Economic Engineering Modeling of Liberalized Electricity Markets: Approaches, Algorithms, and Applications in a European Context

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Abstract

This dissertation focuses on selected issues in regard to the mathematical modeling of electricity markets. In a first step the interrelations of electric power market modeling are highlighted a crossroad between operations research, applied economics, and engineering. In a second step the development of a large-scale continental European economic engineering model named ELMOD is described and the model is applied to the issue of wind integration. It is concluded that enabling the integration of low-carbon technologies appears feasible for wind energy. In a third step algorithmic work is carried out regarding a game theoretic model. Two approaches in order to solve a discretely-constrained mathematical program with equilibrium constraints using disjunctive constraints are presented. The first one reformulates the problem as a mixed-integer linear program and the second one applies the Benders decomposition technique. Selected numerical results are reported.

If you want to build a ship, don't drum up people to collect wood and don't assign them tasks and work, but rather teach them to long for the endless immensity of the sea.

Antoine de Saint-Exupéry

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List of Abbreviations

AC	Alternating Current
BE	Binary Expansion
BeNeLux	Belgium, The Netherlands, Luxembourg
BFR	Border Flow Right
bn	Billion
CA	California
CAPM	Capital Asset Pricing Model
CCGT	Combined Cycle Gas Turbine
CFD	Contract(s) for Differences
CGE	Computable General Equilibrium
CHP	Combined Heat and Power Plants
CO_2	Carbon Dioxide
comp	Competitive
CPU	Central Processing Unit
CTP	Centralized Transmission Planning
DC	Direct Current
DCLF	DC Load Flow Model
DEWI	Deutsches Windenergie Institut
DWD	Deutscher Wetterdienst
EC	European Commission
EDF	Electricité de France
EEX	European Energy Exchange
ELMOD	European Electricity Market Model
EnBW	Energie Baden-Württemberg
EPEC	Equilibrium Problem with Equilibrium Constraints
ETS	Greenhouse Gas Emission Trading Scheme
EU	European Union
EU ETS	European Union Greenhouse Gas Emission Trading Scheme
EWEA	European Wind Energy Association
ext	Extended

FL	Florida
FTR	Financial Transmission Right
GA	Georgia
GAMS	General Algebraic Modeling System
GB	Gigabyte(s)
GDP	Gross Domestic Product
GHz	Gigahertz
GW	Gigawatt(s)
HVDC	High Voltage Direct Current
IDAE	Instituto para la Diversificación y Ahorro de la Energía
IEA	International Energy Agency
IL	Illinois
ISO	Independent System Operator
k	Kilo
KKT	Karush-Kuhn-Tucker
km	Kilometer(s)
km^2	Square Kilometer(s)
kV	Kilovolt(s)
LCP	Linear Complementarity Problem
LMP	Locational Marginal Pricing
LP	Linear Program
MCP	Mixed Complementarity Problem
MILP	Mixed-Integer Linear Program
MINLP	Mixed-Integer Nonlinear Program
MLCP	Mixed Linear Complementarity Problem
mn	Million
MP	Master Problem
MPCC	Mathematical Program with Complementarity Constraints
MPEC	Mathematical Program with Equilibrium Constraints
MTI	Merchant Transmission Investment
MW	Megawatt(s)
MWh	Megawatt Hour(s)
NJ	New Jersey
NO_x	Nitrogen Oxides
NP	Nondeterministic Polynomial
NUTS	Nomenclature des Unités Territoriales Statistiques
NY	New York
OECD	Organization for Economic Co-operation and Development
OPF	Optimal Power Flow
OR	Operations Research

PA	Pennsylvania
PIPA	Penalty Interior Point Algorithm
PJM	Pennsylvania New Jersey Maryland Interconnection Association
PNP	Proactive Network Planning
PSE	Polskie Sieci Elektroenergetyczne Operator S.A.
PSP	Pumped Storage Plant
PTDF	Power Transfer Distribution Factor
RAM	Random Access Memory
ROI	Return on Investment
s.t.	Subject to
SFE	Supply Function Equilibrium
SMD	Standard Market Design
SO	System Operator
SP	Subproblem
SQP	Sequential Quadratic Programming
strat	Strategic
t	Ton(s)
TCC	Transmission Congestion Contract
TSO	Transmission System Operator
U.S.	United States
UC	Unit Commitment
UCTE	Union for the Co-ordination of Transmission of Electricity
UK	United Kingdom
USA	United States of America
VDEW	Vereinigung Deutscher Elektrizitätswerke e.V.
VDI	Verband Deutscher Ingenieure
VI	Variational Inequality
vs.	Versus
WACC	Weighted Average Cost of Capital
WEO	World Energy Outlook
WF12	Wind Force 12
y	Year
ZEW	Zentrum für Europäische Wirtschaftsforschung

Part I

Overview

Chapter 1

Introduction and Summary

1.1 Introduction

> Electricity was painful to regulate; there is no reason to believe that it can easily be deregulated (Smeers, 2003a, p. 172).

Electricity markets around the world are still in a state of flux, even two decades (the UK market), one decade (for some U.S. markets) or a couple of years (continental Europe) into the reform process. In Europe, the reform momentum has accelerated in the second half of this decade. In fact, the 'Acceleration Directive' (2003/54/EC) has been followed by a more coherent attempt of moving toward a single European market. Yet central reform steps such as vertical unbundling, incentives for cross-border transmission investment, and the integration of large-scale renewable electricity into the network are still in the making. Evidence of this process is provided by the discussions of the '3rd Energy Package' of the European Union, providing energy policy guidelines for the next decade.

In general, the objective of electricity market reforms is to replace monopolistic structures with competition and - where natural monopolies prevail - with more efficient regulation. In Europe, several Directives were issued since 1996 to advance on this reform path. In addition, the discussion of climate change has added further elements to energy policy, such as the European Emissions Trading System (ETS), and the ambitious targets for electricity from renewable energy sources, mainly wind. Thus, Germany and Spain have introduced generous feed-in tariffs for onshore and offshore wind energy that the network operators have to integrate in their network management.

All of the mentioned aspects in the course of restructuring the electricity industry are accompanied by the development of different models that intend to represent a certain aspect of the overall picture. Due to the time lag between the liberalization process in the U.S. and continental Europe, the developments of adequate scientific European market models is also delayed. Many models are 'imported' from the U.S. and adjusted to the European situation. However, some of the continental European specifics such as differing regional congestion management combined with different interregional coordination schemes can not easily be modeled based on experiences from abroad. Moreover, a reconsideration of sophisticated existing models and their usefulness for larger-scale applications currently takes place which is due to technological advances of computation capacities. All in all, there is a strong interest of firms, regulators and scientists in electricity market models taking into account the new challenges of liberalization as well as changing generation and demand structures.
Part II of this thesis develops ELMOD, a large-scale perfect competition economic engineering model of the continental European electricity market taking into account technical restrictions in terms of power plant start-up and load flow calculation (Chapter 3). ELMOD is then applied to the question of transmission expansion in combination with additional wind energy generation (Chapter 4). Also other study results applying ELMOD are reported.
Part III of this thesis turns towards the issues of game theoretic modeling. Two different algorithmic approaches are developed to solve a MPEC problem resulting from a Stackelberg game in an electricity market. The first approaches applies techniques in order to transform the problem in a MIP problem (Chapter 5). The second approach applies Benders decomposition technique (Chapter 6) to the same problem.

1.2 Summary

1.2.1 Classification and Research Area

Chapter 2 provides an introduction to the field of mathematical modeling and the various fields of research in electricity markets. It is shown that modeling electricity markets is a melting pot of researchers from different disciplines that all bring their own methods and skills such as operations research, applied economics, and engineering. According to this result, the work at hand can can be located in the area of deterministic partial equilibrium oriented fundamental market modeling approaches under technical

constraints.
The chapter concludes that under the assumption of perfect competition, there is need for large-scale economic engineering models that includes the entire potentially integrated European market in order to contribute to the ongoing discussions on congestion management and investments. Concerning the modeling under the assumption of imperfect electric power markets, there is need for new algorithms that contribute to overcome numerical difficulties in solving medium- and larger-scale strategic models. Hence, the thesis takes on some of this challenges by firstly developing a large-scale economic engineering network model of the EU named ELMOD and secondly carrying out algorithmic work in order to solve a discretely constrained MPEC representing a Stackelberg game in an electric power market including transmission constraints.

1.2.2 Large-Scale Perfect Competitive Economic Engineering Modeling

ELMOD - A Model of the European Electricity Market

Throughout Chapter 3, a structured procedure is elaborated to establish a well working electric power market model that produces meaningful scientific results. First of all, an economic market model is defined that allows to analyze the identified research objectives. The objective function ELMOD is welfare maximization. Then, the representation of the technical specifics are included which constrain the economic model in some way. The technical constraints defined the required technical data. ELMOD is capable of including various types of technical constraints such as transmission constraints (using the DC Load Flow model) and unit-commitment of power plants. Third, the degree of detail had to be defined. On the one hand, the degree of aggregation of data is supposed to be detailed enough to produce meaningful results for a defined research objective. On the other hand, the technical effort to solve the complete model decreases with the degree of aggregation which is an incentive to keep the model as small as possible. In order to thoroughly include all regions of a potentially integrated European electricity market, ELMOD embraces the entire UCTE grid and gathers data that is required to do this reliably. Hence, the demand is modeled in the detail of EU NUTS 2 and 3, respectively, varying by country. Consumption is divided in three load types: households, small businesses, and industry. The demand distribution is conducted by weighting different regions with their GDP and applying standard load profiles. Supply is represented by

power plants above 100 MW installed capacity as well as a detailed data set of existing wind turbines.
On the basis of selected existing studies using ELMOD, it is shown that ELMOD is a valuable tool in terms of analyzing the effect of offshore wind power on the North-West European electricity market, and the effects of congestion between countries and within the German grid. In addition, ELMOD can also be applied to generation investment issues, namely, the siting of new power plants under grid constraints.

When the Wind Blows over Europe

Chapter 4 provides a large-scale application of ELMOD to the issue of wind expansion in Europe. The ELMOD model is complemented by a grid expansion algorithm based on economic principles.
The analysis shows that efforts to prepare Europe's high voltage grid for large amounts of wind generation appear to be rather modest. Developing the network at existing bottlenecks - mainly cross-border connections - should be encouraged by regulatory authorities. With a more moderate wind expansion of 114.5 GW, the optimal grid investments are smaller. However, if the additional wind capacity becomes too great (181 GW), the needed grid extensions will increase compared to the actual situation. 'Greening the grid', i.e. enabling the integration of low-carbon technologies, appears feasible for wind energy.

1.2.3 Game Theoretic Economic Engineering Modeling

Solving Discretely-Constrained MPEC Problems Using Disjunctive Constraints and Discrete Linearization

Chapter 5 moves away from the assumption of a perfect competitive European electricity market. It argues that several models and algorithms have been developed in order to simulate the outcomes of imperfect electricity markets. These models include approaches using game theory. It is stated that existing modeling efforts have achieved some success but there is still room to handle larger-scale or more realistic models as might be found in the EU, North America or in other parts of the world. Hence, a new approach to solve two-stage Stackelberg games with one leader based on disjunctive constraints and discrete linearization is presented. The approach replaces the equilibrium constraints of a mathematical problem with equilibrium constraints by integer restrictions in the form of disjunctive constraints. Also, a bilinear objective function of an electricity market model stemming from the

product of both price and generation variables is linearized using additional binary and continuous variables and new constraints. The result is that the mathematical problem with equilibrium constraints can be replaced by a mixed-integer linear program. This allows for a host of important applications such as: discrete generation levels, fixed cost problems involving binary variables, if-then logic relative to ramping constraints, discrete investment levels, and so on. A second advantage of using the presented method is to be able to solve larger-scale problems in electric power markets than previously attempted. Lastly, a detailed formulation of the DC load flow model is used in order to model physical flows which facilitates a greater flexibility for changing the network topology.
The numerical results on two illustrative problems are promising. Particularly, it is shown that network effects have a significant effect on the strategic behavior of a generator. If the strategic output decision produces too high prices, those competitive fringe companies have an incentive to produce at marginal cost that are located at the same nodes as the strategic player. Otherwise, competitive players can be excluded via gaming over network effects. However, for the time being the calculation times for a 15-node network can become high depending on the number of discrete/binary variables.

Solving Discretely-Constrained MPEC Problems Using Disjunctive Constraints and Benders Decomposition

Chapter 6 starts from the same assumptions, mathematical model, and network as used in Chapter 5. The aim is now to find a new algorithmic approach that can handle larger models computationally better than the one presented in Chapter 5. The approach taken in this chapter applies the Benders decomposition technique. In order to apply the Benders technique, the mathematical problem with equilibrium constraints is decomposed into a linear master problem and a mixed integer linear subproblem. These two problems are then solved sequentially in order to find a solution to the original problem. This approach is applied to a three-node network also used in a first step of Chapter 5. First calculations are promising. Both approaches can be calibrated to produce the same results. However, due to the subproblem structure of the considered Benders approach, the convergence cannot be guaranteed in general. An enumeration approach is applied for illustrative reasons to evaluate the robustness of the numerical example results. Enumeration is normally not doable for larger-scale problems. Hence, the development of a (dynamic) domain decomposition method is required to make the approach work in general which is, however, out of the scope of this thesis.

1.2.4 Outlook

There are several directions for valuable further research. The most obvious one is the further development of the Benders decomposition approach for solving a mathematical problem with equilibrium constraints based on Chapter 6. Speeding up the problem of Chapter 5 could contribute to the solving of large-scale strategic models. Also, one of these approaches might help to solve equilibrium problems with equilibrium constraints. On the perfect competition side, it could be worthwhile to include stochastic elements into ELMOD in order to make the modeling of wind input more realistic.

Chapter 2

Literature on Modeling Electricity Markets

This chapter addresses the issue that energy market research cannot be easily classified according to the standard disciplines by simply looking at its research objectives and methods. Hence, the different disciplines involved in electricity market research are discussed building a general framework for the present work.

2.1 General Framework

2.1.1 Modeling

Research is a way to create or apply new knowledge. In his survey on the research theoretic rationale of modeling, Wierzbicki (2007) argued that the world economy is on a way towards a knowledge-based economy. Building on this trend, the verification and structuring of information become increasingly important but is very often still an unsolved issue. According to Wierzbicki (2007), there are several forms of representing knowledge. Apart from mathematical and computerized models, there are the traditional form of a text with illustrations, and the more contemporary multimedia form. Out of these three forms, particularly mathematical modeling has gained disproportionate greater importance over the last decades.
According to Murthy and Rodin (1987, p. 17), the nature of a model can be defined as following:

> A model is a representation of a system (or object, or phenomenon). The model is called an adequate one if it is appropri-

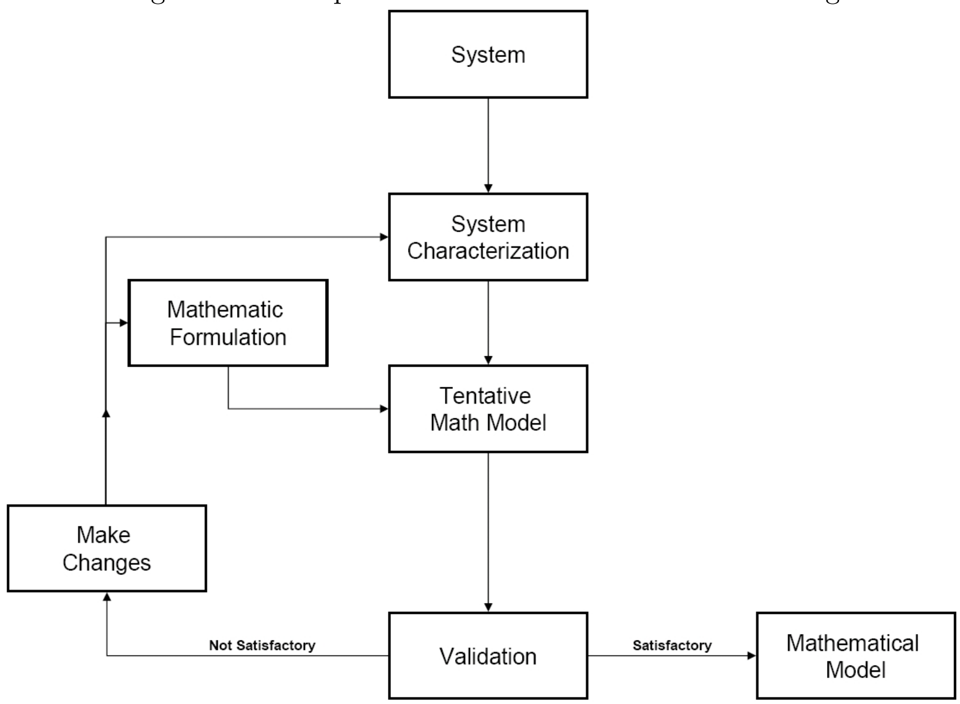
> ate for the purpose (or goal) in the mind of the model builder. Otherwise it is called an inadequate model.

Moreover, a mathematical model makes use of patterns and logic rules (complemented by other elements) in order to serve its purpose. Hence, a mathematical model is a representation of a real system (or object, or phenomenon) that is described using the mathematical methodology. A mathematical model is made up of a symbolic model representation including an abstract mathematical formulation (Murthy and Rodin, 1987). Figure 2.1 shows in a simplified manner the process of mathematical model building. An important issue is that the system to be considered has to be characterized first in terms of variables, parameters and relationships. These relationships describe interdependencies between the variables of a system, e.g. in the form of mathematical functions. Then a mathematical model can be formulated by means of abstract mathematical formulations - if necessary in an iterative procedure - that is able to adequately represent the system under observation.
The link between mathematical modeling and electricity market research, becomes evident considering the literature described subsequently. There, mathematical models of various types play a major role. A large part of the literature specified in Section 2.2 builds, uses or applies mathematical models; reviews different models and their applications; develops and tests algorithms for solving mathematical problems in order to produce adequate model results.

2.1.2 Delimitation

There are three groups of researchers involved in the research on electricity markets: mathematicians, engineers, and economists. In order to understand how the different disciplines interact, one can make use of the idea of linear one-point perspective in painting. The challenge of an adequate perspective is to project a three dimensional real object onto a two dimensional canvas. There is one point in which all lines intersect and that is the position from which the object is beheld. However, if the painter changes its position, the painting can look very different even though the object itself has not changed. The similar is true for electricity market research. The research object is the same but it is approached from different disciplines which have different viewpoints. Furthermore, all researchers involved have to cope with the challenge to represent the (higher dimensional) reality by a simplifying (lower dimensional) model.

Figure 2.1: The process of mathematical model building



Source: Murthy and Rodin (1987, p. 18).

Mathematics

Applied Mathematics is a branch of mathematics that transforms recent methodical and systematic mathematical knowledge into algorithms and models for real applications. Accordingly, the notion, methods and tools developed in applied mathematics are widely used in other disciplines. A relevant field within applied mathematics is Operations Research[1] (OR). The definitions of OR are manifold. A general common denominator is that OR applies scientific methods in order to facilitate decision making (Hillier and Lieberman, 1986; Neumann and Morlock, 1993; Ravindran, 2007). For this purpose, OR develops and applies appropriate algorithms. Murthy and Rodin (1987) structured the topics associated with building an applicable

[1] The term Operational Research is a synonym for Operations Research and can be used interchangeably. Furthermore, the author could not find significant differences between the fields of Management Science and Operations Research with regard to research contents and methodologies (compare Ravindran, 2007). Hence, the same argumentation provided for OR also applies for the field of Management Science.

mathematical model and emphasize two topics: different types of mathematical formulations and the analysis of mathematical formulations. They identify three different types of formulations:

- Nondynamic formulations (deterministic/probabilistic)
- Dynamic deterministic formulations
- Dynamic stochastic formulations

whereby the term dynamic indicates whether the formulation includes a process, i.e. time interdependencies (Gellert et al., 1975). Furthermore, stochastic formulations include the uncertainty of parameters within the model, e.g., via probability distributions. The types of the mathematical formulations result in different mathematical problems steaming from real applications, e.g., from the electricity industry. Throughout this thesis, a certain terminology for mathematical problems will be maintained that bases upon the mathematical and OR terminology:

LP

Linear programs (LP) are optimization problems that aim to minimize[2] a linear function with n variables constrained by a set of linear equalities and/or inequalities (Castillo et al., 2002).

Definiton 2.1 (Following Castillo et al., 2002) *Let F be a mapping from $\mathbb{R}^n$ into $\mathbb{R}$. The general form of a linear programming problem is to minimize*

$$F(x) = \sum_{j=1}^{n} c_j x_j \tag{2.1a}$$

subject to

$$A_1 x = b_1 \tag{2.1b}$$
$$A_2 x \leq b_2 \tag{2.1c}$$

where A_1, A_2 are matrices of suitable size conformal with the vector x and right-hand sides b_1, b_2. ■

[2] Note that each minimization problem can be converted into a maximization problem by multiplication with -1. The same is true to convert a '$\geq$' into a '$\leq$' constraint. See Castillo et al. (2002, chap. 13) for more conversions of this type.

NLP

Nonlinear programs (NLP) are the general form of optimization problems that aim to minimize a mathematical function constrained by a set of equalities and/or inequalities. For the problem to be nonlinear, at least one of the functions involved in the formulation must be nonlinear (Castillo et al., 2002).

Definiton 2.2 (Following Castillo et al., 2002) *Let F be a mapping from $\mathbb{R}^n$ into $\mathbb{R}$. The general form of a mathematical programming problem is to minimize*

$$F(x) \tag{2.2a}$$

subject to

$$H(x) = 0 \tag{2.2b}$$

$$G(x) \leq 0 \tag{2.2c}$$

where x is the vector of decision variables, $F(x)$ is the objective function, $H(x)$ are the equality constraints, and $G(x)$ are the inequality constraints. ■

MIP

Mixed-integer programs (MIP) can be linear or nonlinear optimization problems including continuous and discrete (integer) variables. However, mixed-integer nonlinear programs (MINLP) are normally unfavorable in their numerical behavior and are, thus, often subject to reformulations and simplifications. Hence, here the focus will be on mixed-integer linear programs (MILP).

Definiton 2.3 (Following Castillo et al., 2002) *Let F be a mapping from $\mathbb{R}^n$ into $\mathbb{R}$. The general form of a mathematical mixed-integer programming problem is to minimize*

$$F(x) = \sum_{j=1}^{n} c_j x_j \tag{2.3a}$$

subject to

$$A_1 x = b_1 \tag{2.3b}$$

$$A_2 x \leq b_2 \tag{2.3c}$$

$$x_j \in \mathbb{N}; \textit{for at least one } j = 1, ..., n \tag{2.3d}$$

where $\mathbb{N}$ is the set of natural numbers $\{0, 1, 2, ...\}$. ■

MCP

A mixed complementarity problem (MCP) is a special case of a complementarity problem which in turn is a special case of a variational inequality (VI).[3]

Definiton 2.4 (Facchinei and Pang, 2003, chap. 1) *Let G and H be two mappings from $\mathbb{R}^{n_1} \times \mathbb{R}_+^{n_2}$ into $\mathbb{R}^{n_1}$ and $\mathbb{R}^{n_2}$, respectively. The MCP(G, H) is to find a pair of vectors (u, v) belonging to $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ such that*

$$G(u, v) = 0,\ u \text{ (free)} \tag{2.4a}$$

$$0 \leq v \perp H(u, v) \geq 0 \tag{2.4b}$$

where the notation $\perp$ in (2.4b) means 'perpendicular' which determines the complementarity of the elements of the two vectors v and $H(u, v)$:

$$0 \leq v \tag{2.5a}$$

$$H(u, v) \geq 0 \tag{2.5b}$$

$$v_i H_i(u, v) = 0,\ \forall i = 1, ..., n_2. \tag{2.5c}$$

■

KKT

The Karush-Kuhn-Tucker (KKT) optimality conditions are necessary optimality conditions in mathematical programming. They are the basis for the development of many computational solution algorithms (Castillo et al., 2002).

Definiton 2.5 (Castillo et al., 2002, chap. 8) *The vector $\bar{x} \in \mathbb{R}^n$ satisfies the KKT conditions for the NLP (2.3) if there exists a pair*

[3] The following definitions can be found in Facchinei and Pang (2003, chap. 1): Given a subset K of the Euclidean n-dimensional space $\mathbb{R}^n$ and a mapping $F : K \to \mathbb{R}^n$, a VI is to find a vector $x \in K$ such that $(y - x)^T F(x) \geq 0,\ \forall y \in K$. When K is a cone, a complementarity problem is to find a vector $x \in K$ satisfying the following conditions: $K \ni x \perp F(x) \in K^*$, where K^* is the dual cone of K defined as: $K^* \equiv \{d \in \mathbb{R}^n : v^T d \geq 0,\ \forall v \in K\}$; that is, K^* consists of all vectors that make a non-obtuse angle with every vector in K.

of vectors $\mu \in \mathbb{R}^{n_1}$ *and* $\lambda \in \mathbb{R}^{n_2}$ *such that*

$$\nabla F(\bar{x}) + \sum_{k=1}^{n_2} \lambda_k \nabla H_k(\bar{x}) + \sum_{j=1}^{n_1} \mu_j \nabla G_j(\bar{x}) = 0 \tag{2.6a}$$

$$H_k(\bar{x}) = 0, \ \forall k \tag{2.6b}$$

$$G_j(\bar{x}) \leq 0, \ \forall j \tag{2.6c}$$

$$\mu_j G_j(\bar{x}) = 0, \ \forall j \tag{2.6d}$$

$$\mu_j \geq 0, \ \forall j \tag{2.6e}$$

■

MPEC

A mathematical program with equilibrium constraints (MPEC) is an optimization problem with two different sets of variables where one set of these variables is a solution to another mathematical problem. In other words, a MPEC is an optimization problem that is constrained by another mathematical problem. The actual optimization is called the upper problem or upper level problem whereas the constraining problem is called lower problem, lower level problem, or inner problem (Luo et al., 1996). Specifically, a MPEC is an optimization problem that is constrained by a VI. More formally:

Definiton 2.6 (Following Luo et al., 1996; Ehrenmann, 2004) *Let* F *be a mapping from* $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ *into* $\mathbb{R}$. *The general form of a MPEC is to minimize*

$$F(x, y) \tag{2.7a}$$

subject to

$$(x, y) \in Z \tag{2.7b}$$

$$y \in S(x) \tag{2.7c}$$

where $F(x, y)$ *is the objective function that depends on a vector of design variables* x *and a vector of state variables* y. Z *is a nonempty and closed set describing the feasible space of the upper level while* $S(x)$ *is the solution set of a VI* $(Q(x, \cdot), C(x))$. Q *is the equilibrium function of the lower level and* C *is a set valued mapping that maps* x *onto a non-empty closed convex subset of* $\mathbb{R}^{n_2}$. ■

The KKT system (2.6) has the same structure as a complementarity problem laid out in (2.5). Hence, an optimization problem can be expressed as a set of complementarity conditions (Hobbs and Helman, 2004). Consequently, one can replace the VI constraints in the MPEC by a complementarity problem which is a special case of a VI. Ehrenmann (2004) refers to the resulting problem as a mathematical program with complementarity constraints (MPCC). However, as this is a special case of a MPEC, in this thesis the term MPCC is not used.

EPEC

Equilibrium problems with equilibrium constraints (EPEC) can be understood as mathematical problems where more than one MPEC has to be solved at the same time. Hence, the aim is to find an equilibrium point that solves several MPECs that share the same equilibrium constraints. Refer for example to Ehrenmann (2004) for a deeper discussion.

The descriptions of different types of mathematical problems above are solely definitions. However, in order to challenge real applications, one must also analyze and solve these problems. Murthy and Rodin (1987) distinguished that the analysis of the mathematical formulations can make use of three different types of methods:

- Analytical methods
- Computational methods
- Simulation methods

whereby the required method is often already determined by the type of the mathematical problem. Hence, not all methods can be applied to solve all possible problem types. Particularly analytical methods require functions that can be solved in closed form otherwise numerical methods have to be applied (e.g., Castillo et al., 2002). These numerical methods split up into computational and simulation methods. However, the notation is not always consistent. Neumann and Morlock (1993) defined simulation as the imitation of the reality using computers. According to this definition a differentiation between computational and simulation methods is fuzzy. This definition describes the broader and rather common sense of the term simulation which could also be described as producing numerical model results. In contrast, Hillier and Lieberman (1986) provided a narrower definition of simulation. In their sense a simulation model describes the overall behavior of a complex system in terms of the individual components of a system and their interrelationships. One can conceive these components as a type of black boxes (for the simulation itself) that receive certain inputs which are transformed into certain outputs. The transformation processes can be described by mathematical functions. Hence, simulation in the narrow sense is to define interesting states of the system and simulate them in terms of inputs and outputs which can be seen as a sampling of experimental results. The latter is necessary if the entire system is too complex to compute or enumerate all required solutions.[4]

Engineering

Christie et al. (2000) stated that still in the year 1988 basically all electricity markets around the world were structured in the same way as regional monopolies: a centralized utility operated the entire power system (generation, transmission, and distribution) within a fixed geographic area. According to Stoft (2002), this monopolistic structures stemmed from the pioneering days of electricity at the end of the 19th century and were at that time more efficient than competition. During these 100 years of 'efficient monopolies', *Engineers* have defined the agenda for modeling needs and topics in the energy industry. The engineering disciplines involved in energy modeling are mainly mechanical and electrical engineering whereas one can roughly distinguish that mechanical engineers focus on generation aspects, and electrical engineers focus on the transmission aspects. The entire chain from electricity production to serving the final customer is subsumed under the term power systems including mechanical and electrical engineering topics (compare Bergen and Vittal, 2000; Machowski et al., 2008) as well as economic considerations (Stoft, 2002). The engineering research objectives include a wide range concerning the description, development and modeling of apparatuses, machines and components required to maintain and operate the power system based upon physical laws. Specific foci are (Bergen and Vittal, 2000):

- Physical parameters of technical components
- Transmission-line modeling
- Transformer modeling

[4]Due to a huge variety, specific algorithms and applications shall not be discussed here. They will be mentioned throughout the thesis where applicable.

- Generator modeling
- Voltage control systems
- Network calculations and power flow analysis
- System stability and protection

As the electricity is in a constant state of flux - for several reasons such as the permanent requirement to balance supply and demand as well as the oscillating nature of alternating current (AC) - the modeling of a power system is characterized by dynamic formulations. Thereby, Machowski et al. (2008) distinguished different states of system operation. The steady state describes the state of normal operation (at a constant frequency), the transient and the subtransient states describe oscillating processes with decreasing amplitudes over time at high and very high frequencies that interfere with the normal operation. Accordingly, one branch of engineering models of power systems emphasizes the modeling of the effect of unforeseen events on the steady state, and the technical management of these effects. For this purpose, engineers must often make use of real-time models of the system in order to maintain system stability. As these models include dynamic elements and trigonometric functions, engineering models for realistically large applications can normally not be solved analytically. Therefore, engineers and mathematicians have developed a variety of reduction and transformation techniques to simplify power system models (Machowski et al., 2008, chap. 14) such that they can be solved by computation and simulation methods (Machowski et al., 2008, chaps. 11-13). These modeling efforts require a deep understanding of the technical and physical foundations of electricity.

Economic aspects in this context were subsumed under the rather simple idea of a cost minimization and cost reduction. One day ahead, a system cost minimizing unit commitment (UC) was carried out by the central utility. Unit commitment describes the process of predefining which generation units will be connected to the grid in a certain point of time taking into account the technical specifics of different plant types and forecasted load and network situations (Wood and Wollenberg, 1996). After the UC had taken place, the economic dispatch could then be defined assigning an optimal output level to each plant that is online. Long-term cost reductions were primarily expected to be achieved by technological progress leading to higher degrees of efficiency. However, with the liberalization of electricity sectors around the world, other economic aspects became increasingly important. One major change was the shift from a cost focus towards a price focus. Hence, the cost-based UC of a single entity had to be replaced by a price-based UC of several different companies which in turn raised new requirements regrading system security and price forecasting (Shahidehpour et al., 2002).

Economics

As pointed out in the previous subsection, engineers had to understand the concepts of economics and the meaning of price signals in the course of electricity market liberalization. Of course, the same is true for *Economists* that had to include technical specifics into their models, too. Early economic energy models were initiated after the oil crises in the 1970s and focused on the aspects of scarce resources and resource pricing within energy systems (Bergman, 1988). These models were mainly policy models considering the dependencies on certain energy sources. Amongst these models, Bergman (1988) distinguished partial equilibrium and computable general equilibrium (CGE) models. Partial equilibrium models focus on a single sector of an economy whereas CGE models take into account the mutual interdependencies of the single sector and the rest of the economy. This differentiation is still valid for the time being. Hence, it should be stated that this thesis and the here reviewed literature solely focuses on a partial equilibrium modeling of the electricity sector which allows a more detailed modeling in terms of the representation of the single components of the chosen sector.

Economists were the actual driver of the deregulation process starting in the late 1980s in the United Kingdom (UK) by questioning whether the monopolistic structures were still efficient (Christie et al., 2000). Economists argued that the power system had undergone some change and particularly improvements in transmission had removed the natural monopoly character of the wholesale electricity markets in many locations (Stoft, 2002). However, electricity networks largely still remained local natural monopolies. The ideal deregulation was supposed to liberalize the competitive links of the value chain and regulate the networks as monopolistic bottlenecks.[5] Anyway, it occurred that network aspects remain an essential ingredient of electricity market models as will be shown in Section 2.2. As laid out earlier the focus of the electricity companies shifted from cost-based to price-based approaches. Associated with this shift several mathematical models borrowed

[5]See Jamasb and Pollitt (2005) for an overview of the history and elements of the liberalization process in the EU. In addition, Sioshansi (2006) reported policy conclusions of different deregulation processes from a global perspective. The single steps of deregulation based upon the corresponding theories of competition and regulation are not subject of the work at hand.

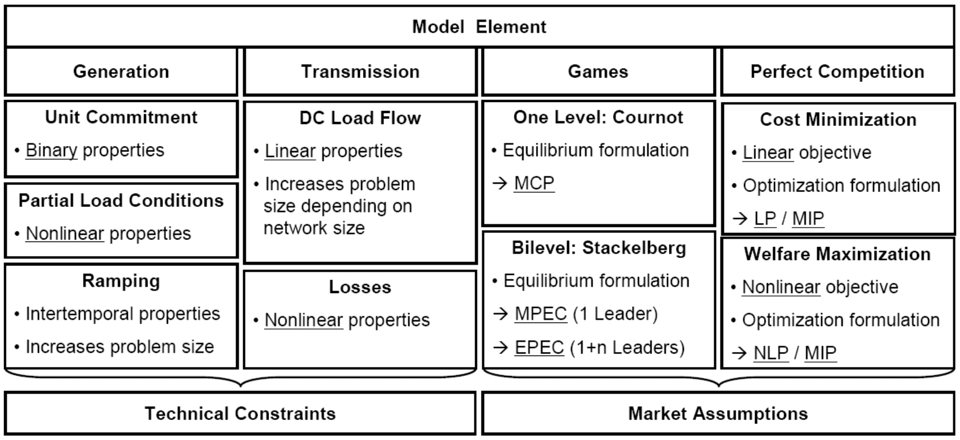
from finance, financial mathematics, and econometrics were introduced for the purpose of modeling and forecasting of electricity prices. Burger et al. (2007) distinguished these statistics oriented models from fundamental market models. Fundamental market models use cost-based bid and offer curves in order to calculate market equilibrium prices. For this purpose, fundamental market models require a sufficiently detailed modeling of each link of the value chain (compare Section 3). In contrast to statistical models, the aim of fundamental models is not necessarily the forecasting of prices but rather to gain insights into fundamental price drivers and market mechanisms (Burger et al., 2007).

Conclusions
Based on the explanations above, one can now range the work at hand into the area of deterministic partial equilibrium oriented fundamental market modeling approaches under technical constraints that interacts within the described creative interdisciplinary tension context of applied economics, engineering and operations research. However, one must be aware that within this context, there is no clear line of demarcation. For example, there are publications of operations researchers dealing with the same topics and using the same methods as work carried out in economics literature. Hence, the approach taken here is further referred to as economic engineering modeling.

2.2 Modeling Liberalized Electricity Markets

Concerning the modeling of electricity market, there are some recurring issues along which one can structure the relevant literature. Important model assumptions to classify a model are: the network representation, the type of competition, the assumed market architecture, short-term vs. long-term considerations, deterministic vs. stochastic models. Figure 2.2 illustrates some of these interrelations.

Figure 2.2: Fundamental features of electric power market equilibrium and optimization models



Source: Own presentation.

2.2.1 Background

Although the need for a deregulation of electricity markets is nowadays widely accepted, the controversies of the structure and required elements of a functional liberalized market are still ongoing. According to Stoft (2002) these controversies center on the following topics:

1. Bilateral vs. centralized market organization
2. Exchanges vs. pools
3. Zonal pricing vs. nodal pricing

The first point (bilateral vs. central markets) concerns the role of the system operator (SO).[6] Certain ancillary services must be provided in electricity markets in order to maintain system reliability. A system operator must ensure that these services are provided but the question is whether the SO provides them itself or buys them on the market. The second point (exchanges vs. pools) focuses on the question which market player should be

[6]One can distinguish an independent system operator (ISO) and a Transco. The ISO is a nonprofit system operator that operates but does not own the network (Stoft, 2002). Hence, the ISO must only be regulated minimally as it does not have the incentive to abuse its monopoly power. A Transco is a for-profit system operator that normally owns the grid and is subject to extensive regulation (Stoft, 2002).

responsible for the UC and dispatch. In an exchange-based system, each of the players assigns the status of it's plants based on the market clearing results of the exchange. This may lead to inefficient results and a lack of reliability (Stoft, 2002). Within a pool-based system, the market players provide the SO with information about their available generation capacities and costs. Based on this information, the SO conducts a centralized market clearing. One weakness of this system is that it produces nontransparent results (Stoft, 2002) which is similar to the situation prior to the liberalization.

The third point concerns the efficient management of the existing grid infrastructure and is described as congestion management. Schweppe et al. (1988) showed that efficient electricity prices differ by location and over time due to the network's physical characteristics and the different demand situations. Their seminal work defining nodal pricing or locational marginal pricing (LMP) has since then become an essential ingredient. Based on Hogan's work (1992) on contract networks, LMP is used as a pricing tool for several types of market studies.

LMP guarantees theoretically and practically the highest utilization of an existing grid because both generation and transmission constraints are considered when calculating electricity prices. The price for energy at a node represents the incremental cost incurring for delivering one more MWh of energy to exactly this node. The energy price is, thus, a scarcity signal for electricity at a specific node incorporating the marginal cost of generation, the scarcity price of generation, the marginal cost of transmission losses, and the scarcity price for transmission capacity.[7] Thus, LMP prices normally vary from node to node. The largest LMP-based market worldwide is currently PJM[8] in the USA which manages more than 8000 different nodes. In Europe, however, the introduction of efficient congestion management is somewhat delayed; greater market integration requires coordination from several sovereign countries that tend to emphasize national interests. Hence, in Europe other concepts are still on the agenda that might be considered outdated elsewhere. In general, the European markets finally move away from non flow-based towards flow-based congestion management methods.[9]

[7]Some of these summands can become zero at a certain point in time. Hence, prices are equal for each node in case of no congestion. In case of congested lines, transmitting electricity becomes a scarce commodity. The price for this commodity produces an extra charge on top of the energy price according to supply and demand at each node.

[8]Compare: http://www.pjm.com/.

[9]Not flow-based methods do not take into account physical power flows that result from commercial power transactions. This can lead to a distorted utilization of the physically

Boucher and Smeers (2002) analyzed the future organization of cross-border trade in the European market and concluded that the economic principles proposed by the European Commission in 2001 were not sufficient. Ehrenmann and Smeers (2005) presented the following different approaches in the European context. Within the market splitting approach, injections and withdrawals of several nodes are represented by a single zone. For this zone, there is only one energy price. Zones can be interpreted as sub-markets that, ideally, form according to network congestion. In most cases, however, zones are defined by political borders. Moreover, there are two alternative ways to merge nodes together as a zone. Within the first method, the entire network is considered physically; including the lines within a zone. The difference is that there is an additional constraint that forces the prices for the nodes within a zone to be equal. Within the second method the inner-zone network is neglected. The nodes of a zone are treated as if they were located upon one big copper plate. Hence, congestion within a zone is not regarded. Zones are linked by interconnections. Ehrenmann and Smeers (2005) referred to the first alternative as the ideal market splitting, whereas, the second one is a second-best approach. In addition, there is the market coupling approach. This approach assumes that sub-markets already exist and cannot be merged to one integrated market in a short- or medium-term. Therefore, market coupling tries to interlink sub-markets as far as possible. Both market splitting and market coupling can be referred to as zonal pricing approaches.

Another important issue in designing and modeling competitive electricity markets is the combination of short-run and long-run efficiencies. The basic assumption in modeling short-run markets is that the locational investment decisions have been made (Hogan, 2007). Hence, the efficient congestion management is that customers pay and generators receive the short-run marginal cost which have to be determined according to the time and location dependent load and congestion situation within the network. However, there is an ongoing debate whether marginal cost pricing leads to sufficient investment incentives, too (e.g., Hogan, 2002; Joskow, 2008). Joskow and Tirole (2005) pointed out that transmission investment decisions are amongst others influenced by the congestion management scheme. Hence, both issues should not be considered independently.

In order to align the issues of transmission and generation investment with efficient congestion management and due to the existence of a great variety

available capacity (Chao et al., 2000). In contrast, flow-based methods take into account the physical flows that result from commercial transactions.

of market designs both Hogan (2003) and Ma et al. (2003) described the development towards a standard market design proposed and used in various regions (e.g., already implemented in PJM). Market designs and electricity market models drifted in into two independent directions: on the one hand reliability-driven and on the other hand pricing-driven. After this partial co-existence an optimal Standard Market Design (SMD) was proposed claiming a coordinated spot market for energy and ancillary services. The SMD framework included bid-based, security-constrained, economic dispatch implementing LMP, and in particular the introduction of financial transmission rights (Hogan, 2002). Joskow (2005) argued in a similar manner that pure economic models have to be expanded to take the complexity of electrical constraints accurately into account. Adding a European focus Pérez-Arriaga and Olmos (2006) examined the compatibility of investment signals in transmission and generation. They emphasized that agents should face the real network cost incurred by their location decision and propose to apply beside the nodal energy price a locational transmission tariff, which should serve as long-term signal for network users. They provided criteria to how such a nodal transmission tariff should be determined.

2.2.2 Perfect Competition Modeling

The concept of perfect competition is one of the fundamental concepts of economics. A perfect competitive market is characterized by the fact that market players are price takers which means that they cannot influence the market price by individual decisions. Hence, price equals marginal cost at the equilibrium point. In contrast to that in the case of market power, players can charge mark-ups on top of marginal costs by individual decisions. Perfect competition models can either focus on aggregated measures such as social welfare or on individual objectives. In the latter case a player is price taker but its decisions are in some way constrained, e.g., by uncertainty.

Smeers (1997) considered perfect competition models as the technically easiest approach. He considered them useful in order to analyze markets. Perfect competition models can be used for an ex post appraisal of deviation of real market results from the economic efficient point. Hence, they can be used to assess market imperfections. According to Smeers (1997), these imperfections in a European context often result from quantitative restrictions and market power. Quantitative restrictions limit the market result by restricting arbitrage opportunities. They can be included in perfect competition models by using an equilibrium formulation. The same is true for including market power if the strategic mark-ups are defined as exogenous parameter which is of course a simplification particularly useful in ex post considerations.

Modeling trends regarding perfect competitive congestion management are manifold. In the U.S., LMP is an accepted concept to efficiently price electricity in a geographically distributed network. Hence, in the U.S. context the focus is to sufficiently complete the mapping of the various technical complexities of the industry, e.g., in terms of UC modeling and security constraints. Sioshansi et al. (2008) focused on the different payoffs to individual generators resulting from the fact that ISOs use different algorithms to solve their large-scale UC models (MIP formulations). In detail, they compared Lagrangian Relaxation and Branch & Bound algorithms. Fisher et al. (2008) developed a cost minimization MIP for a 118-node network in order to examine the effect of optimal transmission switching and found that savings of 25% in system dispatch cost can be achieved. Hedman et al. (2008) used the same approach and conducted a sensitivity analysis showing that changing the topology to cut costs results in lower load payments and higher generation rents for their test network. O'Neill et al. (2008) moved away from the paradigm of economic engineering models using a DC approximation of the AC flow and present a new market design including admittance pricing.[10]

Particularly in Europe, however, there is an ongoing debate about which pricing concept should be implemented. Hence, European models focus on the analysis of advantages and disadvantages of the different proposals applying stylized and larger-scale network models. Two studies of Smeers (Smeers, 2003a,b) analytically examined the impact of incompleteness of regional electricity market designs on market results using a VI formulation. Smeers (2003a) first focused on the forward market and concluded that an energy market without a market for transmission services is incomplete and that nodal and flowgate models can complete the market. Smeers (2003b) then turned to a two stage process where first the forward market is cleared and then the real time (spot) trades take place. In this case nodal pricing provided more efficient results than the flowgate approach. However, Smeers (2003b) stated that the market is still financially incomplete for market players cannot fully trade the risks that they are subject to. Completing the market by defining the right set of tradable financial transmission contracts could make the nodal and flowgate approaches equivalent. Ehrenmann and Smeers (2005) analyzed EU Regulation 1228/2003 using welfare maximizing NLP formulations applying a stylized six-node network and implemented

[10] The need for reactive prices was already advocated by Hogan (1993).

examples for nodal pricing, market splitting, market coupling, and explicit auctions concluding that European market integration is entirely possible, but that making allowances for political reasons will result in economic inefficiencies.

Stigler and Todem (2005) provided one of the very few large-scale economic engineering (cost minimizing) optimization model in the European context including 165 nodes and 136 lines for Austria which is solved as a sequence of a MIP and a NLP problem. The MIP formulation determined the UC whereas the NLP model calculated nodal prices based on an economic dispatch including transmission losses. They used their calculation in order to show that the construction of a planned 380 kV line contributes to overcome congestion problems in terms of high price differences between South and North Austria. The calculated annual costs of managing the congestion were higher than the expected annual costs for the extension project. Another application of congestion management on a realistic data set is the work of Green (2007) who calculated nodal prices for a stylized medium-scale 13-node network of England and Wales using a NLP formulation. Green (2007) showed that nodal prices increase social welfare and is less vulnerable to market power than uniform and a type of zonal pricing. Leuthold et al. (2008a) confirmed the result that nodal pricing is more efficient than uniform pricing for a large-scale NLP model of the German and BeNeLux electricity market using a parent of the ELMOD model presented in Chapter 3. It seems noteworthy to state that all of the mentioned models used some form of the DC load flow model (DCLF) for calculating the optimal power flow (OPF) as introduced to the economic engineering literature by Schweppe et al. (1988).[11]

A major focus of investment models is the aspect of risk and uncertainty and ways to hedge those. Concerning efficient transmission expansion analytical models prevail for the time being. A major research debate is about who should carry out the investment: a regulated entity (centralized transmission planning, or CTP), or the market (merchant transmission investment, or MTI). The objective of a standard CTP approach is to maximize (expected) social welfare, whereas under MTI the investor should be incentivized by positive return on investment (ROI). Also, the investor should participate in the effect that the investment has in the light of network externalities, thus, the question how to deal with the risk that comes with a transmission investment for both the new investor and the existing transmission owner

[11] A description of the DCLF and its adequacy for economic engineering modeling can be found in Section 3.2.3.

is still unanswered. Bushnell and Stoft (1996b) distinguished contracts for differences (CFD) to hedge temporal price risks and transmission congestion contracts (TCC) that pay the owner the locational price difference between the two nodes specified in the contract. Bushnell and Stoft based their analysis on a contract network regime as proposed by Hogan (1992) using a three-node network and showed analytically that in this case TCCs provided the correct incentives for network investments.

Chao and Peck (1996) used the nodal pricing methodology for a three-node network and designed tradable transmission capacity rights that are able to combine a competitive market for transmission services and a competitive spot market for electricity. They suggested a trading rule for these transmission capacity rights that combine a Coasian property right approach to transmission congestion and the Pigouvian principle to account for network externalities. Numerical modeling efforts in the field of transmission expansion look at an investor's risk associated with a MTI decision under uncertainty using simulation models and (stochastic) dynamic programming (e.g., Saphores et al., 2004; Salazar et al., 2007).

Taking a look at generation investment models, one can state that they are also mainly concerned with the uncertainty aspect of the investment from an investor's point of view. Several optimization and simulation models have been applied to model these stochastic aspects focusing amongst others on real options (e.g., Wickart and Madlener, 2007; Roques, 2008; Kumbaroglu et al., 2008; Auerswald and Leuthold, 2009) and mean variance portfolio (e.g., Roques et al., 2008) approaches. Pokharel and Ponnambalam (1997) develop a straightforward cost minimization model in order to analyze the planning for power plant expansion under deterministic and stochastic demand. They find that under the assumption of a deterministic demand, the installed capacity would be higher than if stochasticity in demand is taken into account. In contrast, Smeers (2006) is concerned with the discrete nature of generation location decisions and suggests a multi-part tariff in the context of the regional market within Europe. He developed a model incorporating a separation of short- and long-term prices which is analyzed analytically. As these prices were discriminatory, he concluded that the three criteria economic efficiency, cost reflectiveness and non-discrimination cannot be achieved at the same time and some trade-off has to be made. Accordingly, there is some literature on generation investment that includes the transmission aspect of electrical networks. Bushnell and Stoft (1996a) stated that efficient TCCs support generation investments. Rious et al. (2008) analyzed the impact of a two-part tariff in order to manage electricity networks efficiently in a short- and in the long-run perspective using an agent-based (simulation) model. They find that a joint implementation of nodal pricing and the average participation tariff is the best combination to coordinate the generation and transmission investments as efficiently as possible. However, the optimal set of generation and transmission investments may not be carried out because of transmission lumpiness.

2.2.3 Imperfect Competition Modeling

Ventosa et al. (2005) provided a detailed overview of electricity market modeling tendencies. They defined three trends: optimization models, equilibrium models and simulation models. Optimization models can either apply a profit maximization of a single firm or a welfare maximization approach under perfect competition (compare Section 2.2.2). Ventosa et al. (2005) distinguished two types of models for a single-firm optimization problem: either the price is an exogenous parameter (perfect competition) or determined via a function of the demand supplied by the firm (imperfect competition). The partial equilibrium electricity market modeling literature has emerged to emphasize the latter type of modeling as market power is a serious concern in most electric power markets (Smeers, 1997).

One approach to research how individuals or groups of people interact introduces game theory to electricity sector modeling (compare Shahidehpour et al., 2002). The term strategic behavior or strategic interaction (Hogan, 1997) describes the fact that one or many market incumbent(s) do not act as price taker(s). Hence, these players are able to influence the market equilibrium by their decisions. Strategic interaction in the present context is mostly modeled as noncooperative game which means that each firm behaves in its own self-interest (compare Tirole, 1988). According to Fudenberg and Tirole (1991), the basic solution concept in game theory is the Nash equilibrium which is the equilibrium point from which a firm cannot deviate without being worse off given that the decisions of all other firms are fixed. Smeers (1997) distinguished the Cournot and the Bertrand paradigms. In a Cournot competition firms compete via quantity decisions. In a Bertrand competition the price is the strategic information. The latter concept was used at times arguing that electricity cannot be stored which leads to short-term price competition (Hobbs, 1986). However, strategic behavior in electricity markets is often understood as Cournot competition with few oligopolistic firms (Willems et al., 2009). Another type of strategic interaction is the assumption of Stackelberg games. A Stackelberg game is a two-stage game[12]

[12] The term bilevel game can be alternatively used to describe two-stage games.

where at the first stage output decisions are made by a leader (or a group of leaders) which in turn are taken as fixed for the decision on the second stage (Day et al., 2002; Metzler et al., 2003). The described types of games allow for a large variety of different game theoretic models that are often referred to as equilibrium problems as they can not be solved as mathematical programs in contrast to most perfect competition models (compare Day et al., 2002, and Section 2.1.2).

As in Section 2.2.2, one can start to focus on short-run markets where investment decisions have been made already. In a classical arbitrage-free Cournot game for a LMP-based market, generation firms behave as strategic (oligopolistic) players each assuming that the generation decision of the rivals is fixed (Metzler et al., 2003). Thereby, the SO is assumed to act as disinterested efficient entity that does not interfere with the game (Hobbs and Helman, 2004). These models can be formulated and solved as MCPs (Hobbs, 2001; Hobbs and Helman, 2004) in order find a Nash-Cournot equilibrium. Hobbs (2001) complemented such a Cournot game by arbitrage - for an exemplary three-node network - where generators recognized that arbitragers use price differences for trades where they exceed the cost of transmission. Hobbs (2001) found that in the presence of arbitrage, Cournot competition in a bilateral market leads to the same equilibrium as Cournot competition among generators in a pool system.

The pure Cournot assumption can be adjusted in some ways. Day et al. (2002) presented the approach of a conjectured supply function. The underlying assumption is there that the generators do not take the competitors' decisions as fixed but conjecture their reaction to an own output decision. Day et al. (2002) solved a conjectured supply function model for a 13-node network model as MCP. However, one drawback of this method is that appropriate parameters for the conjectures are not observable. Furthermore, the assumption that a generator acts strategically with regard to the output decision of its competitors but acts as price-taker with respect to transmission prices seems to be unrealistic. Taking into account that the generators are able to game over the transmission prices, too, leads to a Stackelberg type of competition with the strategic players as leaders and the SO as follower. This results in a MPEC problem for each strategic player with the players' objective in the upper level and the ISO problem in the lower level (Hobbs et al., 2000). Hence, if there are more than only one strategic player, the entire problem becomes an EPEC (Daxhelet and Smeers, 2001; Hobbs and Helman, 2004) which is inherently nonconvex and difficult to solve (Neuhoff et al., 2005). (Hobbs et al., 2000) solved such an EPEC (on a 30-node network) using a Penalty Interior Point Algorithm (PIPA) for the single MPECs and a diagonalization algorithm for the multi-firm problem. However, there are some problems associated with the PIPA which are discussed in Chapter 5. In addition, Hu and Ralph (2007) promisingly solved an EPEC for a 39-node network with a similar setting by transforming the single MPECs into NLPs from which they determined the KKT conditions and, thus, replace an EPEC by a complementarity problem.

Another example for the extension of the classical Cournot that can lead to a Stackelberg problem structure is if the strategic suppliers anticipate the trades of arbitragers due to the strategic output decisions (Metzler et al., 2003). Moreover, Daxhelet and Smeers (2007) applied a two-stage approach to the question of cross-border trade of electricity in Europe. In contrast to the models described above, they assume that the regulators are on the upper-level and the energy market game is on the lower-level. In case the regulators cooperate this leads to a MPEC formulation. In case they do not, it becomes an EPEC. In order to solve the latter Daxhelet and Smeers (2007) also used a diagonalization approach where the MPECs of a single regulator is solved separately, holding the decision variables of all other players fixed; then solve and conduct the same for each other player until the sequence converges. The MPECs are solved by rewriting them as NLPs.

Other types of equilibrium models that attempt to solve similar problems as described above are Supply Function Equilibrium (SFE) models. Due to the fact of having few oligopolistic players and short-term demand elasticities, Cournot mark-ups on marginal costs can become very high (Willems et al., 2009). Thus, within the SFE approaches generators compete by bidding complete supply functions instead of one single quantity as in the case of Cournot which is supposed to provide a more realistic view on real market results (Green and Newbery, 1992). Demand uncertainty is a crucial element of this assumption. However, the results of Willems et al. (2009) indicated that Cournot and SFE models can be calibrated in order to produce the same results. Other types of models shall also only be mentioned in the following. There is a significant and growing number of simulation models using agent-based modeling (Metzler et al., 2003). Weidlich and Veit (2008) conducted an extensive survey on these types of models. Agent-based computational economics focuses on the simulation of individual and collective behavior under the assumption of learning. These models simulate marketers' behavior in an iterative process. From iteration to iteration the agents learn how to improve their objective. One example for these simulation types are generic algorithms. Agent-based models can be classified according to the learning function of their agents (Weidlich and Veit, 2008). Concerning transmission investment decisions and imperfect competition,

Joskow and Tirole (2000) focused on the concept of transmission capacity withholding. They distinguished two types of tradable rights: financial transmission rights (FTRs)[13] and physical transmission rights. FTRs are financial instruments that entitle or oblige the holder to receive or make payments in case of congestion. Physical rights give the holder the right to transmit electricity even in congestion scenarios. The two authors found analytically that in instances of loop flow effects for a three-node network physical rights can be withheld and thus are likely to be misused in order to exert market power. Thus, they favored FTRs where physical withholding is not possible.[14] Borenstein et al. (2000) also focused on the interaction between imperfect markets and transmission capacity. Based on an analytical analysis and a numerical example, they concluded that transmission constraints are a major driver of market problems.
Léautier (2001) analytically confirmed these result on a three-node network looking at strategic behavior of generators. Brunekreeft and Newbery (2006) focused on the welfare effects of a must-offer provision of physical line capacity in the case of MTI. They exploited their model analytically concluding that the regulatory instrument of a must-offer provision, has positive short-term welfare effects but may lead to underinvestment in network assets. They did not recommend to apply must-offer provisions. Among the CTP approaches, Vogelsang (2001) analyzed transmission cost and demand functions assuming rather general properties assuming that an ISO intends to maximize its individual profit. He adopted regulatory adjustment processes based on a two-part tariff cap for transmission. Hogan et al. (2007) and Rosellón and Weigt (2008) extended this two-part tariff approach accounting for loop-flow properties of an electricity network by building and solving a small-scale MPEC.
In addition, Sauma and Oren (2006) analyzed analytically and computationally a three-period proactive network planning (PNP) model and compared it to a combined generation-transmission operation and investment planning as well as to a transmission-only planning model. They concluded that PNP can correct some of the shortcomings of transmission-only planning and claimed that it is a valuable economic assessment methodology. They were able to construct and solve a 30-node MPEC. The expected social

[13] According to Joskow and Tirole (2005), the terms TCC and FTR are interchangeable.

[14] Baldick (2007) picked up the issue of financial transmission rights and argued that border flow rights (BFRs) make FTRs dispensable. He stated that BFRs resolve the property-rights issues for existing and new transmission capacity arising from new investments. However, in the paper it remained unclear whether this ia a general result or only true for the case of perfect competition.

gains by the PNP methodology should be distributed to all market players through side payments (Sauma and Oren, 2007). Taking - again a brief - look at generation-only investments under imperfect competition, one can state there are only few models combining generation investment and network congestion. Murphy and Smeers (2005) provided a valuable overview of this generation investment literature not taking into account transmission congestion and focused on the effects of imperfectly competitive markets on generation investment decisions. They extensively analyzed analytically different types of Cournot models and found that the complexity of the investment decision process increases along with the complexity of the electricity market structure. Zoettl (2008) models optimal investment decisions of strategic firms in a liberalized electricity market. He concluded using analytical and numerical methods that under imperfect competition firms have strong incentives to invest into capacities with low marginal costs taking into account the effect of the generation expansion on the output decision of the competitor. At the same time, the total capacities are chosen too low from a welfare point of view.

2.3 Conclusions

Modeling electricity markets is a melting pot of researcher from different disciplines that all bring their own methods and skills. Concerning the modeling under the assumption of perfect competition, there is need for large-scale economic engineering models that include the entire potentially integrated European market in order to contribute to the ongoing discussions on congestion management and investments. Concerning the modeling under the assumption of imperfect electric power markets, there is room and need for new algorithms that contribute to overcome numerical difficulties in solving medium- and larger-scale strategic models. This thesis takes on some of the challenges described above by firstly developing a large-scale economic engineering network model of the EU named ELMOD (Part II) and secondly carrying out algorithmic work in order to solve a discretely constrained MPEC representing a Stackelberg game in an electric power market including transmission constraints (Part III).

Part II

Large-Scale Perfect Competitive Economic Engineering Modeling

Chapter 3

ELMOD - A Model of the European Electricity Market

3.1 Introduction

Taking a closer look at large-scale modeling as valuable research topic identified in Part I, the need to develop a model that is capable to represent the European electricity market becomes obvious. Therefore, in the course of this thesis, a structured procedure was elaborated in order to establish a well working model that produces meaningful scientific results while taking into account that particularly simulations of short-term (i.e. spot) market equilibria in electricity have to consider technical specifics due to the nature of the electricity industry (Stoft, 2002). First of all, an economic (i.e. market) model had to be defined that allowed to analyze the identified research objectives. The underlying economic model defined which economic data were actually needed. Then, the representation of the technical specifics had to be included which normally constrain the economic model in some way. The technical constraints defined the required technical data. Third, the degree of detail had to be defined. On the one hand, the degree of aggregation of data was supposed to be detailed enough to be able to produce meaningful results for a defined research objective. On the other hand, the technical effort to solve the complete model normally decreases with the degree of aggregation which is an incentive to keep the model as small as possible. This chapter describes the scientific work carried out with the goal to develop the large-scale economic engineering model ELMOD (European Electricity Market Model) which was initiated by the author of this thesis. As mentioned earlier this chapter summarizes the current structure of

ELMOD and provides an in-depth description of model assumptions and specifics. Section 3.2 starts with the technical and economic details of ELMOD. Section 3.3 presents the used data and sources as well as the underlying assumptions. In Section 3.4 an overview about existing research results is given including congestion management issues and generation capacity extension. Section 3.5 concludes the chapter and sketches out possible topics for further developments.

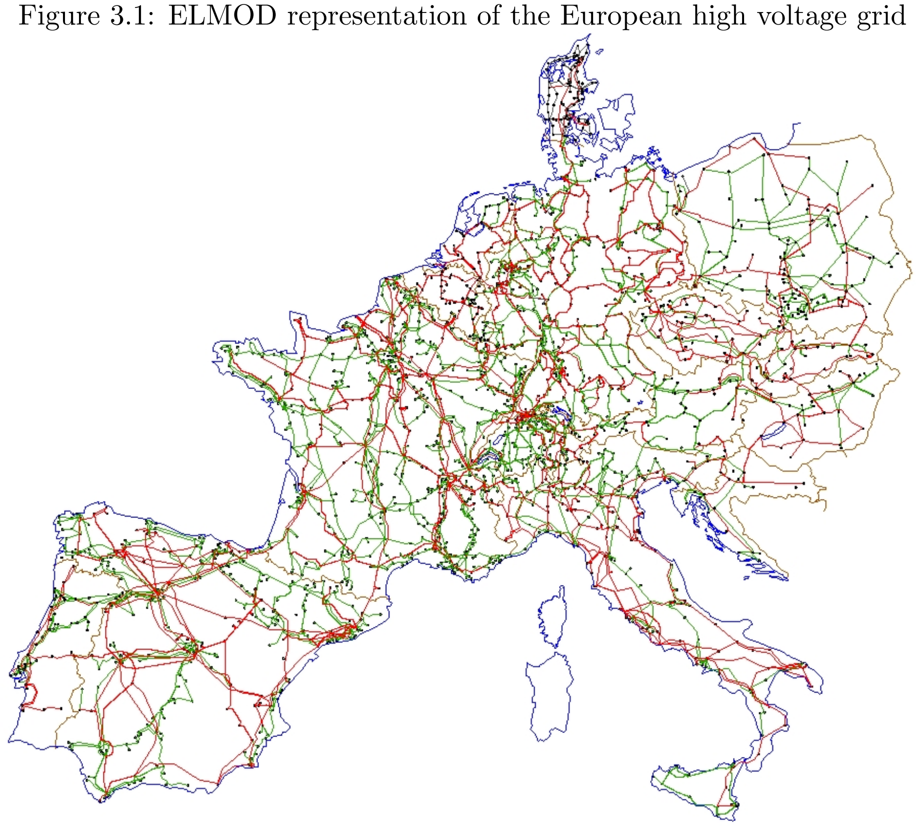
In addition, it must be stated that large-scale models are subject to permanent adjustments and refinements until they are fully applicable. In the course of this process, several different studies were carried out and helped to improve the functioning of ELMOD (Figure 3.1). First modeling steps were reported by Leuthold et al. (2005) for the German electricity market. Weigt et al. (2006) continued this work and extended the model by including France, Benelux, Western Denmark, Austria and Switzerland. Weigt (2006) broadened the scope to a time-frame of 24 hours to simulate variable demand and wind input as well as unit commitment, start-up and pumped storage issues. The model was subsequently extended to cover the entire European UCTE electricity markets (essentially Central and Western Europe).[1]

3.2 Model Description

In its basic formulation, ELMOD can be classified as a non-linear optimization model maximizing social welfare under the assumption of perfect competition taking into account technical constraints. It is solved in GAMS. ELMOD is based conceptually on the work of Schweppe et al. (1988) and Stigler and Todem (2005). Subsequently, first the objective function and the constraints are explained in more detail. Then the DCLF and further modeling specifics such as the representation of demand, time constraints, and unit commitment are elaborated.

[1]The subsequent chapter relies on the working paper Leuthold et al. (2008b).

Figure 3.1: ELMOD representation of the European high voltage grid



Source: Own presentation.

3.2.1 Mathematical Notation

Indices:

$l \in L$	line within the network
$n, j, k \in N$	nodes within the network
$s \in S$	power plant unit
$t, \tau \in T$	time periods

Sets:

L	set of all lines
N	set of all nodes
S	set of all power plants
T	set of all time periods

Parameters:

B_{jk}	series susceptance between two adjacent nodes j and k
$\overline{g}_{ns}$	available maximum generation level of plant s at node n
$\underline{g}_{ns}$	required minimum generation level of plant s at node n
G_{jk}	series conductance between two adjacent nodes j and k
h	height above ground
$\overline{ps}_n$	maximum working capacity of a PSP at node n
$\overline{P}_l$	maximum available power flow capacity over line l
R_{jk}	series resistance between two adjacent nodes j and k
$\lvert\underline{U}_{nt}\rvert$	absolute value of the complex voltage vector at node n in time period t
wi_{nt}	wind input at node n in time period t
$\nu(h)$	wind speed depending on the hight above ground h
$\overline{\vartheta}_s$	minimum online duration of plant type s
$\underline{\vartheta}_s$	minimum offline duration of plant type s

Variables:

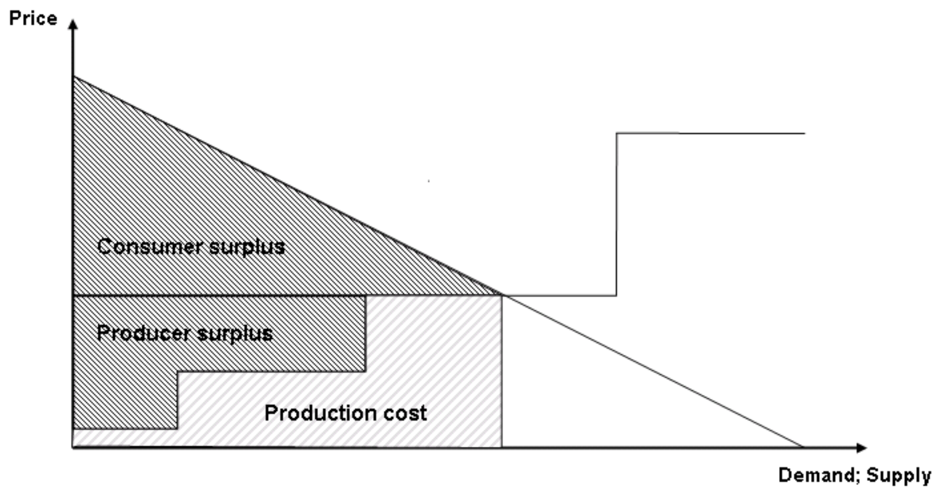
$c_{nst}(g_{nst})$	generation cost of plant s at node n in time period t as a function of g_{nst}
g_{nst}	generation of plant s at node n in time period t
L_{jkt}	transmission losses between two adjacent nodes j and k in time period t
ni_{nt}	net grid input at node n in time period t
on_{nst}	binary variable describing status of plant s at node n in time period t
$p_{nt}(q_{nt})$	linear inverse demand function at node n in time period t
$pslevel_{nt}$	filling level of a PSP at node n in time period t
$\tilde{P}_{jkt}$	power flow between two adjacent nodes j and k in time period t
P_{lt}	power flow over line l in time period t
$\overleftarrow{ps}_{nt}$	energy produced by a PSP at node n in time period t
$\overrightarrow{ps}_{nt}$	energy demanded to fill a PSP at node n in time period t
q_{nt}	demand quantity at node n in time period t
Θ_{jkt}	phase angle difference between two adjacent nodes j and k in time period t

3.2.2 Optimization Problem

ELMOD uses a welfare maximizing approach taking into account line flow, energy balance and generation constraints. Welfare is obtained using a linear demand and a supply function and is calculated subtracting the cost of generation from the area below the demand function (Figure 3.2 and Equation (3.1)). At each node reference demand, reference price and elasticity (see Section 3.3.3) are estimated in order to identify demand via a linear demand function. Generation cost are determined by an individual cost function for each node. The actual generation costs depend on external parameters such as the fuel price or different efficiency levels of plants which in turn are determined by the age or construction type of the power plant, the actual level of output and others.

In order to include technical network limits, a line flow constraint, an energy balance, and a generation constraint are integrated into the model. Through the line flow constraint (Equation (3.2)), a maximum amount of power P_{lt} transported on line l during period t is determined, constrained by the thermal limit of each line $\overline{P}_l$ given a 20% reliability margin. The reliability margin indicates that a line can only be loaded up to 80% of the thermal line capacity thus implementing the so-called (N-1)-criterion in a simpli-

Figure 3.2: Welfare in an electricity market



Source: Own presentation based on Todem (2004).

fied manner. The energy balance (Equation (3.3)) states that at a node n the difference between total generation and demand has to be balanced by injections into or withdrawals from the grid, respectively, adjusted by the amount of incurring losses. Generation consist of the sum of fossil generation $\sum_s(g_{nst})$ and wind input wi_{nt}. Pumped storage plant generation is added if the pumped storage plant generates electricity $\overleftarrow{ps}_{nt}$. If the pumped storage needs to be filled with water this required electricity $\overrightarrow{ps}_{nt}$ is subtracted (see also Section 3.2.5). Generation equals all withdrawals made up of demand q_{nt} and net input ni_{nt} defining whether a node injects or withdraws energy from the grid. The generation constraint in Equation (3.4) assures on the one hand that a power plant s will be turned off if generation is below a minimum generation $\underline{g}_{ns}$ necessary to obtain workable technical conditions and on the other hand that it does not exceed its maximum capacity $\overline{g}_{ns}$. Each of the constraints must hold for each hour t. Welfare is derived over all hours:

$$\max_{g_{nst},q_{nt}} \left\{ W = \sum_{n,t} \left(\int_0^{q_{nt}} p_{nt}(q_{nt})\,\mathrm{d}q_{nt} - \sum_s g_{nst}c_{nst}(g_{nst}) \right) \right\} \tag{3.1}$$

subject to

$$|P_{lt}| \leq \overline{P}_l \quad \forall\, l, t \tag{3.2}$$

$$\sum_s g_{nst} + wi_{nt} + \overleftarrow{ps}_{nt} - \overrightarrow{ps}_{nt} - q_{nt} - ni_{nt} = 0 \quad \forall\, n, t \tag{3.3}$$

$$on_{nst}\underline{g}_{ns} \leq g_{nst} \leq on_{nst}\overline{g}_{ns} \quad \forall\, n, s, t \tag{3.4}$$

$$c_{nst} \geq 0 \quad \forall n, s, t \tag{3.5}$$

$$g_{nst} \geq 0 \quad \forall\, n, s, t \tag{3.6}$$

$$\overleftarrow{ps}_{nt} \geq 0 \quad \forall\, n, t \tag{3.7}$$

$$\overleftarrow{ps}_{nt} \geq 0 \quad \forall\, n, t \tag{3.8}$$

$$q_{nt} \geq 0 \quad \forall\, n, t \tag{3.9}$$

$$on_{nst} \in \; [0, 1] \quad \forall\, n, s, t \tag{3.10}$$

3.2.3 DC Load Flow Model

Schweppe et al. (1988) showed that the DCLF can be used for an economic analysis of electricity networks. They apply it to their nodal price approach for electricity pricing. Overbye et al. (2004) come to the conclusion that the DCLF is adequate for modeling nodal prices even though there are some buses with a certain price deviation. The latter occurs particularly on lines with high reactive power and low real power flows. Stigler and Todem (2005) describe the way from the physical fundamentals to the DCLF equations. Equation (3.11) of the so-called 'decoupled' AC model builds the foundation of all further assumptions and calculations. Power flow[2] $\tilde{P}_{jkt}$ depends on the conductance G_{jk}, the susceptance B_{jk}, and the voltage angle difference Θ_{jkt} between nodes j and k as well as on the voltage magnitudes $|\underline{U}_{jt}|$ and $|\underline{U}_{kt}|$:

$$\tilde{P}_{jkt} = G_{jk}\, |\underline{U}_{jt}|^2 - G_{jk}\, |\underline{U}_{jt}|\, |\underline{U}_{kt}|\, \cos\Theta_{jkt} + B_{jk}\, |\underline{U}_{jt}|\, |\underline{U}_{kt}|\, \sin\Theta_{jkt} \tag{3.11}$$

Schweppe et al. (1988) assume that the voltage angle difference Θ_{jkt} is very small and that the voltage magnitudes $|\underline{U}|$ can be standardized to per unit

[2]The power flow P_{lt} on a line l can be derived from the power flow $\tilde{P}_{jkt}$ between two nodes j and k using a mapping, e.g. a network incidence matrix, stating which lines l connect nodes j and k. For a more detailed description see Schweppe et al. (1988).

calculations.[3] $|\underline{U}_{jt}|$ and $|\underline{U}_{kt}|$ are thus assumed to be equally 1 at each node n during all time periods t. Using the first order terms of the Taylor series approximation the following simplification can be made:

$$\cos\Theta_{jkt} \approx 1 \tag{3.12}$$

$$\sin\Theta_{jkt} \approx \Theta_{jkt} \tag{3.13}$$

Equation (3.11) can then be simplified to become:

$$\tilde{P}_{jkt} = B_{jk}\Theta_{jkt} \tag{3.14}$$

Line losses have not been considered, yet. However, in real networks the sum of total generation does not equal the sum of total demand due to transmission losses. Thus, transmission lines are stressed by demand plus losses. In order to approximate the losses on a line, Equation (3.12) must be complemented by the second order term of the Taylor series approximation:

$$\cos\Theta_{jkt} = 1 - \frac{(\Theta_{jkt})^2}{2} \tag{3.15}$$

Then, after some further assumptions and conversions following Stigler and Todem (2005) transmission losses can be calculated via the power flow $\tilde{P}_{jkt}$ and the line series resistance R_{jk}:

$$L_{jkt} = R_{jk}(\tilde{P}_{jkt})^2 \tag{3.16}$$

[3] If the model includes more than one voltage level as it is the case within ELMOD, the standardization works by choosing a reference voltage level and then convert all other line parameters by a conversion factor. For example, the factor to express 220 kV parameters in 380 kV terms would be approximately 0.58. Hence, resistances and reactances of the 220 kV lines would have been divided by this factor. However, as the maximum power capacity has to be converted, too, one could also define a reference power flow level, relate a line's power capacity to this predefined level, and convert all parameters accordingly. In any case, regarding the conversion factors, one has to be aware that the power capacity is a quadratic function of the voltage magnitude.

3.2.4 Time Constraints, Unit Commitment, and Optimal Dispatch

In order to model electricity markets various idiosyncracies have to be considered. Electricity cannot be stored at a large-scale. Therefore demand and generation always have to equal each other. Demand is not constant over time, but varies in the course of the day, the week and the season. In Europe, demand is higher in winter than in summer mainly influenced by the weather. On workdays more electricity is consumed than on weekends because of a decrease of industrial demand and changed household behavior. To incorporate those characteristics ELMOD is capable to model a 24 hours time-frame.

To respond to the varying demand pattern over a day, power plants are divided into three types according to their load type: base load plants supply the grid with a constant output covering thus the base load which is always demanded. Medium load plants provide the increasing electricity demand during the day and are switched on in the morning hours and shut down during the night. Peak load plants are crucial to satisfy various demand peaks during the day. Peak load plants can be turned on within a short time frame.

Unit commitment describes the decision process on whether and when a power plant is running in order to contribute to the satisfaction of demand. Unit commitment identifies those plants available for the following dispatch process in which the output of each plant is determined exante according to the actual electricity demand, technical needs and the plants cost function. As plants need time to be launched ranging from some minutes for small gas turbines up to several days for large nuclear plants, timing is essential for obtaining a cost minimal dispatch as well as maintaining system stability. ELMOD solves unit commitment within the social welfare optimization process. The optimal output for each plant is determined taking into account the minimal output level to be reached to put a plant online and a certain time for starting up the plant. This introduces a binary variable on_{nst} to the calculation process to determine whether a plant is online or offline. Following Takriti et al. (1998), a minimum online and offline constraint can then be defined:

$$on_{nst} - on_{ns(t-1)} \leq on_{ns\tau}, \quad \tau = t+1, ..., min\{t + \overline{\vartheta}_s, T\} \tag{3.17}$$

$$on_{ns(t-1)} - on_{nst} \leq 1 - on_{ns\tau}, \quad \tau = t+1, ..., min\{t + \underline{\vartheta}_s, T\} \tag{3.18}$$

Equations (3.17) and (3.18) link the hours of the day in order to include online and offline constraints for power plants, respectively. Since the time increment in ELMOD is one hour, it is reasonable to only include the offline constraint (Equation (3.18)). Hence, it is assumed that each plant can be shut down after the end of each hour. Once a plant was shut down, it cannot be turned on again immediately depending on the plant type. Therefore, conditions are introduced to keep plants switched off for a certain time interval $\underline{\vartheta}_s$. Further, in order to reduce the calculation effort, each plant is assigned to one group out of three possible groups following Voorspools and D'haeseleer (2003): the must-run units, the peak units and the test group for which the unit commitment process is crucial. Within the 24 hour model, base load plants such as nuclear and lignite plants are constantly producing at least their minimum required output which basically means that they are by definition online over all time periods. Hydro plants and gas turbines are supposed to be able to go online within one hour. Hence Equation (3.18) is not binding for them. For the remaining plant types (hard coal plants, oil and gas steam plants, and combined cycle gas turbine plants) the start-up decisions are made endogenously during the optimization.
Start-up can be distinguished in cold, warm and hot start-up, according to the time since the last shut down. If a plant has recently gone offline, it can be started much faster than a 'cold' plant. This is due to the remaining heat level in the plant, while a 'cold' plant has to entirely build up the necessary starting heat.
For the time being, the maximum considered time period within the model is one day (24 hours). Therefore the necessary information to decide on the right kind of start-up may not be available. Also, the calculation effort increases as logic operations have to be considered. Thus for those plants where Equation (3.18) applies, the start-up is supposed to be a warm start-up. For gas plants, all start-ups are supposed to be cold start-ups.[4] The start-up times $\underline{\vartheta}_s$ are based on Schröter (2004). Taking these constraints into account, the model calculates the status and the output for each plant in each hour.

3.2.5 Modeling Pumped Storage and Wind Energy Plants

Pumped storage hydro plants (PSP) as well as wind energy plants cannot be modeled as normal thermal plants. In the case of PSPs it has to be considered that energy can either be injected to or withdrawn from the

[4] This is irrelevant for the time constraint but important for the cost estimation.

grid. The peculiarity of wind energy is its priority in feed-in. Subsequently, the implementation of these production types in ELMOD is explained in further detail.

Regarding PSPs, it is important to state that PSPs constitute the only way to store larger amounts of electrical energy. These plants can run either in pumping mode, filling a storage basin by using electricity, or in generation mode, using the stored water like a classical hydro plant. The electrical energy is thus actually stored in form of potential energy within the water. These PSP facilities are crucial for system stability, as they can start-up rapidly and therefore cancel out fluctuations. In general they pump water during night time and weekends and start producing electricity generation during the peak periods. Within the model, PSPs can either demand electricity $\overrightarrow{ps}_{nt}$ and fill their storage or use the stored energy and generate electricity $\overleftarrow{ps}_{nt}$. In the model, PSPs start with an empty storage at 8pm. An overall degree of efficiency is implemented in Equation (3.19) by only adding 75% of that energy $\overrightarrow{ps}_{nt}$ to the storage $pslevel_{nt}$ that is actually withdrawn from the network. In return, the model treats pump storage production as lossless. Consequently, pump storage plants are assumed to have an overall degree of efficiency of 75% for pumping and generating, together[5]:

$$pslevel_{nt} = 0.75\,\overrightarrow{ps}_{nt} - \overleftarrow{ps}_{nt} + pslevel_{n(t-1)} \tag{3.19}$$
$$\overrightarrow{ps}_{nt} + \overleftarrow{ps}_{nt} \leq \overline{ps}_{n} \tag{3.20}$$
$$\overleftarrow{ps}_{nt} \leq pslevel_{n(t-1)} \tag{3.21}$$

Equations (3.20) and (3.21) define the capacity constraints of the storages. The pumped or generated amount is limited by the plant's working capacity $\overline{ps}_n$. Moreover, the storage level $pslevel_{n(t-1)}$ of a PSP facility at the end of a previous period $t-1$ defines the upper bound for the available generation from that facility $\overleftarrow{ps}_{nt}$ for the current period t.[6]

[5]According to Müller (2001), modern PSPs have an average efficiency between 70% and 80%.

[6]Since only one day is simulated, the storage behavior may not be properly modeled, as the storage process largely takes place at weekend nights. Also, the hourly interval may result in a biased representation of PSPs, as one of their main tasks is to react in case of rapidly changing conditions. Since these short time situations are not modeled for the time being, their importance may be underestimated in the model output. However, these shortcomings can be included within the model framework easily by extending the time-frame beyond 24 hours.

Turning towards wind energy it should be mentioned that wind has become a major part of renewable energy produced in the German generation mix with 20.6 GW installed capacity by the end of 2006 (DEWI, 2007). Also on the European level, wind energy is the fastest growing renewable energy source with 48 GW installed in 2006 (EWEA, 2007). Due to the dependence of wind turbines upon wind speed, there is no active control of energy output like in a fossil plant. Only by setting a turbine offline, a minimal active control can be achieved. Because of the feed-in guarantees provided by the Renewable Energy Act in Germany, wind energy has to be accepted as priority energy source by the TSO and is thus a fixed exogenous parameter for the model. Wind speeds change over time according to meteorological conditions and so does the energy input from wind turbines. In times of high generation by wind turbines, fossil plants must reduce output, while in times of low wind input fossil plants have to compensate the shortfall. A consequence could be additional line flows within the transmission grid, particularly in times of high wind input and low demand.

Wind forecasts play a major role in determining the wind input and therefore the plant schedule for the next hours or day. The differences between forecasted wind input and realized input have to be compensated in order to maintain system stability. The operating reserve that must be provided is not considered in the model. While fossil plants are running in constant mode at an optimal load level whenever possible, wind turbines often run in partial load mode and can change output within hours up to 100%. These changes cause an increased need of backup plants to be able to start-up or reduce output according to the wind input. Within the model, the wind input is calculated for each hour and node and given as an external parameter included in the energy balance (see Equation (3.2)).

This constraint can become critical if the grid is not capable of transporting all wind energy. Then the only way to fulfill the energy balance constraint is the increase of local demand even if prices become negative. For the time being, in reality other measures are taken in order to avoid such situations. Possibilities in order to manage such extreme cases are the shut-down of certain wind parks and other technical measures. Such short-term measures are not included in ELMOD.

3.3 Data

3.3.1 Network

The underlying network is based on the European high voltage grid (UCTE, 2004; VGE, 2005). Substations, line voltage level and line length were uploaded into a digital map, making it possible to add and remove additional lines and nodes. An underestimation of line length can occur, since altitude differences have not been considered. Since no data about the system state is publicly available, all lines connected to a node are assumed to be connected with one another. Also, no information about the transformation capacities of the substations is available. Security constraints are considered by a 20% transmission reliability margin. Thus, no line within the modeled grid will be stressed with more than 80% of their thermal capacity limit.

Germany

The most detailed region mapped in the model is Germany with 365 nodes: 336 regular nodes representing substations and 29 auxiliary nodes. Three different reference line characteristics, one for each voltage level, are considered based on Fischer and Kießling (1989). Three main technical factors are included: maximum thermal limit, line resistance and line reactance. The values differ significantly for the three voltage levels. To obtain the values for lines with more circuits, the impedances have been calculated according to a parallel combination. Thus, the interaction of multiple circuits has been neglected. The data source for the line characteristics is based on the UCTE-network map (UCTE, 2004). As cross-border flows and transactions play an important role in electricity markets, nine country nodes are added, representing the neighboring countries and 81 cross-border nodes to simulate the import and export, as well as cross-border flows. The model contains 271 lines of the 220 kV and 309 lines of the 380 kV level as well as six lines with 110 kV. In addition, 50 country tie-lines with unlimited capacity are included, connecting the cross-border nodes with the country node and representing the grid of the respective country. Cross-border lines between countries are modeled according to their length and voltage level.[7]

[7]It must be noticed that the implementation of neighboring countries has an impact on the welfare calculation. As they are part of the overall optimization problem, their demand and generation adds to the total system welfare. Due to energy exports and imports, it is not possible to calculate the welfare for Germany only when including neighboring countries. This must be taken into account while regrading welfare effects. However, as long as only Germany is modeled in detail and the other countries are aggregated to a few

The European Grid

The European UCTE-grid is modeled in a similar way, though with a slightly lower level of detail concerning demand estimations, installed generation capacity, and wind facilities. The entire high voltage grid in Europe is contained in ELMOD based upon the UCTE-network map (UCTE, 2004) as well. The model then covers Portugal, Spain, France, the Netherlands, Belgium, Luxembourg, Western Denmark, Germany, Switzerland, Austria, Italy, Poland, Czech Republic, Slovakia, Hungary and Slovenia. This accounts for about 2120 substations (nodes) and about 3150 lines of the three highest voltage levels. Regarding line characteristics, the same assumptions as for Germany are made.

3.3.2 Generation

Capacities

Generation is divided into eight plant types: nuclear, lignite, coal, oil and gas steam plants, combined cycle gas turbines plants, hydro, pumped storage and combined heat power plants. Wind capacity is addressed separately in a paragraph subsequently (Section (3.3.2)). Power plant capacities are based on VGE (2005). The current database includes all active plants for 2006 with a generation capacity greater than 100 MW. Each plant is assigned to one node. In the case of unclear grid integration, plants are allocated to the geographically closest node. A node can have more than one plant feeding into the grid.
Since thermal plants need a certain heat level to produce electricity, a minimal capacity is defined for each plant class according to DEWI et al. (2005). These values are identical for every thermal power plant. If output drops below this level, the plant has to be turned off. These values are used for defining the binary plant condition variable indicating whether the plant is on- or offline.
Combined heat and power plants (CHPs) often deliver long-distance heat or are integrated in a thermal production process in industries, thus producing electricity as a byproduct. These cogeneration plants were grouped corresponding to their primary output in heat- and power-operated plants. Due to legal guidelines an additional must-run condition was implemented in ELMOD to take into account that energy produced by this type of plant has to be fed-in prior to other energy types. The generation behavior of the

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nodes, the values should largely reflect changes in Germany.

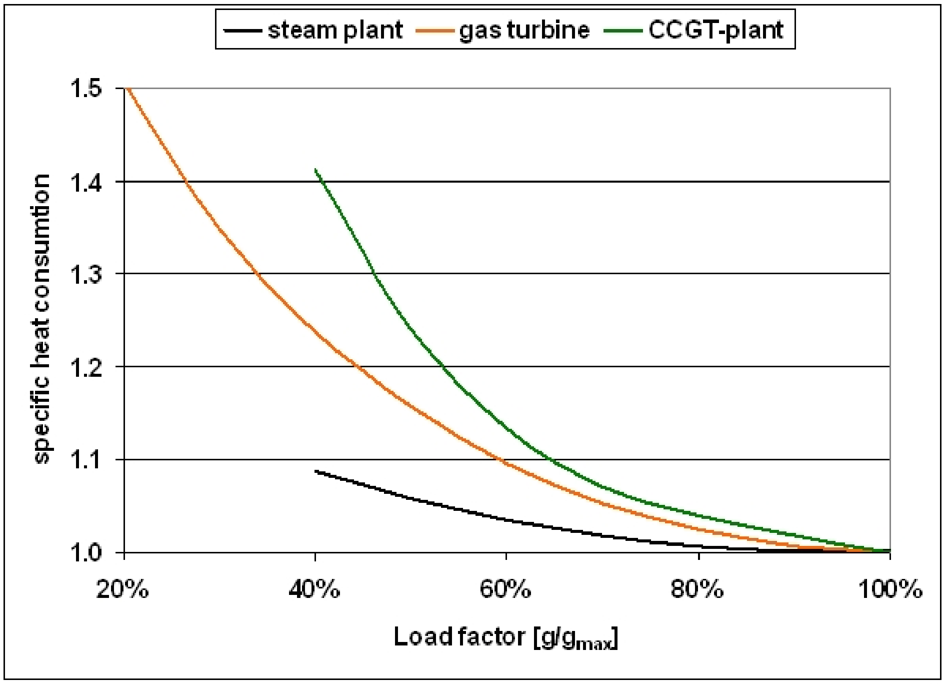
'heat-operated' power plants follows the same criteria as other power plants of the same type but they are assumed to be like base load plants in terms of unit commitment. Thus they constantly produce at least at their minimum output levels which is assumed to correspond to the specific required heat levels.[8] This may lead to an overestimation of output during night times and an underestimation during day times.

Costs

For each plant type a reference efficiency value and marginal cost are estimated based on different fuel types. Depending on the output level a mark-up is added if the output is lower than the reference efficiency value in order to allow for efficiency losses. The mark-ups have been transformed into quadratic polynomials. An additional cost block is added if a thermal plant has to start-up. Hence, cost functions vary between the different plant classes. Also, costs of plants from the same type differ since efficiency levels are not identical. In general, modern plants have a higher efficiency than older ones. However, the construction of the power plant cycle, the actual level of output and external conditions like cooling water availability influence the efficiency as well.
The actual generation costs are calculated on a marginal cost basis. If the output is lower than maximal output, a mark-up is considered to account for efficiency losses. Three mark-ups are defined: one for steam plants, one for combined cycle gas turbine (CCGT) plants and one for gas turbines. The mark-ups depend on the output level in relation to the maximal output. The increase of specific heat consumption due to operating below the optimal output is referred to as partial load conditions (Figure 3.3). Efficiency can be represented by specific heat consumption.
The impact is rather low for classical steam plants, but becomes important for peak load units like gas turbines and therefore is crucial in times of rapidly changing wind input conditions. The mark-up for CCGT-plants is based on VDI (2000) assuming reference efficiency at maximum output of 52.5 % (Müller, 2001). The efficiency of gas and oil fired gas turbines depend on the compressor inlet temperature. Based on a reference efficiency of 34.5% (Müller, 2001) and a temperature level of 15 °C, the partial load efficiency is taken from Kehlhofer et al. (1984). For steam plants, a functional interrelationship of specific heat consumption and partial load can be obtained from Baehr (1985). Nuclear plants may have additional drawbacks

[8] Heat demand curves are not included in ELMOD. The actual output is approximated via seasonal factors.

Figure 3.3: Partial load efficiency



Source: Own presentation based on Kehlhofer et al. (1984), Baehr (1985), and VDI (2000).

due to the necessary security constraints that are not considered within the model formulation.
Based on the above described assumptions it is possible to estimate the impact of varying wind energy on the total system costs. Although wind energy has no marginal generation costs inherently, it causes fossil plants to reduce generation and therefore operate under partial load conditions thus increasing their costs.[9] ELMOD uses the simplified partial load curves in

[9] A simple example reveals the impact: Assume a 1000 MW fossil plant with generation costs of 10 €/MWh that has to reduce its output because 200 MW wind energy are available and need to be fed into the grid. Running at 80% of optimal output causes the efficiency to drop and thereby the costs to rise to 10.07 €/MWh. The cost reduction therefore is not 2000 €/h, but only 1944 €/h. The difference could be considered as the indirect marginal cost of wind energy. In reality, a clear cost allocation of wind energy is not possible, because changes in demand modify the operation of the fossil plants. Furthermore, the indirect cost of wind generation is not constant but changes with the

order to calculate the cost of wind energy and neglects further wind specific additional costs. Nonetheless the overall impact on welfare is considered. Moreover, prices for CO_2 allowances are included into the generation costs. Therefore the plant specific CO_2 emissions are calculated based on efficiency and plant type according to Gampe (2004). Prices for CO_2 allowances are exogenous to the model are adjusted according to most recent data for different applications of ELMOD (compare Chapter 4).
Additional costs occur if a thermal plant has to start-up or go offline. Fossil plants generate electrical energy through transforming heat energy. This heat has to reach a certain level before generation can start and has to be cooled down in a controlled process after generation is stopped. The cool-down phase is assumed to be mainly affected by fixed cost parameters. Since ELMOD uses a marginal cost approach, it does not take into account cooling down specifically in its optimization. The start-up costs are mainly driven by fuel prices, as a certain amount of fuel has to be consumed before the heat level is high enough to start electricity generation. The cost estimations for start-up are taken from DEWI et al. (2005). These costs are added as a cost block within the hour of start-up. As base load plants are assumed to be must-run plants they do not have start-up costs.[10]

Wind

Since wind turbines have relatively small installed capacities, not all of them can be considered individually. To obtain a realistic distribution of wind capacities in Germany a map representing the installed capacity based on 10 km^2 squares is used (ISET/IWES, 2002). Each square - meaning a certain capacity value - is attached to the geographical closest node. This has been done for each federal state separately to obtain a percentage distribution which can then be updated with the actual wind capacities of the federal state. This distribution mechanism makes it also possible to increase the installed capacities without the necessity to reallocate each node individually assuming that installed capacities represent the suitability of a region for the use of wind turbines. As wind input depends on the wind speeds and largely differs between regions, a simplified classification scheme is used. Therefore six different wind zones have been defined using hourly wind speed information covering the time from 2002 to 2004 from seven representative stations (DWD, 2005). Since these reference stations are located approx-

load situation of the fossil power plants.

[10]This may lead to biased results in the long run, but should not influence the price and welfare calculation within the modeled reference time-frame.

imately 10 meter above ground (h_{ref}), an approximation about the speed values in the turbine height is applied. In general, wind speed and height follow a logarithmic function (Hau, 2003):

$$\nu(h) = \nu_{ref} \frac{ln\left(\frac{h}{z_0}\right)}{ln\left(\frac{h_{ref}}{z_0}\right)} \tag{3.22}$$

Wind speed $\nu(h)$ depends on the absolute height of the turbine above ground h and the local conditions like the building density, hillsides or forests that influence the roughness length z_0. To obtain average values a roughness length of 0.2, representing farm land with trees and bushes but without surrounding buildings, is defined for all nodes. The height of all turbines is assumed to be 60 meters, based on average values for mid-sized turbines. Calculating the speed values for all zones shows a clear separation between the coastal area in the North and the Southern areas.
For wind capacities in Europe, the World Energy Outlook (IEA, 2007) and the Wind Force 12 study (Greenpeace International and EWEA, 2005) are chosen. Although both studies analyze the energy sector developments on a global level and for different time horizons it is possible to extract data for continental Europe. Further data are derived from EMD (2005), EWEA (2005), IGW (2005), and WSH (2005). Wind capacities are allocated according to to federal states or similar administrative areas taking into account political, geographical and meteorological framework conditions.

3.3.3 Demand

In order to derive a node-specific demand, ELMOD assumes a positive correlation between economic income and total electricity demand. This relation is modeled in greatest detail for Germany, where demand is differentiated into consumption of industries, services and households: electricity is consumed to around 46% by the industrial sector, 27% by households and 21% by services (Eurostat, 2004).[11] Standard load profiles for households (H0) and services (G0) as specified in VDEW (1999) are applied and calculated for typical winter and summer workdays. Since various different load profiles exist in the industry sector, the industry consumption is approximated by

[11]The remaining electricity consumption is used by agriculture, transportation, the energy sector and others. Since these sectors amount only for a small part of the overall consumption, they are not taken into account separately.

taking real electricity consumption of a typical winter and summer workday from UCTE (2006) and discount power of households and services according to the standard load profiles. Consequently, the difference indicates the industry consumption. Load profiles are calculated on an hourly basis and are normalized to the overall consumption of electricity made by each sector as stated above.

To weight the sector specific consumption with the amount of this sector on a specific node, the gross value added of industry and services and the gross domestic product of households are used. The gross value added is available at Euro NUTS 3 level for larger countries and Euro NUTS 2 for smaller countries. Each district is assigned to a node. In case there are different nodes in one district, the entire gross value is divided by the number of nodes. In case there is no node in the district, the gross value added is distributed to all neighboring districts with nodes. The share of a node of the whole gross value added is calculated and applied to the overall electricity consumption by industry and services, respectively. Regarding the node-specific consumption of households, they are deduced distributing the inhabitants of an administrative district to the node in the same manner as the gross value added for industry and services are assigned to. In a second step, the annual energy consumption of the households is assigned to the nodes according to the node's share in the whole gross domestic product. This, subsequently, yields a reference demand per node. On the basis of this reference demand, a reference price (e.g. average EEX price for Germany) and the assumption of a demand elasticity at this reference point[12], a linear demand function can be estimated.

For the remainder of Europe, demand is based on UCTE data. For ELMOD applications with focus on Germany the neighboring countries are condensed in single nodes, thus a separation of demand according to industry, commerce and residential is not necessary. Reference prices are taken from the national electricity exchanges.[13] A linear demand behavior is obtained in the same way as for Germany. For calculations covering more countries a node specific demand is derived by using the gross value added as key for a distribution of load to different districts. Thus, a separation of household, service and industrial demand is not considered for the rest of Europe.

[12]Green (2007) includes different assumptions about demand point elasticities in his nodal pricing analysis of a simplified network of the UK. Based on his study, the default demand elasticity in ELMOD is -0.25. However, this value can be altered easily for different model applications - normally between 0 and -0.25.

[13]In case no national price is available, a European average price is calculated based on the existing national prices.

3.4 Applications of ELMOD

3.4.1 Network Constraints and Offshore Wind

ELMOD was initially used in order to study different congestion management schemes for the German electricity market, particularly the problem of integrating large scale offshore wind projects as presented in DEWI et al. (2005). Leuthold et al. (2005) demonstrated that nodal pricing is superior to uniform pricing and conclude that when using nodal pricing, 8 GW offshore wind capacities can be implemented without grid extension and additional 5 GW if the North West German grid will be extended. As the underlying model is time static, varying demand and wind input are considered through different reference cases. Also, cross-border flows and unit commitment decisions are neglected. Weigt et al. (2006) continued the work and extended the model by including France, Benelux, Western Denmark, Austria and Switzerland in order to examine cross-border flow issues. They point out that even under status quo conditions, the price situation in Benelux is affected by high wind input in Germany. This situation is bound to aggravate if the planned wind capacity extension will be realized without proper grid adjustments. The work of Weigt et al. (2006) was the first approach to model the effects of nodal pricing in combination with increased wind energy on the North-Western European grid. Weigt (2006) extended the model by including a time-frame of 24 hours to simulate variable demand and wind input as well as unit commitment, start-up and pump storage issues. He shows that for the German market a nodal pricing system would yield significantly lower prices during peak times on average. The impact of wind energy under current conditions is mainly predictable and leads to price decreases in North and East Germany. However, in specific load and wind input cases congestion situations can lead to price increases in South Germany. The planned wind capacity extensions based on a forecast for 2010 led to significant price reductions in North Germany but increased price differences particularly between the Netherlands and Germany as well as between South and North Germany. Leuthold et al. (2008a) built on the aforementioned work in order to recommend nodal pricing as a valuable tool for electricity market analyses particularly also in a European context.

The problem of grid extensions due to increased wind input is taken up by Jeske (2009). He analyzed the possibility of integrating large scale offshore capacities using high voltage direct current (HVDC) lines in order to transport the energy to demand centers in the South and West of Germany. He found that when applying welfare criteria and considering congestion, the HVDC-approach is more efficient than other grid extension measures. Another application by Leuthold et al. (2009) focusing on large-scale wind integration is presented in Chapter 4 of this thesis.

3.4.2 Locating Generation Investments

Dietrich et al. (2009) applied ELMOD in order to model optimal investment behavior up to the year 2012 based on realistic data of planned generation investments. They represent an average year in terms of demand and wind levels. Twelve cases are defined to simulate off-peak, mid-load and peak demand in winter and summer as well as high and average wind input. Analyzing locations of plants yielded different results for different grid extension scenarios. While the projected locations were mainly along the North-Sea coastline and the Ruhr area, the optimal model results for locations varied significantly with assumptions regarding the grid situation. To put it in a nutshell, Dietrich et al. (2009) showed that transmission expansion is a critical condition for generation investment locations, particularly in a European context.

3.5 Conclusions

In this chapter, ELMOD an economic engineering model of the European electricity market was presented. ELMOD is based on a DC Load Flow approach and captures the essentials of the European electricity markets, even though it lacks some idiosyncrasies of some national markets. On the basis of existing studies using ELMOD, it could also be shown that ELMOD is a valuable tool in terms of analyzing the effect of offshore wind power on the North-West European electricity market, and the effects of congestion between countries and within the German grid. Additionally, ELMOD can also be applied to generation investment issues namely the siting of new power plants under grid constraints.

Chapter 4

When the Wind Blows over Europe: An Application of ELMOD

4.1 Introduction

An essential issue in electricity markets is network congestion. As stated earlier, network congestion impacts on the market equilibrium according to geographical characteristics. In case of network congestion, transmission capacity becomes a scarce resource for which scarcity rents apply. The ELMOD model described in Chapter 3 is able to capture these effects based on regional demand and supply data as well as the relevant technical parameters of an electricity grid.

While being a potential source that can influence the congestion situation within the network, renewable energy sources are also a major driver in the debate about the future development of electricity networks. An often-stated objective in the discussion about integrating renewable energy into the transmission network which is referred to as 'greening the grid'. Particularly wind energy is considered to provide significant amounts of electricity in a lower-carbon world. Given the ambitious, politically-driven objectives for wind energy in some U.S. states, in Europe, and elsewhere, an adequate regulatory framework is required to provide the proper incentives for additional generation capacities and network expansion. Some instruments, such as uniform pricing of network access, simply do not provide adequate signals for investment and usage.

Initially, the issue of additional wind generation was discussed in purely political terms. Proponents hailed it as a clean, sustainable resource, while opponents insisted on the infeasibility of integrating intermittent wind into regular dispatch of electricity. However, recent experience with feeding-in large amounts of wind has shown that the operational issues are manageable, and the resource can be addressed with less ideological pathos, since regulatory decisions mandate wind's role as a key player in the development of European electricity markets. Another aspect of the new debate concerns how additional wind capacities may be efficiently integrated. In the past, wind generation was decentralized and its impacts on the grid were generally quite minor. However, factoring in onshore and, more importantly, projected construction of offshore capacities, gives rise to questions about wind's growing impacts, especially whether the existing grid is still capable of reliably securing energy supply in the integrated network or whether an expansion of the existing network capacities is necessary.

This in turn raises the question how efficient transmission expansion can be carried out. A debate that is reviewed in detail in Section 2.2. However, the existing expansion literature has not yet explicitly included increasing renewable energy production as driver for grid investments. To date, the research on electricity network investments and renewables is dominated by technical issues related to network integration and expansion. A study commissioned by the German Energy Agency (DEWI et al., 2005) that analyzed the costs of integrating additional wind capacity in the German grid found that extensions to resolve emerging network bottlenecks would be cost-intensive. Other technical studies with similar results looked at Poland (PSE, 2003), France (Verseille, 2003), the Netherlands (Hondebrink et al., 2004), Austria (Haidvogl, 2002), Denmark (Woyte et al., 2005, 2007) and Spain (IDAE, 2005).

Additionally, the economic investment literature in Section 2.2 focuses on rather small two- or three-node example networks. On contrast, the mentioned technical reports have a larger view. However, results from large-scale economic engineering models on network investments due to wind input have not yet been reported to the knowledge of the author. Thus, rather than modeling two- or three-node networks, this chapter considers the larger scale. An economic algorithm to calculate the optimal extension of electricity networks taking into account additional capacities of wind energy (onshore and offshore) is developed and implemented into ELMOD. Within the present chapter, the CTP approach is taken and it is assumed that a centralized network operator desires to maximize welfare under perfect competition. Furthermore, nodal pricing is suggested to be the adequate regulatory framework to facilitate the integration of wind, because it provides price signals

and indicates potential congestion; thus, the impact of additional wind energy can be estimated by analyzing price situations. Strong price differences between neighboring nodes help to identify highly congested lines in different scenarios. A special grid-extension algorithm allows the model to extend the grid incrementally until an economically optimal grid status is identified that is capable of carrying the additional wind.
The remainder of this chapter is structured as follows: Section 4.2 presents the model based on the ELMOD model of Chapter 3. The network topology, the data set and the scenarios are described in Section 4.3.[1] Section 4.4 presents the results of the model runs, scenario analysis and offers according interpretations. The analysis is able to identify socially valuable investment locations. Interestingly, most of the necessary line extensions are not due to the large increase of wind energy in the next decade but are necessary to overcome already existing bottlenecks particularly at the country borders. The overall investment amount required to cope with increasing wind energy is low compared to the resulting welfare gain. Furthermore, a more equalized increase of wind capacities in the European countries can help to cancel out current local network problems. The chapter concludes in Section 4.5 that the efforts to prepare the European grid for large amounts of wind generation capacities appear rather modest.[2]

4.2 The Model

4.2.1 Mathematical Notation

Indices:

$it \in I$	iteration
$l \in L$	line within the network
$n, j \in N$	nodes within the network
$s \in S$	power plant type

[1]Choosing Europe as the study area is bases particularly on data availability and the ambitious goals for wind expansion in many European countries, such as Germany, Spain, the UK, and France.

[2]The subsequent chapter is based on the working paper Leuthold et al. (2009).

Sets:

I	set of all iterations
L	set of all lines
N	set of all nodes
S	set of all power plants

Parameters:

ANF	annuity factor
AN_{nj}	annuity for upgrading the line between n and j
E	equity
$\overline{g}_{ns}$	available maximum generation level of plant s at node n
$Icost$	cumulated annuity for conducted grid investments
k	given period for grid investment
$\overline{P}_l$	maximum available power flow capacity over line l
r	weighted average cost of capital (WACC)
r_{DC}	interest on debt capital
r_E	equity yield rate
t	tax rate
TC	total capital
u_{nj}	number of circuits of the line between n and j
wi_n	wind input at node n

Variables:

c_{ns}^{it}	generation cost of plant s at node n after iteration it
g_{ns}^{it}	generation of plant s at node n after iteration it
ni_n^{it}	net grid input at node n after iteration it
$p_n^{it}(q_n^{it})$	linear inverse demand function at node n after iteration it as a function of q_n^{it}
P_l^{it}	power flow over line l after iteration it
q_n^{it}	demand quantity at node n after iteration it

4.2.2 Assumptions

This chapter examines the impact of wind energy on the grid for the forecasted scenarios in 2020. It is a ceteris paribus consideration which assumes that the conventional power plant fleet does not change from today to 2020. The research focus is to simulate feeding-in the forecasted wind into the existing electricity system. Therefore, a feed-in guarantee for wind is assumed.

Feed-in guarantees are the dominant schemes applied in Europe (e.g. in Germany and Spain). An optimization problem is formulated that calculates the welfare for the electricity system regardless of country or state borders. This presumes either a single entity managing the grid or perfect coordination between different entities, and neglects imperfect market functioning (i.e. a perfect competition approach).

4.2.3 Optimization Problem

To calculate the scenarios, ELMOD - a model of the European electricity market - described in Chapter 3 is applied. However, compared to the model description in Equations (3.1)-(3.10), ELMOD is adjusted within this chapter by disregarding the time component t. There are two reasons for this simplification. First, the extension algorithm described in Section 4.2.4 requires several welfare runs (one per iteration it). Hence, the model must be as simple as possible in order to achieve manageable calculation times. Second, the focus of the analysis is on wind extensions for the year 2020. Thus, short-term inter-temporal constraints such as start-up conditions for plants are negligible. The time component can be replaced by different load and wind scenarios (compare Section 4.2.4). The objective functions for all considered scenarios is maximizing social welfare W that equals total consumer benefit minus the cost of generation needed to satisfy demand (Equation 4.1a). There is a liner inverse demand function $p_n(q_n)$ for each node where p_n is the nodal price at node n and q_n is the demand quantity at node n. Optimal dispatch is determined respecting physical laws and technical conditions, namely an energy balance (Equation 4.1b), a line capacity (Equation 4.1c), and a maximum generation capacity (Equation 4.1d) constraint:

$$\max_{g_{ns}^{it}, q_n^{it}} \left\{ W^{it} = \sum_n \left(\int_0^{q_n^{it}} p_n^{it}(q_n^{it}) \mathrm{d}q_n^{it} - \sum_s (c_{ns} g_{ns}^{it}) \right) - Icost^{it} \right\} \tag{4.1a}$$

subject to

$$\sum_s g_{ns}^{it} + wi_n - q_n^{it} - ni_n^{it} = 0 \quad \forall n, it \tag{4.1b}$$

$$|P_l^{it}| \leq \overline{P}_l \quad \forall l, it \tag{4.1c}$$

$$g_{ns}^{it} \leq \overline{g}_{ns} \quad \forall n, s, it \tag{4.1d}$$

$$g_{ns}^{it} \geq 0 \quad \forall\, n, s, it \tag{4.1e}$$

$$q_n^{it} \geq 0 \quad \forall\, n, t, it \tag{4.1f}$$

Another simplification compared to the model in Chapter 3 is the assumption that marginal costs c_{ns} are constant for each generation g_{ns} at a node n depending on plant type s. Additional costs, such as those arising from network operation and maintenance as well as start-up costs and ramping conditions are not considered. Power flow P_l and transmission losses are obtained using the DC Load Flow network model (DCLF) described by Schweppe et al. (1988) and Stigler and Todem (2005). Transmission losses are included by splitting them between the start and end nodes of a line l as presented by Todem (2004). Hence, losses are represented within the net input that defines the amount of energy that is injected or withdrawn from the network at node n. To account for the (N-1)-constraint, a transmission reliability margin of 20% is introduced; thus the $\overline{P}_l$ of each line l is 80% of the full thermal limit. The reference period is one hour. Since the approach is time static, the analysis includes different scenarios in order to simulate changing external conditions. The optimization is coded in GAMS and solved on an Intel Xeon CPU E5420 (8 cores) with 16 GB RAM.

4.2.4 Grid Extension Algorithm

The objective of this heuristic algorithm is to approximate the amount of necessary grid extensions to cope with increasing wind energy inputs. The developed algorithm gradually extends the grid (upgrading existing lines). In a first step the model calculates the weighted average nodal prices for each node out of four representative standard load and wind generation cases (low wind and low load, low wind and high load, high wind and low load, high wind and high load) for each extension scenario: high load corresponds to the average value of the highest 33% of hourly demand in 2006 and low load to the average level of the remaining 67%. High wind corresponds to a wind input level of 80% of available installed capacities and low wind to a level of 20%.

Next, the model identifies the most severely congested line (identifying the line between the two nodes with the highest price difference). This line is then extended by adding another circuit of the same kind at the same link, simulating a line extension in the form of adding one additional parallel line to an existing connection. It is assumed that this type of extension measure is possible on each circuit of the model. However, our model does not allow for more than four parallel circuits on one connection. If this constraint becomes binding, the line with the second highest price difference is extended and so on.
After each extension step it, the model performs a new run and determines the new welfare value and the welfare change $\Delta W^{it} = W^{it} - W^{it-1}$. The model then compares the welfare change to the investment effort required to implement the respective grid extension. If the costs are higher than the change of welfare, the line is not considered for further extensions. The model automatically stops if no welfare gain is obtained for 50 different extension steps.

- **Step 0: Initialization.** Set iteration counter $\nu_1 = 0$ and stopping counter $\nu_2 = 0$. $Icost = 0$. Solve the optimization problem (4.1). $W = W^{it}$.

- **Step 1: Grid extension.** Set iteration step it to ν_1. Determine prices p_n^{it} and price differences Δp_{nj}^{it} between all interconnected nodes. Determine $\Delta p_{n^*j^*}^{it} = \max(|\Delta p_{nj}^{it}|)$. If $u_{n^*j^*}^{it} < 4$ set $u_{n^*j^*}^{it} = u_{n^*j^*}^{it} + 1$. Set $Icost = Icost + AN_{n^*j^*}$. Recalculate all line parameters and go to next step. Otherwise fix $\Delta p_{n*j*}^{it} = 0$ and repeat Step 1.

- **Step 2: Economic dispatch.** Update iteration counter $\nu_1 = \nu_1 + 1$. Solve the optimization problem (4.1). $W = W^{it}$. If $W^{it} \geq W^{it-1}$ go to Step 1. Otherwise undo changes ($Icost = Icost - AN_{n^*j^*}$, $u_{n^*j^*}^{it} = u_{n^*j^*}^{it} - 1$, and $W^{it} = W^{it-1}$) and fix $\Delta p_{n^*j^*}^{it} = 0$; set stopping counter $\nu_2 = \nu_2 + 1$. If $\nu_2 = 50$, stop. Otherwise continue with Step 1.

4.2.5 Investment Costs

According to the procedure described by standard finance literature (e.g. Buckley et al., 1998), the model calculates the discounted value of the annual depreciation of the investment costs for the particular extension measure. The discounted annual depreciation value is received by multiplying the initial costs for the particular extension measure with the annuity factor ANF:

$$ANF = \frac{r(1+r)^k}{(1+r)^k - 1} \tag{4.2}$$

where r represents the weighted average capital costs (WACC) and k the given period regarding the invested commodity. The WACC is calculated as:

$$r = \left(\frac{E}{TC}\right) r_E + \left(\frac{DC}{TC}\right) r_{DC}(1-t) \tag{4.3}$$

with E for equity, TC for total capital, r_E for equity yield rate, DC for debt capital, r_{DC} for interest on debt capital and t for tax rate. The rate r_E is determined using the Capital Asset Pricing Model (CAPM):

$$r_E = r_f + (\mu_m - r_f)\beta \tag{4.4}$$

with r_f for the risk-free rate of return, μ_m the market rate of return and β the risk factor. The time value of money is thereby represented by the risk-free rate r_f and compensates the investors for dedicating money to any investment over a period of time. The second term of the equation represents risk and calculates the amount of compensation the investor needs for taking on additional risk. This is calculated by applying a risk measure β that compares the return of the asset to the return of the market over a certain period of time to the market premium, i.e. the differential of average market returns μ_m and the risk-free rate. Thereby, β is defined as function of the covariance between the considered portfolio (investment) and the market portfolio, divided by the variance of this market portfolio. The CAPM says that the expected return of a security or a portfolio equals the rate on a risk-free security plus a risk premium. For the analysis in this chapter, the following values were chosen: a given period (k) of 12 years, 25% equity ratio $\left(\frac{E}{TC}\right)$, 6% interest on debt capital (r_{DC}), 40% tax rate (t), 3.5% risk-free rate of return (r_f), 13% market rate of return (μ_m) and 0.9 as risk factor (β). The latter is a conservative estimation based on the assumption that network investments are carried out by large grid companies (compare Table 4.1). Based on the mentioned assumptions, the annuity factor ANF yields 11.75%.

	RWE	EON	EnBW
β	0.74	0.85	0.80

Table 4.1: Beta factors of selected German energy companies
Source: Own calculations based on EnBW (2008), EON (2008), and RWE (2008).

According to DEWI et al. (2005), the investment required to upgrade a 150kV/220kV and a 380kV is 70,000 €/km and 120,000 €/km respectively. The capital expenditure for upgrades to be compared with the welfare increase is calculated by multiplying the specific price per km with the length of the upgraded line divided by 8760 hours.

4.3 Data and Scenarios

4.3.1 Data

The model data is based on the UCTE extra high voltage grid (UCTE, 2004) of the European Union and Switzerland. It includes Portugal, Spain, France, the Netherlands, Belgium, Luxemburg, Denmark, Germany, Switzerland, Austria, Italy, Poland, the Czech Republic, Slovakia, Hungary and Slovenia. The basic model consists of 2120 substations (nodes) and 3143 lines. Three voltage levels - 380kV, 220kV and 150kV - are considered.
In order to apply the DCLF, different line parameters are required: voltage levels, thermal limits, line resistances and line reactances. For each voltage level a reference line type is selected, neglecting impacts of the wide range of different lines. For 380 kV, four cables per wire, for 220 kV, two cables per wire, and for 150 kV, one cable per wire are assumed and the thermal limits are derived accordingly (Fischer and Kießling, 1989). In the model those maximal power flows are multiplied by the number of circuits, neglecting impacts of influence between multiple circuits. Values for the resistances and reactances of high voltage circuits are subject to empirical experience. Therefore, average values based on Fischer and Kießling (1989) are adopted. Generation capacities are based on VGE (2005). Eight types of conventional power plants are classified and each plant is assigned to one class according to the main fuel type (Table 4.2). Base case wind capacity information is derived from several sources. For Germany a pro rata distribution is used for the nodes in each federal state based on ISET/IWES (2002). For other countries the wind capacity distribution is based on publicly available information, mainly from national wind energy associations. For Italy, Portugal

and the new European Union member states, apart from Poland, no such data is available. Hence, the regional allocation of existing wind energy capacity is approximated, taking geographical and meteorological conditions into account. For Poland, the Polish Wind Energy Association (PWEA) provides detailed information about the locations of the existing 150 MW installed wind capacity (PWEA, 2006).

The node-specific generation costs are calculated on the basis of marginal costs, including fuel costs, but not accounting for operating and service costs. Wind power generation is assumed to have no marginal generation costs. Thus indirect costs of stochastic wind input causing higher balancing and response power costs are neglected. Pumped storage is assumed to store during night hours (from 8 p.m. to 8 a.m.) by purchasing electricity on the stock exchange. Marginal costs of conventional plant types are adopted for two reference cases (see Section 4.3.2).

To obtain a node-specific reference demand, the regional GDP (Eurostat, 2005) is used as proxy for electricity demand. It is assumed that provinces with high economic output - and, respectively, with a high share in the countries' GDP - have a high electricity demand. Consequently, the total electricity consumption is divided according to the GDP share. Within a province, the demand is distributed equally over all nodes.

Fuel	**Installed Capacity [GW]**	**Fuel**	**Installed Capacity [GW]**
Coal	99.2	Natural gas	49.0
Lignite	44.2	Fuel oil	62.4
Nuclear power	107.3	Water	36.0
CCGT	13.7	Pumped storage	23.3
		Total	435.1

Table 4.2: Conventional power plant capacities in Europe
Source: VGE (2005).

4.3.2 Three Scenarios

Three scenarios are considered: Benchmark, WEO (World Energy Outlook), and WF12 (Wind Force 12). The Benchmark scenario uses 2006's installed wind energy capacity according to DEWI (2006). WEO applies the wind extensions according to the World Energy Outlook 2006 (IEA, 2007) of 114 GW. WF12 includes the alternative wind extension scenario according to the study WF12 (Greenpeace International and EWEA, 2005). Although both extension studies analyze the energy developments on a global level and for different time horizons, it is possible to extract data for Europe. The studies use the same geographical sectioning, namely OECD Europe which includes the EU-15 countries as well as the Czech Republic, Hungary, Iceland, Norway, Switzerland and Turkey. The countries of OECD Europe that are not included in the UCTE grid are Finland, Sweden, Norway, Greece, Ireland, Iceland, Norway, Turkey, and the UK. Taking into account political, geographical and meteorological conditions, it is assumed that 22% of the forecasted wind energy capacities will be installed in the countries that are not in the UCTE grid (with a high amount being allocated to high wind resource countries, e.g., the UK and Scandinavia). The time horizon is not identical in both studies; WF12 projects to 2020 and WEO to 2015 and 2030. In the latter case a linear growth is assumed in order to make a linear interpolation possible. In 2020, Greenpeace International and EWEA (2005) forecast 180 GW total installed wind capacities and IEA (2007) forecasts 114 GW total installed wind capacities under the described assumptions. Since the forecast studies do not give detailed regional information but the model uses accurate regional wind capacities, the additional capacities are allocated onto federal states or similar administrative areas. Table 4.3 shows the obtained wind capacities.

Changes in the demand or generation structure are not considered. Thus the approach is a ceteris paribus analysis. However, two different generation cost cases with respect to the price of emission allowances (EU ETS) are included: an average CO_2 price of 20 €/t_{CO_2} and a high CO_2 price of 50 €/t_{CO_2}. Since wind generation has approximated marginal costs of zero the welfare effect increases with higher costs for those fossil fuels that are replaced by wind input. Respectively, with higher CO_2 prices it is expected to observe a larger amount of economic feasible grid extensions. Table 4.4 summarizes the applied generation costs.

4.4 Results and Interpretation

4.4.1 Price Results

First, the results assuming an average emission allowance price of 20 €/t_{CO_2} are presented. Given the modeled situation of 2006 an intermediate price level in Central and Eastern Europe, low prices in France due to the large share of nuclear generation, and high prices in Italy and the Iberian Peninsula can be observed. If the grid is extended a general price convergence in Europe takes place. However, now the low-price regions encounter higher

		Installed Wind Capacity [GW]		
		Benchmark	**WEO**	**WF12**
Country	Austria	1.0	5.2	8.0
	Belgium	0.2	0.2	0.3
	Czech Republic	0.0	3.3	5.0
	Denmark	3.1	2.6	4.0
	France	1.6	11.7	18.0
	Germany	20.6	31.3	48.0
	Hungary	0.1	6.5	10.0
	Italy	2.1	9.8	15.0
	Luxembourg	0.0	0.1	0.1
	Netherlands	1.6	3.6	6.0
	Poland	0.2	10.7	18.5
	Portugal	1.7	3.3	5.0
	Slovakia	0.0	1.3	2.0
	Slovenia	0.1	0.7	1.0
	Spain	11.6	24.1	40.0
	Switzerland	0.0	0.1	0.1
	Total	**43.9**	**114.5**	**181.0**

Table 4.3: Wind generation capacities in 2020 forecasts per scenario
Source: Greenpeace International and EWEA (2005), EWEA (2006), IEA (2007), and own calculations.

prices, which is particularly striking for France (Figure 4.1). The first extension scenario WEO results in lower prices in Europe compared to 2006. On average the projected 115 GW wind capacities have a positive effect on electricity prices. The highest benefit is observed in the Spanish and Portuguese markets. After the grid extension again a price convergence within Europe can be observed, although the price level in Italy remains higher. The same is true for the WF12 scenario. A small further price decrease compared to the WEO case and a price convergence in Central Europe can be observed. Second, the price developments are analyzed, given a high price for emission allowances of 50 €/t_{CO_2}. Starting with the modeled situation in 2006 the results show a similar price pattern as in the 20 €/t_{CO_2} case. However, the absolute price level is about 10 to 20 €/MWh higher, corresponding to the increase in generation costs. After the grid extension the prices tend towards a more common level in Europe, although regional differences remain. In the two wind extension scenarios the price level further decreases particularly

Fuel	**CO_2 Price 20€/t_{CO_2}**	**CO_2 Price 50€/t_{CO_2}**
Nuclear Power	15.15	15.15
Natural Gas	67.00	83.50
Lignite	37.14	67.14
Fuel oil	94.71	114.21
Coal	37.54	61.54
Running Water	0.00	0.00
CCGT	42.64	53.14
Pumped Storage	40.00	40.00

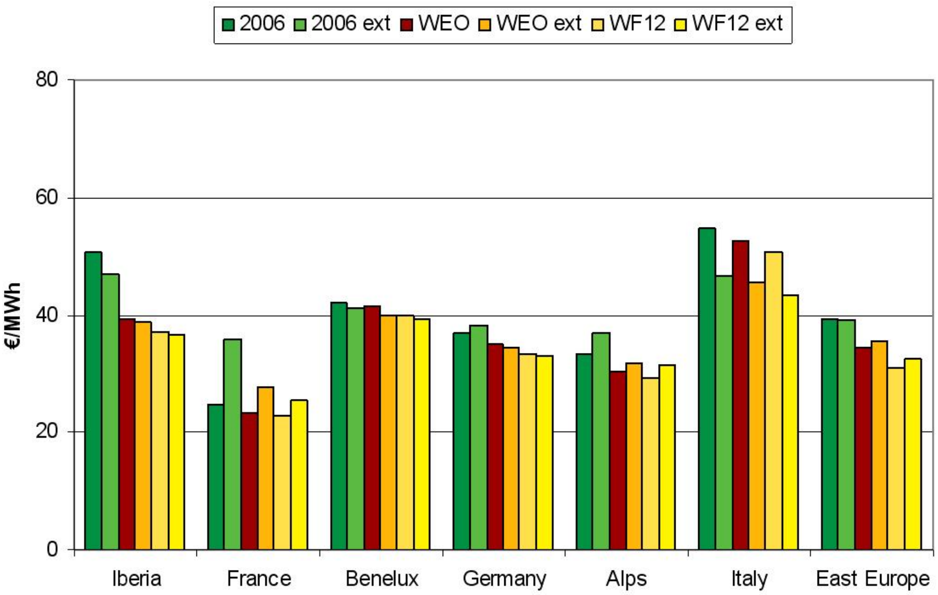
Table 4.4: Generation costs in € per MWh
Source: BAFA (2008).

in Spain and Portugal, with a reduction of about 20 €/MWh. Also in East Europe a significant reduction is observable (Figure 4.2). The extended grid results are quite similar to the case with the lower emission price leading to further reduced prices in southern Europe and a more equalized price level in central Europe. Given the price developments it can be concluded that an increase of installed wind capacity in the years ahead leads to electricity price reductions as wind partially replaces conventional generation. This is of particular concern in Spain and Portugal where a doubling of the current installed wind capacity significantly reduced prices. It can be noted that the benefits of increased wind are less striking in the remaining countries. However, grid expansion will not lead to a reduced price level in all European countries. The present situation is characterized by congestion at the borders and a market separated into several price zones. If increased network capacity removes some bottlenecks and brings prices closer together, formerly low-price regions (e.g., France) will likely encounter higher prices.

4.4.2 Comparison

The results of the scenario runs are presented in Table 4.5. The extension of wind capacities leads to a lower average electricity price. In the case of an average emission allowance price the reduction is about 5 to 8 €/MWh, depending on the installed wind capacity. If a higher emission price is assumed, the positive price impact of wind increases to 18 and 22 €/MWh. However, as already noted, this does not mean that each region profits from increased wind input in a similar fashion. One surprising outcome is that the benchmark model shows the highest amount of grid extension for both

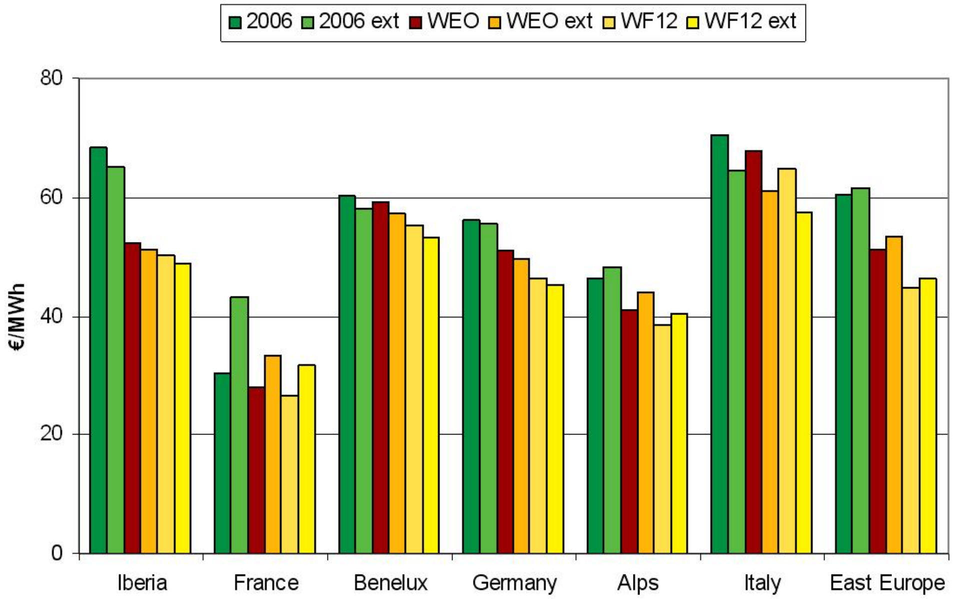
Figure 4.1: Average prices before and after the network extension, 20 €/t_{CO_2}



Source: Own presentation.

CO_2 price scenarios. One would expect that the increased wind capacities in 2020 lead to a greater need for grid extensions due to an increase in the transmission volume and unintended loop flows. However, given the model setting, the results show the opposite. The current grid conditions already show a high level of congestion which makes an ambitious extension schedule necessary. The increase in future wind generation appears to support the overall power flow pattern. This may stem from the fact that in 2006 wind capacity clusters in Germany, Denmark, and Spain, whereas in the two 2020 scenarios France, Italy and Poland have significant installed wind capacity, too. Accordingly, the need for transporting wind might decrease. The model does not differentiate for wind speeds. Thus for the high wind input cases, wind generation is increased equally in all countries, leading to the possibility of counter injections (e.g., between France and Germany) reducing actual load. However, this benefit depends on the amount of wind capacity installed. In the WF12 scenario the total grid extension is similar to the benchmark case, while in the WEO scenario the amount is significantly lower. Hence, the positive effect of opposing wind injections may be offset by localized problems in the case of the large capacity increase in

Figure 4.2: Average prices before and after the network extension, 50 €/t_{CO_2}



Source: Own presentation.

the WF12 scenario. The welfare properties of the extensions are generally positive in all cases. Given the relatively low extension costs of about 500 million Euro (less than 0.5% of the total welfare) a significant welfare increase of more than 2.5 billion Euro per year is obtainable. Furthermore, a large fraction of this welfare gain is already achieved by the first extensions. In the benchmark case with 20 €/t_{CO_2}, a welfare gain of 1 billion Euro is already achieved after 20 line extensions totalling less than 50 million Euro. The relatively low investment costs are a result of the model restriction to line upgrades and the assumption that each line can be upgraded to four circuits which may not always be feasible. Another remarkable outcome is that in the case of higher emission allowance prices, the grid extension is lower. Given the higher costs for conventional power plants one would assume that an increase of wind reduces the need for expensive fossil fuels and thus increases the welfare gain by wind integration. Altogether, the resulting prices are higher because of the more expensive fossil plants that in turn lead to a lower demand in the entire system given the modeled linear demand function. Thus, there are less extension requirements as the transmission volume decreases. The important number in the high CO_2

price case is the welfare gain induced by grid extensions. Table 4.5 shows that the gain is higher as the price differences between the costs of wind energy and fossil production increase. The presented approach calculates the weighted average of four representative hours. The separated observation of one worst-case hour with very strong wind (all wind generation capacities produce maximum power) may lead to collapse even with grid expansion. Because this situation will occur rarely (less than 2 to 4% of time on average), economic considerations tend to accept this 'threat' since additional investment costs are not justified. Such extreme events should be managed with technical measures other than line upgrades, such as active wind farm management, extensive grid monitoring, and others.

4.5 Conclusions

From the perspective of economics, this chapter shows that efforts to prepare Europe's high voltage grid for large amounts of wind generation appear to be rather modest. Developing the network at existing bottlenecks - mainly cross-border connections - should be encouraged by regulatory authorities. With a more moderate wind expansion of 114.5 GW, the optimal grid investments are smaller. However, if the additional wind capacity becomes too great (181 GW), the needed grid extensions will increase compared to the actual situation. 'Greening the grid', i.e. enabling the integration of low-carbon technologies, appears feasible for wind energy. Further research should address issues of stochasticity, and apply similar analysis to other renewables, e.g., solarthermal and photovoltaic. A study of the transferability between Europe and U.S. experiences also appears fruitful. This conclusion suggests that other research might examine the relationship between fostering renewable energy production and the design of efficient contract networks, e.g., resolving issues of priority network access for renewables and transmission rights.

	ETS: 20 €/t_{CO_2}			**ETS: 50 €/t_{CO_2}**		
	Bench-mark	**WEO**	**WF12**	**Bench-mark**	**WEO**	**WF12**
Wind capacity [GW]	43.9	114.5	181.0	43.9	114.5	181.0
Upgraded circuits	159	139	138	124	100	116
Total length [km]	6220	5330	6440	5200	4500	5300
Additional line capacity [GW]	143	106	114	102	85	92
Total costs [mn€]	580	475	560	460	400	445
Average price [€/MWh]	40.1	34.6	32.7	65.2	47.0	43.4
Welfare [bn€/y]	230	238	242	218	229	235
Welfare gain due to extension [bn€/y]	2.9	2.6	2.9	3.9	3.2	3.9

Table 4.5: Results overview Chapter 4
Source: Own calculations.

Part III

Game Theoretic Economic Engineering Modeling

Chapter 5

Solving Discretely-Constrained MPEC Problems Using Disjunctive Constraints and Discrete Linearization with an Application in an Electric Power Market

5.1 Introduction

It is widely accepted that many of the national European Union (EU) electricity markets are characterized by market power of a single or only a few companies. In France, Electricité de France (EDF) has a market share of over 80%. In Germany, the two biggest players own together 55% of the generation capacities, the biggest four player have together 85% market share (Hirschhausen et al., 2007). These numbers show that there are still nearly monopolistic or oligopolistic market structures prevailing in Europe. Moreover, these shortcomings in national competition also have an impact on competition within the entire EU electricity market, particularly because the national markets are separated from each other by limited transport capacities which might enable national players to use their domestic power against foreign competitors.

In light of these facts, several models and algorithms have been developed to simulate the outcomes of imperfect electricity markets. These models include a broad range of approaches using game theory. Existing modeling efforts - discussed in the subsequent literature sections - have achieved some success but there is still room to handle larger-scale or more realistic models as might be found in the EU, North America or in other parts of the world. At a first glance, it seems that one-stage Nash-Cournot models can be solved robustly for large-scale problems. If it comes to two-stage games involving either a mathematical program with equilibrium constraints (MPEC) or an equilibrium program with equilibrium constraints (EPEC), most algorithms are for the most part, still in the development stage when one considers large-scale or integer-constrained formulations.

In this chapter, a new approach to solve two-stage Stackelberg games with one leader based on disjunctive constraints and discrete linearization is presented. The approach replaces the equilibrium constraints of the MPEC by integer restrictions in the form of disjunctive constraints (Fortuny-Amat and McCarl, 1981; Gabriel et al., 2009). Also, a bilinear objective function of an electricity market model stemming from the product of both price and generation variables is linearized using additional binary and continuous variables and new constraints. The result is that the MPEC can be replaced by a mixed-integer linear program (MILP). This allows for a whole host of important applications such as: discrete generation levels, fixed cost problems involving binary variables, if-then logic relative to ramping constraints (Winston, 1994), discrete investment levels, and so on. A second advantage of using the presented method in this context is to be able to solve larger-scale problems in electric power markets than previously attempted. Lastly, a detailed formulation of the DC load flow model is used in order to model physical flows which facilitates a greater flexibility for changing the network topology. The numerical results on two illustrative problems are promising.

The rest of this chapter is organized as follows. In Section 5.2 the existing literature concerning two-stage modeling and its applications in electricity markets is discussed. Section 5.3 presents the general mathematical formulation for the two-stage problem. The upper level is a quadratic program and the lower level is a welfare maximization problem for an independent system operator (ISO). Next Section 5.4 describes numerical results of the developed approach for two illustrative examples. The chapter is concluded with summary remarks and future directions in Section 5.5.[1]

5.2 Literature

This chapter presents a new method for solving two-level planning problems with applications in electric power markets. The upper level involves generation decisions for the Stackelberg leader and the lower level depicts the rest of the market and the ISO problem. These two-level problems are known as Stackelberg games or more generally as mathematical programs with equilibrium constraints (MPEC) (Facchinei and Pang, 2003) which are NP-complete (Jeroslow, 1985). These problems have applications in a number of important planning areas such as: electric power management (Hobbs and Nelson, 1992; Hobbs et al., 2000); taxation and optimal highway pricing (Labbé et al., 1998), the government and the agricultural sector (Bard et al., 2000); chemical process engineering (Raghunathan and Biegler, 2003); engineering safety factors for a rubble mound breakwater and a bridge crane (Castillo et al., 2003); NO_x allowances markets in electric power production (Chen et al., 2004); traffic planning (Codina et al., 2006), and network design in transportation (Gao et al., 2004); see the annotated bibliography (Vicente and Calamai, 1994) for related works.
Recently there has been a fair amount of research devoted to solving MPECs with continuous-valued variables. Some examples include: implicit nonsmooth approaches (Outrata et al., 1998), piecewise sequential quadratic programming (SQP) methods (Luo et al., 1996), and perturbation and penalization methods (Scholtes, 2001; Dirkse et al., 2002; Leyffer et al., 2006). As noted by Leyffer (2003), these approaches require more computational effort than standard nonlinear programming methods. However, within the last five years, there have been some algorithmic successes. For example, Fletcher and others (Fletcher et al., 2002; Fletcher and Leyffer, 2002, 2004) have shown that SQP methods have good numerical results as well as some advantageous convergence properties (Anitescu, 2000). Additionally, as noted by Leyffer (2003), interior-point methods while less reliable than SQP are still able to solve about 80% of the mathematical programs considered with complementarity constraints (MPCCs) using default settings. Further improvements can be gained by applying a relaxation of the complementarity constraints typically found from incorporating the lower-level optimality conditions in the upper-level (Liu and Sun, 2004; Raghunathan and Biegler, 2002; Ralph and Wright, 2004; De Miguel et al., 2004, 2005),

[1] The subsequent chapter is based on the research paper Gabriel and Leuthold (2009).

or penalizing these constraints (e.g., PIPA; Luo et al., 1996; Hu and Ralph, 2004; Anitescu, 2004). However, in the latter case, it has been shown that the standard PIPA method (Luo et al., 1996) can fail to converge (Leyffer, 2005) to a stationary point in some cases. Lastly one can also make use of a trust region approach applied to the nonlinear bilevel programming problem as described in Marcotte et al. (2001).

The proposed methodology by contrast can incorporate integer and continuous-valued variables. A few others have also considered methods for solving MPECs with integer restrictions. These other methods can be grouped into three categories: application-specific approaches, integer-programming methods, or nonlinear programming-based algorithms. Methods specific to applications include for example, a branch and bound version for a flow shop bilevel problem (Karlof and Wang, 1996), shortest path and transshipment algorithms for a modified network for special cases of a bilevel taxation and optimal high pricing formulation (Labbé et al., 1998), and a tree search approach (Scaparra and Church, 2008) to analyze critical infrastructure planning. Integer (or linear) programming methods include for example, a Grid Search Algorithm (Bard, 1983), a simplex-like method (Bialas and Karwan, 1984) or branch and bound (Bard, 1988; Moore and Bard, 1990; Bard and Moore, 1990; Karlof and Wang, 1996), Tabu Search (Wen and Huang, 1996), or genetic algorithm methods (Hejazi et al., 2002; Nishizaki et al., 2003). For discretely-constrained bilevel optimization problems, Moore and Bard (1990) point out that it is not always possible to get tight upper bounds using common relaxation methods and also two of three standard fathoming rules employed used in branch and bound cannot be fully used. Thus, the more traditional approaches may not work for these sorts of problems.

Regarding the approach presented in the subsequent sections, it should be noted that Barroso et al. (2006) develop a similar framework in order to simulate Nash equilibria in strategic bidding for short-term electricity markets which they denote as ‘binary expansion’ (BE). However, a major contribution difference is the additional consideration of the physical transmission network in the model at hand.

5.3 General Mathematical Formulation

Before describing an application in power planning, a more stylized version of the problem at hand is presented. The general form of the problem to be solved by the leader (e.g., strategic generator) is as follows where $x \in$

$R^n, y \in R^m$ are respectively, the upper and lower-level vectors of variables:

$$\min_{x,y} \left\{ \begin{pmatrix} d_x \\ d_y \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2} \begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \right\} \tag{5.1a}$$

subject to

$$A_1 y + B_1 x = b_1 \ (\beta_1) \tag{5.1b}$$
$$A_2 x = b_2 \ (\beta_2) \tag{5.1c}$$
$$A_3 x \leq b_3 \ (\beta_3) \tag{5.1d}$$
$$A_4 y = b_4 \ \ (\beta_4) \tag{5.1e}$$
$$A_5 y \leq b_5 \ \ (\beta_5) \tag{5.1f}$$
$$x_i \in Z_+, i = 1, \ldots, n_1 \tag{5.1g}$$
$$x_i \in R, i = n_1 + 1, \ldots, n \tag{5.1h}$$
$$y \in S(x) \tag{5.1i}$$
$$y_j \geq 0, j = 1, \ldots, m_1 \tag{5.1j}$$
$$y_2 \text{ (free)}, \ j = m_1 + 1, \ldots, m \tag{5.1k}$$

where $A_1, A_2, A_3, A_4, A_5, B_1$ are matrices of suitable size conformal with the vectors x, y and right-hand sides b_1, b_2, b_3, b_4, b_5. The vectors d_x, d_y contain coefficients for x and y, and $M_{xx}, M_{xy}, M_{yx}, M_{yy}$ are the submatrices referring to the quadratic terms of the objective function. The objective function (5.1a) is quadratic in both the upper and lower-level variables which in the particular power application described in Section 5.4 will involve pairwise products of variables (e.g., generation times price) as well as linear terms (e.g., generation times costs). Equation (5.1b) is the set of joint constraints linking the upper and lower-level variables with β_1 representing the dual variables to these constraints (similar notation for dual variables for the other constraints). Equations (5.1c) and (5.1d) are the constraints that only involve the upper-level variables x whereas (5.1e) and (5.1f) are the counterparts for the lower-level variables y. Equations (5.1g) and (5.1h) indicate that a subset of the upper-level variables are integer-valued whereas constraint (5.1i) stipulates that y must be a solution to the lower-level problem given x. Lastly, the vector y is partitioned into a nonnegative subvector (y_1) and the remaining variables (y_2) free as shown in the last two constraints.
The lower-level problem will typically be either a convex, quadratic program whose necessary and sufficient Karush-Kuhn-Tucker conditions or a Nash-

Cournot game can be expressed as a mixed linear complementarity problem (MLCP) (Facchinei and Pang, 2003) given as follows:

$$\begin{aligned} 0 &\leq c_1(x) + M_{11}(x)y_1 + M_{12}(x)y_2 \perp y_1 \geq 0 && (5.2a) \\ 0 &= c_2(x) + M_{21}(x)y_1 + M_{22}(x)y_2 \quad y_2 \text{ (free)} && (5.2b) \end{aligned}$$

where the dependence on the upper-level variables can be in the vector $c = \begin{pmatrix} c_1(x)^T & c_2(x)^T \end{pmatrix}^T$ and/or the matrix

$$M = \begin{pmatrix} M_{11}(x) & M_{12}(x) \\ M_{21}(x) & M_{22}(x) \end{pmatrix}$$

Having a sufficiently large constant K, the complementarity conditions (5.2) can be converted to disjunctive constraints (Fortuny-Amat and McCarl, 1981; Gabriel et al., 2009) as

$$\begin{aligned} 0 &\leq c_1(x) + M_{11}(x)y_1 + M_{12}(x)y_2 \leq Kr && (5.3a) \\ 0 &\leq y_1 \leq K(1-r) && (5.3b) \\ 0 &= c_2(x) + M_{21}(x)y_1 + M_{22}(x)y_2 \quad y_2 \text{ (free)} && (5.3c) \end{aligned}$$

where r is a vector of binary variables. In general finding a reasonable constant K may take trial and error. However, in specific instances such as the case study described below, a suitable value can be found; see Section 5.4.3 and Appendix A.2 for further guidance on how to obtain such a constant. Replacing (5.1i) by (5.3) leads to the overall problem expressed in disjunctive form:

$$\min_{x,y} \left\{ \begin{pmatrix} d_x \\ d_y \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2} \begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \right\} \qquad (5.4a)$$

subject to

$$A_1 y + B_1 x = b_1 \ (\beta_1) \tag{5.4b}$$

$$A_2 x = b_2 \ (\beta_2) \tag{5.4c}$$

$$A_3 x \leq b_3 \ (\beta_3) \tag{5.4d}$$

$$A_4 y = b_4 \quad (\beta_4) \tag{5.4e}$$

$$A_5 y \leq b_5 \quad (\beta_5) \tag{5.4f}$$

$$x_i \in Z_+, i = 1, \ldots, n_1 \tag{5.4g}$$

$$x_i \in R, i = n_1 + 1, \ldots, n \tag{5.4h}$$

$$0 \leq c_1\,(x) + M_{11}(x) y_1 + M_{12}(x) y_2 \leq Kr \tag{5.4i}$$

$$0 \leq y_1 \leq K(1 - r) \tag{5.4j}$$

$$0 = c_2\,(x) + M_{21}(x) y_1 + M_{22}(x) y_2 \quad y_2 \text{ (free)} \tag{5.4k}$$

$$y_j \geq 0, j = 1, \ldots, m_1 \tag{5.4l}$$

$$y_2 \text{ (free)}, \ j = m_1 + 1, \ldots, m \tag{5.4m}$$

$$r_i \in \{0, 1\}^{m_1} \tag{5.4n}$$

5.4 Numerical Example for an Electricity Market

In this section, the above described quadratic program with equilibrium constraints is specialized to an electric power market example. As pointed out in Section 2.2.3, a Stackelberg game with only one leader is an example of a MPEC. A Stackelberg game with several leaders is an EPEC (Ralph and Smeers, 2006). Thus, it can be argued that an EPEC formulation is the most appropriate way to model strategic behavior in short-term electricity markets as EPECs allow more than one strategic player. The justification for a MPEC approach, however, can be seen in other studies, e.g., one of the Center of European Economic Research (ZEW) in Germany (Nikogosian and Veith, 2009). They argued that in an increasing integrated market, the distribution of market shares changes which might lead to fewer or even only a single strategic actor. Furthermore, the formulation presented here could include a simple form of a cartel. Ehrenmann (2004) stated that for liberalized electricity markets, there are two types of models that lead to different types of strategic formulations. He distinguished a centralized and a decentralized system. Within the centralized system, the energy and transmission markets are cleared simultaneously, whereas, in the decentralized system, there are separate markets for transmission and energy which requires a different complementarity formulation of the problem; the numerical examples at hand assume a centralized system.

5.4.1 Mathematical Notation

Indices:

$n, k \in N$	nodes in the network
k'	swing bus
$l \in L$	line between n and k
$f \in F$	firms in the market
$s \in F$	firms acting strategically
$j \in F$	competitive fringe firms
$u \in U$	generation units
$i \in I$	discrete generation steps

Sets:

F	set of all firms
L	set of all lines
N	set of all nodes
U	set of all generation units
I	set of possible generation levels

Parameters:

a_n, b_n	intercept and slope of linear demand functions ($a_n, b_n \geq 0$, $\forall n$)
c_{nfu}	generation cost of firm f at node n for unit u ($c_{nfu} \geq 0$, $\forall n, f, u$)
$\bar{g}_{nfu}$	maximum generation capacity of firm f at node n for unit u ($\bar{g}_{nfu} \geq 0$, $\forall n, f, u$)
B_{nk}	network susceptance matrix $n \times k$
H_{lk}	network transfer matrix $l \times k$
lc_l	physical line capacity limit of line l
sw_k	swing bus vector, $sw_k = \begin{cases} 1 & \text{if } k = k\prime \\ 0 & \text{otherwise} \end{cases} \quad \forall k$
$K, \bar{K}, \hat{K}, \tilde{K}, \check{K}$	positive constants used to replace complementarities by disjunctive constraints
M	positive constant

Variables:

d_n	demand at node n
g_{nfu}	generation of firm f at node n for unit u
$q_{nsu,i}, q_n^\lambda$	binary variables for the linearization of the objective function
$q_{nsu,i}^v$	variable for the linearization of the objective function
δ_k	phase angle at node k
λ_n	shadow price for energy at node n
$\overline{\mu}_l$	shadow price for transmission on line l in forward direction
$\underline{\mu}_l$	shadow price for transmission on line l in negative direction
β_{nju}	dual variable of maximum generation constraint per unit u of fringe firm j at node n
γ_k	dual variable for slack bus constraint
$r_n, \bar{r}_{nfu}, \hat{r}_l, \tilde{r}_l, \check{r}_{nfu}$	binary variables used to replace complementarities by disjunctive constraints

5.4.2 Mathematical Formulation

The mathematical problem described subsequently is a specific form of (5.4) specialized to strategic behavior in electric power markets where there is one dominant player at the upper-level and several competitive fringe players included in the lower-level ISO problem.

A similar approach as presented here is also presented by Barroso et al. (2006). They develop a framework to simulate Nash equilibria in strategic bidding for short-term electricity markets which they denote as 'binary expansion'. However, in contrast to Barroso et al. (2006), the model at hand includes the physical transmission network. Considering transmission issues is often regarded as essential feature in order to find short-term market clearing prices for electricity markets (e.g., Stoft, 2002). In addition, the model presented subsequently allows for a quadratic subproblem of the MPEC whereas the model of Barroso et al. (2006) requires a linear subproblem. Quadratic subproblems are useful in order to represent the expected welfare maximizing behavior of an ISO.

Hence, the model description begins with a social welfare maximization model assuming perfect competition (5.5) for an ISO as the lower-level problem. Afterwards the the upper-level problem is described. Given a linear inverse demand function $a_n - b_n d_n$, the term $\left(a_n d_n - \frac{b_n d_n^2}{2}\right)$ describes the area below the (inverse) demand curve for region n, that is, the gross sur-

plus. The production cost for region n is given by $(g_{nfu}c_{nfu})$ which is then subtracted from the first part after appropriately summing both terms.
In order to calculate the physical grid utilization, electric power market models mostly apply the DC load flow model (compare Chapters 2 and 3) in order to obtain the line flows resulting from a certain generation-load combination (compare Chapter 3). There are basically two different characteristics of the DCLF. One characteristic is a network PTDF matrix (Christie et al., 2000; Delarue et al., 2007). A PTDF matrix contains factors that quantify the impact of an injection or withdrawal at a certain location on all lines within the network. The PTDF can be derived from the network transfer matrix H_{lk} and network susceptance matrix B_{nk}. However, as shown earlier one can directly use the product of network susceptance matrix and the voltage angle δ (5.5b) and the product of the matrix H and the voltage angle δ_k (5.5c-5.5d) in the mathematical problem (Schweppe et al., 1988; Stigler and Todem, 2005) which leads to a greater flexibility, e.g., when considering models where B_{nk} and H_{lk} are not constant. In the present approach, Equation (5.5b) represents the energy balance at node n with $\sum_k (B_{nk}\delta_k)$ corresponding to the net withdrawal/injection which must match the net demand/supply $d_n - \sum_f \sum_u g_{nfu}$. Inequalities (5.5c) and (5.5d) correspond to constraining the line flows determined by $\sum_k (H_{lk}\delta_k)$. Constraint (5.5e) provides an upper bound on generation relating to installed capacity and lastly (5.5f) defines a slack bus. The following ISO problem is the starting point for the presented modeling approach and is the precursor to the equilibrium constraints for the subsequent MPEC.

$$\min_{d_n, g_{nfu}, \delta_k} \left\{ \sum_n \left(-a_n d_n + \frac{b_n d_n^2}{2} \right) + \sum_n \sum_f \sum_u (g_{nfu} c_{nfu}) \right\} \tag{5.5a}$$

subject to

$$d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu} = 0, \forall n \quad (\lambda_n) \tag{5.5b}$$

$$-lc_l + \sum_k (H_{lk}\delta_k) \leq 0, \forall l \quad (\bar{\mu}_l) \tag{5.5c}$$

$$-lc_l - \sum_k (H_{lk}\delta_k) \leq 0, \forall l \quad (\underline{\mu}_l) \tag{5.5d}$$

$$-\overline{g}_{nfu} + g_{nfu} \leq 0, \forall n, f, u \quad (\beta_{nfu}) \tag{5.5e}$$

$$-sw_k\delta_k = 0, \forall k \quad (\gamma_k) \tag{5.5f}$$

$$d_n \geq 0, \forall n \tag{5.5g}$$

$$g_{nfu} \geq 0, \forall n, f, u \tag{5.5h}$$

In order to write the problem in a way that it can be applied to the approach at hand, first the KKT conditions (5.6) of the problem are written out (compare Section 2.1.2).

$$0 \leq -a_n + b_n d_n + \lambda_n \perp d_n \geq 0, \forall n \tag{5.6a}$$

$$0 \leq c_{nfu} - \lambda_n + \beta_{nfu} \perp g_{nfu} \geq 0, \forall n, f, u \tag{5.6b}$$

$$\begin{aligned} 0 = & \sum_n (B_{nk}\lambda_n) + \sum_l (H_{lk}\overline{\mu}_l) - \sum_l \left(H_{lk}\underline{\mu}_l\right) \\ & - \begin{cases} \gamma_k & \text{if } k = k\prime \\ 0 & \text{otherwise} \end{cases} \qquad \delta_k \text{ (free) }, \forall k \end{aligned} \tag{5.6c}$$

$$0 = d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu}, \lambda_n \text{ (free)}, \forall n \tag{5.6d}$$

$$0 \leq lc_l - \sum_k (H_{lk}\delta_k) \perp \overline{\mu}_l \geq 0, \forall l \tag{5.6e}$$

$$0 \leq lc_l + \sum_k (H_{lk}\delta_k) \perp \underline{\mu}_l \geq 0, \forall l \tag{5.6f}$$

$$0 \leq -g_{nfu} + \overline{g}_{nfu} \perp \beta_{nfu} \geq 0, \forall n, f, u \tag{5.6g}$$

$$0 = -sw_k\delta_k, \quad \gamma_k \text{ (free)}, \forall k \tag{5.6h}$$

Then the KKTs are replaced by disjunctive constraints (5.7) as described in Section 5.3. The purpose of this disjunctive form is to have mixed-integer linear constraints at hand.

$$
\begin{aligned}
0 &\leq -a_n + b_n d_n + \lambda_n \leq K r_n, \forall n && (5.7a)\\
0 &\leq d_n \leq K(1 - r_n), \forall n && (5.7b)\\
0 &\leq c_{nfu} - \lambda_n + \beta_{nfu} \leq \bar{K} \bar{r}_{nfu} && (5.7c)\\
0 &\leq g_{nfu} \leq \bar{K}(1 - \bar{r}_{nfu}), \forall n, f, u && (5.7d)\\
0 &= \sum_n (B_{nk}\lambda_n) + \sum_l (H_{lk}\overline{\mu}_l) - \sum_l \left(H_{lk}\underline{\mu}_l\right) && \\
&\quad - \begin{cases} \gamma_k & \text{if } k\prime = k \\ 0 & \text{otherwise} \end{cases} \qquad \delta_k \text{ (free) }, \forall k && (5.7e)\\
0 &= d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu}, \quad \lambda_n \text{ (free)}, \forall n && (5.7f)\\
0 &\leq lc_l - \sum_k (H_{lk}\delta_k) \leq \hat{K}\hat{r}_l, \forall l && (5.7g)\\
0 &\leq \overline{\mu}_l \leq \hat{K}(1 - \hat{r}_l), \forall l && (5.7h)\\
0 &\leq lc_l + \sum_k (H_{lk}\delta_k) \leq \tilde{K}\tilde{r}_l, \forall l && (5.7i)\\
0 &\leq \underline{\mu}_l \leq \tilde{K}(1 - \tilde{r}_l), \forall l && (5.7j)\\
0 &\leq -g_{nfu} + \overline{g}_{nfu} \leq \check{K}\check{r}_{nfu}, \forall n, f, u && (5.7k)\\
0 &\leq \beta_{nfu} \leq \check{K}(1 - \check{r}_{nfu}), \forall n, f, u && (5.7l)\\
0 &= -sw_k\delta_k, \quad \gamma_k \text{ (free)}, \forall k && (5.7m)
\end{aligned}
$$

$$r_n, \bar{r}_{nfu}, \hat{r}_l, \tilde{r}_l, \check{r}_{nfu} \in \langle 0, 1 \rangle \quad \forall n, f, u, l$$

For the purpose of including strategic behavior in the problem, it is now assumed that the set of firms f is partitioned into two subsets. Subset s corresponds to the firms that act strategically. Subset j is for the firms that act as price-takers. It is assumed that the firms s decide first on their output decisions in order to maximize individual profits which means that their quantities are exogenous to the ISO problem. These firms know that they influence the market equilibrium with their decisions. The output decision of fringe firms j is determined by the ISO. The latter can be interpreted as a pool system. The entire problem is known as Stackelberg game where firms s are the leaders and the ISO (deciding on the quantities of firms j) is the follower. In the problem at hand, it is assumed that only one player acts strategically resulting in the MPEC (5.8).

$$\min_{d_n, g_{nsu}, \beta_{nju}, \delta_k, \gamma_k, \lambda_n, \overline{\mu}_l, \underline{\mu}_l} \left\{ \sum_n \sum_s \sum_u (c_{nsu} - \lambda_n) g_{nsu} \right\} \tag{5.8a}$$

subject to

$$\begin{aligned}
0 &\leq \overline{g}_{nsu} - g_{nsu}, \ \forall n, s, u && (5.8b)\\
0 &\leq -a_n + b_n d_n + \lambda_n \perp d_n \geq 0, \forall n && (5.8c)\\
0 &\leq c_{nju} - \lambda_n + \beta_{nju} \perp g_{nju} \geq 0, \forall n, j, u && (5.8d)\\
0 &= \sum_n (B_{nk} \lambda_n) + \sum_l (H_{lk} \overline{\mu}_l) - \sum_l (H_{lk} \underline{\mu}_l) \\
&\quad - \begin{cases} \gamma_k & \text{if } k = k\prime \\ 0 & \text{otherwise} \end{cases} \quad \delta_k \text{ (free)}, \forall k && (5.8e)\\
0 &= d_n + \sum_k (B_{nk} \delta_k) - \sum_f \sum_u g_{nfu}, \lambda_n \text{ (free)}, \forall n && (5.8f)\\
0 &\leq lc_l - \sum_k (H_{lk} \delta_k) \perp \overline{\mu}_l \geq 0, \forall l && (5.8g)\\
0 &\leq lc_l + \sum_k (H_{lk} \delta_k) \perp \underline{\mu}_l \geq 0, \forall l && (5.8h)\\
0 &\leq -g_{nju} + \overline{g}_{nju} \perp \beta_{nju} \geq 0, \forall n, j, u && (5.8i)\\
0 &= -sw_k \delta_k \quad \gamma_k \text{ (free)}, \forall k && (5.8j)
\end{aligned}$$

The correspondence between (5.8) and the more general problem (5.1) is shown in Table 5.1. As stated above the fringe firms' output decisions are determined by the ISO. Hence, the strategic generator takes into account their reaction in terms of the equilibrium problem (5.8c-5.8j) of the ISO within his profit maximization problem.

One computational difficulty is the bilinear terms $\lambda_n g_{nsu}$ in the objective function. In order to deal with the bilinear objective function in (5.8), valid generation levels for the strategic generation $\bar{g}_{nsu,i}$ are defined. One can think of this as selecting a discrete set of possible generation levels. Thereby, $q_{nsu,i}$ are taken as indicator binary variables that equal 1 when the fixed´generation level $\bar{g}_{nsu,i}$ is selected and zero otherwise. Also, q_n^λ is a binary variable for when the price $\lambda_n > 0$, and $q_{nsu,i}^v$ is a binary variable for the case when the variable $v_{nsu,i} > 0$ where

$$v_{nsu,i} = \begin{cases} \bar{g}_{nsu,i} \lambda_n & \text{if } q_{nsu,i} = q_n^\lambda = 1 \\ 0 & \text{otherwise} \end{cases}$$

Taking into account linearization constraints for the bilinear terms (explained below) as well as the replacement of complementarity conditions by disjunctive constraints (previously described), one gets the following mixed-integer linear problem for the MPEC in question.

$$\min_{d_n,g_{nfu},r_n,\bar{r}_{nju},\hat{r}_l,\tilde{r}_l,\check{r}_{nju},\beta_{nju},\gamma_k,\delta_k,\lambda_n,\overline{\mu}_l,\underline{\mu}_l} \left\{ \begin{array}{l} \sum_n \sum_s \sum_u c_{nsu} g_{nsu} \\ -\sum_i v_{nsu,i} \end{array} \right\} \tag{5.9a}$$

subject to

$$0 \leq \lambda_n \leq M q_n^{\lambda}, \forall n \tag{5.9b}$$

$$g_{nsu} = \sum_i q_{nsu,i} \bar{g}_{nsu,i}, \forall n, s, u \tag{5.9c}$$

$$\sum_i q_{nsu,i} = 1, \forall n, s, u \tag{5.9d}$$

$$\left\{ \begin{array}{c} q^v_{nsu,i} \leq q^{\lambda}_n, \forall n, s, u, i \\ q^v_{nsu,i} \leq q_{nsu,i}, \forall n, s, u, i \\ q_{nsu,i} + q^{\lambda}_n - 1 \leq q^v_{nsu,i}, \forall n, s, u \end{array} \right. \tag{5.9e}$$

$$\left\{ \begin{array}{l} v_{nsu,i} \leq \bar{g}_{nsu,i} \lambda_n, \forall n, s, u, i \\ 0 \leq v_{nsu,i} \leq M q^v_{nsu,i}, \forall n, s, u, i \end{array} \right. \tag{5.9f}$$

$$0 \leq -a_n + b_n d_n + \lambda_n \leq K r_n, \forall n \tag{5.9g}$$

$$0 \leq d_n \leq K(1 - r_n), \forall n \tag{5.9h}$$

$$0 \leq c_{nju} - \lambda_n + \beta_{nju} \leq \bar{K} \bar{r}_{nju}, \ \forall n, j, u \tag{5.9i}$$

$$0 \leq g_{nju} \leq \bar{K}(1 - \bar{r}_{nju}), \forall n, j, u \tag{5.9j}$$

$$\begin{aligned} 0 = {} & \sum_n (B_{nk} \lambda_n) + \sum_l (H_{lk} \overline{\mu}_l) - \sum_l \left(H_{lk} \underline{\mu}_l \right) \\ & - \left\{ \begin{array}{ll} \gamma_k & \text{if } k = k\prime \\ 0 & \text{otherwise} \end{array} \right. \qquad \delta_k \ \text{(free)}, \forall k \end{aligned} \tag{5.9k}$$

$$0 = d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu}, \lambda_n \text{ (free)}, \forall n \quad (5.9l)$$

$$0 \leq lc_l - \sum_k (H_{lk}\delta_k) \leq \hat{K}\hat{r}_l, \forall l \quad (5.9m)$$

$$0 \leq \overline{\mu}_l \leq \hat{K}(1-\hat{r}_l), \forall l \quad (5.9n)$$

$$0 \leq lc_l + \sum_k (H_{lk}\delta_k) \leq \tilde{K}\tilde{r}_l, \forall l \quad (5.9o)$$

$$0 \leq \underline{\mu}_l \leq \tilde{K}(1-\tilde{r}_l), \forall l \quad (5.9p)$$

$$0 \leq -g_{nju} + \overline{g}_{nju} \leq \check{K}\check{r}_{nju}, \forall n, j, u \quad (5.9q)$$

$$0 \leq \beta_{nju} \leq \check{K}(1-\check{r}_{nju}), \forall n, j, u \quad (5.9r)$$

$$0 = -sw_k\delta_k, \gamma_k \text{ (free)}, \forall k \quad (5.9s)$$

$$\begin{aligned} r_n, \bar{r}_{nju}, \hat{r}_l, \tilde{r}_l, \check{r}_{nju}, q_{nsu,i}, q_n^\lambda &\in \{0,1\}, \forall n, s, u, i, j \\ q_{nsu,i}^v &\in [0,1], \forall n, s, u, i \end{aligned}$$

The logic of the constraints (5.9a)-(5.9e) is as follows:

1. By $0 \leq \lambda_n \leq M q_n^\lambda$, when $\lambda_n > 0$, since M is a suitably large positive constant, this means that $q_n^\lambda = 1$. Also, $q_{nsu,i} = 1$ corresponds to the ith discrete generation value $\bar{g}_{nsu,i}$ being selected via the constraints $\sum_i q_{nsu,i} = 1, g_{nsu} = \sum_i q_{nsu,i}\bar{g}_{nsu,i}$. Thus,

$$\begin{cases} q_{nsu,i}^v \leq q_n^\lambda \\ q_{nsu,i}^v \leq q_{nsu,i} \\ q_{nsu,i} + q_n^\lambda - 1 \leq q_{nsu,i}^v \end{cases}$$

ensures that when both $q_n^\lambda = 1$ and $q_{nsu,i} = 1 \Leftrightarrow$ the binary indicator variable $q_{nsu,i}^v = 1$ since by the three constraints above, one has $1 \leq q_{nsu,i}^v \leq 1$. If one or both of q_n^λ and $q_{nsu,i} = 0$, then these constraints would force the nonnegative variable $q_{nsu,i}^v = 0$; see Williams (1999) for this and similar logic constraints.

2. The constraints

$$\begin{cases} v_{nsu,i} \leq \bar{g}_{nsu,i}\lambda_n \\ 0 \leq v_{nsu,i} \leq M q_{nsu,i}^v \end{cases}$$

force $v_{nsu,i} \in [0, \bar{g}_{nsu,i}\lambda_n]$ when $q_{nsu,i}^v = 1$ and $v_{nsu,i} = 0$ when $q_{nsu,i}^v =$

0. Since the objective function has $-\sum_i v_{nsu,i}$, larger values of $v_{nsu,i}$ are always preferred. Thus, $q^v_{nsu,i} = 1 \Rightarrow v_{nsu,i} = \bar{g}_{nsu,i}\lambda_n$ so that part of the objective function matches exactly the bilinear term.

3. Hence one can see the following

$$\begin{aligned} \lambda_n &> 0 \text{ and } \bar{g}_{nsu,i} \text{ selected} \\ &\Rightarrow q^v_{nsu,i} = 1 \\ &\Rightarrow v_{nsu,i} = \bar{g}_{nsu,i}\lambda_n \end{aligned}$$

as desired. It therefore suffices to show that when either $\lambda_n = 0$ or $\bar{g}_{nsu,i}$ is not selected that this implies that $v_{nsu,i} = 0$. Clearly, $v_{nsu,i} \leq \bar{g}_{nsu,i}\lambda_n$ forces $v_{nsu,i} = 0$ when $\lambda_n = 0$. On the other hand, if $\lambda_n > 0$ then $q^\lambda_n = 1$. But if $\bar{g}_{nsu,i}$ is not selected, then $q_{nsu,i} = 0$ so that in

$$\begin{cases} q^v_{nsu,i} \leq q^\lambda_n \\ q^v_{nsu,i} \leq q_{nsu,i} \\ q_{nsu,i} + q^\lambda_n - 1 \leq q^v_{nsu,i} \end{cases},$$

one can see that $q^v_{nsu,i} \leq \min\{0,1\} = 0$ and $q^v_{nsu,i} \geq 0$, so that $q^v_{nsu,i} = 0$. Ergo, by $0 \leq v_{nsu,i} \leq Mq^v_{nsu,i}$, $v_{nsu,i} = 0$.

Hence, by transforming the mathematical problem (5.8) into (5.9), it can be achieved to replace a bilinear term $\lambda_n g_{nsu}$ in the objective function by the linear term $\sum_i v_{nsu,i}$ and some more binary constraints. Thereby, Steps 1-3 above describe how these additional constraints work. In particular, they ensure that the model is only able to pick one of the parameterized given output levels. Altogether, through this procedure a MINLP is replaced by a MILP which promises better numerical behavior.

General Form	**Specific Form**
Equation (5.1a)	Equation (5.8a)
Equation (5.1d)	Equation (5.8b)
Equation (5.1i)	Equations (5.8c)-(5.8j)
all other constraints	vacuous

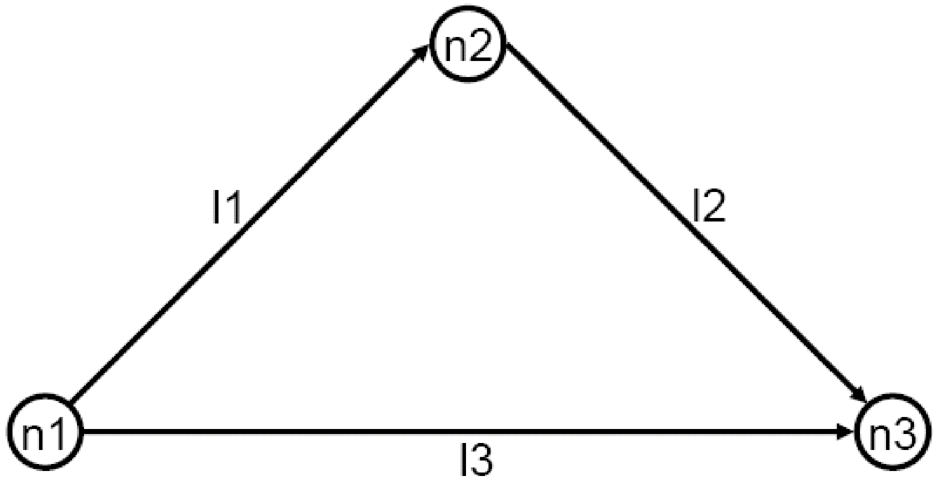
Table 5.1: Correspondence between general formulation and specific case study formulation

5.4.3 The Disjunctive Constraints Constants

An important issue is how to choose the constants $K, \bar{K}, \hat{K}, \tilde{K}$, and $\check{K}$ for the disjunctive constraints as well as the constant M in Equations (5.9b) and (5.9f). These K-constants are critical for replacing KKTs by disjunctive constraints. However, there is no general formula how to obtain these values and they are specific to the application that is to be solved. As pointed out above, it is assumed that the price-quantity relation can be modeled as a linear inverse demand function $p_n = a_n - b_n d_n$ per node with intercept a_n and slope b_n. Intercept a_n is the point of intersection of the linear demand function and the vertical price axis. Hence it is called the "prohibitive price" in economics as at this point demand d_n falls to 0. The point of intersection of the linear demand function and the horizontal quantity axis is referred to as the "market saturation" quantity which shall be denoted as D_n. That is the maximum quantity that consumers would buy at a nonnegative price.

Keeping these basics in mind, the derivation of the constant values is economic commonsense. The constants always refer to primal (quantity) and dual (price) information. The constant K for example refers to the demand function price information (5.9g) and demand quantity information (5.9h). Binary variables r_n are 0 as long as there are positive demands d_n. If demand d_n equals 0 for one of the nodes, its r_n can either be 0 or 1. Economically, zero demand means that the nodal price λ_n is greater than or equal to the prohibitive price a_n. For this result, it does not matter how big λ_n becomes as long as $\lambda_n \geq a_n$. It is economically reasonable to assume for example that $\lambda_n \leq 2a_n$ which leads us according to Equation (5.9g) to the first candidate for K: $K_1 = \max_n(a_n)$. Analogously, the second candidate for K can be found by looking at the inequality $d_n \leq K(1 - r_n)$ for $r_n = 0$. A K for this case is identified by assuming that an economically meaningful demand quantity can not exceed the market saturation quantity of the entire system $K_2 = \sum_n(D_n)$. Thus, K can be defined as the maximum of these two values $K = \max(K_1, K_2)$. The other constants can be derived in a similar manner. Some advice on how to compute these disjunctive constraints constants in a more general setting for a linear program with upper-level variables appearing in the right-hand side of the constraints is provided in Appendix A.2.

Figure 5.1: Three-node test network



Source: Own representation.

5.4.4 Computational Results[2]

Three-node network

The objective of this section is to apply two models of electric power markets. The first tests are carried out with a simple three-node example as depicted in Figure 5.1[3]. Afterwards the computational results for a fifteen-node network of the Western European grid are reported. In both bases, the models were coded in GAMS and used the CPLEX solver.

In order to run the test for the three-node example, the network parameters have to defined. Two types of parameters can be distinguished. One type describes the topology of the network such as the characteristics of the lines, the information which nodes are interconnected, and so on.[4] The other type refers to the electricity market itself. The latter parameters describe the demand, supply, and respective locations within the network. The parameters a_n and b_n for the linear inverse demand function described

[2] All calculations refer to hourly values. Hence, one MW of generation is one MWh of energy delivered.

[3] Note that the arrows define the positive direction for the line flows. If the values for the line flows are negative, the flow goes in the opposite direction.

[4] For this small three-node network, the reactance and resistance of all lines are taken to be equal.

in Section 5.4.2 of the form $a_n - b_n d_n$ are displayed in Table B.1. The demand structure is constructed such that there is a load center at node $n3$. The supply structure is specified by the marginal cost per plant and a maximum generation capacity for this plant. In order to carry out tests with the model, the supply side and the network parameters were varied for five different test cases (Table B.2). It is assumed that the generation centers are located at nodes $n1$ and $n2$, thus there is an electricity transport required in order to balance demand and generation. For the first test cases (Test1-Test4 in Table B.2), the network parameters are chosen in a way that congestion is not expected. For Test5, there is congestion expected on line $l2$ as the line capacity for line $l2$ (lc_{l2} in Table B.2) is decreased from 10 to 4 in this scenario.

As the presented model is supposed to simulate the result of strategic behavior, a benchmark case is defined against which the impact of this behavior ('strat') can be compared. The benchmark is the case of a perfect competition ('comp') model that is solved as an MCP (compare Equation (5.6)). The results for Test1-Test3 shown in Tables 5.3 and 5.2 are easiest to follow: The strategic generator only produces with its cheapest plant $u2$ in both the perfect competition and the strategic gaming case. However, in perfect competition, this generator has a profit of 0 whereas the profit is positive for the strategic runs (Table 5.2). The strategic generator manages to increase its profit by holding back generation. This leads to a decrease in demand and production but maximizes its profit. However, its output decision is constrained by the cheapest plant of the fringe (Table B.2). It cannot hold back too much capacity since otherwise the fringe would have an incentive to produce which would lower the profit of the strategic player. Accordingly, if the marginal cost of the cheapest fringe generator increases (Table B.2), the strategic firm decreases its output to the point of maximum profit i.e., 4.5 MWh. Comparing the changes in strategic output and profits from Test1 to Test2 and from Test2 through Test4, it can be seen that the strategic result moves from producing 7 MWh to 4.5 MWh (Test1 vs. Test2). Furthermore, this level of 4.5 MWh is maintained from Test2 to Test4 (Table 5.3) albeit the marginal cost of the fringe still increases (Table B.2). Hence, even increasing marginal cost of the fringe further does not impact the results. The same behavior can be observed for Test5. However, the difference between Test5 and the other test cases is that there is now network congestion. Hence, the strategic player cannot satisfy enough demand with its cheapest plant. By producing with its second (more expensive) unit, it can create counterflows and relieve congestion which in turn facilitates higher generation with the less expensive plant. The output decisions are chosen in the most profitable way (Tables 5.2 and 5.3). Lastly, the problem size and calculation times[5] are summarized in Tables B.3 and B.5.

		Test1	**Test2**	**Test3**	**Test4**	**Test5**
profit$_s$	comp	0.0	0.0	0.0	0.0	0.0
[€]	strat	14.0	20.3	20.3	20.3	12.3

Table 5.2: Profit of strategic player in the three-node network

Fifteen-node network

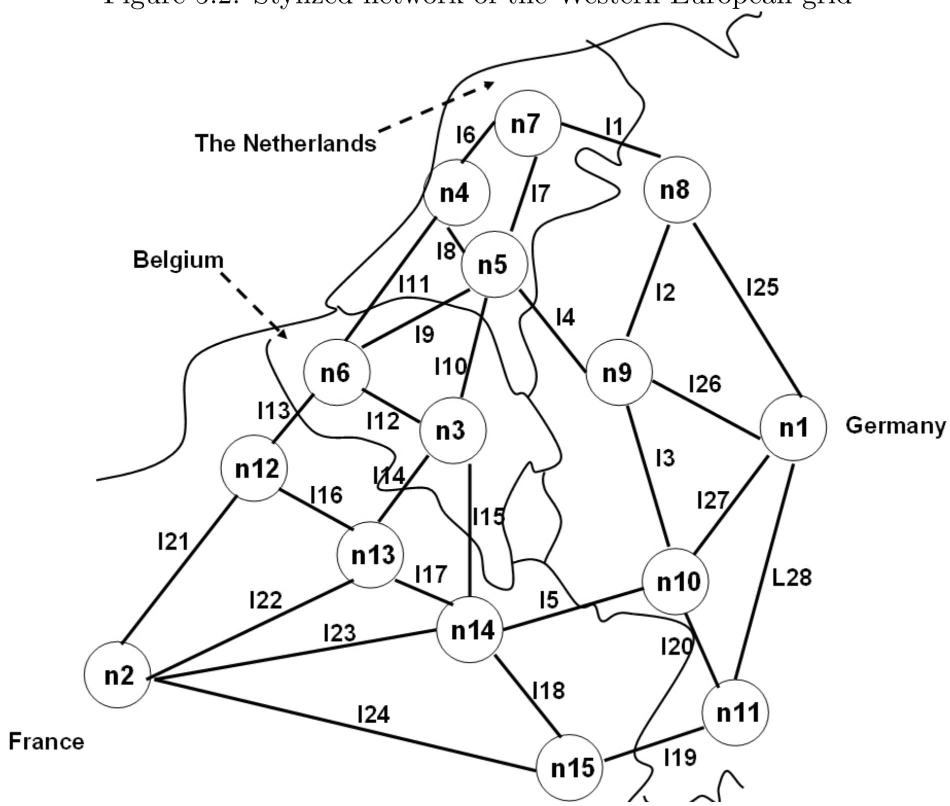
The second example is a more complex fifteen-node network representing a stylized grid of the Western European market based on Neuhoff et al. (2005). In order to obtain a linear inverse demand function, a reference demand and an elasticity for each node in the network were assumed. (For a deeper discussion of the data and the methodology refer to Chapter 3 and Leuthold et al. (2008a)). Demand data base upon UCTE (2006). The network aggregates data for Belgium, France, Germany, and the Netherlands with Germany and France represented by one node each ($n1$ and $n2$, respectively), Belgium by two nodes ($n3$ and $n6$) and the Netherlands by three nodes ($n4$, $n5$, and $n7$). Altogether, there are 15 nodes (Figure 5.2) of which eight are auxiliary without supply and demand (nodes $n8$ to $n15$). These nodes are necessary for the adequate modeling of cross-border flows. Altogether four different tests are carried out. For each of these test runs, a different company is assigned the Stackelberg leader role. In Test_EDF, for example, the French company EDF is the Stackelberg leader and all other companies are fringe players. The same pattern applies for Electrabel (Ebel) of Belgium as well as for EON and RWE of Germany in runs Test_Ebel, Test_EON, and Test_RWE, respectively. Table B.8 displays the installed generation capacities per player.[6] The subsequent results aim to show that the approach presented in this chapter works for a medium-scale test network and it is assumed that the approach is a significant modeling advance.

The numerical example shows that network effects have a significant impact on short-term market equilibria in electricity markets and cannot be neglected. Hence, a preliminary run starts by assuming that there are no

[5] All tests were conducted on a Intel Pentium 4, 3.00 GHz, with 1.00 GB of RAM.

[6] It should be stated that the level of detail of the data and, particularly, of the network is too low in order to draw a conclusion for the real market.

Figure 5.2: Stylized network of the Western European grid



Source: Based on Neuhoff et al. (2005).

network constraints within the entire network. For this case the prices are equal at each node within the network. Also, EDF alone has the potential to lift the price above the competitive level by holding back production and thereby increase its own profit (Table B.9). For all other test cases, the Stackelberg assumption for the respective other players that were tried as Stackelberg leaders does not have a profit-increasing effect.

However, the picture changes significantly if physical network constraints are included. The results of the fifteen-node example are then less intuitive than in case of the previous three-node network. Nonetheless, the outcome follows the same pattern. The Stackelberg leader is able to induce higher prices at the relevant nodes (Table 5.5), i.e., those nodes where it has significant production capacities, by holding back production (Tables 5.4-5.9). As could be observed in Test5 of the three-node example, there is congestion in the larger network, too, evidenced by market prices that differ by node (Table 5.5). For Ebel, EDF, and RWE acting as the only strategic player is profitable (Table 5.4). Particularly for EDF, the potential to increase their individual profit is huge. Presumably, this is due to the fact that in the model EDF has a greater supply of low-cost generation capacity (Table B.8), in particular nuclear power plants, which opens the potential to enact market power.

The important aspect is the issue of network congestion as mentioned earlier. The model is constructed in a way that EDF is the only player at node $n2$ representing France in a single node. Hence, competitive players would have to use the network in order to compete with EDF in node $n2$. However, by strategically choosing output decisions, EDF can influence flow patterns and reap the profits by itself. The situation is different in the case where competitors have generation capacities at the same nodes as the strategic player. In this case, the strategic player has to take into account output decisions of competitors at the nodes with several players. If the prices become too high, the competitive fringe companies still have an incentive to produce at marginal cost and cannot be excluded by network effects (compare the results of the three-node example described earlier). Hence, in the case of the Ebel and RWE, the profit increase is less distinct due to a more competitive situation but also due to smaller power plant fleet of these players.

Furthermore, the results for EON seem to be surprising as EON is not able to increase individual profit (Table 5.4). However, this result is easy to explain: the network is structured such that that EnBW, RWE, and Vattenfall are aggregated within the same node as EON. Hence, withholding of EON does not have an effect as it can be entirely compensated by other competitors.

The latter is of course strongly influenced by the simplified nature of the network representation.
Regarding computational issues, the calculation times vary significantly for the strategic cases. The computation times for Test_EON and Test_RWE are below 20 seconds. The Test_Ebel run takes about four minutes whereas it takes five hours for the Test_EDF case (Table B.11).[7] The computation times depend largely on the number of discrete variables (Table B.10) but also to which extend the position of a companies' generation facilities within the network allows gaming over scarce network resources via its output decisions.

5.5 Conclusions

In this chapter, a mixed-integer linear programming model for a Stackelberg game applicable to network-constrained industries is presented. In order to do so, the equilibrium conditions of the associated MPEC have been converted to disjunctive constraints. This approach is then applied to two different test networks in electricity markets: a three-node example network and a fifteen-node model of the Western European grid. The results show that the approach works well and is promising for solving larger-scale models in the future.

[7] All tests were conducted on a Intel Xeon CPU E5420 (8 cores) with 16 GB RAM.

	Test1		Test2		Test3		Test4		Test5	
	comp	strat	comp	strat	comp	strat	comp	strat	comp	strat
$\mathbf{g}_{n1su1}[MWh]$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.0	1.0
$\mathbf{g}_{n2su2}[MWh]$	9.0	7.0	9.0	4.5	9.0	4.5	9.0	4.5	5.0	5.5
$\mathbf{g}_{n2j1u3}[MWh]$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\mathbf{d}_{n1}[MWh]$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\mathbf{d}_{n2}[MWh]$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\mathbf{d}_{n3}[MWh]$	9.0	7.0	9.0	4.5	9.0	4.5	9.0	4.5	7.0	6.5
$\mathbf{price}_{n1}[€]$	1.0	3.0	1.0	5.5	1.0	5.5	1.0	5.5	2.0	3.25
$\mathbf{price}_{n2}[€]$	1.0	3.0	1.0	5.5	1.0	5.5	1.0	5.5	1.0	3.00
$\mathbf{price}_{n3}[€]$	1.0	3.0	1.0	5.5	1.0	5.5	1.0	5.5	3.0	3.50
$\mathbf{flow}_{l1}[MW]$	-3.0	-2.3	-3.0	-1.5	-3.0	-1.5	-3.0	-1.5	-1.0	-1.5
$\mathbf{flow}_{l2}[MW]$	6.0	4.7	6.0	3.0	6.0	3.0	6.0	3.0	4.0	4.0
$\mathbf{flow}_{l3}[MW]$	3.0	2.3	3.0	1.5	3.0	1.5	3.0	1.5	3.0	2.5

Table 5.3: Model results of the three-node network

		Test_Ebel	Test_EDF	Test_EON	Test_RWE
Profit	comp	82	140	121	94
Leader [k€]	strat	150	1565	121	136
Profit	comp	542	483	514	529
Fringe [k€]	strat	513	547	514	915

Table 5.4: Resulting profits in the fifteen-node Western European network

	Test_Ebel		Test_EDF		Test_EON		Test_RWE	
	comp	strat	comp	strat	comp	strat	comp	strat
$\mathbf{price}_{n1}[€/MWh]$	22.0	22.0	22.0	22.0	22.0	22.0	22.0	29.4
$\mathbf{price}_{n2}[€/MWh]$	10.0	10.0	10.0	41.7	10.0	10.0	10.0	10.0
$\mathbf{price}_{n3}[€/MWh]$	10.0	58.4	10.0	22.0	10.0	10.0	10.0	10.0
$\mathbf{price}_{n4}[€/MWh]$	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0
$\mathbf{price}_{n5}[€/MWh]$	59.3	45.0	59.3	57.2	59.3	59.3	59.3	58.9
$\mathbf{price}_{n6}[€/MWh]$	22.0	52.1	22.0	30.0	22.0	22.0	22.0	22.0
$\mathbf{price}_{n7}[€/MWh]$	41.3	38.2	41.3	41.2	41.3	41.3	41.3	44.8

Table 5.5: Resulting prices in fifteen-node Western European network

	Test_Ebel				
	$\mathbf{g}_{n3Ebelu1}$ [**MWh**]	$\mathbf{g}_{n4Ebelu3}$ [**MWh**]	$\mathbf{g}_{n6Ebelu1}$ [**MWh**]	$\mathbf{g}_{n6Ebelu3}$ [**MWh**]	$\mathbf{g}_{n7Ebelu4}$ [**MWh**]
comp	2548	1000	3000	514	2000
strat	2000	1000	500	2000	1000

Table 5.6: Strategic generation Test_Ebel

	Test_EDF	
	$\mathbf{g}_{n2EDFu7}$ [**MWh**]	$\mathbf{g}_{n2EDFu7}$ [**MWh**]
comp	54209	14000
strat	31000	14000

Table 5.7: Strategic generation Test_EDF

	Test_EON				
	$\mathbf{g}_{n1EONu1}$ [**MWh**]	$\mathbf{g}_{n1EONu2}$ [**MWh**]	$\mathbf{g}_{n1EONu3}$ [**MWh**]	$\mathbf{g}_{n4EONu3}$ [**MWh**]	$\mathbf{g}_{n4EONu5}$ [**MWh**]
comp	8000	1000	7000	3000	303
strat	8000	1000	1000	1000	0

Table 5.8: Strategic generation Test_EON

	Test_RWE	
	$\mathbf{g}_{n1RWEu1}$ [**MWh**]	$\mathbf{g}_{n1RWEu2}$ [**MWh**]
comp	6000	11000
strat	6000	2000

Table 5.9: Strategic generation Test_RWE

Chapter 6

Solving Discretely-Constrained MPEC Problems Using Disjunctive Constraints and Benders Decomposition with an Application in an Electric Power Market

6.1 Introduction and Literature

This chapter presents an alternative approach to solve discretely-constrained MPECs in electric power markets using disjunctive constraints different from the approach presented in Chapter 5. Hence, the basic setting of the problem and the introduction to the topic of modeling strategic behavior in mathematical terms can be found in Sections 5.1 and 5.2. However, in this chapter a new algorithmic approach is presented to solve two-stage Stackelberg games with one leader based on a Benders decomposition technique. Benders decomposition was first reported by Benders (1962)[1] as a partitioning procedure for solving mixed-variables programming problems and was later incorporated into the literature of large scale mathematical program-

[1] Benders (1962) is also available as reprint Benders (2005).

ming by Geoffrion (Geoffrion, 1970a,b, 1972). A more recent rework of the concept can be found in Conejo et al. (2006). Conejo et al. (2006) provide a clear description of Benders decomposition and include exercises in electric power markets and other infrastructure industries that help to comprehend the Benders approach. The work of Conejo et al. (2006) was thus the major starting point for the development of the algorithm introduced in this chapter. The contribution of this work is several fold. First, it establishes an important connection between Benders method and complementarity or MPEC approaches begun by Gabriel et al. (2009) but now extended beyond the linear objective function. Using the Benders approach has the advantage of allowing for a bilinear objective function of the MPEC (here: strategic profit) - that can be decomposed into two linear problems - and the subproblem to be integer-constrained resulting in a discretely-constrained MPEC. For a discussion of possible applications of this type of problems refer to Section 5.2.

This chapter discusses a new decomposition method for solving two-level planning problems with applications in electric power. The upper-level involves generation decisions for the Stackelberg leader and the lower-level depicts the rest of the market and the ISO problem. One of the advantages of the approach combining disjunctive constraints (Fortuny-Amat and McCarl, 1981; Gabriel et al., 2009) and Benders decomposition (Benders, 1962; Geoffrion, 1970a,b, 1972; Floudas, 1995; Conejo et al., 2006) is that it easily allows for integer restrictions on the upper-level variables, for example, allowing discrete generation levels for the producer, if-then logic on generation across time periods, or generation costs consistent with the fixed cost problem (Winston, 1994). In this case, the integer restrictions appear in the master problem which is itself a mixed-integer linear problem. In the present problem the integer variables appear in the subproblem. Hence another advantage of the Benders approach is exploited. This other advantage is that it decomposes a complicated problem into two easier problems which promises better computational behavior particularly for medium- and large-scale problems. It is shown that this approach can potentially speed-up the solving times compared to the pure MIP approach of Chapter 5.

All that is needed is that the optimal value function for the lower-level problem (α) be piecewise convex (compare Section 6.2.1). The reason is that within each segment, a standard Benders approach is applied which itself will need this function to be convex. In the applications in Gabriel et al. (2009), the α-function was shown to be piecewise linear (hence piecewise convex) since the lower-level problems considered were linear programs or linear complementarity problems (LCPs). By contrast, in the current chapter,

the lower-level problem has a bilinear objective function with polyhedral constraints. The difficulty of the nonconvex objective function is avoided since it involves fixed upper-level variables as coefficients and from that perspective, a two-pass process is used to overcome numerical difficulties. A few others have also considered methods for solving MPECs with integer restrictions but have generally not made use of the Benders approach which naturally lends itself to this type of problem via a master and subproblem which separate key variables. These other methods can be grouped into three categories: application-specific approaches, integer-programming methods, or nonlinear programming-based algorithms (compare Chapter 5).
The rest of this chapter is organized as follows. In Section 6.2 the general concept of the Benders decomposition technique is described (Section 6.2.1) which is further applied to the general problem setting introduced in Chapter 5 (Section 6.2.2). The upper-level is a linear program and the lower-lower is a welfare maximization problem for an ISO. Next in Section 6.3, numerical results of a Benders approach for an illustrative electricity market example is described. The numerical results are promising. However, I also point out the limitations of the existing algorithm which leads to according summary remarks and future directions in Section 6.4.[2]

6.2 General Mathematical Formulation

6.2.1 Benders Decomposition: A Primer

This section provides an introduction to the Benders decomposition technique. An example is presented in which both master and subproblem are linear programs. Furthermore, it is assumed that the subproblem is always feasible. One must be aware that these assumptions do not necessarily hold for the application presented in the second part of this chapter.
According to Conejo et al. (2006) there are two different problem structures for which decomposition techniques can be exploited: programs with complicating constraints and programs with complicating variables. The first type can be solved applying the Dantzig-Wolfe decomposition algorithm which will not be considered in detail here. The latter type can be solved applying Benders decomposition. The subsequent example is taken from Conejo et al. (2006) in order to introduce the Benders decomposition concept before it will be applied to the problem of a two-stage game for an electricity

[2] The chapter is based on work carried out jointly with Prof. Dr. Steven A. Gabriel (University of Maryland).

market in Sections 6.2.2 and 6.3.

$$\min_{z_i,\tilde{z}_j} \left\{ \sum_i c_i z_i + \sum_j d_j \tilde{z}_j \right\} \tag{6.1a}$$

subject to

$$\sum_i a_{li} z_i + \sum_j e_{li} \tilde{z}_j \leq b^{(l)};\ l = 1, ..., q \tag{6.1b}$$

$$0 \leq z_i \leq z_i^{up};\ i = 1, ..., n \tag{6.1c}$$

$$0 \leq \tilde{z}_i \leq \tilde{z}_i^{up};\ j = 1, ..., m \tag{6.1d}$$

Within problem (6.1), there are complicating variables z_i and other variables $\tilde{z}_j$. The characteristic of the complicating variables is that if they are fixed to given values, the problem becomes substantially simpler (Conejo et al., 2006). Hence, it could be favorable to determine meaningful values for the z_i variables, fix them and solve the remaining problem. Obviously, there are two challenges. First, a way to determine meaningful z_i variables has to be defined. Second, there has to be a method to adjust the fixed z_i variables such that the algorithm actually arrives at an optimal solution to the overall problem. An algorithm that is able to manage the mentioned challenges is Benders decomposition. Benders method defines a master and a subproblem. The master problem includes the complicating variables and an additional function that cuts out inferior solutions after each iteration (the so-called α-function). The subproblem is made up of a particular instance of the original problem where the complicating variables are parameterized. Accordingly, problem (6.1) can be rewritten as follows:

$$\min_{z_i} \left\{ \sum_i c_i z_i + \alpha(z_i) \right\} \tag{6.2a}$$

subject to

$$0 \leq z_i \leq z_i^{up};\ i = 1, ..., n \tag{6.2b}$$

where

$$\alpha(z_i) = \min_{\tilde{z}_j} \left\{ \sum_j d_j \tilde{z}_j \right\} \tag{6.3a}$$

subject to

$$\sum_j e_{li}\tilde{z}_j \leq b^{(l)} - \sum_i a_{li} z_i;\ l = 1, ..., q \tag{6.3b}$$

$$0 \leq \tilde{z}_i \leq \tilde{z}_i^{up};\ j = 1, ..., m \tag{6.3c}$$

Based on this reformulation, one can define Benders master and subproblem. The master problem (MP) can be deduced from problem (6.2):

$$\min_{z_i,\alpha} \left\{ \sum_i c_i z_i + \alpha \right\} \tag{6.4a}$$

subject to

$$0 \leq z_i \leq z_i^{up};\ i = 1, ..., n \tag{6.4b}$$

$$\alpha \geq \alpha^{down} \tag{6.4c}$$

$$\sum_j d_j \tilde{z}_j^{(k)} + \sum_i \lambda_i^{(k)} (z_i - z_i^{(k)}) \leq \alpha;\ k = 1, ..., \nu \tag{6.4d}$$

where α^{down} is a bound that is determined exogenously according to characteristics of the analyzed real-world application and k is the iteration step.[3] The values for $\tilde{z}_j^{(k)}$ and $\lambda_i^{(k)}$ result from the subproblem (SP) which can be deduced from (6.3):

$$\min_{\tilde{z}_j} \left\{ \sum_j d_j \tilde{z}_j \right\} \tag{6.5a}$$

subject to

$$\sum_j e_{li}\tilde{z}_j \leq b^{(l)} - \sum_i a_{li} z_i;\ l = 1, ..., q \tag{6.5b}$$

$$0 \leq \tilde{z}_i \leq \tilde{z}_i^{up};\ j = 1, ..., m \tag{6.5c}$$

$$z_i = z_i^{(k)}\ (\lambda_i^{(k)});\ i = 1, ..., n \tag{6.5d}$$

[3] The constraints that are added to the master problem after each iteration through inequalities (6.4d) are called Benders cuts (Conejo et al., 2006).

Within the SP (6.5), the complicating variables z_i are fixed. Hence, it is assumed that the SP is much easier to solve than the original problem (Conejo et al., 2006).[4] Constraint (6.5d) forces the problem to hold the complicating variables fixed. The dual variables $\lambda_i^{(k)}$ of (6.5d) then provide the required information for an additional Benders cut in the MP.
In addition to the MP (6.4) and the SP (6.5), one needs a stopping criterion in order to complete the Benders decomposition algorithm. In order to define this criterion, the following bounds are computed:

$$lob^{(k+1)} = \sum_i c_i z_i^{(k+1)} + \alpha^{(k+1)} \tag{6.6a}$$

$$upb^{(k+1)} = \sum_i c_i z_i^{(k+1)} + \sum_j d_j \tilde{z}_j^{(k+1)} \tag{6.6b}$$

On the one hand, the MP defines a relaxed version of the original problem (Conejo et al., 2006). Hence, the optimal objective value of the MP displayed in Equation (6.6a) defines the lower bound *lob* of the optimal value of the original problem. The SP, on the other hand, is a version of the original problem with additional restrictions. Hence, the optimal objective value of the SP displayed in Equation (6.6b) defines the upper bound *upb* of the optimal value of the original problem. Under the assumption of convexity of the α-$function$[5], *lob* can not be greater than *upb* and an optimal solution is found as soon as $lob = upb$. However, in most applications the algorithm stops if the convergence gap between *upb* and *lob* is below a predefined threshold ε.
Based on the above described procedures, the Benders decomposition algorithm can, thus, be implemented as follows (compare Conejo et al., 2006):

- **Step 0: Initialization.** Set iteration counter $\nu = 1$. Solve the master problem (6.4) initially disregarding constraints (6.4d). This problem has the trivial solution $\alpha^{(1)} = \alpha^{down}$, and either $z_i^{(1)} = 0$ if $c_i \geq 0$ or $z_i^{(1)} = z_i^{up}$ if $c_i < 0$.

[4] In this example, there is only one subproblem. However, in some larger problems, there can be several separated SPs. Benders decomposition can also be applied in this case. However, the Benders cuts then have to be modified slightly. Refer to Conejo et al. (2006, p. 118) for a deeper discussion.

[5] Compare Conejo et al. (2006) for a further discussion.

- **Step 1: Subproblem solution.** Set iteration step k to ν. Solve the subproblem (6.5). This problem has the primal variable solution vector $\tilde{z}_j^{(\nu)}$ and a dual variable solution vector $\lambda_i^{(\nu)}$.

- **Step 2: Convergence checking.** Compute $lob^{(\nu)}$ and $upb^{(\nu)}$ according to (6.6a) and (6.6b), respectively. If $upb^{(\nu)} - lob^{(\nu)} < \varepsilon$, stop; the optimal solution vectors are $z_i^{(\nu)}$ and $\tilde{z}_j^{(\nu)}$. Otherwise, continue with the next step.

- **Step 3: Master problem solution.** Update iteration counter $\nu = \nu + 1$. Solve the master problem (6.4) for $k = 1, ..., \nu - 1$. The solution of this problem is the vector $x_i^{(\nu)}$ and $\alpha^{(\nu)}$. Go to Step 1.

6.2.2 General Problem Setting

This section provides a more stylized general formulation of the specific problem at hand. As the problem itself is the same as described in Section 5.3, the beginning of this section is copied from there in order to maintain readability. Starting from this problem formulation, the Benders decomposition technique described in Section 6.2.1 is then applied to the problem in the final part of this section.

The general form of the problem to be solved by the leader (e.g., strategic generator) is as follows where $x \in R^n, y \in R^m$ are respectively, the upper and lower-level vectors of variables:

$$\min_{x,y} \left\{ \begin{pmatrix} d_x \\ d_y \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2} \begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \right\} \tag{6.7a}$$

subject to

$$A_1 y + B_1 x = b_1 \; (\beta_1) \tag{6.7b}$$
$$A_2 x = b_2 \; (\beta_2) \tag{6.7c}$$
$$A_3 x \leq b_3 \; (\beta_3) \tag{6.7d}$$
$$A_4 y = b_4 \;\; (\beta_4) \tag{6.7e}$$
$$A_5 y \leq b_5 \;\; (\beta_5) \tag{6.7f}$$
$$x_i \in Z_+, i = 1, \ldots, n_1 \tag{6.7g}$$
$$x_i \in R, i = n_1 + 1, \ldots, n \tag{6.7h}$$
$$y \in S\,(x) \tag{6.7i}$$
$$y_j \geq 0, j = 1, \ldots, m_1 \tag{6.7j}$$
$$y_2 \text{ (free)}, \; j = m_1 + 1, \ldots, m \tag{6.7k}$$

where $A_1, A_2, A_3, A_4, A_5, B_1$ are matrices of suitable size conformal with the vectors x, y and right-hand sides b_1, b_2, b_3, b_4, b_5. The vectors d_x, d_y contain coefficients for x and y, and $M_{xx}, M_{xy}, M_{yx}, M_{yy}$ are the submatrices referring to the quadratic terms of the objective function. The objective function (6.7a) is quadratic in both the upper and lower-level variables which in the particular power application described in Section 6.3 will involve pairwise products of variables (e.g., generation times price) as well as linear terms (e.g., generation times costs). Equation (6.7b) is the set of joint constraints linking the upper and lower-level variables with β_1 representing the dual variables to these constraints (similar notation for dual variables for the other constraints). Equations (6.7c) and (6.7d) are the constraints that only involve the upper-level variables x whereas (6.7e) and (6.7f) are the counterparts for the lower-level variables y. Equations (6.7g) and (6.7h) indicate that a subset of the upper-level variables are integer-valued whereas constraint (6.7i) stipulates that y must be a solution to the lower-level problem given x. Lastly, the vector y is partitioned into a nonnegative subvector (y_1) and the remaining variables (y_2) free as shown in the last two constraints. The lower-level problem will typically be either a convex, quadratic program whose necessary and sufficient Karush-Kuhn-Tucker conditions or a Nash-Cournot game can be expressed as a mixed linear complementarity problem (MLCP) (Facchinei and Pang, 2003) given as follows:

$$0 \;\leq\; c_1\,(x) + M_{11}(x) y_1 + M_{12}(x) y_2 \perp y_1 \geq 0 \tag{6.8a}$$
$$0 \;=\; c_2\,(x) + M_{21}(x) y_1 + M_{22}(x) y_2 \quad y_2 \text{ (free)} \tag{6.8b}$$

where the dependence on the upper-level variables can be in the vector $c = \begin{pmatrix} c_1(x)^T & c_2(x)^T \end{pmatrix}^T$ and/or the matrix

$$M = \begin{pmatrix} M_{11}(x) & M_{12}(x) \\ M_{21}(x) & M_{22}(x) \end{pmatrix}$$

Having a sufficiently large constant K, the complementarity conditions (6.8) can be converted to disjunctive constraints (Fortuny-Amat and McCarl, 1981; Gabriel et al., 2009) as

$$0 \leq c_1(x) + M_{11}(x)y_1 + M_{12}(x)y_2 \leq Kr \tag{6.9a}$$
$$0 \leq y_1 \leq K(1-r) \tag{6.9b}$$
$$0 = c_2(x) + M_{21}(x)y_1 + M_{22}(x)y_2 \quad y_2 \text{ (free)} \tag{6.9c}$$

where r is a vector of binary variables. In general finding a reasonable constant K may take trial and error. However, in specific instances such as the case study described below, a suitable value can easily be found; see Section 5.4.3 and Appendix A.2 for further guidance on how to obtain such a constant.

Replacing (6.7i) by (6.9) leads to the overall problem expressed in disjunctive form:

$$\min_{x,y} \left\{ \begin{pmatrix} d_x \\ d_y \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2} \begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \right\} \tag{6.10a}$$

subject to

$$A_1y + B_1x = b_1 \ (\beta_1) \tag{6.10b}$$
$$A_2x = b_2 \ (\beta_2) \tag{6.10c}$$
$$A_3x \leq b_3 \ (\beta_3) \tag{6.10d}$$
$$A_4y = b_4 \ \ (\beta_4) \tag{6.10e}$$
$$A_5y \leq b_5 \ \ (\beta_5) \tag{6.10f}$$
$$x_i \in Z_+, i = 1, \ldots, n_1 \tag{6.10g}$$
$$x_i \in R, i = n_1 + 1, \ldots, n \tag{6.10h}$$
$$0 \leq c_1\,(x) + M_{11}(x)y_1 + M_{12}(x)y_2 \leq Kr \tag{6.10i}$$
$$0 \leq y_1 \leq K(1-r) \tag{6.10j}$$
$$0 = c_2\,(x) + M_{21}(x)y_1 + M_{22}(x)y_2 \quad y_2 \text{ (free)} \tag{6.10k}$$
$$y_j \geq 0, j = 1, \ldots, m_1 \tag{6.10l}$$
$$y_2 \text{ (free)},\ j = m_1 + 1, \ldots, m \tag{6.10m}$$
$$r_i \in \{0, 1\}^{m_1} \tag{6.10n}$$

The problem can then be expressed solely in terms of the upper-level variables as follows:

$$\min_{x,y} d_x^T x + \frac{1}{2} x^T M_{xx} x + a(x) \tag{6.11a}$$

subject to

$$A_2x = b_2 \ (\beta_2) \tag{6.11b}$$
$$A_3x \leq b_3 \ (\beta_3) \tag{6.11c}$$
$$x_i \in Z_+, i = 1, \ldots, n_1 \tag{6.11d}$$
$$x_i \in R, i = n_1 + 1, \ldots, n \tag{6.11e}$$

where $\alpha(x)$ is the optimal objective function to

$$\min_y \left\{ d_y^T y + \frac{1}{2} x^T M_{xy} y + \frac{1}{2} y^T M_{yx} x + \frac{1}{2} y^T M_{yy} y \right\} \tag{6.12a}$$

subject to

$$A_1 y + B_1 x = b_1 \ (\beta_1) \tag{6.12b}$$
$$A_4 y = b_4 \quad (\beta_4) \tag{6.12c}$$
$$A_5 y \leq b_5 \quad (\beta_5) \tag{6.12d}$$
$$0 \leq c_1(x) + M_{11}(x) y_1 + M_{12}(x) y_2 \leq K r \tag{6.12e}$$
$$0 \leq y_1 \leq K(1 - r) \tag{6.12f}$$
$$0 = c_2(x) + M_{21}(x) y_1 + M_{22}(x) y_2 \quad y_2 \text{ free} \tag{6.12g}$$
$$y_1 \geq 0, y_2 \text{ free} \tag{6.12h}$$
$$r_i \in \{0, 1\} \tag{6.12i}$$

The key is to solve a sequence of subproblem and master problems in the manner of Benders method (compare Section 6.2.1) for solving (6.10). The MP will be (6.11) but with $\alpha(x)$ replaced by a variable α and Benders cuts (as well as a lower bound on the α variable) used to approximate $\alpha(x)$. The SP will be (6.12) but with an additional fixing constraint of the form $x = x^{fixed}$.[6] The key is to estimate or know in advance, the subdomains for $\alpha(x)$ for which this function is convex. Then, a Benders method for each segment is applied with the overall solution being the best from each of the finite subdomains. In the case study below, it is required to be shown that for this specific instance of (6.11), $\alpha(x)$ is piecewise linear so that the procedure is guaranteed to converge in a finite number of steps assuming that the identification of the subdomains where $\alpha(x)$ is convex is accurate (Gabriel et al., 2009).

6.3 Numerical Example for an Electricity Market

To apply the decomposition method described above, the three nodes network (Figure 5.1) and the corresponding data set known from Section 5.4.4 is used to provide a comparability for the results.

[6]Conejo et al. (2006) show for LPs that the dual variable vector to the fixing constraints corresponds to the subgradients for the function $\alpha(x)$. In addition, Conejo et al. (2006) state that the convergence of the Benders method for any type of a mathematical program is guaranteed as long as the envelope of the α-function is convex.

6.3.1 Mathematical Notation

Indices:

$n, k \in N$	nodes in the network
k'	swing bus
$l \in L$	line between n and k
$f \in F$	firms in the market
$s \in F$	firms acting strategically
$j \in F$	competitive fringe firms
$u \in U$	generation units
$it \in I$	iteration counter within Benders decomposition algorithm

Sets:

F	set of all firms
L	set of all lines
N	set of all nodes
U	set of all generation units
I	set of all iteration steps

Parameters:

a_n, b_n	intercept [€/MWh] and slope [€/MWh^2] of linear demand functions ($a_n, b_n \geq 0$, $\forall n$)
c_{nfu}	generation cost [€/MWh] of firm f at node n with unit u ($c_{nfu} \geq 0$, $\forall n, f, u$)
$\bar{g}_{nfu}$	maximum generation capacity [MW] of firm f at node n with unit u ($\bar{g}_{nfu} \geq 0$, $\forall n, f, u$)
B	network susceptance matrix $n \times k$
H	network transfer matrix $l \times k$
lc_l	physical line capacity limit of line [MW] l
sw_k	swing bus vector, $sw_k = \begin{cases} 1 & \text{if } k = k\prime \\ 0 & \text{otherwise} \end{cases} \quad \forall k$
$K, \bar{K}, \hat{K}, \tilde{K}, \check{K}$	constants in order to replace complementarities by disjunctive constraints
$\underline{\alpha}$	alpha down for Benders decomposition

Variables:

d_n	demand at node n
g_{nfu}	generation of firm f at node n with unit u
δ_k	phase angle at node k
λ_n	shadow price for energy at node n
$\overline{\mu}_l$	shadow price for transmission on line l due to binding line flow constraint in positive direction
$\underline{\mu}_l$	shadow price for transmission on line l due to binding line flow constraint in negative direction
β_{nju}	dual variable of maximum generation constraint per unit u of fringe firm j at node n
γ_k	dual variable for slack bus constraint
$r_n, \bar{r}_{nfu}, \hat{r}_l, \tilde{r}_l, \check{r}_{nfu}$	binary variables in order to replace complementarities by disjunctive constraints
α	alpha for Benders decomposition

6.3.2 Mathematical Formulation

The two formulations (6.13) and (6.14) are already known from Section 5.4. They are repeated in this section in order to maintain readability. The Benders technique is then applied afterwards. Problem (6.13) describes the welfare maximization problem of an ISO assuming that all generators bid at marginal cost. In contrast, problem (6.14) assumes that there is one Stackelberg leader that bids strategically taking into account the ISO's reaction.

$$
\begin{aligned}
0 &\leq -a_n + b_n d_n + \lambda_n \leq K r_n, \forall n && \text{(6.13a)}\\
0 &\leq d_n \leq K(1 - r_n), \forall n && \text{(6.13b)}\\
\\
0 &\leq c_{nfu} - \lambda_n + \beta_{nfu} \leq \bar{K}\bar{r}_{nfu} && \text{(6.13c)}\\
0 &\leq g_{nfu} \leq \bar{K}(1 - \bar{r}_{nfu}), \forall n, f, u && \text{(6.13d)}\\
\\
0 &= \sum_n (B_{nk}\lambda_n) + \sum_l (H_{lk}\overline{\mu}_l) - \sum_l \left(H_{lk}\underline{\mu}_l\right) \\
&\quad - \begin{cases} \gamma_k & \text{if } k\prime = k \\ 0 & \text{otherwise} \end{cases} \qquad \delta_k \text{ (free) } , \forall k && \text{(6.13e)}\\
0 &= d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu}, \quad \lambda_n \text{ (free)}, \forall n && \text{(6.13f)}\\
\\
0 &\leq lc_l - \sum_k (H_{lk}\delta_k) \leq \hat{K}\hat{r}_l, \forall l && \text{(6.13g)}\\
0 &\leq \overline{\mu}_l \leq \hat{K}(1 - \hat{r}_l), \forall l && \text{(6.13h)}\\
\\
0 &\leq lc_l + \sum_k (H_{lk}\delta_k) \leq \tilde{K}\tilde{r}_l, \forall l && \text{(6.13i)}\\
0 &\leq \underline{\mu}_l \leq \tilde{K}(1 - \tilde{r}_l), \forall l && \text{(6.13j)}\\
\\
0 &\leq -g_{nfu} + \overline{g}_{nfu} \leq \check{K}\check{r}_{nfu}, \forall n, f, u && \text{(6.13k)}\\
0 &\leq \beta_{nfu} \leq \check{K}(1 - \check{r}_{nfu}), \forall n, f, u && \text{(6.13l)}\\
\\
0 &= -sw_k\delta_k, \quad \gamma_k \text{ (free)}, \forall k && \text{(6.13m)}
\end{aligned}
$$

$$
r_n, \bar{r}_{nfu}, \hat{r}_l, \tilde{r}_l, \check{r}_{nfu} \in \langle 0, 1 \rangle \quad \forall n, f, u, l
$$

For the purpose of including strategic behavior in the problem, it is assumed that the set of firms f is partitioned into two subsets. Subset s corresponds to the firms that act strategically. Subset j is for the firms that act as price-takers. It is assumed that the firms s decide first on their output decisions in order to maximize individual profits which means that their quantities are exogenous to the ISO problem. These firms know that they influence the market equilibrium with their decisions. The output decision of fringe firms j is determined by the ISO. The latter can be interpreted as a pool system. The entire problem is known as Stackelberg game where firms s are the leaders and the ISO (deciding on the quantities of firms j) is the follower. In the problem at hand, it is assumed that only one player acts strategically resulting in the MPEC (6.14).

$$\min_{d_n, g_{nsu}, \beta_{nju}, \delta_k, \gamma_k, \lambda_n, \overline{\mu}_l, \underline{\mu}_l} \left\{ \sum_n \sum_s \sum_u (c_{nsu} - \lambda_n) g_{nsu} \right\} \tag{6.14a}$$

subject to

$$\begin{aligned}
0 &\leq \overline{g}_{nsu} - g_{nsu}, \forall n, s, u && (6.14b)\\
0 &\leq -a_n + b_n d_n + \lambda_n \perp d_n \geq 0, \forall n && (6.14c)\\
0 &\leq c_{nju} - \lambda_n + \beta_{nju} \perp g_{nju} \geq 0, \forall n, j, u && (6.14d)\\
0 &= \sum_n (B_{nk}\lambda_n) + \sum_l (H_{lk}\overline{\mu}_l) - \sum_l \left(H_{lk}\underline{\mu}_l\right) \\
&- \begin{cases} \gamma_k & \text{if } k = k\prime \\ 0 & \text{otherwise} \end{cases} \quad \delta_k \text{ (free)}, \forall k && (6.14e)\\
0 &= d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu}, \lambda_n \text{ (free)}, \forall n && (6.14f)\\
0 &\leq lc_l - \sum_k (H_{lk}\delta_k) \perp \overline{\mu}_l \geq 0, \forall l && (6.14g)\\
0 &\leq lc_l + \sum_k (H_{lk}\delta_k) \perp \underline{\mu}_l \geq 0, \forall l && (6.14h)\\
0 &\leq -g_{nju} + \overline{g}_{nju} \perp \beta_{nju} \geq 0, \forall n, j, u && (6.14i)\\
0 &= -sw_k\delta_k \quad \gamma_k \text{ (free)}, \forall k && (6.14j)
\end{aligned}$$

As stated earlier the fringe firms' output decisions are determined by the ISO. Hence, the strategic generator takes into account their reaction in terms of the equilibrium problem (6.14c-6.14j) of the ISO within his profit maximization problem. Now the two approaches described above are applied. First, the KKTs are transformed into disjunctive constraints. Then, the problem is decomposed according to Benders approach as presented in Section 6.2.2 which yields the following master (6.15) and subproblem (6.16).

$$\min_{g_{nsu},\alpha} \left\{ \sum_n \sum_u (c_{nsu} g_{nsu}) + \alpha \right\} \tag{6.15a}$$

subject to

$$\underline{\alpha} - \alpha \leq 0 \tag{6.15b}$$

$$\sum_{n,s,u} -g_{nsu}\lambda_n + \sum_{n,s,u} \psi_{nsu}^{(it)}(g_{nsu} - g_{nsu}^{(it)}) - \alpha \leq 0, \ \forall it \geq 2 \tag{6.15c}$$

$$g_{nsu} - \overline{g}_{nsu} \leq 0, \ \forall n, s, u \tag{6.15d}$$

$$g_{nsu} \geq 0, \ \forall n, s, u \tag{6.15e}$$

According to the description in Section 6.2.1, the algorithm is initialized by first solving the MP which then provides the first vector of fixed master problem variables g_{nsu} for the SP. Hence, for the first MP a Benders cut is not available as the Benders cuts result from the dual variable values $\psi_{nsu}^{(it)}$ which are part of the SP. This explains why constraint 6.15c is not included into the MP for $it = 1$. Furthermore, the objective function of the subproblem (6.16a) is bilinear as two factors are multiplied that are both defined variables for the problem. Additionally, there are binary variables in the constraints. This leads to a MINLP:

$$\min_{d_n, g_{nfu}, r_n, \bar{r}_{nju}, \hat{r}_l, \tilde{r}_l, \check{r}_{nju}, \beta_{nju}, \gamma_k, \delta_k, \lambda_n, \overline{\mu}_l, \underline{\mu}_l} \left\{ \sum_n \sum_s \sum_u -g_{nsu}\lambda_n \right\} \tag{6.16a}$$

subject to

$$0 = g_{nsu}^{fix} - g_{nsu}, \ \psi_{nsu} \text{ (free)}, \forall n, s, u \tag{6.16b}$$

$$0 \leq -a_n + b_n d_n + \lambda_n \leq K r_n, \forall n \tag{6.16c}$$

$$0 \leq d_n \leq K(1 - r_n), \forall n \tag{6.16d}$$

$$0 \leq c_{nju} - \lambda_n + \beta_{nju} \leq \bar{K}\bar{r}_{nju}, \ \forall n, j, u \tag{6.16e}$$

$$0 \leq g_{nju} \leq \bar{K}(1 - \bar{r}_{nju}), \forall n, j, u \tag{6.16f}$$

$$0 = -sw_k\delta_k, \quad \gamma_k \text{ (free)}, \forall k \tag{6.16g}$$

$$0 = d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu}, \lambda_n \text{ (free)}, \forall n \tag{6.16h}$$

$$0 = \sum_n (B_{nk}\lambda_n) + \sum_l (H_{lk}\overline{\mu}_l) - \sum_l \left(H_{lk}\underline{\mu}_l\right) - \begin{cases} \gamma_k & \text{if } k = k\prime \\ 0 & \text{otherwise} \end{cases} \quad \delta_k \ (free), \forall k \tag{6.16i}$$

$$0 \leq lc_l - \sum_k (H_{lk}\delta_k) \leq \hat{K}\hat{r}_l, \forall l \tag{6.16j}$$

$$0 \leq \overline{\mu}_l \leq \hat{K}(1 - \hat{r}_l), \forall l \tag{6.16k}$$

$$0 \leq lc_l + \sum_k (H_{lk}\delta_k) \leq \tilde{K}\tilde{r}_l, \forall l \tag{6.16l}$$

$$0 \leq \underline{\mu}_l \leq \tilde{K}(1 - \tilde{r}_l), \forall l \tag{6.16m}$$

$$0 \leq -g_{nju} + \overline{g}_{nju} \leq \check{K}\check{r}_{nju}, \forall n, j, u \tag{6.16n}$$

$$0 \leq \beta_{nju} \leq \check{K}(1 - \check{r}_{nju}), \forall n, j, u \tag{6.16o}$$

$$r_n, \bar{r}_{nju}, \hat{r}_l, \tilde{r}_l, \check{r}_{nju} \in \langle 0, 1 \rangle$$

This mixed-integer nonlinear subproblem which is hard to solve. Hence, two-pass approach is used in order to solve the subproblem described in (6.16). First, the set of Equations is transformed using an equivalent transformation integrating Equations (6.16b) into the objective function (6.16a). In order to do so, variables g_{nsu} in (6.16a) are replaced by the parameters g_{nsu}^{fix}. The resulting equivalent subproblem looks as follows:

$$\min_{d_n, g_{nju}, r_n, \bar{r}_{nju}, \hat{r}_l, \tilde{r}_l, \check{r}_{nju}, \beta_{nju}, \gamma_k, \delta_k, \lambda_n, \overline{\mu}_l, \underline{\mu}_l} \left\{ \sum_n \sum_s \sum_u -g^{fix}_{nsu} \lambda_n \right\} \tag{6.17a}$$

subject to

$$
\begin{aligned}
0 &\leq -a_n + b_n d_n + \lambda_n \leq K r_n, \forall n && \text{(6.17b)}\\
0 &\leq d_n \leq K(1 - r_n), \forall n && \text{(6.17c)}\\
0 &\leq c_{nju} - \lambda_n + \beta_{nju} \leq \bar{K} \bar{r}_{nju},\ \forall n, j, u && \text{(6.17d)}\\
0 &\leq g_{nju} \leq \bar{K}(1 - \bar{r}_{nju}), \forall n, j, u && \text{(6.17e)}\\
0 &= -sw_k \delta_k, \quad \gamma_k \text{ (free)}, \forall k && \text{(6.17f)}\\
0 &= d_n + \sum_k (B_{nk} \delta_k) - \sum_j \sum_u g_{nju} \\
&\quad - \sum_s \sum_u g^{fix}_{nsu}, \lambda_n \text{ (free)}, \forall n && \text{(6.17g)}\\
0 &= \sum_n (B_{nk} \lambda_n) + \sum_l (H_{lk} \overline{\mu}_l) - \sum_l \left(H_{lk} \underline{\mu}_l \right) \\
&\quad - \begin{cases} \gamma_k & \text{if } k = k\prime \\ 0 & \text{otherwise} \end{cases} \qquad \delta_k \ (free), \forall k && \text{(6.17h)}
\end{aligned}
$$

$$
\begin{aligned}
0 &\leq lc_l - \sum_k (H_{lk} \delta_k) \leq \hat{K} \hat{r}_l, \forall l && \text{(6.17i)}\\
0 &\leq \overline{\mu}_l \leq \hat{K}(1 - \hat{r}_l), \forall l && \text{(6.17j)}\\
0 &\leq lc_l + \sum_k (H_{lk} \delta_k) \leq \tilde{K} \tilde{r}_l, \forall l && \text{(6.17k)}\\
0 &\leq \underline{\mu}_l \leq \tilde{K}(1 - \tilde{r}_l), \forall l && \text{(6.17l)}\\
0 &\leq -g_{nju} + \overline{g}_{nju} \leq \check{K} \check{r}_{nju}, \forall n, j, u && \text{(6.17m)}\\
0 &\leq \beta_{nju} \leq \check{K}(1 - \check{r}_{nju}), \forall n, j, u && \text{(6.17n)}
\end{aligned}
$$

$$r_n, \bar{r}_{nju}, \hat{r}_l, \tilde{r}_l, \check{r}_{nju} \in \langle 0, 1 \rangle$$

This problem can be solved as a MILP. However, the MILP does not provide the necessary information for solving the master problem in the set of Equations (6.15). In order to make Benders decomposition work, Benders cuts (6.15c) are needed. For these cuts, the dual variable ψ_{nsu} from the SP constraint (6.16b) is needed. However, the set of conditions (6.17) do not provide the required information. This information is received through a second run of the subproblem. For the second run, the binary variables $r_n, \bar{r}_{nju}, \hat{r}_l, \tilde{r}_l, \check{r}_{nju}$ are fixed to the values taken in the first run. Thus, the binary variables in (6.16) are replaced by the parameters $r_n^{fix}, \bar{r}_{nju}^{fix}, \hat{r}_l^{fix}$, $\tilde{r}_l^{fix}, \check{r}_{nju}^{fix}$. The resulting bilinear subproblem is as follows:

$$\min_{d_n, g_{nju}, \beta_{nju}, \gamma_k, \delta_k, \lambda_n, \overline{\mu}_l, \underline{\mu}_l} \left\{ \sum_n \sum_s \sum_u -g_{nsu}\lambda_n \right\} \tag{6.18a}$$

subject to

$$0 = g_{nsu}^{fix} - g_{nsu}, \;\; \psi_{nsu} \text{ (free)}, \forall n, s, u \tag{6.18b}$$

$$0 \leq -a_n + b_n d_n + \lambda_n \leq K r_n^{fix}, \forall n \tag{6.18c}$$

$$0 \leq d_n \leq K(1 - r_n^{fix}), \forall n \tag{6.18d}$$

$$0 \leq c_{nju} - \lambda_n + \beta_{nju} \leq \bar{K}\bar{r}_{nju}^{fix}, \;\; \forall n, j, u \tag{6.18e}$$

$$0 \leq g_{nju} \leq \bar{K}(1 - \bar{r}_{nju}^{fix}), \forall n, j, u \tag{6.18f}$$

$$0 = -sw_k\delta_k, \quad \gamma_k \text{ (free)}, \forall k \tag{6.18g}$$

$$0 = d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu} = 0, \lambda_n \text{ (free)}, \forall n \tag{6.18h}$$

$$0 = \sum_n (B_{nk}\lambda_n) + \sum_l (H_{lk}\overline{\mu}_l) - \sum_l \left(H_{lk}\underline{\mu}_l\right) - \begin{cases} \gamma_k & \text{if } k = k\prime \\ 0 & \text{otherwise} \end{cases} \quad \delta_k \; (free), \forall k \tag{6.18i}$$

$$0 \leq lc_l - \sum_k (H_{lk}\delta_k) \leq \hat{K}\hat{r}_l^{fix}, \forall l \tag{6.18j}$$

$$0 \leq \overline{\mu}_l \leq \hat{K}(1 - \hat{r}_l^{fix}), \forall l \tag{6.18k}$$

$$0 \leq lc_l + \sum_k (H_{lk}\delta_k) \leq \tilde{K}\tilde{r}_l^{fix}, \forall l \tag{6.18l}$$

$$0 \leq \underline{\mu}_l \leq \tilde{K}(1 - \tilde{r}_l^{fix}), \forall l \tag{6.18m}$$

$$0 \leq -g_{nju} + \overline{g}_{nju} \leq \check{K}\check{r}_{nju}^{fix}, \forall n, j, u \tag{6.18n}$$

$$0 \leq \beta_{nju} \leq \check{K}(1 - \check{r}_{nju}^{fix}), \forall n, j, u \tag{6.18o}$$

This problem is solved as a NLP. However, as the two problems (6.17) and (6.18) solve mathematically equivalent problems, the set of solutions is exactly the same (compare Appendix C for a mathematical justification). Additionally, the nonlinear subproblem formulation provides the necessary information for Benders decomposition in form of the dual variables ψ_{nsu} for the master problem.[7]

6.3.3 Computational Results[8]

The objective of this section is to show that the approach can be applied to problems that occur in electric power markets using a simple three-node example as depicted in Figure 5.1 with the demand structure (Table B.1). As starting point, the data set of Chapter 5 is used (Table 6.1). Test1 in Table 6.2 shows that under the simple assumption of sufficient transmission capacities the Benders approach yields the same results as in Test1 of Table 5.3 using the MIP approach. The maximum profit amounts to 14.0 € and the strategic generation is 7 MWh (Table 6.2) which is exactly that amount of production that induces a price at which the competitive fringe player does not produce. As displayed in Table 6.1, all of the existing plants have a maximum generation limit of 15 MW and the transmission lines have a limit of 10 MW. As the resulting flows are well below 10 MW, none of the transmission constraints is binding; also the maximum generation constraints for the plants are not binding at the calculated production levels (compare Table 6.2). However, carrying out a sensitivity analysis shows that the results change if the maximum generation capacities of the plants $\bar{g}_{nfu}$ are varied from 15 to 10 (Test2 in Tables 6.1 and 6.2). This should not have any impact as the maximum generation constraint should not become binding if the limit is at 10 MWh but the level is at 7 MWh. Hence, one would assume that the results for Test2 are exactly the same as for Test1. However, this is not the case. The strategic generation decreases to 6.3 MWh and the competitive fringe at the same node starts producing 0.7 MWh (Test2 in Table 6.2). This result is surprising as the strategic profit decreases to 12.6 €. Hence, the result for Test1 is superior as the profit there was 14.0 €.

[7]The reason to apply the two-pass process is a numerical one. The implementation in GAMS did not allow to solve the problem (6.16) as MILP although the problem has the same set of solutions as (6.17).

[8]All calculations refer to hourly values. Hence, one MW of generation is one MWh of energy delivered.

	Test1	**Test2**
$\mathbf{c}_{n1su1}$	2	2
$\mathbf{c}_{n2su2}$	1	1
$\mathbf{c}_{n2j1u3}$	3	3
$\bar{\mathbf{g}}_{n1su1}$	15	10
$\bar{\mathbf{g}}_{n2su2}$	15	10
$\bar{\mathbf{g}}_{n2j1u3}$	15	10
$\mathbf{lc}_{l1}$	10	10
$\mathbf{lc}_{l2}$	10	10
$\mathbf{lc}_{l3}$	10	10

Table 6.1: Data for tests of the three-node example using Benders decomposition

	Test1	**Test2**
$\mathbf{g}_{n1su1}$	0.0	0.0
$\mathbf{g}_{n2su2}$	7.0	6.3
$\mathbf{g}_{n2j1u3}$	0.0	0.7
$\mathbf{d}_{n1}$	0.0	0.0
$\mathbf{d}_{n2}$	0.0	0.0
$\mathbf{d}_{n3}$	7.0	7.0
$\mathbf{price}_{n1}$	3.0	3.0
$\mathbf{price}_{n2}$	3.0	3.0
$\mathbf{price}_{n3}$	3.0	3.0
$\mathbf{flow}_{l1}$	-2.3	-2.3
$\mathbf{flow}_{l2}$	4.7	4.7
$\mathbf{flow}_{l3}$	2.3	2.3
$\mathbf{profit}_{s}$	14.0	12.6
iterations	7	4

Table 6.2: Results for tests of the three-node example using Benders decomposition

In order to further examine this problem, the different Benders iterations are displayed in more detail. For both Test1 (Table 6.3) and Test2 (Table 6.4) the first iterations of the Benders algorithm behave similarly. In iteration it1, the production of the two strategic units is at the lower bound of 0. The entire demand of 7 is served by the fringe unit. Accordingly, the negative multipliers ψ_{nsu} add a Benders cut for iteration it2 such that the strategic generation is increased to the upper bound of 15 or 10, respectively. This, however, leads to a negative strategic profit. Positive multipliers ψ_{nsu} indicate that the strategic production should be decreased which takes place stepwise iteration by iteration. In Test1 the algorithm arrives at the optimal solution of $g_{n2su2} = 7$ MWh and a strategic profit of 14 € after constantly decreasing strategic output first with the more expensive unit u1 and then with unit u2 during iterations it2-it7. In Test2, a similar behavior can be observed during iterations it2-it3. After iteration it3, the output of unit u2 has been decreased below the optimal solution of Test1 ($g_{n2su2} = 7$ MWh) to $g_{n2su2} = 6.3$ MWh and profit of 12.6 €. Hence, it would be profitable for the strategic player to increase its profit again to 7 MWh which is also indicated by the change of sign of the multipliers ψ_{nsu} (Table 6.4). However, it appears that this is not possible any more as the MP is already too restricted such that the production of 6.3 MWh is the best it can do with the existing Benders cuts.

	Test1 Iteration step						
	it1	**it2**	**it3**	**it4**	**it5**	**it6**	**it7**
$\mathbf{g}_{n1su1}$	0.0	15.0	0.8	0.0	0.0	0.0	0.0
$\mathbf{g}_{n2su2}$	0.0	15.0	15.0	8.7	7.3	7.0	7.0
$\mathbf{g}_{n2j1u3}$	7.0	0.0	0.0	0.0	0.0	0.0	0.0
ψ_{n1su1}	-3.0	16.0	6.5	7.4	4.6	4.0	0.1
ψ_{n2su2}	-3.0	16.0	6.5	7.4	4.6	4.0	0.1
lob	-10^5	-45.0	-30.8	-17.4	-14.6	-14.0	-14.0
upb	0.0	225.0	36.5	-2.4	-12.5	-13.9	-14.0
$\mathbf{profit_s}$	0.0	-225.0	-36.5	2.4	12.5	13.9	14.0

Table 6.3: Results per iteration for Test1 of three-node example using Benders decomposition

	Test2 Iteration step			
	it1	**it2**	**it3**	**it4**
$\mathbf{g}_{n1su1}$	0.0	10.0	0.8	0.0
$\mathbf{g}_{n2su2}$	0.0	10.0	10.0	6.3
$\mathbf{g}_{n2j1u3}$	7.0	0.0	0.0	0.7
ψ_{n1su1}	-3.0	9.3	3.2	-3.0
ψ_{n1su1}	-3.0	9.3	3.2	-3.0
lob	-10^5	-30.0	-20.8	-12.6
upb	0.0	83.3	7.3	-12.6
$\mathbf{profit_s}$	0.0	-83.3	-7.3	12.6

Table 6.4: Results per iteration for Test2 of three-node example using Benders decomposition

Beyond the tests above, additional tests with a differing data set are conducted where the marginal costs of generators vary from each other by a smaller extent (Table 6.5). Applying the Benders approach to this new data set again provides interesting insights into the functioning of the algorithm.[9] Both algorithms, the MIP approach of Chapter 5 and the Benders approach of the present chapter are applied to the tests in Table 6.5. However, the results of the MIP runs are not displayed here. For Test3-5 in Table 6.6 the results of the both algorithms are identical: the strategic generator holds back generation in order to increase the price to the level of the marginal cost of the competitive player which leads to a positive profit in the absence of binding maximum transmission and generation constraints. However, for Test6-9, the results differ. In the case that playing over network effects is excluded because the transmission constraints are not binding, it can be seen that for the present example there are generally multiple solutions if both generators are located at the same node and have the same marginal costs. For this instance the strategic generator does not have the opportunity to bid strategically as its expected profit is 0 independent from his output. This is the case for Test6. In the MIP run the strategic generator satisfies the entire demand of 9 MWh whereas in the Benders approach the competitive fringe satisfies the entire demand. Nonetheless, both results are valid solutions. Beyond this result, particularly Test9 uncovers an implausible behavior. While the price at node $n1$ increases up to 7 €/MWh, the strategic unit with a marginal cost of 1.13 €/MWh located there does not

[9]The results produced can be compared if applying the MIP approach of Chapter 5 to the same data set.

start to produce while the strategic generator has a profit of 0.

	Test3	Test4	Test5	Test6	Test7	Test8	Test9
$\mathbf{c}_{n1su1}$	1.25	1.20	1.25	1.25	1.25	1.13	1.13
$\mathbf{c}_{n2su2}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\mathbf{c}_{n2j1u3}$	1.13	1.10	1.13	1.00	1.13	1.13	1.00
$\bar{\mathbf{g}}_{n1su1}$	15	15	15	15	15	15	15
$\bar{\mathbf{g}}_{n2su2}$	15	15	5	15	15	15	15
$\bar{\mathbf{g}}_{n2j1u3}$	15	15	15	15	15	15	15
$\mathbf{lc}_{l1}$	10	10	10	10	2	2	2
$\mathbf{lc}_{l2}$	10	10	10	10	10	10	10
$\mathbf{lc}_{l3}$	10	10	10	10	10	10	10

Table 6.5: Data for further tests of three-node example using Benders decomposition

	Test3	Test4	Test5	Test6	Test7	Test8	Test9
$\mathbf{g}_{n1su1}$	0.00	0.00	0.00	0.00	0.78	1.50	0.00
$\mathbf{g}_{n2su2}$	8.87	8.90	5.00	0.00	6.98	7.37	0.00
$\mathbf{g}_{n2j1u3}$	0.00	0.00	3.87	9.00	0.00	0.00	6.00
$\mathbf{d}_{n1}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\mathbf{d}_{n2}$	0.00	0.00	0.00	0.00	0.20	0.00	0.00
$\mathbf{d}_{n3}$	8.87	8.90	8.87	9.00	7.56	8.87	6.00
$\mathbf{price}_{n1}$	1.13	1.10	1.13	1.00	4.08	1.13	7.00
$\mathbf{price}_{n2}$	1.13	1.10	1.13	1.00	0.80	1.13	1.00
$\mathbf{price}_{n3}$	1.13	1.10	1.13	1.00	2.44	1.13	4.00
$\mathbf{flow}_{l1}$	-2.96	-2.97	-2.96	-3.00	-2.00	-1.96	-2.00
$\mathbf{flow}_{l2}$	5.91	5.93	5.91	6.00	4.78	5.41	4.00
$\mathbf{flow}_{l3}$	2.96	2.97	2.96	3.00	2.78	3.46	2.00
$\mathbf{profit}_s$	1.15	0.89	0.65	0.00	0.81	0.96	0.00
iterations	5	5	2	2	20	12	4

Table 6.6: Results for further tests of three-node example using Benders decomposition

In order to explain the instable behavior of the Benders algorithm, the problem is reformulated using an integer version of the master problem in order to enumerate the actual solution to the problem. To derive a mixed-integer linear version of the master problem, the generation quantity variables g_{nsu} in problem (6.15) are defined as integer variables. Thus, the master problem becomes the following:

$$\min_{g_{nsu},\alpha} \left\{ \sum_n \sum_u (c_{nsu} g_{nsu}) + \alpha \right\} \tag{6.19a}$$

subject to

$$\underline{\alpha} - \alpha \leq 0 \tag{6.19b}$$

$$\sum_{n,s,u} -g_{nsu}\lambda_n + \sum_{n,s,u} \psi_{nsu}^{(it)}(g_{nsu} - g_{nsu}^{(it)}) - \alpha \leq 0,\ \forall it \geq 2 \tag{6.19c}$$

$$g_{nsu} - \overline{g}_{nsu} \leq 0,\ \forall n, s, u \tag{6.19d}$$

$$g_{nsu} \geq 0,\ \forall n, s, u \tag{6.19e}$$

$$g_{nsu} \in \mathbb{Z}_+\ , \forall n, s, u \tag{6.19f}$$

For this problem, the result can be found by enumeration. In order to enumerate the feasible values, the possible combinations of g_{n1su1} and g_{n2su2} are defined and for each pair, the subproblem is solved. Based on the nonnegativity assumption and the maximum generation constraint, the following is true for TestInt1 (Table 6.7): $0 \leq g_{nsu} \leq 15$ while $g_{nsu} \in \mathbb{Z}$. This leads to $16 * 16 = 256$ possibilities. For TestInt2, the following is true (Table 6.7): $0 \leq g_{nsu} \leq 10$ while $g_{nsu} \in \mathbb{Z}$. This leads to $11 * 11 = 121$ possibilities. By enumerating the problem, it can be shown which solutions are optimal (Table 6.8) solutions to the overall problem (under the assumption of integer strategic generation levels) as one solves 256 (or 121, respectively) independent MILP for which optimality is proven by the standard solver in GAMS.

When applying the Benders approach to the mixed-integer linear master problem, the same problem occurs as in Test1 and Test2 of the linear MP above (Table 6.9). Test1Int arrives at the optimal solution because the 'communication' between MP and SP via the multipliers ψ_{nsu} works properly (Table 6.10). However, as shown in Table 6.11, TestInt2 does not even convergence. The problem gets stuck from iteration it5 onwards. The reason for this is that the lower bound *lob* becomes greater than the upper bound *upb* which can never happen in case of an convex envelope of the α-function (Conejo et al., 2006). Hence, this is an indication that for the present problem structure, the Benders approach is not straightforward applicable.

	TestInt1	**TestInt2**
$\mathbf{c}_{n1su1}$	2	2
$\mathbf{c}_{n2su2}$	1	1
$\mathbf{c}_{n2j1u3}$	3	3
$\bar{\mathbf{g}}_{n1su1}$	15	10
$\bar{\mathbf{g}}_{n2su2}$	15	10
$\bar{\mathbf{g}}_{n2j1u3}$	15	10
$\mathbf{lc}_{l1}$	10	10
$\mathbf{lc}_{l2}$	10	10
$\mathbf{lc}_{l3}$	10	10

Table 6.7: Data for tests of the integer version of three-node example using Benders decomposition

	TestInt1	**TestInt2**
$\mathbf{g}_{n1su1}$	0.0	0.0
$\mathbf{g}_{n2su2}$	7.0	7.0
$\mathbf{g}_{n2j1u3}$	0.0	0.0
$\mathbf{d}_{n1}$	0.0	0.0
$\mathbf{d}_{n2}$	0.0	0.0
$\mathbf{d}_{n3}$	7.0	7.0
$\mathbf{price}_{n1}$	3.0	3.0
$\mathbf{price}_{n2}$	3.0	3.0
$\mathbf{price}_{n3}$	3.0	3.0
$\mathbf{flow}_{l1}$	-2.3	-2.3
$\mathbf{flow}_{l2}$	4.7	4.7
$\mathbf{flow}_{l3}$	2.3	2.3
$\mathbf{profit}_{s}$	14.0	14.0

Table 6.8: Results for enumeration of the integer version of three-node example using Benders decomposition

	TestInt1	**TestInt2**
$\mathbf{g}_{n1su1}$	0.0	0.0
$\mathbf{g}_{n2su2}$	7.0	6.0
$\mathbf{g}_{n2j1u3}$	0.0	0.0
$\mathbf{d}_{n1}$	0.0	0.0
$\mathbf{d}_{n2}$	0.0	0.0
$\mathbf{d}_{n3}$	7.0	7.0
$\mathbf{price}_{n1}$	3.0	3.0
$\mathbf{price}_{n2}$	3.0	3.0
$\mathbf{price}_{n3}$	3.0	3.0
$\mathbf{flow}_{l1}$	-2.3	-2.3
$\mathbf{flow}_{l2}$	4.7	4.7
$\mathbf{flow}_{l3}$	2.3	2.3
$\mathbf{profit}_{s}$	14.0	12.0
iterations	5	X

Table 6.9: Results for tests of the integer version of three-node example using Benders decomposition

	Test1Int Iteration step				
	it1	**it2**	**it3**	**it4**	**it5**
$\mathbf{g}_{n1su1}$	0.0	15.0	0.0	0.0	0.0
$\mathbf{g}_{n2su2}$	0.0	15.0	15.0	8.0	7.0
$\mathbf{g}_{n2j1u3}$	7.0	0.0	0.0	0.0	0.0
ψ_{n1su1}	-3.0	16.0	6.0	6.0	4.0
ψ_{n2su2}	-3.0	16.0	6.0	6.0	4.0
lob	-10^5	-45.0	-30.0	-16.0	-14.0
upb	0.0	225.0	30.0	-8.0	-14.0
$\mathbf{profit_s}$	0.0	-225.0	-30.0	8.0	14.0

Table 6.10: Results per iteration for Test1Int of three-node example using Benders decomposition

	Test2Int Iteration step			
	it1	**it2**	**it3**	**it4**
$\mathbf{g}_{n1su1}$	0.0	10.0	0.0	0.0
$\mathbf{g}_{n2su2}$	0.0	10.0	10.0	6.0
$\mathbf{g}_{n2j1u3}$	7.0	0.0	0.0	1.0
ψ_{n1su1}	-3.0	9.3	2.7	-3.0
ψ_{n2su2}	-3.0	9.3	2.7	-3.0
lob	-10^5	-30.0	-20.0	-11.3
upb	0.0	83.3	3.3	-12.0
$\mathbf{profit_s}$	0.0	-83.3	-3.3	12.0

Table 6.11: Results per iteration for Test2Int of three-node example using Benders decomposition

6.3.4 Discussion

The results for the Benders decomposition approach are promising. However, the robustness of the approach is insufficient. The Benders technique requires a convex envelope of the α-function which is not the case in the existing MIP subproblem (Equation 6.17). One possibility to overcome the problems arising from the nonconvexity of the α-function is to apply an appropriate domain decomposition approach. As mentioned in Section 6.2.2, domain decomposition works by breaking apart a nonconvex problem into convex pieces (subdomains). For each of these subdomains the optimal solution is determined. Comparing the single subdomain results yields the optimal solution to the overall problem.

A domain decomposition technique for a Benders approach (with a piecewise linear α-function) is presented in Gabriel et al. (2009). Their approach is static which means that they define the subdomains before solving the problem. However, they develop a sophisticated heuristic in order to avoid the enumeration of all possible combinations. Nonetheless, using this heuristic is computationally impossible for larger-scale problems as they are common in electricity markets. The approach of Gabriel et al. (2009) is still associated with a significant computational effort which might not not even be conducive for the problem at hand. Hence, dynamic decomposition algorithms could be a valuable alternative. These dynamic techniques aim to find and define subdomains only if it is necessary during the solving procedure.

However, for the time being only little application literature for these techniques exists in an electric power market context. The development of such an algorithm requires significant further research concerning a new research area as presented here. Thus, the further development of the presented approaches would be out of the scope of the thesis at hand but indicates the direction of possible related future research.

6.4 Conclusions

This chapter presents a promising approach in order to decompose discretely-constrained MPEC problems for electric power markets using Benders decomposition technique. The developed algorithm can be applied to simple problems of strategic behavior of generators. Furthermore, it can be simplified such that domain decomposition techniques can be applied. However, the algorithm not yet fully applicable to capture gaming over network effects due to an insufficient robustness of the computational behavior.

Bibliography

Anitescu, Mihai. 2000. "On Solving Mathematical Programs with Complementarity Constraints as Nonlinear Programs." Technical report, Preprint ANL/MCS-P864-1200, MCS Division, Argonne National Laboratory, Argonne, IL, USA.

Anitescu, Mihai. 2004. "Global Convergence of an Elastic Mode Approach for a Class of Mathematical Programs with Complementarity Constraints." Technical report, Preprint ANL/MCS-P1143-0404, MCS Division, Argonne National Laboratory, Argonne, IL, USA.

Auerswald, Danny and Florian Leuthold. 2009. "Generation Portfolio and Investment Timing until 2030: A Real Options Approach for Germany." Dresden University of Technology, Chair of Energy Economics and Public Sector Management, Working Paper WP-EM-33. Internet: http://www.tu-dresden.de/wwbwleeg/publications/wp_em_33_Auerswald_Leuthold_real_options_generation_2030.pdf. Accessed: 29 April 2009.

Baehr, Reinhardt. 1985. "Konzeption und Aufbau von Dampfkraftwerken." In: Bohn, Thomas (ed.), *Handbuchreihe Energie Band 5*, Cologne, Germany: Technischer Verlag Reschl/Verlag TÜV Rheinland.

BAFA. 2008. "Energie." Federal Office of Economics and Export Control (BAFA), Eschborn, Germany. Internet: http://www.bafa.de/bafa/de/. Accessed: 06 February 2009.

Baldick, Ross. 2007. "Border Flow Rights and Contracts for Differences of Differences: Models for Electric Transmission Property Rights." *IEEE Transactions on Power Systems* 22(4):1495–1506.

Bard, Jonathan F. 1983. "An Efficient Point Algorithm for a Linear Two-Stage Optimization Problem." *Operations Research* 31(4):670–684.

Bard, Jonathan F. 1988. "Convex Two-Level Optimization." *Mathematical Programming* 40(1):15–27.

Bard, Jonathan F. and James T. Moore. 1990. "A Branch and Bound Algorithm for the Bilevel Programming Problem." *SIAM Journal on Scientific and Statistical Computation* 11(2):281–292.

Bard, Jonathan F., John Plummer and Jean-Claude Sourie. 2000. "A Bilevel Programming Approach to Determining Tax Credits for Biofuel Production." *European Journal of Operational Research* 120(1):30–46.

Barroso, Luiz A., Rafael D. Carneiro, Sérgio Granville, Mario V. Pereira and Marcia H. C. Fampa. 2006. "Nash Equilibrium in Strategic Bidding: A Binary Expansion Approach." *IEEE Transactions on Power Systems* 21(2):629–638.

Benders, J. F. 1962. "Partitioning Procedures for Solving Mixed-Variables Programming Problems." *Numerische Mathematik* 4:238–252.

Benders, J. F. 2005. "Partitioning Procedures for Solving Mixed-Variables Programming Problems." *Computational Management Science* 2(1):3–19.

Bergen, Arthur R. and Vijay Vittal. 2000. *Power Systems Analysis*. Upper Saddle River, NJ: Prentice-Hall, 2nd edition.

Bergman, Lars. 1988. "Energy Policy Modeling: A Survey of General Equlibrium Approaches." *Journal of Policy Modeling* 10(3):377–399.

Bialas, Wayne F. and Mark J. Karwan. 1984. "Two-Level Linear Programming." *Management Science* 30(8):1004–1020.

Borenstein, Severin, James Bushnell and Steven Stoft. 2000. "The Competitive Effects of Transmission Capacity in a Deregulated Electricity Industry." *RAND Journal of Economics* 31(2):294–325.

Boucher, Jacqueline and Yves Smeers. 2002. "Towards a Common European Electricity Market - Paths in the Right Direction...Still Far From an Effective Design." *Journal of Network Industries* 3(4):375–424.

Brunekreeft, Gert and David Newbery. 2006. "Should Merchant Transmission Investment Be Subject to a Must-Offer Provision?" *Journal of Regulatory Economics* 30(3):233–260.

Buckley, Adrian, Stephen A. Ross, Randolph W. Westerfield and Jeffrey F. Jaffe. 1998. *Corporate Finance Europe*. London, UK: McGraw-Hill.

Burger, Markus, Bernhard Graeber and Gero Schindlmayr. 2007. *Managing Energy Risk*. Chichester, West Sussex, UK: John Wiley & Sons.

Bushnell, James and Steven Stoft. 1996a. "Transmission and Generation Investment in a Competitive Electric Power Industry." University of California Energy Institute, Working Paper PWP-030. Internet: http://www.ucei.berkeley.edu/ucei/PDF/pwp030.pdf. Accessed: 29 April 2009.

Bushnell, James B. and Steven Stoft. 1996b. "Electric Grid Investment Under a Contract Network Regime." *Journal of Regulatory Economics* 10(1):61–79.

Castillo, Enrique, Antonio J. Conejo, Roberto Minguez and Carmen Castillo. 2003. "An Alternative Approach for Addressing the Failure Probability-Safety Factor Method with Sensitivity Analysis." *Reliability Engineering & System Safety* 82(2):207–216.

Castillo, Enrique, Antonio J. Conejo, Pablo Pedregal, Ricardo Garciá and Natalia Alguacil. 2002. *Building and Solving Mathematical Programming Models in Engineering and Science*. New York, NY: John Wiley & Sons.

Chao, Hung-Po and Stephen Peck. 1996. "A Market Mechanism for Electric Power Transmission." *Journal of Regulatory Economics* 10(1):25–59.

Chao, Hung-Po, Stephen Peck, Shmuel Oren and Robert Wilson. 2000. "Flow-Based Transmission Rights and Congestion Management." *The Electricity Journal* 13(8):38–58.

Chen, Yishu, Benjamin F. Hobbs, Sven Leyffer and Todd S. Munson. 2004. "Leader-Follower Equilibria for Electric Power and NOx Allowances Markets." Technical report, Preprint ANL/MCS-P1191-0804, Mathematics and Computer Science Division, Argonne National Laboratory, Argonne, IL, USA 60439.

Christie, Richard D., Bruce F. Wollenberg and Ivar Wangensteen. 2000. "Transmission Management in the Deregulated Environment." *Proceedings of the IEEE* 88(2):170–195.

Codina, Esteve, Ricardo Garcia and Angel Martin. 2006. "New Algorithmic Alternatives for the O-D Matrix Adjustment Problem on Traffic Networks." *European Journal of Operational Research* 175(3):1484–1500.

Conejo, Antonio J., Enrique Castillo, Roberto Mínguez and Raquel García-Bertrand. 2006. *Decomposition Techniques in Mathematical Programming.* Berlin, Heidelberg: Springer.

Daxhelet, Olivier and Yves Smeers. 2001. “Variational Inequality Models of Restructured Electric Systems.” In: Ferris, Michael C., Olvi L. Mangasarian and Jong-Shi Pang (eds.), *Complementarity: Applications, Algorithms and Extensions*, Dordrecht, The Netherlands: Kluwer Academic Publishers.

Daxhelet, Olivier and Yves Smeers. 2007. “The EU Regulation on Cross-border Trade of Electricity: A Two-stage Equilibrium Model.” *European Journal of Operational Research* 181(3):1396–1412.

Day, Christopher J., Benjamin F. Hobbs and Jong-Shi Pang. 2002. “Oligopolistic Competition in Power Networks: A Conjectured Supply Function Approach.” *IEEE Transactions on Power Systems* 17(3):597–607.

De Miguel, Angel V., Michael P. Friedlander, Francisco J. Nogales and Stefan Scholtes. 2004. “An Interior-Point Method for MPECs Based on Strictly Feasible Relaxations.” Technical report, London Business School, London, UK.

De Miguel, Angel V., Michael P. Friedlander, Francisco J. Nogales and Stefan Scholtes. 2005. “A Two-Sided Relaxation Scheme for Mathematical Programs with Equilibrium Constraints.” *SIAM Journal on Optimization* 16(2):587–609.

Delarue, Erik, David Bekaert, Ronnie Belmans and William D’haeseleer. 2007. “Development of a Comprehensive Electricity Generation Simulation Model Using a Mixed Integer Programming Approach.” *International Journal of Electrical, Computer, and Systems Engineering* 1(2):92–97.

DEWI. 2006. “WindEnergy Study 2006 - Market Assessment of the Wind Energy Industry up to the Year 2014.” Technical report, German Wind Energy Institute (DEWI), Wilhelmshaven, Germany.

DEWI. 2007. “Windenergienutzung in Deutschland - Stand 31.12.2006.” *DEWI Magazin* 30.

DEWI, E.ON Netz, EWI, RWE Transportnetz Strom and VE Transmission. 2005. “Energiewirtschaftliche Planung für die Netzintegration von Windenergie in Deutschland an Land und Offshore bis zum Jahr 2020." Technical report, German Energy Agency (dena), Berlin, Germany. Internet: http://www.dena.de/de/themen/thema-kraftwerke/projekte/projekt/netzstudie-i/. Accessed: 31 January 2009.

Dietrich, Kristin, Florian Leuthold and Hannes Weigt. 2009. "Will the Market Get it Right? The Placing of New Power Plants in Germany." In: *Proceedings 6th Internationale Energiewirtschaftstagung an der TU Wien*, February 11-13, 2009, Vienna, Austria.

Dirkse, Steven P., Michael C. Ferris and Alexander Meerhaus. 2002. "Mathematical Programs with Equilibrium Constraints: Automatic Reformulation and Solution via Constraint Optimization." Technical report, NA-02/11, Oxford University Computing Laboratory, UK.

DWD. 2005. "Datenabgabe 439/05, Wind Speed Information about 8 Stations." German Weather Service (DWD).

Ehrenmann, Andreas. 2004. *Equlibrium Problems with Equlibrium Constraints and their Application to Electricity Markets*. Ph.D. thesis, Fitzwilliam College, University of Cambridge, Cambridge, UK.

Ehrenmann, Andreas and Yves Smeers. 2005. "Inefficiencies in European Congestion Management Proposals." *Utilities Policy* 13(2):135–152.

EMD. 2005. "DK Turbine Positions." EMD International, Aalborg, Denmark. Internet: http://www.emd.dk/EMD%20online/DK%20Turbine%20Positions/. Accessed: 06 February 2009.

EnBW. 2008. "Geschäftsbericht 2007." Energie Baden-Württemberg, Karlsruhe, Germany. Internet: http://www.enbw.com/content/de/investoren/_media/_pdf/gb_2007.pdf. Accessed: 19 January 2009.

EON. 2008. "Geschäftsbericht 2007." E.ON, Düsseldorf, Germany. Internet: http://www.eon.com/de/downloads/EON_Geschaeftsbericht_2007.pdf. Accessed: 19 January 2009.

Eurostat. 2004. "Versorgung, Umwandlung, Verbrauch - Elektrizität - Jährliche Daten." Statistical Office of the European Communities. Internet: http://epp.eurostat.ec.europa.eu. Accessed: 20 January 2007.

Eurostat. 2005. "National Accounts (Including GDP)." Statistical Office of the European Communities. Internet: http://epp.eurostat.ec.europa.eu. Accessed: 09 January 2006.

EWEA. 2005. "Large Scale Integration of Wind Energy in the European Power Supply: Analysis, Issues and Recommendations." The European Wind Energy Association, Brussels, Belgium. Internet: http://www.ewea.org/fileadmin/ewea_documents/documents/publications/grid/051215_Grid_report.pdf. Accessed: 09 January 2006.

EWEA. 2006. "Powering Change - EWEA 2006 Annual Report." Technical report, The European Wind Energy Association, Brussels, Belgium. Internet: http://www.ewea.org/fileadmin/ewea_documents/documents/publications/reports/ewea-report2006.pdf. Accessed: 06 February 2009.

EWEA. 2007. "Wind Power Installed in Europe by the End of 2006." The European Wind Energy Association, Brussels, Belgium.

Facchinei, Francisco and Jong-Shi Pang. 2003. *Finite-Dimensional Variational Inequalities and Complementarity Problems: Volumes I and II*. New York, NY: Springer.

Fischer, Reinhard and Friedrich Kießling. 1989. *Freileitungen - Planung, Berechnung, Ausführung*. Berlin, Heidelberg: Springer Verlag.

Fisher, Emily B., Richard P. O'Neill and Michael C. Ferris. 2008. "Optimal Transmission Switching." *IEEE Transactions on Power Systems* 23(3):1346–1355.

Fletcher, Roger and Sven Leyffer. 2002. "Numerical Experience with Solving MPECs as NLPs." Technical report, NA/210, Department of Mathematics, University of Dundee, Dundee, UK.

Fletcher, Roger and Sven Leyffer. 2004. "Solving Mathematical Programs with Complementarity Constraints as Nonlinear Programs." *Optimization Methods and Software* 19(1):15–40.

Fletcher, Roger, Sven Leyffer, Daniel Ralph and Stefan Scholtes. 2002. "Local Convergence of SQP Methods for Mathematical Programs with Equilibrium Constraints." Technical report, NA/209, Department of Mathematics, University of Dundee, Dundee, UK.

Floudas, Christodoulos A. 1995. *Nonlinear and Mixed-Integer Optimization Fundamentals and Applications*. New York, NY: Oxford University Press.

Fortuny-Amat, José and Bruce McCarl. 1981. "Representation and Economic Interpretation of a Two-Level Programming Problem." *The Journal of the Operational Research Society* 32(9):783–792.

Fudenberg, Drew and Jean Tirole. 1991. *Game Theory*. Cambridge, MA: The MIT Press.

Gabriel, Steven A. and Florian U. Leuthold. 2009. "Solving Discretely-Constrained MPEC Problems with Applications in Electric Power Markets." *Energy Economics* forthcoming.

Gabriel, Steven A., Yohan Shim, Antonio J. Conejo, Sebastian de la Torre and Raquel García-Bertrand. 2009. "A Benders Decomposition Method for Discretely-Constrained Mathematical Programs with Equilibrium Constraints with Applications in Energy." *Journal of the Operational Research Society* forthcoming.

Gampe, Uwe. 2004. "Umweltaspekte von Energieanlagen." Dresden University of Technology, Chair of Thermal Power Machinery and Plants.

Gao, Ziyou, Huijun Sun and Lian L. Shan. 2004. "A Continuous Equilibrium Network Design Model and Algorithm for Transit Systems." *Transportation Research Part B: Methodological* 38(3):235–250.

Gellert, Walter, Herbert Küstner, M. Hellwich and Herbert Kästner. 1975. *Mathematics at a Glance: A Compendium*. Leipzig: VEB Bibliographisches Institut.

Geoffrion, Arthur M. 1970a. "Elements of Large-Scale Mathematical Programming Part I: Concepts." *Management Science* 16(11):652–675.

Geoffrion, Arthur M. 1970b. "Elements of Large Scale Mathematical Programming Part II: Synthesis of Algorithms and Bibliography." *Management Science* 16(11):676–691.

Geoffrion, Arthur M. 1972. "Generalized Benders Decomposition." *Journal of Optimization Theory and Applications* 10(4):237–260.

Green, Richard. 2007. "Nodal Pricing of Electricity: How Much Does It Cost to Get It Wrong?" *Journal of Regulatory Economics* 31(2):125–149.

Green, Richard J. and David M. Newbery. 1992. "Competition in the British Electricty Spot Market." *Journal of Political Economy* 100(5):929–953.

Greenpeace International and EWEA. 2005. "Wind Force 12 – A Blueprint to Achieve 12% of the World's Electricity from Wind Power by 2020." Technical report, Global Wind Energy Council, Brussels, Belgium. Internet: http://www.gwec.net/fileadmin/documents/Publications/wf12-2005.pdf. Accessed: 06 February 2009.

Haidvogl, Herbert. 2002. “Netzanbindung von Windenergieerzeugungsanlagen.” *Elektrizitätswirtschaft* 20–21.

Hau, Erich. 2003. *Windkraftanlagen, Grundlagen, Technik, Einsatz, Wirtschaftlichkeit.* Berlin: Springer.

Hedman, Kory W., Richard P. O’Neill, Emily Bartholomew Fisher and Shmuel S. Oren. 2008. “Optimal Transmission Switching-Sensitivity Analysis and Extensions.” *IEEE Transactions on Power Systems* 23(3):1469–1479.

Hejazi, S. Reza, Azizollah Memariani, G. Jahanshaloo and Mohammad M. Sepehri. 2002. “Linear Bilevel Programming Solution by Genetic Algorithm.” *Computers and Operations Research* 29:1913–1925.

Hillier, Frederick S. and Gerald J. Lieberman. 1986. *Introduction to Operations Research.* Oakland, CA: Holden-Day, 4th edition.

Hirschhausen, Christian von, Hannes Weigt and Georg Zachmann. 2007. “Price Formation and Market Power in Germany’s Wholesale Electricity Markets.” Dresden University of Technology, Chair of Energy Economics and Public Sector Management, Working Paper WP-EM-15b. Internet: http://www.tu-dresden.de/wwbwleeg/publications/wp_em_15b_hirschhausen_weigt_zachmann_marktet_power_germany.pdf. Accessed: 02 January 2009.

Hobbs, Benjamin F. 1986. “Network Models of Spatial Oligopoly with an Application to Deregulation of Electricity Generation.” *Operations Research* 34(3):395–409.

Hobbs, Benjamin F. 2001. “Linear Complementarity Models of Nash-Cournot Competition in Bilateral and POOLCO Power Markets.” *IEEE Transactions on Power Systems* 16(2):194–202.

Hobbs, Benjamin F., Carolyn B. Metzler and Jong-Shi Pang. 2000. “Strategic Gaming Analysis for Electric Power Systems: An MPEC Approach.” *IEEE Transactions on Power Systems* 15(2):638–645.

Hobbs, Benjamin F. and Sushil K. Nelson. 1992. “A Nonlinear Bilevel Model for Analysis of Electric Utility Demand-Side Planning Issues.” *Annals of Operations Research* 34(1-4):255–274.

Hobbs, Benjamin J. and Udi Helman. 2004. "Complementarity-Based Equilibrium Modeling for Electric Power Markets." In: Bunn, Derek W. (ed.), *Modelling Prices in Competitive Electricty Markets*, Chichester, West Sussex, England: John Wiley & Sons.

Hogan, William W. 1992. "Contract Networks for Electric Power Transmission." *Journal of Regulatory Economics* 4(3):211–242.

Hogan, William W. 1993. "Markets in Real Electric Networks Require Reactive Prices." *The Energy Journal* 14(3):171–201.

Hogan, William W. 1997. "Computable Equilibrium Models and the Restructuring of the European Electricity and Gas Markets." *The Energy Journal* 18(4):107–142.

Hogan, William W. 2002. "Electricty Market Restructuring: Reforms of Reforms." *Journal of Regulatory Economics* 21(1):103–132.

Hogan, William W. 2003. "Transmission Market Design." Harvard University, KSG Working Paper No. RWP03-040. Internet: http://ssrn.com/abstract=453483m. Accessed: 01 April 2007.

Hogan, William W. 2007. "Market-based Transmission Investments and Competitive Electricty Markets." In: Kleit, Andrew N. (ed.), *Electric Choices: Deregulation and the Future of Electric Power*, Plymouth, UK: Rowman & Littlefield Publishers.

Hogan, William W., Juan Rosellón and Ingo Vogelsang. 2007. "Toward a Combined Merchant-Regulatory Mechanism for Electricity Transmission Expansion." In: *Proceedings 9th IAEE European Energy Conference*, June 10-13, 2007, Florence, Italy.

Hondebrink, J. P., B. A. Wilbrink, R. de Bruijne and J. L. 't Hooft. 2004. "Connect 6000 MW." Technical report, Ministry of Economic Affairs, The Netherlands. Internet: http://www.senternovem.nl/offshorewindenergy/background_information/integration_electricity_grid/final_report_connect_6000_mw.asp. Accessed: 31 January 2009.

Hu, Xinmin and Daniel Ralph. 2004. "Convergence of a Penalty Method for Mathematical Programming with Complementarity Constraints." *Journal of Optimization Theory and Applications* 123(2):365–390.

Hu, Xinmin and Daniel Ralph. 2007. "Using EPECs to Model Bilevel Games in Restructured Electricity Markets with Locational Prices." *Operations Research* 55(5):809–827.

IDAE. 2005. "Plan de Energías Renovables." Technical report, Institute for Diversification and Saving of Energy, Madrid, Spain. Internet: http://www.idae.es/uploads/documentos/documentos_PER_2005-2010_8_de_gosto-2005_Completo.(modificacionpag_63)_Copia_2_301254a0.pdf. Accessed: 01 February 2009.

IEA. 2007. *World Energy Outlook 2006*. Paris: OECD/IEA.

IGW. 2005. "Landkarte." Austrian Wind Energy Association (IG Windkraft), St. Pölten, Austria. Internet: http://igwindkraft.at/index.php?xmlval_ID_KEY[0]=1055. Accessed: 06 February 2009.

ISET/IWES. 2002. "Räumliche Verteilung der Installierten Nennleistung (Planflächen)." Institue of Solar Energy Supply Technology, Kassel, Germany. Internet: http://reisi.iset.uni-kassel.de/pls/w3reisidad/www_reisi_page.show_menu?p_name=121017&p_lang=ger. Accessed: 09 January 2006.

Jamasb, Tooraj and Michael Pollitt. 2005. "Electricty Market Reform in the European Union: Review and Progress Toward Liberalization & Integration." *The Energy Journal* Special Issue:11–41.

Jeroslow, Robert G. 1985. "The Polynomial Hierarchy and a Simple Model for Competitive Analysis." *Mathematical Programming* 32(2):146–164.

Jeske, Till. 2009. *Economic Aspects of Wind Energy and Integration of Wind in the Electricity System*. Ph.D. thesis, Faculty of Business and Economics, Dresden University of Technology, Dresden, Germany.

Joskow, Paul and Jean Tirole. 2000. "Transmission Rights and Market Power on Electric Power Networks." *RAND Journal of Economics* 31(3):450–487.

Joskow, Paul and Jean Tirole. 2005. "Merchant Transmission Investment." *The Journal of Industrial Economics* 53(2):233–264.

Joskow, Paul L. 2005. "Patterns of Transmission Investment." MIT, Joint DAE-CMI Working Paper 0527. Internet: http://www.econ.cam.ac.uk/electricity/publications/wp/ep78.pdf. Accessed: 08 August 2006.

Joskow, Paul L. 2008. "Lessons Learned from Electricity Market Liberalization." *The Energy Journal* 29(Special Issue 2):9–42.

Karlof, John K. and Wei Wang. 1996. "Bilevel Programming Applied to the Flow Shop Scheduling Problem." *Computers and Operations Research* 23(5):443–451.

Kehlhofer, Rolf, Norbert Kunze, Jochen Lehmann and Karl-Heinz Schüller. 1984. "Gasturbinenkraftwerke, Kombikraftwerke, Heizkraftwerke und Industriekraftwerke." In: Bohn, Thomas (ed.), *Handbuchreihe Energie Band 7*, Cologne, Germany: Technischer Verlag Reschl/Verlag TÜV Rheinland.

Kumbaroglu, Gürkan, Reinhard Madlener and Mustafa Demirel. 2008. "A Real Options Evaluation Model for the Diffusion Prospects of New Renewable Power Generation Technologies." *Energy Economics* 30(4):1882–1908.

Labbé, Martine, Patrice Marcotte and Gilles Savard. 1998. "A Bilevel Model of Taxation and Its Application to Optimal Highway Pricing." *Management Science* 44(2):1608–1622.

Léautier, Thomas-Olivier. 2001. "Transmission Constraints and Imperfect Markets for Power." *Journal of Regulatory Economics* 19(1):27–54.

Leuthold, Florian, Till Jeske, Hannes Weigt and Christian von Hirschhausen. 2009. "When the Wind Blows Over Europe: A Simulation Analysis and the Impact of Grid Extensions." Dresden University of Technology, Chair of Energy Economics and Public Sector Management, Working Paper WP-EM-31. Internet: http://www.tu-dresden.de/wwbwleeg/publications/wp_em_31_Leuthold_etal_EU_wind.pdf. Accessed: 17 February 2009.

Leuthold, Florian, Ina Rumiantseva, Hannes Weigt, Till Jeske and Christian von Hirschhausen. 2005. "Nodal Pricing in the German Electricity Sector – A Welfare Economics Analysis, with Particular Reference to Implementing Offshore Wind Capacities." Dresden University of Technology, Chair of Energy Economics and Public Sector Management, Working Paper WP-EM-08a. Internet: http://www.tu-dresden.de/wwbwleeg/publications/wp_em_08a_leuthold_rumiantseva_weigt_et%20al_nodal_%20pricing_germany.pdf. Accessed: 26 November 2007.

Leuthold, Florian, Hannes Weigt and Christian von Hirschhausen. 2008a. "Efficient Pricing for European Electricity Networks - The Theory of Nodal Pricing Applied to Feeding-In Wind in Germany." *Utilities Policy* 16(4):284–291.

Leuthold, Florian, Hannes Weigt and Christian von Hirschhausen. 2008b. "ELMOD - A Model of the European Electricity Market." Dresden University of Technology, Chair of Energy Economics and Public Sector Management, Working Paper WP-EM-00. Internet: http://www.tu-dresden.de/wwbwleeg/publications/wp_em_00_ELMOD.pdf. Accessed: 04 May 2009.

Leyffer, Sven. 2003. "Mathematical Programs with Complementarity Constraints." *SIAG/OPT Views-and-News* 14(1):15–18.

Leyffer, Sven. 2005. "The Penalty Interior Point Method Fails to Converge." *Optimization Methods and Software* 20(4-5):559–568.

Leyffer, Sven, Gabriel Lopez-Calva and Jorge Nocedal. 2006. "Interior Methods for Mathematical Programs with Complementarity Constraints." *SIAM Journal on Optimization* 17(1).

Liu, Xinwei and Jie Sun. 2004. "Generalized Stationary Points and an Interior Point Method for Mathematical Programs with Equilibrium Constraints." *Mathematical Programming* 101(1):231–261.

Luo, Zhi-Quan, Jong-Shi Pang and Daniel Ralph. 1996. *Mathematical Programs with Equilibrium Constraints*. Cambridge, UK: Cambridge University Press.

Ma, Xingwang, David I. Sun and Kwok W. Cheung. 2003. "Evolution Toward Standardized Market Design." *IEEE Transactions on Power Systems* 18(2):460–469.

Machowski, Jan, Janusz W. Bialek and James R. Bumby. 2008. *Power System Dynamics*. Chichester, West Sussex, UK: John Wiley & Sons, 2nd edition.

Marcotte, Patrice, Gilles Savard and Daoli L. Zhu. 2001. "A Trust Region Algorithm for Nonlinear Bilevel Programming." *Operations Research Letters* 29(4):171–179.

Metzler, Carolyn, Benjamin F. Hobbs and Jong-Shi Pang. 2003. "Nash-Cournot Equilibria in Power Markets on a Linearized DC Network with Arbitrage: Formulations and Properties." *Networks and Spatial Economics* 3(2):123–150.

Müller, Leonhard. 2001. *Handbuch der Elektrizitätswirtschaft, Technische, Wirtschaftliche und Rechtliche Grundlagen*. Berlin: Springer, 2nd edition.

Moore, James T. and Jonathan F. Bard. 1990. "The Mixed Integer Linear Bilevel Programming Problem." *Operations Research* 38(5):911–921.

Murphy, Frederic H. and Yves Smeers. 2005. "Generation Capacity Expansion in Imperfectly Competitive Restructured Electricity Markets." *Operations Research* 53(4):646–661.

Murthy, D. N. Prabhakar and Ervin Y. Rodin. 1987. "A Comparative Evaluation of Books on Mathematical Modelling." *Mathematical Modelling* 9(1):17–28.

Neuhoff, Karsten, Julian Barquin, Maroeska G. Boots, Andreas Ehrenmann, Benjamin F. Hobbs et al. 2005. "Network-Constrained Cournot Models of Liberalized Electricity Markets: The Devil is in the Details." *Energy Economics* 27(3):495–525.

Neumann, Klaus and Martin Morlock. 1993. *Operations Research*. München, Wien: Hanser.

Nikogosian, Vigen and Tobias Veith. 2009. "Strategic Pricing and Competition in Retail Electricty Markets." In: *Proceedings 6th Internationale Energiewirtschaftstagung an der TU Wien*, February 11-13, 2009, Vienna, Austria.

Nishizaki, Ichiro, Masatoshi Sakawa and Tetsuya Kan. 2003. "Computational Methods through Genetic Algorithms for Obtaining Stackelberg Solutions to Two-Level Integer Programming Problems." *Electronics and Communications in Japan (Part III: Fundamental Electronic Science)* 86(6):1251–1257.

O'Neill, Richard P., Emily Bartholomew Fisher, Benjamin F. Hobbs and Ross Baldick. 2008. "Towards a Complete Real-time Electricity Market Design." *Journal of Regulatory Economics* 34(3):220–250.

Outrata, Jiri V., Michal Kocvara and Jochem Zowe. 1998. *Non-Smooth Approach to Optimization Problems with Equilibriuim Constraints*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Overbye, Thomas J., Xu Cheng and Yan Sun. 2004. "A Comparison of the AC and DC Power Flow Models for LMP Calculations." In: *Proceedings of the 37th Hawaii International Conference on System Sciences.*

Pokharel, Shaligram and Kumaraswamy Ponnambalam. 1997. "Investment Planning for Electricity Generation Expansion." *International Journal of Energy Research* 21(2):185–194.

Pérez-Arriaga, Ignacio and Luis Olmos. 2006. "Compatibility of Investment Signals in Distribution, Transmission, and Generation." In: Lévêque, Francois (ed.), *Competitive Electricity Markets and Sustainability*, Cheltenham, UK: Edward Elgar.

PSE. 2003. "Study of Integration Possibilities of Wind Energy with the Polish Power Grid." In: *Proceedings European Wind Energy Conference 2003*, June 16-19, 2003, Madrid, Spain.

PWEA. 2006. "Wind Farms in Poland." Polish Wind Energy Association, Szczecin, Poland. Internet: http://www.pwea.pl/elektrownie_wiatrowe_w_polsce.htm. Accessed: 12 January 2007.

Raghunathan, Arvind and Lorenz T. Biegler. 2002. "Barrier Methods for Mathematical Programs with Complementarity Constraints (MPCCs)." Technical report, Carnegie Mellon University, Department of Chemical Engineering, Pittsburgh, PA, USA.

Raghunathan, Arvind U. and Lorenz T. Biegler. 2003. "Mathematical Programs with Equilibrium Constraints (MPECs) in Process Engineering." *Computers and Chemical Engineering* 27(10):1381–1392.

Ralph, Daniel and Yves Smeers. 2006. "EPECs as Models for Electricty Markets." In: *Proceedings Power Systems Conference and Exposition (PSCE)*, October 29 – November 1, 2006, Atlanta, GA, USA.

Ralph, Daniel and Stephen J. Wright. 2004. "Some Properties of Regularization and Penalization Schemes for MPECs." *Optimizations Methods and Software* 19(5):527–556.

Ravindran, A. Ravi. 2007. *Operations Research and Management Science Handbook*. Boca Raton, FL: CRC Press.

Rious, Vincent, Philippe Dessante and Yannick Perez. 2008. "Is Combination of Nodal Pricing and Average Participation Tariff the Best Solution to Coordinate the Location of Power Plants with Lumpy Transmission Investments?" In: *Proceedings 5th Conference on "The Economics of Energy Markets"*, June 20-21, 2008, Toulouse, France.

Roques, Fabien, David M. Newbery and William J. Nuttall. 2008. "Fuel Mix Diversification Incentives in Liberalized Electricity Markets: A Mean–Variance Portfolio Theory Approach." *Energy Economics* 30(4):1831–1849.

Roques, Fabien A. 2008. "Technology Choices for New Entrants in Liberalized Markets: The Value of Operating Flexibility and Contractual Arrangements." *Utilities Policy* 16(4):245–253.

Rosellón, Juan and Hannes Weigt. 2008. "A Dynamic Incentive Mechanism for Transmission Expansion in Electricity Networks – Theory, Modeling and Application." Dresden University of Technology, Chair of Energy Economics and Public Sector Management, Working Paper WP-EM-26. Internet: http://www.tu-dresden.de/wwbwleeg/publications/wp_em_26_Rosellon_Weigt_Transmission_Expansion.pdf. Accessed: 16 March 2009.

RWE. 2008. "Geschäftsbericht 2007." RWE, Essen, Germany. Internet: http://www.rwe.com/web/cms/contentblob/114616/data/2279/blob.pdf. Accessed: 19 January 2009.

Salazar, Harold, Chen-Ching Liu and Ron F. Chu. 2007. "Decision Analysis of Merchant Transmission Investment by Perpetual Options Theory." *IEEE Transactions on Power Systems* 22(3):1194–1201.

Saphores, Jean-Daniel, Eric Gravel and Jean-Thomas Bernard. 2004. "Regulation and Investment under Uncertainty: An Application to Power Grid Interconnection." *Journal of Regulatory Economics* 25(2):169–186.

Sauma, Enzo E. and Shmuel S. Oren. 2006. "Proactive Planning and Valuation of Transmission Investments in Restructured Electricity Markets." *Journal of Regulatory Economics* 30(3):261–290.

Sauma, Enzo E. and Shmuel S. Oren. 2007. "Economic Criteria for Planning Transmission Investment in Restructured Electricity Markets." *IEEE Transactions on Power Systems* 22(4):1394–1405.

Scaparra, Maria L. and Richard L. Church. 2008. "A Bilevel Mixed-Integer Program for Critical Infrastructure Protection Planning." *Computers and Operations Research* 35(6):1905–1923.

Scholtes, Stefan. 2001. "Convergence Properties of Regularization Schemes for Mathematical Programs with Complementarity Constraints." *SIAM Journal on Optimization* 11(4):918–936.

Schröter, Jochen. 2004. *Auswirkungen des Europäischen Emissionshandelssystems auf den Kraftwerkseinsatz in Deutschland.* Master's thesis, Berlin University of Technology, Institute of Power Engineering, internet: http://basis.gruene.de/bag.energie/papiere/eeg_diplarbeit_schroeter_lang.pdf. Accessed: 01 September 2005.

Schweppe, Fred C., Michael C. Caramanis, Richard D. Tabors and Roger E. Bohn. 1988. *Spot Pricing of Electricity.* Boston: Kluwer.

Shahidehpour, Mohammad, Hatim Yamin and Zuyi Li. 2002. *Market Operations in Electric Power Systems.* New York: John Wiley & Sons.

Sioshansi, Fereidoon P. 2006. "Electricty Market Refom: What Has the Experience Thaught Us so Far." *Utilities Policy* 14(2):63–75.

Sioshansi, Ramteen, Richard P. O'Neill and Shmuel S. Oren. 2008. "Economic Consequences of Alternative Solution Methods for Centralized Unit Commitment in Day-Ahead Electricity Markets." *IEEE Transactions on Power Systems* 23(2):344–352.

Smeers, Yves. 1997. "Computable Equilibrium Models and the Restructuring of the European Electricity and Gas Markets." *The Energy Journal* 18(4):1–31.

Smeers, Yves. 2003a. "Market Incompleteness in Regional Electricty Transmission. Part I: The Forward Market." *Networks and Spatial Economics* 3(2):151–174.

Smeers, Yves. 2003b. "Market Incompleteness in Regional Electricty Transmission. Part II: The Forward and Real Time Market." *Networks and Spatial Economics* 3(2):175–196.

Smeers, Yves. 2006. "Long-term Locational Prices and Investment Incentives in the Transmission of Electricity." In: Lévêque, Francois (ed.), *Competitive Electricity Markets and Sustainability*, Cheltenham, UK: Edward Elgar.

Stigler, Heinz and Christian Todem. 2005. "Optimization of the Austrian Electricity Sector (Control Zone of VERBUND APG) under the Constraints of Network Capacities by Nodal Pricing." *Central European Journal of Operations Research* 13(2):105–125.

Stoft, Steven. 2002. *Power System Economics*. Piscataway, NJ: IEEE Press.

Takriti, Samer, Benedikt Krasenbrink and Lilian S.-Y. Wu. 1998. "Incorporating Fuel Constraints and Electricity Spot Prices into the Stochastic Unit Commitment Problem." *Operations Research* 48(3):268–280.

Tirole, Jean. 1988. *The Theory of Industrial Organization*. Cambridge, MA: The MIT Press.

Todem, Christian. 2004. *Methoden und Instrumente zur Gesamtsystemischen Analyse und Optimierung Konkreter Problemstellungen im Liberalisierten Elektrizitätsmarkt*. Ph.D. thesis, Department of Electricity Economics and Energy Innovation, Graz University of Technology, Graz, Austria.

UCTE. 2004. "Interconnected Network of UCTE." Dortmund, Abel Druck.

UCTE. 2006. "Consumption Data." Internet: http://www.ucte.org/statistics/onlinedata/consumption/e_default.asp. Accessed: 19 January 2007.

VDEW. 1999. *Lastprofile*. Frankfurt (Main): VDEW.

VDI. 2000. *Energietechnische Arbeitsmappe*. Berlin, Heidelberg, New York: Springer.

Ventosa, Mariano, Álvaro Baíllo, Andrés Ramos and Michel Rivier. 2005. "Electricity Market Modeling Trends." *Energy Policy* 33(7):897–913.

Verseille, J. 2003. "Growth and Grids – Panel Discussion on Issues of Grid Extension, Supply Predictability and Power Quality." In: *Proceedings European Wind Energy Conference 2003*, June 16-19, 2003, Spain, Madrid.

VGE. 2005. *Jahrbuch der Europäischen Energie- und Rohstoffwirtschaft 2006*. Essen: Verlag Glückauf.

Vicente, Luis N. and Paul H. Calamai. 1994. "Bilevel and Multilevel Programming: A Bibliography Review." *Journal on Global Optimization* 5(3):291–306.

Vogelsang, Ingo. 2001. "Price Regulation for Independent Transmission Companies." *Journal of Regulatory Economics* 20(2):141–165.

Voorspools, Kris R. and William D. D'haeseleer. 2003. "Long-term Unit Commitment Optimisation for Large Power Systems: Unit Decommitment versus Advanced Priority Listing." *Applied Energy* 76:157–167.

Weidlich, Anke and Daniel Veit. 2008. "A Critical Survey of Agent-based Wholesale Electricity Market Models." *Energy Economics* 30(4):1728–1759.

Weigt, Hannes. 2006. "A Time-Variant Welfare Economic Analysis of a Nodal Pricing Mechanism in Germany." Dresden University of Technology, Chair of Energy Economics and Public Sector Management, Working Paper WP-EM-11. Internet: http://www.tu-dresden.de/wwbwleeg/publications/wp_em_11_weigt_nodal_%20pricing_germany_time.pdf. Accessed: 17 February 2009.

Weigt, Hannes, Karen Freund and Till Jeske. 2006. "Nodal Pricing of the European Electricity Grid - A Welfare Economic Analysis for Feeding-in Offshore Wind Electricity." Dresden University of Technology, Chair of Energy Economics and Public Sector Management, Working Paper WP-EM-10. Internet: http://www.tu-dresden.de/wwbwleeg/publications/wp_em_10_freund_weigt_jeske_nodal_%20pricing_nw_europe.pdf. Accessed: 26 November 2007.

Wen, Ue-Pyng and A. D. Huang. 1996. "A Simple Tabu Search Method to Solve the Mixed-Integer Linear Bilevel Programming Problem." *European Journal of Operational Research* 88:563–571.

Wickart, Marcel and Reinhard Madlener. 2007. "Optimal Technology Choice and Investment Timing: A Stochastic Model of Industrial Cogeneration vs. Heat-only Production." *Energy Economics* 29:934–952.

Wierzbicki, Andrzej P. 2007. "Modelling as a Way of Organising Knowledge." *European Journal of Operational Research* 176(1):610–635.

Willems, Bert, Ina Rumiantseva and Hannes Weigt. 2009. "Cournot versus Supply Functions: What does the Data tell us?" *Energy Economics* 31(1):38–47.

Williams, H. Paul. 1999. *Model Building in Mathematical Programming.* New York: John Wiley & Sons, 4th edition.

Winston, Wayne L. 1994. *Operations Research: Applications and Algorithms*. Belmont, CA: Duxbury Press.

Wood, Allen J. and Bruce F. Wollenberg. 1996. *Power Generation Operation and Control*. New York, NY: John Wiley & Sons, 2nd edition.

Woyte, Achim, Paul Gardner and Helen Snodin. 2005. "Concerted Action for Offshore Wind Energy Deployment (COD). Work Package 8: Grid Issues." Technical report, European Commission. Internet: http://www.3e.be/library/XDVW648E.pdf. Accessed: 01 February 2009.

Woyte, Achim, Paul Gardner and Helen Snodin. 2007. "European Concerted Action on Offshore Wind Energy Deployment: Inventory and Analysis of Power Transmission Barriers in Eight Member States." *Wind Energy* 10(4):357–378.

WSH. 2005. "All About Wind Energy." Wind Service Holland, Woudsend, The Netherlands. Internet: http://home.kpn.nl/windsh/english.html. Accessed: 06 February 2009.

Zoettl, Gregor. 2008. "Investment Decisions in Liberalized Electricity Markets: A Framework of Peak Load Pricing with Strategic Firms." In: *Proceedings 5th Conference on "The Economics of Energy Markets"*, June 20-21, 2008, Toulouse, France.

Appendix A

Mathematical Proofs and Remarks (Chapter 5)

A.1 Result that Shows that $q^v_{nsu,i} \in [0,1]$ is Valid

First it is shown that the variable $q^v_{nsu,i}$ need not be specified as binary but rather constrained to be in the range $[0, 1]$. The result is presented in a slightly more general setting.[1]

Theorem A.1 *The solution set to (A.1) and (A.2) are the same where*

$$\begin{aligned}
z &\leq x && \text{(A.1a)}\\
z &\leq y && \text{(A.1b)}\\
x + y - 1 &\leq z && \text{(A.1c)}\\
x, y &\in \{0, 1\} && \text{(A.1d)}\\
z &\in \{0, 1\} && \text{(A.1e)}
\end{aligned}$$

and

$$\begin{aligned}
z &\leq x && \text{(A.2a)}\\
z &\leq y && \text{(A.2b)}\\
x + y - 1 &\leq z && \text{(A.2c)}\\
x, y &\in \{0, 1\} && \text{(A.2d)}\\
z &\in [0, 1] && \text{(A.2e)}
\end{aligned}$$

[1] This section is based on findings presented in the research paper Gabriel and Leuthold (2009).

Proof. Let $(\bar{x}, \bar{y}, \bar{z})$ be a solution to (A.1). Since $z \in \{0, 1\} \Rightarrow z \in [0, 1]$ and (A.1a)-(A.1d) exactly match (A.2a)-(A.2d) this means that $(\bar{x}, \bar{y}, \bar{z})$ is a solution to (A.2). Now let $(\hat{x}, \hat{y}, \hat{z})$ be a solution to (A.2). If $z \in \{0, 1\}$ then we are done. Thus, assume that $z \in (0, 1)$. But by (A.1a) and (A.1b) since $0 < z\ < 1 \Rightarrow x = y = 1$. Then by (A.1c) $z \geq 1$ which is a contradiction. ■

A.2 Formal Determination of the Disjunctive Constants

Having fixed values for the the upper-level vector x, the lower-level linear programming subproblem from Chapter but with upper bounds y^{up} on the y variables added is of the form[2]:

$$\begin{array}{rrcl} & \min e^T y & & \\ s.t. & My & \geq & k - Nx \\ & y & \leq & y^{up} \\ & y & \geq & 0 \end{array} \tag{A.4}$$

The necessary and sufficient KKT conditions are to find vectors (y, z, w) such that

$$0 \leq e - M^T z + Iw \perp y \geq 0 \tag{A.5}$$

$$0 \leq My + Nx - k \perp z \geq 0 \tag{A.6}$$

$$0 \leq y^{up} - y \perp w \geq 0 \tag{A.7}$$

First note that all variables are nonnegative so that one needs only consider upper bounds on y, z, w. Also, it is assumed that the upper level variable x is bounded above in the upper-level problem, a reasonable assumption given that this variable will relate to a physical quantity. Also, by design, we have $0 \leq y \leq y^{up}$ so that y is bounded from above. What about bounds on the "dual" variables z and w? First, suppose that there exists a positive constant C^w such that $0 \leq w_i \leq C^w, \forall i$. Depending on the application, the existence of positive constants C^w and C^y (which can be taken greater than or equal to y^{up}) may be a reasonable assumption. For example, bounds on the primal variables y are often employed if these variables relate to real, physical quantities (e.g., power generation). Bounds on the dual variables w are also reasonable in that they relate to the shadow price of capacity constraints on the y variables. Typically these prices correspond to the marginal cost of one more unit of capacity which itself should be bounded due to physical considerations. Having made the assumption of bounds on the y and w variables, the next result shows a reasonable condition to

[2]This section is based on findings presented in the research paper Gabriel and Leuthold (2009).

generate a bound C^z on the other dual variables z. This condition states that there must be at least one column of the matrix M with all positive entries.

Theorem A.2 *Suppose that there exists a column j of M such that $M_j^{\min} = \min_i \{M_{ij}\} > 0$. Then,*

$$z \le \min_{j:M_j^{\min}>0} \left\{ \left(\frac{e_j + C^w}{M_j^{\min}} \right) \right\} = C^z, \ \textit{where } C^z > 0.$$

Proof. From (A.5)one can see that for 1 the vector of all ones,

$$0 \le e - M^T z + Iw \Rightarrow M^T z \le e + Iw \le e + C^w 1$$

or

$$M_j^{\min} \sum_i z_i \le \sum_i M_{ij} z_i \le e_j + C^w, \forall j$$

where $M_j^{\min}$ is the minimum value of M_{ij} for column j. Now if there is a column j where $M_j^{\min} > 0$, one can see that

$$\sum_i z_i \le \frac{e_j + C^w}{M_j^{\min}}$$

Moreover, this has to hold for each column j where $M_j^{\min} > 0$ hence

$$\sum_i z_i \le \min_{j:M_j^{\min}>0} \left\{ \left(\frac{e_j + C^w}{M_j^{\min}} \right) \right\}$$

If $e_j < 0$ then, without loss of generality, C^w can be taken sufficiently large so that $\min_{j:M_j^{\min}>0} \left\{ \left(\frac{e_j+C^w}{M_j^{\min}} \right) \right\} = C^z > 0 \Rightarrow z_j \le C^z$ for all j in light of the fact that $z \ge 0$. ■

Remark The condition that there exists a column j of M such that $M_j^{\min} = \min_i \{M_{ij}\} > 0$ may be satisfied for a large class of matrices.

Remark The other condition in (A.6) which involves the vector z namely, $(My + Nx - k)^T z = 0$ does not impose any additional bounds on z.

Remark Getting an appropriate value for the disjunctive constraints constant is then straightforward given the above result. For example in

$$0 \leq e - M^T z + Iw \perp y \geq 0$$

one can see that for row i we have

$$\begin{aligned} 0 &\leq e_i - \sum_j (M^T)_{ij} z_j + w_i \\ &\leq e_i + \sum_j \max_i \left\{ (M^T)_{ij} \right\} C^z + C^w \leq K_i \end{aligned}$$

So one can take any value K_i such that the above inequality holds. Then a valid value for the disjunctive constraints constant is $K = \max\{\max_i \{K_i\}, C^y\} > 0$. A similar line of reasoning applies for the other complementarity constraints $0 \leq My + Nx - k \perp z \geq 0$ in light of the fact that both y and x are bounded variables.

Appendix B

Input Data and Further Model Results of the Numerical Examples (Chapter 5)

In order to improve readability in the main part some of the input data and result tables of Chapter 5 were shifted this appendix.[1]

	n1	**n2**	**n3**
$\mathbf{a}_n$	1	1	10
$\mathbf{b}_n$	1	1	1

Table B.1: Demand structure in the three-node network

[1] This section is based on findings presented in the research paper Gabriel and Leuthold (2009).

	Test1	**Test2**	**Test3**	**Test4**	**Test5**
$\mathbf{c}_{n1su1}[€/MWh]$	2	2	2	2	2
$\mathbf{c}_{n2su2}[€/MWh]$	1	1	1	1	1
$\mathbf{c}_{n2j1u3}[€/MWh]$	3	7	8	9	3
$\bar{\mathbf{g}}_{n1su1}[MW]$	10	10	10	10	10
$\bar{\mathbf{g}}_{n2su2}[MW]$	10	10	10	10	10
$\bar{\mathbf{g}}_{n2j1u3}[MW]$	10	10	10	10	10
$\mathbf{lc}_{l1}[MW]$	10	10	10	10	10
$\mathbf{lc}_{l2}[MW]$	10	10	10	10	4
$\mathbf{lc}_{l3}[MW]$	10	10	10	10	10

Table B.2: Parameters of three-node network

	Tests 1-5
Problem sizes	comp $\left\{ \begin{array}{c} \text{52 continuous variabes} \\ \text{0 discrete variables} \end{array} \right\}$ strat $\left\{ \begin{array}{c} \text{2801 continuous variables} \\ \text{939 discrete variables} \end{array} \right\}$

Table B.3: Model statistics for three-node network

	Line capacity [MW]	**Reactance [Ω]**
l1	2970	12
l2	1840	69
l3	1840	43
l4	900	28
l5	1330	25
l6	1840	33
l7	1840	50
l8	1840	29
l9	640	61
l10	640	42
l11	940	34
l12	1840	31
l13	900	55
l14	1210	45
l15	270	156
l16	2760	22
l17	1840	27
l18	3330	38
l19	1280	11
l20	3330	41
l21	∞	46
l22	∞	46
l23	∞	46
l24	∞	46
l25	∞	46
l26	∞	46

Table B.4: Parameters of fifteen-node Western European network

		Test1	Test2	Test3	Test4	Test5
Computation times	comp	3 s	1 s	1 s	1 s	1 s
	strat	15 s	26 s	31 s	33 s	17 s

Table B.5: Computational issues for the three-node network

	n1	**n2**	**n3**	**n4**	**n5**	**n6**	**n7**
$\mathbf{a}_n$	130	130	130	130	130	130	130
$\mathbf{b}_n$	0.002	0.002	0.033	0.017	0.017	0.050	0.033

Table B.6: Demand structure in the fifteen-node network for Western Europe

	Nuclear u1	**Lignite u2**	**Coal u3**	**CCGT u4**	**Gas u5**	**Oil u6**	**Hydro u7**	**Pump u8**
Marginal Cost [€/**MWh**]	10	20	22	30	45	60	0	35

Table B.7: Marginal costs per unit type in the fifteen-node network for Western Europe

	u1	u2	u3	u4	u5	u6	u7	u8
n1.EON	7628	1125	6720	0	3536	2729	124	380
n1.RWE	6379	11410	3334	2091	1978	765	0	150
n1.ENBW	4302	0	2683	915	132	319	0	0
n1.VAT	2031	7785	1627	415	554	1336	0	1544
n1.FriGER	0	833	14600	4337	4434	368	1147	3030
n2.ENBW	0	0	0	0	0	0	150	0
n2.EDF	58288	0	11685	0	124	11130	11552	3408
n2.FriFR	0	580	4137	0	0	0	2679	0
n3.EBEL	2713	0	2474	350	460	373	0	1164
n3.FriBE	0	0	0	0	115	187	0	144
n4.EON	0	0	1040	0	828	0	0	0
n4.EBEL	0	0	602	0	0	0	0	0
n4.ESSENT	449	0	1696	0	460	0	0	0
n4.NUON	0	0	630	249	2506	0	0	0
n4.FriNL	0	0	0	0	1078	111	0	0
n5.ESSENT	0	0	0	0	1510	0	0	0
n5.FriNL	0	0	253	0	0	0	0	0
n6.EBEL	2618	0	1134	460	1306	1675	0	0
n6.FriBE	0	0	0	350	126	190	0	0
n7.EBEL	0	0	0	1705	2340	0	0	0
n7.FriNL	0	0	0	0	428	0	0	0

Table B.8: Generation capacities of fifteen-node Western European network

		Test_Ebel	Test_EDF	Test_EON	Test_RWE
Profit	comp	72	1006	98	94
Leader [k€]	strat	72	1111	98	94
Profit	comp	1310	376	1296	1288
Fringe [k€]	strat	1310	1032	1296	1288

Table B.9: Resulting profits in the fifteen-node Western European network without line constraints

		Test_Ebel		Test_EDF
Problem sizes	comp	582 continuous variabes 0 discrete variables	comp	582 continuous variabes 0 discrete variables
	strat	3309 continuous variables 2006 discrete variables	strat	22029 continuous variables 7406 discrete variables
		Test_EON		**Test_RWE**
	comp	582 continuous variabes 0 discrete variables	comp	582 continuous variabes 0 discrete variables
	strat	4389 continuous variables 2726 discrete variables	strat	5109 continuous variables 1766 discrete variables

Table B.10: Model statistics for fifteen-node Western European network

		Test_Ebel	Test_EDF	Test_EON	Test_RWE
Computation	comp	2 s	2 s	2 s	2 s
times	strat	4 min	5 h	15 s	19 s

Table B.11: Computation times for fifteen-node Western European network

Appendix C

Two-Pass Process: Mathematical Justifications (Chapter 6)

It can be shown that the subproblem (6.18) solved in two passes matches the solution of the original one.[1] Here the first pass relates to fixing the generation values $g_{nsu} = g_{nsu}^{fix}$ and substituting the constant value g_{nsu}^{fix} into the objective function (i.e., $\sum_{n,s,u} - g_{nsu}^{fix}\lambda_n$). For clarity of presentation, the subproblem is re-expressed in somewhat more stylized form as follows:

$$\min\{-g^T\lambda\} \tag{C.1a}$$

[1]This section is based work carried out jointly with Prof. Dr. Steven A. Gabriel (University of Maryland).

subject to

$$0 \leq Ad + Bg + Cy + Dz - b \leq Kr \tag{C.1b}$$

$$0 \leq \begin{pmatrix} d \\ g \\ y \end{pmatrix} \leq K(1-r) \tag{C.1c}$$

$$\bar{A}d + \bar{B}g + \bar{C}y + \bar{D}z = \bar{b} \tag{C.1d}$$

$$g = g^{fix} \tag{C.1e}$$

$$g, d \geq 0 \tag{C.1f}$$

$$y \geq 0 \tag{C.1g}$$

$$z \text{ (free)} \tag{C.1h}$$

$$r_i \in \{0, 1\}, \forall i \tag{C.1i}$$

where variables are expressed in vector form with $y = (\beta^T, \mu^T, \bar{\mu}^T)^T \geq 0, z = (\gamma^T, \delta^T, \lambda^T)^T$, K is a vector of suitably large constants and r is a vector of binary variables. As stated (C.1) is a mixed-integer, bilinear program. The two-pass approach first replaces g by g^{fix} and then solves

$$\min\{-g^{fixT}\lambda\} \tag{C.2a}$$

subject to

$$0 \leq Ad + Bg + Cy + Dz - b \leq Kr \tag{C.2b}$$

$$0 \leq \begin{pmatrix} d \\ g \\ y \end{pmatrix} \leq K(1-r) \tag{C.2c}$$

$$\bar{A}d + \bar{B}g + \bar{C}y + \bar{D}z = \bar{b} \tag{C.2d}$$

$$g = g^{fix} \tag{C.2e}$$

$$g, d \geq 0 \tag{C.2f}$$

$$y \geq 0 \tag{C.2g}$$

$$z\text{(free)} \tag{C.2h}$$

$$r_i \in \{0, 1\}, \forall i \tag{C.2i}$$

which is a mixed-integer linear program. Then, the optimal binary variables are fixed at these values (e.g., $\bar{r}$) and solved in the following somewhat easier problem to solve.

$$\min\{-g^T\lambda\} \tag{C.3a}$$

subject to

$$0 \leq Ad + Bg + Cy + Dz - b \leq K\bar{r} \tag{C.3b}$$

$$0 \leq \begin{pmatrix} d \\ g \\ y \end{pmatrix} \leq K(1-\bar{r}) \tag{C.3c}$$

$$\bar{A}d + \bar{B}g + \bar{C}y + \bar{D}z = \bar{b} \tag{C.3d}$$

$$g = g^{fix} \tag{C.3e}$$

$$g, d \geq 0 \tag{C.3f}$$

$$y \geq 0 \tag{C.3g}$$

$$z(\text{free}) \tag{C.3h}$$

Theorem C.1 *Considering problems (C.1), (C.2), and (C.3), taking the optimal binary variables' values from (C.2) and then fixing them in (C.3) and taking the optimal values for the other variables in (C.3) corresponds to a solution of (C.1).*

Proof. Problems (C.1) and (C.2) have the same feasible region. Thus, if w^1 is a solution to (C.1) and w^2 is a solution to (C.2) with respectively, objective function values f^1 and f^2, then

$$\begin{array}{rcll} f^1\left(w^1\right) & \leq & f^1\left(w^2\right) & \left[\text{by optimality of } w^1 \text{ and feasibility of } w^2 \text{ in (C.1)}\right] \\ \text{and} & & & \\ f^2\left(w^2\right) & \leq & f^2\left(w^1\right) & \left[\text{by optimality of } w^2 \text{ and feasibility of } w^1 \text{ in (C.2)}\right] \end{array}$$

but $f^1(w) = f^2(w)\ \forall$ feasible w. Thus,

$$\begin{array}{rcl} f^1\left(w^1\right) & \leq & f^1\left(w^2\right) \\ & = & f^2\left(w^2\right) \\ & \leq & f^2\left(w^1\right) \\ & = & f^1\left(w^1\right) \end{array}$$

so that $f^1\left(w^1\right) = f^2\left(w^2\right)$ so that the solution sets of (C.1) and (C.2) are the same. Since the solution sets of (C.1) and (C.2) were shown to be the same, it suffices to show that problems (C.2) and (C.3) have the same set of solutions. If w^3 is a solution to (C.3) with corresponding objective function f^3 then

$$\begin{aligned} f^2\left(w^2\right) &\leq f^3\left(w^3\right) \\ &\quad \text{[since the binary variables were fixed and } w^3 \text{ is} \\ &\quad \text{feasible in (C.2)]} \\ &\leq f^1\left(w^1\right) \\ &\quad \text{[since } g \text{ is not fixed in the objective function of (C.3)]} \\ &\leq f^2\left(w^2\right) \\ &\quad \text{[from above]} \end{aligned}$$

Thus, the solution sets of (C.2) and (C.3) are the same as desired. ■

Appendix D

GAMS Codes

This appendix provides a print out of example implementations of the different models presented in the main part within GAMS. Using adequate data sets the subsequently presented codes can directly typed into GAMS and will run without producing errors. Data and additional codes, however, will not be displayed here. Please refer to the electronically provided supplements.

D.1 GAMS Code for Chapter 3

```
*------------------------------------------------------------------------*
*                     declaration of scalars                             *
*------------------------------------------------------------------------*
SCALARS

        MVABase         for p.u. calculation [MVA]                   / 500 /
        VoltageBase1    for p.u. calculation [kV]                    / 380 /
        VoltageBase2    for p.u. calculation [kV]                    / 220 /
        VoltageBase3    for p.u. calculation [kV]                    / 110 /
        ReferenzBus     swing bus for calculation                    /   1 /
        TRM             transmission reliability margin [%]          /   0.2 /
        epsilon         demand elasticity                            / -0.1 /
;

Scalars
        nuclearp         nuclear price Euro per MWh                  /3/
        lignitep         lignite price Euro per MWh                  /6.3/
        coalp            coal price Euro per MWh                     /7.3/
        gasp             gas price Euro per MWh                      /20.5/
        oilp             oil price Euro per MWh                      /27.2/
;

*------------------------------------------------------------------------*
*                     declaration of sets                                *
*------------------------------------------------------------------------*

SETS
        c               colums in excel-data-sheets   / c1 * c30        /
        t               time                          / t1*t24          /
        l               lines in the network          / Line1 * Line631/
        n               nodes in the network          / 1 * 421         /
```

```
        s               plants in the market        / s1 * s461      /

        snonGER(s)      plants in other countries
        ausland(n)      foreign countries           / 1,85,215,271,351,413,
                                                      411,263,133 /

        z0(n)           single zone for GER
        z1(n)           zone1 as in dena
        z2(n)           zone2 as in dena
        z3(n)           zone3 as in dena
        z4(n)           zone4 as in dena
        z5(n)           zone5 as in dena
        z6(n)           zone6 as in dena

        l0(l)           lines representing foreign countries
        l6(l)           lines between zones and foreign countries

        nuc(s)          nuclear plants
        lig(s)          lignite plants
        coal(s)         coal plants
        steam(s)        oil and gas steam plants
        ccgt(s)         ccgt plants
        gt(s)           gas turbines
        hydro(s)        hydro plants
        pump(s)         pump storage plants
        kwk(s)          combined heat and power plants

        tfirst(t)       first time periode
        tlast(t)        last time periode
;

ALIAS (L,LL), (N, NN);

tfirst(t) = yes$(ord(t) eq 1);
tlast(t)  = yes$(ord(t) eq card(t));

parameter         slack(n)
/1        1/
;

*----------------------------------------------------------------------------*
*----------------------------------------------------------------------------*
*                          PARAMETER Section                                 *
*----------------------------------------------------------------------------*
*----------------------------------------------------------------------------*

*............................................................................*
*                          1. Declaration                                    *
*............................................................................*


PARAMETERS

*data upload...
        LineData(l,c)        line data
        NodeData(s,c)        node data (generation capacities)
        d_ref(n,t)           reference demand
        p_ref(n,t)           reference prices
        WindGen(n,t)         wind energy data
        Zones
        Connect

*...and the rest

        FromBus(l)           starting node of line L
        ToBus(l)             end node of line L
        LineVoltage(l)       voltage level of line L (110 220 380)
        Resistance(l)        Resistance of line L
        Reactance(l)         Reactance of line L
        ThermalLimit(l)      Max. current of line L[A]
        PowerFlowLimit(l)    power flow limit of line L[MW]
        Incidence(l,n)       incidence matrix of the system
        IncidenceTest(l)     checking the incidence matrixfor errors
        H(l,n)               flow sensivity matrix
        B(n,nn)              network susceptance matrix
        BVector(l)
```

```
        GVector(l)

        gmax(n,s)           max generation capacity
        gmin(n,s)           min output to run a plant
        pumpmax(n)          max pump capacity
        marginalcosts(n,s)  reference marginal costs at max output
        pcost1(n,s)         first partial load function factor
        pcost2(n,s)         second partial load function factor
        pcostfix(n,s)       fix partial load function factor
        startup(n,s)        strat up costs

        a(n,t)              intersection demand function
        m(n,t)              slope demand function
;
*---------------------------------------------------------------------------*
*---------------------------------------------------------------------------*
*                           Objective Section                               *
*---------------------------------------------------------------------------*
*---------------------------------------------------------------------------*

*---------------------------------------------------------------------------*
*                           Variables                                       *
*---------------------------------------------------------------------------*

VARIABLES
        w                         social welfare in the system
        consur                    gross consumer surplus
        costs                     generation costs of the system
        startupcosts(n,s,t)       cost due to starting of a plant
        netinput(n,t)             net input at n in t
        lineflow(l,t)             line flow on l in t
        delta(n,t)                voltage angle differenc at n in t
        up0(t)                    uniform price single zone GER
        up1(t)                    uniform price zone1
        up2(t)                    uniform price zone2
        up3(t)                    uniform price zone3
        up4(t)                    uniform price zone4
        up5(t)                    uniform price zone5
        up6(t)                    uniform price zone6
;

POSITIVE VARIABLES
        PSPdown(n,t)              Pump storage generation
        PSPup(n,t)                Pump storage "pump it up"
        PSP                       Pump Storage storage level
        g(n,s,t)                  generation at n of plant s in t
        q(n,t)                    demand at n in t
;

Binary variables
        on(n,s,t)                 plant condition variable
;
*---------------------------------------------------------------------------*
*                           Equations                                       *
*---------------------------------------------------------------------------*

EQUATIONS
objective
grossconsumer
linearcosts
generationcapacity1
generationcapacity2
input
linearinput
flow
linecap_pos
linecap_neg
slackfunct
energybalance
shutdown_co_1h
shutdown_co_2h
shutdown_co_3h
shutdown_st_1h
shutdown_st_2h
shutdown_cc_1h
PSPcapacity
```

```
PSPcapacitystart
PSPupdown
PSPupdown2
uniform0
linecap_pos_up0
linecap_neg_up0

;




objective..          w =e= consur - costs
;

* costs and Co.
grossconsumer..      consur =e= sum ((n,t)$d_ref(n,t),
                                (a(n,t)*q(n,t)+0.5*m(n,t)*sqr(q(n,t)))
                                / 1000000 )
;
linearcosts..        costs =e= sum ((n,s,t), marginalcosts(n,s)*g(n,s,t))
                               /1000000
;

*generation restrictions
generationcapacity1(n,s,t)$gmax(n,s)..    g(n,s,t) =l= on(n,s,t)*gmax(n,s)
;
generationcapacity2(n,s,t)$gmax(n,s)..    g(n,s,t) =g= on(n,s,t)*gmin(n,s)
;

* line flow euqtions
input(n,t)..            NetInput(n,t)
                        - SUM((nn),  B(n,nn)*Delta(nn,t)) * MVABase
                        - 0.5 *SUM(L$Incidence(L,N), Resistance(L)
                          * SQR( LineFlow(L,T) * Incidence(L,N)))* MVABase
                        =E= 0
;
linearinput(n,t)..      NetInput(n,t)
                        - SUM((nn),  B(n,nn)*Delta(nn,t)) * MVABase
                        =E= 0
;

flow(l,t)..             LineFlow(l,t) - SUM(N$H(l,n), H(l,n) * Delta(n,t) )
                        =E= 0
;
linecap_pos(l,t)..      LineFlow(l,t) * MVABase  =L= + PowerFlowLimit(l);
linecap_neg(l,t)..      LineFlow(l,t) * MVABase  =G= - PowerFlowLimit(l);

slackfunct(n,t)$Slack(N)..        Slack(N) * Delta(N,T) =E= 0
;

* the one and only energy balance
energybalance(n,t)..    sum(s$gmax(n,s),g(n,s,t)) + windgen(n,t)
                        + PSPdown(n,t)$pumpmax(n) - PSPup(n,t)$pumpmax(n)
                        - q(n,t)$d_ref(n,t) - NetInput(n,t) =e= 0
;

*coal plant has to be offline for 4h

shutdown_co_1h(n,coal,t)$gmax(n,coal)..       on(n,coal,t-1)- on(n,coal,t) =l=
                                              1-on(n,coal,t+1) ;
shutdown_co_2h(n,coal,t)$gmax(n,coal)..       on(n,coal,t-1)- on(n,coal,t) =l=
                                              1-on(n,coal,t+2) ;
shutdown_co_3h(n,coal,t)$gmax(n,coal)..       on(n,coal,t-1)- on(n,coal,t) =l=
                                              1-on(n,coal,t+3) ;

*oil/gas steam plants has to be offline for 3h

shutdown_st_1h(n,steam,t)$gmax(n,steam)..      on(n,steam,t-1)- on(n,steam,t)
                                               =l= 1-on(n,steam,t+1) ;
shutdown_st_2h(n,steam,t)$gmax(n,steam)..      on(n,steam,t-1)- on(n,steam,t)
                                               =l= 1-on(n,steam,t+2) ;

*ccgt plants has to be offline for 2h
```

```
shutdown_cc_1h(n,ccgt,t)$gmax(n,ccgt)..        on(n,ccgt,t-1)- on(n,ccgt,t) =l=
                                               1-on(n,ccgt,t+1) ;


*PSP mechanism

PSPcapacity(n,t+1)$pumpmax(n)..              PSP(n,t+1) =e= PSP(n,t)
                                             + 0.75*PSPup(n,t)-PSPdown(n,t)
;
PSPcapacitystart(n,tfirst)$pumpmax(n)..      PSP(n,tfirst) =e= 0
;
PSPupdown(n,t)$pumpmax(n)..                  PSPup(n,t)+PSPdown(n,t) =l=
                                             pumpmax(n)
;
PSPupdown2(n,t)$pumpmax(n)..                 PSPdown(n,t) =l= PSP(n,t)
;

*uniform pricing
uniform0(z0,t)$d_ref(z0,t)..                (a(z0,t) + m(z0,t)*q(z0,t)) - up0(t)
                                            =e= 0;

linecap_pos_up0(l0,t)..                      LineFlow(l0,t) * MVABase  =L=
                                             + PowerFlowLimit(l0);
linecap_neg_up0(l0,t)..                      LineFlow(l0,t) * MVABase  =G=
                                             - PowerFlowLimit(l0);

*-----------------------------------------------------------------------------*
*-----------------------------------------------------------------------------*
*                      PARAMETER Section (continued...)                       *
*-----------------------------------------------------------------------------*
*-----------------------------------------------------------------------------*

*.............................................................................*
*                                 Data upload                                 *
*.............................................................................*

$call "GDXXRW [Directory]\24_hours\Input_Data\Linetable_ohneDENA2010.xls
par=linedata rng=GAMS!a2:g633 cdim=1 rdim=1"
$gdxin Linetable_ohneDENA2010.gdx
$load linedata

$call "GDXXRW [Directory]\24_hours\Input_Data\Plants_sommer.xls
par=NodeData rng=Plants!b2:g463 cdim=1 rdim=1"
$gdxin Plants_sommer.gdx
$load NodeData
;

$call "GDXXRW [Directory]\24_hours\Input_Data\LoadSommer.xls
par=d_ref rng=Load!a1:Y431 cdim=1 rdim=1"
$gdxin LoadSommer.gdx
$load d_ref
;

$call "GDXXRW [Directory]\24_hours\Input_Data\PriceSommer.xls
par=p_ref rng=Price!a1:Y431 cdim=1 rdim=1"
$gdxin PriceSommer.gdx
$load p_ref
;

$call "GDXXRW [Directory]\24_hours\Input_Data\Wind_sommer_storm.xls
par=WindGen rng=wind!a1:Y371 cdim=1 rdim=1"
$gdxin Wind_sommer_storm.gdx
$load WindGen
;

$call "GDXXRW [Directory]\24_hours\Input_Data\Zonen.xls
par=Zones rng=GAMS!a2:b354 cdim=1 rdim=1"
$gdxin zonen.gdx
$load Zones
;

$call "GDXXRW [Directory]\24_hours\Input_Data\Konnektoren.xls
par=connect rng=GAMS!a1:b100 cdim=1 rdim=1"
$gdxin konnektoren.gdx
$load connect
;
```

```
*---------------------------------------------------------------------------------*
*                                  Net Topology                                   *
*---------------------------------------------------------------------------------*

PARAMETERS

         ZBase1          Base 1 (380 kV) for p.u. calculation
         ZBase2          Base 2 (220 kV) for p.u. calculation
         ZBase3          Base 3 (110 kV) for p.u. calculation
;
         ZBase1 = (VoltageBase1 *1E3)**2 / (MVABase * 1E6);
         ZBase2 = (VoltageBase2 *1E3)**2 / (MVABase * 1E6);
         ZBase3 = (VoltageBase3 *1E3)**2 / (MVABase * 1E6);


*---------------------------------------------------------------------------------*
         FromBus(L)      =       LineData(L, 'c1')
;
         ToBus(L)        =       LineData(L, 'c2')
;
         LineVoltage(L)  =       LineData(L, 'c3')
;
         Resistance(L)   = LineData(L, 'c4') /
                              (  ZBase1$(LineVoltage(L) eq 380)
                               + ZBase2$(LineVoltage(L) eq 220)
                               + ZBase3$(LineVoltage(L) eq 110))
;
        Reactance(L)    = LineData(L, 'c5') /
                              (   ZBase1$(LineVoltage(L) eq 380)
                                + ZBase2$(LineVoltage(L) eq 220)
                                + ZBase3$(LineVoltage(L) eq 110))
;
        ThermalLimit(L) = LineData(L, 'c6')
;
         BVector(L) =  Reactance(L)  / (SQR(Reactance(L))+SQR(Resistance(L)))
;
         GVector(L) =  Resistance(L) / (SQR(Reactance(L))+SQR(Resistance(L)))
;
         PowerFlowLimit(L) = SQRT(3)*LineVoltage(L)*ThermalLimit(L)
                             *(1 - TRM)/1E3
;

z0(n)$(Zones(n,'c1') ne 0) = yes;

z1(n)$(Zones(n,'c1') eq 1) = yes;
z2(n)$(Zones(n,'c1') eq 2) = yes;
z3(n)$(Zones(n,'c1') eq 3) = yes;
z4(n)$(Zones(n,'c1') eq 4) = yes;
z5(n)$(Zones(n,'c1') eq 5) = yes;
z6(n)$(Zones(n,'c1') eq 6) = yes;

l0(l)$(connect(l,'c1') eq 1) = yes;

*---------------------------------------------------------------------------------*
*                               Incidence Matrix                                  *
*---------------------------------------------------------------------------------*

         Incidence(L,N) = 0;
         IncidenceTest(L) = 0;

Loop(L,
       Loop(N$(ORD(N) eq FromBus(L)), Incidence(L,N) = Incidence(L,N) + 1 );
       Loop(N$(ORD(N) eq ToBus(L)),   Incidence(L,N) = Incidence(L,N) - 1 );
);

*display Incidence;

Loop(L, IncidenceTest(L) = SUM(N, Incidence(L,N) )
);
*display IncidenceTest;
Loop(L,
ABORT$(IncidenceTest(L) ne 0) "Incidence not Balanced" ;
);
```

```
*------------------------------------------------------------------------------*
*                    System Admittance Matrix  B(N,NN)                         *
*------------------------------------------------------------------------------*

Loop(L,
         Loop(N, H(L,N) = BVector(L) * Incidence(L,N))
         ;)
;

         B(N,NN) = SUM(L, Incidence(L,N) * H(L,NN) )
;

*display H,B,Incidence,Bvector;

*------------------------------------------------------------------------------*
*                              Generation                                      *
*------------------------------------------------------------------------------*

Loop(s,
       Loop(n$(ORD(n) eq NodeData(s,'c1')), gmax(n,s) = NodeData(s,'c2'));
);

Loop(s,
       Loop(n$(ORD(n) eq NodeData(s,'c1')),
            marginalcosts(n,s) = NodeData(s,'c3'));
);


nuc(s)$( nodedata(s,'c4') eq 1) = yes;
lig(s)$( nodedata(s,'c4') eq 2) = yes;
coal(s)$( nodedata(s,'c4') eq 3) = yes;
steam(s)$( nodedata(s,'c4') eq 4) = yes;
ccgt(s)$( nodedata(s,'c4') eq 5) = yes;
gt(s)$( nodedata(s,'c4') eq 6) = yes;
hydro(s)$( nodedata(s,'c4') eq 7) = yes;
pump(s)$( nodedata(s,'c4') eq 8) = yes;
kwk(s)$( nodedata(s,'c5') eq 1) = yes;



*aus der Dena
gmin(n,nuc)=0.4*gmax(n,nuc);
gmin(n,lig)=0.4*gmax(n,lig);
gmin(n,coal)=0.38*gmax(n,coal);
gmin(n,steam)=0.38*gmax(n,steam);
gmin(n,ccgt)=0.33*gmax(n,ccgt);
gmin(n,gt)=0.2*gmax(n,gt);

gmin(n,kwk)$gmax(n,kwk)=0.3*gmax(n,kwk);

*aus der Dena
startup(n,nuc)$gmax(n,nuc)=16.7*nuclearp;
startup(n,lig)$gmax(n,lig)=6.2*lignitep;
startup(n,coal)$gmax(n,coal)=6.2*coalp;
startup(n,steam)$gmax(n,steam)=6.2*oilp;
startup(n,ccgt)$gmax(n,ccgt)=3.5*gasp;
startup(n,gt)$gmax(n,gt)=1.1*gasp;

on.fx(n,nuc,t)$gmax(n,nuc)=1;
on.fx(n,lig,t)$gmax(n,lig)=1;
on.fx(n,kwk,t)$gmax(n,kwk)=1;
g.l(n,kwk,t)$gmax(n,kwk)=gmin(n,kwk);
on.fx(n,pump,t)=0;

pumpmax(n)=sum(pump,gmax(n,pump));
gmax(n,pump)=0;


snonGER(s) = yes;
Loop(z0,
Loop(s$gmax(z0,s),
snonGER(s) = no;
););
```

```
*----------------------------------------------------------------------------*
*                           demand function                                   *
*----------------------------------------------------------------------------*


* parameters a and b of the demand function:        p(q) = a + m*q

m(n,t)$d_ref(n,t) = p_ref(n,t)/(epsilon*d_ref(n,t))   ;

a(n,t)$d_ref(n,t) = p_ref(n,t)-d_ref(n,t)*m(n,t)      ;

*----------------------------------------------------------------------------*
*                           Clear it!!!                                       *
*----------------------------------------------------------------------------*

option kill=NodeData;
option kill=LineData;
option kill=p_ref;
option kill=FromBus;
option kill=ToBus;
option kill=LineVoltage;
option kill=Reactance;
option kill=ThermalLimit;
option kill=BVector;
option kill=GVector;

*----------------------------------------------------------------------------*
*                           Solve it!!!                                       *
*----------------------------------------------------------------------------*


q.fx(n,t)$d_ref(n,t)=1.02*d_ref(n,t);
g.l(n,nuc,t)=0.9*gmax(n,nuc);
g.l(n,lig,t)=0.5*gmax(n,lig);
g.l(n,hydro,t)=gmax(n,hydro);


model MIPGermany
/
linearcosts
generationcapacity1
generationcapacity2
linearinput
flow
slackfunct
energybalance
shutdown_co_1h
shutdown_co_2h
shutdown_co_3h
shutdown_st_1h
shutdown_st_2h
shutdown_cc_1h
PSPcapacity
PSPcapacitystart
PSPupdown
PSPupdown2
linecap_pos_up0
linecap_neg_up0
/  ;

MIPGermany.reslim =         1000000000;
MIPGermany.iterlim =        1000000000;
MIPGermany.holdfixed =      1;

solve MIPGermany using mip minimizing costs;

q.lo(n,t)$d_ref(n,t)=0;
q.up(n,t)$d_ref(n,t)=5*d_ref(n,t);

model MINLPGermany
/
objective
grossconsumer
linearcosts
```

```
generationcapacity1
generationcapacity2
linearinput
flow
slackfunct
energybalance
shutdown_co_1h
shutdown_co_2h
shutdown_co_3h
shutdown_st_1h
shutdown_st_2h
shutdown_cc_1h
PSPcapacity
PSPcapacitystart
PSPupdown
PSPupdown2
linecap_pos_up0
linecap_neg_up0
uniform0
/  ;
MINLPGermany.reslim =          1000000000;
MINLPGermany.iterlim =         1000000000;
MINLPGermany.holdfixed =       1;

*solve MINLPGermany using minlp maximizing w;

parameter onfix;
onfix(n,s,t) = on.l(n,s,t);
on.fx(n,s,t)= onfix(n,s,t);



model RMINLPGermany
/
objective
grossconsumer
linearcosts
generationcapacity1
generationcapacity2
input
flow
slackfunct
energybalance
shutdown_co_1h
shutdown_co_2h
shutdown_co_3h
shutdown_st_1h
shutdown_st_2h
shutdown_cc_1h
PSPcapacity
PSPcapacitystart
PSPupdown
PSPupdown2
linecap_pos_up0
linecap_neg_up0
uniform0
/  ;
RMINLPGermany.reslim =          1000000000;
RMINLPGermany.iterlim =         1000000000;
RMINLPGermany.holdfixed =       1;

solve RMINLPGermany using rminlp maximizing w;

parameter genmarket,statusmarket,price;
genmarket(n,s,t)=g.l(n,s,t);
statusmarket(n,s,t)=onfix(n,s,t);
price(n,t)=a(n,t) + m(n,t)* q.l(n,t);

execute_unload "Dispatch_som_storm_fx.gdx" g.L
execute 'gdxxrw.exe Dispatch_som_storm_fx.gdx SQ=N
         var=g.L rng=marketmeritorder!'
execute_unload "Dispatch_som_storm_fx.gdx" price
execute 'gdxxrw.exe Dispatch_som_storm_fx.gdx SQ=N par=price rng=marketprices!'


parameter demfix;
```

```
demfix(n,t) = q.l(n,t);
q.fx(n,t)= 1.03*demfix(n,t);
q.fx(ausland,t)= demfix(ausland,t);
on.lo(n,s,t)= 0;
on.up(n,s,t)= 1;
on.fx(n,nuc,t)$gmax(n,nuc)=1;
on.fx(n,lig,t)$gmax(n,lig)=1;
on.fx(n,kwk,t)$gmax(n,kwk)=1;
on.fx(n,snonGER,t)$gmax(n,snonGER)= on.l(n,snonGER,t);
g.fx(n,snonGER,t)$gmax(n,snonGER)= g.l(n,snonGER,t);


model MIPDispatch
/
linearcosts
generationcapacity1
generationcapacity2
linearinput
flow
slackfunct
energybalance
shutdown_co_1h
shutdown_co_2h
shutdown_co_3h
shutdown_st_1h
shutdown_st_2h
shutdown_cc_1h
PSPcapacity
PSPcapacitystart
PSPupdown
PSPupdown2
/ ;

MIPDispatch.reslim =         1000000000;
MIPDispatch.iterlim =        1000000000;
MIPDispatch.holdfixed =      1;
solve MIPDispatch using mip minimizing costs;

q.fx(n,t)= demfix(n,t);
parameter onfix;
onfix(n,s,t) = on.l(n,s,t);
on.fx(n,s,t)= onfix(n,s,t);



model MINLPDispatch
/
linearcosts
generationcapacity1
generationcapacity2
input
flow
slackfunct
energybalance
shutdown_co_1h
shutdown_co_2h
shutdown_co_3h
shutdown_st_1h
shutdown_st_2h
shutdown_cc_1h
PSPcapacity
PSPcapacitystart
PSPupdown
PSPupdown2
/ ;
MINLPDispatch.reslim =         1000000000;
MINLPDispatch.iterlim =        1000000000;
MINLPDispatch.holdfixed =      1;

solve MINLPDispatch using rminlp minimizing costs;

parameter gendispatch,statusdispatcht;
gendispatch(n,s,t)=g.l(n,s,t);
statusdispatcht(n,s,t)=on.l(n,s,t);
```

```
parameter genchange,statuschange;
genchange(n,s,t)=gendispatch(n,s,t) -genmarket(n,s,t) ;
statuschange(n,s,t)=statusdispatcht(n,s,t)-statusmarket(n,s,t);

parameter dispacthcosts,dispatchstartup;
dispacthcosts(n,s)= sum(t, genchange(n,s,t)*(marginalcosts(n,s)-up0.l(t)));
dispatchstartup(n,s,t)$(statuschange(n,s,t) eq 1) =
gendispatch(n,s,t)*startup(n,s);


display genchange,statuschange,dispacthcosts,dispatchstartup;

execute_unload "Dispatch_som_storm_fx.gdx" g.L
execute 'gdxxrw.exe Dispatch_som_storm_fx.gdx SQ=N
         var=g.L rng=redispatchte_marketmeritorder!'
execute_unload "Dispatch_som_storm_fx.gdx" genchange
execute 'gdxxrw.exe Dispatch_som_storm_fx.gdx par=genchange rng=redispatch_gen!'
execute_unload "Dispatch_som_storm_fx.gdx" statuschange
execute 'gdxxrw.exe Dispatch_som_storm_fx.gdx
         par=statuschange rng=redispatch_stat!'


parameter firstdispatch_low(s,t),firstdispatch_up(s,t);

loop(s,
loop(n$gmax(n,s),
loop(t$(up0.l(t) gt marginalcosts(n,s)),
firstdispatch_low(s,t)= gmax(n,s) - g.l(n,s,t)
););););

loop(s,
loop(n$gmax(n,s),
loop(t$(marginalcosts(n,s) gt up0.l(t)),
firstdispatch_up(s,t)= g.l(n,s,t)
););););

execute_unload "Dispatch_som_storm_fx.gdx" firstdispatch_low
execute 'gdxxrw.exe Dispatch_som_storm_fx.gdx
         par=firstdispatch_low rng=firstdispatch_low!'
execute_unload "Dispatch_som_storm_fx.gdx" firstdispatch_up
execute 'gdxxrw.exe Dispatch_som_storm_fx.gdx
         par=firstdispatch_up rng=firstdispatch_up!'
```

D.2 GAMS Code for Chapter 4

```
*------------------------------------------------------------------------------*
*  sets  = Indices
*------------------------------------------------------------------------------*


sets

n                  node                /1*2120/
s                  plants              /nuc,lig,coal,ccgt,gas,oil,hydro,pump/
l                  line                /line1*line3143/
ld(l)
colums             colums in Ecel      /c1*c20/
sc                 load scenarios      /lsc1*lsc4/
iteration                              /it1*it300/
iter(iteration)
iter1(iteration)

onekv(l)           110kV lines
twokv(l)           220kV lines
fourkv(l)          380kV lines


Alias   (n,nn), (l,ll), (iteration, iterations);

*------------------------------------------------------------------------------*
*                              data upload                                     *
*------------------------------------------------------------------------------*

Parameter nodetable (n,colums);
$call "GDXXRW [Directory]\Input_Data\nodetable.xls
par=nodetable rng=GAMS!a2:m2122 cdim=1 rdim=1"
$gdxin nodetable.gdx
$load nodetable
;

Parameter wind (n,colums);
$call "GDXXRW [Directory]\Input_Data\windtable_2006.xls
par=wind rng=GAMS!a2:b2122 cdim=1 rdim=1"
$gdxin windtable_2006.gdx
$load wind
;

Parameter linedata (l,colums);
$call "GDXXRW [Directory]\Input_Data\linetable.xls
par=linedata rng=GAMS!b2:g3145 cdim=1 rdim=1"
$gdxin Linetable.gdx
$load linedata
;

*------------------------------------------------------------------------------*
*  "variable" reference values (for iterations)  = Scalars
*------------------------------------------------------------------------------*


scalars
p_ref              reference price at reference demand          / 43    /
epsilon            elasticity of demand                          / -0.25 /
MVABase            for p.u. calculation [MVA]                    / 500 /
VoltageBase1       for p.u. calculation [kV]                     / 380 /
VoltageBase2       for p.u. calculation [kV]                     / 220 /
VoltageBase3       for p.u. calculation [kV]                     / 110 /
VoltageBase4       for p.u. calculation [kV]                     / 500 /
windcosts          oportunity costs for wind                     / 4.05 /
;

*------------------------------------------------------------------------------*
*  line parameter
*------------------------------------------------------------------------------*
Parameters
        ZBase1          Base 1 (380 kV) for p.u. calculation
        ZBase2          Base 2 (220 kV) for p.u. calculation
        ZBase3          Base 3 (110 kV) for p.u. calculation
        ZBase4          Base 4 (500 kV) for p.u. calculation  aux.lines
```

```
;
         ZBase1 = (VoltageBase1 *1E3)**2 / (MVABase * 1E6)
;
         ZBase2 = (VoltageBase2 *1E3)**2 / (MVABase * 1E6)
;
         ZBase3 = (VoltageBase3 *1E3)**2 / (MVABase * 1E6)
;
         ZBase4 = (VoltageBase4 *1E3)**2 / (MVABase * 1E6)
;

parameter
                  ref_res(l)
                  ref_rea(l)
                  ref_limit(l)
                  FromBus(l)
                  ToBus(l)
                  LineVoltage(l)
                  length(l)
                  circuits(l)
                  ThermalLimit(l)
                  PowerFlowLimit(l,sc)
                  Resistance_start(l)
                  Reactance_start(l)
                  Resistance(l)
                  Reactance(l)
                  BVector(l)
                  GVector(l)
                  Icost(iteration)
                  Itotal
                  Incidence
                  IncidenceTest
;

Icost('it1') = 0;
Itotal = 0;

onekv(l)$( linedata (l,'c3') eq 110) = yes;
twokv(l)$( linedata (l,'c3') eq 220) = yes;
fourkv(l)$( linedata (l,'c3') eq 380) = yes;

ref_res(onekv)=0.095;
ref_res(twokv)=0.030;
ref_res(fourkv)=0.015;
ref_rea(onekv)=0.205;
ref_rea(twokv)=0.160;
ref_rea(fourkv)=0.130;
ref_limit(onekv)=472;
ref_limit(twokv)=1285;
ref_limit(fourkv)=2582;

FromBus(l)= linedata (l,'c1');
ToBus(l)= linedata (l,'c2');
LineVoltage(l)= linedata (l,'c3');
length(l)=linedata(l,'c4');
circuits(l)=linedata(l,'c5');

Resistance_start(l)=length(l)*ref_res(l)/circuits(l);
Reactance_start(l)=length(l)*ref_rea(l)/circuits(l);

parameter TRM(sc)
/
lsc1      0.2
lsc2      0.2
/;



ThermalLimit(l)= ref_limit(l)*circuits(l);
PowerFlowLimit(L,sc) = SQRT(3)*LineVoltage(L)*ThermalLimit(L)
                        *(1 - TRM(sc))/1E3;

Resistance(L)    = Resistance_start(l) /
                  (  ZBase1$(LineVoltage(L) eq 380)
                   + ZBase2$(LineVoltage(L) eq 220)
                   + ZBase3$(LineVoltage(L) eq 110)
                   + ZBase4$(LineVoltage(L) eq 500))
```

```
;
Reactance(L)      = Reactance_start(l) /
                    (    ZBase1$(LineVoltage(L) eq 380)
                    + ZBase2$(LineVoltage(L) eq 220)
                    + ZBase3$(LineVoltage(L) eq 110)
                    + ZBase4$(LineVoltage(L) eq 500))
;

BVector(L) =  Reactance(L)  / (SQR(Reactance(L))+SQR(Resistance(L)));
GVector(L) =  Resistance(L) / (SQR(Reactance(L))+SQR(Resistance(L)));

Incidence(l,n) = 0;
IncidenceTest(l) = 0;

Loop(L,
       Loop(N$(ORD(N) eq FromBus(L)), Incidence(L,N) = Incidence(L,N) + 1 );
       Loop(N$(ORD(N) eq ToBus(L)),   Incidence(L,N) = Incidence(L,N) - 1 );
);

Loop(L, IncidenceTest(L) = SUM(N, Incidence(L,N) )
);

Loop(L,
ABORT$(IncidenceTest(L) ne 0) "INCIDENCE NOT BALANCED" ;
);

display Incidence;

parameter H(l,n);
H(l,n) = BVector(l) * Incidence(l,n);

parameter B(n,nn);
B(n,nn) = SUM(l, Incidence(l,n) * H(l,nn) ) ;

Parameter Slack(n)       referencebus
/1       1/
;

loop(n,
FromBus(l)$(Incidence(l,n) eq 1)= ORD(n);
ToBus(l)$(Incidence(l,n) eq -1)= ORD(n);
);


*------------------------------------------------------------------------------*
* generation data
*------------------------------------------------------------------------------*
parameter g_max(n,s)          maximal plant capcities;
g_max(n,'nuc')= nodetable (n,'c1')                  ;
g_max(n,'lig')= nodetable (n,'c2')                  ;
g_max(n,'coal')= nodetable (n,'c3')                 ;
g_max(n,'ccgt')= nodetable (n,'c4')                 ;
g_max(n,'gas')= nodetable (n,'c5')                  ;
g_max(n,'oil')= nodetable (n,'c6')                  ;
g_max(n,'hydro')= nodetable (n,'c7')                ;
g_max(n,'pump')= nodetable (n,'c8')                 ;

parameter c(n,s)          marginal generation costs;
c(n,'nuc')= 15                                      ;
c(n,'lig')= 67                                      ;
c(n,'coal')= 62                                     ;
c(n,'ccgt')= 53                                     ;
c(n,'gas')= 84                                      ;
c(n,'oil')= 114                                      ;
c(n,'hydro')= 0                                     ;
c(n,'pump')= 40                                     ;

parameter wi(n)  wind input per node;
wi(n)=wind(n,'c1')
;

parameter windspeed(sc)
/
lsc1      0.2
lsc2      0.2
lsc3      0.8
```

```
lsc4        0.8
/;


parameter weight(sc)
/
lsc1        0.25
lsc2         0.5
lsc3         0.083
lsc4         0.167
/;


*------------------------------------------------------------------------------*
* demand (linear function: p = a + m*q )
*------------------------------------------------------------------------------*

parameter q_ref(n,sc)        average reference demand;
q_ref(n,'lsc1')=nodetable (n,'c9') ;
q_ref(n,'lsc2')=nodetable (n,'c10') ;
q_ref(n,'lsc3')=nodetable (n,'c11') ;
q_ref(n,'lsc4')=nodetable (n,'c12') ;



parameter m(n,sc)    slope of the linear demand function;
m(n,sc)$q_ref(n,sc) = p_ref/(epsilon*q_ref(n,sc));

parameter a(n,sc)    intersection of the linear demand function;
a(n,sc)$q_ref(n,sc) = p_ref-q_ref(n,sc)*m(n,sc);



*------------------------------------------------------------------------------*
* problem formulation
*------------------------------------------------------------------------------*

variables
w                                 welfare
delta(n,sc)                       voltage angle difference
costs                             total generation costs
sur                               area below demand function
ni(n,sc)                          net input at node n
lf(l,sc)                          line flow on l
;

positive variables
q(n,sc)                    demand at node n
g(n,s,sc)                  generation at node n of planttype s
;

*p.fx(n)=30;
*q.fx(n)=0.2*q_ref(n);
*g.l(n,s)=0.5*g_max(n,s);


equations

welfare                           objective function
concumersur                       consumer surplus
prodcosts                         producer costs
Capacity                          capacity limit of generation
netinput                          equation for net input
Energybalance                     energybalance
flow                              equation for line flow
Lineflow_pos                      upper capacity limit of lineflow
Lineflow_neg                      lower capacity limit of lineflow
Slackbus                          delta at reference bus equals zero
consumers
linaerinput
linearbalance
;


welfare..       w =e= (sur-costs)
;
```

```
concumersur..   sur =e= sum ((n,sc)$q_ref(n,sc),
                weight(sc)*(a(n,sc)*q(n,sc)+0.5*m(n,sc)*q(n,sc)*q(n,sc)))
                /1000000
;
prodcosts..     costs =e= sum((n,s,sc)$g_max(n,s),weight(sc)*c(n,s)*g(n,s,sc))
                          /1000000
                          + sum((n,sc)$wi(n),
                          weight(sc)*windspeed(sc)*wi(n)*windcosts)/1000000
                          + Itotal/1000000
;
capacity(n,s,sc)$g_max(n,s)..      g_max(n,s) =g= g(n,s,sc);
;
netinput(n,sc)..          ni(n,sc)
                          - SUM((nn),  B(n,nn)*Delta(nn,sc)) * MVABase
                          - 0.5 *SUM(l$Incidence(l,n), Resistance(l)
                                * SQR( lf(l,sc) * Incidence(l,n)))* MVABase
                          =e= 0
;
energybalance(n,sc)..      windspeed(sc)*wi(n)
                           +sum(s$g_max(n,s),g(n,s,sc))-q(n,sc)-ni(n,sc) =e= 0
;
flow(l,sc)..               lf(l,sc) =e=
                           SUM(n$Incidence(l,n), H(l,n) * Delta(n,sc))
;
Lineflow_pos(l,sc)..       PowerFlowLimit(l,sc) - lf(l,sc)* MVABase =g= 0
;
Lineflow_neg(l,sc)..       PowerFlowLimit(l,sc) + lf(l,sc)* MVABase  =g= 0
;
Slackbus(n,sc)$slack(n).. slack(n)*delta(n,sc) =e= 0
;

*linear model

consumers..             sur =e= sum ((n,sc)$q_ref(n,sc),
                                weight(sc)*(a(n,sc)*q_ref(n,sc)+0.5*m(n,sc)
                                *q_ref(n,sc)*q_ref(n,sc)))/1000000
;
linaerinput(n,sc)..     ni(n,sc)
                        - SUM((nn),  B(n,nn)*Delta(nn,sc)) * MVABase
                        =e= 0
;
linearbalance(n,sc)..    windspeed(sc)*wi(n)+
                         sum(s$g_max(n,s),g(n,s,sc))-0.5*q_ref(n,sc)-ni(n,sc)
                         =e= 0
;


g.l(n,s,sc)= 0.5* g_max(n,s);

model linear
/
prodcosts
Capacity
flow
Lineflow_pos
Lineflow_neg
Slackbus
consumers
linaerinput
linearbalance/;

linear.reslim =         1000000000;
linear.iterlim =        1000000000;
linear.holdfixed =      1;



model test
/welfare
concumersur
prodcosts
Capacity
netinput
Energybalance
flow
```

```
Lineflow_pos
Lineflow_neg
Slackbus
/;

test.reslim =         1000000000;
test.iterlim =        1000000000;
test.holdfixed =      1;


display    LineVoltage, ThermalLimit,PowerFlowLimit,q_ref, g_max;


parameter
nodalprice(n,sc)
pricedifference(l,sc)
averagepricediff(l)
avprice(ll)
maximum
circs
cir
welf
welfdiff
pricediff
dummy
nodal
ausbau
count
;
        scalar leitung;

dummy(l) = 0;
count = 0;
iter(iteration) = no;
iter1(iteration) = no;

Loop(iteration,
If(Ord(iteration) eq 1,
iter(iteration) = yes;
iter1(iteration) = yes;
solve linear minimizing costs using lp;
solve test maximizing w using nlp;
nodalprice(n,sc)=energybalance.m(n,sc)*(-1000000)*1/weight(sc);
pricedifference(l,sc)= abs(sum(n$(ORD(N) eq FromBus(L)),nodalprice(n,sc))
- sum(n$(ORD(N) eq ToBus(L)),nodalprice(n,sc)));
averagepricediff(l) = sum(sc,weight(sc)*pricedifference(l,sc));
loop(ll, Loop(l$(ORD(l) eq ORD(ll)), avprice(ll)=averagepricediff(l));;);
maximum=smax(ll,avprice(ll));

circs(l,iter)=circuits(l);
welf(iter) = w.l;
welfdiff(iter) = welf(iter);
pricediff(l,iter)=averagepricediff(l);
nodal(n,iter)=sum(sc,weight(sc)*nodalprice(n,sc));
ausbau(l,iter)=0;
iter(iteration) = no;
);

If((Ord(iteration) gt 1),
nodalprice(n,sc)=energybalance.m(n,sc)*(-1000000)*1/weight(sc);
pricedifference(l,sc)= abs(sum(n$(ORD(N) eq FromBus(L)),nodalprice(n,sc))
- sum(n$(ORD(N) eq ToBus(L)),nodalprice(n,sc)));
averagepricediff(l) = sum(sc,weight(sc)*pricedifference(l,sc));


loop(ll$(dummy(ll) eq 0), Loop(l$(ORD(l) eq ORD(ll)),
     avprice(ll)=averagepricediff(l));;);
loop(ll$(dummy(ll) ne 0), Loop(l$(ORD(l) eq ORD(ll)), avprice(ll)=0););

loop(ll, Loop(l$(ORD(l) eq ORD(ll)), cir(ll)=circuits(l)););
loop(ll$(cir(ll)  ge 4), Loop(l$(ORD(l) eq ORD(ll)), avprice(ll)=0););

maximum=smax(ll,avprice(ll));
```

```
Loop(ll$(cir(ll) le 3),
If(avprice(ll) eq maximum,
         iter1(iteration) = yes;
         iter(iteration) = yes;
         ld(ll) = yes;
         circuits(ll)=circuits(ll)+1;
         Resistance_start(l)=length(l)*ref_res(l)/circuits(l);
         Reactance_start(l)=length(l)*ref_rea(l)/circuits(l);
         ThermalLimit(l)= ref_limit(l)*circuits(l);
         PowerFlowLimit(L,sc) = SQRT(3)*LineVoltage(L)*ThermalLimit(L)
                                *(1 - TRM(sc))/1E3;

         Resistance(L)   = Resistance_start(l) /
                             (  ZBase1$(LineVoltage(L) eq 380)
                             + ZBase2$(LineVoltage(L) eq 220)
                             + ZBase3$(LineVoltage(L) eq 110)
                             + ZBase4$(LineVoltage(L) eq 500))
;
         Reactance(L)    = Reactance_start(l) /
                             (   ZBase1$(LineVoltage(L) eq 380)
                              + ZBase2$(LineVoltage(L) eq 220)
                              + ZBase3$(LineVoltage(L) eq 110)
                              + ZBase4$(LineVoltage(L) eq 500))
;
         BVector(L) =  Reactance(L)  / (SQR(Reactance(L))+SQR(Resistance(L)));
         GVector(L) =  Resistance(L) / (SQR(Reactance(L))+SQR(Resistance(L)));
         H(l,n) = BVector(l) * Incidence(l,n);
         B(n,nn) = SUM(l, Incidence(l,n) * H(l,nn)) ;
         display Reactance,length;
         Icost(iter) =((8225*length(ll))$(LineVoltage(ll) eq 110)/8760
                    + (8225*length(ll))$(LineVoltage(ll) eq 220)/8760
                    + (14687.5*length(ll))$(LineVoltage(ll) eq 380)/8760);
         Itotal = sum(iter1,Icost(iter1));
         solve test maximizing w using nlp;
         circs(l,iter)=circuits(l);
         welf(iter)=w.l;
         ausbau(ll,iter)=length(ll);
         welfdiff(iteration)=welf(iteration)-welf(iteration -1);
         dummy(ld) = 0;
         leitung=ORD(ll);
         display w.l,welf,welfdiff,ausbau, leitung, icost,Itotal;

If(welfdiff(iteration) le 0,
         Loop(l$(ORD(l) eq ORD(ll)), circuits(l)=circuits(l)-1);
         circs(l,iter)=circuits(l);
         ausbau(ll,iter) = 0;
         Icost(iter) = 0;
         Itotal = sum(iter1,Icost(iter1));
         dummy(ld) = 1;
         ld(ll) = no;
         count = count + 1;
);
);
iter(iteration) = no;
);
);
If(count ge 50,
         execute_unload "2006_highc02.gdx" circs
         execute 'gdxxrw.exe 2006_highc02.gdx par=circs rng=circs!'
         execute_unload "2006_highc02.gdx" welf
         execute 'gdxxrw.exe 2006_highc02.gdx par=welf rng=welf!'
         execute_unload "2006_highc02.gdx" welfdiff
         execute 'gdxxrw.exe 2006_highc02.gdx par=welfdiff rng=welfdiff!'
         execute_unload "2006_highc02.gdx" pricediff
         execute 'gdxxrw.exe 2006_highc02.gdx par=pricediff rng=pricediff!'
         execute_unload "2006_highc02.gdx" nodalprice
         execute 'gdxxrw.exe 2006_highc02.gdx par=nodalprice rng=nodalprice!'
         Abort "NO WELFARE INCREASE";
);
);
execute_unload "2006_highc02.gdx" circs
         execute 'gdxxrw.exe 2006_highc02.gdx par=circs rng=circs!'
         execute_unload "2006_highc02.gdx" welf
         execute 'gdxxrw.exe 2006_highc02.gdx par=welf rng=welf!'
```

```
execute_unload "2006_highc02.gdx" welfdiff
execute 'gdxxrw.exe 2006_highc02.gdx par=welfdiff rng=welfdiff!'
execute_unload "2006_highc02.gdx" pricediff
execute 'gdxxrw.exe 2006_highc02.gdx par=pricediff rng=pricediff!'
execute_unload "2006_highc02.gdx" nodalprice
execute 'gdxxrw.exe 2006_highc02.gdx par=nodalprice rng=nodalprice!'
```

D.3 GAMS Code for Chapter 5

```
Sets
col       columns for output          /Sub1,Sub2,MPstat,SP1stat,SP2stat,zup,zlo/
n         nodes                       /n1*n15/
l         lines                       /l1*l28/
u         plant types                 /u1*u8/
f         firms                       /j1,s1/
s(f)                                  /s1/
j(f)                                  /j1/
i         g sample values             /i1*i59/
;

Alias (n,nn);

*----------------------------------------------------------------------------*
* Some parameters
*----------------------------------------------------------------------------*
Scalars
p_ref            Reference price for demand function           / 30    /
epsilon          Demand elasticity at reference point          / -0.25 /
loadfactor       Factor to define load levels                  / 1.0   /
;


*----------------------------------------------------------------------------*
* Line parameters
*----------------------------------------------------------------------------*

Parameter cap(l)
/l1         2971
l2          1842
l3          1842
l4          896
l5          1326
l6          1842
l7          1842
l8          1842
l9          641
l10         641
l11         936
l12         1842
l13         898
l14         1207
l15         267
l16         2762
l17         1842
l18         3329
l19         1282
l20         3329
l21         20000
l22         20000
l23         20000
l24         20000
l25         20000
l26         20000
l27         20000
l28         20000/
;
cap(l) = cap(l)/1000;
Parameter x(l)
/l1         12.21372516
l2          69.22912356
l3          42.95031339
l4          28.25678513
l5          25.43110662
l6          33.0604386
l7          50.01450968
l8          29.10448868
l9          61.03465588
l10         41.82004199
l11         34.19071
l12         31.08246364
l13         55.38329885
l14         45.2108562
```

```
l15       156.4776426
l16       22.22867097
l17       27.12651372
l18       38.24945533
l19       11.4748366
l20       41.30941176
l21       45.8993464
l22       45.8993464
l23       45.8993464
l24       45.8993464
l25       45.8993464
l26       45.8993464
l27       45.8993464
l28       45.8993464/
;

Table Incidence(l,n)
        n1      n2      n3      n4      n5      n6      n7      n8
l1      0       0       0       0       0       0       -1      1
l2      0       0       0       0       0       0       0       1
l3      0       0       0       0       0       0       0       0
l4      0       0       0       0       -1      0       0       0
l5      0       0       0       0       0       0       0       0
l6      0       0       0       -1       0      0       1       0
l7      0       0       0       0       -1      0       1       0
l8      0       0       0       -1      1       0       0       0
l9      0       0       0       0       1       -1      0       0
l10     0       0       -1      0       1       0       0       0
l11     0       0       0       1       0       -1      0       0
l12     0       0       -1      0       0       1       0       0
l13     0       0       0       0       0       1       0       0
l14     0       0       1       0       0       0       0       0
l15     0       0       1       0       0       0       0       0
l16     0       0       0       0       0       0       0       0
l17     0       0       0       0       0       0       0       0
l18     0       0       0       0       0       0       0       0
l19     0       0       0       0       0       0       0       0
l20     0       0       0       0       0       0       0       0
l21     0       1       0       0       0       0       0       0
l22     0       1       0       0       0       0       0       0
l23     0       1       0       0       0       0       0       0
l24     0       1       0       0       0       0       0       0
l25     1       0       0       0       0       0       0       -1
l26     1       0       0       0       0       0       0       0
l27     1       0       0       0       0       0       0       0
l28     1       0       0       0       0       0       0       0

      n9      n10     n11     n12     n13     n14     n15
l1    0       0       0       0       0       0       0
l2    -1      0       0       0       0       0       0
l3    1       -1      0       0       0       0       0
l4    1       0       0       0       0       0       0
l5    0       1       0       0       0       -1      0
l6    0       0       0       0       0       0       0
l7    0       0       0       0       0       0       0
l8    0       0       0       0       0       0       0
l9    0       0       0       0       0       0       0
l10   0       0       0       0       0       0       0
l11   0       0       0       0       0       0       0
l12   0       0       0       0       0       0       0
l13   0       0       0       -1      0       0       0
l14   0       0       0       0       -1      0       0
l15   0       0       0       0       0       -1      0
l16   0       0       0       1       -1      0       0
l17   0       0       0       0       1       -1      0
l18   0       0       0       0       0       1       -1
l19   0       0       -1      0       0       0       1
l20   0       -1      1       0       0       0       0
l21   0       0       0       -1      0       0       0
l22   0       0       0       0       -1      0       0
l23   0       0       0       0       0       -1      0
l24   0       0       0       0       0       0       -1
l25   0       0       0       0       0       0       0
l26   -1      0       0       0       0       0       0
l27   0       -1      0       0       0       0       0
```

```
l28     0          0          -1          0          0          0          0
;

Parameter H, B;
H(l,n) = 1/X(l) * Incidence(l,n);
B(n,nn) = SUM(l, Incidence(l,n) * H(l,nn));

Parameter Slack(n)        Slack bus
/n1       1/
;

*------------------------------------------------------------------------*
* Generation capacit per type and firm
*------------------------------------------------------------------------*


Table gen_max(n,f,u)
                u1          u2          u3          u4          u5
n1.j1     20340.000   21153.000   28964.000    7758.000   10656.000
n2.s1     58288.000     580.000   15822.000                 124.000
n3.j1      2713.000                2474.000     350.000     575.000
n4.j1       449.000                3968.000     249.000    4872.000
n5.j1                               253.000                1510.000
n6.j1      2618.000                1134.000     810.000    1432.000
n7.j1                                          1705.000    2768.000

n1.j1          u6            u7          u8
n2.s1     5752.000      1271.000    5954.000
n3.j1    11130.000     14381.000    3408.000
n4.j1      560.000                  1308.000
n5.j1      111.000
n6.j1
n7.j1     1865.000
;

gen_max(n,f,u) = round(gen_max(n,f,u)/1000);
Parameters gen_bar(n,s,u,i);
gen_bar(n,s,u,'i1') = 0;
loop((n,s,u,i)$(ord(i) le gen_max(n,s,u)),
gen_bar(n,s,u,i+1) = gen_bar(n,s,u,i) + 1;);

*------------------------------------------------------------------------*
* Generation costs per fuel type and location
*------------------------------------------------------------------------*

table uc(n,u)          Erzeugungskosten der Kraftwerkstypen
*        Nuc      Lig      Coal     CCGT     Gas      Oil      Hydro    Pump
         u1       u2       u3       u4       u5       u6       u7       u8
n1       10       20       22       30       45       60       0        35
n2       10       20       22       30       45       60       0        35
n3       10       20       22       30       45       60       0        35
n4       10       20       22       30       45       60       0        35
n5       10       20       22       30       45       60       0        35
n6       10       20       22       30       45       60       0        35
n7       10       20       22       30       45       60       0        35
;
parameter c(n,f,u);
loop((n,f,u)$gen_max(n,f,u),c(n,f,u) = uc(n,u));
display c;

*------------------------------------------------------------------------*
* Reference demand  (linear demand fucntion p = a + m*q )
*------------------------------------------------------------------------*

parameter q_ref(n)        Average demand
/        n1        58380
         n2        54474
         n3        2850
         n4        5925
         n5        6240
         n6        2385
         n7        3021
/
;
q_ref(n) = q_ref(n)/1000;
```

```
q_ref(n) = round(q_ref(n));
q_ref('n8') = 0.1;
q_ref('n9') = 0.1;
q_ref('n10') = 0.1;
q_ref('n11') = 0.1;
q_ref('n12') = 0.1;
q_ref('n13') = 0.1;
q_ref('n14') = 0.1;
q_ref('n15') = 0.1;

parameter slp(n)   Slope of demand function;
slp(n)$q_ref(n) = p_ref/(epsilon*loadfactor*q_ref(n));
parameter icept(n)   Intercept of demand function;
icept(n)$q_ref(n) = p_ref-loadfactor*q_ref(n)*slp(n);

slp(n) = Abs(slp(n));
icept(n) = Abs(icept(n));

Scalars K1 /10000/, K2 /10000/, K3 /10000/, K4 /10000/,
        K5 /10000/, Kl /10000/, Kv /10000/;

Variables
du_engy(n) dual energy balance constraint,
Delta(nn) voltage angle at node nn,
du_del(nn) dual reference bus constraint,
obj;

Positive Variables
cons(n) consumption at node n,
gen(n,f,u) generation at node n of firm f,
du_fpos(l) dual line flow limit forward direction,
du_fneg(l) dual line flow limit backward direction,
du_capf(n,j,u) dual generation capacity limit,
du_cap(n,f,u), v(n,s,u,i);

Binary Variables
r1(n) BV for KKT_Cons,
r2(n,j,u) BV for KKT_Genf,
r3(l) BV for KKT_Fpos,
r4(l) BV for KKT_Fneg,
r5(n,j,u) BV for KKT_Capf,
ql(n), qv(n,s,u,i), qq(n,s,u,i);

Equations OF, DISJ_Cons, DISJ_Cons2, DISJ_Cons3, DISJ_Genf, DISJ_Genf2,
          DISJ_Genf3, DISJ_Delta, DISJ_Engy, DISJ_Fpos, DISJ_Fpos2,
          DISJ_Fpos3, DISJ_Fneg, DISJ_Fneg2, DISJ_Fneg3, DISJ_Capf,
          DISJ_Capf2,DISJ_Capf3, DISJ_Slack, New_Con1, New_Con2,
          New_Con3, New_Con4, New_Con5, New_Con6, New_Con7,
          New_Con8, New_Con9, New_Con10;

*==============================================================================
*Perfect competition pre-run
*==============================================================================

Equations PC_Cons, PC_Gen, PC_Delta, PC_Engy, PC_Fpos, PC_Fneg, PC_Cap, PC_Slack;

PC_Cons(n)..         -icept(n) + (slp(n)*cons(n)) + du_engy(n) =g= 0;
PC_Gen(n,f,u)..      c(n,f,u) - du_engy(n) + du_cap(n,f,u) =g= 0;
PC_Delta(nn)..       -SUM(n,B(n,nn)*du_engy(n)) + SUM(l,H(l,nn)*du_fpos(l))
                     - SUM(l,H(l,nn)*du_fneg(l)) + (du_del(nn)*slack(nn)) =e= 0;
PC_Engy(n)..         cons(n) - SUM((f,u),gen(n,f,u))
                     - SUM(nn,B(n,nn)*delta(nn)) =e= 0;
PC_Fpos(l)..         cap(l) - SUM(n, H(l,n) * Delta(n)) =g= 0;
PC_Fneg(l)..         cap(l) + SUM(n, H(l,n) * Delta(n)) =g= 0;
PC_Cap(n,f,u)..      gen_max(n,f,u) - gen(n,f,u) =g= 0;
PC_Slack(n)..        slack(n)*delta(n) =e= 0;

Model PC_mcp
/PC_Cons.cons, PC_Gen.gen, PC_Delta.delta, PC_Engy.du_engy, PC_Fpos.du_fpos,
PC_Fneg.du_fneg, PC_Cap.du_cap, PC_Slack.du_del /
;
PC_mcp.optfile=0;

Solve PC_mcp using mcp;

parameter profit_comp(f), prices_comp, gen_comp, flow_comp;
```

```
profit_comp(f) = sum((n,u),gen.l(n,f,u)*du_engy.l(n)-gen.l(n,f,u)*c(n,f,u));
prices_comp(n) = du_engy.l(n);
gen_comp(n,f,u) = gen.l(n,f,u);
flow_comp(l) = SUM(n, H(l,n) * Delta.l(n));
display flow_comp, gen_comp, prices_comp, profit_comp;

*==========================================================================
*MIP
*==========================================================================

OF..                obj =e= SUM((n,s,u),(c(n,s,u)*gen(n,s,u))-SUM(i,v(n,s,u,i)));

DISJ_Cons(n)..      -icept(n) + (slp(n)*cons(n)) + du_engy(n) =g= 0;
DISJ_Cons2(n)..     K1*r1(n) + icept(n) - (slp(n)*cons(n)) - du_engy(n) =g= 0;
DISJ_Cons3(n)..     K1*(1-r1(n)) - cons(n) =g= 0;
DISJ_Genf(n,j,u)..  c(n,j,u) - du_engy(n) + du_capf(n,j,u) =g= 0;
DISJ_Genf2(n,j,u).. K2*r2(n,j,u) - c(n,j,u) + du_engy(n) - du_capf(n,j,u) =g= 0;
DISJ_Genf3(n,j,u).. K2*(1-r2(n,j,u)) - gen(n,j,u) =g= 0;
DISJ_Slack(nn)..    (-1)*slack(nn)*delta(nn) =e= 0;
DISJ_Engy(n)..      cons(n) + SUM(nn,B(n,nn)*delta(nn))
                    - SUM((f,u),gen(n,f,u)) =e= 0;
DISJ_Delta(nn)..    SUM(n,B(n,nn)*du_engy(n)) + SUM(l,H(l,nn)*du_fpos(l))
                    - SUM(l,H(l,nn)*du_fneg(l)) - (du_del(nn)*slack(nn)) =e= 0;
DISJ_Fpos(l)..      cap(l) - SUM(nn, H(l,nn) * Delta(nn)) =g= 0;
DISJ_Fpos2(l)..     K3*r3(l) - cap(l) + SUM(nn, H(l,nn) * Delta(nn)) =g= 0;
DISJ_Fpos3(l)..     K3*(1-r3(l)) - du_fpos(l) =g= 0;
DISJ_Fneg(l)..      cap(l) + SUM(nn, H(l,nn) * Delta(nn)) =g= 0;
DISJ_Fneg2(l)..     K4*r4(l) - cap(l) - SUM(nn, H(l,nn) * Delta(nn)) =g= 0;
DISJ_Fneg3(l)..     K4*(1-r4(l)) - du_fneg(l) =g= 0;
DISJ_Capf(n,j,u)..  -gen(n,j,u) + gen_max(n,j,u) =g= 0;
DISJ_Capf2(n,j,u).. K5*r5(n,j,u) + gen(n,j,u) - gen_max(n,j,u) =g= 0;
DISJ_Capf3(n,j,u).. K5*(1-r5(n,j,u)) - du_capf(n,j,u) =g= 0;

New_Con1(n)..       Kl*ql(n) - du_engy(n) =g= 0;
New_Con2(n,s,u)..   gen(n,s,u) - SUM(i,qq(n,s,u,i)*gen_bar(n,s,u,i)) =e= 0;
New_Con3(n,s,u)..   SUM(i,qq(n,s,u,i)) - 1 =e= 0;
New_Con4(n,s,u,i).. ql(n) - qv(n,s,u,i) =g= 0;
New_Con5(n,s,u,i).. qq(n,s,u,i) - qv(n,s,u,i) =g= 0;
New_Con6(n,s,u,i).. qv(n,s,u,i) - qq(n,s,u,i) - ql(n) + 1 =g= 0;
New_Con7(n,s,u,i).. gen_bar(n,s,u,i)*du_engy(n) - v(n,s,u,i) =g= 0;
New_Con8(n,s,u,i).. Kv*qv(n,s,u,i) - v(n,s,u,i) =g= 0;

New_Con9(n)..       du_engy(n) =g= 0;
New_Con10(n,s,u,i).. 1 - qv(n,s,u,i) =g= 0;


Model mpec_mip1
/ OF, DISJ_Cons, DISJ_Cons2, DISJ_Cons3, DISJ_Genf, DISJ_Genf2,
DISJ_Genf3,DISJ_Delta, DISJ_Engy, DISJ_Fpos, DISJ_Fpos2,
DISJ_Fpos3, DISJ_Fneg, DISJ_Fneg2, DISJ_Fneg3, DISJ_Capf,
DISJ_Capf2,DISJ_Capf3, DISJ_Slack, New_Con1, New_Con2, New_Con3,
New_Con4, New_Con5, New_Con6, New_Con7,
New_Con8, New_Con9, New_Con10/
;

mpec_mip1.reslim = 1000000;
mpec_mip1.OptCR = 0;
mpec_mip1.optfile = 1;

Solve mpec_mip1 using mip minimizing obj;

parameter flow_strat(l), profit_strat(f), gen_strat(n,f,u), prices_strat;
flow_strat(l) = SUM(n, H(l,n) * Delta.l(n));
profit_strat(f) = SUM((n,u),(du_engy.l(n)-c(n,f,u))*gen.l(n,f,u));
gen_strat(n,f,u)$gen.l(n,f,u) = gen.l(n,f,u);
prices_strat(n) = du_engy.l(n);
display flow_comp, flow_strat, gen_comp, gen_strat, prices_comp, prices_strat,
        profit_comp, profit_strat,DISJ_Fpos2.m;
```

D.4 GAMS Code for Chapter 6

```
Sets
r         rows output file     /SubMIPv,SubNLPv,MPv_zlo,zup,zdiff,ogens1,ogens2,
                                ogensf, conn3,profsg1,profsg2,proff,pricen1,
                                pricen2, pricen3,MPstat,SPstat,m1,m2/
n         nodes                 /n1*n3/
l         lines                 /l1*l3/
u         units                 /u1*u3/
f         firms                 /s1,f1/
s(f)                            /s1/
j(f)                            /f1/
it        iteration             /it1*it30/
iter(it)
iter2(it)
;

Alias (n,nn), (it,it1);

Table Incidence(l,n)
       n1      n2      n3
l1     1       -1      0
l2     0       1       -1
l3     1       0       -1
;
Parameter slack(n)
/n1    1/
;
Parameter Reactance(l)
/l1    1
l2     1
l3     1/
;
Parameter Resistance(l)
/l1    0.1
l2     0.1
l3     0.1/
;
Parameter cap(l)
/l1   10
l2    10
l3    10/
;
Parameter icept(n)
/ n1   1
  n2   1
  n3  10/
;
Parameter slp(n) Slope of demand functions;
slp(n) = 1;

Parameter BVector Susceptance vector,
          H Network transfer matrix,
          B Network suceptance matrix;

BVector(l) =  Reactance(l)  / (SQR(Reactance(l))+SQR(Resistance(l)));
H(l,n) = BVector(l) * Incidence(l,n);
B(n,nn) = SUM(l, Incidence(l,n) * H(l,nn));

Parameter c(n,f,u) marginal cost of production
/ n1.s1.u1 2
n2.s1.u2  1
n2.f1.u3 3/
;
Parameter gen_max(n,f,u) maximum generation capacity
/n1.s1.u1 15
n2.s1.u2  15
n2.f1.u3  15/
;
Scalars K1 /100/, K2 /100/, K3 /100/, K4 /100/, K5 /100/,
        alpha_d /-1E5/, ep /1E-5/, err /1/;

Parameters gen_fix(n,s,u), gen_old(it,n,s,u), subv(it), du_fix(it,n,s,u),
           zlo, zup, rf1, rf2, rf3, rf4, rf5;
```

```
Variables
du_engy(n) dual energy balance constraint,
Delta(nn) voltage angle at node nn,
du_del(nn) dual reference bus constraint,
sub_mip, sub_nlp, ma, alpha;

Positive Variables
cons(n) consumption at node n,
gen(n,f,u) generation at node n of firm f, genf,
du_fpos(l) dual line flow limit positive direction,
du_fneg(l) dual line flow limit negative direction,
du_capf(n,j,u) dual generation capacity limit;

Binary Variables
r1(n) binary variables for KKT_Cons,
r2(n,j,u) binary variables for KKT_Genf,
r3(l) binary variables for KKT_Fpos,
r4(l) binary variables for KKT_Fneg,
r5(n,j,u) binary variables for KKT_Capf;

Equations MIP_Cons, MIP_Cons2, MIP_Cons3, MIP_Genf, MIP_Genf2, MIP_Genf3,
          MIP_Delta, MIP_Engy, MIP_Fpos, MIP_Fpos2, MIP_Fpos3, MIP_Fneg,
          MIP_Fneg2, MIP_Fneg3, MIP_Capf, MIP_Capf2,MIP_Capf3, MIP_Slack,
          MIP_Obj, NLP_Cons, NLP_Cons2, NLP_Cons3, NLP_Genf, NLP_Genf2,
          NLP_Genf3, NLP_Delta, NLP_Engy, NLP_Fpos, NLP_Fpos2, NLP_Fpos3,
          NLP_Fneg, NLP_Fneg2, NLP_Fneg3, NLP_Capf, NLP_Capf2,NLP_Capf3,
          NLP_Slack, NLP_Obj, Fix, MP_Obj, MP_Cap, MP_alpha, MP_Cuts;

MP_Obj..             ma =e= SUM((n,s,u),gen(n,s,u)*c(n,s,u)) + alpha;
MP_Cap(n,s,u)..        gen(n,s,u) - gen_max(n,s,u) =l= 0;
MP_alpha..           alpha =g= alpha_d;
MP_Cuts(iter)..      alpha =g= subv(iter) + SUM((n,s,u),du_fix(iter,n,s,u)
                              *(gen(n,s,u)-gen_old(iter,n,s,u)));

MIP_Obj..              sub_mip =e= -SUM((n,s,u),gen_fix(n,s,u)*du_engy(n));
MIP_Cons(n)..        -icept(n) + (slp(n)*cons(n)) + du_engy(n) =g= 0;
MIP_Cons2(n)..       K1*r1(n) + icept(n) - (slp(n)*cons(n)) - du_engy(n) =g= 0;
MIP_Cons3(n)..       K1*(1-r1(n)) - cons(n) =g= 0;
MIP_Genf(n,j,u)..      c(n,j,u) - du_engy(n) + du_capf(n,j,u) =g= 0;
MIP_Genf2(n,j,u)..     K2*r2(n,j,u) - c(n,j,u) + du_engy(n) - du_capf(n,j,u)
                       =g= 0;
MIP_Genf3(n,j,u)..     K2*(1-r2(n,j,u)) - gen(n,j,u) =g= 0;
MIP_Slack(nn)..      (-1)*slack(nn)*delta(nn) =e= 0;
MIP_Engy(n)..        cons(n) + SUM(nn,B(n,nn)*delta(nn)) - SUM((j,u),gen(n,j,u))
                     - SUM((s,u),gen_fix(n,s,u)) =e= 0;
MIP_Delta(nn)..      SUM(n,B(n,nn)*du_engy(n)) + SUM(l,H(l,nn)*du_fpos(l))
                     - SUM(l,H(l,nn)*du_fneg(l)) - (du_del(nn)*slack(nn)) =e= 0;
MIP_Fpos(l)..        cap(l) - SUM(nn, H(l,nn) * Delta(nn)) =g= 0;
MIP_Fpos2(l)..       K3*r3(l) - cap(l) + SUM(nn, H(l,nn) * Delta(nn)) =g= 0;
MIP_Fpos3(l)..       K3*(1-r3(l)) - du_fpos(l) =g= 0;
MIP_Fneg(l)..        cap(l) + SUM(nn, H(l,nn) * Delta(nn)) =g= 0;
MIP_Fneg2(l)..       K4*r4(l) - cap(l) - SUM(nn, H(l,nn) * Delta(nn)) =g= 0;
MIP_Fneg3(l)..       K4*(1-r4(l)) - du_fneg(l) =g= 0;
MIP_Capf(n,j,u)..      -gen(n,j,u) + gen_max(n,j,u) =g= 0;
MIP_Capf2(n,j,u)..     K5*r5(n,j,u) + gen(n,j,u) - gen_max(n,j,u) =g= 0;
MIP_Capf3(n,j,u)..     K5*(1-r5(n,j,u)) - du_capf(n,j,u) =g= 0;

NLP_Obj..              sub_nlp =e= -SUM((n,s,u), gen(n,s,u)*du_engy(n));
NLP_Cons(n)..        -icept(n) + (slp(n)*cons(n)) + du_engy(n) =g= 0;
NLP_Cons2(n)..       K1*rf1(n) + icept(n) - (slp(n)*cons(n)) - du_engy(n) =g= 0;
NLP_Cons3(n)..       K1*(1-rf1(n)) - cons(n) =g= 0;
NLP_Genf(n,j,u)..      c(n,j,u) - du_engy(n) + du_capf(n,j,u) =g= 0;
NLP_Genf2(n,j,u)..     K2*rf2(n,j,u) - c(n,j,u) + du_engy(n) - du_capf(n,j,u)
                       =g= 0;
NLP_Genf3(n,j,u)..     K2*(1-rf2(n,j,u)) - gen(n,j,u) =g= 0;
NLP_Slack(nn)..      (-1)*slack(nn)*delta(nn) =e= 0;
NLP_Engy(n)..        cons(n) + SUM(nn,B(n,nn)*delta(nn)) - SUM((f,u),gen(n,f,u))
                     =e= 0;
NLP_Delta(nn)..      SUM(n,B(n,nn)*du_engy(n)) + SUM(l,H(l,nn)*du_fpos(l))
                     - SUM(l,H(l,nn)*du_fneg(l)) - (du_del(nn)*slack(nn)) =e= 0;
NLP_Fpos(l)..        cap(l) - SUM(nn, H(l,nn) * Delta(nn)) =g= 0;
NLP_Fpos2(l)..       K3*rf3(l) - cap(l) + SUM(nn, H(l,nn) * Delta(nn)) =g= 0;
NLP_Fpos3(l)..       K3*(1-rf3(l)) - du_fpos(l) =g= 0;
NLP_Fneg(l)..        cap(l) + SUM(nn, H(l,nn) * Delta(nn)) =g= 0;
NLP_Fneg2(l)..       K4*rf4(l) - cap(l) - SUM(nn, H(l,nn) * Delta(nn)) =g= 0;
NLP_Fneg3(l)..       K4*(1-rf4(l)) - du_fneg(l) =g= 0;
```

```
NLP_Capf(n,j,u)..     -gen(n,j,u) + gen_max(n,j,u) =g= 0;
NLP_Capf2(n,j,u)..    K5*rf5(n,j,u) + gen(n,j,u) - gen_max(n,j,u) =g= 0;
NLP_Capf3(n,j,u)..    K5*(1-rf5(n,j,u)) - du_capf(n,j,u) =g= 0;
Fix(n,s,u)..           gen(n,s,u) =e= gen_fix(n,s,u);


Model SP_MIP
/MIP_Cons, MIP_Cons2, MIP_Cons3, MIP_Genf, MIP_Genf2, MIP_Genf3,
 MIP_Delta, MIP_Engy, MIP_Fpos, MIP_Fpos2, MIP_Fpos3, MIP_Fneg, MIP_Fneg2,
 MIP_Fneg3, MIP_Capf, MIP_Capf2,MIP_Capf3, MIP_Slack, MIP_Obj/;

Model SP_NLP
/NLP_Cons, NLP_Cons2, NLP_Cons3, NLP_Genf, NLP_Genf2, NLP_Genf3,
 NLP_Delta, NLP_Engy, NLP_Fpos, NLP_Fpos2, NLP_Fpos3, NLP_Fneg, NLP_Fneg2,
 NLP_Fneg3, NLP_Capf, NLP_Capf2,NLP_Capf3, NLP_Slack, NLP_Obj, Fix/;

Model MP
/MP_Obj, MP_Cap, MP_alpha, MP_Cuts/;

Model MP1
/MP_Obj, MP_Cap, MP_alpha/;

parameter iteration, zdiff, ogens1, ogens2, ogenf, conn3, profsg1,
          profsg2, proff, outprim, outsec, status_sp_mip,
          status_sp_nlp, status_mp;

iter(it) = no;

Solve MP1 minimizing ma using lp;
zlo = ma.l;
gen_fix(n,s,u) = gen.l(n,s,u);

Loop(it$(err ge ep),
Solve SP_MIP minimizing sub_mip using mip;
rf1(n)=r1.l(n);
rf2(n,j,u)=r2.l(n,j,u);
rf3(l)=r3.l(l);
rf4(l)=r4.l(l);
rf5(n,j,u)=r5.l(n,j,u);
zup = sub_mip.l + SUM((n,s,u),gen_fix(n,s,u)*c(n,s,u));
subv(it) = sub_mip.l;
gen_old(it,n,s,u) = gen_fix(n,s,u);
Solve SP_NLP minimizing sub_nlp using nlp;
du_fix(it,n,s,u) = Fix.m(n,s,u);
err = Abs(zup - zlo);
$include [directory]\outputcommands.gms
display zlo, zup, iteration;
iter(it) = yes;
Solve MP minimizing ma using lp;
zlo = ma.l;
gen_fix(n,s,u) = gen.l(n,s,u);
);

parameter flow, ni, profit;
flow(l) = SUM(nn, H(l,nn) * Delta.l(nn));
ni(n) = SUM(nn,B(n,nn)*delta.l(nn));
profit(f) = sum((n,u), gen.l(n,f,u)*( -c(n,f,u) + du_engy.l(n)));

display flow, ni, profit, outprim, outsec;
```