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The banking firm under ambiguity aversion

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We examine risk taking when the bank's preferences exhibit smooth ambiguity aversion. Ambiguity is modeled by a second-order probability distribution that captures the bank's uncertainty about which of the subjective beliefs govern the financial asset return risk. Ambiguity preferences are modeled by the (second-order) expectation of a concave transformation of the (first-order) expected utility of profit conditional on each plausible subjective distribution of the return risk. Within this framework, the banking firm finds it less attractive to take risk in the presence than in the absence of ambiguity. This result extends to the case of greater ambiguity aversion. Given that the competitive bank's smooth ambiguity preferences exhibit non-increasing absolute ambiguity aversion, imposing a more stringent capital requirement to the bank reduces the optimal amount of loans, if the bank's coefficient of relative risk aversion does not exceed unity. Ambiguity and ambiguity aversion as such have adverse effect on the bank's risk taking.

JEL classification: D01; D81; G11; G12

Keywords: Banking firm; Ambiguity; Ambiguity aversion; Capital requirement

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1 Introduction

Management of risk is an important topic in many fields of economics, banking, finance, insurance and international trade.[1] In the economics of banking, how financial intermediaries (FI) react to changes in random and non-random variables has a long tradition.[2] Most of the literature make use of the expected utility approach. Such a modeling approach rules out the possibility that the banking firm is unable to unambiguously assign a probability distribution that uniquely describes the financial asset's return risk, which gives rise to ambiguity.[3]

Since the seminal work of Ellsberg, ambiguity has been alluded to the violation of the independence axiom, which is responsible for the decision criterion being linear in the outcome probabilities. There are ample experiments (Einhorn and Hogarth, 1986; Sarin and Weber, 1993; Chow and Sarin, 2001) and surveys (Viscusi and Chesson, 1999; Chesson and Viscusi, 2003) that document convincing evidence that individuals prefer gambles with known rather than unknown probabilities, implying that ambiguity aversion prevails.

The purpose of this paper is to incorporate ambiguity and ambiguity aversion into the industrial organization approach (IO) of banking and finance under asset rate uncertainty.[4] Klibanoff, Marinacci and Mukerji (2005) have recently developed a powerful decision criterion known as "smooth ambiguity aversion" that is compatible with ambiguity averse preferences under uncertainty (hereafter referred to as the KMM model). The KMM model features the recursive structure that is far more tractable in comparison to other models of ambiguity such as the pioneering maxmin expected utility (or multiple-prior) model of Gilboa and Schmeidler (1989). Specifically, the KMM model represents ambiguity by a second-order probability distribution that captures the firm's uncertainty about which of the subjective beliefs govern the price risk. The KMM model then measures the bank's expected utility under ambiguity by taking the (second-order) expectation of a concave

[1] See, for example, Baker and Filberg, 2015; Broll and Wong 2015.

[2] See, for example, Freixas and Rochet, 2008; Admati and Hellwig, 2013.

[3] See Deck and Schlesinger, 2014, for a discussion.

[4] For the standard IO approach of banking, see, Wong, 1997; Freixas and Rochet, 2008; Pausch and Welzel, 2013, to name just a few).

transformation of the (first-order) expected utility of profit conditional on each plausible subjective distribution of the price risk. This recursive structure creates a crisp separation between ambiguity and ambiguity aversion, i.e., between beliefs and tastes, which allows us to study these two attributes independently. Another nice feature of the KMM model is that we can apply the conventional techniques in the decision making under uncertainty in the context of ambiguity (Taboga, 2005; Gollier, 2011; Snow, 2010, 2011; Alary et al., 2013; Wong, 2015).

Since financial investment exposes the banking firm to the return risk, the prevalence of ambiguity creates additional risk to the ambiguity-averse banking firm. Hence, the firm finds it less attractive to invest in the financial asset in the presence than in the absence of ambiguity. This result extends to the case of greater ambiguity aversion. Ambiguity and ambiguity aversion as such have adverse effect on the firm's incentive to invest. Given that the competitive bank's smooth ambiguity preferences exhibit non-increasing absolute ambiguity aversion, imposing a more stringent capital requirement to the bank reduces the optimal amount of loans, if the bank's coefficient of relative risk aversion does not exceed unity.

In our model of a bank the supply of deposits is perfectly elastic at a fixed deposit rate. Deposits are insured by a government–funded deposit insurance scheme. The shareholders of the bank contribute equity capital with a required rate of return which is higher than the deposit rate, reflecting the scarcity of shareholders' wealth. We abstract from both incentive contracts and strategic delegation, issues that arise or can arise when bank ownership and control are separated. The impact of these issues on risk taking is well–understood (see The focus of our study is to incorporate ambiguity and ambiguity aversion.

The rest of this paper is organized as follows. Section 2 delineates the model of a competitive banking firm under return rate uncertainty. Section 3 examine how ambiguity and ambiguity aversion affect the bank's optimal financial investment decision. In section 4, we examine the effect of a more stringent capital requirement on the bank's lending decision. The final section concludes.

2 The model

Consider a competitive bank that makes decisions in a single period horizon with two dates, 0 and 1. At date 0, the bank has the following balance sheet:

$$L = D + E, \tag{1}$$

where $L > 0$ is the amount of loans, $D > 0$ is the quantity of deposits, and $E > 0$ is the stock of equity capital. By regulation, the bank is subject to the following capital adequacy requirement:

$$\alpha L \leq E, \tag{2}$$

where $\alpha \in (0, 1)$ is the minimum capital-to-loan ratio.[5]

The bank's loans belong to a single homogeneous class, which mature at date 1. The gross return on loans, $\tilde{R}$, is stochastic and distributed according to an objective cumulative distribution function (CDF), $F^{\circ}(R)$, over support $[\underline{R}, \overline{R}]$ with $0 < \underline{R} < \overline{R}$.[6] The bank's deposits are insured by a government-funded deposit insurance scheme. The supply of deposits is perfectly elastic at the fixed one-plus deposit rate, $R_D \geq 1$. The bank's shareholders contribute equity capital with a required gross return, R_E, on their investment, where $R_E > R_D$, reflecting the scarcity of shareholders' wealth. We assume that the unconditional expected gross return on loans is no less than R_E, and thereby is greater than R_D, so that the bank has incentives to extend loans by raising deposits and equity capital.

The bank's shareholders receive the following net worth at date 1:

$$\tilde{W} = \tilde{R}L - R_D D - R_E E - C(L), \tag{3}$$

where $C(L)$ is the cost function of servicing loans such that $C(0) = C'(0) = 0$ and that $C'(L) > 0$ and $C''(L) > 0$ for all $L > 0$. The bank possesses a von Neumann-Morgenstern

[5] This specification of capital adequacy requirement is consistent with those of Basel I and the standardized approach of Basel II, both of which set $\alpha = 8\%$. See also Wong (1997, 2011).

[6] Throughout the paper, random variables have a tilde ($\tilde{}$) while their realizations do not.

utility function, $u(W)$, defined over the net worth of its shareholders at date 1, W, with $u'(W) > 0$ and $u''(W) < 0$, indicating the presence of risk aversion.

Since $R_E > R_D$, the bank would like to rely on deposits rather than equity capital to finance loans, thereby rendering the capital adequacy requirement to be binding. Substituting the initial balance sheet constraint, Eq. (1), and the binding capital adequacy requirement, Eq. (2), into Eq. (3) yields the following net worth of the bank's shareholders at date 1:

$$\tilde{W} = [\tilde{R} - \alpha R_E - (1-\alpha)R_D]L - C(L). \tag{4}$$

We can interpret $\alpha R_E + (1-\alpha)R_D$ as the bank's weighted average cost of capital, where the weights are based on the capital-to-loan ratio, α, and the deposit-to-loan ratio, $1-\alpha$.

The bank faces ambiguity in that it is uncertain about the objective CDF, $F^\circ(R)$. Succinctly, the bank has a continuum of priors, $\{F(R|\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$, where $F(R|\theta)$ denotes a plausible first-order CDF of $\tilde{R}$ over support $[\underline{R}, \overline{R}]$, which is sensitive to a parameter, θ, whose value is not known ex ante. Based on its subjective information, the bank associates a second-order CDF, $G(\theta)$, over the continuum of priors, i.e., over support $[\underline{\theta}, \overline{\theta}]$, where $\underline{\theta} < \overline{\theta}$. This captures the bank's uncertainty about which of the first-order CDF, $F(R|\theta)$, governs the gross return on loans, $\tilde{R}$. Following Snow (2010, 2011) and Wong (2015), we assume that the bank's ambiguous beliefs are unbiased in the following sense:

$$\int_{\underline{\theta}}^{\overline{\theta}} F(R|\theta)\mathrm{d}G(\theta) = F^\circ(R), \tag{5}$$

for all $R \in [\underline{R}, \overline{R}]$.[7] We denote $\mathrm{E}_F(\cdot|\theta)$, $\mathrm{E}_G(\cdot)$, and $\mathrm{E}_{F^\circ}(\cdot)$ as the expectation operators with respect to the first-order CDF, $F(R|\theta)$, the second-order CDF, $G(\theta)$, and the objective CDF, $F^\circ(R)$, respectively.

The recursive structure of the KMM model implies that we can compute the bank's expected utility under ambiguity in three steps. First, we calculate the bank's expected

[7]This assumption is motivated by the premise that the behavior of an ambiguity-neutral decision maker should be unaffected by the introduction of, or changes in, ambiguity.

utility for each first-order CDF of $\tilde{R}$:

$$U(L,\theta) = \mathrm{E}_F[u(\tilde{W})|\theta], \tag{6}$$

where $\tilde{W}$ is given by Eq. (4). Second, we transform each first-order expected utility obtained in the first step by an ambiguity function, $\phi(U)$, where $\phi'(U) > 0$ and U is the bank's utility level. Finally, we take the expectation of the transformed first-order expected utility obtained in the second step with respect to the second-order CDF of $\tilde{\theta}$. The bank's ex-ante decision problem as such is given by

$$\max_{L\geq 0} \int_{\underline{\theta}}^{\overline{\theta}} \phi[U(L,\theta)]\mathrm{d}G(\theta), \tag{7}$$

where $U(L,\theta)$ is given by Eq. (6). Inspection of the objective function of program (7) reveals that the effect of ambiguity, represented by the second-order CDF, $G(\theta)$, and the effect of ambiguity preferences, represented by the shape of the ambiguity function, $\phi(U)$, can be separated and thus studied independently.

The bank is said to be ambiguity averse if, for a given amount of loans, L, the objective function of program (7) decreases when the bank's ambiguous beliefs, specified by $G(\theta)$, change in a way that induces a mean-preserving spread in the distribution of the bank's expected utility. According to this definition, Klibanoff et al. (2005) show that ambiguity aversion implies concavity for $\phi(U)$, and that a concave transformation of $\phi(U)$ results in greater ambiguity aversion.[8] To see this, we define the ambiguity aversion premium, P, as the solution to the following equation:

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi[U(L,\theta)]\mathrm{d}G(\theta) = \phi\bigg[\int_{\underline{\theta}}^{\overline{\theta}} U(L,\theta)\mathrm{d}G(\theta) - P\bigg]. \tag{8}$$

Hence, P measures the "pain" the bank is willing to suffer in order to get rid of ambiguity. It follows from Eq. (8) and Jensen's inequality that $P > 0$ if, and only if, $\phi''(U) < 0$. To

[8]When $\phi(U) = [1-\exp(-\eta U)]/\eta$, Klibanoff et al. (2005) show that the maxmin expected utility model of Gilboa and Schmeidler (1989) is the limiting case as the constant absolute ambiguity aversion, η, approaches infinity under some conditions.

compare ambiguity aversion, we define the ambiguity aversion premium, P_i, as the solution to the following equation:

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi_i[U(L,\theta)]\mathrm{d}G(\theta) = \phi_i\left[\int_{\underline{\theta}}^{\overline{\theta}} U(L,\theta)\mathrm{d}G(\theta) - P_i\right], \tag{9}$$

for $i = 1$ and 2. It follows from Eq. (9) and Pratt (1964) that $P_1 < P_2$ if, and only if, $-\phi_1''(U)/\phi_1'(U) < -\phi_2''(U)/\phi_2'(U)$, which is equivalent to $\phi_2(U)$ being a concave transformation of $\phi_1(U)$. Throughout the paper, we assume that $\phi''(U) < 0$ so that the bank is ambiguity averse.

The first-order conditions for program (7) are given by

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi'[U(L^*,\theta)]U_L(L^*,\theta)\mathrm{d}G(\theta) = 0, \tag{10}$$

where $U_L(L,\theta) = \mathrm{E}_F\{u'(\tilde{W})[\tilde{R} - \alpha R_E - (1-\alpha)R_D - C'(L)]|\theta\}$, and an asterisk (*) signifies an optimal level. Differentiating the objective function of program (7) twice yields

$$\frac{\partial^2}{\partial L^2}\int_{\underline{\theta}}^{\overline{\theta}} \phi[U(L,\theta)]\mathrm{d}G(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \phi''[U(L,\theta)]U_L(L,\theta)^2\mathrm{d}G(\theta)$$

$$+\int_{\underline{\theta}}^{\overline{\theta}} \phi'[U(L,\theta)]\mathrm{E}_F\{u''(\tilde{W})[\tilde{R} - \alpha R_E - (1-\alpha)R_D - C'(L)]^2|\theta\}\mathrm{d}G(\theta)$$

$$-\int_{\underline{\theta}}^{\overline{\theta}} \phi'[U(L,\theta)]\mathrm{E}_F[u'(\tilde{W})|\theta]C''(L)\mathrm{d}G(\theta) < 0, \tag{11}$$

for all $L \geq 0$, where the inequality follows from the assumed properties of $\phi(U)$, $u(W)$, and $C(L)$. It then follows from Eq. (11) that Eq. (10) is both necessary and sufficient for L^* to the unique maximum solution to program (7).

3 Ambiguity and the bank's risk-taking

In this section, we examine how ambiguity and ambiguity aversion affect the bank's optimal lending decision. To this end, we suppose that the bank becomes more ambiguity averse in that its ex-ante decision problem is given by

$$\max_{L\geq 0} \int_{\underline{\theta}}^{\overline{\theta}} K\{\phi[U(L,\theta)]\}\mathrm{d}G(\theta), \tag{12}$$

when $K(\cdot)$ is a concave function with $K'(\cdot) > 0$ and $K''(\cdot) < 0$. The first-order condition for program (12) is given by

$$\int_{\underline{\theta}}^{\overline{\theta}} K'\{\phi[U(L^{\dagger},\theta)]\}\phi'[U(L^{\dagger},\theta)]U_L(L^{\dagger},\theta)\mathrm{d}G(\theta) = 0, \tag{13}$$

where a dagger (†) indicates an optimal level. It is evident that Eq. (11) remains valid when we replace $\phi(U)$ by $K[\phi(U)]$.

Differentiating the objective function of program (12) with respect to L, and evaluating the resulting derivative at $L = L^*$ yields

$$\frac{\partial}{\partial L}\int_{\underline{\theta}}^{\overline{\theta}} K\{\phi[U(L,\theta)]\}\mathrm{d}G(\theta)\bigg|_{L=L^*}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} K'\{\phi[U(L^*,\theta)]\}\phi'[U(L^*,\theta)]U_L(L^*,\theta)\mathrm{d}G(\theta). \tag{14}$$

If the right-hand side of Eq. (14) is negative (positive), it follows from Eqs. (11) and (13) that $L^{\dagger} < (>)\ L^*$. In the following proposition, we derive sufficient conditions under which the right-hand side of Eq. (14) is negative.

Proposition 1. *Making the ambiguity-averse competitive bank more ambiguity averse reduces the optimal amount of loans, i.e., $L^{\dagger} < L^*$, if the bank's coefficient of relative risk aversion, $\Phi(W) = -Wu''(W)/u'(W)$, does not exceed unity, and if an increase in the parameter, θ, always deteriorates (improves) the subjective cumulative distribution function, $F(R|\theta)$, in the sense of first-order stochastic dominance.*

Proof. Differentiating $K'\{\phi[U(L,\theta)\}$ with respect to θ yields

$$\frac{\partial K'\{\phi[U(L,\theta)]\}}{\partial\theta} = -\int_{\underline{R}}^{\overline{R}} K''\{\phi[U(L,\theta)]\}\phi'[U(L,\theta)]u'(W)LF_{\theta}(R|\theta)\mathrm{d}R, \tag{15}$$

where $F_\theta(R|\theta) = \partial F(R|\theta)/\partial\theta$, and we have used integration by parts. Differentiating $U_L(L,\theta)$ with respect to θ yields

$$\frac{\partial U_L(L,\theta)}{\partial\theta} = \int_{\underline{R}}^{\overline{R}} \{u'(W)[\Phi(W)-1] + u''(W)[C'(L)L - C(L)]\}F_\theta(R|\theta)\mathrm{d}R, \tag{16}$$

where $\Phi(W) = -Wu''(W)/u'(W)$ is the bank's coefficient of relative risk aversion, and we have used integration by parts. Since $C(0)=0$, the strict convexity of $C(L)$ implies that $C'(L) > C(L)/L$ for all $L > 0$. If $\Phi(W) \le 1$ and an increase in θ always deteriorates (improves) $F(R|\theta)$ in the sense of first-order stochastic dominance, Eq. (16) implies that $U_L(L^*,\theta)$ is decreasing (increasing) in θ, and Eq. (15) implies that $K'\{\phi[U(L^*,\theta)]\}$ is increasing (decreasing) in θ.

Consider first the case that an increase in θ always deteriorates $F(R|\theta)$ in the sense of first-order stochastic dominance. Since $\partial U_L(L^*,\theta)/\partial\theta < 0$, there are three possible cases: (i) $U_L(L^*,\theta) > 0$ for all $\theta \in [\underline{\theta},\overline{\theta})$, (ii) $U_L(L^*,\theta) < 0$ for all $\theta \in (\underline{\theta},\overline{\theta}]$, and (iii) there exists a unique point, $\theta_1 \in (\underline{\theta},\overline{\theta})$ such that $U_L(L^*,\theta) > (<)\ 0$ for all $\theta < (>)\ \theta_1$. In case (i), the right-hand side of Eq. (14) must be positive. Using Eq. (10), we can write Eq. (14) as

$$\frac{\partial}{\partial L}\int_{\underline{\theta}}^{\overline{\theta}} K\{\phi[U(L,\theta)]\}\mathrm{d}G(\theta)\bigg|_{L=L^*}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{K'\{\phi[U(L^*,\theta)]\} - K'\{\phi[U(L^*,\overline{\theta})]\}\Big\}\phi'[U(L^*,\theta)]U_L(L^*,\theta)\mathrm{d}G(\theta). \tag{17}$$

Since $\partial K'\{\phi[U(L^*,\theta)]\}/\partial\theta > 0$, we have $K'\{\phi[U(L^*,\theta)]\} < K'\{\phi[U(L^*,\overline{\theta})]\}$ for all $\theta \in [\underline{\theta},\overline{\theta})$. It then follows from $U_L(L^*,\theta) > 0$ for all $\theta \in [\underline{\theta},\overline{\theta})$ that the right-hand side of Eq. (17) is negative, a contradiction. Hence, case (i) cannot hold. Analogously, we can show that case (ii) cannot hold. In case (iii), we can use Eq. (10) to write Eq. (14) as

$$\frac{\partial}{\partial L}\int_{\underline{\theta}}^{\overline{\theta}} K\{\phi[U(L,\theta)]\}\mathrm{d}G(\theta)\bigg|_{L=L^*}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{K'\{\phi[U(L^*,\theta)]\} - K'\{\phi[U(L^*,\theta_1)]\}\Big\}\phi'[U(L^*,\theta)]U_L(L^*,\theta)\mathrm{d}G(\theta). \tag{18}$$

Since $K'\{\phi[U(L^*,\theta)]\} < (>)\ K'\{\phi[U(L^*,\theta_1)]\}$ and $U_L(L^*,\theta) > (<)\ 0$ for all $\theta < (>)\ \theta_1$, the right-hand side of Eq. (18) is negative. It then follows from Eqs. (11) and (13) that $L^\dagger < L^*$.

Consider now the case that an increase in θ always improves $F(R|\theta)$ in the sense of first-order stochastic dominance. Since $\partial U_L(L^*,\theta)/\partial\theta > 0$, there are three possible cases: (i) $U_L(L^*,\theta) > 0$ for all $\theta \in (\underline{\theta},\overline{\theta}]$, (ii) $U_L(L^*,\theta) < 0$ for all $\theta \in (\underline{\theta},\overline{\theta}]$, and (iii) there exists a unique point, $\theta_2 \in (\underline{\theta},\overline{\theta})$ such that $U_L(L^*,\theta) < (>)\ 0$ for all $\theta < (>)\ \theta_2$. In case (i), the right-hand side of Eq. (14) must be positive. Using Eq. (10), we can write Eq. (14) as

$$\frac{\partial}{\partial L}\int_{\underline{\theta}}^{\overline{\theta}} K\{\phi[U(L,\theta)]\}\mathrm{d}G(\theta)\bigg|_{L=L^*}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{K'\{\phi[U(L^*,\theta)]\} - K'\{\phi[U(L^*,\underline{\theta})]\}\Big\}\phi'[U(L^*,\theta)]U_L(L^*,\theta)\mathrm{d}G(\theta). \tag{19}$$

Since $\partial K'\{\phi[U(L^*,\theta)]\}/\partial\theta < 0$, we have $K'\{\phi[U(L^*,\theta)]\} < K'\{\phi[U(L^*,\underline{\theta})]\}$ for all $\theta \in (\underline{\theta},\overline{\theta}]$. It then follows from $U_L(L^*,\theta) > 0$ for all $\theta \in (\underline{\theta},\overline{\theta}]$ that the right-hand side of Eq. (19) is negative, a contradiction. Hence, case (i) cannot hold. Analogously, we can show that case (ii) cannot hold. In case (iii), we can use Eq. (10) to write Eq. (14) as

$$\frac{\partial}{\partial L}\int_{\underline{\theta}}^{\overline{\theta}} K\{\phi[U(L,\theta)]\}\mathrm{d}G(\theta)\bigg|_{L=L^*}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \Big\{K'\{\phi[U(L^*,\theta)]\} - K'\{\phi[U(L^*,\theta_2)]\}\Big\}\phi'[U(L^*,\theta)]U_L(L^*,\theta)\mathrm{d}G(\theta). \tag{20}$$

Since $K'\{\phi[U(L^*,\theta)]\} > (<)\ K'\{\phi[U(L^*,\theta_2)]\}$ and $U_L(L^*,\theta) < (>)\ 0$ for all $\theta < (>)\ \theta_2$, the right-hand side of Eq. (20) is negative. It then follows from Eqs. (11) and (13) that $L^\dagger < L^*$. $\square$

The sufficient conditions derived in Proposition 1 can also ensure the adverse effect of ambiguity on the bank's lending decision. To see this, we consider the benchmark case wherein there is no ambiguity in that the bank knows the objective CDF of $\tilde{R}$, i.e., $F(R|\theta) =$

$F^\circ(R)$ for all $R \in [\underline{R}, \overline{R}]$ and $\theta \in [\underline{\theta}, \overline{\theta}]$. Eq. (10) as such reduces to

$$\mathrm{E}_{F^\circ}\{u'(\tilde{W}^\circ)[\tilde{R} - \alpha R_E - (1-\alpha)R_D - C'(L^\circ)]\} = 0, \tag{21}$$

where a nought ($^\circ$) indicates an optimal level in the absence of ambiguity. Using Eq. (5), we can write Eq. (21) as

$$\int_{\underline{\theta}}^{\overline{\theta}} U_L(L^\circ, \theta)\mathrm{d}G(\theta) = 0. \tag{22}$$

If we set $\phi(U) = U$ and $K(\cdot) = \phi(\cdot)$, then the comparison between L° and L^* is identical to that between L^* and $L^\dagger$. Hence, we have the following proposition.

Proposition 2. *Introducing ambiguity to the ambiguity-averse competitive bank reduces the optimal amount of loans, i.e., $L^* < L^\circ$, if the bank's coefficient of relative risk aversion, $\Phi(W) = -Wu''(W)/u'(W)$, does not exceed unity, and if the parameter, θ, always deteriorates (improves) the subjective cumulative distribution function, $F(R|\theta)$, in the sense of first-order stochastic dominance.*

Since the results of Propositions 1 and 2 have similar underlying reasons, we focus on the intuition for Proposition 2. As in Gollier (2011), Taboga (2005), and Wong (2015), we define the following function:

$$H(\theta) = \int_{\underline{\theta}}^{\theta} \frac{\phi'[U(L^\circ, X)]}{\mathrm{E}_G\{\phi'[U(L^\circ, \tilde{\theta})]\}}\mathrm{d}G(X) = \frac{\mathrm{E}_G\{\phi'[U(L^\circ, \tilde{X})]|\tilde{X} \leq \theta\}}{\mathrm{E}_G\{\phi'[U(L^\circ, \tilde{\theta})]\}}G(\theta), \tag{23}$$

for all $\theta \in [\underline{\theta}, \overline{\theta}]$, where $\mathrm{E}_G(\cdot|\cdot)$ is the conditional expectation operator with respect to the second-order CDF, $G(\theta)$. It is evident from Eq. (23) that $H(\underline{\theta}) = 0$, $H(\overline{\theta}) = 1$, and $H'(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ so that we can interpret $H(\theta)$ as the distorted second-order CDF of $\tilde{\theta}$. From Eqs. (10), (11), (22), and (23), we have $L^* < L^\circ$ if, and only if,

$$\int_{\underline{\theta}}^{\overline{\theta}} U_L(L^\circ, \theta)\mathrm{d}G(\theta) = 0 \;\Rightarrow\; \int_{\underline{\theta}}^{\overline{\theta}} U_L(L^\circ, \theta)\mathrm{d}H(\theta) < 0. \tag{24}$$

We can interpret the left-hand side of condition (24) as the first-order condition for program (7) when the bank is ambiguity neutral. The right-hand side of condition (24) is then the condition that ensures the ambiguity-neutral bank to reduce its optimal amount of loans when the second-order CDF of $\tilde{\theta}$ is shifted from $G(\theta)$ to $H(\theta)$. Hence, the effect of introducing ambiguity on the ambiguity-averse bank's lending decision is identical to the comparative static result of a shift in ambiguity from $G(\theta)$ to $H(\theta)$ on the ambiguity-neutral bank's optimal amount of loans.

Using Leibniz's rule, we have

$$\frac{\partial}{\partial\theta}\mathrm{E}_G\{\phi'[U(L^\circ,\tilde{X})]|\tilde{X}\leq\theta\} = \frac{G'(\theta)}{G(\theta)}\Big\{\phi'[U(L^\circ,\theta)] - \mathrm{E}_G\{\phi'[U(L^\circ,\tilde{X})]|\tilde{X}\leq\theta\}\Big\}, \quad (25)$$

which, from Eq. (15), is positive (negative) if an increase in θ deteriorates (improves) $F(P|\theta)$ in the sense of first-order stochastic dominance, i.e., $F_\theta(R|\theta) > (<)\ 0$ for all $\theta \in [\underline{\theta},\overline{\theta}]$. It then follows from Eqs. (23) and (25) that $H(\theta) < (>)\ G(\theta)$ for all $\theta \in (\underline{\theta},\overline{\theta})$ so that $H(\theta)$ dominates (is dominated by) $G(\theta)$ in the sense of first-order stochastic dominance if $F_\theta(R|\theta) > (<)\ 0$ for all $\theta \in [\underline{\theta},\overline{\theta}]$. In this case, condition (24) holds if, and only if, $U_L(L^\circ,\theta)$ is decreasing (increasing) in θ, which, from Eq. (16), is true if the bank's coefficient of relative risk aversion does not exceed unity, i.e., $\Phi(W) \leq 1$.

4 Capital requirement and the bank's risk-taking

In this section, we examine the effect of a more stringent capital requirement on the bank's lending decision. To this end, we totally differentiate Eq. (10) with respect to α to yield

$$\frac{\mathrm{d}L^*}{\mathrm{d}\alpha} = \Big\{\int_{\underline{\theta}}^{\overline{\theta}}\phi''[U(L^*,\theta)]\mathrm{E}_F[u'(\tilde{W}^*)|\theta]U_L(L^*,\theta)L^*\mathrm{d}G(\theta)$$

$$-\int_{\underline{\theta}}^{\overline{\theta}}\phi'[U(L^*,\theta)]\mathrm{E}_F\{u'(\tilde{W}^*)[\Phi(\tilde{W}^*)-1] + u''(\tilde{W}^*)[C'(L^*)L^* - C(L^*)]|\theta\}\mathrm{d}G(\theta)\Big\}$$

$$\times(R_E - R_D)\Big/\frac{\partial^2}{\partial L^2}\int_{\underline{\theta}}^{\overline{\theta}}\phi[U(L,\theta)]\mathrm{d}G(\theta)\Big|_{L=L^*}. \quad (26)$$

Since $R_D < R_E$, it follows from Eq. (11) that $\mathrm{d}L^*/\mathrm{d}\alpha < (>)\ 0$ if the expression inside the curly brackets on the right-hand side of (26) is positive (negative).

We say that the bank's smooth ambiguity preferences exhibit non-increasing absolute ambiguity aversion if the bank's coefficient of absolute ambiguity aversion, $-\phi''(U)/\phi'(U)$, is a non-increasing function of U. In the following proposition, we derive sufficient conditions under which $\mathrm{d}L^*/\mathrm{d}\alpha < 0$.

Proposition 3. *Given that the competitive bank's smooth ambiguity preferences exhibit non-increasing absolute ambiguity aversion, imposing a more stringent capital requirement to the bank reduces the optimal amount of loans, i.e.,* $\mathrm{d}L^*/\mathrm{d}\alpha < 0$, *if the bank's coefficient of relative risk aversion,* $\Phi(W) = -Wu''(W)/u'(W)$, *does not exceed unity, and if an increase in the parameter,* θ, *always deteriorates (improves) the subjective cumulative distribution function,* $F(R|\theta)$, *in the sense of first-order stochastic dominance.*

Proof. Since $C(0) = 0$, the strict convexity of $C(L)$ implies that $C'(L) > C(L)/L$ for all $L > 0$. It then follows from $\Phi(W) \leq 1$ that the second term inside the curly brackets on right-hand side of Eq. (26) must be positive. It remains to show that the first term inside the curly brackets on right-hand side of Eq. (26) is also positive.

Consider the function:

$$\Psi(\theta) = \frac{\phi''[U(L^*,\theta)]\mathrm{E}_F[u'(\tilde{W}^*)|\theta]}{\phi'[U(L^*,\theta)]}. \tag{27}$$

Differentiating Eq. (27) with respect to θ yields

$$\Psi'(\theta) = -\frac{\phi''[U(L^*,\theta)]}{\phi'[U(L^*,\theta)]}\bigg\{\int_{\underline{R}}^{\overline{R}} u''(W^*)L^*F_\theta(R|\theta)\mathrm{d}R$$

$$+\bigg\{\frac{\phi'''[U(L^*,\theta)]}{\phi''[U(L^*,\theta)]} - \frac{\phi''[U(L^*,\theta)]}{\phi'[U(L^*,\theta)]}\bigg\}\mathrm{E}_F[u'(\tilde{W}^*)|\theta]\int_{\underline{R}}^{\overline{R}} u'(W^*)L^*F_\theta(R|\theta)\mathrm{d}R\bigg\}. \tag{28}$$

Since $\phi(U)$ exhibits non-increasing absolute ambiguity aversion, Eq. (28) implies that $\Psi'(\theta) < (>)\ 0$ if an increase in θ always deteriorates (improves) $F(R|\theta)$ in the sense of first-order stochastic dominance.

Consider first the case that an increase in θ always deteriorates $F(R|\theta)$ in the sense of first-order stochastic dominance. Since $\Phi(W) \leq 1$, Eq. (16) implies that $\partial U_L(L^*,\theta)/\partial\theta < 0$. There are three possible cases: (i) $U_L(L^*,\theta) > 0$ for all $\theta \in [\underline{\theta},\overline{\theta})$, (ii) $U_L(L^*,\theta) < 0$ for all $\theta \in (\underline{\theta},\overline{\theta}]$, and (iii) there exists a unique point, $\theta_1 \in (\underline{\theta},\overline{\theta})$ such that $U_L(L^*,\theta) > (<)\ 0$ for all $\theta < (>)\ \theta_1$. In case (i), the first term inside the curly brackets on right-hand side of Eq. (26) must be negative. Using Eqs. (10) and (27), we can write

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi''[U(L^*,\theta)]\mathrm{E}_F[u'(\tilde{W}^*)|\theta]U_L(L^*,\theta)L^*\mathrm{d}G(\theta)$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} [\Psi(\theta) - \Psi(\overline{\theta})]\phi'[U(L^*,\theta)]U_L(L^*,\theta)\mathrm{d}G(\theta). \tag{29}$$

Since $\Psi'(\theta) < 0$, we have $\Psi(\theta) > \Psi(\overline{\theta})$ for all $\theta \in [\underline{\theta},\overline{\theta})$. It then follows from $U_L(L^*,\theta) > 0$ for all $\theta \in [\underline{\theta},\overline{\theta})$ that the right-hand side of Eq. (29) is positive, a contradiction. Hence, case (i) cannot hold. Analogously, we can show that case (ii) cannot hold. In case (iii), we can use Eqs. (10) and (27) to write

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi''[U(L^*,\theta)]\mathrm{E}_F[u'(\tilde{W}^*)|\theta]U_L(L^*,\theta)L^*\mathrm{d}G(\theta)$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} [\Psi(\theta) - \Psi(\theta_1)]\phi'[U(L^*,\theta)]U_L(L^*,\theta)\mathrm{d}G(\theta). \tag{30}$$

Since $\Psi(\theta) > (<)\ \Psi(\theta_1)$ and $U_L(L^*,\theta) > (<)\ 0$ for all $\theta < (>)\ \theta_1$, the right-hand side of Eq. (30) is positive.

Consider now the case that an increase in θ always improves $F(R|\theta)$ in the sense of first-order stochastic dominance. Since $\Phi(W) \leq 1$, Eq. (16) implies that $\partial U_L(L^*,\theta)/\partial\theta > 0$. There are three possible cases: (i) $U_L(L^*,\theta) > 0$ for all $\theta \in [\underline{\theta},\overline{\theta})$, (ii) $U_L(L^*,\theta) < 0$ for all $\theta \in (\underline{\theta},\overline{\theta}]$, and (iii) there exists a unique point, $\theta_1 \in (\underline{\theta},\overline{\theta})$ such that $U_L(L^*,\theta) < (>)\ 0$ for all $\theta < (>)\ \theta_1$. In case (i), the first term inside the curly brackets on right-hand side of Eq. (26) must be negative. Using Eqs. (10) and (27), we can write

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi''[U(L^*,\theta)]\mathrm{E}_F[u'(\tilde{W}^*)|\theta]U_L(L^*,\theta)L^*\mathrm{d}G(\theta)$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} [\Psi(\theta) - \Psi(\underline{\theta})]\phi'[U(L^*,\theta)]U_L(L^*,\theta)\mathrm{d}G(\theta). \tag{31}$$

Since $\Psi'(\theta) > 0$, we have $\Psi(\theta) > \Psi(\underline{\theta})$ for all $\theta \in (\underline{\theta}, \overline{\theta}]$. It then follows from $U_L(L^*,\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta})$ that the right-hand side of Eq. (31) is positive, a contradiction. Hence, case (i) cannot hold. Analogously, we can show that case (ii) cannot hold. In case (iii), we can use Eqs. (10) and (27) to write

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi''[U(L^*,\theta)]\mathrm{E}_F[u'(\tilde{W}^*)|\theta]U_L(L^*,\theta)L^*\mathrm{d}G(\theta)$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} [\Psi(\theta) - \Psi(\theta_2)]\phi'[U(L^*,\theta)]U_L(L^*,\theta)\mathrm{d}G(\theta). \tag{32}$$

Since $\Psi(\theta) < (>)\ \Psi(\theta_2)$ and $U_L(L^*,\theta) < (>)\ 0$ for all $\theta < (>)\ \theta_2$, the right-hand side of Eq. (32) is positive. □

The intuition for Proposition 3 is as follows. Since $R_E > R_D$, the bank regards equity capital to be more expensive than deposits. Raising the capital-to-loan ratio, α, reduces the net worth of the bank's shareholders at date 1. Since the bank's smooth ambiguity preferences exhibit non-increasing absolute ambiguity aversion, the bank becomes more ambiguity averse when the capital requirement is more stringent. It then follows from Proposition 1 that the bank reduces the optimal amount of loans in response to a more stringent capital requirement.

5. Conclusion

In this paper, we have examined the asset investment decision of a banking firm under uncertainty when the bank's preferences exhibit smooth ambiguity aversion developed by Klibanoff et al. (2005). The KMM model represents ambiguity by a second-order probability distribution that captures the bank's uncertainty about which of the subjective beliefs govern the asset return risk. On the other hand, the KMM model specifies ambiguity preferences by the (second-order) expectation of a concave transformation of the (first-order) expected utility of profit conditional on each plausible subjective distribution of the asset return rate risk. Within this framework and the industrial organization (IO) approach of banking, we have shown that the ambiguity-averse bank finds it less attractive to invest in the financial asset in the presence than in the absence of ambiguity, and with greater ambiguity aversion. The value of financial investment as such decreases when ambiguity and ambiguity aversion prevail.

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