

Capital Flows and Trade in an Integrated World

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For my parents and for the late Herbert.

Contents

Introduction	v
Part 1. Theoretical Inquiries	1
Chapter 1. International Trade, Hedging and the Demand for Forward Contracts	3
1. Introduction	4
2. The model	6
3. Solving the model	9
4. Equilibrium properties	12
5. Conclusion	19
6. Mathematical appendix	22
Chapter 2. International Capital Flows meet Corporate Liquidity Demand	27
1. Introduction	28
2. The workings of the Holmstroem-Tirole model	30
3. Introducing international capital flows	35
4. Empirical implications	46
5. Conclusion	47
6. Mathematical appendix	49
Part 2. Empirical Inquiries	55
Chapter 3. Accounting for Flows	57
1. Introduction	58
2. General remarks	59
3. The evidence from national sources	64
4. The evidence from supranational sources	70
5. The evidence from the foreign exchange market	71
6. Trade versus financial flows	76
7. Disentangling the aggregation bias	79
8. Conclusion	82
Part 3. Auxiliary Inquiries	85
Chapter 4. Expected Utility and Nonlinearly Transformed Random Variables	87
1. Introduction	88

2. The model	88
3. The direct road - evaluating expected values	90
4. The indirect way - restricting the model	96
5. Conclusion	100
6. Mathematical appendix	102
Bibliography	109

Introduction

The world we live in is increasingly integrated. The last decade of the 20th century saw a remarkable increase in global interaction in fields like communication, culture, travel and politics to name just a few examples. For the work of economists, increasing international integration bears a significant importance. A vast body of literature which deals with the topic is proof of that proposition.

International integration of an economy is usually discussed in terms of its trade and capital flows. Although these are not the only aspects of the phenomenon, there is no work on the subject without addressing at least one of these two basic pillars. The present thesis will duly follow the classic setup.

Early economic theory suggests that open economies fare at least as good as closed ones. In other words, trade will almost always be beneficial and hence, from the perspective of this theories there is not much room for discussion of the phenomenon of the increasing volume of international trade - the more, the better. The same applies to capital flows. Fundamental insights have it that international capital flows are beneficial. International capital flows allow countries to share risks and to smooth consumption paths. Following these fundamental insights on capital flows, there is not much room for discussion either - the more of it, the better.

The real world, however, is not without imperfections. Incorporating imperfections into economic models has occupied the profession for the last 50 years and is an ongoing battle. This process brought about new (and now standard) concepts like asymmetric information, incomplete markets, moral hazard and sticky prices. In much the same way that these new advances created new insights in virtually every corner of reality, the existing economic contributions on international integration were subject to qualifications. The mantra - the more, the better - ceased to exist. Today's International Economics resembles a collection of small purpose build models. It lacks a unifying framework that encompasses most (or even all) different fields. This seems to be the price for the surrender of economists ignorance with respect to imperfections.

The present thesis is essentially a work on International Economics. As such it is no exception in that it consists of different chapters, all of which address a different issue of the field. The first two chapters

are theoretical in nature, whereas the third chapter is empirical. The last chapter provides a technical reference to mathematical problems encountered in the first chapter.

The first chapter is concerned with one of the negative aspects of international trade: terms-of-trade uncertainty. In reality, economies face uncertainty with respect to the value of their produce in the world market. The reason for this could be technology shocks, industrial action or policy. It is often argued that this uncertainty should not matter as long the agents have access to well-developed financial markets. In effect they can buy insurance against this risk. Empirically, however, this claim seems to have little support. Studies showed that firms only hedge a small fraction of their exposure to foreign price changes. "International Trade, Hedging and the Demand for Forward Contracts" offers fresh insights as to why agents fail to hedge terms-of-trade uncertainty in the presence of well-developed financial markets. The reason is an imperfection: incomplete markets. Agents simply cannot fully hedge away their risk. The incompleteness in our setup can only be solved with another instrument: options. Hence, the policy implication of the first chapter is to facilitate the use of options.

The second theoretical model is concerned with the second basic pillar: international capital flows. The huge increase of international capital flows is probably the most controversial issue in the discussion on globalization. Several severe financial crises in the eighties and nineties of the last century have created some doubts over the stability of the international financial system in general and the desirability of capital account liberalization in particular. "International Capital Flows meet Corporate Liquidity Demand" adds some new insights to the already huge literature on capital account liberalization. To this end, we extend a standard model on corporate liquidity demand to a two-country world. We show that if a country protects the interests of domestic agents better than those of foreign agents, it will be punished with more fickle foreign investors. Foreign capital then will be the proverbial shy deer. Another important result of the second chapter is that having little capital will severely constrain the economy from borrowing abroad. This result is an important qualification to the neo-classical growth literature that predicts the catch-up of poor countries to the rich via capital flows. Again, an imperfection plays a vital role for our results and the qualification it adds to standard theory: moral hazard.

In the debate on globalization increasing trade and capital flows figure prominently. There are, however, different measures and different data sources for these flows. The third chapter aims to offer an overview over the existing sources for both gross and net flow data on international capital flows. Further, we set out to offer some measures of the development of goods market integration relative to financial

market integration. Using data for the last decade, we find that although gross financial flows were increasing relative to trade flows, net financial flows were not. Hence, we make the claim that goods market integration actually has had roughly the same speed as international financial integration in the last decade. Increasing gross flows point towards lower transaction costs and hence increased efficiency in the global financial markets. In this sense, the integration of the world has indeed deepened.

The last chapter deals with a particular problem economists encounter in models with expected utility maximization. If the first order condition is a nonlinear transformation of the underlying random variable, explicit demand functions are hard to derive. The chapter first gives an overview on the existing mathematical tools we have to tackle the problem and then develops each method in detail using a simplified version of the model of the first chapter. It turns out that deriving explicit demand functions is always possible, as long as the economist is willing to invoke additional restrictions on the parameter space of the model.

The present thesis would not have been possible to write without the support of many people. First and foremost, I owe great debt to Professor Klaus Wälde, my academic supervisor at Technische Universität Dresden. In many instances he was the lighthouse in stormy waters and provided invaluable intellectual impetus. Further thanks are due to my colleagues Ken Sennewald, Benjamin Weigert and Mirko Wiederholt and numerous participants at research seminars and conferences where I had the honour to present parts of my thesis. Financial support of the State of Saxony, the European Commission and the Banca D'Italia is gratefully acknowledged. All this outstanding professional support, however, would have been falling on barren land without the constant encouragement of my family and friends.

Part 1

Theoretical Inquiries

CHAPTER 1

International Trade, Hedging and the Demand for Forward Contracts

One of the main results of the literature on the effects of uncertainty on trade states that uncertainty should not matter in the presence of well developed forward markets. Empirical studies, however, do not support this result. We derive the demand for forward cover in a small open economy with terms of trade uncertainty. Adopting a standard and more realistic decision structure than the one usually used in this literature, we find that risk averse agents will not buy forwards at an unbiased price. Agents treat forward contracts as an asset rather than as an insurance. This is the reason why, when calibrating the model, only 17% of imports are covered by forwards.

1. Introduction

^{1,2}International trade in goods is characterized by uncertainty. Common sense and economic theory suggest that exporters, importers and households should try to hedge against this uncertainty. Natural candidates for hedging instruments are future and forward contracts. In fact, Ethier (1973) introduced the separation theorem and the full hedge theorem under exchange rate uncertainty, showing that demand for forward contracts perfectly compensates uncertainty. Benninga, Eldor and Zilcha (1985) and Kawai and Zilcha (1986) additionally discussed price level uncertainty, obtaining the same results. Recently, this strong result has been subject to some qualifications. Viaene and Zilcha (1998), for example, consider additionally output and cost uncertainty and find that under this setup full-double hedge and separation fail to hold.

Adam-Müller (2000) introduces inflation risk which cannot be hedged away and finds that full-hedge and separation break down if the two sources of risk in the model are not statistically independent. Market structure issues have been addressed as well, examples are Eldor and Zilcha (1987) and Broll and Zilcha (1992).

The empirical literature, though sparse, does not support the strong theoretical predictions of the early literature. As Carse, Williamson and Wood (1980) and others have shown, only roughly one-third of the value of international trade is covered by forward contracts. Even equity flows are only poorly hedged. According to Hau and Rey (2003), only 8% of US equity holdings abroad are hedged against exchange rate risks. Furthermore, there exists a lively debate in the empirical literature as to whether exchange rate volatility depresses trade levels or not. This debate is related to the issue of demand for forwards in that often the argument is made that as long as agents have access to well developed forward markets, the uncertainty should not matter. Strikingly, the evidence is rather mixed and seems to be independent of the existence of well developed forward markets (see Coté (1994) for a survey on the empirical evidence and Wei (1998) for a discussion of the underlying causes).

This chapter reconciles empirical findings with theoretical considerations. We show that by allowing agents to optimally choose their consumption bundle after resolution of price uncertainty - which is in contrast to the literature on forwards but standard in e.g. macro models with uncertainty - forward contracts resemble normal assets rather

¹Together with Klaus Wälde.

²Thanks are due to Jacques Olivier, Philipp Hartmann, Lucie White, Christian Gollier and seminar participants at the Universities of Bonn, Konstanz and Toulouse for discussion and helpful comments.

than insurance contracts. As forward contracts tend to have lower returns than e.g. physical capital, agents do not hold many of those assets and trade flows are only purely hedged.

We build an infinite horizon small open economy model where one good is domestically produced with capital and labour, another good is imported. Both goods are consumed. Capital is accumulated and risk averse households hedge optimally against terms of trade uncertainty.³ One forward contract allows (and obliges) them to buy one import good in the next period at a fixed price \bar{p}^Y .

We first study the determinants of demand for forwards. We show that the exogenous internationally given forward price \bar{p}^Y is the crucial determinant of demand for forwards. When this forward price equals the expected price of the import good, i.e. when the forward price is unbiased, risk averse households do not want to buy any forwards - they would actually want to sell forwards. When the forward price equals the price at which risk neutral households would be indifferent, risk averse households demand a positive amount of forward contracts.

Risk averse households want to sell forwards at unbiased prices as their utility function is concave in consumption levels. With consumption levels optimally chosen ex-post, indirect utility functions of individuals exhibit convexity in prices, though still concavity in expenditure. As expenditure is a function of prices as well, overall, the indirect utility function exhibits convexity in prices and households are actually (price-) risk lovers. Positive demand therefore requires a price that is sufficiently low, e.g. the price offered by risk neutral households. Intuitively, we could think of the risk averse households as not willing to commit themselves to a consumption decision when faced with price uncertainty. They do not want to give away the option to adjust their consumption bundles.

We then calibrate the model by using realistic and reasonable parameter values. We find that between 10% and 20% of international trade is covered by forward contracts. The low ratios cited in the empirical literature are therefore not surprising and may reflect the curvature of utility functions of utility maximizing households. Partial equilibrium setups or setups focusing on risk neutral firms should therefore be extended to take this aspect into consideration.

We are not the first that find that full-hedge theorem and separation theorem do not hold. As argued above, there is a substantial literature that finds that these two theorems will not hold as soon as certain conditions are violated. Our result, however, is derived in a

³In contrast to the majority of the literature on that topic, households demand forwards, not firms. This, however, simply follows from the general equilibrium setup we use. Firms are owned by the households, who look "through" them. A similar argument is made in Bacchetta and Wincoop (1998, pp. 18).

completely different manner. The crucial point is the decision structure of our agents. The standard approach assumes that *all* decisions are made *before* the resolution of uncertainty. In contrast, we employ an alternative decision rule, which is commonly used in macro models with uncertainty. In the first period, still before resolution of uncertainty, the agents decide upon their level of hedging and in the second, *after* the uncertainty is resolved, the agents actually make their consumption decision. Following this approach, agents will never be able to eliminate uncertainty from their budgets and hence are faced with a trade-off. As a consequence, risk averse agents will never buy forward cover under unbiased insurance prices.

The chapter is structured as follows: The next section introduces the model, section 3 presents the solution of the model and the subsequent section discusses equilibrium properties and presents results. Section 5 concludes the chapter.

2. The model

2.1. Technologies. We study a small open economy that produces one good X that is internationally traded. It imports a foreign consumption good Y which is not domestically produced. Domestic production requires capital K and labour L , which are non-tradable,

$$X_t = X(K_t, L_t). \quad (1.1)$$

Time is discrete and variables are indexed by t . The production function $X(\cdot)$ has the standard neoclassical properties. Firms produce under perfect competition and factor rewards w_t^L and w_t^K for labour and capital are given by their value marginal productivities,

$$w_t^L = p_t^X \partial X_t / \partial L_t, \quad w_t^K = p_t^X \partial X_t / \partial K_t. \quad (1.2)$$

The number of units of the import good to be exchanged for one unit of the export good, i.e. international terms of trade p_t^X / p_t^Y at a point in time t , are exogenously given to the economy and random. Before any trade in t takes place, prices for period t become common knowledge. Prices for period $t + 1$ are not known in t but the density function $f(p_\tau^X / p_\tau^Y)$ of p_τ^X / p_τ^Y for $\tau > t$ is common knowledge. In what follows, we choose X as numeraire and denote its price by p^X ,

$$p_{t+1}^X = p_t^X \equiv p^X.$$

One can therefore think of the price of the domestic good as a deterministic price and of the price of the foreign good as stochastic.

Domestic output X from the production process (1.1) is used for domestic consumption C_t^X , exports X_t^E and gross investment I_t ,

$$X_t = C_t^X + X_t^E + I_t. \quad (1.3)$$

Letting δ capture depreciation, capital grows according to

$$K_{t+1} = (1 - \delta) K_t + I_t. \quad (1.4)$$

In addition to producing the good Y , foreign agents offer forward contracts. At transaction costs of $\chi \geq 0$ per unit to be paid in t , domestic agents can buy forward contracts from foreign agents. Thus, foreign agents agree in t to sell in $t+1$ one unit of the foreign good at the exogenous internationally given price \bar{p}^Y . This is equivalent to fixing in t next periods terms of trade at p^X/\bar{p}^Y . When forward contracts of total volume D_t are signed, foreign agents agree to sell D_t units of good Y at \bar{p}^Y in $t+1$. Domestic buyers commit to buy in $t+1$ at this price, irrespective of the realization of p_{t+1}^Y .⁴

2.2. Households. The horizon of the economy is infinite. Agents in this economy live for two periods. They work in the first period of their life and consume in the second period. Consumption in the second period comprises both the domestically produced good and the foreign good.

2.2.1. *Preferences and budget constraints.* The utility function of households is given by

$$v = v(u(C_X, C_Y)),$$

where $u(C_X, C_Y)$ is some homothetic utility function and $v(\cdot)$ determines the degree of risk aversion. For illustrating purposes, we will later use

$$u(C_X, C_Y) = C_X^\alpha C_Y^{1-\alpha}, \quad 0 < \alpha < 1 \quad (1.5)$$

$$v(x) = \frac{x^\sigma}{\sigma}, \quad 1 \geq \sigma > 0. \quad (1.6)$$

Note that the utility function (1.5) displays risk aversion towards the consumption *levels*. Risk aversion in total consumption *expenditure* is given for $0 < \sigma < 1$, risk neutrality in consumption expenditure would be represented by $\sigma = 1$.

A household's first period budget constraint equates labor income with savings and transaction costs for financial contracts D_t ,⁵

$$w_t = s_t + \chi D_t. \quad (1.7)$$

Savings are used to buy capital goods s_t/p^X . There is the implicit assumption of a market in which today's old, being the owners of the capital stock sell it to today's young in exchange for consumption good

⁴If, in contrast, D_t represented options, domestic agents would not be obliged to buy and thus only draw on the contract in favorable situations. This will be analysed in section 4.5.

⁵We are grateful to one Referee who pointed out that our setup is similar to an endowment economy: The endowment is given by the wage w_t and agents decide whether to transfer this endowment into the next period by holding capital or buying forward contracts.

X , which in turn constitutes the wage of today's young. The sum over all individual savings equal the current capital stock (i.e. after depreciation) plus additional aggregate investment,

$$K_{t+1} = (1 - \delta) K_t + I_t = \frac{s_t}{p^X} L. \quad (1.8)$$

In the second period, households use all of their wealth and other income for financing consumption expenditure e_{t+1} . End of second period wealth amounts to $p^X (1 - \delta) \frac{s_t}{p^X} = (1 - \delta) K_{t+1}$. Factor rewards for wealth amount to $p^X \frac{\partial X_{t+1}}{\partial K_{t+1}} \frac{s_t}{p^X}$. Income from forward contracts is $(p_{t+1}^Y - \bar{p}^Y) D_t$, which might be negative. Hence

$$e_{t+1} \equiv p^X C^X + p_{t+1}^Y C^Y = (1 + r_{t+1}) p^X \frac{w_t - \chi D_t}{p^X} + (p_{t+1}^Y - \bar{p}^Y) D_t, \quad (1.9)$$

where we defined

$$1 + r_{t+1} \equiv 1 + \frac{\partial X_{t+1}}{\partial K_{t+1}} - \delta \quad (1.10)$$

and savings s_t were replaced by using the first period budget constraint (1.7).

The second period budget constraint (1.9) nicely shows that payoffs $(p_{t+1}^Y - \bar{p}^Y) D_t$ from forward contracts are positive and therefore a second period source of income when the price p_{t+1}^Y of good Y is sufficiently high relative to its exogenous price \bar{p}^Y specified one period before. Forward contracts imply a loss in the case of low price of good Y . Of course, bad terms of trade shocks leading to income and good terms of trade shocks leading to losses from forward contracts are the reason why forwards exist: they insure against terms of trade shocks.

This budget constraint also shows that households can *not* insure fully against terms of trade risk. Forward contracts refer to a certain amount of goods that can be purchased at this fixed price \bar{p}^Y . As the actual amount of goods consumed depends on the realization p_{t+1}^Y of the price, some uncertainty always remains. This is the crucial departure of our model from the classic setups in the hedging literature Ethier (1973, pp. 496) and Benninga et al. (1985, pp. 540). There, firms decide today in t how much they will produce tomorrow in $t + 1$. This allows them to fully insure against uncertainty in the price of their output good. The well-known separation theorem of no uncertainty after hedging results. If our agents knew how much they will consume tomorrow, full hedging would be possible as well. They will never know, however, as price uncertainty has an income effect as well.

2.2.2. A no-bankruptcy constraint. In order to avoid insolvency of households, we have to introduce a no-bankruptcy constraint. Our point of departure is the expenditure equation (1.9). As negative expenditure is not feasible, we argue that the worst that can happen to

the budget of our agents is an expenditure of zero,

$$e_t = (1 + r_{t+1}) w_t + (p_{t+1}^Y - (1 + r_{t+1}) \chi - \bar{p}^Y) D_t = 0.$$

Solving for D_t yields

$$D_t = \frac{(1 + r_{t+1}) w_t}{(1 + r_{t+1}) \chi + \bar{p}^Y - p_{t+1}^Y}.$$

Regarding our forward, the most unfavorable situation for households is $p_{t+1}^Y = 0$. Prudence thus demands that the amount of D_t an agent is allowed to purchase shall never be any greater than

$$D_t \leq \frac{(1 + r_{t+1}) w_t}{(1 + r_{t+1}) \chi + \bar{p}^Y}. \quad (1.11)$$

This condition makes intuitively sense: the greater the contracted price \bar{p}^Y and the greater the costs of forward cover χ , the smaller the amount of forwards the agents are allowed to buy. Similarly, the greater the interest rate and wage income w_t , the greater the amount of D_t the agents can commit to. Note that the interest rate r_{t+1} is deterministic and hence known in period t , since the capital stock is deterministic and there are no technology shocks in the model. The expression $(1 + r_{t+1}) w_t$ then simply gives maximum period $t + 1$ income, computed in period t . The denominator of (1.11) in turn gives the highest possible costs of the forward position, evaluated in period t . This ratio gives the number of forwards D_t an agent can buy such that in the most unfavorable realization of forward prices the agent still has a non-negative expenditure level.

3. Solving the model

3.1. The maximization problem of households. The maximization problem of households consists in choosing the amount D_t of forward contracts and optimal consumption levels C_X and C_Y such that expected utility $E[v(u(C_X, C_Y))]$ is maximized, given the budget constraint (1.9).

Conceptually, maximization can be subdivided into two steps. The second step consists in allocating consumption expenditure to goods X and Y , taking consumption expenditure as given. This second sub-problem is solved after realization of terms of trade. It is therefore a choice under certainty. The Cobb-Douglas specification (1.5) implies

$$C_{t+1}^X = \frac{\alpha e_{t+1}}{p^X}, \quad (1.12)$$

$$C_{t+1}^Y = \frac{(1 - \alpha) e_{t+1}}{p_{t+1}^Y}. \quad (1.13)$$

These equations hold at each point in time and determine consumption levels after uncertainty has been resolved.

The first step consists in choosing the optimal amount D_t of forward contracts that maximizes $E[v(e_{t+1}/P(p^X, p_{t+1}^Y))]$ where $v(e_{t+1}/P(p^X, p_{t+1}^Y))$ is indirect utility where consumption levels in the homothetic utility function $u(C_X, C_Y)$ have been replaced by optimal consumption levels. Utility $u(C_X, C_Y)$ can then be written as expenditure divided by the price index. In the Cobb-Douglas case, the price index reads $P(p^X, p_{t+1}^Y) = \Phi p_X^\alpha p_Y^{1-\alpha}$, where Φ is a constant. Expenditure is given by (1.9).

This two-step solution to our maximization problem is made possible by assuming that consumption takes place only when agents are old. If consumption were to take place in both periods, the consumption choice in the first period would be linked to the saving decision. The system that would have to be analyzed would be more complicated (as an intertemporal consumption rule would have to be added).

The solution to this problem is then given by the first order condition

$$E \left[v' \left(\frac{e_{t+1}}{P(p^X, p_{t+1}^Y)} \right) \frac{p_{t+1}^Y - (1 + r_{t+1}) \chi - \bar{p}^Y}{P(p^X, p_{t+1}^Y)} \right] = 0, \quad (1.14)$$

where the expectations operator refers to the entire bracket $[\cdot]$. This condition consists of two parts. The first is marginal utility $v'(\cdot)$, here expressed in the form of the indirect utility function. Marginal utility is positive but decreasing in consumption levels, or as stated here, increasing in expenditure and decreasing in prices. The denominator of the second term, $p_{t+1}^Y - (1 + r_{t+1}) \chi - \bar{p}^Y$, represents the realized nominal return from the forwards. Its expected value is negative under unbiased (or actuarially fair⁶) forwards, i.e. if $E[p_{t+1}^Y] = \bar{p}^Y$, since the term $(1 + r_{t+1}) \chi$ representing the opportunity costs of entering the forward market enters negatively. If forwards could be obtained without costs, clearly these opportunity costs would vanish and unbiased forwards would have an expected nominal return of zero. Dividing the nominal return by the price index gives the complete second term, the real return of the forward contract.

Now that the meaning of the two components of (1.14) is clear, the intuition of this first order condition is more easy to see. The expectations operator is an integral in our case where terms-of-trade is a continuous random variable. This integral can be split into a

⁶In the literature, actuarially fair refers to a situation where the expected payoff of an insurance is equal to the insurance premium (Kreps, 1990, p. 92 or Dixit, 1990, p. 124). Unbiasedness usually describes a (statistical) property of the forward price, i.e. $E[p^Y] = \bar{p}^Y$ (Zilcha and Broll, 1992, p. 475 or Viaene and Zilcha, 1998, p. 594). As we do not explicitly model how the forward price \bar{p}^Y is determined, we use the expression unbiasedness. Note, however, that the two concepts would be identical if we assumed that the forward price is the outcome of competition among perfectly competitive firms and χ are opportunity cost of buyers of insurance (e.g. shipping cost) and not risk-premia for the insurer.

negative and a positive part. As long as $p_{t+1}^Y < (1 + r_{t+1})\chi + \bar{p}^Y$, the realized price is in the "loss region". Marginal utility is multiplied by a negative number and this interval of the integral contributes negatively. Concavity of the utility function with respect to the amount of forwards implies that as long as (1.14) is negative, the agents have too many forwards and hence should decrease holdings. On the other hand, as soon as $p_{t+1}^Y > (1 + r_{t+1})\chi - \bar{p}^Y$ - the "win region" - marginal utility contributes positively. Again, concavity tells us that as long as (1.14) is positive agents should increase holdings of D_t . However, given positive costs to obtain forward cover, i.e. $\chi > 0$, increasing D_t will increase r_t , hence opportunity costs will rise as well, up to a point where marginal utility will fall in D_t . Hence the optimal amount of D_t is such that the positive and the negative components of the integral just balance.

3.2. Reduced form. The reduced form of the model consists of two equations. The capital stock in the next period is given by savings today times the number L of individuals and divided by the price of one unit of capital and is given by (1.8). With the first-period budget constraint (1.7) giving individual savings, we obtain

$$K_{t+1} = \frac{p_t^X \partial X_t / \partial L - \chi D_t}{p^X} L, \quad (1.15)$$

where the wage rate was replaced by its value marginal product (1.2).

The amount of forward contracts is determined by the first order condition (1.14). Consumption of the old is given by the current capital stock, interest payments on the current capital stock plus income (or losses) from forward contracts. Using the budget constraint (1.9), where wages w_t were replaced by value marginal productivities in (1.2), expenditure in (1.14) therefore equals

$$e_{t+1} = (1 + r_{t+1}) p^X \partial X(K_t, L) / \partial L + (p_{t+1}^Y - (1 + r_{t+1})\chi - \bar{p}^Y) D_t. \quad (1.16)$$

Equilibrium in our economy is therefore described by equations (1.14) and (1.15), given (1.16). These equations determine the two variables K_t and D_t , given an initial capital stock K_0 .

Equation (1.15), determining the evolution of capital, shows that next periods capital is known in t . By contrast, expenditure (1.16) is uncertain when some forward contracts are signed. This makes consumption levels of both goods and exports and imports uncertain. If no forward contracts are signed ($D = 0$), expenditure is deterministic, consumption of good X would be deterministic but consumption of good Y would be stochastic.

3.3. Steady-state. In the steady state, the capital stock is the same in each period. Variables that are constant are printed without a time subscript. All stochastic variables are denoted by a tilde (\sim).

The capital stock is then determined by

$$K = \frac{p^X \partial X / \partial L - \chi D}{p^X} L \quad (1.17)$$

and is therefore a deterministic variable. Domestic production (1.1) is then deterministic as well, $X = F(K, L)$. Steady state expenditure \tilde{e} is given from (1.16) as

$$\tilde{e} = (1+r)p^X \partial X / \partial L + (\tilde{p}^Y - (1+r)\chi - \bar{p}^Y) D \quad (1.18)$$

and remains stochastic. Using (1.18), D follows implicitly from the first order condition (1.14),

$$E \left[v' \left(\frac{\tilde{e}}{P(p^X, \tilde{p}^Y)} \right) \frac{\tilde{p}^Y - (1+r)\chi - \bar{p}^Y}{P(p^X, \tilde{p}^Y)} \right] = 0. \quad (1.19)$$

4. Equilibrium properties

Given the steady state quantities of the capital stock K and forward contracts D as determined in (1.17) and (1.19) with (1.18), will agents want to hold a positive amount of forwards? This will be analyzed in the next subsection. In order to obtain an idea about quantitative predictions, we calibrate the model in the subsequent section and provide numerical results afterwards. We also perform a comparative static analysis and finally introduce options as an alternative to forwards. By deriving several equilibrium properties under options, the properties of forwards will also become clearer.

4.1. The equilibrium demand for forwards. We now present three important results with respect to the existence of interior solutions, i.e. a positive demand for forwards D in the steady state. For simplicity, we set transaction cost equal to zero, $\chi = 0$, in what follows. Note that this implies by (1.17) a capital stock that is independent of the choice of D . All proofs are in app. 6.1.

THEOREM 1.1. *Risk averse agents will not buy forward cover at unbiased prices, i.e. $E[\tilde{p}^Y] = \bar{p}^Y$.*

We illustrate this result in figure 1. It plots expected utility of agents in the steady state, $E[v(\tilde{e}/P(p^X, \tilde{p}^Y))]$, as a function of forwards D , taking expenditure from (1.18) into account.⁷

Figure 1: Expected utility as a function of forwards D

The figure shows how expected utility of households depends only on forwards, provided that they anticipate the choice of consumption levels, and thereby illustrates the maximization problem of section 3.1.

⁷All numerical results were obtained by using Mathematica. The files are available upon request.

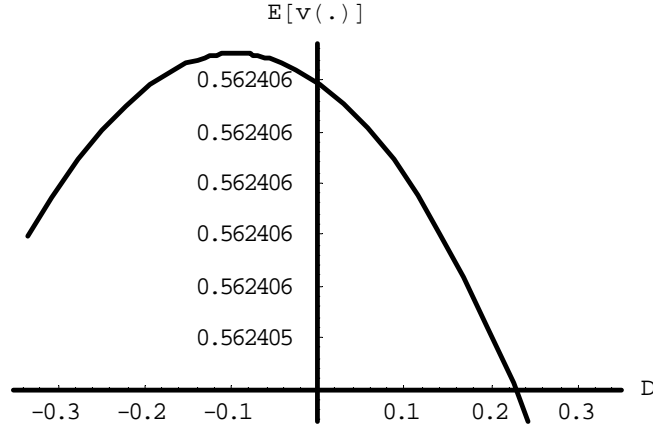


FIGURE 1

Since our objective function is globally concave in D (see app. 6.4), the sign of the first derivative of this function with respect to D at the point $D = 0$ determines whether or not there is an interior solution. As plotted above, expected utility would be maximized at a negative D . Agents therefore do not want to hold forward contracts.

In the light of the existing literature on the topic this result is rather surprising. The standard result states⁸ that if an unbiased forward market exists, agents use this market to avoid all uncertainty, i.e. they obtain full cover for their position. The crucial difference of our model to the literature lies in the timing structure. The main body⁹ of the literature assumes that all decisions are made *before* uncertainty is resolved. In contrast, we assume, as is standard in e.g. stochastic macro models, that although the agents decide on the optimal amount of forward cover before uncertainty is resolved, their consumption decision is made after the resolution of the price uncertainty. Under this setup, buying forward contracts amounts to no less than restricting one's possibilities to adjust to price realizations. Risk averse agents will not give away this opportunity. It is clear that there are some decisions that will be made in advance and for this part the analysis of the existing literature would be appropriate. We believe, however, that most of consumption decisions are made when actual consumption takes place and prices are known.

THEOREM 1.2. *Risk averse agents will only buy forward cover for sufficiently low \bar{p}^Y , i.e. $E[\hat{p}^Y] > \bar{p}^Y$.*

⁸See, inter alia, Ethier (1973), Benninga et al. (1985), Kawai and Zilcha (1986), Eldor and Zilcha (1987), Viaene and de Vries (1992), Zilcha and Broll (1992), Viaene and Zilcha (1998) and Adam-Müller (2000).

⁹There are a few papers that discuss the theoretical possibility of a different timing structure, an example being Perée and Steinherr (1989). We are, however, not aware of any work that explicitly models this.

Note that this result follows from the first theorem. One possible interpretation would be that if \bar{p}^Y is lower than the expected value of the price p_{t+1}^Y , the average return of a forward position will be positive. Thus the agent will be compensated for giving up their possibility to adjust their consumption bundle according to the price realizations in the next period. Hence the agents are willing to hold a forward position.

THEOREM 1.3. *If the exogenous forward price amounts to $\bar{p}^Y = \frac{E[p_Y^\alpha]}{E[p_Y^{\alpha-1}]}$, i.e. the price risk neutral households would offer, risk averse agents will buy forward contracts.¹⁰*

To illustrate the third result, imagine a figure similar to figure 1 for risk-neutral households. Letting the forward price be given by the risk-neutral price $\bar{p}^Y = E[p_Y^\alpha] / E[p_Y^{\alpha-1}]$, the slope of the expected utility at $D = 0$ is zero. The slope of expected indirect utility at this point $D = 0$ can be expressed, for any given value of \bar{p}^Y , as a function of the degree of risk aversion. Theorem 3 essentially states that the more risk averse agents are, the larger the slope becomes. Hence, moving from risk neutrality, i.e. $\sigma = 1$, to risk aversion is equivalent to shifting the whole graph to the right. This in turn implies that the forward price \bar{p}^Y at which the risk neutral agents are just indifferent between buying and selling induces a positive demand by any risk averse agent.

Note that these results may be somewhat surprising, given the "full-hedge theorem" we normally encounter in the literature (see Ethier (1973) and Kawai and Zilcha (1986) for example). The reason for this is that our model differs from the usual models such that agents always face uncertainty through the price-index channel, whereas in the former models there is the possibility to avoid all uncertainty, for agents completely decide upon their plans in period one. Risk averse agents do not want to lose the ability to adjust to price shocks in the next period, whereas risk neutral agents are indifferent towards this opportunity.

Secondly, we have another factor at work here. By buying forward contracts the agents trade one risk against the other. Holding a forward position means that risk now directly affects nominal income. This can be easily seen from (1.9). Risk aversion regarding nominal income and the uncertainty through the price-index channel are the reasons for the agents asking for more than unbiased forwards.

Figure 2: The indirect utility function which is convex in p^Y

The convexity of the indirect utility function with respect to the prices is illustrated in figure 2. It shows indirect utility as a function of the foreign price. Convexity of indirect utility with respect to prices

¹⁰Strictly speaking, we should write $(p^Y)^\alpha$. To simplify notation, we use p_Y^α and deviate from our convention of indicating the type of the good with superscripts.

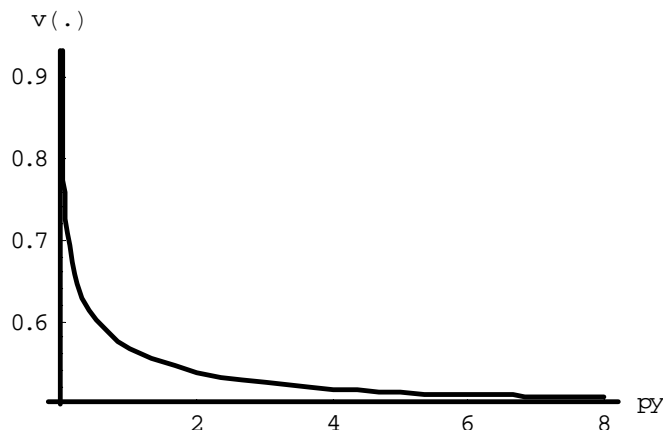


FIGURE 2

implies that agents prefer any linear combination of prices to the average of this linear combination. Hence, agents are in fact risk-lovers with respect to period two price uncertainty. Note that this result entirely hinges on the timing assumption of the consumption decision.

4.2. Calibrating the model. We will now calibrate our model as this allows us to provide quantitative results in the next subsection and perform a comparative static analysis subsequently. We begin with discussing the chosen values. Solving the model numerically involves computing values of both D and K which satisfy (1.17) and simultaneously (1.19). To get numerical results, we need to specify a couple of parameters and the underlying distribution. As far as possible, this is achieved by drawing on real world data.

Parameter	L	α	β	δ	χ	σ	Φ	S	p_X
Value	100	$\frac{4}{5}$	$\frac{3}{10}$	0.54	$\frac{1}{100}$	$\frac{1}{2}$	$\frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}$	1	1

Table 1: Parameter values used for calibrating

As a first step, we specify the production technology by a Cobb-Douglas form, $X(K, L) = SK^\beta L^{1-\beta}$. The scale parameter for the technology, S , is set to one. Equation (1.17) depends on various parameters: P_X is the price of the numeraire good and can thus be set to one. Transaction costs for forwards are captured by χ in our setup. One could think of χ as including some kind of market price of the forward contract or of obtaining forward cover. These two concepts are in fact quite different. In reality, the market price of the forward cover is quite small, whereas the real costs of obtaining forward cover may very well be substantial.¹¹ This leaves some room for determining

¹¹Think of a firm which has to hire expertise to contract such cover and thus may have substantial costs. In terms of transfers, the χD s, think of margin requirements.

the value of χ and thus we will set this value arbitrarily, but close to zero. In our calibration we use $1/100$. The size of the population, as the *TFP* measure S , is just a scale parameter and therefore no further elaboration is necessary. We set $L = 100$. The output elasticity β in our production function reflects relative shares of capital and labour and is commonly found (e.g. Maddison, 1987, p. 658) to be around 0.3.

The second reduced form equation (1.19) and (1.18) require the specification of some parameters as well. Depreciation is assumed to be 2.5% per year. With one period representing 30 years in our two-period OLG setup, we have $\delta = 0.54$.¹² The price \bar{p}^Y is determined by the price at which risk neutral individuals would offer the forwards, i.e.¹³

$$\bar{p}^Y = \frac{E[\tilde{p}_Y^\alpha] - (1+r)\chi E[\tilde{p}_Y^{\alpha-1}]}{E[\tilde{p}_Y^{\alpha-1}]} \quad (1.20)$$

Equation (1.20) is determined by using the first order condition (1.19), setting $\sigma = 1$ and solving for \bar{p}^Y (see app. 6.1). The parameter of the utility function, α , determines the share of domestic in total consumption. Using data from 'Statistisches Bundesamt', the empirically observed share of foreign products in total consumption in Germany is approximately 0.8. To determine the most appropriate distribution, we obtained monthly price index data for both import prices and export prices over the period January 1962 until January 2002, leaving us with 482 observations. Dividing the import index by the export index amounts, in terms of our model, to obtaining the price series p_t^Y . The shape of the histogram suggested choosing a lognormal distribution, which is an assumption commonly made, for example in the finance literature.¹⁴ The parameters of the distribution were obtained by maximum likelihood estimation.¹⁵ The estimates were

Distribution	$E[\tilde{p}^Y]$	$E[\tilde{p}^Y]^2$
lognormal	0.1149	0.0071
underlying normal	1.1261	0.0103

Table 2: Parameters of the lognormal and the underlying normal distribution

¹²This follows from $(1 - 0.025)^{30} \approx 0,46$. Hence 46% of the capital stock remains and 54% are lost after 30 years of constant annual depreciation of 2,5%.

¹³As stated in the model section, \bar{p}_Y is exogenously given by international markets. We use this equation to find a plausible value for \bar{p}_Y . It does not mean that \bar{p}_Y is endogenous in our model.

¹⁴The Black-Scholes formula relies on lognormality of prices. Even in international macro this assumption is often used, see for example Obstfeld and Rogoff (1998).

¹⁵We use R and the function `fitdistr` which is included in the MASS package.

4.3. A numerical solution. We now present a simulation result for a small country. Under lognormal distributed price uncertainty, using the parameter specification we presented above, we found that the economy will buy a total amount of 1.62 units of forward contracts, given the price risk neutral agents would offer. The capital stock and thus GDP of the economy can be calculated and using the mean on the distribution as the realization of the price in period two, the economy will import 9.5 units of good Y . This means that the forward cover to import ratio is in this case approximately 17%. This is in accordance to surveys on the topic. For example Carse et al. (1980) found that firms that import or export and thus face terms of trade risk, only cover between 15-30% of their open positions.

Some caveats are in order here. First, the actual terms of trade variance may well be underestimated with our proxy used. If this is true, the calculated amount of forwards is too high as well. Second, the costs of forwards we used are to some degree arbitrary. They are, however, close to the actual transaction fees charged by banks but would not incorporate such items as information costs and fixed costs for setting up the appropriate institutions, letting alone deliberation costs. To the extent to which the actual costs are higher, our result overestimates the amount of forwards purchased. Third, there is the issue of the degree of risk aversion with respect to wealth. In the literature there is no consensus on that parameter. We choose to set this parameter, $1-\sigma$ in our model, to $1/2$, which is a conservative choice in the sense that a broad range of publications support this choice. It also turns out that this particular parameter is the least influential in altering our results. Lastly, our result is to some extent related to the literature on international portfolio diversification, i.e. the home bias puzzle in equity holdings. One strand of this literature (see for example Baxter and Jerman (1997)) argues that in order to explain actual portfolio holdings quantitatively, one needs to consider multiple sources of uncertainty. Recently, however, other contributions - see as an example Obstfeld and Rogoff (2000) who consider trade costs as the relevant explanation for the observed home bias in equity holdings - have relied on a more parsimonious specification with only one source of uncertainty. We follow here the more parsimonious approach. The aforementioned qualifications notwithstanding, this numerical exercise recapitulates our analytical results and shows that the model is able to fit the actual data for reasonable parameter values.

4.4. Comparative statics. There are a couple of interesting questions arising when changing the parameters. We begin with the terms of trade variance. If there is an exogenously induced increase in the variance of the foreign price, we observe a fall in the demand for forwards. At our calculated equilibrium point, we observe a decrease of

4.7% in demand for forwards if we increase the variance by 1%. This is accordance with the intuition for our results. Risk averse agents are not willing to give up the possibility to adjust themselves to a terms of trade shock. The greater the likelihood of a terms of trade shock, the more they have to be compensated for holding forward contracts.

Next consider the costs of the forwards. If costs decrease, demand will increase. At the point of our interior solution a 1% decrease in the costs would induce a 16% rise in the demand for forward contracts.

Lastly we look at the degree of risk aversion. A society which is more risk averse than another will demand less forward cover than the less risk averse society. A 1% increase of the degree of risk aversion, i.e. a 1% fall in σ , reduces demand for forwards by 0.4%.

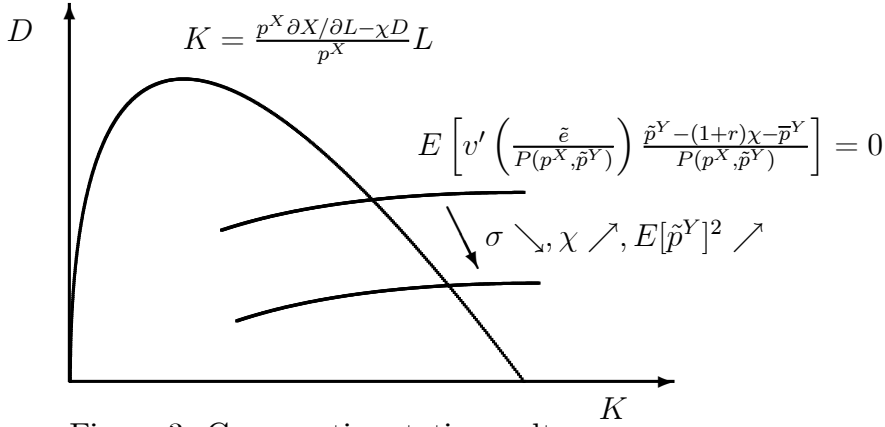


Figure 3: Comparative static results

The comparative static results are summarized in Figure 3. An increase in the variance of p^Y , an increase in the costs χ and an increase in the degree of risk aversion will ceteris paribus decrease the demand for forward cover by shifting the schedule implied by (1.14) downwards. Note that in the case of changing costs χ , the capital schedule will also shift.

4.5. Options. In order to give additional insights into the workings of our model, we will in this section examine what the optimal hedging behavior would be if the agents could buy options instead of forward contracts to insure against the uncertainty regarding the price of the foreign good. A (call) option, as opposed to a forward contract, does not oblige to buy the underlying asset (or commodity), instead the buyer can choose whether or not he will exercise his option. Keeping our notation, we can extend our model very easily to model an option instead of a forward contract by observing that in the event $p_t^Y \leq \bar{p}^Y$, the buyer of that option would simply not exercise it. To model options, we only have to change the expenditure equation into

$$e_{t+1} = \left\{ \begin{array}{l} (1 + r_{t+1})(w_t - \chi D_t) \\ (1 + r_{t+1})(w_t - \chi D_t) + (p_{t+1}^Y - \bar{p}^Y) D_{t+1} \end{array} \right\} \quad \forall p_{t+1}^Y \left\{ \begin{array}{l} \leq \\ > \end{array} \right\} \bar{p}^Y.$$

D_t now denotes the amount of options instead of forward contracts, the strike price being \bar{p}^Y . By buying one option for the price χ , an agent is entitled to buy one unit of good Y in the next period for the price \bar{p}^Y . The first order condition (1.14) now becomes

$$\int_0^{\bar{p}^Y} v' \left(\frac{(1+r_{t+1})(w_t - \chi D_t)}{P(p^X, p_{t+1}^Y)} \right) \frac{-\chi(1+r_{t+1})}{P(p^X, p_{t+1}^Y)} dP^Y + \int_{\bar{p}^Y}^{\infty} v' \left(\frac{e_{t+1}}{P(p^X, p_{t+1}^Y)} \right) \frac{p_t^Y - \bar{p}^Y - \chi(1+r_{t+1})}{P(p^X, p_{t+1}^Y)} dP^Y = 0, \quad (1.21)$$

where P^Y is the cumulative density function of p_{t+1}^Y . Three results emerge for the steady state (see app. 6.3).

THEOREM 1.4. *If options are costless, i.e. $\chi = 0$, the optimal amount is infinity, $D = \infty$.*

This is probably the most straightforward result. Rational agents, being offered a free lunch, will happily accept this. Here the free lunch comes as a free lottery ticket, without any risk of losing. We present this otherwise not very surprising result to make the structure of the decision problem clearer.

THEOREM 1.5. *If agents can choose between options and forwards at the same costs, they will always choose options.*

To facilitate the comparison between forwards and options, we present the second result. It constitutes, again, a standard property of the utility function of the agents. Forwards will always be dominated by options, as long as the price is the same for both.

These two theorems imply that we can replicate the real-world co-existence of options and forwards in our model. This necessitates that either forwards cost less or are more than unbiased (or both).

THEOREM 1.6. *Let transaction costs for options be given by χ . If options are unbiased, i.e. $E[\tilde{p}^Y] = \bar{p}^Y$, agents will demand a positive amount of options.*

Our last results highlights again the difference between forwards and options. In contrast to forward contracts there exist a positive demand, depending on the price χ , of "unbiased options", that is options that have a strike price that equals the expected value of the price in the next period.

5. Conclusion

One largely debated issue in international economics is the question whether or not volatility in exchange rates and terms of trade depresses trade levels. There is an extensive literature on that question, both theoretical and empirical. The main body of the theoretical literature

claims that terms of trade and/or exchange rate uncertainty does not matter as long as well developed forward and futures markets exist. This literature further predicts that agents fully hedge the existing risks. The empirical work done in this field fails to unambiguously support these findings.

We model a small open economy that is subject to terms of trade risk originating entirely from abroad. Agents can buy forward contracts to insure against this uncertainty but can adjust consumption bundles after terms of trade have realized. This small departure from the standard assumption in the hedging literature where consumption can not be adjusted after resolution of uncertainty implies that forward contracts turn into an asset. When forward contracts are unbiased, there is no demand for terms of trade insurance, a direct effect of the convexity of the indirect utility function with respect to prices. Risk aversion with respect to consumption levels and expenditure levels is not a sufficient motive to buy forwards. We derive conditions under which, on part of the risk averters, a positive demand for forwards exists. Again, this demand does not stem from hedging but purely from investment motives.

We calibrate our model with data for Germany to obtain numerical solutions. The equilibrium amount of forwards contracted in relation to the equilibrium amount of imports closely resembles the empirical observed values, thus providing a rationale for the apparent underhedging of domestic agents against price level and/or exchange rate uncertainty. The reason for low hedging lies again in the asset-nature of forwards: As returns for forwards should be lower than returns for e.g. capital, few futures will be held and hedging is low. We also showed that options, in contrast to forwards, will be bought as means of insurance. At unbiased prices, options strictly dominate forward contracts. This may help explain why the market for options has grown exponentially over the last decade or so.

The main contribution of our analysis, however, is that the "price-convexity" effect should be incorporated in the existing models, which could be achieved by giving up the assumption that all plans are irrevocably made in the period which precedes the resolution of the uncertainty. This should alter dramatically the strong theoretical predictions of this literature with respect to forward markets and should thus provide a better understanding of the effects at work here. Since forwards are unattractive and options perhaps too expensive, our analysis may also provide an additional argument *in favor* of international capital flows, and hence capital account liberalization, as a means of insuring the economy.

Our work can be extended in some promising ways. First, to understand the implications of covariance effects so often at work in the hedging process money and thus a nominal exchange rate could be

brought into the model. This would also allow a comparison between our modeling approach and the existing literature that has proceeded with considering multiple sources of risk. Another interesting extension would be to explicitly study the effect of heterogeneity in risk-aversion. This would allow to endogenize the forward price \bar{p}^Y and thereby to confirm (as we would expect) that returns on forwards as assets are low. This would strengthen our explanation that trade coverage is low because forwards are assets.

6. Mathematical appendix

6.1. Proofs of forward theorems.

PROOF OF THEOREM 1. Consider the first-order condition (1.19) and set transaction costs to zero, $\chi = 0$. Analyzing the point $D = 0$, the derivative of expected utility is negative iff

$$E \left[\left(\frac{(1+r)w}{\tilde{p}_Y^{1-\alpha}} \right)^{\sigma-1} \frac{\tilde{p}^Y - \bar{p}^Y}{\tilde{p}_Y^{1-\alpha}} \right] < 0 \Leftrightarrow \quad (1.22)$$

$$E \left[((1+r)w)^{\sigma-1} \left(\tilde{p}_Y^{-\sigma(1-\alpha)} (\tilde{p}^Y - \bar{p}^Y) \right) \right] < 0 \Leftrightarrow \quad (1.23)$$

$$((1+r)w)^{\sigma-1} \{ E[\tilde{p}_Y^{-c}] E[\tilde{p}^Y] + Cov[\tilde{p}_Y^{-c}, \tilde{p}^Y] - \bar{p}^Y E[\tilde{p}_Y^{-c}] \} < 0,$$

where the last equality used the definition $E[\tilde{p}_Y^{-c}] \equiv E[\tilde{p}_Y^{-\sigma(1-\alpha)}]$, i.e.

$$c = \sigma(1-\alpha) > 0. \quad (1.24)$$

Two results emerge. First, since $Cov[\tilde{p}_Y^{-c}, \tilde{p}_Y]$ is negative¹⁶, we have for $E[\tilde{p}_Y] = \bar{p}^Y$:

$$((1+r)w)^{\sigma-1} * Cov[\tilde{p}_Y^{-c}, p_Y] < 0$$

Together with the result from app. 6.4 that $d^2 EU/dD^2 < 0$, we know that there cannot be an interior solution with $D > 0$. \square

PROOF OF THEOREM 2. For any interior solution, we need the first order condition to be fulfilled. This requires, at the point $D = 0$,

$$E[\tilde{p}_Y^{-c}] E[\tilde{p}_Y] + Cov[\tilde{p}_Y^{-c}, \tilde{p}_Y] - \bar{p}^Y E[\tilde{p}_Y^{-c}] = 0.$$

This implies $E[\tilde{p}_Y] + \frac{Cov[\tilde{p}_Y^{-c}, \tilde{p}_Y]}{E[\tilde{p}_Y^{-c}]} = \bar{p}^Y < E[\tilde{p}_Y]$, which is our second result. \square

PROOF OF THEOREM 3. The third result is approached in a slightly different manner. Define a function $\xi(c)$ which gives the sign of the first order condition (1.19) at the point $D = 0$. This function is from (1.23) given by

$$\begin{aligned} \xi(c) &= E \left[\left(\tilde{p}_Y^{-\sigma(1-\alpha)} (\tilde{p}^Y - \bar{p}^Y) \right) \right] = E \left[\tilde{p}_Y^{1-\sigma(1-\alpha)} - \tilde{p}_Y^{-\sigma(1-\alpha)} \bar{p}^Y \right] \\ &= E[\tilde{p}_Y^{1-c}] - \bar{p}^Y E[\tilde{p}_Y^{-c}], \end{aligned}$$

¹⁶This follows from the fact that in our case we have $f'(p_Y) * g'(p_Y) \leq 0 \forall p$ where $f(p_Y) = p_Y$ and $g(p_Y) = p_Y^{-c}$. An application of Chebychevs second inequality brings the result that $Cov(p_Y, p_Y^{-c}) \leq 0$. See Hardy, Littlewood and Polya (1952, pp. 43 and p.168).

where the last step used the definition of c in (1.24). Surely, a price \bar{p}^Y for which an interior solution exists is given by

$$\bar{p}^Y = \frac{E[\tilde{p}_Y^{1-c}]}{E[\tilde{p}_Y^{-c}]}.$$

When we consider risk neutral households, the optimization problem will remain unchanged. Hence, the price at which risk-neutral agents, for which $\sigma = 1$, are indifferent between buying and selling forwards is,

$$\bar{p}^Y = \frac{E[\tilde{p}_Y^\alpha]}{E[\tilde{p}_Y^{\alpha-1}]} \quad (1.25)$$

At this price and for risk-neutral agents, the slope of the expected utility function at $D = 0$ is zero.

Now differentiate $\xi(c)$ with respect to c ,

$$\begin{aligned} \frac{d\xi}{dc} &= -E[\tilde{p}_Y^{1-c} \ln p_Y] + \bar{p}^Y E[\tilde{p}_Y^{-c} \ln \tilde{p}_Y] \\ &= -E[\tilde{p}_Y^{1-c}] E[\ln \tilde{p}_Y] - Cov[\tilde{p}_Y^{1-c}, \ln \tilde{p}_Y] \\ &\quad + \frac{E[\tilde{p}_Y^{1-c}]}{E[\tilde{p}_Y^{-c}]} E[\tilde{p}_Y^{-c}] E[\ln \tilde{p}_Y] + Cov[\tilde{p}_Y^{-c}, \ln \tilde{p}_Y] \\ &= -Cov[\tilde{p}_Y^{1-c}, \ln \tilde{p}_Y] + Cov[\tilde{p}_Y^{-c}, \ln \tilde{p}_Y]. \end{aligned}$$

Observe that c in (1.24) is bounded, $0 < c < 1$. Hence, the derivative is negative, for both covariance terms are negative. This implies that the more agents are risk averse, i.e. the lower c , the more the slope of the first order condition for forwards at $D = 0$ increases.

If forwards are offered at a price $\bar{p}^Y = E[\tilde{p}_Y^\alpha] / E[\tilde{p}_Y^{\alpha-1}]$ for which risk-neutral agents are indifferent, the slope at $D = 0$ is zero for $\sigma = 1$. Hence, the slope at $D = 0$ for risk averse agents, for whom $\sigma < 1$, is positive, expected utility is maximized at $D > 0$. They will buy forward contracts. \square

6.2. Equation (1.20). We start with the first-order condition (1.19), $E\left[\left(\frac{e}{P(\cdot)}\right)^{\sigma-1} \frac{p^Y - (1+r)\chi - \bar{p}^Y}{P(\cdot)}\right] = 0$. Since we consider $\sigma = 1$, the first term within the brackets equals one and the first order condition reads, inserting the Cobb-Douglas price index presented after (1.13), $E\left[\frac{p^Y - (1+r)\chi - \bar{p}^Y}{\Phi p_X^\alpha p_Y^{1-\alpha}}\right] = 0$. As X is the numeraire, dividing by $p_Y^{1-\alpha}$ and simplifying gives $E[p_Y^\alpha - ((1+r)\chi + \bar{p}^Y) p_Y^{\alpha-1}] = 0 \Leftrightarrow E[p_Y^\alpha] = ((1+r)\chi + \bar{p}^Y) E[p_Y^{\alpha-1}]$. Solving for \bar{p}^Y yields equation (1.20) in the text.

6.3. Proofs for option theorems.

PROOF OF THEOREM 4. For $\chi = 0$ and in the steady state, we always have

$$E [v'(\tilde{e}, P(p^X, \tilde{p}^Y))] > 0,$$

regardless of the choice of D . Since utility is increasing in consumption and consumption is increasing in D , it is optimal to demand an infinite amount. \square

PROOF OF THEOREM 5. We prove this by contradiction. First note that for an interior solution to the optimal choice of D , we need to have the first order conditions fulfilled. If we subtract (1.14) from (1.21), all expressed for the steady state, we arrive at the following expression,

$$\begin{aligned} & \int_0^{\bar{p}^Y} v' \left(\frac{(1+r)(w - \chi D)}{P(p^X, \tilde{p}^Y)} \right) \frac{-\chi(1+r)}{P(p^X, \tilde{p}^Y)} dP^Y \\ &= \int_0^{\bar{p}^Y} v' \left(\frac{\tilde{e}}{P(p^X, \tilde{p}^Y)} \right) \frac{(\tilde{p}^Y - \bar{p}^Y) - \chi(1+r_{t+1})}{P(p^X, \tilde{p}^Y)} dP^Y. \end{aligned}$$

It cannot be true for the same set of parameters. This establishes that the two first order conditions cannot hold simultaneously. Moreover, this makes clear that

$$\begin{aligned} & \int_0^{\bar{p}^Y} v' \left(\frac{(1+r)(w - \chi D)}{P(p^X, \tilde{p}^Y)} \right) \frac{-\chi(1+r)}{P(p^X, \tilde{p}^Y)} dP^Y \\ &> \int_0^{\bar{p}^Y} v' \left(\frac{(1+r)(w - \chi D) + (\tilde{p}^Y - \bar{p}^Y) D}{P(p^X, \tilde{p}^Y)} \right) \frac{(\tilde{p}^Y - \bar{p}^Y) - \chi(1+r)}{P(p^X, \tilde{p}^Y)} dP^Y \end{aligned}$$

for the same set of parameters. It follows that

$$\begin{aligned} & \int_0^{\bar{p}^Y} v' \left(\frac{(1+r)(w - \chi D)}{P(p^X, \tilde{p}^Y)} \right) \frac{-\chi(1+r)}{P(p^X, \tilde{p}^Y)} dP^Y + \\ & \int_{\bar{p}^Y}^{\infty} v' \left(\frac{\tilde{e}}{P(p^X, \tilde{p}^Y)} \right) \frac{(\tilde{p}^Y - \bar{p}^Y) - \chi(1+r)}{P(p^X, \tilde{p}^Y)} dP^Y = 0 \\ \implies & E \left[v' \left(\frac{\tilde{e}}{P(p^X, \tilde{p}^Y)} \right) \frac{\tilde{p}^Y - (1+r)\chi - \bar{p}^Y}{P(p^X, \tilde{p}^Y)} \right] < 0 \end{aligned}$$

This, together with concavity of utility in D , establishes the result. \square

PROOF OF THEOREM 7. The proof follows directly from (1.21). The first integral enters negatively, the second positively. In general, there is a χ small enough to render the overall sum zero. \square

6.4. Concavity of expected utility with respect to D . We prove here the concavity of the indirect expected utility function with respect to forwards.

Our first order condition, i.e. the first derivative of indirect expected utility with respect to D is given by

$$G(K_t, D_t, \psi) = E \left[v' \left(\frac{e_{t+1}}{P(p_x, p_Y)} \right) \frac{p_{t+1}^Y - \bar{p}^Y - (1 + r_{t+1})\chi}{P(p_x, p_Y)} \right] = 0.$$

The second derivative is then simply

$$\frac{\partial G}{\partial D} = E \left[v'' \left(\frac{e_{t+1}}{P(p_x, p_Y)} \right) \frac{(p_{t+1}^Y - \bar{p}^Y - (1 + r_{t+1})\chi)^2}{P(p_x, p_Y)} \right].$$

Since by definition $v'' \left(\frac{e_{t+1}}{P(p_x, p_Y)} \right) < 0$, we integrate over negative values and, therefore, $\partial G / \partial D < 0$.

6.5. Balance of payment. This appendix checks consistency of the model by validating that the overall balance of the balance of payments equals zero. Formally, $EX_t - IM_t + CF_t = 0$ must hold. Then $p^X X_t - \alpha e_t L - p^X I_t - (1 - \alpha) e_t L + (p_t^Y - \bar{p}^Y) D_{t-1} L - \chi D_t L = 0 \Leftrightarrow$

$$p^X [X_t + K_t - I_t] - e_t L - \chi D_t L = - (p_t^Y - \bar{p}^Y) D_{t-1} L.$$

Using (1.9) for expenditure implies

$$\begin{aligned} p^X [X_t - I_t] - (1 + r_t) (w_{t-1} - \chi D_{t-1}) L - (p_t^Y - \bar{p}^Y) D_{t-1} L - \chi D_t L \\ = - (p_t^Y - \bar{p}^Y) D_{t-1} L \Leftrightarrow \\ p^X [X_t - I_t] - (1 + r_t) (w_{t-1} - \chi D_{t-1}) L - \chi D_t L = 0. \end{aligned}$$

We complete our proof by replacing X_t and I_t . Nominal investment in our model is by (1.7) and (1.8) simply first period income reduced by first period spending,

$$p^X I_t = (w_t - \chi D_t) L - (1 - \delta) p^X K_t. \quad (1.26)$$

The capital stock in the period after saving is given by (1.4), specifically

$$p^X K_{t+1} = (1 - \delta) p^X K_t + p^X I_t.$$

Noting further that, given constant returns to scale, output in period t can be written as the sum of factor payments, i.e.

$$p^X X_t = w_t^L L + w_t^K K_t, \quad (1.27)$$

we can proceed as follows:

$$\begin{aligned} w_t^L L + w_t^K K_t - (w_t - \chi D_t) L + (1 - \delta) p^X K_t \\ - (1 + r_t) (w_{t-1} - \chi D_{t-1}) L - \chi D_t L = 0 \Leftrightarrow \\ w_t^K K_t + (1 - \delta) p^X K_t - \left(1 + \frac{\partial X_t}{\partial K_t} - \delta \right) (w_{t-1} - \chi D_{t-1}) L = 0 \Leftrightarrow \\ p^X \frac{\partial X_t}{\partial K_t} K_t - \left(\frac{\partial X_t}{\partial K_t} - \delta \right) p^X K_t - \delta p^X K_t = 0 \Leftrightarrow 0 = 0, \end{aligned}$$

where we made use of (1.2), (1.15) and (1.10).

CHAPTER 2

International Capital Flows meet Corporate Liquidity Demand

There is a huge literature on corporate liquidity demand. The implications of this theory for international macro, however, are poorly studied. We extend the Holmstroem-Tirole (1998) model on corporate liquidity demand to a two country world. Introducing dual agency problems as well as differing agency costs allows us to link the share of foreign capital holdings with the extent of liquidity shock resistance of the domestic economy. When capital is scarce, a higher share of foreign capital implies a more vulnerable domestic economy. Thus countries that want to open up their capital account face a trade-off: a higher level of investment versus an increased vulnerability with respect to liquidity shocks. We find that less developed countries are more severely affected by this trade-off and are thus more likely to resort to policy instruments such as capital controls. We further show that domestic capital scarcity will place restrictions on foreign capital inflows. The smaller the domestic capital base, the smaller the amount of foreign capital that can be attracted. This explains why most international capital flows occur between rich countries and offers a fresh view on the missing catch-up predicted in neoclassical growth models.

1. Introduction

¹There is a quite extensive literature on corporate liquidity demand. This literature explains the failure of the financial markets to provide sufficient liquidity in times of crisis using assumptions and modeling devices like asymmetric information, principal agents structure, moral hazard and adverse selection.² In this sense these model have a strong industrial organizations and contract theory background and it is not surprising that most of these models are micro models. Questions that have been posed (and answered) are, for example, "What is the role of the government in supplying liquidity?" Holmstroem and Tirole (1998), "What are the implications of the distribution of wealth for investment?" Holmstroem and Tirole (1997) and "What are asset prices like under liquidity constraints?" Holmstroem and Tirole (2001). On the macro level, however, applications of the main results of this literature are hard to find. The most prominent model here is perhaps the Kyotaki-Moore model (Kiyotaki and Moore (1997)) that does not deal with liquidity demand directly but rather assumes it, by introducing a Leontief technology. Firms thus produce using two types of capital, using the illiquid one as collateral to borrow against. Other examples that use elements of corporate liquidity demand in a macro context are Bernake, Gertler and Gilchrist (1998) and Gertler and Gilchrist (1996). Here liquidity demand is brought into the picture by focusing on the financing decision of a firm that uses two types of capital: fixed installed and a variable form. This demand in turn is used to link micro conditions, i.e. a weak financial sector or a low net-worth, with macroeconomic performance. Overall, however, we find that the implications of the findings of this strand of literature for international macro, although interesting and of high relevance, are poorly studied and rarely used in macro models. In particular, the specific form of liquidity that is modelled by Holmstroem and Tirole has to date, to the best of our knowledge, not been brought into a macro framework.

The present work brings the sound micro structure of the corporate liquidity demand models into a framework that is suited to address macroeconomic questions. To this end, we extend the Holmstroem-Tirole (see Holmstroem and Tirole (1998)) model to allow for international capital movements. First we study the implications for the second-best contract if the home country is in autarky and investment

¹I wish to thank Udo Broll, Guido Lorenzoni, Sara Maioli, Alberto Pozzolo, Uwe Vollmer, Benjamin Weigert and seminar participants at the Ente "Luigi Einaudi, the IWH Halle and the GEP Nottingham for discussion and helpful comments. Most of the paper was written while I was enjoying the hospitality of the Ente "Luigi Einaudi" in Rome.

²Bengt Holmstroem and Jean Tirole have been very important for this literature. Main contributions include Holmstroem and Tirole (1996), (1997), (1998) and (1999).

resources are scarce. This will lead to a redistribution of the net-social surplus and under some additional conditions to a less vulnerable economy with respect to liquidity shocks. We then introduce international capital flows into our model. We use the concept of a dual-agency problem (for an introduction with some empirical evidence see chapter 5 in Tirole (2002)) to distinguish between domestic and foreign investors. Investors in general not only have a contract with the entrepreneurs but also an implicit one with the home government that protects their rights. Domestic interests will usually be better protected than foreign ones. Thus foreign investors face higher agency costs when investing abroad.

It turns out that under this setup the vulnerability of the domestic economy with respect to liquidity shocks increases relative to the autarky case: the more foreign money firms have to raise, the narrower will be the bandwidth of feasible liquidity shocks. If a large enough liquidity shock materializes, foreigners will withdraw all their money from domestic projects, even though home investors would have remained.

Our second result is that domestic capital scarcity can be the reason for limited foreign capital supply. In the context of our analysis this amounts to saying that if the domestic economy does not have a sufficiently large capital base, the level of aggregate investment will be constrained, even if there is foreign capital abundance. This result, again, hinges on the dual-agency structure assumption, i.e. that investments made by foreigners are less well protected than those made by domestic investors. Countries for which this result bears most relevance are, of course, less developed countries that have a relatively small capital base. Using our second result we are able to address a broad range of questions in international economics and growth theory, for example the phenomenon that most capital flows occur between rich countries (Lucas (1990)) or the missing catch-up that is predicted in neoclassical growth models.

We see our work as contribution to the huge literature that discusses the effects of capital account liberalization³. We show that from a corporate liquidity demand perspective the composition of debt matters. In a world where capital is scarce a high ratio of foreign capital to overall domestic investment renders the economy more vulnerable to

³There are a great many contributions to this literature from many different perspectives. The majority appears to take a growth perspective see, inter alia, Klein (2003), Bekaert, Harvey and Lundblad (2004) and Singh (2003). Recent empirical evidence is captured in Eichengreen (2001). Edison, Klein, Ricci and Sloek (2002) provide a thorough survey. Other aspects are discussed in Bacchetta (1992) (for the interaction of capital account liberalization and domestic financial liberalization), Kim (2003) (on how the budget deficit is influenced by opening up the capital account), Gruben and McLeod (2002) (on inflation dampening effects) and Bartolini and Drazen (1997) (the information content of policies regarding the capital account).

liquidity shocks. Thus the good that international capital flows bring about, i.e. in the context of our analysis an increased level of investment, also has some severe side effects. This side effects are especially strong in countries that either receive a lot of foreign capital (relative to domestic capital) or suffer from particularly high agency costs for foreign investors (or both). Natural candidates for this type of countries are, again, less developed countries, which renders the results of our work relevant for the discussion about globalization and may provide a rationale why capital controls are a widespread policy instrument in this group of countries.

The results of our work match quite well with recent findings in the empirical literature on the effects of capital account liberalization. If we were to use the classification of the World Bank for low, middle and high income countries, our results would predict that most of the benefits of capital account liberalization accrue to middle income countries. The reason for this is that low income countries would only have restricted access to foreign capital and high income countries would have a sufficiently large capital base to begin with. This is in accordance to recent findings that point to an inverted U-shape distribution of the gains from capital account liberalization, i.e. low and high income countries gaining insignificantly in the process. This view is voiced, for example, in Klein (2003) and Edison et al. (2002).

The chapter is organized as follows. The first section will shed some light on the workings of the Holmstroem-Tirole (HT) model that serves as our starting and reference point throughout. The second section then extends the HT-model. The first extension deals with the implications of the HT-model under autarky. The following section derives the main results of our work under free capital flows. In the next section we turn to discuss the severity of the assumptions with respect to the investment levels and show that foreign capital scarcity may arise without limits placed by foreigners. Section four then turns to discuss the empirical implications of our model and section five concludes.

2. The workings of the Holmstroem-Tirole model

In what follows we will devote some space to replicate and illustrate some of the results of the original HT-model. Our analysis will draw heavily on this results, therefore we discuss them in some length. We make use of the notation of the original paper. This will facilitate understanding for readers already familiar with this class of models.

2.1. Description of the model. The economy is populated by two types of agents: entrepreneurs (or firms) and investors (or consumers). There is a continuum of entrepreneurs with unit mass. Holmstroem and Tirole assume that all entrepreneurs possess a constant returns technology so the assumption of identical endowments for each

firm can be made. The discussion will then be, of course, in terms of the representative entrepreneur. The model has three periods. In period t_0 the entrepreneurs can begin a project that pays off a return R in period t_2 if it succeeds and nothing in case of failure. To start the project, the entrepreneurs require funds. There are two sources of these funds in the model: inside finance, i.e. the endowment of the entrepreneurs A and outside finance, i.e. what the investors have to add to A in order to reach the overall amount of investment I . In t_2 the project, in the case of success, thus pays out RI . In period t_1 there is a liquidity shock ρ that requires the investors to pay an additional amount ρI for the project to be continued - there is no further endowment the entrepreneurs have, i.e. no second period endowment. The liquidity shock is stochastic and follows a density function $f(\rho)$. If the liquidity shock cannot be paid, the project is terminated and pays off nothing. If, in period t_1 , continuation of the project is decided the entrepreneurs have the choice about the effort they put into the project. This effort in turn results in two distinct success probabilities: p_H if effort is exerted and p_L if they shirk, where $p_H > p_L$. There are private benefits from shirking that accrue to the entrepreneurs of the amount $BI > 0$. A necessary condition for an interior solution to this problem is that the net present value of an investment stream ($t_0 = I; t_1 = \rho I$) is positive when effort is exerted in between periods t_1 and t_2 (otherwise there would not be much point in investing in the first place). Formally this amounts to

$$I \int_0^{\infty} \max(p_H R - \rho, 0) f(\rho) d\rho > I. \quad (2.1)$$

This condition implicitly defines the so-called first best cutoff: $\rho_1 = p_H R$. This cutoff defines a range of liquidity shocks $[0, \rho_1]$ within which continuation of the project generates a surplus and hence is socially desirable. If in turn the entrepreneur shirks, the net present value of the project is negative:

$$I \int_0^{\infty} \max(p_L R + B - \rho, 0) f(\rho) d\rho < I.$$

This two conditions taken together ensure that only contracts that implement the action p_H are feasible.

2.2. Solving the model. It is clear from this setup that the first-best cutoff ρ_1 cannot be reached, since outside liquidity is needed in t_1 and the firms have to be promised a certain share of the expected profits to implement the action p_H . This in turn means that in t_1 not all expected unit profits can credibly be promised to outside investors, rendering the (theoretically possible) dilution of outside claims up to the amount ρ_1 not feasible anymore. We are therefore looking for a second-best solution. The optimal (second-best) contract specifies:

- (1) what the entrepreneurs (and hence investors) will get in case of success: $p_H R_f(\rho)$
- (2) the amount of investment undertaken in period t_0 : I and
- (3) a state contingent rule what to do in period t_1 when the shock materializes: $\lambda(\rho)$.

Formally the second-best contract solves

$$\begin{aligned} \max_{R_f(\rho), I, \lambda(\rho)} I \int_0^\infty p_H R_f(\rho) \lambda(\rho) f(\rho) d\rho - A & \quad (2.2) \\ \text{s.t.} & \\ I \int_0^\infty (p_H [R - R_f(\rho)] - \rho) \lambda(\rho) f(\rho) d\rho \geq I - A & \\ R_f(\rho) p_H \geq B + R_f(\rho) p_L \quad \forall \rho. & \end{aligned}$$

The first condition simply states that the entrepreneur's goal is to maximize his profits. The second condition is the participation constraint for the investors (simply a zero profit condition for the investors). This condition will bind at the optimal solution.⁴ The last inequality is the incentive compatibility constraint. This condition states that in order for the entrepreneur not to shirk, he has to be paid a certain amount. It later turns out that this condition will be binding as well. The optimal contract hence boils down to specify a cutoff value $\bar{\rho}$ ⁵ that can be interpreted in the following way: continue if and only if $\rho \leq \bar{\rho}$.⁶ The resulting objective function of the entrepreneur can be written as follows:⁷

$$\max_{\bar{\rho}} U_f(\bar{\rho}) = m(\bar{\rho}) I \quad (2.3)$$

where $m(\rho) = \int_0^{\bar{\rho}} (\rho_1 - \rho) f(\rho) d\rho - 1$. Equation (2.3) shows the expected share of each unit of investment that the firms can appropriate as a function of the threshold shock, i.e. the expected return per unit of gross investment for the firm. Investors in turn will, on average, only get their money back, which directly follows from the participation constraint holding with equality. It can be seen from (2.3) that the objective function reaches its maximum at the first-best cutoff ρ_1 . Further condition (2.1) implies that $m(\rho_1) > 0$, hence expected profits will be positive. Using again the second equation of (2.2), we can derive the optimal amount of investment:

$$I = k(\bar{\rho}) A \quad (2.4)$$

⁴Note that the assumed underlying utility function $U = c_1 + c_2 + c_3$ is linear and implies no discount for future consumption.

⁵See 6.2 for proof of the optimality of a cut-off probability.

⁶The proof for this is in the appendix 6.2.

⁷The exact derivation is relegated to appendix 6.1.

where

$$k(\bar{\rho}) = \frac{1}{1 + \int_0^{\bar{\rho}} \rho f(\rho) d\rho - F(\bar{\rho}) \rho_0}.$$

Holmstroem-Tirole name the function $k(\bar{\rho})$ the equity multiplier. It holds in general (there is another condition necessary for that, namely $\int_0^{\rho_o} (\rho_o - \rho) f(\rho) d\rho < 1$) that $k(\rho_o) > 1$. This in turn means that the initial investment I is bigger than the endowment of the entrepreneurs at the cutoff ρ_0 , formally that $I(\rho_0) - A > 0$. This cutoff is defined as

$$\rho_o = \left(p_H - \frac{B}{p_H - p_L} \right) R = \left(p_H - \frac{B}{\Delta p} \right) R$$

and marks the *pledgeable date 1 unit return*, i.e. the amount of money that can be in t_1 at most promised to outside investors, given that there are private benefits to the entrepreneurs B that cannot be appropriated by the outside investors. This moral hazard problem creates the short-fall of liquidity provision from the social optimal value ρ_1 . The equity multiplier can also be less than one, if the cutoff value is set sufficiently high. This would in turn imply that in period t_0 we have $I - A < 0$ meaning that in period t_0 the entrepreneurs would not borrow. Total investment, i.e. $I - A + \rho I$ would, of course, still be positive for otherwise the discussion would be pointless. In their analysis Holmstroem-Tirole rule out this possibility, i.e. they assume that $I > A$ with the argument that "it seems natural to have the firm a net borrower in period 0"⁸.

Within this very simple framework it remains to determine the second-best cutoff, i.e. the liquidity shock up to which projects are continued. Intuitively this second best solution should be somewhere between the first-best cutoff ρ_1 and the pledgeable unit return at date t_1 ρ_o . This becomes clear from the following reasoning: as long as the cutoff value of the liquidity shock is chosen $\rho < \rho_o$ both investors and entrepreneurs choose to continue after the shock has materialized. If it is chosen to be above ρ_1 , the net-present value of continuing the project is negative (by condition 2.1), and it would be better to leave the project. Within the interval things are less obvious. By lowering the cutoff the firms can increase the amount of money they can raise, by increasing it, they can withstand higher liquidity shocks and thus raise the marginal expected return on the initial investment, but at the same time they decrease initial outside investment, i.e.

$$\frac{dk(\rho)}{d\rho} < 0, \quad \frac{dm(\rho)}{d\rho} > 0.$$

The firms choose $\bar{\rho}$ to maximize their marginal gain per unit invested, formally

$$\max_{\bar{\rho}} U(\bar{\rho}) = m(\bar{\rho}) k(\bar{\rho}) A. \quad (2.5)$$

⁸See Holmstroem and Tirole (1998, p. 10.), p.10.

This cutoff value $\bar{\rho}$ then is determined by the first order condition⁹:

$$\int_o^{\bar{\rho}} F(\rho) d\rho = 1. \quad (2.6)$$

and hence gives an utility level of

$$U_{HT}(\bar{\rho}) = \frac{\rho_1 - \bar{\rho}}{\bar{\rho} - \rho_o} A. \quad (2.7)$$

The following picture captures the basic properties of the second-best contract of the HT-world:

Liquidity Shock and Cutoff Values

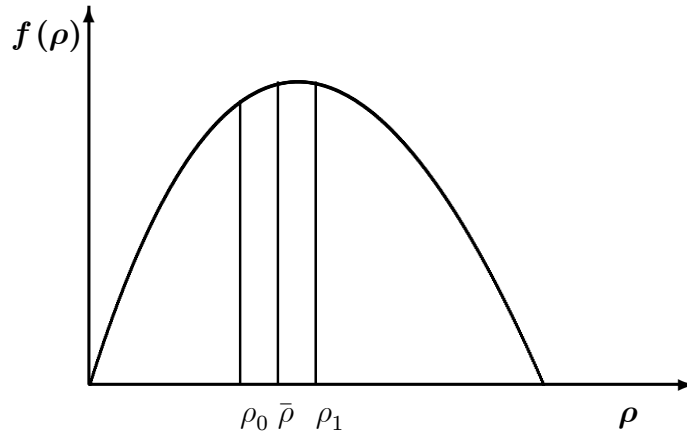


Figure 1: The Holmstroem-Tirole Model graphically

Figure 1 shows as an example a particular distribution of a liquidity shock. Three cutoff values are depicted, ρ_0 , $\bar{\rho}$ and ρ_1 . If no contract would be set up beforehand, entrepreneurs would "overinvest" and could not, once in period t_1 raise more than $\rho_0 I$, i.e. the expected net-worth of the project minus private benefit payments. If negotiations were to take place (and implementation issues were solved) in period t_0 , investment *per project* would be less, however, due to the increased equity quota, the range of feasible liquidity shocks would increase and hence $\bar{\rho} I$ could be raised in period t_1 . However, the first-best outcome ρ_1 cannot be implemented.

2.3. Properties of the second-best contract. From equation (2.7) it becomes clear that the higher the first-best threshold, the higher is the return of investment for the entrepreneurs. Also, higher inside benefits B that accrue only to the entrepreneurs, will reduce their return (via lowering the equity multiplier). These results are more or less standard and not overly surprising. Holmstroem-Tirole point to another, more surprising result: if the distribution of the liquidity shock becomes riskier (in the sense of a mean preserving spread), the cutoff

⁹The necessary steps to derive (2.6) are in the appendix .

will go down. This can be seen from (2.6). Profits, however, will go up. This is a consequence of the fact that the option to terminate the project becomes more valuable when the distribution is more risky.

3. Introducing international capital flows

We are now in the position to extend the HT-analysis to a macro framework. We consider a two open country setup, where both, home and foreign are also Holmstroem-Tirole worlds. This essentially means that the capital allocation is ruled by the same principles as outlined above. Our crucial assumption, however, will be that there are not enough funds in the home country to meet the period t_0 investment demand

$$I_D(\bar{\rho}) = k(\bar{\rho})A$$

where $\bar{\rho}$ is the second-best cutoff in the unconstrained HT-world. This shortfall of investment supply can be thought of as either a result from domestic portfolio optimization or capital scarcity (or both). With free capital flows this shortfall of domestic investment supply will (at least partially) be matched by capital inflows. In contrast, under autarky this is not possible, the optimal solution of our Holmstroem-Tirole world will be different: the cutoff ρ goes up. We derive the behavior of the HT-world under autarky and capital scarcity in detail below. We then turn to analyze our two country world with free capital flows.

3.1. Autarky. A natural starting point for our analysis is to consider the implications of the HT-model under autarky. Autarky itself is only interesting if we have

$$\bar{I}_H \leq I_D(\bar{\rho}) - A = (k(\bar{\rho}) - 1)A \quad (2.8)$$

where \bar{I}_H ¹⁰ is the exogenously given amount of investment home investors are willing (or able) to undertake and I_D denotes the amount of investment that would prevail in an unconstrained world. In principle we could think of the investment-shortfall $\bar{I}_H - I_D(\bar{\rho}) + A$ as also being dependent on the parameter vector of the domestic economy, however, to keep matters simple we simply assume this excess demand for investments. The original HT-world result will now be altered. As long as there were, by assumption, enough funds to meet any arising investment demand, the investors got zero expected returns. This result was brought about by the assumptions of risk neutrality, the absence of time preference of the investors and the zero returns of the outside asset cash. Investors would then compete away any positive profit - hence the net-social surplus accrues to the entrepreneurs. With limited outside funds, however, this result changes. Now the entrepreneurs compete for the limited funds and investors will be able to appropriate

¹⁰Note that in order to distinguish fixed quantities from quantities that are variables we use the bar, hence \bar{I} denotes a fixed amount as opposed to I .

a part or all of the net-social surplus. Formally the optimal contract now solves

$$\begin{aligned} \max_{R_f(\rho), \lambda(\rho), I} \quad & I \int_0^\infty (p_H[R - R_f(\rho)] - \rho) \lambda(\rho) f(\rho) d\rho - I + A \\ \text{s.t.} \quad & R_f(\rho) p_H \geq B + R_f(\rho) p_L \quad \forall \rho \\ & I \int_0^\infty p_H R_f(\rho) \lambda(\rho) f(\rho) d\rho \geq A \\ & I \leq \bar{I}_H + A \end{aligned}$$

Note the similarity of this program to the HT-model optimization problem. First of all, what was the participation constraint of the entrepreneurs now is the objective function. Secondly, the incentive compatibility constraint remains unaltered. There is now a new participation constraint relevant for the investors. The only new part is the capacity constraint, i.e. that investments undertaken cannot be greater than an exogenously given amount $\bar{I}_H + A$. Two cases that arise out of this optimization problem can now be distinguished: $I p_H \frac{B}{\Delta p} F(\rho^A) > A$ and $I p_H \frac{B}{\Delta p} F(\rho^A) \leq A$. We will analyze each case in turn.

3.1.1. $I p_H \frac{B}{\Delta p} F(\rho) > A$. If this condition were to hold, entrepreneurs would be able to get a part of the net social surplus, as they will recover more than their initial investment, implying a rate of interest above zero. The optimization problem then reduces to

$$\max_{\rho^A} I \int_0^{\rho^A} (p_H[R - R_f(\rho)] - \rho) f(\rho) d\rho - I + A.$$

where ρ^A refers to the cutoff value under autarky. It is easy to see that letting $\rho = \rho_o = p_H[R - R_f(\rho)]$ maximizes this expression. However, this case is in the original paper assumed away by the condition

$$\int_0^{\rho_o} (\rho_o - \rho) f(\rho) d\rho < 1,$$

i.e. that investors would realize a negative expected return on their investment. Holmstroem and Tirole in the original paper stress that without this condition investment would be self-financing and the problem under consideration would be trivial. Without this condition, the allowable liquidity shock in t_1 would always be less than the pledgeable income and thus financing it can always be achieved by diluting capital.¹¹ Hence, this case must be excluded from the set of possible solutions.

¹¹Any investor who has a share in the project and is offered the alternative of a complete loss will up to the amount $\rho_o I$ be willing to invest additional funds in order to recover at least a part of his initial investment. This amount to dilution of initial period t_0 claims.

3.1.2. $I p_H \frac{B}{\Delta p} F(\rho) \leq A$. Under this condition entrepreneurs now simply get a zero expected return on their investment A . Since the moral hazard constraint will always be binding, ρ adjusts to ρ^A so as to bring about $\bar{I} p_H \frac{B}{\Delta p} F(\rho^A) = A$. To see this, simply note that entrepreneurs would not start projects with negative expected return but instead keep their initial endowment for themselves. Hence $\bar{I} p_H \int_0^{\rho^A} \frac{B}{\Delta p} f(\rho) d\rho = A$ must always hold. The second-best contract under autarky and capital scarcity will thus specify an optimal cutoff value ρ^A following the program:

$$\begin{aligned} \max_{\rho^A, R_f(\rho), I} & I \left(\int_0^{\rho^A} (p_H R - \rho) f(\rho) d\rho - 1 \right) & (2.9) \\ \text{s.t.} & \\ & I \leq \bar{I}_H + A \\ & \int_0^{\rho^A} R_f(\rho) f(\rho) d\rho = \frac{A}{\bar{I}_H + A} \frac{1}{p_H}. \end{aligned}$$

Note that the solution to this program always implies that the entrepreneurs get an expected return of zero on their investment. Further, if $I \int_0^{\bar{\rho}} p_H \frac{B}{\Delta p} f(\rho) d\rho < A$, there would be in principle two ways to implement the participation constraint of the entrepreneurs: increase the payment to the entrepreneurs, i.e. chose $R_f(\rho) > \frac{B}{\Delta p}$ or increase ρ for $F(\rho)$ is increasing in ρ . Clearly, there is no incentive for the investors to give more away than absolutely necessary, hence ρ will increase and the payments for the entrepreneurs will remain $\frac{B}{\Delta p}$. The investors will be able to appropriate the full net-social surplus. In this sense this solution is exactly the opposite of the original HT-world solution.

From the perspective of the original HT-world solution, two cases have to be distinguished: $I p_H \frac{B}{\Delta p} F(\bar{\rho}) = A$ and $I p_H \frac{B}{\Delta p} F(\bar{\rho}) < A$. In the first case the solution to (2.9) is $\rho^A = \bar{\rho}$, just as in the HT-world. However, due to the restricted amount of investment, the overall utility will be strictly less than in the unrestricted world. We have

$$U_A < U_{HT}^{12}$$

with

$$U_A = \frac{\rho_1 - \bar{\rho}}{\rho_1 - \rho_0} A < \frac{\rho_1 - \bar{\rho}}{\bar{\rho} - \rho_0} A = U_{HT}. \quad (2.10)$$

The solution for the second case is then simply a cutoff value ρ^A that solves:

$$F(\rho^A) = \frac{A}{\bar{I}_H + A} \frac{\Delta p}{B p_H} \quad \forall \rho \leq \rho_1.$$

A different way of stating the results under autarky is the following graphic:

¹²Appendix 6.3 derives this solution in more detail.

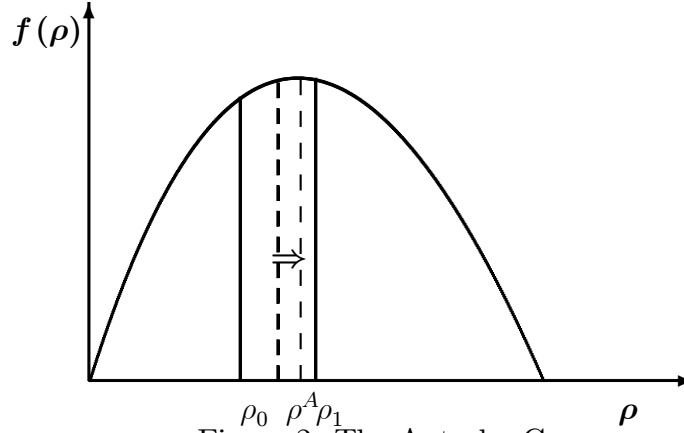


Figure 2: The Autarky Case

Figure 2 above makes clear that forcing the economy to overall invest less, expands the range of admissible liquidity shocks from $\bar{\rho}$ to ρ^A . Thus, even though the economy as a whole invests less, a greater proportion of projects will survive any given liquidity shock. Hence the economy becomes less vulnerable with respect to liquidity shocks.

We conclude the discussion of the autarky solution of the HT-model with the following proposition:

PROPOSITION 1. *In a HT-model under autarky the following holds true: The representative firm with endowment A will invest $I(\rho^A) = k(\rho^A)A < I(\bar{\rho})$. The second-best cutoff under autarky ρ^A will lie in the interval $[\bar{\rho}, \rho_1)$. For sufficiently small values of $\bar{I}_H = I(\rho^A) - A$ we will have $\rho^A > \bar{\rho}$. In that situation the domestic economy will be less vulnerable with respect to liquidity shocks. This comes at the cost of a smaller scale for each and every project. The net-social surplus accrues to the investors. The utility level that can be reached upon implementation of the second-best contract is strictly less than in the original HT-world.*

3.2. Free capital flows. In this section we will introduce the possibility that foreigners can (and will) add some amount I_F to make up for (some of) the shortfall of investment supply to investment demand. Hence we have

$$I_F \leq I_D(\bar{\rho}) - \bar{I}_H - A.$$

Foreign investors will be investing their money under exactly the same conditions as home investors with one exception: in the spirit of a dual agency problem (see Tirole (2002)) we introduce higher agency costs, that is a different B for foreign investors.¹³ The main implication

¹³There is a literature of its own that concludes that the amount B is indeed country specific. For example Porta, Lopez-De-Silanes, Shleifer and Vishny (2002) tries to assess the influence this differences have on corporate valuation and Porta, Lopez-De-Silanes, Shleifer and Vishny (1998) identifies four different legal systems,

of this assumption is that foreigners will face a different pledgeable income in period t_1 than home investors. This can be justified on grounds of several reasons. The first that comes to mind are monitoring costs. Even though not explicitly modelled here, one could think of the amount of private benefits that accrue only to the firm also as being the outcome of a monitoring process - the more efficient the latter the lesser the former. It is reasonable to assume that foreigners possess a less efficient monitoring technology. Secondly, there is the dual agency perspective. Foreign investors do not only have a contract with the firms they invest in, but also, implicitly, with the home government that sets the rules (bankruptcy laws etc.) within its domain. There may be a tendency in case of failure of the firm to protect home interests more efficient than foreign ones. From this we conjecture that $B^F > B$, i.e. that the firm will need more from foreign investors to be kept diligent. Recently there have been contributions that endogenize the choice of B by the government (see for example Tirole (2003)) and in general this would be possible in our model as well. To keep matters simple here, however, we stick with the assumed exogenous nature of the agency costs. The analysis of the economy under free capital movements bears close resemblance with the setups discussed above. To use our previous results we need to clarify and discuss in turn the different cases that can arise when foreign capital is allowed to augment the domestic capital stock. In what follows we will abstract from exchange rate issues or terms of trade uncertainty. Home and foreign thus share the same commodity that is not subject to any change in its price. Further, as outlined above, home and foreign agents have the same utility function, namely $U(c_0, c_1, c_2) = c_0 + c_1 + c_2$. If agents decide to simply hold their endowments without investing their returns will be zero, there is a loss-free storage technology. The minimum expected return for any investor (and entrepreneur) in this model is therefore zero and cannot be less. In order to gain clarity over the implications of the model we distinguish in what follows two different cases. Both cases, of course, involve domestic capital scarcity, but in the first the additional capital that is provided from abroad is sufficient to bridge the gap between demand and supply. In the second case we will discuss the implications if capital scarcity as measured by the gap $\bar{I}_F + \bar{I}_H < I(\bar{\rho}) - A$ remains, even with foreign capital flowing in.

3.2.1. $I_F + \bar{I}_H > I(\bar{\rho}) - A$. If the supply of foreign capital is large enough, capital scarcity no longer exists and, from a contractual point of view we are back in the original HT-world. The net-social surplus

each with a different degree of investor protection. Finally Porta, Lopez-De-Silanes, Shleifer and Vishny (1997) finds that countries with lower investor protection tend to have narrower capital markets.

then accrues to the entrepreneurs. The second-best contract solves

$$\begin{aligned} \max_{R_H(\rho), I_H, I_F, \lambda(\rho), R_F(\rho)} \quad & \alpha I \int_0^\infty p_H R_H(\rho) \lambda(\rho) f(\rho) d\rho \\ & + (1 - \alpha) I \int_0^\infty p_H R_F(\rho) \lambda(\rho) f(\rho) d\rho - A \end{aligned} \quad (2.11)$$

s.t.

$$\alpha I \int_0^\infty (p_H (R - \alpha R_H(\rho) - (1 - \alpha) R_F(\rho)) - \rho) \lambda(\rho) f(\rho) d\rho \geq I_H \quad (2.12)$$

$$(1 - \alpha) I \int_0^\infty (p_H (R - \alpha R_H(\rho) - (1 - \alpha) R_F(\rho)) - \rho) \lambda(\rho) f(\rho) d\rho \geq I_F. \quad (2.13)$$

$$R_H(\rho) p_H \geq B + R_H(\rho) p_L \quad \forall \rho \quad (2.14)$$

$$R_F(\rho) p_H \geq B^F + R_F(\rho) p_L \quad \forall \rho \quad (2.15)$$

$$I_H \leq \bar{I}_H \quad (2.16)$$

where $\alpha = \frac{I_H}{I - A}$ the denotes the share of domestic outside capital relative to total outside capital and the overall level of investment is given by $I = I_F + I_H + A$. Note that in order to make the description of the optimization problem complete we need to introduce two types of returns here: the entrepreneur will get $R_H(\rho)$ for the investment that is financed with domestic outside money and $R_F(\rho)$ for the foreign financed part. If the project is successful, investors will share the proceeds $R - \alpha R_H - (1 - \alpha) R_F$ according to their respective shares in the investment. This is what constitutes the left hand side of equations (2.12) and (2.13). The implicit assumption in writing this type of contract is that every project is financed in part by foreigners and in part by domestic investors (joint-venture). Clearly, an alternative to that assumption is that a certain share of all projects will receive finance from domestic investors and the remainder will receive foreign outside funding (pure-origin finance). Mathematically, both problems are equivalent.¹⁴ In this section we therefore restrict the discussion to the aforementioned case. The implications for the vulnerability of the domestic economy with respect to liquidity shocks, however, are quite different under each setup and will be discussed in more detail below.

The solution to program (2.11) is the same as in the original HT-world and specifies a cutoff value $\bar{\rho}$ ¹⁵. This is not surprising as the only parameter that has changed is the composition of the net-social surplus. Note also that the difference in agency costs does not directly matter here. This, at the first glance, slightly strange result follows from the independence of the optimal cutoff $\bar{\rho}$ from $\rho_0^F < \rho_o < \bar{\rho} < \rho_1$ as long

¹⁴This is shown in the Appendix (6.5).

¹⁵A detailed derivation can be found in the appendix 6.4.1.

as it does not hit one of this boundaries. However, this independence is not complete. The higher agency costs that foreigners face will place a constraint on the equity multiplier - i.e. equation 2.4 - for this type of capital. Depending on the parameters of the economy, especially the amount of domestically provided outside capital and the size of the endowment of the entrepreneurs, there may well be *endogenous foreign capital scarcity*. We will in a later section analyze this case in more detail, i.e. under which conditions the increased B^F is going to induce foreigners to limit their investments.

The level of utility that can be reached is lower than in the original HT-model. This follows directly from the assumption of the higher agency costs foreigners investors face and the implicit restriction on the size of the projects. Formally this is stated below

$$U_{FA} = \frac{\rho_1 - \bar{\rho}}{\bar{\rho} - (\alpha\rho_0 + (1 - \alpha)\rho_0^F)} A < U_{HT}$$

where $\rho_0^F = p_H \left(R - \frac{B^F}{\Delta p} \right) < \rho_0$. Program (2.11) has 5 choice variable but only 4 restrictions, since the second and third equation are equivalent. Hence without a further condition, α remains undetermined within the bounds defined by \bar{I}_H . However, as argued above the size of the domestic capital base restricts the maximum amount of foreign capital that will flow into the domestic economy, especially implying that α will never fall to zero. This together with the result that entrepreneurs will seek a share of outside finance as high as possible provides the last restriction on our optimization problem. This is, of course, an artefact of the linear objective function. For the purposes of this work it is sufficient to note that the composition of outside debt is not vital for our main results as long as we assume parameter values for which there will always be both, foreign and domestic outside finance.

The results of this section are summarized by the following proposition.

PROPOSITION 2. *In a HT-model with domestic capital scarcity defined as $I_D(\bar{\rho}) - A > \bar{I}_H$, international capital flows and foreign capital abundance defined as $\bar{I}_F > I(\rho) - \bar{I}_H - A$ the following holds true: The second-best contract has the same properties as in the original HT-model, namely firms invest $I_D(\bar{\rho}) = k(\bar{\rho})A$, the net-social surplus accrues to the entrepreneurs and investors break even. The utility level that entrepreneurs can reach will be lower than in the original HT-model.*

3.2.2. $\bar{I}_F + \bar{I}_H \leq I(\bar{\rho}) - A$. We now turn to determine the second-best contract under both domestic and foreign capital scarcity. Similar to the analysis under autarky, this situation again calls for a different approach. The first implication is, of course, a redistribution of the domestic net-social surplus in favor of the investors. At the first glance

it seems as if there are now two second-best programs to be solved, one for the domestic investors and the second for the foreign ones.:

$$\max_{R_H(\rho), \lambda^H(\rho), I_H} \alpha I \int_0^\infty (p_H (R - \alpha R_H(\rho) - (1 - \alpha) R_F(\rho)) - \rho) \lambda^H(\rho) f(\rho) d\rho - I_H \quad (2.17)$$

s.t.

$$\begin{aligned} \alpha I \int_0^\infty p_H R_H(\rho) \lambda^H(\rho) f(\rho) d\rho &\geq \left(\frac{I_H}{I - A} \right) A \\ R_H(\rho) p_H &\geq B + R_H(\rho) p_L \quad \forall \rho \\ I_H &\leq \bar{I}_H \end{aligned} \quad (2.18)$$

The program for the foreign capital then reads

$$\max_{R_F(\rho), \lambda^F(\rho), I_F} (1 - \alpha) I \int_0^\infty (p_H (R - \alpha R_H(\rho) - (1 - \alpha) R_F(\rho)) - \rho) \lambda^F(\rho) f(\rho) d\rho - I_F \quad (2.19)$$

s.t.

$$\begin{aligned} (1 - \alpha) I \int_0^\infty p_H R_F(\rho) \lambda^F(\rho) f(\rho) d\rho &\geq \left(\frac{I_F}{I - A} \right) A \\ R_F(\rho) p_H &\geq B^F + R_F(\rho) p_L \quad \forall \rho \\ I_F &\leq \bar{I}_F. \end{aligned}$$

However, merging the two maximization problems into one is mathematically equivalent and leads to the same results. The solution technique employed is the same as in section (3.1.2). First we observe that any meaningful solution must involve binding incentive compatibility constraints, i.e. $R_F = \frac{B^F}{\Delta p}$ and $R_H = \frac{B^H}{\Delta p}$. Secondly the cutoff values have to adjust to fulfill the participation constraint of the entrepreneurs. This implies in general two different cutoff values for home and foreign investment contracts respectively. To see this write out the two participation constraints as

$$F(\rho^H) = \frac{1}{I} \frac{A}{B} \frac{\Delta p}{p_H} \quad (2.20)$$

and

$$F(\rho^F) = \frac{1}{I} \frac{A}{B^F} \frac{\Delta p}{p_H}.^{16} \quad (2.21)$$

The first implication is that contracts with foreigners will always have a lower cutoff value than contracts with domestic investors. In this setup this result is completely driven by the assumption of the higher agency costs that foreigners face and the fulfillment requirement of the participation constraint. The second implication is that under free (but limited) capital inflows the domestic contract cutoff value will be less

¹⁶The exact derivation is again relegated to appendix (6.4.2).

than under autarky as long as foreigners contribute a small fraction of the domestic capital stock. Condition (2.20) makes this immediately clear.

The resulting contracts under all our setups considered are depicted for reference in the table below:

Free Capital Flows	Contract Home Investors
$I_D(\bar{\rho}) - A \leq \bar{I}_H$	$\{\bar{\rho}, I(\bar{\rho}), R_f = \frac{B}{\Delta p}\}$
$I_D(\bar{\rho}) - A > \bar{I}_H, I_D(\bar{\rho}) - A < \bar{I}_H + I_F(\bar{\rho})$	$\{\bar{\rho}, \bar{I}, R_H = \frac{B}{\Delta p}\}$
$I_D(\bar{\rho}) - A > \bar{I}, I_D(\bar{\rho}) - A \geq \bar{I}_H + \bar{I}_F$	$\{\rho^H, \bar{I}, R_H = \frac{B}{\Delta p}\}$
	Contract Foreign Investors
$I_D(\bar{\rho}) - A \leq \bar{I}_H$	-
$I_D(\bar{\rho}) - A > \bar{I}_H, I_D(\bar{\rho}) - A < \bar{I}_H + I_F(\bar{\rho})$	$\{\bar{\rho}, I_F(\bar{\rho}, \bar{I}), R_F = \frac{B^F}{\Delta p}\}$
$I_D(\bar{\rho}) - A > \bar{I}, I_D(\bar{\rho}) - A \geq \bar{I}_H + \bar{I}_F$	$\{\rho^F, \bar{I}_F, R_F = \frac{B^F}{\Delta p}\}$

The implications of our results are straightforward. Foreign capital inflows will render the domestic economy more vulnerable to liquidity shocks. This effect has two sources. The first is the share of foreign capital invested in domestic firms, the second is the higher agency costs that foreign investors face. We can decompose these two sources by comparing the different cutoff values that prevail in an economy with foreign and domestic capital with another with the same level of investment but only domestic capital:

$$\frac{I_F}{I-A}\rho^F + \frac{I_H}{I-A}\rho^H < \rho_{\bar{I}-A=I_H+I_F}^A = \rho^H \quad (2.22)$$

$$|(\rho^F - \rho^H) \frac{I_F}{I-A}| = FCCP > 0.$$

Relationship (2.22) defines a "foreign capital cutoff premium" (FCCP) and makes clear that the higher the share of foreign outside finance in domestic firms, the higher the premium the economy pays in terms of increased vulnerability. What is depicted in (2.22) is thus the effect stemming from the *share* of foreign capital, for any given ratio $\frac{B^F}{B}$ multiplied by $(\rho^F - \rho^H)$ that stands for the direct contribution of the differing agency costs to the higher vulnerability of the domestic economy with respect to liquidity shocks. Note that the impact of a given liquidity shocks also depends on whether firms seek outside finance as a joint-venture or as pure-origin. Under joint venture the impact is the greatest, if we assume that projects cannot be divided, implying that all projects that experience a liquidity shock higher than ρ^F are abandoned. Under pure-origin finance only a fraction $(1 - \alpha)$ of all projects is subject to this lower threshold, the remainder being continued up to ρ^H .

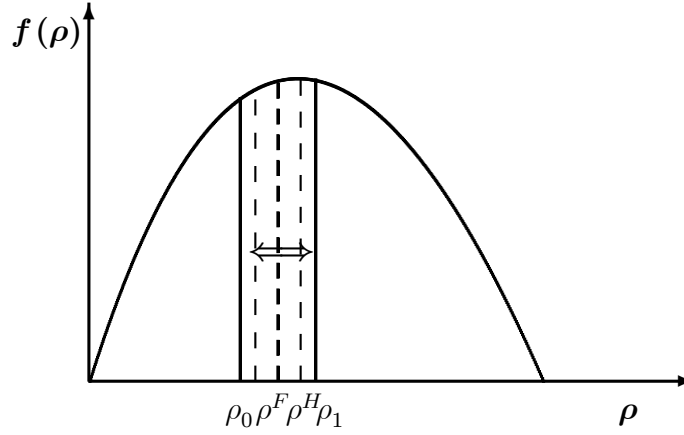


Figure 3: Free Capital Flows

Figure 3 states (one of) the results under free capital flows graphically. The thick dotted line represents the reference point $\bar{\rho}$ of our benchmark HT-world. As the above analysis made clear, international capital flows will, under certain conditions, lead to two distinct cutoff values: ρ^H for domestic outside capital and ρ^F for the foreign counterpart. The part of the project (or the part of the projects, depending on pure origin vs. joint-venture finance) that is financed from abroad will only be continued up to liquidity shocks of ρ^F . This will, in general, be less than $\bar{\rho}$ and thus be worse than in an otherwise comparable HT-world. The domestically financed part will be better protected than the part that is financed from abroad. Whether or not the domestically financed part is even better protected than it would be in the benchmark HT-world (as shown in Figure 3) depends on the parameter vector of the model.

The following proposition sums up the results of this section:

PROPOSITION 3. *In a HT-model with domestic capital scarcity defined as $I_D(\bar{\rho}) - A > \bar{I}_H$, international capital flows and foreign capital scarcity as defined by $I_D(\bar{\rho}) - \bar{I}_H - A > \bar{I}_F$ the following hold true: the representative firm invests $I = \bar{I}_H + \bar{I}_F + A < I_D(\bar{\rho})$, the net-social surplus now accrues to domestic and foreign investors, the second-best contract specifies two distinct cutoff values, one for the foreign investors ρ^F and one for the domestic investors ρ^H where $\rho^F < \rho^H < \rho^A$. The domestic economy is thus more vulnerable to liquidity shocks than under autarky, i.e. a larger part of the firms will have to abandon the project in period t_1 for any given liquidity shock $\rho > \rho_0$.*

3.3. Domestic capital scarcity versus international capital flows. To derive our results, we so far had to make some strong assumptions on the relative size of capital supply to demand without giving a justification for the respective sizes. It turns out, however,

that even if we stick with the exogenous nature of the size of the outside capital, we can say a bit more about the relevant ranges. Consider equation (2.4) that serves as our benchmark throughout the analysis. This equation determines the second-best capital stock in an otherwise unconstrained world. Hence it yields the amount of investment in the HT-model that maximizes the expected (second-best) net-social surplus. The term $\frac{1}{1 + \int_0^{\bar{\rho}} \rho f(\rho) d\rho - F(\bar{\rho})\rho_0}$ is called (by Holmstrom and Tirole) equity multiplier and is in the original HT-model assumed to be greater than 1. Under free capital flows there are two scenarios that are of interest. First we consider domestic capital scarcity and foreign capital abundance. This, as stated in section (3.2.1), implies a contract $\{\bar{\rho}, \bar{I}_H, I_F(\bar{\rho}), R_H = \frac{B}{\Delta p}, R_F = \frac{B^F}{\Delta p}\}$. The properties of this contract are the same as in the HT-model, with the exception that foreigners face a smaller period t_1 pledgeable income as do home investors. In order for this contract to be feasible, we need the following condition to hold

$$\begin{aligned} I_F(\bar{\rho}) &\leq \frac{1}{1 + \int_0^{\bar{\rho}} \rho f(\rho) d\rho - F(\bar{\rho})\rho_0^F} A - A & (2.23) \\ I_F(\bar{\rho}) &\leq I^F(\bar{\rho}) - A, \end{aligned}$$

where $I^F(\bar{\rho})$ denotes the level of investment that would prevail in a world in which all investors were foreigners. Clearly, as $\rho_0^F < \rho_0$ by assumption, we have $I_D(\bar{\rho}) > I^F(\bar{\rho})$. This places a restriction on the term "foreign capital abundance" in the sense that foreign capital cannot indefinitely augment the domestic economy, even if there is no shortage in foreign supply. This in turn renders our results from section (3.2.2), the second case we considered, more relevant: *foreign capital scarcity, as we defined it, will not only follow from a lack of foreign supply but also from an especially low domestic capital base.* To see this, note that the overall level of initial outside investment demand under free capital flows was given by $I_D(\bar{\rho}) - A = \bar{I}_H + I_F(\bar{\rho})$. For small enough values of \bar{I}_H , however, constraint (2.23) will be binding and thus we face domestic *and* foreign capital scarcity even though there is no restriction placed on the level of foreign capital from outside. The reason for this "self-restriction" of the model are the different (and higher) agency costs foreigners face. The higher these costs are, relative to the costs domestic investors face, the earlier this "self-restriction" is going to bind. Further, the higher this relative agency costs are, the greater the "distance", as measured by $\rho^H - \rho^F$, between domestic and foreign contracts. Formally the constraint on the level of domestic outside investment reads

$$\bar{I}_H < (k(\bar{\rho}) - k^F(\bar{\rho})) A \quad (2.24)$$

where $k(\bar{\rho})$ is the equity multiplier of the original HT-model and $k^F(\bar{\rho})$ is the equity multiplier that would prevail in the case in which all outside finance would be stemming from abroad. Condition (2.24) states that if domestic outside capital supply fails in making up for the difference between unconstrained investment demand and maximum foreign outside capital, foreign outside capital will be restricted endogenously. Note that the endowment has a different role. In general, as (2.24) makes clear, the smaller the domestic endowments, the smaller the domestic outside capital supply must be in order to induce foreign capital scarcity (and vice versa). This is, of course, a pure demand effect. The higher the endowment, the higher is the demand for outside capital. The results of this section are summarized in the following proposition.

PROPOSITION 4. *In a HT-model with international capital flows domestic outside capital scarcity may induce foreign capital scarcity. This will be the case when the amount of domestic outside finance investors are willing (or able) to invest falls short of the difference between the investment level that would prevail in an unconstrained world and the amount of outside finance foreigners will be willing to provide at given agency costs. The resulting second-best contract in this situation is the one described in section (3.2.2).*

4. Empirical implications

Our extensions to the original HT model have some interesting empirical implications. According to our results, depending on the domestic capital base, countries may have limited scope for borrowing abroad. Naturally, the poorer the country under consideration, the lower the domestic capital base and hence the more limited its borrowing potential will be. This in turn would imply that the effects of international capital flows on poor countries are ambiguous in our model. As the analysis in section (3.2.2) makes clear the positive effect of augmenting the domestic capital base has the (potentially) negative side effect of rendering the domestic economy more vulnerable to liquidity shocks. The final evaluation thus depends on the distribution of the liquidity shocks. On the other hand, rich countries that are capital abundant to begin with, will not gain anything from opening their capital account, the original HT-contract remains in place unaltered. However, the main effects of international capital flows according to our model will occur in the middle-income group of countries, i.e. those countries that have an insufficient capital base to begin with but that can significantly augment their capital base with the help of foreign capital inflows. If this augmentation is complete (in the sense that the complete excess demand for outside capital can be contracted abroad), only gains will accrue. In the case of incomplete augmentation it remains unclear whether there will be any gains or not.

It is clear that our model does not account for all possible motives of international capital flows. Hence it will, in general, not be able to explain all these flows between countries nor the empirical findings often found in studies on the subject. What is remarkable, however, is the model's ability to rationalize two main findings of the empirical literature in the field. First, recent studies on the effect of international capital flows on growth find, although far from conclusive, evidence that the growth effects of capital account liberalization are distributed in a U-shape manner (see Klein (2003) and Edison et al. (2002)). The main gains accrue to middle-income countries with poor and rich countries benefitting only marginally. This is in line with the predictions of our model. Secondly, it has become a stylized fact that most net-capital flows occur between developed countries, much the opposite to what neoclassical growth theory would predict.¹⁷ Our model offers a very simple explanation for this: Poor countries do not have enough collateral to borrow up to their needs. They are in effect credit constrained.

With respect to income volatility our analysis bears another empirical implication. As a result of foreign capital inflows, the poorest countries *may* witness an (relative) increase in output variability due to the increased vulnerability with respect to liquidity shocks. However, this is only a possibility, depending on the liquidity shocks that hit the economy that may or may not materialize. Surprisingly, although many contributions to the discussion on the effects of capital account liberalization name increasing volatility as a possible negative consequence, empirical studies on this topic are scarce. We are only aware of two such papers, namely Bekaert et al. (2004) and Mukerji (2003). The first (and most recent) study finds that *overall* the impact of financial liberalization in real consumption growth volatility is negligible. However, if we are viewing the effects Bekaert et al. (2004) find at the country level there are significant negative effects for some countries. The second study finds, in accordance with our model, negative effects for less developed countries but negligible output effects for more developed countries.

5. Conclusion

In this work we extended a model of corporate liquidity demand in order to be able to address questions of macroeconomic importance. This was achieved by introducing international capital flows into the model of Holmstroem and Tirole that can augment the domestic capital base. Foreign investors, as opposed to domestic ones, face higher

¹⁷Lucas (1990) in a widely cited paper draws on this point and sparked a literature of its own on that topic. Examples of follow up papers are Kray (2000) and Reinhardt and Rogoff (2004).

agency costs when investing in domestic firms. We justify this assumption in the spirit of a dual-agency problem: principals will not only have a contract with their agents but also an implicit one with the authorities that set the frame for any economic activity. However, we could think of many more possible reasons why agency costs should be higher for foreigners, monitoring costs being one of these reasons. The model then allows to discuss and answer several interesting questions that relate to the field of international macroeconomics, like the phenomenon that net capital flows are substantially higher between developed countries than between developed countries and less developed countries.

If outside capital is scarce domestically the economy will invest less but will also be able to withstand higher liquidity shocks. Foreign capital that flows in then brings about two effects: more investments but at the cost of higher vulnerability with respect to liquidity shocks. This effect would be especially pronounced in less developed countries. This may provide a rationale why several countries from this group chose to introduce capital controls as a policy option.

If outside capital is in limited supply both, from domestic investors and foreign ones, the economy will be even more vulnerable to liquidity shocks as the foreign share of the outside finance cannot sustain liquidity shocks as high as the domestic share. The consequence is that a larger fraction of projects will be abandoned in period t_1 . Foreign outside capital in this sense is less robust than domestic outside capital. This result has some interesting implications. First, countries that have to draw heavily on foreign capital, should try hard to lower agency costs for foreigners. This would mitigate the negative side effects foreign capital inflows have. To the extent that foreign outside capital is rendered scarce by the low domestic capital base our model offers an explanation why most net-capital flows occur between developed countries, i.e. those with a relatively high domestic capital base. Secondly, our model provides a rationale why the catch-up prediction of neoclassical growth models does not work empirically: foreign capital simply cannot augment the domestic capital base indefinitely. There are two reason for this: the higher agency costs that foreign investors end up paying and the credit rationing itself that is already part of the original HT-model.

Our analysis has several shortcomings that could be the subject of further research. First, the crucial point of capital scarcity is exogenous in our model. An extension to endogenize domestic capital supply thus seems natural. Secondly, agency costs may also be made endogenous by introducing a government. Even though our results would not change qualitatively, this step would allow to analyze the effect of different policy regimes on domestic welfare. Lastly an extension of the present framework to an overlapping generation model and thus ultimately the

possibility to answer the question whether or not trading off vulnerability against scale is worth the effort may prove an exiting avenue for further research.

6. Mathematical appendix

6.1. Derivation of the optimal HT-world contract. In this first part of the mathematical appendix we will devote some space to go in some detail through the steps necessary to solve the optimization problem of the original HT-paper. We want to determine $\lambda(\rho)$, I , $R_f(\rho)$ that solve the second-best contract:

$$\begin{aligned} & \max_{R_f(\rho), I, \lambda(\rho)} I \int_0^\infty p_H R_f(\rho) \lambda(\rho) f(\rho) d\rho - A \\ & \quad \text{s.t.} \\ & I \int_0^\infty (p_H[R - R_f(\rho)] - \rho) \lambda(\rho) f(\rho) d\rho \geq I - A \\ & R_f(\rho) p_H \geq B + R_f(\rho) p_L \quad \forall \rho. \end{aligned}$$

Noting first that the second equation must hold with equality for otherwise the solution would be unconstrained and substituting into the objective function we get

$$\begin{aligned} & \max I \int_0^\infty p_H R_f(\rho) \lambda(\rho) f(\rho) d\rho - I \\ & + I \int_0^\infty (p_H[R - R_f(\rho)] - \rho) \lambda(\rho) f(\rho) d\rho \\ \implies & \max I \left(\int_0^\infty (p_H R - \rho) \lambda(\rho) f(\rho) d\rho - 1 \right) \end{aligned}$$

Note that $R_f(\rho)$ disappears. Now making use of the cutoff value $\bar{\rho}$, i.e. rewriting $\lambda(\rho)$ as $\lambda(\rho) = \begin{cases} 1 & \forall \rho \leq \bar{\rho} \\ 0 & \text{otherwise} \end{cases}$ we arrive at

$$\max_{\bar{\rho}} I \left(\int_0^{\bar{\rho}} (p_H R - \rho) f(\rho) d\rho - 1 \right)$$

Further using that $p_H R I = \rho_1 I$ is the social optimal cutoff value we can find the marginal net social return on investment as a more concise objective function of the entrepreneur:

$$\max_{\bar{\rho}} U_f(\bar{\rho}) = m(\bar{\rho}) I.$$

Now using again the second equation of (2.2), we can derive the amount of investment:

$$\begin{aligned} I \int_0^\infty p_H [R - R_f(\rho) - \rho] \lambda(\rho) f(\rho) d\rho &= I - A \\ \frac{1}{1 + \int_0^{\bar{\rho}} \rho f(\rho) d\rho - \rho_o F(\bar{\rho})} A &= I \\ k(\bar{\rho}) A &= I. \end{aligned}$$

To determine the second-best cutoff $\bar{\rho}$ we maximize (2.5) with respect to $\bar{\rho}$. Writing out (2.5) gives

$$\begin{aligned} \max_{\bar{\rho}} U(\bar{\rho}) &= \frac{F(\bar{\rho}) \rho_1 - \int_0^{\bar{\rho}} \rho f(\rho) d\rho - 1}{1 + \int_0^{\bar{\rho}} \rho f(\rho) d\rho - F(\bar{\rho}) \rho_o} A \\ \max_{\bar{\rho}} U(\bar{\rho}) &= \frac{\rho_1 - \left(1 + \int_0^{\bar{\rho}} \rho f(\rho) d\rho\right) \frac{1}{F(\bar{\rho})}}{\left(1 + \int_0^{\bar{\rho}} \rho f(\rho) d\rho\right) \frac{1}{F(\bar{\rho})} - \rho_o} A \\ &= \min_{\bar{\rho}} \left(1 + \int_0^{\bar{\rho}} \rho f(\rho) d\rho\right) \frac{1}{F(\bar{\rho})}. \end{aligned}$$

The first-order condition for this problem is simply $\int_0^{\rho^*} F(\rho) d\rho = 1$, since

$$\frac{d \left(1 + \int_0^{\bar{\rho}} \rho f(\rho) d\rho\right) \frac{1}{F(\bar{\rho})}}{d\bar{\rho}} = \rho f(\bar{\rho}) \frac{1}{F(\bar{\rho})} - \left(1 + \int_0^{\bar{\rho}} \rho f(\rho) d\rho\right) \frac{1}{(F(\bar{\rho}))^2} f(\bar{\rho}) = 0$$

this, together with the result that $F(\rho) \rho - \int_0^{\bar{\rho}} \rho f(\rho) d\rho = \int_0^{\bar{\rho}} F(\rho) d\rho$ gives (2.6). Using this last equality once more in the numerator and the denominator of (2.5) and assuming an interior solution, we finally arrive at (2.7).

6.2. Proof of the optimality of a cutoff rule. Here we prove that $\lambda(\rho)$, the state-contingent continuation policy, takes the form of a cutoff rule, i.e. continue iff $\rho \leq \bar{\rho}$. Suppose we would propose a continuation policy that says continue for $\forall \rho$. In this case we would continue projects that have a negative net present value, hence also those that are socially undesirably. Now suppose we would propose a rule that no project should be continued for any positive ρ . Then, of course, there are projects with positive net present value that are abandoned by this rule. We can improve both situations by imposing a cut-off value ρ upon reaching which projects are terminated. The value of $\bar{\rho}$ is left to be determined endogenously. We can see this result also from the concavity of the objective function of the second-best

program with respect to a cut-off $\bar{\rho}$:¹⁸

$$\max_{\lambda(\rho)} \int_0^{\infty} (p_H R - \rho) \lambda(\rho) f(\rho) d\rho.$$

As long as ρ is increasing but less than $p_H R$ the value of the objective function increases. As soon as $p_H R$ is surpassed, additional contributions will be negative and hence undesirable. As long as the support of the liquidity shock ρ comprises also of $p_H R$ it will be optimal to define a cutoff rule.

6.3. Derivation of the second best contracts under autarky.

We start solving the program (2.9) subject to the condition of the first case: $I \int_0^{\rho^A} p_H \frac{B}{\Delta p} f(\rho) = A$. This condition states that the amount of funds provided by the investors is such that moral hazard constraint just binds. We can then note that this amount of investment is simply given by

$$I = \frac{A}{p_H \frac{B}{\Delta p} F(\rho)}.$$

Inserting this into the first equation yields

$$\max_{\rho^A} \frac{A \left(\int_0^{\rho^A} (p_H R - \rho) f(\rho) d\rho - 1 \right)}{p_H \frac{B}{\Delta p} F(\rho)}. \quad (2.25)$$

This is equivalent to the maximization problem in the original setup, i.e. maximizing (2.25) is equivalent to $\min_{\bar{\rho}} \left(1 + \int_0^{\bar{\rho}} \rho f(\rho) d\rho \right) \frac{1}{F(\bar{\rho})}$. To see this rewrite (2.25) as

$$\begin{aligned} \max_{\rho^A} \frac{\rho_1 F(\rho) - \int_0^{\rho^A} \rho f(\rho) d\rho - 1}{p_H \frac{B}{\Delta p} F(\rho)} A \\ \max_{\rho^A} \frac{\rho_1 - \left(1 + \int_0^{\rho^A} \rho f(\rho) d\rho \right) \frac{1}{F(\rho)}}{p_H \frac{B}{\Delta p}} A. \end{aligned}$$

The solution is thus the same as in the original HT-world, namely $\int_0^{\rho^A} F(\rho) = 1$. Using this and the definition of $\rho_0 = p_H \left(R - \frac{B}{\Delta p} \right)$ we can rewrite utility of the investors as (2.10).

6.4. Derivation of the second-best contracts under free capital flows.

¹⁸This follows from the optimization.

6.4.1. *Domestic capital scarcity, foreign capital abundance.* We start with the program (2.11). First we observe that the program is linear in I and hence participation constraints of the investors have to bind in order to obtain an interior solution. Thus we can consolidate the second and the third equation to

$$\alpha I \int_0^\infty (p_H (R - \alpha R_H(\rho) - (1 - \alpha) R_F(\rho)) - \rho) \lambda(\rho) f(\rho) d\rho \\ + (1 - \alpha) I \int_0^\infty (p_H (R - \alpha R_H(\rho) - (1 - \alpha) R_F(\rho)) - \rho) \lambda(\rho) f(\rho) d\rho = I - A$$

Solving for A and inserting into the objective function yields the familiar expression

$$\max_{\rho} I \left(\rho_1 F(\rho) - \int_0^{\rho^F} \rho f(\rho) d\rho - 1 \right)$$

and thus $\rho^F = \bar{\rho}$.

6.4.2. *Domestic and foreign capital scarcity.* Under this section we derive the form of the participation constraints (2.20) and (2.21). We start out with stating the second equation of program (??) as an example:

$$\left(I_H + \frac{I_H}{I_F + I_H} A \right) \int_0^{\rho^H} p_H \frac{B}{\Delta p} f(\rho) d\rho = \left(\frac{I_H}{I_F + I_H} \right) A.$$

This is equivalent to

$$\left(\frac{I_H (I - A)}{I - A} + \frac{I_H}{I - A} A \right) p_H \frac{B}{\Delta p} F(\rho^H) d\rho = \left(\frac{I_H}{I_F + I_H} \right) A.$$

Simplifying and solving for $F(\rho^H)$ then yields $F(\rho^H) = \frac{1}{I} \frac{A \Delta p}{B p_H}$.

6.5. The equivalence of joint-venture and pure-origin finance. In section (3.2.1) we state the problem for the entrepreneur under the assumption that each project is in part financed by domestic investors and receives also a share from foreign investors. Alternatively the economy could also consist of a share of projects α that is financed entirely by home investors, leaving a share of projects $(1 - \alpha)$ that is financed by foreign investors. This then gives rise to two programs to be solved:

$$\max_{R_H(\rho), \lambda(\rho), I_H} \alpha I \int_0^\infty p_H R_H(\rho) \lambda(\rho) f(\rho) d\rho - \alpha A \\ s.t. \\ \alpha I \int_0^\infty (p_H (R - R_H(\rho)) - \rho) \lambda(\rho) f(\rho) d\rho \geq I_H \\ R_H(\rho) p_H \geq B + R_H(\rho) p_L \quad \forall \rho \\ I_H \leq \bar{I}_H$$

and

$$\begin{aligned} \max_{R_F(\rho), \lambda(\rho), I_F} & (1 - \alpha) I \int_0^\infty p_H R_F(\rho) \lambda(\rho) f(\rho) d\rho - (1 - \alpha) A \\ \text{s.t.} & \\ (1 - \alpha) I \int_0^\infty & (p_H (R - R_H(\rho)) - \rho) \lambda(\rho) f(\rho) d\rho \geq I_F \\ R_F(\rho) p_H & \geq B^F + R_F(\rho) p_L \quad \forall \rho. \end{aligned}$$

Clearly, the two programs can be merged into one by simply adding the objective functions. This yields (2.11).

Part 2

Empirical Inquiries

CHAPTER 3

Accounting for Flows

We offer a survey on the existing sources for both gross and net flow data on international capital flows. Further, we set out to offer some measures of the development of goods market integration relative to financial market integration. Using data for the last decade, we find that although gross financial flows were increasing relative to trade flows, net financial flows were not. Hence, we make the claim that goods market integration actually has had roughly the same speed as international financial integration in the last decade. Increasing gross flows point towards lower transaction costs and hence increased efficiency in the global financial markets. In this sense, the integration of the world has indeed deepened.

1. Introduction

¹The following chapter draws a picture of the amount of international financial and trade flows and develops a common framework for the different sources of data we have on these flows. There are at least three good reasons to do that.

First, in the debate on globalization, much attention is given to the amounts of trade and capital flows, or better their development over the second half of the last century and, of course, their likely future. The huge increase in these flows is a defining element of the phenomenon globalization (Obstfeld and Taylor (2002)). There are voices that say that there are too many international financial transactions, relative to real activity, and others that claim that the increased volatility of the real economy is largely caused by the huge increase in the volume of international financial flows. Related and quite lively issues in current economic research are, for example, the quest for the associated welfare gain by opening the capital account (see for a recent example Gourinchas and Jeanne (2003)) or the potential harm excessive volatility of short term capital flows is inflicting on emerging economies (examples being Edwards (2001) and Rodrik and Velasco (1999)).

Second, closely related to the first point, there is the discussion around the Tobin Tax proposal (see Frankel (1996)). Ever since the introduction of the idea to tax turnover in the international capital markets there has been a lively debate within the profession over the potential merits and dangers of such a tax.²

Third, exchange rate economics seem to have come to a dead end³. At the latest since Meese and Rogoff's influential paper (Meese and Rogoff (1983)) the profession has struggled with the phantom of the unbeatable random walk or, to put it another way, it seems very hard to impossible to come up with a reasonable model for the exchange rate that performs well out of sample. A recent follow-up assessment on the topic provide Cheung, Chinn and Pascual (2004). Assessing the magnitudes of real and financial flows and comparing both to each

¹I wish to thank seminar participants at University of Dresden, Ente "Luigi Einaudi" Rome and Humboldt University Berlin for discussion and helpful comments. Most of the work was done while I enjoyed the hospitality of the University of Toulouse 1.

²Recently there has been new research into that field under another headline. Certain implementations of capital controls work in effect exactly as a Tobin Tax and have only limited success. This, however, proves only that an unilateral Tobin Tax may be not as effective as a global one. See Caballero (2003) and Edwards (2001) for discussion.

³Recent resurrection efforts, however, are quite promising, see Hau and Rey (2003) and indicate the incorporation of capital flows into models of exchange rate determination. Further, to the extent that exchange rate models rely on PPP there seems to be another interesting development with respect to the PPP failure discussion. See Imbs, Mumtaz, Ravn and Rey (2005).

other may help to understand the importance of the international parity conditions on which exchange rate economics so heavily is reliant. Recent attempts to incorporate the idea that capital flows may play a vital role in exchange rate determination are Hau and Rey (2003) and Brook, Edison, Kumar and Slok (2001).

The chapter is organized as follows. The first section introduces the terminology and discusses the various sources of data on international flows, real and financial. The next section is devoted to describe the standard data, i.e. balance of payments, where we have to distinguish between national and supranational reporting. The third part of this chapter describes the data from the Bank of International Settlements (BIS) and its triennial survey on the foreign exchange market. It will turn out that in the same way the balance of payments (gross flow reporting) is a lower boundary measure of international capital flows the foreign exchange markets survey of the BIS is an upper one. We will explore this relationship in more detail in the fourth section. Further, we will present estimates of the ratio of real to financial transactions, based on different sources. The final section concludes the chapter.

2. General remarks

In this section we will give an overview over the existing sources of data on international capital flows (ICF) as well as discuss terminology. We begin with a discussion of the national sources of data on ICFs. As specific examples we will use the cases of Germany and the United States. These sources will mainly be related to the balance of payments (BoP) of the country, a statistic that is normally compiled and supplied by the national central bank. Subsequently, we turn to supra-nationally provided data. The two sources that are publicly available (and widely used in practice) are the International Monetary Fund International Financial Statistics (IMF-IFS) and the World Bank Global Development Finance (GDF). Lastly, we give an overview over the BIS triennial survey on foreign exchange market activity.

Data on real (gross) flows are not difficult to come by and are readily available using either supra-national sources, as for instance the World Bank or the IMF, or national sources as the central banks or governmental statistical offices that provide data on the current account. Moreover, as much discussion there is with respect to ICFs, real flows (and the measurement of it) are a strikingly undisputed issue. In what follows, we therefore restrict our attention to the different sources of capital flows and introduce - in passing - the definition of real flows we will use throughout the chapter.

2.1. Definitions. *International capital flows in the sense we use for our analysis are monetary flows from one country to another that*

result into changes of ownership of physical capital, financial capital or derivative assets.

This is the also definition that is used by virtually all data compiling agencies. When using this definition, it is important to be aware of the difference between capital flows that occur between two countries that have the same unit of account and countries that have different currencies. Which concept should be used depends on both the question under consideration and data availability. Since, in theory, the denomination of the original capital flow does not matter⁴, one should, from a theoretical point of view, prefer the broadest possible measure of ICFs including such that occur inside a currency union. Whereas this seems feasible for the Euro area, it is not straightforward to obtain such data for the United States. Additionally, most negative effects of capital flows are attributed to their influence on nominal exchange rates. As currency unions are a device to credibly fix nominal exchange rates between member countries these negative effects can no longer occur. The member countries have the same nominal interest rates and it remains only a real exchange rate that separates them.

ICFs are usually categorized into foreign direct investment (FDI), foreign portfolio investment (FPI) and other foreign investment (OFI) including bank loans and corporate debt. We will make use of the definitions of the IMF BoP manual (see IMF (1993)).

In contrast to the ICFs stand what we call real flows (RF). Following the standard definition, *every transaction between two entities of different nationality involving the exchange of goods and services will constitute a real flow*⁵. It is clear that, following this definition, for every real transaction, there will be at least one financial transaction. Further, for the sake of consistency and following the lines of the argument made above, we have to distinguish between real transaction within a currency union and such that involve at least two currencies.⁶ In what follows, we will only focus on RFs that involve two different currencies, however, in order to facilitate comparison in some instances we will also report those that occurred within a currency union, namely within the EMU.

2.2. National sources. Under this section we will explore the sources for ICF data we have on the national level. In principle we could use, grace to increased data availability, data from almost all countries of the world. We will, however, confine our attention to data from Germany and the United States. The case of Germany

⁴For example in the context of a neoclassical growth model.

⁵Hence barter transactions are not included in our definition.

⁶There are a couple of reason why a real transaction could provoke more than one currency transaction. The easiest example would be a flow of goods from country A to country B whereas the exchange of monies would use a vehicle currency C.

may be considered as the benchmark for the typical OECD country, whereas the United States deserve special attention for a variety of other reasons. To our knowledge, it is only in the U.S. that data on ICFs can be obtained independently from the BoP accounts and in a more detailed manner. The Treasury International Capital (TIC) data contains monthly gross purchases of long term capital of U.S. entities from foreigners and, vice versa, sales of U.S. assets to foreigners. Long term securities following the definition of the U.S. treasury department are "equities and debt issues with an original maturity of more than one year .. issued by U.S. or foreign-based firms"⁷. Hence, using the TIC data, we are able to obtain an estimate for gross portfolio investment into and out of the most important economy of the world.

At the national level, e.g. for Germany, we will use publicly available BoP figures. The BoP is the systematic account of all transactions of the inhabitants of one country with the rest of the world. It is normally compiled by the central bank or monetary authority of the country. The quality and availability of the data greatly differs between developing and developed countries, yet recently the IMF has tried to promote international standards (see IMF (1993)) to which a growing number of states comply. It is thus natural to employ the capital account of the balance of payments to assess the volume of international financial flows. This attempt, however, is hampered for two reasons. The first is that the capital account only reports differences in the stocks of the assets. We can, of course, overcome this first problem, by simply differencing stocks to obtain flow data.⁸ The second impediment is actually more severe. The capital account of the BoP only reports net flows that occurred in the period under consideration. That means that, *ceteris paribus*, the length of the period determines the degree to which the amount of net flows is an accurate estimator for the amount of gross flows in that period. Ideally we would like to have access to data that covers each transaction and thus gross flow data. Instead we only have, for example, differences in foreign holdings of long term bonds between, at best, the beginning and the end of a month. This would mean, to continue our example that a transaction, involving the purchase and resale of a certain amount of long term bonds within the period would not be reported in the BoP. Also, more generally, capital brought into the country for shorter maturities than the reporting period will not show up in our data. To that extent the BoP statistic systematically underestimates international capital flows. Henceforth we will call this underestimation "aggregation bias", for a main source of error is the aggregation within the reporting interval.

⁷See <http://www.treas.gov> for the description of the data.

⁸This, however, would only be sufficient in a world without depreciation and other valuation effects.

A special case, again, is the US TIC data. Even though this data provides access to gross flows, it suffers heavily from measurement problems.⁹ The existence of survey data to scrutinize the accuracy of the TIC (see Warnock and Cleaver (2003)), however, distinguishes in our view this data from the usual available national sources. We will thus use this source to gain further insights into the size of the aggregation bias.

2.3. Supranational sources. There are several institutions in the world that collect and distribute data on ICFs, the most prominent being the IMF and the World Bank. Naturally, most of these institutions simply present data provided by national central banks in another way. Thus, theoretically, this data should be the same as the data that can be obtained from national sources alone. The crucial difference, however, is the way the data are aggregated and sorted, mostly in a fashion to facilitate comparison or induce consistency (or both) (see IMF (2001)).

In the case of the IMF BoP data the price for consistency seems quite high - where in the majority of cases monthly data is available the IMF reports only with annual frequency. The aforementioned aggregation bias thus may be even more severe with the IMF data. In principle then, we have the alternative of using the IMF annual data - which reports capital flows directly - for a comprehensive set of countries or, for the sake of accuracy, compile a list of countries that report capital account changes monthly and then take each of these countries BoP data to infer the monthly amount of capital flows. Interestingly enough, doing both and compare the results with the data we have from the IMF should enable us to assess the extent of the aggregation bias when switching from monthly to annual reporting. Although doing that for all countries is beyond the scope of our work, this will be our methodology for the two selected cases of Germany and the US.

Another source that deals, indirectly, with the issue of ICFs is the World Bank Global Development Finance (GDF) that is published annually. The GDF data contains flow and stock data. The main focus of the dataset are less developed countries. Because of this focus most of the flows reported are debt related. In terms of aggregation bias the quality of this data is roughly comparable with the IFS data, for the reporting is annually. In what follows we will not make use of this data set, mainly for reasons of consistency.

There is a sizeable literature that makes use of capital flow data for several purposes.¹⁰ Some researchers use publicly available sources

⁹Apart from that it only covers FPI.

¹⁰Examples are the influence of capital account liberalization on welfare Gourinchas and Jeanne (2003), the influence of structure of capital flows on growth Mody and Murshid (2002) and Rodrik and Velasco (1999).

for their work, others have the benefit of having access to confidential data (see Warnock and Cleaver (2003)) and others still make use of data that is very hard to come by or grossly expensive (one example being Rodrik and Velasco (1999)). One of this sources is the data set on ICFs of the Institute of International Finance (IIF) that has a special focus on short term capital flows. This source appears to be quite comprehensive in scope and thus better suitable as the IMF and World Bank data taken together (see Rodrik and Velasco (1999, p.12)). We were, however, not able to obtain this data and will thus proceed without incorporating its implication into our analysis.

2.4. BIS data. Although the Bank for International Settlement is a supranational organization in the sense introduced above, the character of the data warrants its treatment in a separate section. The evidence from the triennial exchange rate market survey (see Bank BIS (2002)), conducted under the guidance of the BIS by the central banks of 52¹¹ participating countries is in some sense the opposite to the BoP data we discussed in the previous sections. The BIS data reports the complete¹² foreign exchange turnover for one selected day of a given month. Since international capital flows must, by definition, go through the foreign exchange market, the data will account for all international capital flows at this date.¹³ Unfortunately, it will also cover other activities than movements of capital. A considerable proportion will be intra dealer transactions and thus not falling under our definition of capital flows. Another source of error is that capital flows often are intermediated through financial centers and thus there may be two flows reported where in reality is only one. The above should make clear that the figures of the foreign exchange market survey systematically overestimates the amount of international capital flows. Problematic may also prove the aggregation we have to undertake in order to compare the daily data with our usual monthly aggregates. By aggregating the flows of one day we have to implicitly assume that all the other days were just the same. This seems to be the compromise we have to make when using this data and we have to be aware that this assumption will lead us astray in case of such extreme events as an international financial crises.

¹¹This is the latest figure for the 2004 survey. The first survey was conducted in 1986 with only 15 countries participating.

¹²Complete here refers to the best we can get. There are qualifications that have to be made. First, not all countries decided to participate in the survey. Second, not all market participants in the countries that decided to take part did respond. Even so, the BIS survey claims to cover roughly 95% of foreign exchange market turnover.

¹³With the possible exception of equity swaps that are increasingly popular to pay for merger and acquisition activities. An estimation of the relative importance of such transactions is presented by Warnock (2002).

3. The evidence from national sources

3.1. Evidence from the US - capital flows.

3.1.1. *BoP data.* In the case of the US there is, to our knowledge, no gross flow data on BoP level. Instead the Bureau of Economic Analysis (BEA), the body that is in charge of providing the BoP figures, reports quarterly net flows. Thus we report our estimates of monthly net capital flows in and out of the US in this section as these are derived from the BoP issuing body, namely the BEA:

BoP figures for the US	Domestic Net	Foreign Net
	Capital Outflow	Capital Inflow
April 1992	9	18
April 1995	40	41
April 1998	47	66
April 2001	32	91
April 2004	37	91

Table 1: US Net Capital Flows 1992 - 2004 (Billions of US-Dollars)¹⁴

Since in the case of the US-BoP we had only access to quarterly data, we divided the reported amounts for the second quarter of the year in question by 3. Note that for 1992 there were no direct investment figures available.

The above figures show that the US is a net borrower in the international capital markets. This is true for all periods we have data on. At the same time, the amount of international diversification was increasing. This can be seen from the rising net capital flows into and out of the US.

3.1.2. *TIC data.* The US Treasury department has its own database for international financial transaction. Originally the reason to start the collection of this data has been to monitor the development of foreign ownership of US assets, thus political considerations were at play here. A decade or so later the reporting was extended to cater also for US holdings of foreign assets. Data collection is done on a regular basis and quality is controlled for by survey studies. From the BEA we can obtain gross direct investment data to complete the picture.

¹⁴We obtained the data from the website of the BEA - http://www.bea.gov/bea/international/bp_web/simple.cfm.

TIC and BEA
figures for the US

	Gross Capital Outflow	Gross Capital Inflow	Daily Turnover
April 1992	242	249	23,4
April 1995	304	312	29,3
April 1998	781	792	74,9
April 2001	1026	1064	99,6
April 2004	1570	1655	154

Table 2: US Gross Capital Flows 1992-2004 (Billions of US-Dollars)¹⁵

Our estimation of the capital outflow in the period under consideration was obtained by using the TIC long term asset gross figures for purchases of US residents from foreign entities plus the amount of US direct investment abroad. Since in this case we had only access to quarterly data, we divided the reported amounts for the second quarter of the year in question by 3. Note that for 1992 there were no direct investment figures available. Implied turnover is calculated as the sum of capital outflows and inflows divided by 21, which is the amount of working days in April. Implicit in that procedure is the notion that transactions are not netted out against each other. Instead each and every transaction adds to turnover.

The US figures are remarkable in some ways. First they reflect the commonly known fact that the US enjoys the benefits of sustained net capital inflows, or two put it differently, is allowed to run sustained current account deficits. This is true for all the different investment categories we have detailed data on and over the entire horizon under consideration. Secondly, from both gross and net flow figures, we can see a pronounced rise in capital movements. This might be interpreted as what is commonly referred to as financial globalization, i.e. the increasing integration of the world capital markets. Some words of caution, however, may be in order here. The increase appears less startling, if we account for both inflation and growth. Whereas this argument may be valid for the increase between April 1992 and April 1995 - around 25% - it is less problematic if we look at the increase between April 1995 and April 2004 for all quarters. All in all, gross capital flows - as measured by our estimates - grew more than fourfold between 1992 and 2001. It thus seems safe to say that on basis of this

¹⁵Data are taken from http://www.treas.gov/tic/s1_99996.txt. In the case of the US we have had only access to gross long term securities transactions and gross direct investment figures. Direct investment was on quarterly basis, gross long term TIC data on monthly basis.

evidence we can assert that financial globalization of the US indeed deepened, at least from 1995 onwards.

3.2. Evidence from the US - trade flows. The BEA provides BoP data for the US. From the website of this institution we were able to obtain the gross trade figures for the US. These figures are shown below:

Real Flows US			
	Import	Exports	Daily Turnover
April 1992	53	50	4,9
April 1995	75	65	6,7
April 1998	91	78	8,0
April 2001	118	87	9,8
April 2004	143	94	11,3

Table 3: US Real Flows 1992-2004 (Billions of US-Dollars)

Table 3 shows a steady increase in trade levels (goods and services) between April 1992 and April 2004, which is, however, less pronounced than the increase in financial flows during the same period. Nevertheless, the growth in trade levels outstrips real GDP growth so that we can, in the light of the above figures, assert that trade integration of the US economy in terms of real flows has increased during the 1990s.

3.3. The case of Germany - capital flows. The Bundesbank collects figures and compiles the balance of payments since 1956. In 1995 the reporting standard was changed to accommodate for the suggestions made by the IMF, i.e. the latest edition of the IMF Balance of Payments Handbook. The monthly publication contains capital account data with fine breakdowns after countries, sectors and instruments. As is standard, the Zahlungsbilanzstatistik contains net flows in and out of the country. In the case of Germany, however, there are also gross figures of flows, at least for the three asset classes direct investment, portfolio investment and other investment. The figures are stemming from fortnightly collected reports that German banks are obliged to send to the Bundesbank. These reports should in theory contain every transaction above a certain threshold level, this at the moment being 12500 Euro (see Kruse (2000, pp.1-3)). We report, both

net and gross flow figures below:

BoP figures for Germany		
	Domestic Net Capital Outflow	Foreign Net Capital Inflow
April 1992	6,0	10
April 1995	11	14
April 1998	27	28
April 2001 with EMU	22	20
April 2001 without EMU	15	9
April 2004 with EMU	30	-5

Table 4: Germany: Net Capital Flows 1992-2004

BoP Figures for Germany			
	Gross Capital Outflows	Gross Capital Inflows	Daily Turnover
April 1992	84	53	6,6
April 1995	182	183	17,4
April 1998	291	287	27,5
April 2001 with EMU	384	363	35,5
April 2001 without EMU	263	270	25,4
April 2004 with EMU	682	668	64,3
April 2004 without EMU	487	454	44,8

Table 5: Germany: Gross Capital Flows 1992-2004

Table 4 and 5 report figures in billions of US-Dollars. If original figures referred to Euro or German Mark we converted this using the average US-Dollar/Euro Rate for the period under consideration: 1.1985 for April 2004, 0.89290 for April 2001, 0.551 (US-Dollar/DM) for April 1998, 0.72483 for April 1995 (Source: www.oanda.com).

Capital outflows, as presented here, are the summation of new direct investment abroad, domestic purchases of foreign held financial assets, liquidation of stock of foreign direct investment and foreign sales of domestic financial assets. All the figures were taken from the Bundesbank (2005). In the years 1998 and 1995 only aggregated data

was available, so an estimate of the monthly figures was obtained by dividing by 12. Capital inflows, as presented here, are the summation of new foreign direct investment, sales of domestic financial assets to foreigners, liquidation of stock of domestic direct investment abroad and purchases of foreigners of domestic financial assets. All figures were taken from Bundesbank (2005). In the years 1998 and 1995 only aggregated data was available, so an estimate of the monthly figures was obtained by dividing by 12. Implied daily turnover is calculated as the sum of capital outflows and inflows divided by 21, which is the amount of working days in April. Implicit in that procedure is the notion that transactions are not netted out against each other but instead each and every transaction adds to turnover. These figures were estimated using the overall estimated amount for April 2001 ICFs including EMU countries subtracting the share EMU countries had in this year in net capital exports or imports. The respective shares we used were 28,55% and 31,15%. This asymmetry explains the reversal of the qualitative result.

The German gross figures confirm the global trend of a rise of international financial integration during the last decade. The increase of capital movements in and out of Germany between 1995 and 2004 was steady and pronounced. Our estimates for the gross monthly capital flows imply a daily turnover at the foreign exchange market which is reported in the last column of table 5. In fact, this will be our vehicle to compare the BoP gross flow data with the BIS foreign exchange turnover data. In doing this calculation we cannot, however, attribute this flows to a certain instrument.

The ICFs in and out of Germany grew more than 8-fold in the last decade if intra EMU capital flows are taken into account. In the Dollar figures used above, this is slightly more than the increase in the US. We finally turn to compare the degree of international financial integration of Germany and the US, using the capital movements per GDP ratio for the two countries:

Financial Openness: Gross Capital Movements
per GDP

	Germany	US
April 1995	0,6	0,1
April 1998	1,1	0,2
April 2001	1,7/1,2	0,2
April 2004	3,2/2,2	0,8

Table 6: Financial Openness Indicator: US and Germany

Capital movements as used in Table 6 are all capital inflows plus all capital outflows of the period under consideration. GDP figure were

obtained from IMF IFS. For the years 2001 and 2004 the first ratio includes intra EMU ICFs, the ratio does not.

Clearly, although the changing Dollar (strong until 2002, then weaker) exerts some influence on this figures, according to this simple calculation Germany started from a much more open position. In general, the German economy is perceived to be more integrated into the world markets than is the US. Lately, however, the US integration into world financial markets has been increasing with higher speed.

3.4. The case of Germany - trade flows. As noted earlier there is no great difficulty obtaining the amount of gross real flows. We report the real flows in and out of Germany following the Bundesbank figures (see for example Bundesbank (2005)):

Real Flows Germany			
	Imports	Exports	Daily Flows
April 1992	46	45	4,3
April 1995	60	59	5,7
April 1998	58	56	5,4
April 2001 with EMU	64	62	6,0
April 2001 without EMU	37	37	3,5
April 2004 with EMU	83	95	8,5
April 2004 without EMU	36	41	3,7

Table 7: Germany: Real flows 1992 - 2004 (Billions of US-Dollars)

Imports (exports) in Table 7 are imports (exports) of goods and services, including income payments. Original figures referred to either DM or Euro. We calculated US-Dollar values using average exchange rates for the period under consideration, specifically: April 1995: 0,72483 USD/DEM, April 1998 0,551 USD/DEM, April 2001 0,8929 USD/EUR, April 2004, 1,1985 USD/EUR (Source: www.oanda.com). Figures are to monthly averages, since we had only access to annual data. In the case of the year 2001, monthly figures were available, yet in order to allow for the exclusion of intra EMU trade we had, once again, resort to annual figures. Breakdown for EMU counterparties is only available for trade in goods. Therefore, services and transfers have been attributed using the trade in goods share.

It may seem surprising that levels of trade have decreased between 1995 and 1998 (and roughly remained constant between 1995 and 2001). The main reason for this, however, is that we report the figures in US-Dollar. Between 1995 and 2001 there has been a steady devaluation of the Deutsche Mark (later Euro) against the US-Dollar. To distinguish between real flows that stipulate the exchange of currencies and such that do not, we report RF's with and without intra EMU flows. As can

be seen from the table, intra EMU trade in 2001 accounted for roughly 50% of the overall trade. This share decreased slightly in 2004.

4. The evidence from supranational sources

4.1. The evidence from the IMF-IFS. Since its birth one clear goal of the IMF has always been to promote an internationally consistent system of accounting. To this end, the IMF sets international standards for the compilation of the BoP that are widely adapted. The publication of the 5th edition of the BoP manual in 1993 by the IMF marks the most recent effort (IMF (1993)). Since then, most of the member states have adopted this set of rules and report BoP data accordingly to the IMF. The result is what the IMF publishes regularly under the heading International Financial Statistic. In order to estimate the amount of ICFs from the IFS we use the aggregate outflows presented under the section balance of payments, especially positions Direct Investment Abroad (78bdd), Portfolio Investment Assets (78bfd), Other Investment Assets (78bhd) and Reserve Assets (79dbd). We excluded Financial Derivatives Assets (78bwd) since the reporting is not consistent (see IMF (2001, p.xxiii)) and data are sparse. Further, we focus here on outflows, since an outflow in country A constitutes an inflow in country B. Therefore we prevent double counting by only using either outflows or inflows.

The data in the IFS will show a negative sign if in the period under consideration the asset position of the country increased (outflow) and a positive sign if the asset position decreased (capital moving back into the economy). For the purposes of calculating the amount of ICFs, however, we will treat capital that is moving back into the economy as an outflow and hence negate the sign of the transaction. In contrast to the numbers reported for the International Investment Position (IIP), depreciation is not accounted for by calculating the BoP figures. In general, the BoP figures only relate to flows that occurred, leaving aside valuation issues of existing stocks (see IMF (1993, p.77)). (There is now an influential literature that picks up exactly this point, see for example Gourinchas and Rey (2005)).

The amount of net capital flows reported in the BoP is underestimating the actual net flows in that reporting is limited in some cases to annual data (see IMF (1993, p.80)). Transactions concerning the same class of assets within the reporting period will be aggregated and thus may net out each other (see IMF (1993, p.80)). Hence, *ceteris paribus*, the greater the reporting interval, the greater will be what we call aggregation bias. On the national level, we have access to monthly BoP data, yet consolidated IFS data is only available at annual frequency, thus, most likely, rendering the aggregation bias worse. We report our estimates in the table below:

Net ICFs IMF-IFS
figures

Year	1992	1995	1998	2001	2004
Capital outflows	1200	1625	2144	2568	n.a.

Table 8: World Capital Outflows 1992-2001 (Billions of US-Dollars)

Following the IMF-IFS data, net flows of capital between countries increased between 1992 and 2001. This result appears to be somewhat in contrast to the findings of the BIS survey, which reported a marked decline of exchange market activity in 2001. However, by plotting the evolution of the ICFs over the whole period we see that capital flows peaked twice in the sample, 1997 and 2000 at both dates being in value higher than the 2001 figure. The diagram below makes this point more clear:

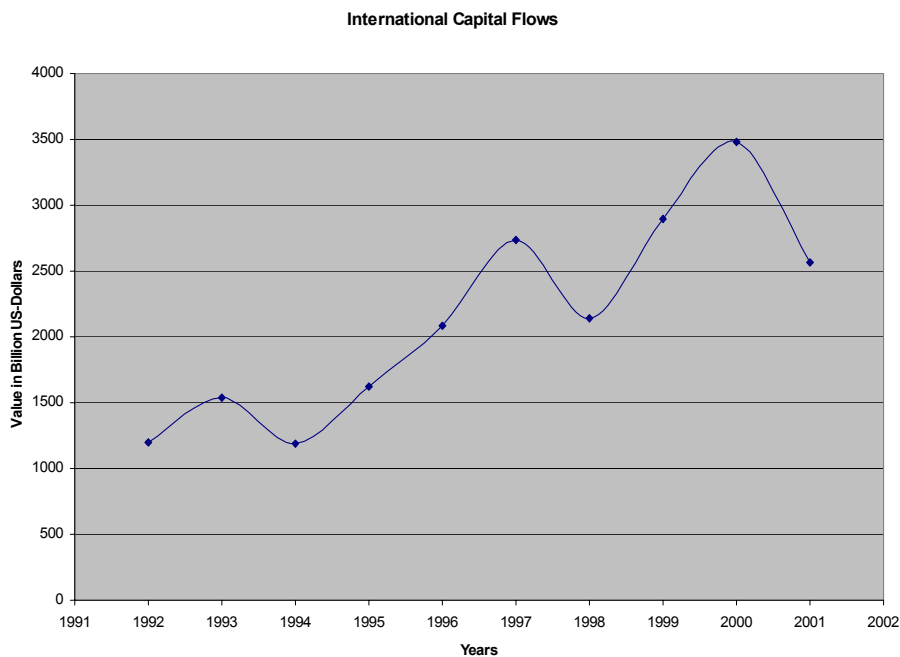


Figure 1. World Capital Outflows 1992-2002

5. The evidence from the foreign exchange market

One way to define international financial flows is to postulate that international financial flows are flows of liquid assets from one currency area to another. This definition surely fails to accommodate for capital flows between, at present for instance, France and Germany. It is, however, the relevant one when the focus is on exchange rate behavior, for capital flows between France and Germany only have an influence on the real exchange rate between the two countries, as the nominal

no longer exists. Sticking with this definition, it is natural to look at the foreign exchange market in order to assess the magnitude of international capital flows falling into the framework given above.

5.1. The triennial survey of the Bank for International Settlements. The perhaps most comprehensive, publicly available source that documents the developments in the foreign exchange market is the "Triennial Central Bank Survey" which is conducted under the aegis of the BIS in Basle. As the name suggests this is a survey study which once in every three years monitors the foreign exchange market for the period of one day. The first survey has been carried out in 1989, the most recent one in 2004. The month under investigation has always been the April of that year, whereas the day differed from study to study due to the influence of bank holidays. At country level the central banks of the participating nations are responsible for gathering the data. These data are then send over to Basle, where the final report is assembled. The number of participants in this survey has increased steadily from only 21 countries in 1989 to 52 in 2004.

5.2. Main findings of the last BIS-Survey. The global daily turnover in the foreign exchange market, as measured in April 2004, was around 1880 billion US Dollars. This constitutes a marked rise in comparison with the previous survey, where the daily turnover was found to equal roughly 1380 billion Dollar and also a rise in comparison with the former "record turnover" reported in April 1998. The following table gives a more detailed view of these results.

Global FX market turnover							
	Year	1989	1992	1995	1998	2001	2004
Spot		317	394	494	568	387	621
Forwards		27	58	97	128	131	208
Swaps		190	324	546	734	656	944
Estimated gaps		56	44	53	60	26	107
Total turnover		650	840	1120	1590	1380	1880

Table 9: Foreign Exchange Market Turnover 1989-2004 (Billions of US-Dollars)

The breakdown is in the classical instruments of the foreign exchange market. Spot transactions are either purchases or sales of foreign exchange today for delivery in two days. Outright forwards are contracts which obligate to purchase or sell a specified amount of foreign exchange at a specified date in the future (The demand for forward contracts in the context of hedging is investigated in chapter one.). Foreign exchange swaps are purchases or sales of foreign exchange today for delivery in two days (spot/forward swap) or delivery at farther away

(forward/forward swap) dates together with the opposite transaction at a date even more in the future. It is clear from the above that outright forwards can also be synthesized by a suitable combination of a spot transaction and a swap. This is the reason for the distinction being made between outright forwards and such that are constructed.

Table 9 shows that between 1998 and 2001 global FX turnover sharply declined and bounced back in 2004. The by far biggest proportion of the secular decline in turnover between 1998 and 2001 occurred in the spot market, followed by the swaps, whereas the volume of outright forwards increased every period. The main reasons behind this observed decline may be the introduction of the Euro, a more widespread use of electronic brooking and the ongoing concentration processes in the banking industry and the corporate sector (BIS2001, p. 6). In fact, additional evidence for this proposition is provided by the national gross figures. According to the US and German figures, capital flows increased, if not for reasons of the unit of account or the introduction of the Euro. The decline in 2001 can thus, in part, also be attributed to the dollar appreciation versus Euro and Yen.

Another interesting aspect of the data is the breakdown of the foreign exchange market activity after type of counterparty. The share of intradealer trading between 1992 and 2004 declined substantially by 20%, the likely causes being the ongoing consolidation of the industry and the introduction of electronic brooking systems. In contrast, the amount traded between banks rose by 20% in the same time. The relative share of the turnover with non-financial customers stayed constant, with a peak in 1998. In the last two rows, table 10 contains a geographical breakdown. Following this, the relative importance of cross-border FX transactions has steadily risen. The findings are summarized below.

FX turnover by counterparty					
Year	1992	1995	1998	2001	2004
with reporting dealers	73	64	64	59	53
with other financial institutions	13	20	20	28	33
with non-financial customers	14	16	17	13	14
local	n.a.	46	46	43	38
cross-border	n.a.	54	54	57	62

Table10: Foreign Exchange Market Turnover, Breakdown After Counterparties (Percentage of global turnover)

There is a simple way to estimate the amount of ICFs from the BIS FX survey data. If we only focus on ICFs that involve capital crossing currency borders, we know that the estimate we are looking for is a subset of the total amount of foreign exchange market turnover. Further we should deduct the amount that was traded between traders.

This implicitly assumes that capital movements from one destination to another just involve one dealer. This assumption may be justified on grounds of efficiency reasoning and the existence of active intradealer trading to overcome information problems (Galati (2001, p. 4)). Further we should also deduct the amount of RFs from this data, since for every RF in the sense we use it here, there should be at least one FX transaction. As mentioned above, this figure would still be an overestimation of actual ICFs but will serve us well as a benchmark.

ICF benchmark -
BIS figures

	1992	1995	1998	1998*	2001	2004
FX without intra dealer	227	302	572	572	566	884
RF of that year	4701,4	6350,6	6833,7	6833,7	7640,4	9298,0
RF adjusted for						
intra euro area trade	4701,4	6350,6	6833,7	6023,4	6677,9	7585,5
RF daily	18,8	25,4	27,3	24,1	26,7	30,3
Benchmark	208,2	276,6	375,7	547,9	625,3	853,7
Ratio	1:11	1:11	1:20	1:23	1:20	1:28

Table 11: ICF Benchmark (Billions of US-Dollars)

Real flow figures refer to world total of exports of merchandise goods of that year plus world total of services exports of that year and are taken from the World Trade Organization (WTO) Trade Statistic. Intra Euro Area data is taken from Eurostat and converted by the average USD/EUR exchange rate of April of that year. Note that for 1998 the exchange rate is based on a synthetic Euro. We divided the annual figure by 250, which is approximately the annual number of working days to get daily figures.

For the years 1992 and 1995 the global aggregate reported by the WTO is not problematic. After that date the introduction of the Euro had already affected the traded volume in the FX market (Galati (2001, p. 5)). Since the Euro has come into effect at the 01.01.1999 we are faced with an ambiguity regarding the year 1998. The convergence to the announced rates may have affected the amount of FX trades well before the 01.01.1999 and hence our sample of 1998 may be biased downwards. For this reason we report two benchmark values for 1998. One with and one without the amount of trade within the Euro area. The case is clear again in 2001, where we deducted the amount of intra Euro area trade. In order to render the annual trade volume comparable to the foreign exchange data, we divided the amount by the number of business days of the year under consideration.

Following our results the ratio of real to financial transactions has increased from 1992 with roughly constant speed and then in 2001 fallen to 1995 levels. The increase is certainly due to a global trend of

increasing financial integration, something that is also often referred to as financial globalization (see for a discussion Eichengreen and Bordo (2002)).

5.3. Results for the US. The Federal Reserve Bank of New York conducts the US-part of the BIS survey. The breakdown is more detailed than in the aggregate study that is provided by the BIS, however, the figures do not contain an estimation of the amount of foreign exchange transactions that were intradealer trades. The US were among the first to join the initiative of the BIS to conduct a FX market survey every three years. Even though the available estimates reach back until 1986, we do not report them here since in most of our other cases we only have data from 1992 onwards.

US FX market turnover					
Year	1992	1995	1998	2001	2004
Spot	95	134	148	104	221
Forward	13	28	37	36	62
Swap	59	83	166	114	178
Total	167	244	351	254	461

Table 12: US: Foreign Exchange Market Turnover after Instruments¹⁶

What can be seen from table 12 is the steady rise in exchange market activity until its first peak in 1998, the sudden fall in overall volume in 2001 and the recovery in 2004. It is apparent that in the case of the US data the intermediate fall in turnover can mainly be attributed to the decrease in spot and swap trading.

5.4. Results for Germany. The Bundesbank collects the figures of the triennial survey for Germany. In April 2004 a total of 33 financial institutions took voluntarily part in the survey. This amounts to some 90% of the whole German foreign exchange market. The global trend of a declining volume of spot transactions in 2001 was also visible in the German data. The interbank dealing fell substantially between 1998 and 2001. The data are shown in the table below:

German FX market turnover					
Year	1992	1995	1998	2001	2004
Spot	30	34	42	26	36
Forward	3	4	8	12	n.a.
Swap	22	38	44	51	82*
Total	55	77	94	88	118

¹⁶Billions of current Dollars.

Table 13: Germany: Foreign Exchange Market Turnover after Instruments (Billions of US-Dollars)

6. Trade versus financial flows

We are now in the position to present four different estimators for the ratio of real to financial transactions. There are two distinct perspectives for the estimators of this ratio: on the world level and on the country level. For each perspective we calculated several ratios. At the world level, the first ratio will be based on the comparison between the amount of ICFs implied by the BIS financial flow data and the RFs reported by WTO trade figures. Naturally, as the above discussion made clear, this will constitute an upper bound for our ratio. Correspondingly, the lower bound measure for the ratio of real to financial flows is based on a comparison between the (net) ICFs reported in the IMF IFS database and the aforementioned WTO trade figures. Our estimates are reported in the table below:

ICF-RF ratio World Level	BISICF	IMF-IFSN _{et} ICF
	$\frac{\text{WorldGrossRF}}{\text{WorldGrossRF}}$	$\frac{\text{WorldGrossRF}}{\text{WorldGrossRF}}$
April 1992	11:1	1:4
April 1995	11:1	1:4
April 1998	23:1	1:3
April 2001	23:1	1:3
April 2004	28:1	n.a.

Table 14: International Financial Integration vs. Goods Market Integration

The picture that emerges is interesting. Based on our upper bound measure, for every dollar worth of real flows, we had something in between 11 and 28 dollar worth gross capital flows. The ratio increased sharply between 1995 and 2004, implying that the foreign exchange turnover was growing at a higher rate than international trade turnover. If we were to accept this turnover data as a measure of international financial integration, we could assert that international financial integration was growing stronger than goods market integration during the period 1992-2004. However, our second measure is pointing to a different interpretation. The ratio of net financial flows to real flows is first constant and then increasing slightly, implying that goods market integration has indeed been growing with roughly comparable speed to financial market integration. For 2004 we could not calculate the ratio due to missing IMF-IFS figures.

At this point it is important to note the different economic content of these two estimators. Both are measures of financial integration relative to goods market integration. The first one could be, broadly, interpreted as a measure of financial market efficiency, whereas the

second one provides a measure for international risk sharing. Behind this lies the reasoning that gross ICFs reflect mainly short term capital movements and hence measure arbitrage, whereas net ICFs, as provided by the IMF-IFS database, reflect structural changes in portfolios.

In this context it is important to remember the fact that net financial flows and net trade flows are connected via an accounting identity. This is the reason why comparing the two would be a pointless exercise. We believe, however, that calculating ratios of gross flows or comparing gross trade flows with net financial flows does convey economic content in the sense discussed above.

The second measure we provide is a country specific estimator based on the ICF estimators derived from the BoP data we obtained from the US and Germany, in both cases relative to the gross trade figures of these two countries versus the rest of the world. At the country level we have two measures of gross ICFs, the country specific BIS FX turnover data and the gross ICFs obtained from TCI (US) and BoP (Germany).

ICF_RF ratio		
Country Level - US		
	$\frac{\text{US (BIS) ICF}}{\text{US RF}}$ (Gross)	$\frac{\text{US (TCI) ICF}}{\text{US RF}}$ (Gross)
April 1992	8:1	5:1
April 1995	12:1	4:1
April 1998	15:1	9:1
April 2001	10:1	10:1
April 2004	18:1	14:1
	$\frac{\text{US (BoP) ICF}}{\text{US RF}}$ (Net)	
April 1992	1:4	
April 1995	1:2	
April 1998	1:2	
April 2001	1:2	
April 2004	1:2	

Table 15: US: International Financial Integration versus Goods Market Integration¹⁷

The picture that emerges is similar to the one we have at the world level. First, the ratio of gross ICFs to gross RFs is very high and increases steadily between 1992 and 1998. After that there is a markedly decline in that ratio, i.e. it goes down a third. In 2004 the ratio is again up to a new record level. Again, following this estimate, we would conclude that in the case of the US, financial market integration has deepened at higher speed during the 1990s than goods market

¹⁷Since in the US data intradealer turnover is not accounted for we calculate the ex intra dealer FX turnover using the international average and subtract this amount from the original reported. Additionally the dollar equivalent of the trade flows is subtracted.

integration. Using the TCI data to measure ICFs into and out of the US, however, a different picture emerges: financial market activity has been increasing, notwithstanding the marked drop in 2001, relative to real activity from 1995 onwards. Lastly, comparing net ICFs with gross RFs, the case of the US roughly resembles our results for the world. The ratio of net ICFs to real flows has been slightly increasing between 1992 and 1995 and from then on has been relative constant - which points towards an interpretation that goods and financial market globalization in the US go with comparable speed.

Our last table in this section report the same type of estimators for Germany. The comparison between BIS data and gross RFs yields qualitatively the same results as in the other two cases. It is noteworthy, however, that Germany constitutes a special case, as it is a member of the EMU. This in turn is very likely to affect our results. The common currency was introduced only in 2001, yet financial markets use the Euro already since 1999. This leads to an ambiguity with respect to the year 2001 figures. Clearly, FX turnover in Germany will have declined, since intra EMU FX trade is not necessary anymore. The same should be true for gross ICFs and, to a lesser extent perhaps, for net ICFs. This is the reason why we report two figures for 2001, the first comparing the respective measures of ICFs with overall gross trade flows, the second comparing it only with extra EMU trade flows.

ICF- RF ratio		
Country Level - Germany		
	$\frac{\text{German (BIS) ICF}}{\text{German RF}}$	$\frac{\text{German (BoP) ICF}}{\text{German RF}}$ (Gross)
April 1992	2:1	2:1
April 1995	4:1	3:1
April 1998	5:1	5:1
April 2001 with EMU	5:1	4:1
April 2001 without EMU	9:1	9:1
April 2004	13:1	12:1
	$\frac{\text{German (BoP) ICF}}{\text{German RF}}$ (Net)	
April 1992	1:6	
April 1995	1:5	
April 1998	1:2	
April 2001 with EMU	1:3	
April 2001 without EMU	1:3	
April 2004	1:2	

Table 16: Germany: International Financial Integration versus Goods Market Integration¹⁸

Again, our results for Germany are broadly in line with the other two cases. First, BIS ICFs have, compared to trade flows increased during the sample period, the only exception being the case where we allow for intra EMU trade to be counted. Note that this seems to absorb the fall we see in the 2001 ratios worldwide and in the US. The same holds true for the comparison between BoP gross ICFs and trade flows, showing the same tendency as the US ratios, again with the exception of allowing for intra EMU trade to be counted. Lastly, the ratio of net ICFs to gross RFs in Germany exhibits a hump-shaped trend, in contrast to what we observe in the US or at the world level. It first rises and then, after 1998 falls and has in 2004 again slightly risen. This may point towards an increased tendency to invest intra EMU, whereas trade levels may not have been affected too much from the introduction of the common currency.

The results presented above make clear that in the public often used comparisons between trade flows and financial flows are to be taken with a grain of salt. First, it is not clear, which ratio should be used. We believe we cannot use one without the other. Second, the assertion that financial markets have completely disconnected from real activity is only partially supported by our results. Using our figures, we find evidence for "exuberance" in the 1998 and 2004 gross ICF/gross RF ratios. Apart from that it seems that there is no secular trend to be observed. If anything, based on our findings, we would report a somewhat stable relationship between gross trade flows and net financial flows and a deepening of financial market integration relative to goods market integration if the focus is on gross flows only.

7. Disentangling the aggregation bias

We will now turn our attention to the concept of aggregation bias. As noted above, we can think of this bias arising whenever a data collecting body such as a central bank or the IMF sums up and hence possibly nets out several positions against each other. There are two sources of this aggregation: aggregation over time and aggregation over instruments. The first occurs whenever data that comes into the reporting body is transformed to meet dissemination standards, for example daily data is transformed into monthly figures. To illustrate this point a bit further we could think of a foreign purchase of a bond and a resale of the same bond the next day. Clearly the gross capital flow - in

¹⁸Since in the German data intra dealership turnover is not accounted for we calculate the ex intradealer FX turnover using the international average and subtract this amount from the original reported. Additionally the dollar equivalent of the trade flows is subtracted.

domestic currency terms - connected with this transaction would be

$$p_t + p_{t+1},$$

yet the daily net transaction would be only

$$\Delta p_t.$$

If the following day the same transaction but with opposite sign takes place, aggregated net flows will be zero, even though there may have been substantial capital movements. This makes clear that the reporting interval determines the degree of accuracy with which reported net figures will deliver a realistic picture of the underlying capital movements.

Ideally, of course, reporting should consist of both net and gross flow figures. To get a complete picture, we would want that the gross figures that are presented in the national accounts are just the sums of all transaction, without regard of the signing. Another source of reporting bias are the thresholds that are set by the central banks. For instance in Germany, the current threshold is 12.500 Euros, meaning that only transactions in the volume of (or exceeding) 12.500 Euros have to be reported. Clearly this leads to a systematic underestimation of the actual amount of gross flows.

The second source of bias occurs when several categories of financial transactions are merged into one, e.g. several maturities and instruments into portfolio investment assets. Here the signs matter and net out each other, so that under normal circumstances we get under-reporting and accuracy only when all sub-categories are of the same sign.

We can try to assess the extent of the aggregation bias we have in gross flow data on the country level. The FX turnover survey data from the BIS can safely be assumed to be as close as we will ever get to true gross flow data. By way of comparing the estimates of ICFs using this data with the gross data we obtained from the national reporting, we can say something about the size of this aggregation bias that will occur here.

7.1. The US. In this section we compare the estimates of gross flows from the TIC with the ICFs that we obtain by using the BIS survey. This is a comparison of gross flows with gross flows and hence a direct measure of aggregation bias, given a couple of underlying assumptions.

	1992	1995	1998	2001	2004
FX	45,1	87,8	126,4	104,1	216,7
- RFs	4,9	6,7	8,0	9,8	11,3
Total	40,2	81,1	118,4	94,4	205,4
Gross flows	23,4	29,3	74,9	99,6	154
AggBias	2:1	3:1	2:1	1:1	2:1

Table 17: US: Gross Flow Aggregation Bias

The above table shows this bias to be between 3 and 1, implying that for every Dollar of gross capital flows that is reported by the TIC dataset, we observe 3 to 1 dollars gross capital flows in the foreign exchange market. To some extent this is not very surprising. First, the TIC system does not cover all gross capital flows that occur. The focus there is on long-term assets, such with a maturity of one year or above. Secondly, the data provided by the BIS survey overstates capital flows. To obtain the BIS measure for capital flows using this data, we controlled for RFs, transfers and intradealer flows. There may be, however other activities we did not control for in the data, one example being pure hedging activities or other speculative positions. Further, it must be said that, even though we are controlling for intradealer positions it remains unclear to what extent interbank dealership that is included in our measure of gross capital flows reflects just the same activities, namely trading of currencies without underlying capital movements. It seems safe to assume that at least a fraction of the turnover between banks is of this sort, but there seems no easy way to estimate this fraction.

7.2. Germany. The picture that emerges for Germany is somewhat similar. The main difference is that in the case of Germany the gross capital flow data that is reported in the BoP should be more accurate an estimator as in the US case, for no maturities are omitted. The ratios we obtain are smaller and hence support this assertion. A possible interpretation for the remaining aggregation bias is that the 12500 Euro threshold is at work here. In any case, what can be seen from the data is that the bias is somewhat narrowing, both in the German data as in the US case. There may be several reasons for that, more accurate reporting being only one. Other that come to mind would be decreasing share of intradealer alike trades between banks that are included in our measure of gross ICFs, or increasing of average transaction size and thus rendering the threshold bias less problematic. We report our findings below:

	1992	1995	1998	2001	2004
FX	14,8	27,7	33,8	36,6	55,5
- RFs	4,3	5,7	5,4	6,0	3,7
Total	10,6	22,2	28,4	32,6	51,8
Gross flows	6,6	17,4	27,5	35,4	44,8
AggBias	1,5:1	1,3:1	1:1	1,3:1	1,2:1

Table 18: Germany Aggregation Bias

The information of table 18 and the trend contained is summarized in the picture below:

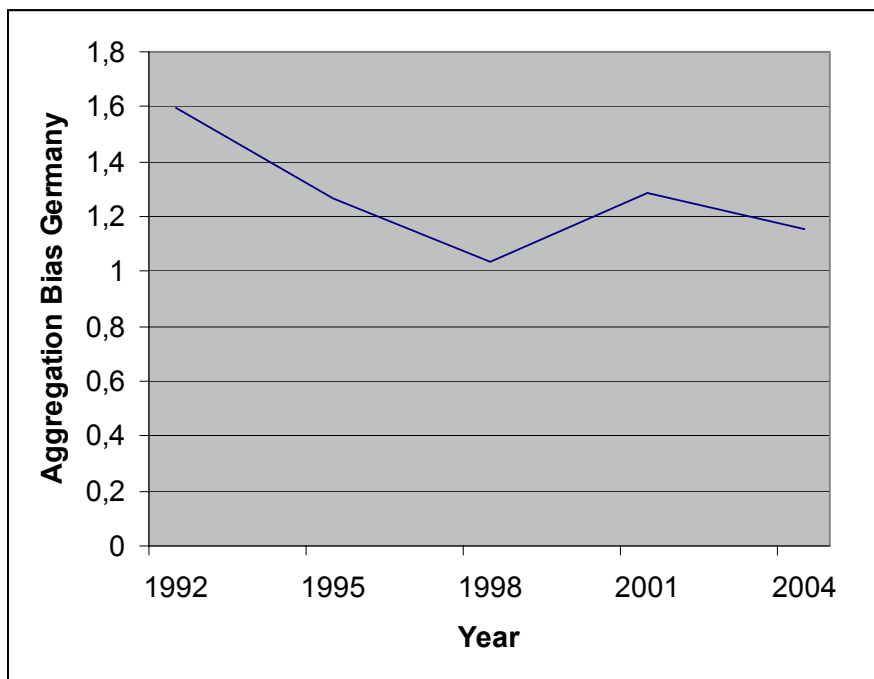


Figure 2: Aggregation Bias for Germany

8. Conclusion

In this chapter we gave an overview over the existing sources of data on international flows of capital and goods. Data on the latter are easy to come by, both on the national and at the supra-national level. The former is discussed in more detail.

First of all, the distinction between gross and net flows is of importance. Whereas at the national level, we typically find both types, at the supra-national level we only find net flow data. Further, the accuracy of the net flow data decreases from the national to the supra-national level. This is what we call aggregation bias. This bias also exists within gross flow data. To obtain an estimate of the aggregation bias in the case of gross flows, we compared BIS FX turnover data with

the reported gross flows of Germany and the US. It turns out that national reporting may underestimate gross capital flows up to a factor of three in the US and up to a factor of one and a half in Germany. This difference mainly reflects the more comprehensive German reporting. In both cases we observe a decreasing trend in this bias.

The often asserted disconnect of real and financial flows (the first reason to study international financial flows), can, if anything, only be attributed to the ratio of gross financial flows to gross real flows. At the world level and in the US, this ratio has increased between 1992 and 2005 by a factor of around two. In the case of Germany this factor is around six. However, if we look at the ratio of net financial flows to gross real flows, we see that goods market integration is actually growing with roughly the same speed as financial market integration.

The results also show that the gross flow measure of the foreign exchange market dominates all other measures of capital flows in size. This suggests that there is some room for additional theories on the issue of exchange rate determination. In any case, given the huge amount of trading that is not related to fundamentals (e.g. intra dealer trading), it is not overly surprising that models of the exchange rate that are solely based on fundamentals cannot track exchange rates in a satisfactory way. It also may indicate why the proposal of a global turnover tax (i.e. a Tobin Tax - the second reason to study international financial flows) has been around for so long now.

A more subtle result from our study is that, depending on the question under consideration, either national net flow data or national gross flow data should be used. In contrast, IMF-IFS data is highly aggregated net data that should only be used if the effort to construct a comprehensive dataset from the national reporting of the member states proves fruitless.

Part 3

Auxiliary Inquiries

CHAPTER 4

Expected Utility and Nonlinearly Transformed Random Variables

The chapter deals with a particular problem economists encounter in models with expected utility maximization. If the first order condition is a nonlinear transformation of the underlying random variable, explicit demand functions are hard to derive. We first give an overview on the existing mathematical tools we have to tackle the problem and then develop each method in detail using a simplified version of the model of the first chapter. It turns out that deriving explicit demand functions is always possible, as long as the economist is willing to invoke additional restrictions on the parameter space of the model.

1. Introduction

The following chapter is concerned with a particular problem arising out of standard economic modeling under uncertainty. When invoking expected utility maximization as proposed, for example, by Arrow (1974) and Rothschild and Stiglitz (1971) economists do touch several delicate fields of probability theory (and topology). When dealing with random variables in general, it holds that as long as linearity in transformations - e.g. stemming from equilibrium conditions - is preserved no great difficulties arise.¹ If, however, the structure of the model is more complex, especially with non-linear transformations of the random variable (i.e. the underlying uncertainty), the problem of finding explicit solutions to the utility maximization problem becomes more difficult and in most cases analytically intractable. At the same time it is highly desirable to obtain explicit solutions. For one thing econometric specification then becomes more rigorous, for another there are questions that simply cannot be answered without having an explicit solution at hand. The aim of the following chapter is first, to present a summary on the possible methods we have and, second, en passant, provide a toolkit to tackle nonlinearly transformed random variables.

It will become clear that in order to obtain explicit (demand) functions, certain restrictions on the underlying model have to be made. It has to be decided on a case by case basis whether or not these restriction are a reasonable price to pay. However, even without gaining explicit demand functions - and thus only invoking very few restrictions - we can always obtain indirect demand schedules which make comparative statics analysis feasible.

The chapter is organized as follows. Section one briefly outlines a model which generates a non-linear transformation of the underlying random variable. This model will serve as a reference point throughout. The subsequent section discusses whether and when it will be possible to evaluate the expected value of an nonlinearly transformed random variable directly. In the following section we turn to discuss indirect approaches that involve, for example, reversing the problem at hand or integration by parts. The fourth section concludes the chapter.

2. The model

We start with the description of a simple OLG model (see for an introduction to this class of models Blanchard and Fischer (1989, Chapter 3) or Azariadis (1993, Part II)). The example used throughout in this section is a simplified version of the model of chapter one.

As usual there are two generations, young and old. Agents in our model live for two periods, work when young and consume when old.

¹These cases are covered to greath length in most textbooks on statistics. Examples are Feldman and Fox (1991, p. 338ff.) and Whittle (2000, p.141ff)

Consumption is split up into two goods, X which is produced domestically and Y , a foreign good. Supply is governed by a constant returns technology and perfect competition using capital and labour as inputs. There is uncertainty about the second period price of the foreign good which is assumed to be stochastic. This is a small open economy model, hence domestic demand does not influence foreign prices. In the first period of their live, period t , domestic agents can buy forwards to hedge against price uncertainty of the foreign good which is resolved at time of consumption, i.e. in period $t + 1$. Using this very simple model the first order condition for utility maximization reads²

$$E \left(\frac{p_{t+1}^Y - \bar{p}^Y - (1 + r_{t+1}) \chi}{(1 + r_{t+1}) (w_t - \chi D_t) + (p_{t+1}^Y - \bar{p}^Y) D_t} \right) = 0. \quad (4.1)$$

where \bar{p}^Y is the price domestic agents can fix for period $t + 1$ consumption when entering in a forward contract in t , r_{t+1} denotes the real interest rate and χ the cost per unit of forward cover D_t . The random variable in this context is P_{t+1}^Y . Unlike in economic models with a timing structure that has both forward purchase and consumption decision of the agents in period t , the more realistic assumption that forward purchase and consumption decision take place in different periods, the first order condition, i.e. equation (4.1), has the random variable both in numerator and denominator. This in turn means that the convenient linear properties of the expectations operator will be of limited use. In general, making inferences about the mathematical properties of (4.1) is not possible without evaluating the expected value, i.e. being more specific about the distribution of the random variable.

We consider the term inside the brackets as a transformation Y of the underlying random variable P_{t+1}^Y . It will be convenient, in what follows, to rewrite this transformation as stated below:

$$Y = \frac{X - a}{bX + c_1 - ab} \quad (4.2)$$

where we defined

$$X \equiv P_{t+1}^Y$$

(being our original random variable) and

$$a \equiv \bar{p}^Y + (1 + r_{t+1}) \chi = \text{const.}$$

$$b \equiv D_t$$

$$c \equiv (1 + r_{t+1}) (w_t - \chi D_t) - \bar{p}^Y D_t = c_1 - ab$$

being constants in the context of finding an expected value, i.e. with respect to the variable of integration (or summation respectively). The control variable of the agents for their utility maximization problem is

²A more detailed description of the model can be found in app. (6.1).

D_t , the amount of forward cover to be obtained. The solution to (4.1) will then be a demand function

$$D_t = D_t(\phi, K_t)$$

where ϕ is a vector of the parameters of the model and K_t is the capital stock. The difficulty, however, in the light of a nonlinear transformation of the random variable, is to obtain this demand schedule explicitly, that is in a more spelled out form than (4.1). Barring the calculation of an exact solution, obtaining an explicit solution is not always necessary to derive analytically the equilibrium properties of the model. In fact, as is shown in chapter one many of the equilibrium properties can be derived without explicit demand functions. However, having at hand an explicit demand schedule facilitates econometric specification (and thus empirical tests) and allows comparative statics analysis of the model. Further, deriving analytical results without an explicit demand schedule is more often than not very tedious and without making additional assumptions on the structure of the solution almost always impossible. Having at hand only first-order condition 4.1 the comparative statics behavior cannot be inferred without drawing on numerical methods. However, obtaining analytical results is clearly preferable, especially since calibration exercises often lack accuracy (an example is the recent discussion of second order vs. first order approximation, as discussed for example in Kim and Kim (2003)) and introduce additional sources of errors like ranges of relevant parameters.

In what follows we therefore investigate the possible remedies we have for this situation. The remaining sections will briefly introduce and discuss a host of different approaches to our search for an explicit demand schedule. In doing so, we will always make reference to our benchmark transformation introduced above, but the methods used are readily applicable to any kind of problem with a similar structure, i.e. in economic models with choice under uncertainty and nonlinear transformations of the random variable(s).

3. The direct road - evaluating expected values

In principle we can derive expected values of random variables in two ways. Depending on the character of the underlying distribution, we either calculate the probability weighted sum over a certain amount of points of the support of the random variable or we integrate with respect to the random variable weighted by the probability density function (in short p.d.f.). These two ways reflect the two distinct cases: discrete vs. continuous support. This section will follow this structure. We first take a glimpse at the discrete case that is often found in economic models as benchmark and example alike. We then turn to the more general continuous support case. It will become clear that under the discrete support assumption there is always an explicit demand

schedule, as long as we have a finite number of points to consider. Contrary to that, the continuous support case is more complicated. Although in most cases the integrals do exist, most of the time we cannot derive closed expressions and thus cannot obtain explicit demand functions. We will, however, always be able to make inferences about the comparative statics behavior of the model by invoking the implicit function theorem. In this sense, the solutions we obtain represent a trade-off. Neglecting the increased costs of computing specific integrals, to make inferences about the comparative statics behavior we need to add the assumption of a specific distribution for the random variable of the model. In general it will depend on the model and the character of the underlying random variable, whether or not invoking the additional constraint is worthwhile.

3.1. The simplest case: discrete support. In this section we will derive an explicitly demand schedule for our small model, assuming that the underlying price uncertainty has a discrete support. More specifically we assume the following distribution governing our random variable $P_{t+1}^Y = X$:

$$\begin{aligned} p_{t+1}^Y &= p_L \text{ with probability } \alpha \\ p_{t+1}^Y &= p_H \text{ with probability } 1 - \alpha \end{aligned}$$

The expected value (4.1) is then simply given by

$$\begin{aligned} E(Y) &= \alpha \left(\frac{p_L - \bar{p}^Y - (1 + r_{t+1})\chi}{(1 + r_{t+1})(w_t - \chi D_t) + (p_L - \bar{p}^Y) D_t} \right) + \\ &\quad (1 - \alpha) \left(\frac{p_H - \bar{p}^Y - (1 + r_{t+1})\chi}{(1 + r_{t+1})(w_t - \chi D_t) + (p_H - \bar{p}^Y) D_t} \right) \end{aligned}$$

and yields, after collecting terms and using additionally the first-order condition $E(Y) = 0$, the following explicit demand function

$$\begin{aligned} D_t &= \frac{(1 + r_{t+1}) w_t [((1 - \alpha) p_H + \alpha p_L) - (1 + r_{t+1})\chi - \bar{p}^Y]}{(p_H - \bar{p}^Y - (1 + r_{t+1})\chi) ((1 + r_{t+1})\chi + \bar{p}^Y - p_L)} \\ \implies D_t &= \frac{(1 + r_{t+1}) w_t [E(p_{t+1}^Y) - \bar{p}^Y - (1 + r_{t+1})\chi]}{f(\text{Var}(p_{t+1}^Y))} \quad (4.3) \end{aligned}$$

with $f(\text{Var}(p_{t+1}^Y)) = \text{Var}(p_{t+1}^Y)$ iff $p_L = 4(\alpha^2 - 2\alpha^3 + \alpha^4)$. From 4.3 we can now derive the basic equilibrium properties of our model, i.e. comparative statics behavior and conditions for an interior solution to exist. We first note that the demand function displays the same basic properties as found (for the continuous case, using numerical methods) in chapter one, namely that increasing the variance of the underlying uncertainty decreases demand for forward cover (since $f(\cdot)$ is increasing in $\text{Var}(p_{t+1}^Y)$) and that agents need to be offered more than unbiased

(i.e. $E(p_{t+1}^Y) = \bar{p}^Y$) forwards in order to demand a positive amount of forwards. Further, demand is proportional to total wage income and sensitive to increasing costs - either directly, i.e. via χ or indirectly via the opportunity costs, i.e. the interest rate. Increasing costs, of course, decrease demand for forward cover.

This section made clear that using discrete support is a useful starting point in order to obtain first results in a setting as the above. It is obvious that the more points the support has, the more complicated the resulting function, i.e. in our case the demand function, will be. The demand schedule could be derived at the expense of an extremely simplified assumption on part of the underlying uncertainty. A more realistic modeling approach, however, will have to include uncertainty with continuous support. We therefore turn in the remainder of this section (and for the rest of the chapter) our attention to this case.

3.2. Continuous support - using the transformation theorem I . Consider again the non-linear transformation of our random variable X as described by (4.2), namely:

$$Y = \frac{X - a}{bX + c_1 - ab}$$

This transformation was introduced in section 2. The distribution of X is not specified in general and may be chosen in any particular context as deemed convenient. For our purpose here it is sufficient to note that the distribution of X is known to the agents of our model. We are looking for the expected value of Y , specifically for the first order condition

$$E(Y) = 0.$$

Since $E(Y)$ is by definition

$$E(Y) = \int_S Y f_Y(y) dY$$

where S denotes the support and f_Y the density of the random variable, our problem amounts to finding the p.d.f. of Y . The most straightforward way to tackle this is by drawing on the transformation theorem. The theorem is stated below:

THEOREM 4.1 (Transformation Theorem (Pitman (1995, p.304))). *Let X be a random variable with density $f_X(x)$ on the range (l, u) . Let $Y = g(X)$ where g is either strictly increasing or strictly decreasing on (l, u) . The range of Y is then an interval with endpoints $g(l)$ and $g(u)$. The density of Y on this interval is*

$$f_Y(y) = f_X(X(y)) \left| \frac{dY}{dX} \right|^{-1}, \quad (4.4)$$

where the absolute value of the derivative has to be used.³

There are some additional conditions⁴ under which the theorem holds. However, it is relatively easy to verify that these are met in our case.⁵ Turning to our model, it is easy to see that X as a function of Y is simply given by

$$X = \frac{(c_1 - ab)Y + a}{1 - bY}$$

and thus, in our case we have:

$$\frac{dX}{dY} = \frac{c_1}{(1 - bY)^2}$$

which is, given the structure of our underlying model, always positive. Therefore we can dispense with the absolute value operator in (4.4). Upon insertion into (4.4) we find

$$f_Y(y) = f_X\left(\frac{(c_1 - ab)Y + a}{1 - bY}\right) \frac{c_1}{(1 - bY)^2} \quad (4.5)$$

and finally we arrive at an expression for the expected value using the transformed density

$$E(Y) = \int_{H(S)} Y f_X\left(\frac{cY + a}{1 - bY}\right) \frac{c_1}{(1 - bY)^2} dY \quad (4.6)$$

where $H(S)$ stands for the transformed range of integration over which we have to evaluate the integral. We will now illustrate the application of the Transformation Theorem with two examples.

EXAMPLE 1. *Uniform Distribution*

In the first example we will use (4.6) together with the assumption of an underlying uniform distribution. We chose this distribution, since it is particularly easy to deal with. It is well known that the p.d.f. of an uniform distribution is given by

$$f_{UD} = \frac{1}{u - l}$$

where u and l denote the upper and the lower bound of the range of the distribution. The expected value of an uniformly distributed variable is then simply given by

$$E(X|X \sim UD) = \int_l^u x \frac{1}{u - l} dx = \frac{1}{u - l} \int_l^u x dx = \frac{1}{u - l} \left[\frac{1}{2} x^2 \right]_l^u = \frac{u + l}{2},$$

³Note that in our case we only deal with univariate densities. Then the term $|\frac{dY}{dX}|$ does refer to a simple absolute value, whereas in the multivariate context it refers to the determinant of the Jacobian.

⁴These are first continuity of the transformed density and secondly that our transformation employed generates a one to one mapping.

⁵See app. (6.2) for this.

where $E(X_{|X \sim UD})$ reads: expected value of the random variable X , given that X follows a uniform density. Applied to our problem at hand, i.e. equation (4.6), this leads to the following formula

$$E(Y_{|X \sim UD}) = \int_{l^*}^{u^*} Y \frac{1}{u-l} \frac{c_1}{(1-bY)^2} dY = \frac{c_1}{u-l} \int_{l^*}^{u^*} \frac{Y}{(1-bY)^2} dY,$$

where we made use of the "instruction" laid out in (4.6) and appropriately took care of the altered support of the new random variable, which changes according to the transformation and hence reads:

$$l^* = \frac{l-a}{bl+c_1-ab}; u^* = \frac{u-a}{bu+c_1-ab}.$$

The integral as such can be obtained using the result (see Bronstein and Semendjajew (1987, p. 35) for reference)

$$\int \frac{x}{(hx+j)^2} dx = \frac{j}{h^2(hx+j)} + \frac{1}{h^2} \ln |(hx+j)| + C$$

where applied to our case, we have $h = -b$ and $j = 1$. This in turn leads to

$$E(Y_{|X \sim UD}) = \frac{c_1}{u-l} \times \left[\frac{1}{b^2(1-bY)} + \frac{1}{b^2} \ln |(1-bY)| \right]_{l^*}^{u^*} \quad (4.7)$$

and upon evaluation finally yields⁶

$$E(Y_{|X \sim UD}) = \frac{1}{b} + \frac{c_1}{u-l} \frac{1}{b^2} \ln \frac{c_1-ab+bl}{c_1-ab+bu}. \quad (4.8)$$

In the last step, we now use the constraint $E(Y_{|X \sim UD}) = 0$. Thus (4.8) becomes

$$b = -\frac{c_1}{u-l} \ln \frac{c_1-ab+bl}{c_1-ab+bu}.$$

It is now time to go back to our original model. We simply reinsert the structure of the model and derive the demand function by solving for D_t . To this end, we need to specify the range of the uniform distribution. Since in our context we consider price uncertainty, natural candidates would be 0 for the lower bound, since nominal prices cannot be negative and - as a normalization - 1 for the upper bound. The demand function is then - implicitly - given by

$$D_t = (1+r_{t+1}) w_t \ln \frac{(1+r_{t+1})(w_t - \chi D_t) - \bar{p}^Y D_t + D_t}{(1+r_{t+1})(w_t - \chi D_t) - \bar{p}^Y D_t}. \quad (4.9)$$

It is obvious from (4.9) that no explicit solution for D_t can be obtained, for the equation contains linear and logarithmic terms in D_t together. Nevertheless, we can derive basic properties, e.g. the comparative static behavior of our demand function with respect to the parameters of

⁶The omitted steps can be found in app. 6.3.

the model using the implicit function theorem. This, for example, yields $\frac{dD_t}{dx} < 0$, a result that could not be obtained by using the same technique with the f.o.c., i.e. equation (4.1).⁷ Hence we can summarize that although with (4.9) we were not able to achieve our ultimate goal, i.e. an explicit demand function for D_t , we are still left with a structure that is more convenient to analyze than our point of departure (4.1).

EXAMPLE 2. *Symmetric Triangular Distribution*

The symmetric triangular distribution will be used in our second example, because it is relatively straightforward to handle and makes, given its shape, perhaps for a bit more realistic an example. The p.d.f. of the symmetric triangular distribution with support $[l, u]$ is given by

$$f_{TD} = \frac{2}{u-l} \left(1 - \frac{2}{u-l} \left| x - \frac{l+u}{2} \right| \right).$$

The expected value is then found to be

$$E(X|X \sim TD) = \int_l^u \left(x \frac{2}{u-l} \left(1 - \frac{2}{u-l} \left| x - \frac{l+u}{2} \right| \right) \right) dx$$

After some algebra and further manipulations this can be condensed to

$$E(X|X \sim TD) = \frac{a+b}{2}.$$

Turning again to our problem at hand, this time using the triangular distribution, we have to solve the following integral:

$$E(Y|X \sim TD) = \int_{l^*}^{u^*} Y \frac{2}{u-l} \times \left(1 - \frac{2}{u-l} \left| \frac{(c_1 - ab)y + a}{1 - by} - \frac{l+u}{2} \right| \right) \frac{c_1}{(1 - by)^2} dy. \quad (4.10)$$

Again we have to specify the support of the underlying random variable X . For our now familiar range of 0 to 1 equation (4.10) yields

$$E(Y|X \sim TD) = \int_{l^*}^{u^*} \left(2y \left(1 - 2 \left| \frac{(c_1 - ab)y + a}{1 - by} - \frac{1}{2} \right| \right) \frac{c_1}{(1 - by)^2} \right) dy. \quad (4.11)$$

To solve this integral we may first note that Y starts in the negative domain and, depending on the magnitude of a , remains there or not. Further, the integral has to be split up into two parts, to cater for the

⁷The result is derived in app. (6.3).

absolute value operator:

$$E(Y_{|X \sim TD}) = \int_{l^*}^{\frac{1-2a}{2(c_1-ab)+b}} \left(\frac{4y^2(c_1-ab) + 4ya}{1-by} \right) \frac{c_1}{(1-by)^2} dy \\ + \int_{\frac{1-2a}{2(c_1-ab)+b}}^{u^*} \left(4y - \frac{4y^2(c_1-ab) + 4ya}{1-by} \right) \frac{c_1}{(1-by)^2} dy$$

Note that no singularity can occur as the model we introduced in section (2) always implies $1-by > 0$. We can now evaluate the integral. We document this lengthy and somewhat tedious procedure in more detail in the appendix (6.4). The result reads

$$E(Y_{|X \sim TD}) = \frac{1}{b^3} (b^2 + 4c_1[(2c_1m_3) \ln m_3 + (c_1m_2) \ln m_2 + (c_1m_1) \ln m_1])$$

It remains to use the constraint $E(Y_{|X \sim TD}) = 0$, hence we set $0 = b^2 + 4c_1[(2c_1m_3) \ln m_3 + (c_1m_2) \ln m_2 + (c_1m_1) \ln m_1]$. Reinserting the variables of our underlying model then gives

$$D^2 = 4(1+r)w \left(\begin{aligned} & (2(1+r)(w-\chi D) - 2\bar{p}^Y D + D) \ln \frac{2(1+r)w}{2(1+r)(w-\chi D) - 2\bar{p}^Y D + D} \\ & + ((1+r)(w-\chi D) - \bar{p}^Y D + D) \ln \frac{(1+r)w}{(1+r)(w-\chi D) - \bar{p}^Y D + D} \\ & + (1+r)(w-\chi D) \ln \frac{(1+r)w}{(1+r)(w-\chi D)} \end{aligned} \right) \quad (4.12)$$

where we omitted the time subscripts due to space constraints. Just as in the first example above, equation (4.12) contains linear and logarithmic terms in D_t together. Therefore, we cannot solve for D_t and have to be content with an indirect demand function. The problem lies in the structure of the resulting integrals. Due to the transformation of the random variable as given by (4.2) and the structure of the expectations operator, we will in most cases encounter integrals that have a polynomial structure and these in turn leave us with both linear and logarithmic expressions in D_t in the same equation. Therefore, we can conclude from this section that using the Transformation Theorem will almost always result in not explicitly solvable demand functions. As the first example has shown, however, we will be able to make comparative statics inferences more easily. In the remainder of the chapter we will explore several possibilities to avoid having to deal with overly complicated integrals.

4. The indirect way - restricting the model

4.1. Continuous support - using the transformation theorem II . The two examples in the previous section have made clear that trying to obtain an explicit function for the transformed density may be too much of a good thing. In most cases we are only interested

in expected values of transformed random variables (and not in the density itself). For this particular question, another form of the Transformation Theorem may be more appropriate (see Pitman (1995, pp. 304)). The formula is given by

$$E(Y) = \int_S Y(x) f_X(x) dx.$$

where $f_X(x)$ refers to the density of the underlying random variable X . This formula, when applied to our problem at hand indeed delivers the same results as derived in the previous sections. As such it is not more interesting than the original Transformation Theorem (4.4). However, an interesting application of this formula consist in choosing a density in such a way that there are only very simple integrals left. In the context of our transformation (4.2) this amounts to using a distribution which has, for example, a structure as given below

$$f_X(x) = \frac{(bx + c_1 - ab)^2}{x - a}.$$

Using this density and our now familiar transformation formula, we arrive at

$$\begin{aligned} E(Y) &= \int_S (bx + c_1 - ab) dx \\ &= \left[\frac{1}{2}bx^2 + (c_1 - ab)x \right]_l^u \\ &= \frac{1}{2}b(u^2 - l^2) + (c_1 - ab)(u - l) = 0 \\ \implies b &= \frac{c_1}{\frac{1}{2}(u + l) - a} \end{aligned}$$

which implies, given the structure of our underlying model, that

$$D_t = \frac{(1 + r_{t+1}) w_t}{\frac{1}{2}(u + l) - (1 + r_{t+1}) \chi - \bar{p}^Y}. \quad (4.13)$$

Equation (4.13) represents an explicit demand function with reasonable economic properties. Note, however, that for this demand function to be valid, another condition has to be met, namely

$$\int_l^u f_X(x) dx = 1,$$

which ensures that the density we chose is regular. In our example this yields a relatively complicated expression⁸ that, in effect, represents a mapping from the parameter space of our model to the support of the

⁸A detailed derivation is shown in app. (6.3).

distribution of the underlying uncertainty. More formally we have

$$D_t = \frac{(1 + r_{t+1}) w_t}{\frac{1}{2}(u + l) - (1 + r_{t+1}) \chi - \bar{p}^Y} \quad (4.14)$$

s.t. $\xi(\phi, u, l) = 0$

where ϕ represents the parameter vector of our model. This formulation makes clear that the approach chosen comes at a well defined cost. We can either chose to restrict the parameter values of our model in order to satisfy a certain assumption about the support of the random variable, or restrict the support in order to satisfy the constraints the model levies on the parameter values. In any event, if such constraints are feasible, i.e. will not damage the economic content of the underlying model, this is a straightforward way, as shown above, to derive an explicit demand function.

4.2. Continuous support - using integration by parts. Another alternative that is closely related to the approach discussed in section (4.1) is to tackle the problem the other way around and to define a distribution which suits the purpose. This is the approach to be taken in this section. Suppose we are given a bijective transformation, formally

$$X(Y) = Y^{-1}(Y).$$

This can, for example, accommodate for our equation (4.2). Then we can establish the following line of reasoning

$$\begin{aligned} E(Y) &= \int_S Y f_Y(y) dY = \int_S Y f_X(X(y)) \left| \frac{dX}{dY} \right| dY \\ &= \int_S Y h(Y) dY \end{aligned}$$

which makes use of (4.4) and defines

$$h(Y) = f_X(X(y)) \left| \frac{dX}{dY} \right| = \frac{dF(X(y))}{dY},$$

where $F(X(y))$ denotes the cumulative density function (in short c.d.f.) of $f_X(X(y))$. A well known result in calculus⁹, often referred to as integration by parts, is that

$$\int uv' dy = uv - \int u'v dy.$$

We can apply this readily to our problem at hand, resulting in

$$E(Y) = \int Y \frac{dF(X(Y))}{dY} dY = YF(X(Y)) - \int F(X(Y)) dY,$$

⁹See any good textbook on analysis, e.g. Rudin (1993)

and thus converting our problem to the mere integration of a c.d.f.. If we employ a distribution which has a closed range, we can equivalently state

$$E(Y) = [YF(X(y))]_{l^*}^{u^*} - \int_{l^*}^{u^*} F(X(y)) dY. \quad (4.15)$$

The first term on the right hand side of equation (4.15) is simply a c.d.f. evaluated at the lower and upper bounds. This in turn must yield 0 and 1 respectively. Hence, expression (4.15) simplifies to

$$E(Y) = u^* - \int_{l^*}^{u^*} F(X(y)) dY.$$

Again, as discussed in section (3.2), we have to be careful about our boundaries: those we used above refer to the already converted. The expected value we are looking for has, since it is a first order condition of an optimization problem, to be zero. This condition further simplifies the algebraic expression for $E(Y)$ to

$$u^* = \int_{l^*}^{u^*} F(X(y)) dY = [H(X(y))]_{l^*}^{u^*} = H(u^*) - H(l^*) \quad (4.16)$$

where $H(X(y))$ is mathematically equivalent to the integral of a c.d.f.. In principle, we are now free to assume any distribution (and thus c.d.f.) that suits our purpose here. To present a relatively accessible example, we will in what follows work with the assumption that, first, the *transformed* random variable $Y(X)$ has an uniform density and second that the *original* random variable X has as its support the non-negative real line. It is well known that the c.d.f. of an uniform distribution with support $[l, u]$ is simply given by

$$F(x)_{UD} = \left\{ \begin{array}{l} 0 \text{ if } x \leq l \\ \frac{x-l}{u-l} \text{ if } l < x < u \\ 1 \text{ if } x \geq u \end{array} \right\} \quad (4.17)$$

Using formula (4.16) together with (4.17) leaves us with

$$H(y) = \int_{l^*}^{u^*} \frac{y-l^*}{u^*-l^*} dy = \frac{1}{2} (u^* - l^*).$$

¹⁰We then use the support of the underlying random variable X , insert into (4.2) (and evaluate limits where appropriate). This yields $u^* = \frac{1}{b}$ and $l^* = -\frac{a}{c_1-ab}$. These results together with (4.16) bring about $b = \frac{c_1}{2a}$ which in turn, upon reinserting our model structure yields the demand function

$$D_t = \frac{(1+r_t)w_t}{2((1+r_t)\chi + \bar{p}^Y)}. \quad (4.18)$$

Note that this demand function makes intuitively sense. The demand for forward cover increases with wage income and decreases with the

¹⁰See app. (6.6.1) for the derivation.

contracted forward price and rising opportunity costs. However, we do not know anything about the properties of the underlying distribution other than that its support is the non-negative real line. We therefore have no way of knowing whether the implicit assumption about the distribution of foreign price uncertainty matches empirical findings or not. It thus depends mainly on the problem under consideration whether or not we can make use of this approach.

We will now turn to a second example to illustrate the "Integrations-by-parts-approach". The triangular distribution will again serve as the second simplest distribution. The c.d.f. of the symmetric triangular distribution is given by

$$F(x)_{TD} = \left\{ \begin{array}{l} 0 \text{ if } x \leq l \\ \frac{2(l-x)^2}{(l-u)^2} \text{ if } l < x \leq \frac{u+l}{2} \\ \frac{1}{2} + \frac{(u+l-2x)(3u+2x-3u+l)}{2(l-u)^2} \text{ if } \frac{u+l}{2} < x < u \\ 1 \text{ if } x \geq u. \end{array} \right\} \quad (4.19)$$

Using equation (4.16) then yields $\frac{7}{19} \frac{c_1}{a} = b^{11}$. Reinserting the structure of our model another explicit demand function emerges, this time based on the assumptions that the transformed density is triangular and the support of the underlying uncertainty is the non-negative real line:

$$D_t = \frac{7(1+r_t)w_t}{19((1+r_t)\chi + \bar{p}^Y)}. \quad (4.20)$$

The two resulting demand functions of this section, i.e. equation (4.18) and (4.20), are in fact identical up to a scalar transformation and hence share the same intuitively appealing properties. This, however, is not overly surprising, since the uniform and the triangular distributions are very similar in nature. Natural questions that then arise are: What other distributions can be used to yield explicit demand functions using the "Integrations-by-parts-approach"? and, related to this: What are the properties of the demand functions under different distributional assumptions?. These simple extensions, however, bring the approach of this section to its limits. For example, the assumption of an underlying exponential distribution already results in demand functions without explicit solution. As a rule, we can state that as long as we have exponential (or logarithmic) and linear terms together, the approach of this section (just like the others) will fail to deliver.

5. Conclusion

The present chapter dealt with the question of how to derive explicit demand schedules under expected utility maximization when there is a nonlinear transformation of the underlying random variable. We first set out a simple OLG model in which we encountered a nonlinearly

¹¹See app (6.6.2) for detailed derivation.

transformed random variable as first order condition. The subsequent discussion showed that under discrete and finite support, there will always be an explicit demand schedule. We then turned to the more general case of a random variable with continuous support. With continuous random variables, there are three different possible approaches to our problem.

The first and most direct, using the transformation theorem, does not normally lead to an explicit demand function. This is due to the complications encountered when solving the resulting integrals. However, using the direct approach we are always able to obtain indirect demand schedules that are better suited for tasks like comparative statics analysis than the "raw" first order condition of the model.

The second approach we introduced uses a variant of the Transformation Theorem. The trick here is to choose a distribution for the underlying random variable X on grounds of easy mathematical handling. This might be, in terms of economic content, questionable, a thorough discussion of the properties of the new born distribution surely being in order. This approach will normally lead to explicit demand functions, but at the cost of additional restrictions on the parameter space of the model. In that sense this approach represents a trade-off: more restrictions against an explicit demand schedule.

The last road to be taken goes a completely different way. Using the integration by parts result from calculus, we defined a transformed density, without having to draw on the underlying distribution. Using this device, we were able to obtain explicit demand functions for sufficiently simple densities like uniform and triangular. The resulting demand functions are economically meaningful and bear structurally close resemblance to the those derived in the other sections of the chapter. The same criticism as for the other "economically ignorant" method applies here in a more general way: the distribution of the transformed random variable Y is chosen, thus implying a distribution for the underlying random variable X . Therefore a check what kind of underlying distribution this approach creates seems to be in order. This check, however, may not always be feasible.

From our discussion of the different tools we have to approach nonlinearly transformed random variables in a framework of expected utility maximization one result clearly stands out: the more restrictions we can invoke on the underlying model, the closer we get to an explicit demand schedule. The question then is, of course, whether or not the trade-off is worthwhile. In general this will depend on the economic model under consideration and the character of the underlying uncertainty.

6. Mathematical appendix

This Appendix is concerned with the derivation of the results used in the text. In most cases it contains the bits and pieces of everyday math which are easy to follow but demand too much space to be left in the main text.

6.1. The details of the model. This appendix briefly lines out the optimization problem of the agents that leads to (4.1) in the text. The model further features the capital accumulation equation and an equilibrium of our model will be a vector (D, K) . The equilibrium condition for the capital stock is not relevant for the scope of the chapter and will therefore be omitted in what follows. The choice problem of an agent born in t is given by

$$\max_{C_X, C_Y, D_t} U(C_X, C_Y)$$

where C_X and C_Y represent consumption levels in the second period of the life of an agent. Under the additional assumption of a log utility function, (as proposed by Arrow (1974, Chapter 3)), namely

$$U(C_X, C_Y) = \alpha \ln C_X + (1 - \alpha) \ln C_Y$$

we know the optimal levels of consumption under certainty to be

$$C_X = \frac{\alpha e}{p_X}; C_Y = \frac{(1 - \alpha) e}{p_Y}$$

where e is our budget constraint. Now reinserted in our utility maximization problem, we arrive at

$$\max_D E \left(\alpha \ln \frac{\alpha e}{p_X} + (1 - \alpha) \ln \frac{(1 - \alpha) e}{p_Y} \right)$$

as the "new" problem the agent faces in t knowing the structure of his decision in $t+1$. This is a period t decision and under uncertainty about the price of the foreign good in $t+1$. The solution to this problem is a first order condition

$$E \left(\frac{1}{e} \frac{\partial e}{\partial D} \right) \stackrel{!}{=} 0$$

and together with

$$e = (1 + r_{t+1})(w_t - \chi D_t) + (p_{t+1}^Y - \bar{p}^Y) D$$

establishes equation (4.1) used in the text throughout.

6.2. The applicability of the Transformation Theorem .

This appendix verifies that the transformations theorem can be used for our transformation (4.2). Following theorem (4.4) we need that our

transformation is monotonous on the support of the underlying random variable X , i.e. $[l, u]$. For this it is sufficient to note that

$$\frac{dY}{dX} = \frac{c + ab}{(bX + c)^2} > 0 \quad \forall x$$

the transformation is strictly increasing on $[l, u]$. The second, implicit, condition is bijectivity of the transformation, i.e. the existence of $Y(X)^{-1}$. This can be easily shown by solving (4.2) for Y . This yields a unique solution, $X = \frac{cY+a}{1-bY}$. Therefore we can state that our transformation is bijective.

6.3. The uniform distribution example . We start out the result (4.7) for the integral of the uniform density in the text, namely

$$\begin{aligned} E(Y_{|X \sim UD}) &= \left(\frac{c_1}{u-l} \right) \frac{1}{b^2} \times \{ \\ &\quad \frac{1}{\left(1 - b \frac{u-a}{bu+c_1-ab}\right)} + \ln \left| \left(1 - b \frac{u-a}{bu+c_1-ab}\right) \right| \\ &\quad - \frac{1}{\left(1 - b \frac{l-a}{bl+c_1-ab}\right)} - \ln \left| \left(1 - b \frac{l-a}{bl+c_1-ab}\right) \right| \}. \end{aligned}$$

This expression has two distinct parts, one linear L the other one involving logs LG , hence $E(Y_{|X \sim UD}) = L + LG$. The linear part reads:

$$\begin{aligned} L &= \frac{c_1}{u-l} \frac{1}{b^2} \left(\frac{1}{\left(1 - b \frac{u-a}{bu+c_1-ab}\right)} - \frac{1}{\left(1 - b \frac{l-a}{bl+c_1-ab}\right)} \right) \\ &= \frac{1}{b}. \end{aligned}$$

The remaining part of our integral is then given by

$$\begin{aligned} LG &= \frac{c_1}{u-l} \frac{1}{b^2} \left(\ln \left| \left(1 - b \frac{u-a}{bu+c_1-ab}\right) \right| - \ln \left| \left(1 - b \frac{l-a}{bl+c_1-ab}\right) \right| \right) \\ &= \frac{c_1}{u-l} \frac{1}{b^2} \ln \frac{bl+c_1-ab}{bu+c_1-ab}. \end{aligned}$$

Note that in order to arrive at this last equation the absolute value operator had to be dispensed with. It is straightforward to see that neither expression can become negative, especially since $c_1 - ab$ will never be any smaller than zero. Merging the two results together yields (4.8) in the text.

The comparative static behavior of (4.9) with respect to a change in χ can be obtained by calculating

$$\frac{dD}{d\chi} = - \frac{\frac{\partial F}{\partial \chi}}{\frac{\partial F}{\partial D}}$$

where $F = D - (1+r)w \ln \frac{(1+r)(w-\chi D) - \bar{p}^Y D + D}{(1+r)(w-\chi D) - \bar{p}^Y D}$. The partial derivatives with respect to the costs of forward cover χ is given by

$$\frac{\partial F}{\partial \chi} = \frac{k_1 D}{w} \left(\frac{-D}{(n_1 + D) n_1} \right) < 0$$

with $k_1 = (1+r)w$ and $n_1 = k_1 \left(1 - \frac{\chi D}{w}\right) - \bar{p}^Y D$. By virtue of the no bankruptcy condition, n_1 is positive. Therefore the derivative is negative. The partial derivative with respect to D reads

$$\frac{\partial F}{\partial D} = 1 - k_1 \left(\bar{p}^Y + \frac{k_1}{w} \chi \right) \left(\frac{1}{n_1} - \frac{1}{n_1 + D} \right) - \frac{k_1}{n_1 + D} < 0,$$

where $\frac{k_1}{n_1 + D} > 1$ (due to $k_1 > n_1 + D$), rendering the derivative negative.

6.4. The triangular distribution example . We start with the analytical expression for the expected value, equation (4.11):

$$E(Y_{|X \sim TD}) = \int_{l^*}^{u^*} 2y \left(1 - 2 \left(\left| \frac{(c_1 - ab)y + a}{1 - by} - \frac{1}{2} \right| \right) \frac{c_1}{(1 - by)^2} \right) dy$$

where $l^* = -\frac{a}{c_1 - ab}$ and $u^* = \frac{1-a}{c_1 - ab + b}$. Due to the absolute value operator within (4.11) we have to split the integral into two, depending on $\frac{(c_1 - ab)y + a}{1 - by} \leq \frac{1}{2}$. We verify this simply by setting

$$\frac{(c_1 - ab)y + a}{1 - by} = \frac{1}{2}$$

which yields

$$\bar{y} = m^* = \frac{1 - 2a}{2c_1 - 2ab + b}.$$

This is clearly below our upper boundary of integration as

$$\frac{1 - a}{c_1 - ab + b} - \frac{1 - 2a}{2c_1 - 2ab + b} = \frac{c_1}{(c_1 - ab + b)(2c_1 - 2ab + b)} > 0$$

shows. Hence we split (4.11) into two integrals as follows:

$$\begin{aligned} E(Y_{|X \sim TD}) &= \int_{l^*}^{m^*} 2y \left(1 - 2 \left(\frac{1}{2} - \frac{(c_1 - ab)y + a}{1 - by} \right) \frac{c_1}{(1 - by)^2} \right) dy \\ &\quad + \int_{m^*}^{u^*} 2y \left(1 - 2 \left(\frac{(c_1 - ab)y + a}{1 - by} - \frac{1}{2} \right) \frac{c_1}{(1 - by)^2} \right) dy \end{aligned}$$

Collecting terms and sorting the remaining sections into manageable parts, we end up with the following three basic integrals to be evaluated:

$$\begin{aligned} E(Y_{|X \sim TD}) &= 4(c_1 - ab) c_1 \int_{l^*}^{m^*} \frac{y^2}{(1 - by)^3} dy + 4ac_1 \int_{l^*}^{m^*} \frac{y}{(1 - by)^3} \\ &\quad + 4c_1 \int_{m^*}^{u^*} \frac{y}{(1 - by)^2} dy - 4(c_1 - ab) c_1 \int_{m^*}^{u^*} \frac{y^2}{(1 - by)^3} dy \\ &\quad - 4ac_1 \int_{m^*}^{u^*} \frac{y}{(1 - by)^3} dy. \end{aligned}$$

This in turn can concisely be written as

$$\begin{aligned} E(Y_{|X \sim TD}) &= k(2F(m^*) - F(u^*) - F(l^*)) \\ &\quad + \frac{ak}{c_1 - ab}(2G(m^*) - G(u^*) - G(l^*)) \\ &\quad + \frac{k}{c_1 - ab}(H(u^*) - H(m^*)) \end{aligned}$$

where we defined $k = 4c_1(c_1 - ab)$; $F(\cdot) = \int \frac{y^2}{(1-by)^3} dy$; $G(\cdot) = \int \frac{y}{(1-by)^3} dy$ and $H(\cdot) = \int \frac{y}{(1-by)^2} dy$. Analytical solutions for each of these integrals exist. They read

$$\begin{aligned} F(\cdot) &= \frac{1}{2b^3(1-by)^2} - \frac{2}{b^3(1-by)} - \frac{\ln(1-by)}{b^3} \\ G(\cdot) &= \frac{1}{2b^2(1-by)^2} - \frac{1}{b^2(1-by)} \\ H(\cdot) &= \frac{1}{b^2(1-by)} + \frac{\ln(1-by)}{b^2} \end{aligned}$$

and can be obtained by trial and error procedures or from integral tables as found in (Bronstein and Semendjajew (1987)). The remaining algebra is tedious but not complicated. The result reads:

$$\begin{aligned} E(Y_{|X \sim TD}) &= \frac{1}{2} \frac{k}{b^3} (2m_3^2 - m_2^2 - m_1^2) \\ &\quad - \frac{k}{b^3} (2 \ln m_3 - \ln m_2 - \ln m_1) \\ &\quad + \frac{1}{2} \frac{ak}{(c_1 - ab) b^3} (2m_3^2 - m_2^2 - m_1^2) \\ &\quad + \frac{k}{c_1 - ab} \left(\frac{1}{2bc_1} + \frac{1}{b^2} (\ln m_3 - \ln m_2) \right) \\ &= \frac{1}{b^3} (b^2 + 4c_1 [(2c_1 m_3) \ln m_3 + (c_1 m_2) \ln m_2 + (c_1 m_1) \ln m_1]) \end{aligned}$$

where we set $m_1 = \frac{c_1 - ab}{c_1}$, $m_2 = m_1 + \frac{b}{c_1}$ and $m_3 = m_1 + \frac{b}{2c_1}$. Using the first order condition $E(Y_{|X \sim TD}) = 0$ we can simplify further and arrive at

$$b^2 = 4c_1 \left((2c_1 - 2ab + b) \ln m_3^{-1} + (c_1 - ab + b) \ln m_2^{-1} + (c_1 - ab) \ln m_1^{-1} \right).$$

We now reinsert the underlying variables of our model and will finally be left with:

$$D^2 = 4(1+r)w \left(\begin{aligned} & (2(1+r)(w - \chi D) - 2p_Y D + D) \ln \frac{2(1+r)w}{2(1+r)(w - \chi D) - 2p_Y D + D} \\ & + ((1+r)(w - \chi D) - p_Y D + D) \ln \frac{(1+r)w}{(1+r)(w - \chi D) - p_Y D + D} \\ & + (1+r)(w - \chi D) \ln \frac{(1+r)w}{(1+r)(w - \chi D)} \end{aligned} \right).$$

This is (4.12) in the text.

6.5. The Transformation Theorem II. This appendix provides an example for the type of restrictions encountered when using the approach introduced in section (4.1). There an explicit demand function is derived subject to that another condition, $\int f_X dx = 1$, has to be met. The assumed density is given by $f_X = \frac{(bx + c_1 - ab)^2}{x - a}$. Hence the condition reads

$$\int_l^u \frac{(bx + c_1 - ab)^2}{x - a} dx = 1.$$

The integral can be found with the help of integral tables and is our case equal to

$$[bx(2c_1 - ab) + \frac{b^2 x^2}{2} + c_1^2 \ln |x - a|]_l^u = 1.$$

Evaluating yields

$$b(u - l)(2c_1 - ab) + \frac{b^2(u^2 - l^2)}{2} + c_1^2 \ln \left| \frac{u - a}{l - a} \right| = 1.$$

For any given support of the distribution $[l, u]$ this will result in a mapping from one parameter of the model to the other, restricting the degrees of freedom in choosing parameters for our underlying model. For example, if we would assume $[l, u] = [0, 1]$ we would get

$$b(2c_1 - ab) + \frac{b^2}{2} + c_1^2 \ln \left| \frac{1 - a}{-a} \right| = 1.$$

Inserting the structure of our model, we find

$$D_t \left((1 + r_{t+1})(w_t - \chi D_t) - \bar{p}^Y D_t \right) + \frac{D_t^2}{2} + (1 + r_{t+1}) w_t \ln \left| \frac{1 - (1+r)\chi + \bar{p}^Y}{-(1+r)\chi + \bar{p}^Y} \right| = 1.$$

Finally, replacing D_t with (4.14) then yields

$$\frac{(1+r_{t+1})w_t}{\frac{1}{2} - (1+r_{t+1})\chi - \bar{p}^Y} \left(\begin{array}{c} (1+r_{t+1}) \left(w_t - \chi \frac{(1+r_{t+1})w_t}{\frac{1}{2} - (1+r_{t+1})\chi - \bar{p}^Y} \right) \\ -\bar{p}^Y \frac{(1+r_{t+1})w_t}{\frac{1}{2} - (1+r_{t+1})\chi - \bar{p}^Y} \end{array} \right) \\ + \left(\frac{(1+r_{t+1})w_t}{\frac{1}{2} - (1+r_{t+1})\chi - \bar{p}^Y} \right)^2 \frac{1}{2} + (1+r_{t+1})w_t \ln \left| \frac{1 - (1+r)\chi - \bar{p}^Y}{-(1+r)\chi - \bar{p}^Y} \right| = 1$$

which may be satisfied, for example, by a suitable choice of χ .

6.6. The "Integration-By-Parts-Approach".

6.6.1. *The uniform distribution example*. Consider equation (4.16). In order to solve it we need to evaluate u^* and l^* and integrate the c.d.f. (4.17). We start with the boundaries. By assumption, we have $[l, u] = [0, \infty)$. Our transformation (4.2) is such that $u^* = \lim_{x \rightarrow \infty} \frac{x-a}{bx+c_1-ab} = \frac{1}{b}$ and $l^* = -\frac{a}{c_1-ab}$. These are the boundaries used in the text. The integral of the c.d.f. is given by $\int_{l^*}^x \frac{x-l^*}{u^*-l^*} dx = \frac{1}{2} [\frac{x^2-2xl^*}{u^*-l^*}]_{l^*}^{u^*} = \frac{1}{2} (u^* - l^*)$ which is used in the text.

6.6.2. *The triangular distribution example*. Equation (4.16) is again our point of departure. The integral of the c.d.f. (4.19) is given by $\int_{l^*}^x \frac{2(l-x)^2}{(l-u)^2} dx + \int_{\frac{u^*+l^*}{2}}^x \frac{1}{2} + \frac{(u+l-2x)(3u+2x-3u+l)}{2(l-u)^2} dx$. The two resulting integrals read

$$H_L = \frac{2(l^* - x)^2 (x - l^*)}{3(l^* - u^*)^2}$$

and

$$H_U = \frac{(2x - l^* - u^*) (5l^{*2} - u^{*2} + 10u^*x - 4x^2 - 2l^* (4u^* + x))}{12(l^* - u^*)^2}.$$

Equation (4.16) then reads

$$\frac{1}{b} = [H_L]_{l^*}^{\frac{u^*+l^*}{2}} + [H_U]_{\frac{u^*+l^*}{2}}^{u^*}.$$

Evaluating these expressions is straightforward and yields

$$\frac{1}{b} = \frac{5}{12} \left(\frac{1}{b} + \frac{a}{c_1 - ab} \right).$$

Finally, solving for b results in

$$b = \frac{7}{19} \frac{c_1}{a}$$

which is used in the text.

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