	$C_{\rm GFDM} = N^2$
	$\hat{\mathbf{d}} = \mathbf{B}\mathbf{y}$	
GFDM	$\mathbf{x} = \sqrt{M} \mathbf{W}_N^{\mathrm{H}} \sum_{k=0}^{K-1} \mathbf{P}^{(k)} \mathbf{GR}^{(K,M)} \mathbf{W}_M \mathbf{d}_k$	$C_{\rm GFDM, ZF} = C_{\rm DFT, N} + K(N + C_{\rm DFT, M})$
time conv.	$\hat{\mathbf{d}}_{k} = \sqrt{M} \mathbf{W}_{M}^{\mathrm{H}} \left(\mathbf{R}^{(K,M)} \right)^{\mathrm{T}} \mathbf{\Gamma} \left(\mathbf{P}^{(k)} \right)^{\mathrm{T}} \mathbf{W}_{N} \mathbf{y}$	
GFDM	$\mathbf{x} = \sqrt{K} \sum_{m=0}^{M-1} \mathbf{P}^{(m)} \operatorname{diag}(\boldsymbol{g}) \mathbf{R}^{(M,K)} \mathbf{W}_{K}^{H} \mathbf{d}_{m}$	$C_{\rm GFDM} = M(N + C_{\rm DFT,K})$
freq. conv.	$\hat{\mathbf{d}}_{m} = \sqrt{K} \mathbf{W}_{K} \left(\mathbf{R}^{(M,K)} \right)^{\mathrm{T}} \operatorname{diag}(\boldsymbol{\gamma}) \left(\mathbf{P}^{(m)} \right)^{\mathrm{T}} \mathbf{y}$	7

 Table 3.1: Number of multiplications of modulation models. Modulation and demodulation complexity are the same with this criterion

The GFDM modulator and demodulator based on time convolution require the complexity of the M-point DFT algorithm with additional K times complex multiplications over repeated chunks of M complex samples. This approach has the advantage to easily incorporate the frequency domain equalization (FDE) operations, but requires strict block alignment on the receiver operation due the DFT operations and is outside the focus of this thesis.

A graphical analysis for the complexity formulas in Table 3.1 is presented in Fig. 3.15. As an example, K = 128 is chosen and M and N = MK are changeable. For small values of M the complexity of the model using convolution in frequency is more attractive. The advantage is reduced as M increases, particularly when approaching the number of subcarriers K. Nevertheless, for practical cases, it is considered that M is smaller than K.

When the equalization aspects are relaxed, as is common when time and frequency alignment are not yet achieved, the demodulator scheme based on frequency domain convolution brings the advantage of a less complex implementation compared to the time convolution model and allows the design of pipeline multiplier structures operating directly on the time domain, loosening the strict block alignment requirements imposed by DFT blocks implemented with FFT algorithms. For instance, when performing a data aided synchronization operation, the properties of a training sequence can be tracked directly in time domain. To perform this task (3.35) could be directly used to track a given feature in the training signal, avoiding the need of the DFT operation in (3.34), as will be shown in Section 5.2.2.



Figure 3.15: GFDM implementation complexity according to Table 3.1. K = 128, L = K,N = KM.

3.3 GFDM as a flexible framework

The main goal of this section is to highlight the GFDM advancements proposed in this chapter. To do so, a framework that represents other major waveform candidates for fifth generation (5G) is proposed. A parameterizable table will be presented, summarizing the configurations that allow other waveforms to be implemented as corner cases of GFDM.

First, a new time and frequency grid will be discussed to include faster than Nyquist signaling (FTN) principles. Basically, the GFDM time and frequency grid can be rewritten considering that the subcarrier and subsymbol spacing are not only critically defined. Here the subcarrier is pulse-shaped by a filter impulse response with S = RT samples, where T is the number of periods of the filter and R is the number of samples per period.

Accordingly, the GFDM equation can be modified as follows

$$x[n] = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} d_{k,m} g\left[(n-m\nu_t R) \mod S\right] \exp\left(j2\pi \frac{kQ}{S}n\right), \quad n = 0, 1, \dots, S-1, \quad (3.46)$$

where P is the spacing between adjacent subsymbols and Q is the distance between adjacent subcarriers.

The subsymbol and subcarrier spacing factors are given by

$$\nu_{\rm t} = \frac{P}{R},\tag{3.47}$$

and

$$\nu_{\rm f} = \frac{Q}{T},\tag{3.48}$$

respectively, which allows to rewrite (3.46) as

$$x[n] = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} d_{k,m} g\left[(n - m \,\nu_t R) \, \text{mod} \, S \right] \exp\left(j2\pi \frac{k \,\nu_f T}{S} n\right), \quad n = 0, 1, \dots, \, S - 1.$$
(3.49)

The terminology is summarized in Table 3.2.

Variable	Meaning
R T	samples per period in the filter
S = RT	total number of samples in the signal
$P \\ Q$	subsymbol spacing in the time domain subcarrier spacing in the frequency domain
$\nu_{\rm t} = P/R$ $\nu_{\rm f} = Q/T$	subsymbols distance factor subcarriers distance factor
$K = RT/Q = R/\nu_{\rm f} = S/Q$ $M = TR/P = T/\nu_{\rm t} = S/P$ $N = KM$	subcarriers per block subsymbols per block number of data symbols per block

 Table 3.2:
 Terminology for the GFDM framework

When $\nu_t < 1$ the overlapping between the subsymbols increases, leading to a higher ISI and better spectrum efficiency. For $\nu_t > 1$, the subsymbols are taken apart from each other, reducing the time overlapping and decreasing the spectrum efficiency. The same reasoning is valid for ν_f , but now in the frequency domain. The Fig. 3.16 illustrated this aspects, where the "critically sampled GFDM" is equivalent to (2.12).

The advanced GFDM features can be used as a flexible framework for future physical layer (PHY) applications, serving as a platform to emulate diverse multicarrier waveforms. For this purpose, the parameters and properties of GFDM that are needed to describe the complete waveform setup achieved with cyclic filtered multicarrier systems are given in Table 3.3. All components derived from Table 3.3 have common roots in the filtered multicarrier systems proposed by [Cha66] and [Sal67]. Although this framework covers all most known waveforms, note that some candidates are still not covered. For example,

conventional CP-OFDM combined with band-pass filter and UFMC [G. 14], which applies separated linear filtering to sets of subcarriers in zero-padding (ZP)-OFDM [MWG⁺02], are techniques that can still be combined later on with the presented block filtered schemes, replacing the OFDM block.

In the context of this framework, the different waveforms are characterized by two aspects. First, parameters related to the dimensions of the underlying resource grid are explored. This includes the number of subcarriers K and subsymbols M in the system. The scaling factor in time $\nu_{\rm t}$ and frequency $\nu_{\rm f}$ can theoretically take values of any rational number larger than zero, while numbers close to one are meaningful because they relate to critically sampled Gabor frames. Additionally, the option to force specific data symbols in a block to carry the value zero, that is, the so-called 'guard subsymbols' [MMG⁺14], with $M_{\rm s}$ being a number between 0 and M - 2, is relevant for some candidates. The second set of features is related to the properties of the signal. Here, the choice of the pulse shaping filter is a significant attribute and the presence or absence of circularity constitutes a characteristic feature. Moreover, the use of OQAM is needed for some waveforms, aiming to achieve higher flexibility. Further, some waveforms rely on a cyclic prefix (CP) to allow transmission of a block based frame structure in a time dispersive channel, while others don't use CP in order to achieve higher spectrum efficiency.



Figure 3.16: Influence of distance factors ν_t and ν_f in time and frequency.

The family of classical waveforms includes OFDM, block OFDM, single-carrier with frequency domain equalization (SC-FDE) and single-carrier frequency division multiplexing (SC-FDM). Particularly OFDM and SC-FDM have been relevant for the development of the fourth generation (4G) cellular standard Long Term Evolution (LTE). All four waveforms in this category have in common that $\nu_{\rm f} = 1$ and $\nu_{\rm t} = 1$, which allows to meet the Nyquist criterion. Silent subsymbols are not employed, the CP and regular QAM are used in the default configuration. OFDM and block OFDM are corner cases of GFDM, where a rectangular pulse is used. Additionally, OFDM is restricted to one subsymbol, while block OFDM constitutes the concatenation of multiple OFDM symbols in time to create a block with a single common CP. Similarly, SC-FDE and SC-FDM can also be considered as corner cases of GFDM. However, here a Dirichlet pulse (see Appendix A) is used and analogously, the number of subcarriers in SC-FDE is K = 1, while SC-FDM is a concatenation in frequency of multiple SC-FDE signals. All waveforms in this category share property of orthogonality, but with different sensitivities towards various radio frequency (RF) imperfections, for instance SC-FDE is well known for its low peak-to-average power ratio (PAPR), which greatly benefits the mobile stations (MS) in terms of transmit power efficiency and reduced cost of the power amplifier.

The family of *filter bank waveforms* revolves around filtering the subcarriers in the system and still retaining orthogonality. As the names suggest, FBMC-OQAM [BBC⁺14] and its cyclic extension FBMC-COQAM [LS14] rely on offset modulation, while in FBMC-FMT and cyclic block filtered multitone (CB-FMT) [TG14] the spacing between the subcarriers is increased such that they do not overlap, i.e., $\nu_{\rm f} > 1$. Also, a separation between cyclic and non-cyclic prototype filters can be made. In this context, silent subsymbols become relevant. The best spectral efficiency is achieved with $M_{\rm s} = 0$, while $M_{\rm s} > 0$ helps to improve the spectral properties of the signal. Using a sufficiently large number of silent subsymbols at the beginning and the end of a block allows to emulate non-cyclic filters from a cyclic prototype filter response, in order to generate FBMC-OQAM and FBMC-FMT bursts. More precisely, $M_{\rm p}$ is the length of the prototype filter and $M_{\rm s} = M_{\rm p}$. Lastly, the CP is only compatible with cyclic filters.

Generally, the waveform can become non-orthogonal depending on the use of specific filters and for a given value of $\nu_{\rm f}$ and $\nu_{\rm t}$. This is addressed in the final category that consists of the non-orthogonal multicarrier techniques FTN [BBC⁺14] and spectrally efficient frequency division multiplexing (SEFDM) [KCRD09]. The key property of FTN is $\nu_{\rm t} < 1$, which reflects in increment of the subsymbol data rate. The isotropic orthogonal transform algorithm (IOTA) pulse, in combination with OQAM, has been proposed in order to avoid the need for a CP. Since the impulse response of the filter is not cyclic, $M_{\rm p}$ subsymbols are silent. Analogously, the idea of SEFDM is to increase the density of subcarriers in the available bandwidth, i.e., $\nu_{\rm f} < 1$. Here, M = 1 because each block consists of a single subsymbol that is filtered with a rectangular pulse and a CP is prepended to combat multipath propagation. In this case, regular QAM is employed. Clearly, the amount of squeezing without severely impacting the error rate performance is limited. The Mazo limit states that this threshold is around 25% for both schemes. Assuming that non-linear receivers are to be used for the demodulation of GFDM. e.g., [MGZF15a, ZMM⁺15], and that high signal-to-noise ratio (SNR) regimes can be assured in small cells, this additional gain can boost the performance of the system. On the other hand, if very simple receivers are to be used and high throughput is not a target, then it is better to use the sub-critical grid to mitigate interference.

3.3 GFDM as a flexible framework

design space	GFDM	OFDM	block OFDM	SC-FDE	SC-FDM	FBMC OQAM	FBMC FMT	FBMC COQAM	CB-FMT	FTN	SEFDM
# subcarriers	K	K	K	1	K	K	K	K	K	K	K
# subsymbols	M	1	M	M	M	M	M	M	M	M	1
scaling freq.	$ u_{\rm f} $	1	1	1	1	1	> 1	1	> 1	1	< 1
scaling time	$ u_{\rm t} $	1	1	1	1	1	1	1	1	< 1	1
silent subsym.	$M_{\rm s}$	-	-	-	-	$M_{\rm p}$	$M_{\rm p}$	-	-	$M_{\rm p}$	-
filter imp. resp.	cyclic	rect	rect	Dirichlet	Dirichlet	$\sqrt{Nyquist}$	$\sqrt{Nyquist}$	cyclic	cyclic	IOTA	rect
offset mod.	(yes)	(yes)	no	no	no	yes	no	yes	no	yes	no
cyclic prefix	yes	yes	yes	yes	yes	no	no	yes	yes	no	yes
orthogonal	(yes)	yes	yes	yes	yes	yes	yes	yes	yes	no	no
application scenarios	all	legacy systems	bitpipe	IoT/MTC	IoT/MTC	WRAN, bitpipe	WRAN	tactile Internet	tactile Internet	bitpipe	bitpipe
beneficial features	flex.	orth.	small CP overhead	low PAPR	low PAPR	low OOB	low OOB	no filter tail	no filter tail	spectral eff.	spectral eff.

Table 3.3: GFDM as a framework of 5G waveform candidates

3.4 Summary

In this chapter GFDM advancements were presented. The issues of prohibitive self-interference and matrix ill-conditioning for even number of subsymbols have been contoured with the proposition of OQAM for GFDM. As one main contribution, FS-OQAM-GFDM has been proposed and a corresponding new short pulse shape termed Meyer RRC were introduced, with great benefits in terms of time and frequency localization, and without penalties in the SER performance in FSC.

Another main contribution is the proposition of an efficient GFDM implementation based on time domain processing. With this proposal the modulator is now more suitable to handle odd number of subsymbols. On the demodulator side, the proposed design is more attractive to be used in early stages of synchronization, supporting pipeline inner receiver implementation for embedded training sequences, as it will be shown in the synchronization chapter.

The chapter also contributes with the proposal of a waveform framework based on the advanced GFDM. With proper parameterization, other prominent waveform candidates, such as FBMC-OQAM, FBMC-filtered multi tone (FMT), FBMC-cyclic OQAM (COQAM), FTN, and SEFDM, are achieved based on a unified design.

The main contributions of this chapter have been published in [GMM⁺15a, GNN⁺13, GMM⁺15b, GMM⁺15c], while more implementation details can be found in [GNMF13, Cal15, DMG⁺15b, DMG⁺15a].

Details can be summarized as follows:

- ▷ The main benefit of using OQAM combined with GFDM is the real-orthogonality in the time-frequency grid, where a matched filter receiver can be used without self-interference, as shown in the analysis of the eye pattern and ambiguity function.
- ▷ OQAM can be applied to GFDM by using frequency-shifts instead of time-shifts. This procedure can bring further advantages, such as easy use of null subsymbols to reduce the out-of-band emission, reduction of the impulse response length, and lower implementation complexity. It is an interesting proposal to be used in case of low latency transmission with challenging constraints in spectral emission.
- ▷ The Meyer RRC pulse proposed for FS-OQAM-GFDM has good time and frequency localization. This property can be particularly beneficial for isolated preamble design, as it will be explored in Chapter 5.2.1.
- ▷ The conversion of the time-shift OQAM scheme to the frequency-shift one is conducted by means of inverse Fourier transform. However, the analysis performed in the new OQAM model is not limited to Fourier transform. It can be straightforward

extended to any unitary transformation.

- ▷ The GFDM modulator and demodulator can be described with low-complexity multiplication in the frequency domain, using the convolution theorem in time, or in the time domain, using the convolution theorem in frequency.
- ▷ Both frequency domain processing and time domain processing present advantages. The former can be easily combined with FDE on the demodulator side. The latter is useful on the transmitter side, avoiding the use of odd-length DFTs, and can be useful on the receiver side, supporting early stages of demodulation as synchronization and future channel estimation approaches.
- ▷ Considering that sparse pulses can be defined either in frequency, e.g. RC, or in time, e.g., Meyer RRC, further complexity reduction can be achieved in both time and frequency domain because of the limited number of non-zero filter coefficients used in the pulse shape multiplication.
- ▷ GFDM is also presented as a baseline for a software-defined waveform (SDW), serving as a framework to describe other prominent multicarrier waveforms. For this purpose, the parameters and properties of GFDM that are necessary to emulate various waveforms are given. All contestants have common roots in the filtered multicarrier systems proposed by [Cha66] and [Sal67].

Chapter 4

Coarse Synchronization Strategy for GFDM

Synchronization represents one of the most challenging issues in communication systems and plays a major role in the physical layer design. The goal of this chapter is to introduce the synchronization problem for generalized frequency division multiplexing (GFDM) based systems. A fundamental tool for assessing the sensitivity to time and frequency misalignments is provided. Also, non-data aided synchronization schemes for coarse synchronization are addressed based on the well-established state of the art methods developed for OFDM. In particular, a new proposal for frequency estimation based on inter-carrier interference (ICI) is designed.

4.1 Problem formalism

A general system model for the transmitted signal is illustrated in Fig. 4.1(a). On the transmitter side, this model considers a source of complex data d[n] modulated as a GFDM symbol x[n]. Additionally, a guard interval with optional windowed cyclic prefix (CP) is used to prevent inter-block interference, as detailed in Sec. 2.6.

For simplicity, the presented model bypasses some details that occurs in practice. These include the fact that the transmitted sequence is converted from discrete to continuous time domain through processes of interpolation and digital-to-analog (D/A) convertion. Also, non-linear impairments on the radio frequency (RF) chain, as upconversion and amplification, are neglected. After transmission through the wireless channel, similar assumptions are made on the corresponding reverse operations performed by the receiver.

Although near perfect RF chains is an oversimplified assumption, where linear and non-linear distortions are often modeled as signal-dependent noise [FLP⁺07], it is reasonable to consider a precise sampling rate at the analog-to-digital (A/D) output. Today, accuracy of less than 10 parts per million (ppm) precision can be obtained with

inexpensive crystal oscillators [KW88]. Therefore, the sampled version of the received signal can be accurately represented in terms of block length, bandwidth, and intermediate frequency (IF) frequency.

Since fine timing errors can be jointly compensated with channel equalization, it is sufficient to locate the beginning of each received block within one sampling period. It is a common practice to model the timing error as a multiple of the sampling period and consider the remaining fractional error as part of the channel impulse response (CIR) [MKP07], while the frequency offset is usually normalized to the subcarrier spacing.



Figure 4.1: GFDM synchronization system model

The transmitted signal under the influence of multipath, as indicated in Fig. 4.1(b), is given by

$$s[n] = \sum_{l=0}^{L-1} h_l e^{j2\pi f_l n} \tilde{x}[n-\tau_l], \qquad (4.1)$$

where h_l represents the complex coefficients of a CIR of a time-varying frequency selective channel (FSC). This tapped-delay line model [PSY02] represents each *l*-tap of the CIR with a discrete delay, τ_l , and a Doppler frequency, f_l , [AHG11].

Usually, h_l values are described as statistically independent complex Gaussian random variables with zero mean, i.e., Rayleigh fading. Additionally, a power delay profile (PDP) is applied over these variables. As is typical in an urban environment channel [Fai89], the exponential decaying PDP will be adopted in most of the simulations in this thesis. The expectation of this PDP is modeled as

$$\mathcal{E}\left\{\left|h_{l}\right|^{2}\right\} = \beta_{\mathrm{CIR}} \exp\left(\frac{-\mu l}{L}\right),\tag{4.2}$$

where β_{CIR} is a suitable factor chosen to normalize the average energy of the CIR to unity, and μ defines the decay rate.



Figure 4.2: Realization of a time-varying FSC with exponential decaying gain.

With a proper choice of the symbol length, the CIR can be considered as a constant within the duration of one GFDM block, i.e., when channel coherence time is much larger than the block duration. In this case, the CIR is time-variant only when observed over the time interval of several symbols. Note that the CIR can encompass not only the physical channel, but also the analog transmit/receive filters which are not explicitly included in the model.

The received signal, r[n] as shown in Fig. 4.1(b), can be represented by discrete baseband samples collected with an accurate sampling rate, given by

$$r[n] = s[n-\theta]e^{j2\pi\frac{z}{K}n} + w[n], \qquad (4.3)$$

where $\theta \in \mathbb{Z}$ is the symbol time offset (STO), $\varepsilon \in \mathbb{R}$ is the carrier frequency offset (CFO) normalized to the subcarrier bandwidth, and w[n] denotes the additive white Gaussian noise (AWGN). The STO represents the unknown time of arrival of the GFDM signal. While the CFO represents in practice the effects of Doppler shifts, and also oscillator instabilities and inaccuracy between the received carrier and the local IF used for signal demodulation.

The unknown STO and CFO must be properly compensated to avoid severe degradation on the system performance. Therefore, the synchronization task refers to the signal processing carried on the receiver side to estimate θ and ε . This task allows the removal of an existing cyclic prefix, compensating for carrier frequency offset, and generating the aligned received signal y[n]. In the following, a metric based on the GFDM matrix model is proposed for assessing the impact of time and frequency offsets in a linear demodulation process. Then, the properties of the GFDM signal are also exploited in non-data aided synchronization approaches.

4.2 Sensitivity to interference

The synchronization algorithms used to estimate STO and CFO might result in residual errors, which are termed here as θ_r and ε_r , respectively.

In the following analysis, it is assumed that coarse synchronization would lead to y'[n], Fig. 4.1(b), where θ_r is within the guard interval and ε_r is within one subcarrier range.



Figure 4.3: Illustration of (a) time and (b) frequency offsets in a received signal.

The time and frequency misalignment can be modeled as

$$\boldsymbol{\Theta} = \frac{1}{N} \boldsymbol{\Theta}_{\text{freq}} \mathbf{W}^{\text{H}} \boldsymbol{\Theta}_{\text{time}} \mathbf{W}$$
(4.4)

where **W** is the Fourier matrix, and

$$\boldsymbol{\Theta}_{\text{freq}} = \begin{pmatrix} e^{j2\pi\frac{0\varepsilon_{\text{r}}}{K}} & 0 & \cdots & 0\\ 0 & e^{j2\pi\frac{\varepsilon_{\text{r}}}{K}} & \vdots\\ \vdots & & \ddots\\ 0 & \cdots & e^{j2\pi\frac{(N-1)\varepsilon_{\text{r}}}{K}} \end{pmatrix}$$
(4.5)

is the matrix representing the frequency shifts, and

$$\Theta_{\text{time}} = \begin{pmatrix} e^{j2\pi\frac{\theta_{\text{r}}}{N}} & 0 & \cdots & 0 \\ 0 & e^{j2\pi\frac{\theta_{\text{r}}}{N}} & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & e^{j2\pi\frac{(N-1)\theta_{\text{r}}}{N}} \end{pmatrix}$$
(4.6)

is the matrix representing the time shifts in the frequency domain. The received signal, considering residual STO and CFO in AWGN channel, can be defined as $\mathbf{y}' = \boldsymbol{\Theta} \mathbf{y}$, where \mathbf{y} is the vector representation of y[n]. Fig. 4.3 depicts the received signal \mathbf{y}' , the time offset $\theta_{\mathbf{r}}$, the CP interval, and the residual frequency offset $\varepsilon_{\mathbf{r}}$. methods. Note that in (4.4), it is assumed that Doppler effects are limited such that the spectrum will generally fall inside the receiver filter window, without causing power loss. Such issues can reasonably be ignored because of the assumption that STO and CFO are residual (small).

4.2.1 Analysis of self-interference

The estimated symbols with misalignment on the receiver side can be linearly modeled as

$$\hat{\mathbf{d}} = \mathbf{B}\mathbf{y}' = \mathbf{B}\mathbf{\Theta}\mathbf{y},\tag{4.7}$$

where **B** is the GFDM receiver matrix.

Considering an ideal channel, where $\Theta y = \Theta x$, the estimated symbols can be described as $\hat{d} = By' = B\Theta Ad = Gd$, where G represents the overall system matrix including the transmitter matrix, A, the misalignment matrix, Θ , and the receiver matrix, B.

Considering a linear detection, if $\mathbf{G} = \mathbf{I}_N$ the recovered symbols equal the transmitted symbols. Hence, the mean squared error (MSE) between \mathbf{G} and the identity matrix can be seen as a measure of the degradation introduced by the overall system matrix. In short, this metric is capable of modeling the self-generated interference in a GFDM system with residual time and frequency offsets.

The self-interference metric presented in [GMMF14] is proposed to evaluate the penalty caused by residual synchronization errors and is defined as

$$\Delta_{\mathbf{I}} = \frac{1}{2N} \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} |[\mathbf{I}]_{m,k} - [\mathbf{G}]_{m,k}|^2, \qquad (4.8)$$

which is equivalent to calculating the MSE between the system matrix G and the identity matrix.

Observe that in case of perfect synchronization, $\Theta = \mathbf{I}_N$, the metric can also be used to evaluate the prototype filter g[n] and the performance of the system given by **BA**. For instance, Fig. 4.4 exhibits the influence of matched filter (MF) receiver with raised cosine (RC) and root raised cosine (RRC) [MMGF14] as a function of the roll-off factor α , as well as for the proposed Meyer RC and Meyer RRC (Section 3.1.4), assuming perfect synchronization.

From Fig. 4.4, it is clear that a small roll-off factor leads to smaller interference. When the roll-off is equal to zero the self-interference is absent, i.e., all considered filters turn into a rectangular filter in frequency domain. This is a corner case where GFDM implements the orthogonal concept of the localized single-carrier frequency division multiple access (SC-FDMA) scheme [MLG06].

Particularly, the RC filter outperforms RRC for any value of α . This is an interesting counterintuitive result. The interference introduced by the MF with RC, which is composed of inter-symbol interference (ISI) and ICI, is smaller than the ICI obtained with RRC, which is a Nyquist pulse and thus ISI-free [MMGF14]. Similar behavior is observed between the Meyer RRC and Meyer RC. Notably, both the Meyer RRC and RC produce less self-induced ISI and ICI than traditional RRC and RC.

Note that the pulse shapes can be defined either in the time or in the frequency domain. However, the self-interference will remain constant in any case. That is, the combined effect remains the same once ICI and ISI are interchanged.



Figure 4.4: Impact of the pulse-shape in the value of the proposed metric. Prototype filters: RRC and RC with variable roll-off factor α .

Since the degradation introduced by self-generated interference depends only on the transmission and receiver matrices, the metric proposed in (4.8) can be combined with the noise enhancement ξ defined in (2.21). This is, when $\mathbf{y} = \mathbf{x} + \mathbf{w}$, the estimated data then is $\hat{\mathbf{d}} = \mathbf{B}\mathbf{y}' = \mathbf{B} (\boldsymbol{\Theta}\mathbf{A}\mathbf{d} + \mathbf{w}) = \mathbf{G}\mathbf{d} + \mathbf{B}\mathbf{w}$ and approximately $\hat{\mathbf{d}} = \mathbf{G}\mathbf{d} + \xi\mathbf{w}$, where $\xi = \sum_{i=0}^{N-1} |[\mathbf{B}]_{k,i}|^2$ as presented in (2.21) in Section 2.6.

The impact of the prototype filter and synchronization errors can be combined as [GMMF14]

$$\xi_{\Delta_{\mathrm{I}}} = \frac{E}{E\Delta_{\mathrm{I}} + \xi N_0},\tag{4.9}$$

where E is the average energy of the signal and N_0 is the noise power spectral density. In (4.9), the denominator combines the self-interference energy, $E\Delta_{\rm I}$, with the enhanced noise, ξN_0 , to calculate the effective signal-to-noise ratio (E-SNR), $\xi_{\Delta_{\rm I}}$.

The evaluation of residual frequency offset is presented in Fig. 4.5 as an example of signal-to-noise ratio (SNR) degradation for GFDM signals in different parameterizations. The orthogonal frequency division multiplexing (OFDM) case, where M = 1 and g[k] is a rectangular prototype filter, is presented in order to be compared with the bound in [Moo94], given by

$$\xi_{\rm I} \ge \frac{E/N_0 {\rm sinc}^2(\varepsilon)}{1 + 0.5947 E/N_0 {\rm sin}(\pi\varepsilon)^2}.$$
(4.10)

Fig. 4.5 shows that the metric (4.8) is consistent with the lower bound proposed in [Moo94] for the OFDM case, although GFDM has further SNR decrement because of self-interference. Furthermore, if the impact of self-generated interference on the metric value is known, then (4.8) can be used to evaluate the impact of synchronization errors on the system performance.

Notice that the self-interference degradation can be easily obtained from (4.8) by assuming perfect CFO synchronization, as shown in Fig. 4.4 when $\varepsilon = 0$. When using zero-forcing (ZF), the noise enhancement ξ is smaller when employing the RC filter instead of RRC.

Although the Meyer RRC has been proposed for FS-OQAM-GFDM (Section 3.1.3), the Meyer RC can also be considered for QAM-GFDM, replacing RC with a smaller penalty for higher roll-off. Hence, the proposed metric is useful as a synthesis tool, allowing for evaluating different GFDM configurations and estimating its performance under presence of time and frequency offsets.



Figure 4.5: SNR degradation versus relative frequency offset in ZF receiver in an AWGN channel.

In the following sections, coarse synchronization approaches will be discussed.

4.3 Non-data aided search approaches

The benefit of non-data aided synchronization schemes compared to data aided schemes is that no extra information has to be sent, which means that the spectral efficiency is not affected [MMF97, HTPD01]. There exist several blind estimation techniques for OFDM, e.g., [TL98, GG98, Böl01]. One class of techniques to estimate CFO is developed by minimizing the ICI in OFDM caused by the frequency misalignment. However, the non-orthogonality of GFDM in general makes a direct use of such derived techniques impossible. Therefore, this thesis considers techniques that are based on identifying repetitions in the time domain signal [HTPD01, SCM97, vdBSB97] and reverts it to the frequency domain perspective. That is, an equivalent approach for the GFDM is designed in the frequency domain based on the existence of ICI.

4.3.1 Sliding window for energy detection

In this subsection the detection concept will be exploited to bring attention to a basic idea that is dominant in several estimation approaches, namely the double sliding window principle [HTPD01].

Signal detection is a task on which the rest of the synchronization process is dependent. The detection can be performed to coarsely estimate the start of an incoming transmission burst. Signal detection can be described as a binary hypothesis test, i.e., consisting of two complementary statements about a parameter of interest. These statements are called the detection hypothesis and null hypothesis and assert whether a signal is present or not, respectively. The actual presence of a signal is defined based on a decision metric and a predefined threshold value. The performance of the packet detection algorithm can be summarized with two probabilities: probability of detection (PD) and probability of false alarm (PFA). PD is the probability of detecting a packet when it is truly present, thus high PD is a desirable quality for the test. PFA is the probability that the test incorrectly decides that a packet is present, when actually there is none, thus PFA should be as small as possible.

In general, increasing PD increases PFA and decreasing PFA decreases PD, hence the algorithm must be settled for a balanced compromise between these two conflicting goals. The general hypothesis test problem is discussed in details in [Kay98] and it has been extensively investigated in the context of GFDM in [Dat14].

The simplest algorithm for signal detection is to measure the received signal energy [Urk67]. When there is no signal being transmitted, the received signal consists only of noise and should be classified as the null hypothesis. In case transmission starts, the received energy is increased, thus the beginning of a burst can be detected as a change in the received energy level. This energy is accumulated over a window with length N' to reduce sensitivity to large individual noise samples or impulsive interference,

$$P_{\rm E}[n] = \sum_{k=0}^{N'-1} r^*[n-k]r[n-k] = \sum_{k=0}^{N'-1} |r[n-k]|^2.$$
(4.11)

Calculation of this metric can be interpreted as a moving average of the received signal energy, also called a sliding window sum. The rationale for the name is that at every time instant one new value enters the sum and one old value is discarded. This principle can be explicitly used to simplify the computation of (4.11), calculating the moving sum recursively as

$$P_{\rm E}[n+1] = P_{\rm E}[n] + |r[n+1]|^2 - |r[n-N'+1]|^2, \qquad (4.12)$$

allowing the number of complex multiplications to be reduced at the cost of using more memory to store all the values in the window.

The energy detection metric provided by (4.11) suffers from a significant drawback, the value of the threshold depends absolutely on the received signal energy. If the level of the noise power is unknown or there are other interference sources, the receiver needs to adjust its decision settings. These factors make it difficult to set a fixed threshold to decide when an incoming signal starts. An improvement that alleviates the threshold selection problem is achieved with a double sliding window energy detection metric [HTPD01], defined by

the ratio metric

$$M_{\rm R}[n] = \frac{P_{\rm E}[n]}{P_{\rm E}[n+N']}.$$
(4.13)

When only uncorrelated noise is received, the response of the double sliding window is flat. That is, both windows contain ideally the same amount of noise energy and the output value is unitary. When the first window starts to cover a signal edge, the average energy there gets higher until the point where the start of the packet is totally contained inside the window. This point corresponds to the peak of the metric and its sample index reveals the arrival time of the incoming signal. After this point the second window also starts to collect signal energy, and when it is also completely inside the received packet, the response of the composed metric becomes flat again.

The response of the metric can be interpreted as a differentiator, in that its value is large when the input energy level changes rapidly. The detection hypothesis is declared when the value in the metric crosses over a threshold limit value, defined according to the PFA criterion [Dat14].

At the peak value the metric can be used to estimate the SNR once

$$M_{\rm R}[n_{\rm peak}] = \frac{P_{\rm E}[n_{\rm peak}]}{P_{\rm E}[n_{\rm peak} + L]} = \frac{\sigma_s^2 + \sigma_w^2}{\sigma_w^2} = \frac{\sigma_s^2}{\sigma_w^2} + 1 = SNR + 1,$$
(4.14)

where σ_s^2 and σ_w^2 are, respectively, the variance of the multipath signal s[n] and the additive noise w[n]. This basic metric can eventually be used as a component of more elaborated maximum likelihood synchronization methods, which are addressed in the next subsection.

The basic principle presented so far can be summarized as a decision variable that expresses the metric as a ratio of the total energy contained inside the two windows. This is the concept that will be revisited in all following schemes considered in this thesis, but the energy will be weighted by autocorrelation and correlation criteria to achieve higher estimation accuracy.

4.3.2 STO and CFO estimation based on repetitive patterns

A coarse timing synchronization scheme for CP-OFDM has been proposed in [SCM97] using the redundancy of repeated signals. The metric performs a simple difference operation between two segments of the received signal, transmitted N samples apart. If these two segments match, as a result of the repeated signal, the metric has a minimum

$$\hat{\theta} = \arg\min_{n} \{\Lambda[n]\} = \arg\min_{n} \left\{ \sum_{k=n}^{n+N_{\rm CP}-1} |r[k+N] - r[k]|^2 \right\} .$$
(4.15)

Thus, this metric also employs a double sliding window to provide meaningful coarse information about time offset acquisition. Interesting to observe that (4.15) is a least squares metric. That is, when the quadratic equation is expanded in the form

$$\Lambda[n] = \sum_{k=n}^{n+N_{\rm CP}-1} \left(|r[k+N]|^2 + |r[k]|^2 \right) - 2 \left| \sum_{k=n}^{n+N_{\rm CP}-1} r^*[k+N]r[k] \right| , \qquad (4.16)$$

the components can be seen as the outcomes of calculating the signal power of two segments of the received signal and the correlation among them. The correlation dependence makes this metric more accurate than energy detection metric presented in the previous subsection.

A similar result has been encountered by Beek et al. in [vdBSB97] using a joint maximum likelihood estimator (MLE) for both STO and CFO estimation. This metric is based on the use of CP in OFDM and is asymptotically a minimum variance unbiased estimator (MVUE) for an AWGN channel (Appendix D).

Based on the assumption that the transmitted data values $d_{k,m}$ are independent, [vdBSB97] assumes that the OFDM signal is approximately complex Gaussian distributed, with real and imaginary values being independent. This process is not white due to the existence of the CP, which yields a correlation between some pairs of samples, which are located N samples apart. Defining the index sets

$$D_1 = \theta, \dots, \theta + N_{\rm CP} - 1 \tag{4.17}$$

$$D_2 = \theta + N, .., \theta + N + N_{\rm CP} - 1, \qquad (4.18)$$

the MLE metric is given by

$$\Lambda(\theta,\varepsilon) = \ln p(\mathbf{r}|\theta,\varepsilon) \tag{4.19}$$

$$= \ln \left(\prod_{n \in D_1} p(r[n], r[n+N]) \prod_{n \notin D_1 \cup D_2} p(r[n]) \right) \quad . \tag{4.20}$$

After some algebraic calculations, restricting the time offset to be integer valued, the STO and CFO estimators are derived as

$$\hat{\theta} = \arg \max_{n} \{ |P_{\mathcal{A}}[n]| - \rho R_{\mathcal{A}}[n] \}$$
(4.21)

$$\hat{\varepsilon} = \frac{\operatorname{angle}\left\{P_{A}[\theta]\right\}}{2\pi}$$
(4.22)

with

$$P_{\rm A}[n] = \sum_{k=n}^{n+N_{\rm CP}-1} r^*[k+N]r[k]$$
(4.23)

$$R_{\rm A}[n] = \frac{1}{2} \sum_{k=n}^{k+N_{\rm CP}-1} |r[k+N]|^2 + |r[n]|^2$$
(4.24)

$$\rho = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2} \quad . \tag{4.25}$$

The techniques presented so far are directly applicable to CP-GFDM and, without any change, can achieve similar results obtained with CP-OFDM.

Nevertheless the more flexible concept of GFDM allows that a similar principle of exploring repetitive patterns can be applied in the frequency domain, as will be presented next.

4.3.3 CFO estimation based on ICI in GFDM

A single subcarrier in the GFDM transmit signal can be modeled as the modulation of a K-times oversampled version of $d_{k,m}$, (3.24). The time domain data of one subcarrier equals the circular convolution of the prototype filter g[n] and the oversampled data stream. This operation can also be represented by a multiplication of the filter in the frequency domain with the discrete Fourier transform (DFT) of the data samples. The zero stuffing operation in the time domain causes a K repetition of the spectrum of the M data samples, Fig 4.6. The repetitive pattern observed in a filtered subcarrier causes the ICI to its neighbors, but also carries redundancy. The redundancy can be enhanced by increasing the bandwidth of the filter g[n], e.g., increasing the roll-off factor α . Note, that opposed to this effect, the greater the bandwidth of the filter is, more interference to this area will be injected by the neighboring subcarrier. Thus the performance of using this redundancy is limited. Nevertheless, the metric can also be improved by averaging over several GFDM blocks.

Because the redundancy is observed in the frequency domain, the DFT of the signal has to be calculated in order to evaluate this information. This can only be done correctly if a time synchronization is applied before, for instance, using the CP based scheme [vdBSB97] discussed previously.



Figure 4.6: Multiplication of prototype filter with data of one subcarrier in the frequency domain. L, R represents the left and the right half of the data in the frequency domain, which is repeated because of the zero stuff operation performed in the time domain.

The N-point DFT of the received samples with known time offset θ can be expressed as

$$Y[\ell] = \mathcal{F}\{r[n+\theta]\}, \quad \ell = 0, ..., N-1$$
(4.26)

The correlation of the two redundancy parts of the subcarrier spectrum can be calculated in analogy to (4.23) as

$$p_{\mathcal{A}}[\ell] = \sum_{k=\ell}^{\ell+M/2-1} Y^*[k+M]Y[k], \qquad (4.27)$$

where the redundant parts in the frequency domain have a distance of M samples and length $\frac{M}{2}$, Fig. 4.6.

An averaged ICI correlation can be obtained combining the metric of several GFDM blocks as

$$\bar{p}_{A}[\ell] = \frac{\sum_{b=1}^{B_{on}} p_{A}^{(b)}[\ell]}{B_{on}} , \qquad (4.28)$$

where B_{on} represents the number of considered blocks. The averaged correlation produces K_{on} peaks in the metric $\bar{p}_{A}[\ell]$, one for every active subcarrier.

To use the statistical information of all peaks, a new metric is proposed. It uses the fact, that the distance between the peaks equals M samples, which means that every subcarrier produces a peak in the frequency domain, as illustrated in Fig. 4.7.

The proposed metric exploring ICI is then given by

$$M_{\rm IC}[\ell'] = \sum_{k=0}^{K_{\rm on}-1} |\bar{p_{\rm A}}'[\ell' + kM]|$$
(4.29)

with $\ell' = 0, ..., N - 1$ and $\bar{p_A}'[\ell']$ is assumed to have a repeated sequence concatenated, so that $\bar{p_A}'[\ell'] = \bar{p_A}[\ell' \mod N]$. The metric $M_{\rm IC}[\ell']$ adds up all samples in distance M of the metric $\bar{p_A}'[\ell']$ for one value of ℓ .

The frequency shift based on ICI can be consequently derived as

$$\hat{\varepsilon} = \frac{\arg\max_{\ell'} \{M_{\rm IC}[\ell']\} - \lceil M/2 \rceil}{M} . \tag{4.30}$$

The range of the estimation is $-K/2, ..., \varepsilon, ..., +K/2-1/M$. Because large errors can occur at the border, the range is limited to $-K/2+1, ..., \varepsilon, ..., +K/2-1$. The accuracy is limited to be a multiple of 1/M because there are M sampling points between subcarriers in the frequency domain.

For simulation, a GFDM setup according to Tab. 4.1 is chosen with the motivation of having a minimal set of subcarriers in a total bandwidth of 180 kHz (as in a LTE resource block). This setup is flexible and can be accommodated in a small gap in the spectrum, e.g., in applications using opportunistic spectrum for sensor utilization. The used RC
filter with roll-off factor $\alpha = 1$ produces large side lobes in frequency and thus maximum redundancy of data.

GFDM setup	K	$K_{\rm on}$	M	α
	24	12	3	1
channel	1st tap	2nd tap	3rd tap	$ au_{rms}$
power delay profile	0 dB	-2.5 dB	-5 dB	$5.1 \mathrm{ms}$

 Table 4.1: Used parameter sets for ICI based frequency estimation experiment



Figure 4.7: Metrics for CFO estimation using ICI in GFDM. $K = 24, K_{on} = 12, M = 3, \alpha = 1$, averaging of 20 GFDM blocks with random data for a frequency shift of $\varepsilon = 1$

As performance criteria the MSE [Kay93, p.19] has been chosen. It is equal to the variance in the case of an unbiased estimator and a common quality indicator for synchronization measurements [RCZZ05, GLG08]. The frequency offset is fixed to be a multiple of 1/Mto offer the possibility of perfect synchronization.

In Fig. 4.8, the simulation results show the CFO estimation performance in the proposed multipath environment. With 20 blocks, the achieved CFO MSE is in the order of $6 \cdot 10^{-2}$

at high SNRs. By increasing the number of blocks to 200, a CFO MSE of $4 \cdot 10^{-3}$ can be achieved at high SNRs. Note that the proposed channel can result in deep notches in the frequency domain, affecting the correlation of the spectrum, which results in higher and large fluctuations of the CFO MSE when compared to the AWGN channel performance.



Figure 4.8: CFO MSE in AWGN and multipath channel (average of 20 and 200 GFDM blocks). The frequency offset is uniform distributed in the range of $+K/2 - 1, ..., \varepsilon, ..., -K/2 + 1$, Monte-Carlo simulation with 375 000 realizations.

The performance of the estimator can be improved by increasing the amount of blocks to be considered in the estimation. With the proposed multipath channel, the increment of the number of GFDM blocks by one decade results, approximately, in a one decade lower MSE.

This method can be useful for large bursts and streaming applications, without affecting spectral efficiency. As a complement to the direct use of the CP-OFDM method [vdBSB97] in GFDM, the presented non-data aided scheme is capable to convey a wider range of CFO. This feature can be beneficial for applications where inexpensive oscillators used in low cost devices would lead to CFO in the order of the bandwidth of several subcarriers. Besides that, the proposed method can also be used for tracking CFO changes after an initial preamble based estimation.

4.4 Summary

In this chapter the synchronization problem has been presented in the context of GFDM. A tool to access the sensitivity to misalignments and a non-data aided scheme based on ICI have been proposed.

The main contributions of this chapter have been published in [GMMF14, KGMF14].

Details can be summarized as follows:

- ▷ The problem formalism of STO and CFO estimation has been presented based on a system model. The effects of synchronization errors have been quantified by introducing a self-interference metric based on the GFDM matrix model.
- ▷ As a corner case example, the proposed metric has been used to reproduce known results achieved with OFDM. Also, the impact on the pulse shape choice in a perfectly synchronized system or with residual frequency offset have been presented. In particular, it has been shown that the Meyer RRC and Meyer RC filters produce less self-interference than RRC and RC.
- ▷ Non-data-aided search methods based on dual sliding window have been introduced to exploit energy detection and cyclostationary properties, e.g., due the use of CP.
- ▷ A novel non-data aided method for coarse CFO estimation has been proposed for GFDM based on ICI. The redundant information that leaks from the sidelobes of the prototype filter is exploited. The estimation can be done while transmitting payload data only. The spectral efficiency is not affected.
- ▷ Simulations have shown that an acquirement of 20 symbols is sufficient to get useful results in a multipath scenario with deep notches in the frequency domain. Also the wide estimation range of a CFO is achieved. The performance of the estimation can be improved by increasing the number of GFDM blocks, e.g., in large bursts or streaming applications.

In the following chapter, the focus will be redirected towards the use of data-aided schemes for GFDM fine synchronization.

Chapter 5

Fine Synchronization Strategy for GFDM

This chapter proposes data aided approaches for fine synchronization of generalized frequency division multiplexing (GFDM). For instance, data aided synchronization schemes are essential in burst transmission because they provide a fast and accurate estimation of symbol time offset (STO) and carrier frequency offset (CFO). Classic data aided schemes used in orthogonal frequency division multiplexing (OFDM) are briefly introduced as a basis for the development of a proper scheme for GFDM. Then, isolated preamble and embedded midamble training sequences in scenarios such as wireless metropolitan area networks are discussed. Additionally, pseudo-circular pre/post-amble (PCP) instead of cyclic prefix (CP) is introduced aiming at more robust synchronization in car-2-x scenarios.

5.1 Data aided search approaches used in OFDM

In one of the most cited papers about data aided synchronization in OFDM, by Schmidl & Cox [SC97], the proposed scheme exploits the use of autocorrelation in a preamble containing repeated sequences. The preamble symbol consists of a pseudo noise (PN) sequence on the even subcarriers and zeros on the odd subcarriers, or vice versa. This procedure results in a multicarrier symbol with two identical halves in the time domain, and the autocorrelation between them can be exploited as a metric to identify the STO and CFO.

The autocorrelation of these two repeated halves allows the estimation of STO and CFO similar to (4.21) and (4.22) [vdBSB97], respectively. However, the length and distance

between the two repeated halves are different and given by

$$P_{\rm A}[n] = \sum_{k=n}^{n+N/2-1} r^*[k]r[k+N/2], \qquad (5.1)$$

$$R_{\rm A}[n] = \sum_{k=n}^{n+N/2-1} |r[k+N/2]|^2.$$
(5.2)

The autocorrelation (5.1) considers a training sequence of length $\frac{N}{2}$ while (4.23) computes the CP autocorrelation with length $N_{\rm CP}$. Differently from (4.24), in (5.2) the estimation of the energy of the received signal is simplified to the computation of a single half, of length $\frac{N}{2}$ samples. The subtraction of the log-likelihood terms in (4.21) is replaced by a linear division and does not consider the signal-to-noise ratio (SNR) normalization, ρ , resulting in the metric [SC97]

$$M_{\rm S}[n] = \frac{|P_{\rm A}[n]|^2}{(R_{\rm A}[n])^2}.$$
(5.3)

Note that the squared operation can be avoided in implementation as it changes the shape of the metric but does not affect the criterion of peak search.

Because of the existence of CP, the metric forms a plateau with length $N_{CP} + 1$, see Fig. 5.1. In additive white Gaussian noise (AWGN) channels, this plateau leads to an uncertainty regarding the start of the frame because there may be no single maximum. In frequency selective channel (FSC), each channel tap leads to a new plateau and the uncertainty is even higher [SC97].



Figure 5.1: CP influence over the metrics proposed by Schmidl & Cox [SC97] and Minn et. al [MZB00] in an ideal channel. M = 1, K = 1024, $N_{\rm CP} = 128$. A plateau effect can be observed in the S&C metric, it has the length of the CP. This plateau is mitigated in the Minn metric using a integration approach.

Two proposals are suggested by [SC97]. The first one simply finds the maximum of the timing metric, $M_{\rm A}[n]$, ignoring the plateau. The second further search for the points which are 90% of the maximum, at the left and right sides, and average these two points to find

the symbol timing estimate. According to [SC97], the rationale behind this second method is that the best timing points typically lie in the plateau region. By trying to determine the center of this plateau, it is more likely that the estimate will not fall off the plateau. Once $\hat{\theta}$ is obtained by the first or second approach, the CFO estimation is given by

$$\hat{\varepsilon} = \frac{\operatorname{angle}\left\{P_{\mathrm{A}}[\hat{\theta}]\right\}}{\pi},\tag{5.4}$$

where the normalized frequency offset has to be in the range $\varepsilon \in [-1, 1]$, which is twice the range achievable with the CP approach in (4.22) [vdBSB97]. Although the frequency offset estimator operates near the Cramér-Rao lower bound (CRLB) in AWGN case, the plateau in the metric causes high variance in the time estimation [SC97].

To avoid the plateau issue, [MZB00] suggests the use of an additional integration operation in the metric proposed by [SC97], Fig. 5.1. Alternatively, [MZB00] also suggests the use of a preamble with four identical quarters instead of two halves in the time domain. In this approach, a phase shift of π is applied to the last two quarters, i.e., they are multiplied by -1. The signal inversions improve the metric, but create discontinuity that enhances the out-of-band (OOB) emission.

In [AKE08], a multistage synchronization scheme further exploits the metrics proposed by [SC97] and [MZB00]. After coarse STO estimation and compensation of the fractional CFO in the received signal, the autocorrelation metric is combined with a correlation metric [Mas72] to achieve fine STO estimation. As a final step, the algorithm performs an additional search for the first channel path, which is not necessarily the strongest one in non-line-of-sight (NLOS) transmission. A threshold criterion proposed by [KE06] is used in this operation. More details will be presented in the context of GFDM in the following sections.

5.2 Training sequence designs for GFDM

Synchronization can be achieved in GFDM on a block basis, allowing adaptation of fundamental OFDM solutions to estimate STO and CFO as given in [vdBSB97], [SC97]. For instance, the use of a preamble sequence that consists of certain periodic structure allows for the design of an isolated training sequence, independent from the data payload. However, further attention must be spent on OOB issue, this is an important aspect to be addressed with GFDM. In most of the literature, synchronization techniques for OFDM neglect the effects of the OOB in the preamble design. Therefore, a main contribution in this chapter is the investigation of solutions for the GFDM synchronization that do not increase the OOB emission and have a performance at least comparable to the solutions designed for OFDM [SC97, MZB00, AKE08, KE06].

As a first approach, an isolated windowed preamble is proposed for GFDM to achieve low OOB emission [GMMF14]. This solution is illustrated in Fig. 5.2, where the training sequence and the data are conveniently separated. The training and data signals are represented by two trapezoids in Fig. 5.2, $x_p[n]$ and x[n], respectively. These signals are in fact two GFDM blocks with independent parameterization, which can be obtained by pinching the block boundaries as described in (2.15) in Chapter 2. For instance, different transitions lengths can be applied to the preamble and data blocks in order to achieve a desired emission mask, i.e., N_{Wp} and N_{Wd} , respectively.



Figure 5.2: Windowed preamble preceding a GFDM data block.

As a second approach, the training sequence is embedded as a midamble [GF15] (Fig. 5.3). This solution is illustrated in Fig. 5.3, where the training sequence and the data compose a self-contained packet. The training and data signal are represented by two overlapping trapezoids in Fig. 5.2. These signals belong to the same GFDM blocks and are obtained by reserving the central time slots for the midamble and using the remaining time slots for data. Here, the smooth transition of the central trapezoid should be interpreted as inter-symbol interference (ISI). Although a windowed CP is considered in Fig. 5.3, the use of guard symbols (see Fig. 2.7 in Chapter 2) is also an attractive solution.



Figure 5.3: Embedded midamble in a GFDM data block.

As a third approach, a pseudo-circular pre/post-amble (PCP) is proposed [GFF15]. This solution is illustrated in Fig. 5.4, where the CP is explicitly removed and the corner subsymbols are defined as pilots. With this approach, soft concatenation of blocks can be achieved in continuous transmissions. This idea is illustrated by three trapezoids; a larger one at the center, representing the data, and two smaller ones at the corners. The solid half-trapezoids represents the PCP defined by the first subsymbol, $d_{k,0}$. The dashed and dotted half-trapezoids at the corners represent the soft concatenation obtained from adjacent GFDM blocks.



Figure 5.4: Pseudo-circular pre/post-amble PCP in a GFDM data block.

All three proposals can use the principles of transmitting repeated halves. This simple property will be used in the next sections as a strategy to achieve robust metrics based on [vdBSB97, SC97, MZB00, AKE08, KE06], while improving the OOB in comparison to OFDM.

5.2.1 Preamble synchronization

This proposal considers a preamble block that consists of a GFDM signal with M = 2 subsymbols and K subcarriers. In this block, a PN sequence $\vec{c} = (c[0], \ldots, c[K-1])^{\mathrm{T}}$ of length K is transmitted twice, i.e., the preamble carries the data vector [SC97]

$$\mathbf{d}_{\mathrm{P}} = \begin{pmatrix} c[0] & \dots & c[K-1] & c[0] & \dots & c[K-1] \end{pmatrix}^{\mathrm{T}},$$
 (5.5)

leading to the signal $\mathbf{x}_{\mathrm{P}} = \mathbf{A}\mathbf{d}_{\mathrm{p}}$. In \mathbf{A} , the filter g[n] can be defined as a rectangular function covering exactly the duration of one subsymbol. As the two subsymbols carries the same data sequence, there will be a continuous transition between them.

With inclusion of CP according to (2.15), and without applying windowing, the complete preamble signal $\tilde{\mathbf{x}}_{\rm P}$ is exactly the same considered in [AKE08]. In order to control the OOB emissions, windowing can be applied according to (2.15), which leads to the windowed preamble $\tilde{\mathbf{x}}_{\rm Pw}$. The ramp up and ramp down functions in (2.16) are designed as a variation of the Tukey window [Blo00]. The proposal is similar to (3.22) and defined as

$$p_{\rm UP}[n] = \frac{1}{2} - \frac{1}{2} \cos\left(\pi v \left(\frac{n + N_{\rm W} + N_{\rm CP}}{N_{\rm W}}\right)\right),$$
 (5.6)

$$p_{\rm DW}[n] = \frac{1}{2} + \frac{1}{2} \cos\left(\pi v \left(\frac{n-N}{N_{\rm W}}\right)\right), \qquad (5.7)$$

where v(n) is again the Meyer wavelet auxiliary function [Mey90] (see Section 3.1.4).

The obtained non-windowed and windowed preambles are plotted in Fig. 5.5. Half of the subcarriers are disabled to highlight their difference in terms of the OOB emissions.



(b) Power spectrum density of the windowed and non-windowed preamble.

Figure 5.5: Proposed preamble. M = 2, $K_p=128$, $N_W = N_{CP} = 32$

While the non-windowed preamble $\mathbf{x}_{\rm P}$ produces non-negligible spectrum side-lobe of -35 dBc, the proposed window in $\mathbf{x}_{\rm Pw}$ is able to reduce the OOB by several orders of magnitude.

On the receiver side, a set of samples r[n] containing the preamble is collected. In order to estimate time and frequency offsets, an autocorrelation of the sequence r[n] is performed according to (5.3) [SC97]. The presence of CP creates a plateau effect in the metric, but this ambiguity can be resolved by integrating over the length of CP [MZB00], which yields

$$M_{\rm M}[n] = \frac{1}{N_{\rm CP} + 1} \sum_{k=-N_{\rm CP}}^{0} M_{\rm S}[n+k].$$
(5.8)

With this result, the coarse STO estimation is obtained by searching for the peak $\hat{n}_{\rm M}$ of (5.8), i.e.,

$$\hat{n}_{\mathrm{M}} = \underset{n}{\mathrm{arg\,max}} \quad (M_{\mathrm{M}}[n]) \,. \tag{5.9}$$

The angle of $P_{\rm A}[\hat{n}_{\rm M}]$ reveals effects of phase rotation $\hat{\varepsilon}$ and is used to estimate the CFO, hence,

$$\hat{\varepsilon} = \frac{\text{angle}\left\{P_{\mathrm{A}}[\hat{n}_{\mathrm{M}}]\right\}}{\pi}.$$
(5.10)

The CFO information can be used to correct the frequency offset in the received signal, yielding

$$r'[n] = r[n]^* \exp\left(-j2\pi \frac{\hat{\varepsilon}}{N}n\right).$$
(5.11)

This operation prepares r'[n] to be used in a sharper metric, employing cross-correlation with the known transmitted preamble, which is given by

$$P_{\rm C}[n] = \frac{1}{N} \sum_{k=0}^{N-1} r'^*[n+k] x_{\rm p}[k], \qquad (5.12)$$

where $x_{\rm p}$ contains the two halves of the known preamble.

The metrics (5.8) and (5.12) can be combined in order to suppress side peaks that result from the repeating parts inside the preamble [AKE08], which leads to

$$M_{\rm A}[n] = M_{\rm M}[n] \cdot |P_{\rm C}[n]|.$$
(5.13)

The complete idea is depicted in Fig. 5.6. This figure presents a realization of $M_{\rm S}[n]$, $M_{\rm M}[n]$, $|P_{\rm C}[n]|$, and $M_{\rm A}[n]$, and illustrates the metric evolution in the schemes from [SC97] to [MZB00] and [AKE08]. Without windowing, the plateau effect in $M_{\rm S}[n]$ is eliminated in $M_{\rm M}[n]$ and this metric is then multiplied with $|P_{\rm C}[n]|$ to remove its side peaks.



Figure 5.6: Metric realization for a perfect channel without and with the use of windowing in the preamble design.

With windowing the final result $M_{\rm M}[n] \cdot |P_{\rm C}[n]|$ differs only slightly, while the main difference is an alteration in the shape of $M_{\rm S}[n]$. By choosing the window length to be within the CP region, the plateau is suppressed. So with a proper time offset adjustment it would be possible to combine $M_{\rm S}[n]$ directly with $|P_{\rm C}[n]|$ and further simplify the algorithm without computing $M_{\rm M}[n]$.

The more accurate STO estimation is obtained by searching for the peak value $\hat{n}_{\rm A}$ of (5.13) around the range $[\hat{n}_{\rm M} - N/2, \hat{n}_{\rm M} + N/2]$:

$$\hat{n}_{\rm o} = \arg\max_{\hat{n}_{\rm M} - \frac{N}{2} \le n \le \hat{n}_{\rm M} + \frac{N}{2}} \quad (M_{\rm A}[n]) \,. \tag{5.14}$$

Although the impulsive shape of $M_{\rm A}[n]$ results in precise STO estimation in single path channels, in time-variant FSC the first multipath peak might not be found using only the maximum value principle. Fading propagation conditions can affect the initial channel tap gain such that it can be lower than one of the other echoes.

Finding the position of the first multipath is addressed in [KE06] with a threshold criterion proposed to identify other multipath peaks before the maximum peak with a given probability of false alarm (PFA), $p_{\rm FA}$. Based on the central limit theorem, samples that do not correspond to the preamble echoes in the cross-correlation output are interpreted as complex Gaussian random samples and the threshold is derived from a Rayleigh cumulative density function (CDF), resulting in [KE06]

$$T_{\rm Th} = \sqrt{-\frac{4}{\pi} \ln(p_{\rm FA})} \left(\sum_{k=-\frac{N}{2}+\lambda}^{-\lambda} \frac{|P_{\rm C}[\hat{n}_{\rm o} + k]|}{\frac{N}{2} - 2\lambda} \right),$$
(5.15)

where λ is an adjustable parameter defined according to the channel impulse response characteristics, e.g., $\lambda < N_{\rm CP}$.

For frequency selective channels, the estimation of the first multipath is then performed in the range $(\hat{n}_A - \lambda, \hat{n}_A]$ and is given by

$$\hat{n}_f = \underset{\hat{n}_o - \lambda < n \le \hat{n}_o}{\operatorname{argfirst}} \left(|P_{\mathcal{C}}[n]| > T_{\mathrm{Th}} \right).$$
(5.16)

In order to analyze the performance of the proposed windowed preamble, the synchronization technique is evaluated in a time-variant FSC. The unknown time offset θ , fixed to a given integer value, and frequency offset $\varepsilon \in \mathbb{R}$, uniformly distributed within [-1, 1] will be estimated considering a channel impulse response with 32 taps. The tap gains vary linearly in decibel scale from 0 to -13 dB. Each tap is multiplied by a Rayleigh distributed random variable with variance $\sigma_w^2 = 1/2$. The system parameters used for the simulation are listed in Table 5.1.

Parameter	Value
Mapping	16-QAM
Transmit Filter	RC
Roll-off (α)	$0.1 \ / \ 0.9$
Number of subcarriers (K)	128
Number of subsymbols (M)	9
CP length $(L_{\rm CP})$	32 samples
CS length $(L_{\rm CS})$	16 samples
length of ramp up/down (W)	16 samples

 Table 5.1: Configuration of the GFDM data block using isolated preamble.

The mean error and the mean squared error (MSE) of CFO and STO estimations for non-windowed and windowed preambles have been evaluated for 1000 realizations at each value of SNR and are presented in Fig. 5.7. The CRLB, computed according to [SC97], is included as a reference. The gap observed in the CFO estimation, which occurs for both non-windowed and windowed preambles, is due to the overlap between the segments of the preamble introduced by the time variant FSC [MM99]. The MSE of the STO estimation is within 10^{-1} as the SNR is increased for the non-windowed and the windowed preambles.

For the particular parameters used in the simulation, it is observed that the mean value of the STO presents a positive offset behavior that decreases with the increment of the SNR. This is a consequence not only of the power delay profile (PDP) but the threshold criterion used for searching the first multipath. The threshold is proportional to the noise level and, if a low false alarm probability is specified, the chance to ignore a severely attenuated first path gets higher.

An illustration of positive residual STO estimation is provided by Fig. 5.8. It contains one realization of the synchronization process for the windowed preamble at 0 dB SNR. Although $|M_{\rm M}[n]|$ is not sharp, this metric can coarsely identify the region where the preamble is present and provides the CFO estimation. Once the CFO is compensated, the cross-correlation $|P_{\rm C}[n]|$ is obtained. The search for the highest peak performed using the metric $M_{\rm A}[n]$ does not return the first multipath, which is located in the time sample position 0. Two regions are explored for a refined search for the position of the first multipath in $|P_{\rm C}[n]|$: Region 1 (\mathcal{R}_1) defines the range used to compute the threshold value; Region 2 (\mathcal{R}_2) defines the range where peaks higher than the threshold are considered as previous multipaths. The zoomed curve (the top right of Fig. 5.8) shows the range $\hat{n}_{\rm o} - \lambda \leq n \leq \hat{n}_{\rm o}$. In this example h[0] << h[1] and the threshold criterion indicates that h[1] should be considered as the first multipath, resulting in a positive offset error.



Figure 5.7: Mean error and MSE of the (a) CFO and (b) STO estimation with non-windowed and windowed isolated preamble.



Figure 5.8: Analysis of the metrics in a multipath channel assuming SNR = 0 dB and K = 128.

The realization in Fig. 5.8 suggests that, even when the realization of the PDP has a weak first tap and the SNR is low (0 dB), the residual estimation error of the position of the primary path is small. Furthermore, under higher SNR conditions the impact of losing a weak multipath is not as severe as a potential ISI from an adjacent transmitted sequence in a bit pipe communication system, which might happen if a negative residual error is introduced.

A simulation was performed with both non-windowed and windowed preamble assuming perfect knowledge of the channel impulse response in the reception. Fig. 5.9(a) presents the symbol error rate (SER) performance curve considering a perfect synchronization as a reference and also the synchronization obtained with the use of the preamble.

In the realization of the PDP in Fig. 4.2, the first multipath is not the highest value, e.g., it can represent an indoor reception with a receiver device in a fixed position. Under this circumstance, both non-windowed and windowed algorithms perform without noticeable STO estimation errors and the SER performance curves are similar for both approaches.

Considering a different realization of the PDP, the influence of the roll-off is illustrated in 5.9(b). It is noticeable that when $\alpha = 0.9$, with higher noise enhancement, the SER curves are deviated to the right. On average, the difference between $\alpha = 0.9$ and $\alpha = 0.1$ is near 2.5dB at higher SNRs, in accordance with Fig. 4.5 when the residual CFO is smaller.



Figure 5.9: SER analysis over time-invariant FSC for a GFDM data block appended to the preamble.

The main difference between the SER obtained assuming the synchronization approaches and the curve assuming perfect synchronization is that the high variance of the CFO in low SNR regime leads to a performance gap when the synchronization techniques are considered. However, all curves tend to match each other for high SNR, since in this case the CFO variance is smaller, as shown in Fig. 5.7(b).

5.2.2 Midamble synchronization

The midamble design is an interesting option in order to create a self-contained packet, with possible use of guard subsymbols on the edges of the GFDM block. The midamble can be placed in the center of the block to minimize its average time difference to any subsymbol, and can be potentially exploited for channel estimation, as in second generation (2G) [GH96]. One challenge in this approach is that, depending on the pulse shape design, data and training sequences overlap in time, which affects the synchronization performance.

The midamble design is illustrated in Fig. 5.10. Observe that, due to ISI, the midamble energy spreads along the adjacent subsymbols, but keeps higher intensity in the center part of the symbol.



Figure 5.10: GFDM terminology and midamble design.

To assess the impact of the overlap effect, the midamble will be designed with a similar configuration as used in the isolated windowed preamble, with the two repeated training parts structure, and evaluated in a single-shot approach with a typical wireless channel model, e.g., in scenarios like the wireless metropolitan area networks [MFP03]. The midamble concept will now be detailed and the algorithm presented in the previous isolated preamble section will be adapted accordingly.

Considering that the same set of K pilots c[k] is allocated in two subsymbols in the center of the GFDM symbol, $d_{k,\lfloor M/2 \rfloor} = d_{k,\lfloor M/2 \rfloor+1} = c[k]$, where $\lfloor . \rfloor$ stands for rounding towards negative infinity (floor function), the GFDM midamble can be expressed as in (3.27), i.e.,

$$x_{\rm p}[n] = \sqrt{K} \sum_{m=\lfloor M/2 \rfloor}^{\lfloor M/2 \rfloor + 1} g[(n - mK) \bmod N] \mathcal{F}_K^{-1}\{d_{(\cdot),m}\}|_n,$$
(5.17)

or in the vector form, yelding

$$\mathbf{x}_{\mathrm{p}} = \sqrt{K} \sum_{m=\lfloor M/2 \rfloor}^{\lfloor M/2 \rfloor+1} \mathbf{P}^{(m)} \mathrm{diag}(\boldsymbol{g}) \mathbf{R}^{(M,K)} \mathbf{W}_{K}^{H} \mathbf{d}_{m}.$$
(5.18)

The information coefficient vector (ICV) concept is defined here to represent the K-point inverse discrete Fourier transform (IDFT) of the data in (5.20) as follows

$$\boldsymbol{\S}_m = \mathbf{W}_K^H \mathbf{d}_m. \tag{5.19}$$

The idea is to highlight the ICV as a key component for the synchronization algorithm, once the search is performed on the corresponding training sequence obtained in the time domain. Therefore, the midamble can be obtained as discussed in Section 3.2.3, i.e.,

$$\mathbf{x}_{\mathrm{p}} = \sqrt{K} \sum_{m = \lfloor M/2 \rfloor}^{\lfloor M/2 \rfloor + 1} \mathbf{P}^{(m)} \mathrm{diag}(\boldsymbol{g}) \mathbf{R}^{(M,K)} \mathbf{S}_{m}.$$
 (5.20)

If the filter g is configured as a rectangular pulse with the duration of one subsymbol, the obtained GFDM midamble can be interpreted as the concatenation of two identical OFDM signals. This rectangular midamble can be tracked using the same double sliding window approach of [SC97]. However, for other pulse shapes, the same approach might result in performance loss. This is because the double sliding window approach is actually a matched filter, that is, a moving average performed with a rectangular filter. If the received signal is not based on a rectangular pulse, then there is a mismatch.

To contour this issue and support more general pulse shapes, this thesis proposes the use of an estimator that recovers the ICV component. The idea is to revert the pulse shape operation and isolate the ICV using (3.33) as discussed in Section 3.2.2.

First, using the classic double sliding window metric [Moo94, vdBSB97, SC97], the autocorrelation metric is set as

$$P_{\rm A}[n] = \sum_{k=n}^{n+K-1} r[k+K]^* r[k], \qquad (5.21)$$

where the argument $\hat{n}_{A} = \underset{n}{\arg \max} |P_{A}[n]|$ is taken as a coarse STO, and the CFO is estimated as

$$\hat{\varepsilon} = \text{angle} \{ P_{\mathcal{A}}[\hat{n}_{\mathcal{A}}] \} / \pi.$$
 (5.22)

Then, using $\hat{\varepsilon}$ to adjust the CFO in the received sequence $r'[n] = r[n]e^{-j2\pi(\hat{\varepsilon}/K)n}$, the recovered ICV is obtained by using equation (3.35) considering r'[n] as input. For instance, by defining an N-length buffer vector

$$\mathbf{r}'[n] = \left(r'[n - (N - 1)] \quad r'[n - (N - 2)] \quad \dots \quad r'[n]\right)^{\mathrm{T}}.$$
(5.23)

The ICV estimation can be obtained as

$$\hat{\mathbf{S}}_{\lceil \frac{M}{2}\rceil}[n] = \sqrt{K} \left(\mathbf{R}^{(M,K)} \right)^{\mathrm{T}} \operatorname{diag}(\boldsymbol{\gamma}) \left(\mathbf{P}^{\left(\lceil \frac{M}{2} \rceil \right)} \right)^{\mathrm{T}} \mathbf{r}[n]'.$$
(5.24)

This operation can be performed continuously using a parallelized demodulation block diagram, see Fig. 5.11. For every new time sample, the first block receives the buffered signal, performs the filtering operation regarding the m-th midamble subsymbol, and applies the M-fold summation as discussed in Section 3.2.3.



Figure 5.11: Proposed block diagram to recover the midamble information.

A similar operation would be performed to isolate $\hat{S}_{k,\lceil \frac{M}{2}\rceil+1}$, and the enhanced autocorrelation is finally achieved by

$$P'_{\mathcal{A}}[n] = \left(\hat{\mathbf{S}}_{\lceil \frac{M}{2} \rceil}[n]\right)^{\mathcal{H}} \hat{\mathbf{S}}_{\lceil \frac{M}{2} \rceil+1}[n].$$
(5.25)

Now, following the approach of [AKE08] introduced in Section 5.2.1, the enhanced autocorrelation metric is combined with correlation to achieve an optimized metric. Also, a fine search using a threshold criteria of [KE06] is applied to estimate the first multipath over a set of received samples r[n].

The cross-correlation operation is performed by (5.12), i.e., $P_{\rm C}[n] = \frac{1}{N} \sum_{k=0}^{N-1} r'[n+k] x_{\rm p}[k]$, where $x_{\rm p}[k]$ represents now the known training sequence embedded as a midamble into the GFDM block.

To suppress side peaks that arise from the partial alignment of the two segments, $P_{\rm C}[n]$ is multiplied with $P'_{\rm A}[n]$. The optimized estimate of the STO is obtained by searching the peak value in the range around the coarse STO estimation $[\hat{n}_{\rm c} - K, \hat{n}_{\rm c} + K]$ that is

$$\hat{n}_{\rm o} = \operatorname*{argmax}_{\sim} \left(|P'_{\rm A}[n]| \cdot |P_{\rm C}[n]| \right).$$
 (5.26)

A final search before \hat{n}_{o} is performed to find the primary peak \hat{n}_{f} according to (5.16) using the threshold criteria (5.15).

The metrics $P_{\rm A}[n]$, $P'_{\rm A}[n]$, and $P_{\rm C}[n]$, are illustrated in Fig. 5.12. The adopted pulse shape g[n] is a Dirichlet pulse [MMGF14] (see Appendix A). Note that the maximum value of $P_{\rm A}[n]$ is smaller than $P'_{\rm A}[n]$, this is because the latter metric actually considers the contribution of the recovered ICV obtained through the *M*-fold summation.



Figure 5.12: Midamble autocorrelation and correlation metric (ideal channel).

In Fig. 5.12, the side peaks of the correlation metric $P_{\rm C}[n]$ are aligned with low values of the autocorrelation metric. These side peaks could induce synchronization errors at low SNRs, but they practically disappear after being multiplied by the autocorrelation metric, as already shown in Fig. 5.8.

The performance of the midamble synchronization is plotted in Fig. 5.13. The simulation is performed using the same time-variant FSC adopted the isolated preamble in the previous subsection. The unknown STO and CFO are estimated considering a channel impulse response with 32 taps. The tap gains vary linearly in decibel scale from 0 to -13 dB. Each tap is multiplied by a Rayleigh distributed random variable with variance $\sigma_w^2 = 1/2$. The GFDM configuration using the midamble approach is shown in Table 5.2.

Parameter	Value
Transmit filter	Dirichlet
Number of subcarriers (K)	128
Number of subsymbols (M)	11
Number of data subsymbols (M)	9
Number of training subsymbols (M)	2
Guard interval (samples)	32
Probability of false alarm $(p_{\rm FA})$	10^{-6}
Multipath search parameter (λ)	16

 Table 5.2: GFDM parameters with embedded midamble



Figure 5.13: Mean error and MSE of the CFO and STO estimation with midamble. Dashed lines represents the CRLB [SC97].

The mean error and the MSE of CFO and STO estimations for the midamble using a Dirichlet pulse shape have been evaluated for 1,000 realizations at each value of SNR, with a given integer θ and a uniformly distributed $\varepsilon \in [-1, 1]$, and are presented in Fig. 5.13.

In Table 5.2, the PFA is chosen with a relative low value and this is not related to the detection of the overall GFDM signal, but only a criterion for the first multipath. The lower the $p_{\rm FA}$, the higher is the threshold, which implies that only relatively strong first echo will be considered. This observation leads to an interesting behavior where the STO residual error is typically non symmetric around zero, tending to be positive as observed in [GMMF14]. The impact of losing weak initial paths is not as severe as a potential ISI from an adjacent transmitted sequence, which might happen if a negative residual error is introduced.

Due to the overlap between the segments of the training sequence introduced by the time-variant FSC the CRLB in Fig. 5.13, computed according to [SC97], is not reached for both isolated and embedded synchronization approaches.

In the embedded midamble, an error floor is observed in the CFO estimation. This is caused by the additional data interference. It is at the order of magnitude of 10^{-2} to 10^{-3} , and this can be later compensated in the detection process [MMF15].

The MSE of the STO estimation at high SNR is very similar for both isolated and the embedded midamble approach, mainly influenced by the dominant correlation metric influence.

At low SNR the STO estimation is more sensible to the coarse metric performance and it can be seen that the use of the ICV approach enhances considerably its useful range, as shown in Fig. 5.12.

5.2.3 Pseudo-circular pre/post-amble synchronization

This section explores the use of the first subsymbol of the GFDM symbol as a pseudo circular preamble. As data and training sequence overlap in one GFDM symbol, an approach for isolating the preamble information from the data is presented. The concept allows for adaptation of state-of-the-art techniques developed for OFDM in order to estimate time offset at every GFDM symbol. The section studies the proposal in the context of vehicular communication scenarios and assesses the performance of single-shot data-aided estimation of time offset. Both, line-of-sight (LOS) and NLOS scenarios with doubly dispersive wireless channels are considered and compared with the CP-based method from the IEEE 802.11p standard.



Figure 5.14: GFDM terminology and pseudo-circular pre/post-amble (PCP) design.

Particularly in vehicular communication scenarios, two effects can cause CFO and significant STO variations: (i) Vehicles driving at high velocity create Doppler shifts of the signal; (ii) Signal reflections at the road surface and buildings cause multipath propagation. Although the average CFO can be obtained with the acquisition of an initial preamble or by using high-quality oscillators referenced to the global navigation satellite system (GNSS) clock, the combination of multipath effects and Doppler (doubly-dispersive channel) makes the STO estimation at the receiver a challenging task.

For vehicular communication, the IEEE 802.11p standard¹ operating in the 5.9 GHz frequency band has been adopted as part of the protocol stacks [Ken11, Fes14]. Being based on the 'a' version of IEEE 802.11, the 'p' standard defines a conventional preamble that is used for the estimation of CFO and STO at the beginning of the frame. With fast varying channel conditions, as it is common in vehicular communication scenarios, STO of the strongest multipath component changes during the reception of the frame and can considerably obscure the correct estimation of the first multipath. The preamble-based synchronization in combination with pilots and cyclic prefix does not allow easy tracking

 $[\]overline{\ }^{1}$ The p-amendment has been integrated into IEEE 802.11-2012. ITS-G5 is the European variant of IEEE 802.11p.

of the signal variations during the complete transmission of the frame, which degrades the SER performance.

A specific solution for the estimation CFO and STO for OFDM in vehicular communication has been proposed in [AHG11], which requires the complex use of reliable soft decoded data information as pilots. However, the iterative procedure of channel estimation introduces an additional processing delay. This approach is appropriate at the physical layer for the target latency in the tens of milliseconds order, but may not be acceptable for physical layer (PHY) frame processing with even faster timing requirements, e.g., in the order of milliseconds for pre-crash applications.

This section presents a synchronization scheme based on a pseudo-circular training sequence that combines training and pilot symbols together with payload symbols into a single block. The proposed scheme efficiently tracks the STO of the first multipath at every symbol over the complete transmission block and hence follows the fast varying channel conditions. The use of a PCP is a specific property of GFDM. In this section, a new parametrization is proposed to replace the traditional use of CP with known cyclic data-aided information, the PCP, which can potentially be used in the future to also estimate the channel impulse response.

The contributions of the section in this context are as follows: (i) Propose an alternative to the use of preamble and CP in OFDM, which allows for one-shot synchronization while reducing data overhead and out-of-band radiation;(ii) Evaluate the proposal in a vehicular communication scenario using different channel models as examples.

The main requirement for STO synchronization in the single-shot burst is then to search for the first multipath component of the signal at every GFDM symbol. More specifically, this section will evaluate the performance of the proposed PCP in LOS and NLOS scenarios for two vehicles approaching an urban intersection or on a highway. The scenarios consider signal blocking from other road traffic as well as reflections at buildings and fences [ETS14].

The CFO estimation problem can be reduced to Doppler effects – hence, mitigating other effects – if CFO is minimized by the use of stable external clock references, on both the transmitter and receiver sides. Alternatively, CFO accuracy can be achieved at the beginning of the frame by the use of a dedicated first GFDM symbol as known information, as discussed in Section 5.2.1.

The PCP is designed with the insertion of known pilots c[k] in the first subsymbol, i.e., $d_{k,0} = c[k]$, and is denoted as

$$\mathbf{x}_{\mathrm{p}} = \sqrt{K} \mathrm{diag}(\boldsymbol{g}) \mathbf{R}^{(M,K)} \mathbf{S}_{0}, \qquad (5.27)$$

where $\mathbf{S}_0[n]$ is the ICV at the first subsymbol, as defined in (5.19).

Using a similar approach proposed for the midamble (Section 5.2.2), an estimator for the received ICV, $\hat{\mathbf{S}}_0[n]$, is used to revert the ISI introduced by pulse shaping. The block

diagram in Fig. 5.11 can be used to perform this operation by setting m = 0, i.e.,

$$\hat{\mathbf{S}}_{0}[n] = \sqrt{K} \left(\mathbf{R}^{(M,K)} \right)^{\mathrm{T}} \operatorname{diag}(\boldsymbol{\gamma}) \left(\mathbf{P}^{(0)} \right)^{\mathrm{T}} \mathbf{r}[n]'.$$
(5.28)

Under this consideration, a commonly accepted metric that can reveal the STO of the strongest multipath is the correlation of the received signal \hat{S}_0 with the known transmitted information coefficient vector \hat{S}_0 , [Mas72], given by

$$P_{\rm C}[n] = \left(\hat{\mathbf{S}}_0[n]\right)^{\rm H} \mathbf{S}_0[n], \qquad (5.29)$$

with

$$\hat{n}_{\rm o} = \underset{n}{\operatorname{argmax}} |P_{\rm C}[n]|. \tag{5.30}$$

Again, a final search before \hat{n}_{o} is performed to find the primary peak \hat{n}_{f} according to (5.16) using the threshold criteria (5.15).

The performance of the estimation of STO using the proposed PCP is evaluated in the wireless channel models described by Tables C.1 – 5.5 [ETS14]. In the LOS case, Table C.1, the first multipath is a dominant component that follows a Rician distribution with a K-factor equal 10. In the NLOS cases, Tables 5.4 and 5.5, each tap is multiplied by a Rayleigh distributed random variable with parameter $\sigma_w = 1/\sqrt{2}$. In the simulation, the channel taps are normalized for a unitary gain.

Table 5.3: Urban Approach LOS, 119 km/h differential speed

Delay [ns]	0	117	183	333
Power [dB]	0	-8	-10	-15
Doppler[Hz]	0	236	-157	492

Table 5.4: Street Crossing NLOS, 126 km/h differential speed

Delay [ns]	0	267	400	533
Power [dB]	0	-3	-5	-10
Doppler[Hz]	0	295	-98	591

Table 5.5: Highway NLOS, 252 km/h differential speed

Delay [ns]	0	200	433	700
Power [dB]	0	-2	-5	-7
Doppler[Hz]	0	689	-492	886

The GFDM signal is configured according to the parameters from IEEE 802.11p standard, with a bandwidth of 10 MHz per channel and an overall symbol duration of $8\mu s$. Compared to OFDM, the CP interval is replaced by the PCP. With K = 16 subcarriers, M = 5 subsymbols and using a Dirichlet pulse shape [MMGF14] (see Appendix A), the PCP creates a common guard signal at the boundaries of the GFDM symbol. This particular property allows for a smooth concatenation of symbols, drastically reducing the out of band radiation, as shown in Fig. 5.15.



Figure 5.15: GFDM signal with PCP in the time and frequency domains. Despite extremely short GFDM symbols, the OOB is improved by 20 dB in comparison with OFDM.

The detection rate and the MSE of STO estimations have been evaluated for 1,000 realizations at different SNR values. The conventional OFDM case, as used in IEEE 802.11p employing CPs [vdBSB97], has been added as reference. The detection

rate is defined as the proportion of STO estimations obtained within the range of the channel impulse response (CIR) length.

On the top part of Fig. 5.16, the detection rates of GFDM and OFDM are compared over a SNR range from 0 to 15 dB. As can be seen on the bottom part of Fig. 5.16, the MSE of the STO estimation is within tenths of a sample for the urban approach channel with LOS communication, outperforming the reference case over the entire SNR range. In the two NLOS scenarios, it can be observed that the MSE of the STO increases considerably compared to the LOS scenario, but is still within the amount of a few samples and outperforms the reference case at low SNR.

The correlation approach of PCP for single-shot synchronization shows a considerably better performance for LOS conditions compared to the CP-based auto-correlation in OFDM. Under NLOS conditions, we observe benefits of PCP for low SNR values. For safety applications, a better synchronization performance effectively enables an extended communication range in both LOS and NLOS conditions.



Figure 5.16: Detection rate and MSE of the STO estimation with PCP.

It is worth noting that there exists also a proposal to replace CP in OFDM, which is termed unique word (UW)-OFDM [HOH11, HHH12]. In this solution, a group of redundant pilots is modulated such that a deterministic sequence is obtained in the end of the OFDM symbol. A comparison between the proposed PCP-GFDM and UW-OFDM is an open point that should be addressed in future works.

5.3 Summary

In this chapter, data aided synchronization approaches were proposed for GFDM. Their performances were evaluated with respect to different types of training sequences, including isolated preamble, embedded preamble and pseudo-circular pre/post-amble (PCP). In particular, the performance of PCP based approach was compared with that achieved by the sole use of CP in vehicular communications scenarios with and without line-of-sight. Furthermore, the design of training sequences targets at lower OOB for a more efficient use of the spectrum.

The main contributions of this chapter have been published in [GMMF14, GFF15, GF15]. Details of the preamble subsection can be summarized as follows

- ▷ Aiming at low OOB emission, an isolated windowed preamble has been proposed for GFDM synchronization. The proposed windowing function combines the classic Tukey window with the Meyer auxiliary function. This window is able to provide a considerable reduction in the spectrum side-lobes, several orders of magnitude below what can be achieved with a standard preamble designed for OFDM, preserving the advantageous spectral properties of the GFDM transmission.
- ▷ As the CP is shaped by windowing, the plateau effect commonly observed in the OFDM preamble no longer exists. Therefore, the autocorrelation metric is simplified once it does not need additional integration, as performed in [MZB00, AKE08] to solve the plateau ambiguity.
- ▷ As an extension to previous works on the original non-windowed OFDM preamble, the mean value of the residual time offset has been investigated. The results show that the error is always positive. This implies that ISI from the previous symbol is avoided and the error of estimating the block start should be within the CP. Notably, when operating at low SNRs, this leaves the potential to improve the estimation quality with a feedback loop.
- ▷ The overall performance of the system using the low OOB windowed preamble scheme is simulated in terms of SER vs. SNR in a time-invariant FSC. As an overall result, the proposed scheme exhibited no degradation in performance when compared to the standard method used in OFDM, yet keeping spectrum side-lobes several decibels below the one presented by the non-windowed preamble.

Details of the midamble subsection can be summarized as follows

- ▷ An embedded midamble has been proposed as a synchronization approach in order to obtain a self-contained GFDM subsymbol that can use silent subsymbols to explore advantageous reduction of spectral side lobes.
- ▷ Pulse shape effects have been considered while adapting classic OFDM synchronization algorithms to track the midamble.
- ▷ Using a receiver model based on element-wise time domain multiplication, a block diagram that can run continuously has been proposed to track the subsymbols containing the training sequence.
- ▷ An improved coarse synchronization metric, based on the pulse shape properties of GFDM, has been presented, which is able to improve the performance of the scheme on low SNR conditions and present a comparable performance to the case using isolated preamble.

Details of the PCP subsection can be summarized as follows

- ▷ PCP is proposed as an alternative to the use of preamble and CP in OFDM 802.11p, which allows for one-shot synchronization while reducing data overhead and out-of-band radiation.
- ▷ The pseudo circular preamble provides smoother concatenation of blocks and achieve 20 dB better OOB than the OFDM case for 802.11p case - which is particularly challenging once the number of employed subcarrier is very small. These results can be further improved if the number of subcarriers is increased, i.e., using longer GFDM blocks.
- ▷ An improved coarse synchronization metric, based on the pulse shape properties of GFDM, has been presented, which is able to improve the performance of the scheme under low SNR conditions and present a comparable performance to the case of using isolated preamble.
- ▷ Simulations were carried out considering vehicular communication scenarios using different channel models as corner cases. The performance improvements compared to the OFDM case with CPs enables GFDM to effectively extend the communication range between two approaching vehicles.

Chapter 6

Conclusions and Future Works

In this thesis waveform advancements and synchronization techniques have been presented for generalized frequency division multiplexing (GFDM). GFDM properties, both time and frequency domains, have been extensively explored to bring new perspectives in the use of offset quadrature amplitude modulation (OQAM) schemes, low complexity signal processing chains, as well as to design proper non-data and data aided synchronization approaches.

The main contributions of this thesis have been published in [GNN⁺13, GNMF13, MMG⁺14, GMM⁺15a, GMM⁺15b, GMM⁺15c, GMMF14, KGMF14, GFF15, GF15]. A complete list of other relevant publications can be found at the end of the thesis.

Details can be summarized as follows:

The advancements on GFDM

In Chapter 2, several waveform characteristics have been discussed, such as orthogonal frequency division multiplexing (OFDM), single-carrier with frequency domain equalization (SC-FDE), filter bank multicarrier (FBMC), faster than Nyquist signaling (FTN), universally filtered multicarrier (UFMC), and GFDM. In Chapter 3, advancements have been proposed for GFDM, culminating into a waveform framework design. Backward compatibilities and the exploitation of new degrees of freedom to address future wireless systems have been highlighted.

▷ The investigations started with the development of a new grid approach for the GFDM signal generation with a double transceiver model to support OQAM. A compact matrix model based on the modulation matrix has been used to represent the traditional use of time-shift OQAM. With the introduction of an original idea of using unitary transformation, the concept has been extended to a frequency-shift OQAM version. A Fourier transform is used to design a reverse pulse shape with

short duration in the time domain.

- ▷ A new pulse shape design, termed Meyer raised cosine (RC), has been introduced to achieve a half-Nyquist filter with smooth transitions in the time domain. With a filter span of only 2 subsymbols, an out-of-band (OOB) emission of -40dBc and -60dBc is achieved at the 2nd and 3rd neighbor subcarriers. When considering short bursts and a large number of subcarriers, the proposed frequency-shift OQAM is more localized in time and frequency than time-shift OQAM. Simulations have been performed to show that there is no penalty in performance on additive white Gaussian noise (AWGN) or frequency selective channel (FSC) channels.
- \triangleright A new signal processing chain for GFDM has been proposed based on convolution in the frequency domain. On the transmitter side, the approach allows for KMelement-wise multiplication of M-repeated blocks of processed subcarriers in the time domain, which is equivalent to zero stuff interpolation (upsampling) in the frequency domain. On the receiver side, M-fold accumulation in the time domain performs an equivalent decimation (downsampling) of the GFDM spectrum.
- ▷ The new GFDM signal processing models in the frequency and time domains allow for a later introduction of an inter-carrier interference (ICI) based on non-data aided synchronization and also on an information coefficient vector (ICV) concept for data aided synchronization. Frequency domain processing can be used to estimate large carrier frequency offset (CFO), while the ICV can be used to support fine symbol time offset (STO) synchronization.
- ▷ A framework has been proposed using the advanced GFDM as a baseline to reproduce not only classic OFDM and SC-FDE waveforms as corner cases, but several other prominent candidates proposed as new 5G modulation formats.

GFDM Synchronization

In Chapter 4 and Chapter 5, the problem of synchronization has been formalized. Relevant state-of-the-art solutions designed for OFDM have provided the insights to develop appropriated solutions for GFDM. The relationship between misalignments in time and frequency and the different training sequence design possibilities have been covered.

▷ A matrix model has been used to describe the unknown STO and CFO of the received GFDM signal. With this matrix model, a metric for self-interference analysis has been proposed as a tool to evaluate the impact of time and frequency misalignments. The matrix model can be parameterized to reproduce known results in literature, e.g., signal-to-noise ratio (SNR) degradation in OFDM. The matrix

model can also be used to evaluate the impact of the pulse shape design in GFDM when no misalignment is present. Moreover, by discarding proper rows and columns, the concept can be adapted to consider the use of guard subsymbols and subcarriers in GFDM and other waveforms derived from it.

- ▷ The well-established time correlation approach used in repetitive patterns in OFDM has been revisited. The particular case when GFDM presents a large ICI has been exploited as a non-data aided source of information for coarse CFO estimation with a wide frequency range. It has been shown that correlation can be performed in the frequency domain to highlight the repetitive aspects of the subcarriers' spectrum.
- ▷ As data aided solutions, the use of an isolated preamble with good spectral properties has been investigated for the GFDM waveform. Results comparable to state-of-the-art are achieved when a block composed of a repetitive pattern is shaped by a proper window. The windowed preamble and data form a double pinching pattern transmission signal and BER simulations endorse the approach.
- ▷ As an alternative to the isolated preamble, an embedded solution using a midamble has been proposed. In this case, the repetitive pattern is designed using the subsymbols in the center of the GFDM block, while the corner subsymbols can be disabled to produce low OOB. In this case a single block can be transmitted with all the required information for proper receiver operation.
- ▷ A new concept of a pseudo pre/post-amble can be used when several blocks are to be transmitted in a frame. The scenarios of car-to-car communication have been used, considering challenging double dispersive channel models, representing intersections in both line and non-line of sight. Performance results show that better time synchronization can be achieved at lower SNR conditions. This result can increase the prediction and reaction range in car safety applications.
- ▷ When searching for the embedded training sequence, the idea of an ICV is proposed to avoid the strict block alignment required in discrete Fourier transform (DFT) based operations. This idea allows the use of parallelized processes operating in continuous mode to improve the synchronization performance under low SNR conditions.

Future Work

To strengthen the use of GFDM as a future wireless waveform, the natural continuation of the research effort is necessary. With respect to the problems treated in this thesis, the following aspects are particularly interesting:

- ▷ The advancements in GFDM have been evaluated with the help of uncoded linear receivers. The use of coded non-linear receivers is an appealing topic and must be investigated. Although coded non-linear iterative receivers usually require additional complexity and a processing delay, large gains in symbol error rate (SER) performance can be achieved from the combination of estimation of likelihood information and detection approaches. The comparisons via information rates rather than SERs would also provide more insight into the GFDM performance.
- ▷ A more extensive comparison considering modulation and coding targeting diverse QoS requirements, such as rate, spectral efficiency, reliability, with major parameters and scenarios, still are subject of investigation – and classic bandpass filtering method can still be applied on top of it.
- ▷ To pave the way to a practical implementation, not only synchronization must be achieved, but also a method for channel estimation needs to be developed. In this context, pilot and frame design pose important research questions. While there is a large number of techniques available for OFDM, the self-interference in GFDM can be a challenge.
- ▷ Analyzing the distribution of the OQAM symbols can also be a source of a new sharp metric for synchronization. Additionally, the specific use of guard symbols in the frequency-shift QAM can also provide an interesting and effective approach to isolate data and training sequences. Open questions regarding an efficient channel estimation and implementation in the context of OQAM remain.
- ▷ A comparison between the proposed pseudo-circular pre/post-amble (PCP)-GFDM and unique word (UW)-OFDM is an open point that would lead to interesting studies in future. The two techniques can be evaluated in terms of spectrum efficiency, OOB, complexity and performance under time varying FSC.
- ▷ This thesis has focused on SISO systems, whereas multiple input multiple output (MIMO) is a relevant technique that uses beamforming and diversity techniques to improve the system in terms of reliability, data rates, and multiple access capabilities. A solution considering the various forms of MIMO techniques constitutes another very promising topic.

Appendix A

Filter Functions

This appendix presents supplementary information for the Chapter 2, describing pulse shape prototypes that can be used in filtered multicarrier systems.

The description of the filter functions are given in the continuous domain. The parameter T corresponds to the duration of one period, M is the number of periods, m is the period index, f is the frequency index, and t is the time index, and $sinc(x) = \frac{sin(\pi x)}{\pi x}$.

The critically sampled discrete model assumed in this thesis is obtained by sampling the interval, for instance T = K, and by replacing t and f by corresponding discrete sample indexes.

Rectangular

$$G_{\rm rect}(f) = \begin{cases} 1 & \text{for } f = 0\\ \operatorname{sinc}(f) & \text{otherwise} \end{cases}$$
(A.1)

$$g_{\rm rect}(t) = \begin{cases} 1 & \text{for } |t| < \frac{T}{2} \\ \frac{1}{2} & \text{for } |t| = \frac{T}{2} \\ 0 & |t| > \frac{T}{2} \end{cases}$$
(A.2)

Note that, by inverting the domains of the above definitions, i.e., if the rectangular pulse is defined in the frequency domain, the Dirichlet pulse is obtained in the time domain. In the discrete case, the Dirichlet pulse is defined by a perfect rectangular function in the frequency domain with the width of M frequency bins that are located around the DC bin, so that the time domain response is a Dirichlet kernel [BBT97, MMGF14].

Near perfect reconstruction (NPR)

 Table A.1: NPR filter coefficients in the frequency domain

M	G_1	G_2	G_3	background noise
2	$\frac{\sqrt{2}}{2}$			$<-35~\mathrm{dB}$
3	0.911438	0.411438		<-44 dB
4	0.971960	$0.\frac{\sqrt{2}}{2}$	0.235147	<-65 dB
4	0.971900 g _{NP} 1	$\frac{0.\frac{1}{2}}{1}$	$\frac{0.233147}{2\sum^{M-1}G_m \cos^2 \theta}$	$\sim -05 \text{ dB}$

Raised Cosine (RC)

$$G_{\rm rc}(f) = \begin{cases} T, & \text{for } |f| \leq \frac{1-\alpha}{2T} \\ \frac{T}{2} \left[1 + \cos\left(\frac{\pi T}{\alpha} \left(|f| - \frac{1-\alpha}{2T}\right)\right) \right], & \text{for } \frac{1-\alpha}{2T} < |f| \leq \frac{1+\alpha}{2T} \end{cases}$$
(A.4)
$$0, & \text{otherwise} \end{cases}$$
$$g_{\rm rc}(t) = \begin{cases} 1, & \text{for } t = 0 \\ \sinc\left(\frac{1}{2\alpha}\right), & \text{for } |t| = \frac{T}{2\alpha} \\ \sinc\left(\frac{t}{T}\right) \cdot \frac{\cos(\alpha \pi t/T)}{1-4\alpha^2 t^2/T^2}, & \text{otherwise} \end{cases}$$
(A.5)

Root Raised Cosine (RRC)

$$G_{\rm rrc}(f) = \sqrt{|G_{\rm rc}(f)|} \tag{A.6}$$

$$g_{\rm rrc}(t) = \begin{cases} \frac{1}{\sqrt{T}} \left(1 - \alpha + 4\frac{\alpha}{\pi}\right), & \text{for } t = 0\\ \frac{\alpha}{\sqrt{2T}} \left[\left(1 + \frac{2}{\pi}\right) \cdot \sin\left(\frac{\pi}{4\alpha}\right) + \left(1 - \frac{2}{\pi}\right) \cos\left(\frac{\pi}{4\alpha}\right) \right], & \text{for } |t| = \frac{T}{4\alpha} \\ \frac{\sin\left(\pi\frac{t}{T}\left(1 - \alpha\right)\right) + 4\alpha\frac{t}{T}\cos\left(\pi\frac{t}{T}\left(1 + \alpha\right)\right)}{\pi\frac{t}{\sqrt{T}}\left(1 - \left(4\alpha\frac{t}{T}\right)^2\right)}, & \text{otherwise} \end{cases}$$
(A.7)

Appendix B

GFDM Basic Experiments Using LabView

In order to demonstrate and validate basic properties of the GFDM waveform in Chapter 2, selected features were implemented in an interactive graphical user interface (GUI) using the software LabView. The configuration is limited to a fixed number of 32 active subcarriers, each one with 16 subsymbols, which are pulse shaped with a RC filter.

The experiment can be summarized with the figures below:



Figure B.1: In the 'GFDM spec' tab, the allocated subcarriers of GFDM can be enabled/disabled, updating the exhibited spectrum and CCDF plots.




Figure B.2: Other tabs exhibit (a) filter construction, (b) subcarrier construction, (c) GFDM vs OFDM spectrum, (d) GFDM cyclic autocorrelation in the frequency domain.

The source files and more detailed description are available at [Gas15a]. These public domain files can be used to verify the results of this thesis and motivate further studies.

Appendix C

GFDM PSD Experiments Using LabView/Mathscript

With the combination of LabView and Mathscript, a more advanced experiment has been prepared to validate the GFDM power spectral density (PSD) properties. By including the GFDM advancements addressed in Chapter 3, the experiment presents a rich waveform engineering, allowing the configuration of several degrees of freedom. The range is limited to a maximum number of 256 subcarriers, each one with a maximum of 64 subsymbols. A large set of impulse responses and windowing are available.

The experiment can be summarized with the figures below:



Figure C.1: In the 'PSD' tab, the parameters of GFDM can be tuned to produce diverse configurations.

Κ	Number of samples per sub-symbol
Kon	Number allocated subcarriers (disable corner subcarriers)
Rnd	Produce null subcarriers randomly (non-continuous spectrum)
Sngl	Enable a single subcarrier only
Full	Enable all active subcarriers
Impulse response	Pulse shaping filter
a	Rolloff of the pulse shaping filter
oQAM	offset QAM Modulation (apply time-shift or frequency-shift)
М	Number of sub-symbols
Mon	Number of active sub-symbols (disable corner subsymbols)
Ncp	Number of cyclic prefix samples
windowing	Window that multiplies the GFDM block
b	Window factor (rolloff of the window in samples)

Table C.1: GFDM parameters legend (as used in the experiment)











Figure C.2: The many configurations allow for instance: (a) subcarrier pulse shaping analysis, (b) non-continuous spectrum construction, (c) impact of individual subsymbols impulse response, (d) Diverse pulse shaping designs, (e) classic OFDM, and (f) general windowing design.

The source files and more detailed description are available at [Gas15b]. These public domain files can be used to verify the results of this thesis and motivate further studies.

Appendix D

Fundamentals of Estimation Theory

In Chapter 4 and Chapter 5, synchronization can be seen as an estimation problem of the time offset θ and the frequency offset $f_{\epsilon} = \frac{\Delta f}{f_{sc}N} = \frac{\epsilon}{N}$, where $\epsilon = \frac{\Delta f}{f_{sc}}$ denotes the normed frequency offset. Appropriate estimators have to be applied and the performance criterion for the estimators will be defined in the following.

Parameter estimation in signal processing

For following considerations, the unknown, estimated value is considered to be deterministic.

For explanation, a data set is considered, which can be described with

$$x[n] = \gamma + w[n] \tag{D.1}$$

with x[n] being the received, observed data. γ denotes a constant variable, which has to be estimated and w[n] is a zero mean Gaussian noise process with n = 0...N - 1. If the expectation value of an estimator equals the value of the estimated parameter

$$\mathcal{E}\{\gamma - \hat{\gamma}\} = 0 \quad \forall \gamma \tag{D.2}$$

it is called *unbiased*.

Minimum variance unbiased estimator

The existence of several intuitive unbiased estimators for γ as, for example,

$$\hat{\gamma} = \frac{1}{N} \sum_{n=0}^{N-1} x[n]$$
 (D.3)

$$\mathcal{E}\{\hat{\gamma}\} = \frac{1}{N} \mathcal{E}\left\{\sum_{n=0}^{N-1} x[n]\right\} = \frac{1}{N} \sum_{n=0}^{N-1} \mathcal{E}\{x[n]\} = \gamma$$
(D.4)

109

and

$$\hat{\gamma} = x[0] \tag{D.5}$$

$$\mathcal{E}\{\hat{\gamma}\} = \mathcal{E}\{x[0]\} = \mathcal{E}\{\gamma + w[n]\} = \gamma, \tag{D.6}$$

motivates the question for an adequate performance criterion beyond (D.2). It is found as the variance, which denotes the amount of spreading of the observed values around the mean. Following the examples, (D.3) and (D.5), the variance makes a big difference in the proposed estimators. For the estimator described with eq. (D.3), the variance

$$\operatorname{var}\left\{\frac{1}{N}\sum_{n=0}^{N-1}x[n]\right\} = \frac{1}{N^2}\sum_{n=0}^{N-1}\operatorname{var}\left\{x[n]\right\}$$
(D.7)

$$= \frac{\sigma^2}{N} \tag{D.8}$$

is smaller than the variance of the second estimator (eq.(D.5))

$$\operatorname{var} \{x[0]\} = \operatorname{var} \{x[0]\}$$
 (D.9)

$$= \sigma^2 \qquad . \tag{D.10}$$

While the unbiased property allows the easy combination of N independent estimators to

$$\hat{\gamma} = \frac{1}{N} \sum_{i=1}^{N} \hat{\gamma}_i, \qquad (D.11)$$

with

$$\mathcal{E}\{\hat{\gamma}\} = \gamma \tag{D.12}$$

$$\operatorname{var}\{\hat{\gamma}\} = \frac{1}{N^2} \sum_{i=1}^{N} \operatorname{var}\{\gamma_i\},$$
 (D.13)

the minimum variance defines the 'closeness' to the expectation value and the performance of the estimator. An estimator with these properties is called minimum variance unbiased estimator (MVUE).

Because the variance can be derived only analytically, a more practical definition of the performance has to be found for performance evaluation with simulations. This is given by the mean squared error (MSE)

$$\operatorname{mse}\left\{\hat{\gamma}\right\} = \mathcal{E}\left\{\left(\hat{\gamma} - \gamma\right)^{2}\right\}$$
(D.14)

$$= \mathcal{E}\left\{ \left[\left(\hat{\gamma} - \mathcal{E}\{\hat{\gamma}\} \right) + \left(\mathcal{E}\{\hat{\gamma}\} - \gamma \right) \right]^2 \right\}$$
(D.15)

$$= \operatorname{var}\left\{\hat{\gamma}\right\} + b^2(\hat{\gamma}) \tag{D.16}$$

[Kay93, p.19], which is equal to the variance in the case of an unbiased estimator with $\hat{\gamma} = 0$.

Because the MVUE does not exist in every case ([Kay93, p.20]), the challenge is to find the estimator with the best *possible* performance. A lower bound is given by the Cramér-Rao lower bound (CRLB), which defines the minimum achievable variance for an estimator. If an estimator has the same value of the variance as the CRLB for every value of γ , this estimator is the MVUE and is called *efficient*. Even if no estimator reaches this variance, nevertheless a MVUE may exist, which means that an existing MVUE does not necessarily has to be *efficient*.

Cramér-Rao lower bound

Based on the statements above, the CRLB is an important parameter and can be seen as a benchmark against which the variance of any derived unbiased estimator can be compared.

For example, if the probability density function (pdf) of the obtained signal x is derived as Gaussian, where its mean value is dependent of the parameter to estimate, γ , the problem from eq. (D.1) can be formulated as [Kay93, p.28]

$$p(x[n]|\gamma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\left(-\frac{1}{2\sigma^2}(x[n]-\gamma)^2\right)\right),\tag{D.17}$$

which can also be generally described with $p(\mathbf{x}|\gamma)$. By intending the parameter γ to be the unknown parameter, this function is also called *likelihood function*. The 'sharpness' of this likelihood function is one important measure to determine how accurately the parameter can be estimated. No information will be lost if the likelihood function is logarithmized. For exponential-like pdf, the log-likelihood analysis is more mathematically tractable.

The *Fisher information* gives a measurement for the 'sharpness' by deriving the curvature of the log-likelihood function [Kay93, p.34]

$$I_{Fisher}(\gamma) = -\mathcal{E}\left\{\frac{\partial^2 \ln p(\mathbf{x}|\gamma)}{\partial \gamma^2}\right\} .$$
(D.18)

Based on this, the CRLB is defined as [Kay93, p.30]

$$\operatorname{var}\left\{\hat{\gamma}\right\} > \frac{1}{I_{Fisher}(\gamma)} = \frac{1}{-\mathcal{E}\left\{\frac{\partial^2 \ln p(\mathbf{x}|\gamma)}{\partial \gamma^2}\right\}} . \tag{D.19}$$

If the pdf satisfies the regularity condition

$$\mathcal{E}\left\{\frac{\partial p(\mathbf{x}|\gamma)}{\partial \gamma}\right\} = 0 \quad \forall \gamma , \qquad (D.20)$$

the MVUE is efficient and reaches the CRLB and in eq. (D.19) the 'greater', >, becomes a 'greater-equal', \geq .

Maximum likelihood estimator

If the MVUE is not existing , cannot be found, or is too complex to derive or implement, the alternative is the use of the maximum likelihood estimator (MLE). The MLE estimator is a good choice because it is asymptotically optimal. That is, the MLE approximates the MVUE for large data sets ([Kay93, p.157 and p.183]) and it is easier to implement.

The MLE maximizes the *likelihood function* $p(\mathbf{x}|\gamma)$ to find a good estimation of γ and is defined as

$$\hat{\gamma} = \operatorname*{arg\,max}_{\gamma} p(\mathbf{x}|\gamma) \;. \tag{D.21}$$

An analytical expression of the pdf of the MLE is very hard in general ([Kay93, p.164]). Therefore, computer simulations are an appropriate instrument to measure the performance of this estimator.

The MLE can be extended to a joint estimator for different parameters as

$$\vec{\hat{\gamma}} = \underset{\vec{\gamma}}{\arg\max} \ln p(\mathbf{x}|\gamma) \quad ,$$
 (D.22)

which also has the same asymptotically property as in the scalar case.

If the first and second order derivatives of (D.22) are well defined and (D.20) is fulfilled, the MLE is asymptotically efficient and has a Gaussian pdf. That is,

$$\gamma \stackrel{a}{\sim} \mathcal{N}\left(\vec{\gamma}, \mathbf{I}_{Fisher}^{-1}(\gamma)\right),$$
 (D.23)

where $\mathbf{I}_{Fisher}(\gamma)$ denotes the Fisher information matrix evaluated at the true value of γ ([Kay93, p.183]).

Acronyms

2G second generation **3GPP** Third Generation Partnership Project 4G fourth generation **5G** fifth generation A/D analog-to-digital **AWGN** additive white Gaussian noise Car-2-x car-to-car and car-to-infrastructure communication **CB-FMT** cyclic block filtered multitone **CDF** cumulative density function **CFO** carrier frequency offset **CIR** channel impulse response COQAM cyclic OQAM **CP** cyclic prefix **CR** cognitive radio **CRLB** Cramér-Rao lower bound D/A digital-to-analog **DFT** discrete Fourier transform **DVB-T** terrestrial digital video broadcasting **FBMC** filter bank multicarrier FDE frequency domain equalization **FDM** frequency division multiplex **FFT** fast Fourier transform **FMT** filtered multi tone **FSC** frequency selective channel FS-OQAM-GFDM frequency-shift OQAM-GFDM **FTD** fractional time delay **FTN** faster than Nyquist signaling GFDM generalized frequency division multiplexing **GNSS** global navigation satellite system **GS** guard symbols

GUI graphical user interface **ICI** inter-carrier interference **ICV** information coefficient vector **IDFT** inverse discrete Fourier transform **IF** intermediate frequency **IoT** Internet of Things **IOTA** isotropic orthogonal transform algorithm **ISDB-T** terrestrial integrated services digital broadcasting **ISI** inter-symbol interference LOS line-of-sight LTE Long Term Evolution MF matched filter **MIMO** multiple input multiple output MLE maximum likelihood estimator **MMSE** minimum mean squared error **MS** mobile stations MSE mean squared error **MVUE** minimum variance unbiased estimator **NEF** noise enhancement factor NLOS non-line-of-sight **NPR** near-perfect reconstruction **OFDM** orthogonal frequency division multiplexing **OFDMA** orthogonal frequency division multiple access **OOB** out-of-band **OQAM** offset quadrature amplitude modulation **PAPR** peak-to-average power ratio **PCP** pseudo-circular pre/post-amble **PD** probability of detection **pdf** probability density function **PDP** power delay profile **PFA** probability of false alarm **PHY** physical layer **PN** pseudo noise **ppm** parts per million **PSD** power spectral density **QAM** quadrature amplitude modulation QoS Quality of Service RC raised cosine

RF radio frequency **RRC** root raised cosine SC-FDE single-carrier with frequency domain equalization SC-FDM single-carrier frequency division multiplexing SC-FDMA single-carrier frequency division multiple access **SDW** software-defined waveform **SEFDM** spectrally efficient frequency division multiplexing **SER** symbol error rate **SISO** single-input and single-output **SNR** signal-to-noise ratio E-SNR effective signal-to-noise ratio ${\bf STFT}\,$ short-time Fourier transform **STO** symbol time offset **TFL** time-frequency localization TS-OQAM-GFDM time-shifted OQAM-GFDM UFMC universally filtered multicarrier UW unique word

ZF zero-forcing

 ${\bf ZP}\,$ zero-padding

List of Figures

1.1	5G scenarios [G. 14]. \ldots	1
2.1	Example of frequency division multiplex (FDM) transmissions	6
2.2	Rectangular pulse shaped subcarriers	8
2.3	OFDM symbol in the time and frequency domains	10
2.4	FBMC symbol in the time and frequency domains	14
2.5	UFMC sub-band symbol in the time and frequency domains	16
2.6	GFDM terminology [GMM ⁺ 15c]	18
2.7	GFDM symbol in the time and frequency domains	20
2.8	Reciprocal condition number for GFDM matrix	22
3.1	GFDM eye pattern	26
3.2	TS-OQAM-GFDM real-orthogonality	28
3.3	GFDM time-shifted OQAM-GFDM (TS-OQAM-GFDM) eye pattern	29
3.4	Block structure of the time-shift or frequency-shift OQAM-GFDM	32
3.5	Nyquist and half-Nyquist pulse design in the time and frequency domains .	33
3.6	GFDM OQAM time signal	34
3.7	Meyer RRC pulse with different roll-offs	35
3.8	FS-OQAM-GFDM vs TS-OQAM-GFDM in time and frequency	36
3.9	GFDM OQAM spectrum	37
3.10	GFDM terminology and signal processing chains	40
3.11	GFDM matrix modulator, (3.39), with frequency domain processing	43
3.12	GFDM matrix demodulator, (3.43), with frequency domain processing	44
3.13	GFDM matrix modulator, (5.18), with time domain processing	44
3.14	GFDM matrix demodulator, (3.45), with time domain processing	45

3.15	GFDM implementation complexity	47
3.16	Influence of distance factors ν_t and ν_f in time and frequency	49
4.1	GFDM synchronization system model	56
4.2	Realization of a time-varying FSC	57
4.3	Illustration of (a) time and (b) frequency offsets in a received signal	58
4.4	Self-interference impact of the pulse-shape	60
4.5	SNR degradation versus relative frequency offset	62
4.6	Multiplication of prototype filter with data	66
4.8	CFO MSE in AWGN and multipath channel	69
5.1	cyclic prefix (CP) influence over metrics	72
5.2	Windowed preamble preceding a GFDM data block	74
5.3	Embedded midamble in a GFDM data block.	74
5.4	Pseudo-circular pre/post-amble PCP in a GFDM data block	75
5.5	Proposed preamble	76
5.6	Metric realization for a perfect channel	77
5.7	Mean error and MSE of the (a) CFO and (b) STO	80
5.8	Analysis of the metrics in a multipath channel	81
5.9	SER analysis over time-invariant FSC	82
5.10	GFDM terminology and midamble design.	83
5.11	Proposed block diagram to recover the midamble information	85
5.12	Midamble autocorrelation and correlation metric.	86
5.13	Mean error and MSE of the CFO and STO	87
5.14	GFDM terminology and PCP design	89
5.15	GFDM signal with PCP in the time and frequency domains	92
5.16	Detection rate and MSE of the STO estimation with PCP	93
B.1	GFDM spectrum simulation using LabView (main interface) 1	103
B.2	GFDM spectrum simulation using LabView (other interfaces) $\ldots \ldots \ldots$	104
C.1	GFDM PSD simulation using LabView (main interface)	105
C.2	GFDM PSD simulation using LabView (configurations)	107

List of Tables

3.1	Number of multiplications of modulation models.	46
3.2	Terminology for the GFDM framework	48
3.3	GFDM as a framework of 5G waveform candidates	51
4.1	Used parameter sets for ICI based frequency estimation experiment $\ . \ . \ .$	68
5.1	Configuration of the GFDM data block using isolated preamble	79
5.2	GFDM parameters with embedded midamble	86
5.3	Urban Approach line-of-sight (LOS), 119 km/h differential speed	91
5.4	Street Crossing non-line-of-sight (NLOS), 126 km/h differential speed $\ .$.	91
5.5	Highway NLOS, 252 km/h differential speed $\ldots \ldots \ldots \ldots \ldots \ldots$	91
A.1	NPR filter coefficients in the frequency domain	102
C.1	GFDM parameters legend (as used in the experiment)	106

Publications and Patents

Book chapters

- Gerhard Wunder, Thorsten Wild, Prof. Gerhard Fettweis, Ivan Gaspar, Nicola Michailow, Maximilian Matthé, Luciano Mendes, Dimitri Ktenas, Slawomir Pietrzyk. Towards 5G: Applications, Requirements and Candidate Technologies. Chapter: New physical layer waveforms for 5G. John Wiley & Sons Ltd. Publisher, 2015
- [2] Luciano Mendes, Nicola Michailow, Maximilian Matthé, Ivan Gaspar, Dan Zhang and Gerhard Fettweis. 5G Networks: A Research and Development Perspective Chapter: *GFDM: Providing Flexibility for the 5G Physical Layer*. Taylor & Francis LLC, CRC Press, 2015.
- [3] Maximilian Matthé, Ivan Gaspar, Luciano Mendes, Nicola Michailow, Dan Zhang and Gerhard Fettweis. 5G Mobile Communication Chapter: Generalized Frequency Division Multiplexing: A Flexible Multi-carrierWaveform for 5G Springer, 2015.

Patents

- [4] **Ivan Gaspar**, and Gerhard Fettweis. Pseudo Circular Preamble for GFDM, December 2013. US Patent 20,150,172,010.
- [5] Ivan Gaspar, Maximilian Matthé, and Gerhard Fettweis. Generalized Frequency Division Multiplexing Radio Transmission using Frequency Domain Offset-QAM, December 2014. EU Patent App, pending.
- [6] Maximilian Matthé, Ivan Gaspar, and Gerhard Fettweis. Orthogonal Multicarrier Transmission system using Conjugate-Root Offset-QAM, December 2014. EU Patent App, pending.
- [7] Luciano Mendes, Ivan Gaspar, Maximilian Matthé, Andreas Festag, and Gerhard Fettweis. Walsh-Hadamard Transformed GFDM Radio Transmission, April 2014. EU and USA Patent App, pending.

Journal publications

- [8] Ivan Gaspar, Maximilian Matthé, Nicola Michailow, Luciano Leonel Mendes, Dan Zhang, and Gerhard Fettweis. Frequency-shift Offset-QAM for GFDM. In *IEEE Communication Letters*, 2015.
- [9] Ivan S Gaspar, Luciano L Mendes, Nicola Michailow, and Gerhard Fettweis. A synchronization technique for generalized frequency division multiplexing. In EURASIP Journal on Advances in Signal Processing, 2014(1):67, 2014.
- [10] Gerhard Wunder, Peter Jung, Martin Kasparick, Thorsten Wild, Frank Schaich, Yejian Chen, Stephan ten Brink, Ivan Gaspar, Nicola Michailow, Andreas Festag, et al. 5GNOW: Non-Orthogonal, Asynchronous Waveforms for Future Mobile Applications. In *IEEE Communications Magazine*, 52(2):97–105, 2014.
- [11] Nicola Michailow, Maximilian Matthé, Ivan Simões Gaspar, Ainoa Navarro Caldevilla, Luciano Leonel Mendes, Andreas Festag, and Gerhard Fettweis. Generalized Frequency Division Multiplexing for 5th Generation Cellular Networks. In *IEEE Transactions on Communications*, 2014.
- [12] Nicola Michailow, Luciano Leonel Mendes, Maximillian Matthé, Ivan Gaspar, Andreas Festag, and Gerhard Fettweis. Robust WHT-GFDM for Next Generation of Wireless Networks. In *IEEE Communications Letters*, 2015.
- [13] Maximilian Matthé, Luciano Leonel Mendes, Ivan Gaspar, Nicola Michailow, Dan Zhang, and Gerhard Fettweis. Multi-User Time-Reversal STC-GFDMA for Future Wireless Networks. In EURASIP Journal on Wireless Communications and Networking, 2015.
- [14] Dan Zhang, Luciano Leonel Mendes, Maximilian Matthé, Ivan Gaspar, Nicola Michailow, and Gerhard Fettweis. Expectation Propagation for Near-Optimum Detection of MIMO-GFDM Signals In *IEEE Transactions on Wireless* Communications, 2015.

Reports

- [15] Ivan Gaspar, Maximilian Matthé, Nicola Michailow, Luciano Leonel Mendes, Dan Zhang, and Gerhard Fettweis. GFDM modulation and demodulation using low-complex multiplication in the time domain. Report on arxiv. org/pdf/1506.03350.
- [16] Ivan Gaspar, Luciano Leonel Mendes, Maximilian Matthé, Nicola Michailow, Dan Zhang, Antonio Alberti, and Gerhard Fettweis. GFDM – A Framework for Virtual PHY Services in 5G Networks. Report on arxiv. org/abs/1507.04608.

Conference publications

- [17] Ivan Gaspar, and Gerhard Fettweis. An Embedded Midamble Synchronization approach for Generalized Frequency Division Multiplexing. In *IEEE Global Communications Conference (Globecom)*, 2015.
- [18] Ivan Gaspar, Andreas Festag, and Gerhard Fettweis. Synchronization using a Pseudo-Circular Preamble for Generalized Frequency Division Multiplexing in Vehicular Communication. In *IEEE 82nd Vehicular Technology Conference (VTC Spring)*, 2015.
- [19] Ivan Gaspar, Luciano Mendes, Maximilian Matthé, Nicola Michailow, Andreas Festag, and Gerhard Fettweis. LTE-compatible 5G PHY based on Generalized Frequency Division Multiplexing. In *IEEE 11th International Symposium on Wireless* Communication Systems (ISWCS), 2014.
- [20] Ivan Gaspar, Nicola Michailow, Ainoa Navarro, Eckhard Ohlmer, Stefan Krone, and Gerhard Fettweis. Low complexity GFDM receiver based on sparse frequency domain processing. In *IEEE 77th Vehicular Technology Conference (VTC Spring)*, 2013.
- [21] Ivan Gaspar, Ainoa Navarro Caldevilla, Nicola Michailow, and Gerhard Fettweis. FPGA implementation of Generalized Frequency Division Multiplexing transmitter using NI LabVIEW and NI PXI platform. In NI VIP 2013 Virtuelle Instrumente in der Praxis (NI VIP), 2013.
- [22] Maximilian Matthé, Ivan Gaspar, Dan Zhang, and Gerhard Fettweis. Short Paper: Reduced Complexity Calculation of LMMSE Filter Coefficients for GFDM In *IEEE 82nd Vehicular Technology Conference (VTC Spring)*, 2015.
- [23] Maximilian Matthé, Ivan Gaspar, Dan Zhang, and Gerhard Fettweis. Short Paper: Near-ML Detection for MIMO-GFDM In *IEEE 82nd Vehicular Technology* Conference (VTC Spring), 2015.
- [24] Martin Danneberg, Nicola Michailow, Ivan Gaspar, Dan Zhang and Gerhard Fettweis. Short Paper: Flexible GFDM implementation in FPGA with support to run-time reconfiguration. In *IEEE 82nd Vehicular Technology Conference (VTC Spring)*, 2015.
- [25] Martin Danneberg, Nicola Michailow, Ivan Gaspar, Maximilian Matthé, Dan Zhang, Luciano Leonel Mendes and Gerhard Fettweis. Implementation of a 2 by 2 MIMO-GFDM Transceiver for Robust 5G Networks. Submitted to IEEE International Symposium on Wireless Communication Systems (ISWCS), 2015.

- [26] Nicola Michailow, Ivan Gaspar, Stefan Krone, Michael Lentmaier, and Gerhard Fettweis. Generalized Frequency Division Multiplexing: Analysis of an alternative multi-carrier technique for next generation cellular systems. In *IEEE International* Symposium on Wireless Communication Systems (ISWCS), 2012.
- [27] Tobias Kadur, Ivan Gaspar, and Nicola Michailow, and Gerhard Fettweis. Non-data aided frequency synchronization exploiting ICI in non-orthogonal systems. In *IEEE* 80th Vehicular Technology Conference (VTC Fall), 2014.
- [28] Maximilian Matthé, Nicola Michailow, Ivan Gaspar, and Gerhard Fettweis. Influence of pulse shaping on bit error rate performance and out of band radiation of Generalized Frequency Division Multiplexing. In *IEEE International Conference* on Communications Workshops (ICC), 2014.
- [29] Gerhard Wunder, Martin Kasparick, Thorsten Wild, Frank Schaich, Yejian Chen, Marcin Dryjanski, Mateusz Buczkowski, Slawomir Pietrzyk, Nicola Michailow, Maximilian Matthé, Ivan Gaspar, et al. 5GNOW: Intermediate frame structure and transceiver concepts. In Globecom 2014 Workshop - Telecommunications Standards - From Research to Standards (GC14 WS - TCS), 2014.
- [30] Thorsten Wild, Gerhard Wunder, Frank Schaich, Yejian Chen, Martin Kasparick, Marcin Dryjanski, Slawomir Pietrzyk, Nicola Michailow, Maximilian Matthé, Ivan Gaspar, et al. 5GNOW: Intermediate Transceiver and Frame Structure Concepts and Results. In European Conference on Networks and Communications (EUCNC), 2014.
- [31] Gerhard Wunder, Martin Kasparick, Stephan Ten Brink, Frank Schaich, Thorsten Wild, Yejian Chen, Ivan Gaspar, et al. System-level interfaces and performance evaluation methodology for 5G physical layer based on non-orthogonal waveforms. In *IEEE Asilomar Conference on Signals, Systems and Computers*, 2013.
- [32] Gerhard Wunder, Martin Kasparick, Stephan ten Brink, Frank Schaich, Thorsten Wild, Ivan Gaspar, et al. 5GNOW: Challenging the LTE Design Paradigms of. Orthogonality and Synchronicity. In *IEEE 77th Vehicular Technology Conference* (VTC Spring), 2013.
- [33] Gerhard Wunder, Martin Kasparick, Thorsten Wild, Frank Schaich, Yejian Chen, Stephan Ten Brink, Ivan Gaspar, Nicola Michailow, Ainoa Navarro Caldevilla, Gerhard Fettweis, et al. 5GNOW: Application Challenges and. Initial Waveform Results. In Future Network and Mobile Summit (FNMS'13), 2013.
- [34] Bruno Alves, Luciano Mendes, Dayan Guimarães, and Ivan Gaspar. Performance of GFDM over Frequency-Selective Channels. In International Workshop on Telecommunications (IWT), 2013.

Bibliography

- $[ABC^+14]$ J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. Soong, and J. C. Zhang. What will 5G be? Selected Areas in Communications, *IEEE Journal on*, 32(6):1065–1082, 2014. [AHG11] P. Alexander, D. Haley, and A. Grant. Cooperative intelligent transport systems: 5.9-GHz field trials. Proceedings of the IEEE, 99(7):1213–1235, July 2011. [AKE08] A. Awoseyila, C. Kasparis, and B. G. Evans. Improved Preamble-Aided Timing Estimation for OFDM Systems. *IEEE Communications Letters*, 12(11):825-827, November 2008. [AMGaG13] B. M. Alves, L. L. Mendes, D. A. Guimãraes, and I. S. Gaspar. Performance of GFDM over Frequency-Selective Channels -Invited Paper. Revista Telecomunicações, 2013. [Ant05] A. C. Antoulas. Approximation of large-scale dynamical systems, volume 6. Siam, 2005.
- [ARO13] J. B. Anderson, F. Rusek, and V. Owall. Faster-than-Nyquist signaling. Proceedings of the IEEE, 101(8):1817–1830, 2013.
- [AZS11] H. Abdzadeh-Ziabari and M. G. Shayesteh. Robust timing and frequency synchronization for OFDM systems. Vehicular Technology, IEEE Transactions on, 60(8):3646–3656, 2011.
- [BÖ3] H. Bölcskei. Orthogonal Frequency Division Multiplexing Based on Offset QAM. In H. Feichtinger and T. Strohmer, editors, Advances in Gabor Analysis, Applied and Numerical Harmonic Analysis, pages 321–352. Birkhäuser Boston, 2003.
- [BBC⁺14] P. Banelli, S. Buzzi, G. Colavolpe, A. Modenini, F. Rusek, and A. Ugolini. Modulation Formats and Waveforms for 5G Networks: Who Will Be the Heir of OFDM?: An overview of alternative modulation schemes for improved spectral efficiency. *Signal Processing Magazine*, *IEEE*, 31(6):80–93, Nov 2014.

[BBT97]	A. M. Bruckner, J. B. Bruckner, and B. S. Thomson. <i>Real Analysis.</i> Prentice Hall PTR ISBN: 9780134588865, 1997.
[Bel10]	M. Bellanger. FBMC physical layer: a primer. Technical report, PHYDYAS, 2010.
[Bin90]	J. A. Bingham. Multicarrier modulation for data transmission: An idea whose time has come. <i>Communications Magazine, IEEE</i> , 28(5):5–14, 1990.
[Blo00]	P. Bloomfield. Fourier Analysis of Time Series: An Introduction. Wiley-Interscience, New York, 2000.
[BLRR+10]	M. Bellanger, D. Le Ruyet, D. Roviras, M. Terré, J. Nossek, L. Baltar, Q. Bai, D. Waldhauser, M. Renfors, T. Ihalainen, et al. FBMC physical layer: a primer. <i>PHYDYAS, January</i> , 2010.
[BLY13]	E. Bala, J. Li, and R. Yang. Shaping spectral leakage: a novel low-complexity transceiver architecture for cognitive radio. <i>Vehicular Technology Magazine, IEEE</i> , 8(3):38–46, 2013.
[BM67]	E. O. Brigham and R. Morrow. The fast Fourier transform. <i>Spectrum</i> , <i>IEEE</i> , 4(12):63–70, 1967.
[Böl01]	H. Bölcskei. Blind estimation of symbol timing and carrier frequency offset in wireless OFDM systems. <i>Communications, IEEE Transactions on</i> , 49(6):988–999, 2001.
[BZ97]	J. Benedetto and G. Zimmermann. Sampling multipliers and the Poisson Summation Formula. <i>Journal of Fourier Analysis and Applications</i> , 3(5):505–523, 1997.
[Cal15]	A. N. Caldevilla. GFDM Basic FPGA Implementation Using LabView. https://cloudstore.zih.tu-dresden.de/public.php?service= files&t=47f87282249f26a1039a297090ed4986, 2015.
[Cha66]	R. W. Chang. Synthesis of Band-Limited Orthogonal Signals for Multichannel Data Transmission. <i>The Bell Systems Technical Journal</i> , 45(10):1775–1796, 1966.
[CSBM06]	J. Coon, M. Sandell, M. Beach, and J. McGeehan. Channel and noise variance estimation and tracking algorithms for unique-word based single-carrier systems. <i>Wireless Communications, IEEE Transactions on</i> , 5(6):1488–1496, 2006.
$[D^+92]$	I. Daubechies et al. Ten lectures on wavelets, volume 61. SIAM, 1992.

- [Dat14] R. Datta. Generalized Frequency Division Multiplexing in Cognitive Radio. PhD thesis, Fakultät Elektrotechnik und Informationstechnik, Technischen Universität Dresden, 2014.
- [DBCK14] J.-B. Doré, V. Berg, N. Cassiau, and D. Kténas. FBMC receiver for multi-user asynchronous transmission on fragmented spectrum. EURASIP Journal on Advances in Signal Processing, 2014(1):1–20, 2014.
- [DMG⁺15a] M. Danneberg, N. Michailow, I. Gaspar, M. Matthé, D. Zhang, L. L. Mendes, and G. Fettweis. Implementation of a 2 by 2 MIMO-GFDM Transceiver for Robust 5G Networks. In 2015 International Symposium on Wireless Communication Systems (ISWCS), 2015.
- [DMG⁺15b] M. Danneberg, N. Michailow, I. Gaspar, D. Zhang, and G. Fettweis. Short Paper: Flexible GFDM implementation in FPGA with support to run-time reconfiguration. In Vehicular Technology Conference, 2015. VTC Fall 2015, IEEE 82nd, 2015.
- [DW01] N. Dinur and D. Wulich. Peak-to-average power ratio in high-order OFDM. Communications, IEEE Transactions on, 49(6):1063–1072, 2001.
- [ETS14] ETSI. Intelligent Transport Systems (ITS); Access Layer; ITS-G5 Channel Models and Performance Analysis Framework (ETSI TS 103 257 V0.0.1), June 2014.
- [FA14] G. Fettweis and S. Alamouti. 5G: Personal mobile internet beyond what cellular did to telephony. *IEEE Communications Magazine*, 2(52):140–145, 2014.
- [FABSE02] D. Falconer, S. L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson. Frequency domain equalization for single-carrier broadband wireless systems. *Communications Magazine*, *IEEE*, 40(4):58–66, 2002.
- [Fai89] M. Failli. Digital Land Mobile Radio Communications. COST 207. EC, 1989.
- [FB11] B. Farhang-Boroujeny. OFDM Versus Filter Bank Multicarrier. *IEEE Signal Processing Magazine*, 28(3):92–112, May 2011.
- [Fes14] A. Festag. Cooperative Intelligent Transport Systems (C-ITS) Standards in Europe. *Communication Magazin*, 12(52):166–172, December 2014.
- [Fet12] G. P. Fettweis. A 5G wireless communications vision. Microwave Journal, 55(12):24–36, 2012.

[Fet14]	G. P. Fettweis. The tactile internet: Applications and challenges. <i>Vehicular Technology Magazine</i> , <i>IEEE</i> , 9(1):64–70, 2014.
[FKB09]	G. Fettweis, M. Krondorf, and S. Bittner. GFDM - Generalized Frequency Division Multiplexing. In VTC Spring 2009 - IEEE 69th Vehicular Technology Conference, pages 1–4. IEEE, April 2009.
[FLP+07]	G. Fettweis, M. Löhning, D. Petrovic, M. Windisch, P. Zillmann, and W. Rave. Dirty RF: a new paradigm. <i>International Journal of Wireless Information Networks</i> , 14(2):133–148, 2007.
[G. 14]	G. Wunder et al. 5GNOW: non-orthogonal, asynchronous waveforms for future mobile applications. <i>IEEE Communications Magazine</i> , 52(2):97–105, February 2014.
[Ga10]	D. A. Guimãraes. Digital Transmission: A Simulation-Aided Introduction with VisSim/Comm. Springer Science & Business Media, 2010.
[Gas15a]	<pre>I. Gaspar. GFDM Basic Experiments Using LabView. https: //cloudstore.zih.tu-dresden.de/public.php?service=files&t= ba2c249a6880192f9d003959defbc9f3, 2015.</pre>
[Gas15b]	I. Gaspar. GFDM PSD Experiments Using LabView/Mathscript. https://cloudstore.zih.tu-dresden.de/public.php?service= files&t=ba2c249a6880192f9d003959defbc9f3, 2015.
[GF15]	I. Gaspar and G. Fettweis. An Embedded Midamble Synchronization Approach for Generalized Frequency Division Multiplexing. In <i>IEEE Globecom'15</i> , 2015.
[GFF15]	I. Gaspar, A. Festag, and G. Fettweis. Synchronization using a Pseudo-Circular Preamble for Generalized Frequency Division Multiplexing in Vehicular Communication. In <i>Vehicular Technology</i> <i>Conference, 2015. VTC Fall 2015, IEEE 82nd</i> , 2015.
[GG98]	F. Gini and G. B. Giannakis. Frequency offset and symbol timing recovery in flat-fading channels: a cyclostationary approach. <i>Communications, IEEE Transactions on</i> , 46(3):400–411, 1998.
[GH96]	W. Gerstacker and J. Huber. Improved equalization for GSM mobile communications. In <i>Proceedings of International Conference on Telecommunications (ICT)</i> , 1996.
[GLG08]	Y. Guo, G. Liu, and J. Ge. A novel time and frequency synchronization scheme for OFDM systems. <i>IEEE Trans. Consumer Electronics</i> , 54(2):321–325, 2008.

- [GMM⁺14] I. Gaspar, L. L. Mendes, M. Matthé, N. Michailow, and G. Fettweis. LTE-compatible 5G PHY based on Generalized Frequency Division Multiplexing. In Proceedings of the The Eleventh International Symposium on Wireless Communication Systems (ISWCS'14), 2014.
- [GMM⁺15a] I. Gaspar, M. Matthé, N. Michailow, L. Mendes, D. Zhang, and G. Fettweis. Frequency-shift Offset-QAM for GFDM. 2015.
- [GMM⁺15b] I. Gaspar, M. Matthé, N. Michailow, L. Mendes, D. Zhang, and G. Fettweis. GFDM Transceiver using Precoded Data and Low-complex Multiplication in Time Domain. arXiv:1506.03350, 2015.
- [GMM⁺15c] I. Gaspar, L. Mendes, M. Matthé, N. Michailow, D. Zhang, A. Albertiy, and G. Fettweis. GFDM - A Framework for Virtual PHY Services in 5G Networks. arXiv:1507.04608, 2015.
- [GMMF14] I. S. Gaspar, L. L. Mendes, N. Michailow, and G. Fettweis. A synchronization technique for generalized frequency division multiplexing. *EURASIP Journal on Advances in Signal Processing*, 2014(1):1–10, 2014.
- [GNMF13] I. Gaspar, A. Navarro, N. Michailow, and G. Fettweis. FPGA implementation of Generalized Frequency Division Multiplexing transmitter using NI LabVIEW and NI PXI platform. In NI VIP 2013 Virtuelle Instrumente in der Praxis (NI VIP), 2013.
- [GNN⁺13] I. S. Gaspar, M. N., A. Navarro Caldevilla, E. Ohlmer, S. Krone, and G. Fettweis. Low Complexity GFDM Receiver Based On Sparse Frequency Domain Processing. In Vehicular Technology Conference, 2013. VTC Spring 2013, IEEE 77th, 2013.
- [GW13] I. Gaspar and G. Wunder. 5GNOW D2.1 5G Cellular Communications Scenarios and System Requirements. Technical report, 5GNOW, 2013.
- [HHH12] M. Huemer, C. Hofbauer, and J. B. Huber. Non-systematic complex number RS coded OFDM by unique word prefix. Signal Processing, IEEE Transactions on, 60(1):285–299, 2012.
- [HJB84] M. T. Heideman, D. H. Johnson, and C. S. Burrus. Gauss and the history of the fast Fourier transform. *ASSP Magazine*, *IEEE*, 1(4):14–21, 1984.
- [HOH11] M. Huemer, A. Onic, and C. Hofbauer. Classical and Bayesian linear data estimators for unique word OFDM. Signal Processing, IEEE Transactions on, 59(12):6073–6085, 2011.
- [HTPD01] J. Heiskala and J. Terry Ph D. OFDM wireless LANs: A theoretical and practical guide. Sams, 2001.

[Hun72]	B. Hunt. A matrix theory proof of the discrete convolution theorem. <i>IEEE Transactions on Audio and Electroacoustics</i> , pages 285–288, 1972.
[HWH03]	M. Huemer, H. Witschnig, and J. Hausner. Unique word based phase tracking algorithms for SC/FDE-systems. In <i>Global Telecommunications Conference, 2003. GLOBECOM'03. IEEE</i> , volume 1, pages 70–74. IEEE, 2003.
[ITU14]	Itu-r bt.1206-2: Spectrum limit masks for digital terrestrial television broadcasting, 2014.
[Kay93]	S. M. Kay. Fundamentals of statistical signal processing: estimation theory Volume I. Prentice-Hall, Inc., 1993.
[Kay98]	S. M. Kay. Fundamentals of statistical signal processing: detection theory. 1998.
[KCRD09]	I. Kanaras, A. Chorti, M. R. Rodrigues, and I. Darwazeh. Spectrally efficient FDM signals: bandwidth gain at the expense of receiver complexity. In <i>Communications, 2009. ICC'09. IEEE International Conference on</i> , pages 1–6. IEEE, 2009.
[KE06]	C. Kasparis and B. G. Evans. A cross-correlation approach for improved timing estimation in OFDM broadcasting systems. In 24th AIAA International Communications Satellite Systems Conferences (ICSSC), pages 1039–1048, San Diego, USA, June 2006.
[Ken11]	J. B. Kenney. Dedicated Short-Range Communications (DSRC) Standards in the United States. <i>Proc. IEEE</i> , 12(7):1162–82, July 2011.
[KGMF14]	T. Kadur, I. Gaspar, N. Michailow, and G. Fettweis. Non-Data Aided Frequency Synchronization exploiting ICI in Non-Orthogonal Systems. In <i>Proceedings of the IEEE 80th IEEE Vehicular Technology Conference</i> (VTC Fall'14), 2014.
[KW88]	J. Krim and A. Widom. Damping of a crystal oscillator by an adsorbed monolayer and its relation to interfacial viscosity. <i>Physical Review B</i> , $38(17)$:12184, 1988.
[LAB95]	B. Le Floch, M. Alard, and C. Berrou. Coded orthogonal frequency division multiplex [TV broadcasting]. <i>Proceedings of the IEEE</i> , 83(6):982–996, June 1995.
[LCJ97]	X. Li and L. J. Cimini Jr. Effects of clipping and filtering on the performance of ofdm. In <i>Vehicular Technology Conference</i> , 1997, IEEE 47th, volume 3, pages 1634–1638. IEEE, 1997.

- [LG03] A. D. Liveris and C. N. Georghiades. Exploiting faster-than-Nyquist signaling. *Communications, IEEE Transactions on*, 51(9):1502–1511, 2003.
- [LK00] N. Lashkarian and S. Kiaei. Class of cyclic-based estimators for frequency-offset estimation of OFDM systems. *Communications, IEEE Transactions on*, 48(12):2139–2149, 2000.
- [LS14] H. Lin and P. Siohan. Multi-carrier modulation analysis and WCP-COQAM proposal. EURASIP Journal on Advances in Signal Processing, 2014(1):79, 2014.
- [LSL08] C. Lélé, P. Siohan, and R. Legouable. 2 dB better than CP-OFDM with OFDM/OQAM for preamble-based channel estimation. In *Communications, 2008. ICC'08. IEEE International Conference on*, pages 1302–1306. IEEE, 2008.
- [MADS13] M. Mirahmadi, A. Al-Dweik, and A. Shami. BER Reduction of OFDM Based Broadband Communication Systems over Multipath Channels with Impulsive Noise. *IEEE Transactions on Communications*, 61(11):4602–4615, November 2013.
- [Mas72] J. L. Massey. Optimum frame synchronization. Communications, IEEE Transactions on, 20(2):115–119, 1972.
- [Maz75] J. Mazo. Faster-than-Nyquist Signaling. Bell System Technical Journal, 54(6):1451–1462, Oct 1975.
- [MC58] R. Mosier and R. Clabaugh. Kineplex, a bandwidth-efficient binary transmission system. American Institute of Electrical Engineers, Part I: Communication and Electronics, Transactions of the, 76(6):723–728, 1958.
- [Mey90] Y. Meyer. Ondelettes et opérateurs: Ondelettes. Actualités mathématiques. Hermann, 1990.
- [MFP03] A. F. Molisch, J. R. Foerster, and M. Pendergrass. Channel models for ultrawideband personal area networks. *Wireless Communications, IEEE*, 10(6):14–21, 2003.
- [MGK⁺12] N. Michailow, I. Gaspar, S. Krone, M. Lentmaier, and G. Fettweis. Generalized frequency division multiplexing: Analysis of an alternative multi-carrier technique for next generation cellular systems. In 2012 International Symposium on Wireless Communication Systems (ISWCS), pages 171–175. IEEE, August 2012.

[MGZF15a]	M. Matthé, I. Gaspar, D. Zhang, and G. Fettweis. Short Paper: Near-ML Detection for MIMO-GFDM. In Vehicular Technology Conference, 2015. VTC Fall 2015, IEEE 82nd, 2015.
[MGZF15b]	M. Matthé, I. Gaspar, D. Zhang, and G. Fettweis. Short Paper: Reduced Complexity Calculation of LMMSE Filter Coefficients for GFDM. In <i>Vehicular Technology Conference, 2015. VTC Fall 2015, IEEE 82nd</i> , 2015.
[Mic15]	N. Michailow. Generalized Frequency Division Multiplexing Transceiver Principles. PhD thesis, Fakultät Elektrotechnik und Informationstechnik, Technischen Universität Dresden, 2015.
[MKP07]	M. Morelli, C. J. Kuo, and MO. Pun. Synchronization techniques for orthogonal frequency division multiple access (OFDMA): A tutorial review. <i>Proceedings of the IEEE</i> , 95(7):1394–1427, 2007.
[MLG06]	H. Myung, J. Lim, and D. Goodman. Single carrier FDMA for uplink wireless transmission. <i>Vehicular Technology Magazine</i> , <i>IEEE</i> , 1(3):30–38, Sept 2006.
[MM99]	M. Morelli and U. Mengali. An improved frequency offset estimator for OFDM applications. In <i>Communication Theory Mini-Conference</i> , 1999, pages 106–109. IEEE, 1999.
[MMF97]	H. Meyr, M. Moeneclaey, and S. Fechtel. Digital Communication Receivers: Synchronization and Channel Estimation Algorithms, 1997.
[MMF14]	M. Matthé, L. L. Mendes, and G. Fettweis. GFDM in a Gabor Transform Setting. <i>IEEE Communications Letters</i> , 2014.
[MMF15]	M. Matthé, L. L. Mendes, and G. Fettweis. Asynchronous multi-user uplink transmission with generalized frequency division multiplexing. In <i>Communication Workshop (ICCW), 2015 IEEE International Conference on</i> , pages 2269–2275. IEEE, 2015.
[MMG ⁺ 14]	N. Michailow, M. Matthé, I. Gaspar, A. Navarro Caldevilla, L. L. Mendes, A. Festag, and G. Fettweis. Generalized Frequency Division Multiplexing for 5th Generation Cellular Networks. <i>IEEE Transactions on Communications</i> , 2014.
[MMGF14]	M. Matthé, N. Michailow, I. Gaspar, and G. Fettweis. Influence of Pulse Shaping on Bit Error Rate Performance and Out of Band Radiation of Generalized Frequency Division Multiplexing. In <i>ICC'14 - Workshop on 5G Technologies (ICC'14 WS - 5G)</i> , pages 43—-48, Sydney, Australia, 2014.

- [MMM⁺15] N. Michailow, L. Mendes, M. Matthé, I. Gaspar, A. Festag, and G. Fettweis. Robust WHT-GFDM for the Next Generation of Wireless Networks. Communications Letters, IEEE, 19(1):106–109, 2015.
- [Moo94] P. H. Moose. A technique for orthogonal frequency division multiplexing frequency offset correction. *Communications, IEEE Transactions on*, 42(10):2908–2914, Oct 1994.
- [MTGB01] X. Ma, C. Tepedelenlioğlu, G. B. Giannakis, and S. Barbarossa. Non-data-aided carrier offset estimators for OFDM with null subcarriers: identifiability, algorithms, and performance. Selected Areas in Communications, IEEE Journal on, 19(12):2504–2515, 2001.
- [MWG⁺02] B. Muquet, Z. Wang, G. B. Giannakis, M. De Courville, and P. Duhamel.
 Cyclic prefixing or zero padding for wireless multicarrier transmissions?
 Communications, IEEE Transactions on, 50(12):2136–2148, 2002.
- [MZB00] H. Minn, M. Zeng, and V. Bhargava. On Timing Offset Estimation for OFDM Systems. *IEEE Communications Letters*, 4(7):242–244, July 2000.
- [NBS92] K. Nayebi, T. P. Barnwell, and M. J. Smith. Time-domain filter bank analysis: A new design theory. Signal Processing, IEEE Transactions on, 40(6):1412–1429, 1992.
- [NP00] R. v. Nee and R. Prasad. *OFDM for wireless multimedia communications*. Artech House, Inc., 2000.
- [OBB⁺14] A. Osseiran, F. Boccardi, V. Braun, K. Kusume, P. Marsch, M. Maternia,
 O. Queseth, M. Schellmann, H. Schotten, H. Taoka, et al. Scenarios for
 5G mobile and wireless communications: the vision of the METIS project.
 Communications Magazine, IEEE, 52(5):26–35, 2014.
- [OSB⁺89] A. V. Oppenheim, R. W. Schafer, J. R. Buck, et al. *Discrete-time signal* processing, volume 2. Prentice-hall Englewood Cliffs, 1989.
- [Pir14] P. Pirinen. A brief overview of 5G research activities. In 1st International Conference on 5G for Ubiquitous Connectivity, pages 17–22, Nov 2014.
- [Pro01] J. G. Proakis. Intersymbol Interference in Digital Communication Systems. Wiley Online Library, 2001.
- [PSY02] M. Pätzold, A. Szczepanski, and N. Youssef. Methods for modeling of specified and measured multipath power-delay profiles. Vehicular Technology, IEEE Transactions on, 51(5):978–988, 2002.

[RCZZ05]	G. Ren, Y. Chang, H. Zhang, and H. Zhang. Synchronization Method Based on a New Constant Envelop Preamble for OFDM Systems. <i>IEEE TRANSACTIONS ON BROADCASTING</i> , 51(1), March 2005.
[Sal67]	 B. Saltzberg. Performance of an Efficient Parallel Data Transmission System. Communication Technology, IEEE Transactions on, 15(6):805-811, 1967.
[SC97]	T. Schmidl and D. Cox. Robust Frequency and Timing Synchronization for OFDM. <i>IEEE Transactions on Communications</i> , 45(12):1613–1621, Dec 1997.
[SCM97]	M. Speth, F. Classen, and H. Meyr. Frame synchronization of OFDM systems in frequency selective fading channels. In <i>In Proceedings of the Vehicular Technology Conference (VTC'97</i> , pages 1807–1811, 1997.
[SIVR10]	T. H. Stitz, T. Ihalainen, A. Viholainen, and M. Renfors. Pilot-based synchronization and equalization in filter bank multicarrier communications. <i>EURASIP Journal on Advances in Signal Processing</i> , 2010:9, 2010.
[TG14]	A. M. Tonello and M. Girotto. Cyclic block filtered multitone modulation. <i>EURASIP Journal on Advances in Signal Processing</i> , 2014(1):1–18, 2014.
[TL98]	U. Tureli and H. Liu. Blind carrier synchronization and channel identification for OFDM communications. In <i>IEEE INTERNATIONAL CONFERENCE ON ACOUSTICS SPEECH AND SIGNAL PROCESSING</i> , volume 6, pages VI–3509. INSTITUTE OF ELECTRICAL ENGINEERS INC (IEE), 1998.
[Urk67]	H. Urkowitz. Energy detection of unknown deterministic signals. <i>Proceedings of the IEEE</i> , 55(4):523–531, 1967.
[vdBSB97]	J. van de Beek, M. Sandell, and P. Borjesson. ML estimation of time and frequency offset in OFDM systems. <i>IEEE Transactions on Signal</i> <i>Processing</i> , 45(7):1800–1805, July 1997.
[VWS ⁺ 13]	V. Vakilian, T. Wild, F. Schaich, S. ten Brink, and JF. Frigon. Universal-filtered multi-carrier technique for wireless systems beyond LTE. In 9th International Workshop on Broadband Wireless Access (BWA) @ IEEE Globecom'13, 2013.
[WKTB ⁺ 13a]	G. Wunder, M. Kasparick, S. Ten Brink, F. Schaich, T. Wild, Y. Chen, I. Gaspar, N. Michailow, G. Fettweis, D. Ktenas, et al. System-level interfaces and performance evaluation methodology for 5G physical layer

based on non-orthogonal waveforms. In Signals, Systems and Computers, 2013 Asilomar Conference on, pages 1659–163. IEEE, 2013.

- [WKtB⁺13b] G. Wunder, M. Kasparick, S. ten Brink, F. Schaich, T. Wild, I. Gaspar,
 E. Ohlmer, S. Krone, N. Michailow, A. Navarro, et al. 5GNOW: Challenging the LTE design paradigms of orthogonality and synchronicity. In Vehicular Technology Conference (VTC Spring), 2013 IEEE 77th, pages 1–5. IEEE, 2013.
- [WKW⁺14] G. Wunder, M. Kasparick, T. Wild, F. Schaich, Y. Chen, M. Dryjanski,
 M. Buczkowski, S. Pietrzyk, N. Michailow, M. Matthé, et al. 5GNOW: Intermediate frame structure and transceiver concepts. In *Globecom Workshops (GC Wkshps)*, 2014, pages 565–570. IEEE, 2014.
- [ZJC⁺15] X. Zhang, M. Jia, L. Chen, J. Ma, and J. Qiu. Filtered-ofdm enabler for flexible waveform in the 5th generation cellular networks. In 2015 IEEE Global Communications Conference (GLOBECOM), pages 1–6, Dec 2015.
- [ZMM⁺15] D. Zhang, L. Mendes, M. Matthe, I. Gaspar, N. Michailow, and G. Fettweis. Expectation Propagation for Near-Optimum Detection of MIMO-GFDM Signals. Wireless Communications, IEEE Transactions on, PP(99):1–1, 2015.