Buffalo Law Review

Volume 47 | Number 2 Article 6

4-1-1999

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Recommended Citation
Michael J. Meurer, *Fair Division*, 47 Buff. L. Rev. 937 (1999).
Available at: https://digitalcommons.law.buffalo.edu/buffalolawreview/vol47/iss2/6

Fair Division

MICHAEL J. MEURER†

INTRODUCTION

Fair division is a fundamental issue of legal policy. The law of remedies specifies damages and rules of contribution that apportion liability among multiple defendants.[1] Probate law specifies how assets from an estate are divided when the intentions of a decedent are unclear.[2] Family law specifies how assets from a dissolved marriage are apportioned.[3] Similarly, partnership law specifies how assets from a dissolved partnership are apportioned.[4] Bankruptcy law specifies how the assets of a debtor are apportioned among creditors.[5] In civil actions in which the plaintiff is successful but failed to mitigate, the damages are apportioned between the defendant and plaintiff.[6] When courts or legislatures create or modify property rights they implicitly make choices about wealth allocation.[7] For some public

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1. *See generally* William M. Landes & Richard A. Posner, *Joint and Multiple Tortfeasors: An Economic Analysis,* 9 J. LEGAL STUD. 517 (1980).

2. *See generally* Trust Decisions *Alleged Oral Agreement Among Heirs as to Division of Estate Could be Enforceable without any Requirement of Court Approval,* Hennessey v. Froehlich, 464 S.E.2D 246 (Ga. Ct. App. 1995) 113 BANKING L.J. 848 (1996).

3. *See generally* Amy L. Wax, *Bargaining in the Shadow of the Market: Is There a Future for Egalitarian Marriage?* 84 VA. L. REV. 509 (1998); Honorable Willis J. Zick, *Divorce Law: Exclusions and Disproportionate Divisions of the Marital Estate,* 76 MARQ. L. REV. 519 (1993).

4. *See generally* Alan R. Bromberg, *Partnership Dissolution – Causes, Consequences, and Cures,* 43 TEX. L. REV. 631 (1965).

5. *See generally* 1 NORTON BANKR. L. & PRAC. 2D §3:12 (providing an overview of liquidation).

6. *See generally* Robert Cooter, *Unity in Tort, Contract, and Property: The Model of Precaution,* 73 CAL. L. REV. 1 (1985).

7. *See generally* Guido Calabresi & A. Douglas Melamed, *Property Rules, Liability Rules, and Inalienability: One View of the Cathedral,* 85 HARV. L. REV. 1089 (1972).

projects the government apportions the cost of the projects among users.[8] Despite the importance of this issue, law professors have largely ignored[9] a small, but flourishing, band of economic theorists who study questions of fair division.[10] In this book review I suggest a starting point for such analysis.

Hervé Moulin, the author of *Cooperative Microeconomics: A Game-Theoretic Introduction,*[11] and H. Peyton Young, the author of *Equity: In Theory and Practice,*[12] are two of the economic theorists working to move the study of fairness into the mainstream of economic analysis. In the introduction to his book, Moulin laments: "To the majority of economists today, the ethical choices of distributive justice are alien to economic analysis . . . The standard view simply incorporates . . . concerns about justice (distributive and otherwise) in the description of individual characteristics: some of us derive utility from giving to the needy, some of us do not."[13] It is no surprise that fairness does not play a significant role in law and economics since it does not play much of a role in any area

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8. *See infra* text accompanying note 55.

9. *See e.g.,* RICHARD A. POSNER, ECONOMIC ANALYSIS OF LAW 461-63 (1992) (devoting only two pages to a discussion of distributive justice); NICHOLAS MERCURO AND STEVEN G. MEDEMA, ECONOMICS AND THE LAW: FROM POSNER TO POST-MODERNISM 188-89 (1997) (arguing that the efficiency focus of economics carries over to law and economics).

10. Four Nobel prize winning economists who have studied fairness issues are: AMARTYA K. SEN, COMMODITIES AND CAPABILITIES (1985) (discussing the moral status of preferences in welfare economics); Kenneth Arrow, *Rational Choice Functions and Orderings,* 26 ECONOMETRICA 121 (1959) (noting the aggregation of individual preferences by a social preference ordering); John Harsanyi, *Cardinal Utility in Welfare Economics and in the Theory of Risk-Taking,* 61 J. POL. ECON. 434 (1953); John Harsanyi, *Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisions of Utility,* 63 J. POL. ECON. 302 (1955) (foreshadowing Rawls with a theory of distributive justice based on average expected utility maximization behind a veil of ignorance); and John Nash, *The Bargaining Problem,* 18 ECONOMETRICA 155 (1950) and *Two-Person Cooperative Games,* 21 ECONOMETRICA 128 (1953) (discussing fairness in bargaining problems).

11. HERVÉ MOULIN, COOPERATIVE MICROECONOMICS: A GAME-THEORETIC INTRODUCTION 8 (1995).

12. H. PEYTON YOUNG, EQUITY: IN THEORY AND PRACTICE (1994).

13. *See* MOULIN, *supra* note 11, at 8. *Cf.* Henrik Lando, *An Attempt to Incorporate Fairness into an Economic Model of Tort Law,* 17 INT'L REV. L. & ECON. 575 (1997) (building taste for fairness into the preferences of injurers and victims).

of applied economics.[14] I hope my review of these two books will pique some interest in this subject and promote legal scholarship that incorporates fairness into the economic analysis of law.[15]

The main topic of the two books is the microeconomic analysis of methods for fairly allocating benefit and cost. The authors are eclectic and do not insist on a single preferred mode of fairness analysis. Moulin advocates a three-pronged approach that combines some notion of end-state justice with procedural requirements related to voluntary participation and voluntary disclosure of personal information.[16] Young also discusses procedural and end-state justice in various economic models, but does not suggest a unified theory.[17] In both books, procedural questions are subsidiary to end-state questions. Procedure is usually studied in the following terms: Given method X achieves desirable results in terms of end-state justice, can we find a way to implement X that allows the affected parties to participate voluntarily, subject to reasonable

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14. *See generally* EDWARD ZAJAC, POLITICAL ECONOMY OF FAIRNESS 76 (1995) (noting that economists split efficiency from fairness analysis and leave the consideration of fairness to policy-makers).

15. My Westlaw search found no citations to the book by Moulin. The book by Young was cited by the following five authors: Richard O'Brooks, *Legal Realism, Norman Williams, and Vermont's Act* 250, 20 VT. L. REV. 699 (1996); Kirsten Engel, *Reconsidering the National Market in Solid Waste: Trade-offs in Equity, Efficiency, Environmental Protection, and State Autonomy*, 73 N.C. L. Rev. 1481 (1995); Ugo Mattei, *Efficiency as Equity: Insights from Comparative Law and Economics* 18 HASTINGS INT'L & COMP. L. REV. 157 (1994); Larry T. Garvin, *Disproportionality and the Law of Consequential Damages: Default Theory and Cognitive Reality,* 59 OHIO ST. L.J. 339 (1998); and Amy L. Wax, *Bargaining in the Shadow of the Market: Is There a Future for Egalitarian Marriage?* 84 VA. L. REV. 509 (1998). I frequently attend meetings of the American Law and Economic Association and I have never witnessed a presentation that used the cooperative microeconomic analysis that I will describe in this book review.

16. *See* MOULIN, *supra* note 11, at 3 ("I submit that cooperation between selfish economic agents can be conceived in three fundamental "modes," namely, direct agreement, justice, and decentralized behavior.") He also con-tends that an ideal cooperative mechanism should be just, stable, and imple-mentable via a unique equilibrium. *Id. at* 4.

17. Young argues that allocation methods tend to follow one of three conceptions of fairness (he uses the term equity): parity, proportionality, or priority. *See* YOUNG, *supra* note 12 at 8. Parity corresponds to equal treatments of equals. Priority is invoked as the fair way to address the allocation of indivisible goods. The good is given to the party with the highest priority based on some measure of merit. *See id.*

constraints on the government's behavior?[18]

As in Moulin and Young, most of my comments in this review concern end-state justice. The two simple allocation methods that serve as the starting point for the analysis of end-state justice find their origin in the Aristotelian theory of distributive justice: *equal* cost (or benefit) sharing and *proportional* cost (or benefit) sharing.[19] Consider the following illustration. A man wins the lottery and decides to make a gift of $1 million to his divorced parents. He could choose equality and give one-half million dollars to each, or he could choose proportionality and give $600,000 to his mother in recognition that her life expectancy is fifty percent longer.

Moulin and Young consider many variations on both themes and insist that there is no single fair method of division. Each new context demands a new analysis of the advantages and disadvantages of different methods. For example, if we change the context of the son's fair division problem by awarding him a car rather than cash in the lottery, his new division problem is more difficult. The car is not divisible. He might sell the car and divide the proceeds as above. But that approach is not always acceptable. No market exists for some items, such as organs available for transplant.[20] Furthermore, the market may not capture the idiosyncratic valuation of an item.[21] Suppose that the car is a unique antique that the mother would dearly love to own. Also suppose the father would prefer cash. What's a poor son to do? Should he try to equalize the monetary value of

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18. Fair process is discussed in Part V.

19. According to Aristotle, equal treatment is required when individuals are the same in terms of merit or blame. When individuals differ in a morally relevant way (for example, some work harder, or some are more costly to serve) then benefit or cost should be allocated in proportion to some factor that captures the difference. *See* YOUNG, *supra* note 12, at 64; ZAJAC, *supra* note 14, at 105.

20. Young devotes two chapters to the fair allocation of indivisible benefits. He chooses topics like the allocation of organs for transplants and the apportionment of legislative seats to illustrate his discussion. *See* YOUNG, *supra* note 12, at 20-62.

21. *See* W. KIP VISCUSI, JOHN M. VERNON, AND JOSEPH E. HARRINGTON, ECONOMICS OF REGULATION AND ANTITRUST 629-52 (1992) (developing methods for valuing life and other non-market commodities). *Cf.* Margaret Jane Radin, *Market Inalienability,* 100 HARV. L. REV. 1849 (1987) (discussing problems with commodification and assignment of market value); Margaret Jane Radin, *Compensation and Commensurability,* 43 DUKE. L. J. 56 (1993) (noting problems with market valuation of bodily integrity).

his gifts or their utility value? Is it possible to compare the happiness of his mother and father? What if he cannot afford a large cash gift to his father if he gives the car to his mother?

As the son's quandary suggests, several difficulties plague the analysis of fair division. There are many ways to formulate the rule that a fair share should be responsive to relevant differences. The correct definition of proportional sharing is not obvious when there are heterogeneous individuals who differ in many morally relevant respects.[22] The correct definition is also not obvious when there are many choices about how to measure differences.[23]

Once a fair end-state is chosen there are still more difficulties. First, methods that yield attractive end-states might be associated with coercive or undemocratic procedures.[24] Second, methods that are attractive in a world of full information might not be feasible in a world where participants cannot be induced or trusted to disclose relevant private information.[25] Finally, methods that give fair end-states through fair and feasible processes may generate perverse economic incentives.[26] Moulin and Young persevere despite these difficulties. They show the reader how to devise satisfactory division methods that cope with the features of a particular context.

One of the most attractive features of these books is that both authors focus on local rather than global issues of fairness.[27] Other economic analyses of fairness pursue global questions such as the nature of a just society or the ideal social contract.[28] In contrast, Young states that "this

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22. *See* YOUNG, *supra* note 12, at 9 (indicating one problem with a proportional rule is that there are different possible scales that can be used for measurement).

23. *See id.*

24. *See* Hervé Moulin, *Procedural cum Endstate Justice: An Implementation Viewpoint,* unpublished manuscript 11-12 (March 1997) (on file with *Buffalo Law Review*).

25. *See e.g.,* HAL R. VARIAN, MICROECONOMIC ANALYSIS 256-59 (1984).

26. *See infra* text at Part IV.

27. *See generally* MICHAEL WALZER, SPHERES OF JUSTICE 6 (1983) (stating "I want to argue . . . that the principles of justice are themselves pluralistic in form; that different social goods ought to be distributed for different reasons, in accordance with different procedures, by different agents; and that all these differences derive from different understandings of the social goods themselves—the inevitable product of historical and cultural particularism.").

28. *See* YOUNG, *supra* note 12, at xi. A central problem for political philosophers is the social contract and a just social order. Much of the

book. . .is about the meaning of equity in concrete situations that we meet every day."[29] He goes on to criticize global theories by observing "theories of justice in the large have little to say about what it means in the small. They do not tell us how to solve concrete, everyday distributive problems such as how to adjudicate a property dispute, who should get into medical school, or how much to charge for a subway ride."[30] I am attracted to a local theory of justice because it supports a pragmatic attitude toward law and policy-making. Local theories of justice recognize that law and policy are compartmentalized; there are few chances for compensation across different problems of fair division.[31]

In this review I intend to introduce the reader to some valuable tools for analyzing concrete problems of distributive justice. The tools are found in cooperative microeconomics; especially cooperative game theory. I will illustrate the value of these tools by showing how they can be applied to legal policy issues. I hope to convince the reader that these methods deserve greater attention in the practice of law and economics.

There are at least three roles for fairness analysis in

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interesting work in economic theory that explores fairness issues is couched in terms of the social contract, and is responsive to the theories of Hobbes, Rousseau, Nozick, and Rawls. *See generally* ZAJAC *supra* note 12; KEN BINMORE, PLAYING FAIR: GAME THEORY AND THE SOCIAL CONTRACT (1994); KEN BINMORE, JUST PLAYING: GAME THEORY AND THE SOCIAL CONTRACT (1998); DANIEL M. HAUSMAN AND MICHAEL S. MCPHERSON, ECONOMIC ANALYSIS AND MORAL PHILOSOPHY (1996).

The other main strand of economic theory related to fairness arises from the Fundamental Theorems of Welfare Economics. *See e.g.,* VARIAN, *supra* note 25, at 198-203. The First Theorem provides conditions under which a competitive equilibrium is Pareto efficient. The Second Theorem states that any competitive equilibrium can be implemented by the proper specification of property rights. Law and economics scholars rely on the Second Theorem to justify legal analysis that bifurcates efficiency and fairness analysis of the law. The usual attitude is that law should be shaped by efficiency concerns, and the legislature can achieve fairness through taxation and spending policies. *See infra* text at Part IV. I will justify the local analysis of justice developed by Young and Moulin below by arguing that the usual appeals to the Second Theorem are unpersuasive in terms of microeconomics and in terms of political economy. Furthermore, I argue that fairness depends on more than the distribution of income. Economic analysis of the law needs to pay attention to fairness concerns that arise when all of the parties affected by a law have similar income and wealth, but differ in other morally significant dimensions.

29. *See* YOUNG, *supra* note 12, at xi.

30. *Id.* at 6.

31. *See* YOUNG, *supra* note 12, at 6-7.

law and economics:[32] (1) for many cooperative activities, incentive problems are minimal and fairness is the only way to guide the allocation of cost and benefit;[33] (2) when Coasean conditions of low transaction costs prevail,[34] then property rights should be allocated according to fairness criteria;[35] and (3) when policymakers want to consider efficiency and fairness objectives simultaneously, then an explicit description of fairness criteria allows an explicit trade-off.[36] This book review will touch on all of these roles for fairness analysis. I work through two estate division problems that do not present any incentive or efficiency issues in Part I. I move to more complicated problems of fair division in Part II wherein property rights and efficiency issues are introduced. In Part III, I discuss the axiomatic basis of the methods of fair division that are invoked in Part II. I address the question of how traditional efficiency analysis in law and economics can be integrated with fairness analysis in Part IV. I conclude in Part V with comments about procedural fairness and economic analysis.

As one final preliminary matter, I must comment on the accessibility of these books. Young claims "the text is intended to be a 'primer' and does not presuppose any mathematical background, though a taste for logical argument and some familiarity with economic concepts would certainly help,"[37] and Moulin claims "this book is elementary and self-contained."[38] When I was a graduate student in economics I was always skeptical about such comments.[39]

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32. Another role for fairness criteria is prediction of the outcomes of social choices or bargains, especially when fairness can be used as a focal device to choose among multiple equilibria. Since I am interested in normative issues, I do not consider that role. *See* ZAJAC, *supra* note 14, at 7, 102-104 (discussing the positive use of fairness in the study of the political economy of regulation).

33. Part I gives two examples of estate division problems in which no efficiency issue is apparent.

34. *See* Ronald H. Coase, *The Problem of Social Cost,* 3 J. LAW & ECON. 1 (1960).

35. *See* YOUNG, *supra* note 12, at 4-5 (arguing that equity should guide the assignment of rights to newly created property and property that is excluded from the market); MOULIN, *supra* note 11, at 164 (indicating that questions about new property rights recur because of technical and economic change)

36. The trade-off between efficiency and fairness is covered in Part IV.

37. YOUNG, *supra* note 12, at xiii.

38. MOULIN, *supra* note 11, at 4.

39. A formative experience for me in graduate school involved reading the famous *Theory of Value*. The book begins with the reassuring comment: "This chapter presents *all* the mathematical concepts and results which will be used

Of course, since I am urging law professors to read these books, I do believe they are accessible. However, Young is a significantly easier read than Moulin. Most of the economics in Young is no more difficult than the economics that follows in this Review. The same cannot be said about Moulin's book, which is targeted at an audience with significant economics training. Nevertheless, the first chapter of Moulin's book is a delight to read and is accessible to all readers. Readers without strong economics training will have to be content to skim the rest of the book.

I. TWO EXAMPLES OF ESTATE DIVISION

The following two examples of the fair division of an estate illustrate many of the issues central to the economic analysis of fairness. Example one displays three possible fair solutions to the basic problem of dividing money between two parties. The multiplicity of solutions shows that the choice of a reference point for evaluation of fairness is critical. The example also shows that the choice of which factor is morally relevant to a fair distribution will sometimes be sufficient to determine how the money is allocated. Example two modifies example one by replacing money with personal property. This introduces the issues of idiosyncratic valuation and interpersonal utility comparison. I also allude to two problems involved in implementing a fair division: wealth constraints and private information.

A. *Example one*[40]

Xerxes dies and leaves an estate of $300. He has told his priest to divide the estate between his friends Yves and Zack in accordance with instructions in a note he gave the priest. Regrettably, the priest finds out that the instructions direct him to give $200 to Yves and $300 to Zack. The priest considers three possible fair divisions of the estate.[41] First, he could choose an egalitarian solution

later... Its reading requires, in principle, *no* knowledge of mathematics." GERARD DEBREU, THEORY OF VALUE 1 (1959) (italics in original). Needless to say, the chapter and the whole book was a struggle.

40. *See* YOUNG, *supra* note 12, at 67.

41. I assume that the priest cannot discern anything more about Xerxes' intention.

and give an equal amount of $150 to both men.[42] Second, he could give $100 to Yves and $200 to Zack. The justification for this division is that Zack has an exclusive claim to the last $100 in the estate so that $100 should go to Zack. The first $200 in the estate can then be divided equally. The dissatisfaction of Yves and Zack is equalized in the sense that both men are $100 short of their claims.[43] Comparing the first two solutions, it is easy to recognize a shift in a background assumption. The first solution uses a reference point in which Yves and Zack have nothing. The second solution uses a reference point in which both have their claims fully satisfied.[44] As a third possibility, the priest could follow probate law and abate the bequests in proportion to the claims.[45] In such a case, Yves gets (2/5) of $300, or $120 and Zack gets (3/5) of $300, or $180.

B. *Example two*[46]

Xerxes dies and leaves a painting to Yves or Zack. The priest is told to do whatever seems fair. The painting has a market value of $400. Yves attaches a higher value of $1000 to the painting because of sentimental reasons. Zack only cares about the painting for its resale value. The first solution that occurs to the priest is for Yves to receive the painting and give $500 to Zack. The justification is that both men gain $500 in value. The priest reconsiders when Yves complains that he does not have $500 to give Zack. Yves also mutters something about really only valuing the painting at $410. The priest disregards Yves's claim that he only values the painting at $410 because he can see in his eyes that Yves really values it at $1000. The priest decides on a second solution in which Yves receives the painting and gives $200 to Zack. The new justification is that both men suffer a loss of $200 compared to what they would enjoy if they each somehow got an equivalent painting.

42. Equal division would be unattractive if Yves' claim was only $100 in which case equal division would give him more than he was left in the note.

43. *See* YOUNG, *supra* note 12, at 67-69 (noting that this allocation method comes from the Talmud and is called the contest garment rule).

44. *See* YOUNG, *supra* note 12, at 75-76 (noting that assessments of equality depend on the choice of a baseline and a yardstick).

45. *See* WEST'S ANN. CAL. PROB. CODE §21403 (1999) (indicating that pro rata abatement is used within a class of beneficiaries).

46. *See* MOULIN, *supra* note 11, at 20-22.

Yves likes this proposal better and comes up with the $200. Zack grumbles but acknowledges that $200 is half of the market value of the painting. Nevertheless, he still thinks that Yves is getting the better deal.[47]

A division problem as simple as example two raises a host of issues about choosing a fair end-state and also about how to achieve that end-state. An efficiency issue appears—the painting is worth more to Yves than to Zack. Later in this review I will comment on trade-offs between efficiency and fairness,[48] but here I am content to observe that efficiency itself may be a fairness consideration: Yves should get the painting because he values it more. Of course, the very question of how much Yves values the painting is also an important issue. How can a fair end-state be achieved when parties hold private information? I assumed that the priest knew Yves' valuation – normally the judge, jury, arbitrator or legislature will not have all the relevant information they need to choose a fair outcome.[49] The issue of a wealth constraint also surfaces in example two. Is it fair to give the painting to Yves if he cannot or will not make some payment to Zack? Finally, we must confront the classic issue of interpersonal comparison. It is possible to make some headway in resolving fairness questions without making such comparisons,[50] but the exposition is easier and the methods are more powerful if we allow comparisons. Thus my presentation follows the usual welfare economics tradition.[51]

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47. Moulin points to one other fair division in which Yves takes the painting and pays $350 to Zack. This allocation equalizes the gain of risk neutral parties in comparison to a benchmark in which either Yves or Zack gets the painting with one-half probability. *See id.* at x.

48. *See infra* text at Part IV.

49. *See generally* Bengt Holmstrom & Roger Myerson, *Efficient and Durable Decision Rules with Incomplete Information,* 51 ECONOMETRICA 1799 (1983) (discussing a social planner's problem in the face of private information).

50. An allocation is envy-free if no person prefers the allocation given to someone else. *Compare* YOUNG, *supra* note 12, at 11 (stating the role for this notion of equity is limited because it assumes that everyone has an equal claim to whatever is being allocated *with* STEVEN J. BRAMS AND ALAN D. TAYLOR, FAIR DIVISION: FROM CAKE CUTTING TO DISPUTE RESOLUTION 1-2 (1996) (indicating that envy-free methods of fair division are central to their analysis).

51. *See Young, supra* note 12, at 12-13 (rejecting utilitarianism in the sense of comparing people's happiness but says that interpersonal comparisons must be made).

II. COOPERATIVE GAME THEORY

One economic approach to dividing cost and benefit relies on cooperative game theory. A cooperative game consists of a list of players and a characteristic function.[52] The characteristic function indicates what payoff should be assigned to every conceivable coalition of players. A coalition is any set of players, including singletons and the entire set of players. There are different possible interpretations of the characteristic function. I will return to the estate division examples to illustrate some cooperative games and explain their characteristic functions.

The cooperative games that describe the two estate division examples are similar. The list of players is Y and Z (short for Yves and Zack). The characteristic function, v( ), associates a payoff with each coalition: {Y}, {Z}, {Y,Z}, and the empty set 0. One possible characteristic function for example one is:

v(Ø) = 0,
v(Y) = 200,
v(Z) = 300
v(Y,Z) = 300.

The interpretation of v(Y) = 200 is that Yves could get $200 from the estate if Zack did not exist. Similarly, Zack could get $300 if Yves did not exist. Of course, the coalition with no players creates no value. And finally, the pair can cooperate and share the whole estate of $300. The same type of reasoning leads to the following characteristic function for example two:

v(Ø) = 0
v(Y) = 1000
v(Z) = 400
v(Y,Z) = 1000.

Given the specification of a cooperative game, the economist uses a solution concept to determine the fair division of cost or benefit between the players. Different solution concepts are available. The leading solution concepts can be justified in terms of their axiomatic origin. In this Part of the review, I will introduce two solution concepts and in Part III I will explain the axiomatic approach to constructing a solution concept.

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52. *See id.* note, at 85; MOULIN, *supra* note 12, at 402-427; MARTIN SHUBIK, GAME THEORY IN THE SOCIAL SCIENCES 127-130 (1982).

Essentially, a solution concept is a way to generalize the Aristotelian proportionality principle. The two most popular solution concepts are the *Shapley value* and the *nucleolus*. The Shapley value determines the average marginal benefit or cost associated with each player in each possible coalition, and allocates that amount to each player.[53] The nucleolus, instead, picks the allocation that minimizes the dissatisfaction of all possible coalitions of cooperating individuals.[54]

Let me apply the Shapley value solution concept to the first estate division problem. I will start with the allocation to Yves. There are two possible coalitions that include Yves: {Y} and {Y,Z}. The incremental benefit that Yves brings to the coalition {Y,Z} is zero, because Zack alone gets v(Z) = 300 which is the same as the payoff to the pair, i.e., v(Y,Z) = 300. The marginal benefit that Yves brings to the coalition {Y} is 200, because v(Y) = 200 and the payoff is zero to the empty coalition, i.e., v(Ø) = 0. The Shapley value allocation to Yves is the average of 0 and 200 or 100. Similarly, the Shapley value for Zack averages the marginal contributions made by Zack: namely, v(Z) – v(Ø) = 300, and v(Y,Z) – v(Y) = 100. Thus his Shapley value allocation is 200.

The nucleolus gives the same allocation for this problem. A payoff of 100 to Yves and 200 to Zack minimizes the dissatisfaction of the various coalitions. The dissatisfaction of the coalition {Y} is measured by the gap between the payoff specified for Y by the characteristic function v(Y) = 200 and Y's actual allocation of 100. So Yves gets 100 less than he is "entitled to" according to the characteristic function. Similarly, Zack gets 100 less than v(Z) =300. The coalition of Yves and Zack gets a total allocation of $100 + $200 = $300 which just matches the payoff specified by the characteristic function: v(Y,Z) = 300. There is no way to rearrange the allocation of the estate between Yves and Zack so that the dissatisfaction of at least one coalition falls, and no coalition grows more dissatisfied. Thus we have the nucleolus.

Compare these results to the discussion of example one in Part I. In example one, the priest considered giving the uncontested $100 portion of the estate to Zack and dividing

53. *See* YOUNG, *supra* note 12, at 69-71; *See* MOULIN, *supra* note 11, at 417-423.

54. *See* YOUNG, *supra* note 12, at 93-96.

the remaining $200 equally (yielding an allocation of $100 to Yves and $200 to Zack). The discussion of cooperative games shows the same allocation can be derived using the appropriate game and either the Shapley value or the nucleolus solution concept. As the reader might expect, the other allocations considered by the priest can also be derived as solutions to a cooperative game. For example, the allocation that splits the estate equally and gives $150 to both Yves and Zack comes from a cooperative game in which v(Ø) = v(Y) = v(Z) = 0, and v(Y, Z) = 300.[55] The interested reader can also verify that cooperative games can be specified that have solutions that correspond to the two allocations considered by the priest in the second estate division example.[56]

The Shapley value and the nucleolus are not merely obscure inventions by game theorists. The Shapley value has been derived independently by non-economists in two real-life settings. First, civil engineers developed a similar scheme to allocate dam construction costs by the Tennessee Valley Authority.[57] Engineers estimated the costs associated with an actual dam and also with hypothetical dams that would be built to serve the various possible coalitions of users. The actual costs were then allocated to users based on a method that approximated the Shapley value. More surprising perhaps, distributors of used machinery devised an ingenious scheme to allocate the surplus created by their collusion at auctions.[58] After official auctions ended, the

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55. Instead of giving the singleton coalitions the payoff that they would receive if the other claimant did not exist, in this cooperative game I assume that neither claimant can take anything from the estate without the permission of the other. Therefore, v(Y) = v(Z) = 0.

The priest also considered a proportional allocation in example one that gave $120 to Yves and $180 to Zack. A natural specification of the characteristic function also produces that outcome. Suppose that by acting alone Yves could hire a lawyer at a cost of 100 and win a payment of 120 in a probate proceeding. Similarly, Zack could hire a lawyer at a cost of 100 and win a payment of 180. Then v(Y) = 20 and v(Z) = 80, and the Shapley value or nucleolus give $120 to Yves and $180 to Zack.

56. In example two consider the following characteristic function: v(Ø) = 0, v(Y) = 1000, v(Z) = 400, and v(Y,Z) = 1000. The nucleolus and Shapley value both yield an allocation in which Yves gets 800 and Zack gets 200. Alternatively, consider the characteristic function: v(Ø) = 0, v(Y) = 0, v(Z) = 0, and v(Y,Z) = 1000. The nucleolus and Shapley value both yield an allocation in which Yves gets 500 and Zack gets 500.

57. *See* YOUNG, *supra* note 12, at 86.

58. *See* Daniel Graham, Robert C. Marshall, & Jean-François Richard,

colluding bidders gathered and held a series of private (and illegal) auctions open only to ring members to allocate the machinery won by the collusive ring and to allocate the financial gains derived at the expense of the auctioneer. These private auctions were designed in such a way that the collusive gain was allocated to ring members according to the Shapley value.[59]

The nucleolus is a generalization of the contested garment rule that appears in the Talmud as a rule for fair division.[60] The contested garment rule first appears in the Babylonian Talmud nearly 2000 years ago, where its name derives from the following problem: "Two hold a garment; one claims it all, the other claims half. What is an equitable division of the garment?"[61] The answer given in the Talmud is ¾ to the party claiming the whole garment and ¼ to the other. Because there is no dispute that the first claimant is entitled to half, the remaining, disputed portion is divided equally.[62] Other Talmudic writings extend this procedure to estate division and partnership dissolution problems with three claimants.[63]

The Shapley value and the nucleolus give the same allocation in the two-claimant estate division problems in Part I. In general, however, the Shapley value and nucleolus yield different solutions to cooperative games. Take, for example, the problem of apportioning damages between three polluters. Suppose that Xerxes, Yves, and Zack independently took actions that contaminated land owned by Wilma. The pollution destroyed the value of the land and caused a loss of 400 (measured in appropriate units, e.g., $400 million). Zack was responsible for the largest amount of contamination and Xerxes the least. If Zack had not

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Differential Payments within a Bidder Coalition and the Shapley Value, 80 AMER. ECON. REV. 493 (1990). The bidders involved in the collusive ring were ultimately detected and prosecuted. *See* U.S. v. Seville Industr. Mach. Corp., 696 F. Supp. 986 (D. N. J. 1988).

59. *See* Graham et al., *supra* note 58.

60. *See* YOUNG, *supra* note 12, at 93-96

61. *See id.* at 65.

62. I can illustrate the cooperative game associated with this problem by designating the party claiming the whole garment A, and the other claimant B. Let $v(A) = 1$, $v(B) = ½$, and $v(A,B) = 1$. The numbers represent a fraction of the garment. The outcome in the Talmud is the nucleolus *and* the Shapley value for the cooperative game. When there are more than two players the contested garment rule generalizes to the nucleolus *not* the Shapley value.

63. *See* YOUNG, *supra* note 12, at 71.

acted, the pollution by Xerxes and Yves would have caused only 300 units of damage. If Xerxes had acted alone, the harm would have been 100 units. Yves acting alone would have caused 200 units of damage. Zack acting alone would have caused 300 units of damage, and Zack paired with either Xerxes or Yves would have caused a loss of 400. The natural specification of the characteristic function is:

c(X) = 100
c(Y) = 200
c(Z) = c(X,Y) =300
c(X,Z) = c(Y,Z) = c(X,Y,Z) = 400.[64]

First, we find the Shapley value allocation for Yves. An intuitive approach to the analysis is to imagine that the damages are apportioned by means of a lottery. The three polluters are randomly assigned the first, second, or third position in a queue. Starting with the first person in the queue, payments are made until Wilma recovers the 400 total. Payments are limited by the amount of damage that a polluter would have caused if he had acted alone; thus, Yves pays 200 at most. There are six possible outcomes of the lottery: (X,Y,Z), (X,Z,Y), (Y,X,Z), (Y,Z,X), (Z,X,Y), and (Z,Y,X). Suppose the outcomes are equally likely. Yves is first with probability (1/3) and pays 200. He is last with probability (1/3) and pays 0, because Xerxes and Zack would pay the 400 total. Yves is second behind Xerxes with probability (1/6) and pays 200. Finally, Yves is second behind Zack with probability (1/6) and pays 100 (because Zack has already paid 300). These observations allow us to calculate that the Shapley value allocation for Yves is 116(2/3).[65] The Shapley value allocation for Xerxes is 66(2/3) and for Zack it is 216(2/3).

In contrast, the nucleolus yields a payment of 50 by Xerxes, a payment of 125 by Yves, and a payment of 225 by Zack. This is the allocation that minimizes the dissatisfaction (in this case, it is easier to think of *maximizing satisfaction*) of the various coalitions. Under this allocation the least fortunate coalitions ({X} and {Y,Z}) enjoy a surplus of 50. The coalition consisting of Xerxes created a cost of c(X) = 100, but only has to pay 50, and therefore gets a surplus of 50. The coalition of Yves and Zack created a cost

64. I use c( ) rather than v( ) to indicate that a cost not a value is associated with each coalition.

65. (1/3 x 200) + (1/3 x 0) + (1/6 x 200) + (1/6 x 100).

of c(Y,Z) = 400, but only has to pay 350, and therefore gets a surplus of 50. The other coalitions get a higher surplus: {Y} and {Z} both get a surplus of:

75 = C(Y) – 125 = C(Z) – 225

and {X,Y} and {X,Z} both get a surplus of:

125 = C(X,Y) – 175 = C(X,Z) – 275.

Thus each of the least satisfied coalitions have the same complaint—they got a surplus of just 50. But there is no way to shift the payments to give every coalition a surplus above 50. Consider that a smaller payment by Xerxes would lead to a higher combined payment by Yves and Zack—with the result being the surplus of {Y,Z} would fall below 50. Likewise, raising the surplus to {Y,Z} would necessarily reduce the surplus to Xerxes. Notice that the Shapley value is not equivalent to the nucleolus because Xerxes gets a surplus of only 33(1/3) from the Shapley value allocation.

I do not want to pause yet to compare the merits of the two solutions. It is not obvious that either apportionment of damages is fairer. The purpose of the example was to illustrate how the solution concepts are applied. I will move to a more detailed example drawn from nuisance law to set the stage for a normative evaluation of the two solution concepts.

The private nuisance problem is a perennial concern in law and economics.[66] The standard analysis was fashioned by Coase, who observed that parties can bargain to eliminate inefficiencies associated with conflicting uses on neighboring property.[67] The Coase Theorem states that parties will negotiate the efficient solution to a private nuisance problem if transaction costs are low.[68] Further, the solution is independent of the assignment of property rights.[69] I will take the Coasean analysis of private nuisance as a point of departure for the application of cooperative game solution concepts to the question of the fair assignment of property rights.

Let me illustrate the issue with a dispute between three parties: Ann, Bob and Carol. Suppose that Ann is engaged in activity that interferes with the activities of Bob and Carol. For example, Ann operates a gravel pit that generates dust that bothers Bob and Carol in their neighboring

66. *See e.g.*, POSNER, *supra* note 9, at 61-67.
67. *See* Coase, *supra* note 34.
68. *See id.*
69. *See id.*

homes. Alternatively, Ann plays loud music in her home that bothers neighbors Bob and Carol, or Ann uses a powerboat on a public lake in a way that disturbs Bob and Carol while they are fishing.

To make the problem concrete I suppose that the benefit to Ann from her activity is 4. Similarly, the benefit to Bob and Carol from their activities if Ann disturbs them is 4 each. If Bob and Carol are undisturbed, then their benefit rises to 6 each. Ann's net benefit falls to 3 if she modifies her activity so that it does not disturb Bob and Carol. This is enough data to specify a cooperative game and find a solution.[70]

The traditional Coasean argument in law and economics holds that rights should be assigned in a way that maximizes efficiency. Here, efficiency requires that Ann refrain from the activity that diminishes the benefits available to Bob and Carol. If transaction costs are low, it does not matter how the rights are assigned, because the parties will negotiate that outcome. Where transaction costs are high, however, the Coasean argument favors assignment of rights to Bob and Carol. In the following discussion, I assume that transaction costs are sufficiently low so that efficiency will be achieved regardless of how property rights are assigned.

The game specification consists of a list of players: A, B and C, and a characteristic function. The following characteristic function is just one of the possible functions consistent with the data in this problem.[71] The payoff to Ann acting alone is v(A) = 4. Similarly, the payoffs to Bob or Carol acting alone are v(B) = v(C) = 4 in the face of Ann's disruptive activity. If Ann cooperates with either Bob or Carol, then the joint payoff they can achieve is v(A,B) = v(A,C) = 9. Because Ann can mitigate her harm at a cost of 1 while the other party gains 2, the net benefit associated with Ann's activity falls to 3, but the benefit from the other party's activity rises to 6.[72] The coalition of Bob and Carol

70. I use this exercise to build some intuition about how the private nuisance problem should be resolved.

71. I will consider an alternative characteristic function later.

72. A background assumption in this version of cooperative game theory is that utility is transferable between members of a coalition. Ann's sacrifice could be induced within the coalition of Ann and Bob by Bob's offer to transfer a benefit of at least 1 to Ann. The nominal net benefits of 3 to Ann and 6 to Bob are not relevant to the coalition {A,B}; only the payoff v(A,B) = 9 is relevant to

only nets the sum of what they can achieve alone: v(B,C) = 8. Finally, the grand coalition of all three will achieve the highest joint payoff if Ann mitigates her harm. The result is a net benefit of 3 from Ann's activity, and 6 from the activities of Bob and Carol so that v(A,B,C) = 15.

The Shapley value of this game allocates a utility to each player that equals his or her average marginal contribution to each coalition. The Shapley value for Ann is $5\frac{1}{3}$. To check the derivation of this result first notice that Ann contributes 4 to the singleton coalition {A}. She contributes v(A,B) – v(B) = 5 to the doubleton coalition {A, B}. She makes the same contribution to the coalition {A, C}. Finally, she contributes v(A,B,C) – v(B,C) = 7 to the grand coalition {A, B, C}. Ann's Shapley value is the average of 4, 5, and 7, or $5\frac{1}{3}$. Similar calculations for Bob and Carol yield Shapley values of $4\frac{5}{6}$ for each.[73]

The division of benefits given by the Shapley value can be used to guide the assignment of property rights in this nuisance problem. If Bob and Carol are assigned the right to be free from interference by Ann and that right can be enforced at no cost, then Bob and Carol will enjoy their full benefits of 6 each and Ann will get a net benefit of 3. This pattern of benefits departs considerably from the Shapley value calculations that would give $5\frac{1}{3}$ to Ann and $4\frac{5}{6}$ to Bob and Carol. In fact, assigning a right to Ann to choose whatever activity she pleases is suggested by the Shapley value calculation. In this case, efficiency dictates that Bob and Carol make a payment to Ann to induce her to curtail her interference with their activities. The total gain that

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the coalition.

73. The reader might expect a different weighting of the marginal contributions. The reader might guess that since there are four different coalitions that include A (namely, {A}, {A, B}, {A, C}, and {A, B, C}) each of these marginal contributions should get a weight of one-fourth. The best explanation for the weighting used to calculate the Shapley value is based on the following heuristic. Suppose that A, B, and C come through a door in a random order. We calculate A's marginal contribution to the coalition of agents who passed through the door before her. There are six possible orderings: ABC, ACB, BAC, BCA, CAB, and CBA. Each order is equally likely, so the marginal contribution in each case is weighted by one-sixth. For the example in the text: if B comes through the door first (BAC or BCA) the marginal contribution is 4; if B comes through the door second and follows A (ABC) the marginal contribution is 5; if B comes through the door second and follows C (CBA) the marginal contribution is 4; and if B comes through the door third (ACB or CAB) the marginal contribution is 6. Thus, the Shapley value for B is: (1/6)4 + (1/6)4 + (1/6) 5 + (1/6)4 + (1/6)6 + (1/6)6 = 4(5/6).

the parties can share from an efficient arrangement is 3.[74] Ann should get a payment of at least 1 to compensate for the mitigation, and Ann could get as much as 3 in payment if she is an extremely successful bargainer. Thus Ann should enjoy a benefit between 4 and 7 depending on her bargaining skill. An outcome where each party gets 5 seems plausible[75] and is much closer to the Shapley value.

If cooperative game theory in general and the Shapley value in particular are of any real value to law professors, it does not lie in Delphic pronouncements about fair outcomes. Instead, the value must arise from some intu-itions about fairness that come from applying the theory. So far, the analysis supports an allocation of rights in favor of the *polluter* (generally the party called upon to mitigate). This is because Ann, the polluter, offers a valuable contribution to the well being of neighbors Bob and Carol. If Ann forbears from polluting, she offers a significant benefit to either Bob or Carol or the pair, and the Shapley value recognizes that contribution by rewarding her.

It may seem that the Shapley value solution is somehow stacked in favor of the polluter. In fact, it is. The problem, though, is not with the Shapley value solution concept; rather, it lies in my specification of the characteristic function. While my specification was reasonable, it is not the only reasonable specification. I will now revise the characteristic function and recalculate the Shapley value. The result will be a solution that is more favorable to Bob and Carol. It is important to notice that the specifi-cation of the characteristic function incorporates significant value judgments.

In the first specification of the characteristic function, I assumed that Ann's polluting activity was a background condition that could affect any coalition. Now I reverse that assumption and suppose that the absence of pollution is the background condition. Recall that originally the singleton coalition consisting of Ann was assigned a payoff v(A) = 4. Now I reduce that payoff to v(A) = 3. Intuitively, even in isolation Ann cannot choose the polluting activity without permission. Alternatively, we can interpret the payoff v(A)

74. Bob and Carol both get a benefit of 2 while Ann loses 1.

75. This outcome arises when the parties split the gains from cooperation equally. There is no consensus in economics about how bargains of this sort are resolved. *See generally,* MARTIN OSBORNE AND ARIEL RUBINSTEIN, BARGAINING AND MARKETS (1990).

= 3 as meaning that Ann by herself only deserves the payoff associated with her non-polluting activity.[76] I assumed that the singleton coalition of Bob or Carol alone would suffer the harm from Ann's pollution and so also get a payoff of 4. I now change that assumption and assume that Bob and Carol alone get a payoff of 6 (so that v(B) = v(C) = 6). Continuing with the new assumptions, the pair of Bob and Carol would get a payoff of 12 (so that v(B,C) = 12). The payoff to the other coalitions is unchanged (so that v(A,B) = v(A,C) = 9 and v(A,B,C) = 15. The Shapley value for this alternative specification of the characteristic function yields a payoff of 3 to Ann and 6 to Bob and Carol.[77]

How do we evaluate which of the alternative specifications of the characteristic function is correct? In this setting I think that either can be defended. I might choose a payoff to the singleton coalitions that reflects what each party actually can achieve in isolation. That might mean that Bob gets 6 because he can intimidate Ann until she abates the dust from her gravel pit, turns down her music, or keeps her boat from interfering with the fishing. Of course, the opposite might also be true. Descriptive realism is not necessarily required or desirable. I might specify that v(B) = 6 because Bob deserves a benefit of 6 when he stands alone. Finally, I might specify payoffs to coalitions based on either status quo or hypothetical property rights.

Comparing the Shapley values obtained for the two specifications, we see that Bob and Carol do better under the new specification. The payoffs yielded by the new Shapley value match the payoffs that result from giving Bob and Carol the right to be free from interference by Ann. The difference between the two specifications is that Ann's marginal contributions are smaller under the new specification because Bob and Carol do better without Ann.[78] Intuitively, the new analysis models the cooperative

76. Still another interpretation is that in the state of nature Bob and Carol can prevent Ann from choosing the polluting activity by force.

77. Ann makes a marginal contribution of 3 to the singleton coaltion {A}. She makes a marginal contribution of 3 to either doubleton coalition. And she makes a marginal contribution of 3 to the grand coalition. Averaging over all 3's gives an allocation of 3 to Ann. For Bob or Carol, their marginal contributions are 6 to every coalition that they might join, so they each get a Shapley value allocation of 6.

78. Under the new specification Ann contributes 3 to the coalition {A} compared to 4 before, she contributes 3 to the coalitions {A, B} and {A, C}

problem in a way that is more sympathetic to the victims of pollution. As a result, the Shapley value derived from the new specification of the characteristic function gives Bob and Carol a relatively high allocation. Thus, I reiterate the point I made earlier that a critical issue is how the analyst abstracts from the context of the problem.[79] To a large degree, fairness analysis depends on how reference points are chosen. One advantage of this formal approach is that it makes the effect of an analyst's assumption quite clear.

Having just stated that fairness analysis depends on reference points to a large degree, I hasten to add that the choice of solution concept is also very important. The nucleolus is a leading alternative to the Shapley value as a solution concept for cooperative games. In some problems the two solution concepts will give similar outcomes; in other problems they diverge widely. To a large degree, the choice of a solution concept depends on the analyst's moral intuition. Once the analyst understands how different solution concepts work, he or she can make a choice based on which one more closely comports with the analyst's moral intuition.

In Part III, I will compare the axiomatic basis of the nucleolus and the Shapley value, but for now I am content to apply the concepts to the problem at hand and compare the results. Given the first specification of the characteristic function, the nucleolus yields a payoff of 5 for Ann and 5 for both Bob and Carol (compared to $5^1/_3$ for Ann and $4^5/_6$ for Bob and Carol under the Shapley value). Given the second specification, the nucleolus yields a payoff of 3 for Ann and 6 for Bob and Carol (the same as under the Shapley value).

Let me explain the derivation of the nucleolus and why the solution concepts diverge for the first specification and agree for the second. The nucleolus creates a rough parity among all coalitions by maximizing satisfaction experienced by each coalition. I judge satisfaction by comparing the payoffs allocated to the members of a coalition by a potential solution to the payoff specified for that coalition by the characteristic function. For example, the nucleolus associated with the first characteristic function allocates a payoff of 5 to Ann. Her level of satisfaction is the gap

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compared to 5 before, and she contributes 3 to the grand coalition {A, B, C} compared to 7 before.

79. *See* YOUNG, *supra* note 12, at 122 (framing effects in bargaining).

between that payoff, and the payoff given by the characteristic function v(A) = 4. I will say that her satisfaction level is equal to 1. For Bob and Carol the satisfaction level is also equal to 1, because the nucleolus allocates them a payoff of 5 each, whereas the characteristic functions specifies v(B) = v(C) = 4. Now consider the doubleton coalitions. Ann and Bob together get a payoff of 5 + 5 = 10 under the nucleolus compared to the payoff of v(A,B) = 9 specified by the characteristic function. Their satisfaction level is also 1. The same level of satisfaction is achieved by the coalition of Ann and Carol. The coalition of Bob and Carol gets a satisfaction level of 2 because together they get 10 under the nucleolus while v(B,C) = 8. Thus all coalitions get a payoff at least one in excess of their characteristic function payoff.[80] There is no way to rearrange the nucleolus payoffs without forcing some coalition's satisfaction level below 1. Now let's compare the Shapley value. Ann receives a higher payment and Bob and Carol receive lower payments compared to the nucleolus. Obviously, the satisfaction level of the coalitions {B} and {C} fall below 1 to $^5/_6$. Thus the Shapley value allocation decreases the satisfaction level of two of the least satisfied coalitions. Of course, by its nature, the Shapley value rewards parties based on their average marginal contribution to all coalitions. In this problem the Shapley value favors Ann relative to the nucleolus because of Ann's high marginal contributions. Ann's marginal contributions are relatively large because she is needed to abate the pollution in an efficient outcome.

Why do the two solution concepts yield an identical allocation of surplus in the second cooperative game problem? To understand the answer it helps to calculate the satisfaction level of each coalition. First note that all singleton coalitions have a satisfaction level of zero. The reason is that solution payoffs match the payoffs to the singleton coalitions given by the characteristic function: v(A) = 3, and v(B) = v(C) = 6. The doubleton coalitions also each have a satisfaction level of zero. The Shapley value payoffs to Ann and Bob (or Ann and Carol) are 3 + 6 = 9 which matches the characteristic function payoff: v(A,B) = 9

80. Of course, for the grand coalition, {A,B,C}, the sum of the individual nucleolus payoffs is equal to the characteristic function payoff: v(A,B,C) = 15. This is always true for a grand coalition.

(or v(A,C) = 9). Similarly, for Bob and Carol 6 + 6 = 12 = v(B,C). So all coalitions have the same level of zero satisfaction. It is not possible to rearrange payoffs without shifting some coalition's satisfaction level to a negative number. The Shapley value solution and the nucleolus solution coincide here because of the simple additive structure implied by the characteristic function. Each player's marginal contribution to any coalition is constant. Ann brings the value 3 to every coalition, while Bob and Carol bring the value 6 to every coalition. In this second cooperative game, when we reward each player based on his or her marginal contribution, we are also creating parity among all of the possible coalitions.

III. THE AXIOMATIC APPROACH

Reviewing the examples in the last section, it is difficult to choose among the solution concepts—to prefer one as fairer than the other. One way to develop a preference for a particular solution is to see how each of the solutions performs over a broad range of examples. With luck, we will find that one solution consistently gives sensible and intuitively fair solutions, while another generates enough aberrant solutions that we can reject it as a solution method.[81] An interesting, alternative way to choose among solution concepts requires understanding the origin of each concept in terms of its constitutive axioms.[82]

The game theorist specifies a list of axioms that he or she thinks a desirable (for moral, aesthetic or other reasons) solution concept should satisfy. He or she then checks to see whether the axioms are consistent with one another. Assuming that the axioms are consistent, the game theorist checks to see which solution concepts satisfy the axioms. As

81. I am not transgressing my rule against global analysis of fairness issues. I do not mean to suggest that one fairness rule can either answer all questions about a just society or even that one fairness rule is satisfactory for all problems of fair division. Rather, I am suggesting that we should make sure that a fair division method is *robust* in the sense that it performs well for all plausible division problems in a particular class.

82. *See* Donald Wittman, *The Geometry of Justice: Three Existence and Uniqueness Theorems,* 16 THEORY AND DECISION 239 (1984) for a lucid and accessible presentation of the axiomatic approach to fair division between two parties. *Cf.* NORMAN FROLICH AND JOE A. OPPENHEIMER, CHOOSING JUSTICE: AN EXPERIMENTAL APPROACH TO ETHICAL THEORY 156-57 (1992) 156-57 (1992) (expressing skepticism toward an axiomatic approach to distributive justice).

more and stronger axioms are added to the list, the odds grow that no solution concept can satisfy them all. A difficult and happy achievement is to find a min-imally sufficient list of axioms that can be satisfied by only one solution concept. When this axiomatic characterization of a solution concept is achieved, then it makes sense to identify the moral significance of a solution concept with the content of its axioms (and the implicit assumptions hidden in the statement of the problem).

A rough version of the axiomatic characterization of the Shapley value states:[83] it is the unique allocation rule that is (i) impartial, (ii) Pareto optimal, and (iii) satisfies the marginality principle. Impartiality means that the allocation only depends on the relevant specified information. The allocation cannot depend on factors that are morally arbitrary—like whose name is shorter. Pareto optimality simply means that all benefits are allocated; nothing is wasted. In the cost allocation context, Pareto optimality means that the sum of the allocated costs equals and does not exceed the required cost of the project. The marginality principle requires that a player's allocation depends only on that player's marginal contributions to all possible coalitions.[84]

A rough version of the axiomatic characterization of the nucleolus states:[85] it is the unique allocation rule that is (i) impartial, (ii) Pareto optimal, (iii) homogeneous, (iv) separable, and (v) satisfies the consistency principle. The first two axioms also appear in the characterization of the Shapley value. The marginality principle in the Shapley value characterization is replaced in the nucleolus characterization by homogeneity, separability, and consistency. Costs or benefits are separable if they are solely attributable to one player. A solution concept satisfies separability if it allocates the separable portion of cost or benefit to the responsible party. A solution concept is homogeneous when scaling up costs or benefits scales the allocation in the same way.[86] Roughly speaking, a solution concept is consistent if it gives the same allocation over a group of players when the number of players in the game is

83. *See* YOUNG, *supra* note 12, at 200.

84. I will explain this principle further below.

85. *See Young*, *supra* note 12, at 201-204.

86. The Shapley value satisfies the separability and homogeneity axioms, but they are not needed to characterize the Shapley value.

scaled up or down in a regular way.[87]

The essential differences between the Shapley value and the nucleolus can be traced to the difference between the marginality principle used to characterize the former and the consistency principle used to characterize the latter. I will explore the relative merits of the two axioms by explaining how they can be violated. Specifically, I will describe why the nucleolus fails the marginality principle and why the Shapley value fails the consistency principle.

Let me show that the nucleolus violates the marginality principle by returning to the problem of apportioning damages among three polluters. Recall the goal is to divide fairly the 400 unit cost of pollution between Xerxes, Yves, and Zack. The cooperative game specified the following characteristic function: c(0) = 0, c(X) = 100, c(Y) = 200, c(Z) = 300, c(X,Y) = 300, c(X,Z) = 400, c(Y,Z) = 400, and c(X,Y,Z) = 400. The marginality principle is built upon the notion of a player's marginal contribution. A player's marginal contribution is measured by the change in the value of a coalition when the player is removed from the coalition. Thus Xerxes makes a marginal contribution of 100 to the coalition {X,Y} because c(X,Y) − c(Y) = 300 − 200 = 100. Similarly, Xerxes makes a marginal contribution of 100 to the coalitions {X, Z} and {X}. Finally, Xerxes makes a marginal contribution of 0 to the coalition {X, Y, Z} because c(X,Y,Z) − c(Y,Z) = 400 − 400 = 0. The marginality principle dictates that a solution concept gives the same allocation to a player if that player makes the same marginal contributions in two different cooperative games.[88]

To apply the marginality principle, I will modify the pollution cost sharing game in a way that keeps Xerxes' marginal contributions constant. The new cooperative game has the same characteristic function, except c(Y) is reduced from 200 to 100, and c(X,Y) is reduced from 300 to 200.[89] The marginal contributions by Xerxes are the same in the two games.[90] As required, the new Shapley value satisfies

87. As a precise definition is difficult to understand, I will illustrate the consistency principle with an example below.

88. The set of players is the same in the two games.

89. In the new game Yves' costs are symmetric with Xerxes' costs.

90. In the new problem Xerxes' marginal contribution to the coalition {X,Y} is still 100 because c(X,Y) − c(Y) = 200 − 100. The marginal contributions of Xerxes to the coalitions {X}, {X,Z} and {X,Y,Z} are unchanged in the two problems, because c(0), c(X), c(X,Z), c(Z), c(Y,Z), and c(X,Y,Z) are all unchanged.

the marginality principle because Xerxes' cost allocation does not change. The Shapley value allocation to Xerxes remains 66(2/3), the allocation to Yves falls from 116(2/3) to 66(2/3), and the allocation to Zack rises from 216(2/3) to 266(2/3). The nucleolus violates the marginality principle because the cost allocated to Xerxes rises despite the fact his marginal contributions are unchanged. The nucleolus in the new problem allocates a cost of 66(2/3) to Xerxes compared to 50 in the original problem.[91] This example provides intuitive support for the marginality axiom because it seems unfair to make Xerxes pay more when his marginal contributions are unchanged, and the total damage of 400 is unchanged.

The consistency principle is also intuitively appealing, but it points toward the nucleolus rather than the Shapley value. The consistency principle requires that an allocation rule yields consistent results when it is applied to one cooperative game that is derived from another by changing the number of players. I can illustrate the consistency principle by returning to the first estate division problem.[92] Yves had a claim of 200 and Zack had a claim of 300 to an estate with 300 in assets. Recall that both the nucleolus and the Shapley value yielded an allocation of 100 for Yves and 200 for Zack.[93] Now consider a similar problem with everything doubled: there are four claimants instead of two; the assets in the estate are doubled to 600; Yves and new player Yvette each have a claim of 200; and Zack and new player Zeke each have a claim of 300. As one might expect, the nucleolus allocates 100 to both Yves and Yvette, and 200 to both Zack and Zeke – not so the Shapley value. The Shapley value allocation gives 116(2/3) to both Yves and Yvette, and 183(1/3) to both Zack and Zeke.[94] The consistency principle is violated by the Shapley value as applied to these two fair division problems.

To recapitulate the discussion so far in Part III, the marginality principle and the consistency principle offer alternative axiomatic bases for constituting a rule of fair division. If an analyst finds the marginality principle more attractive, then he or she should choose the Shapley value

91. The allocation to Yves falls from 125 to 66(2/3), and the allocation to Zack rises from 225 to 266(2/3).

92. *See* text at notes 40-45.

93. Given the characteristic function with v(Y) = 200 and v(Z) = 300.

94. *See* YOUNG, *supra* note 12, at 71.

as a solution concept. If the analyst finds the consistency principle more attractive, then he or she should choose the nucleolus.

I will conclude this part by describing two desirable properties of fair division rules: one property is satisfied only by the Shapley value, and the other is not satisfied by the Shapley value at all.

The first property, which holds only for the Shapley value, is the monotonicity condition.[95] Roughly speaking, when two cooperative games are compared, if the payoffs of all coalitions are greater in the second game than the first, then the payoffs to all players must be greater in the second game.[96] The nucleolus sometimes violates this condition. It is possible that external changes that increase the surplus available to everyone actually reduce some player's nucleolus payoff. Not only does monotonicity seems intuitively desirable, it also has political-economic significance. To the extent that legal policy represents a fair bargain among interest groups, it is desirable that the bargain does not have to be reworked in the face of exogenous social or economic changes. An exogenous change that increases total surplus may reduce some player's nucleolus payoff. Such an event would provoke demands for revision of the sharing rule to protect the player that suffers.[97] This type of problem is avoided by the Shapley value.

The second desirable property concerns the temptation of coalitions to defect from a joint enterprise. The nucleolus is preferable to the Shapley value on the grounds that the nucleolus is more resistant to threats of defection. The nucleolus is designed to minimize the dissatisfaction of each coalition. It gives every coalition a payoff greater than the coalition's characteristic function payoff – whenever that is possible.[98] The same cannot be said of the Shapley value.

95. The marginality principle is the key to assuring that the monotonicity condition is satisfied.

96. *See* YOUNG, *supra* note 12, at 200.

97. *See* ZAJAC, *supra* note 14, at 121-22.

98. An economic concept called the core is useful here. The core is defined to be the set of feasible allocations that cannot be blocked by the objection of some coalition. Any coalition that can do better on its own that it does with a suggested allocation is allowed to object. Intuitively, the core is the set of allocations that are resistant to defection. *See* MOULIN, *supra* note 11, at 403-06; YOUNG, *supra* note 12, at 85. Unfortunately, the core does not exist for all cooperative games. In other words, for some cooperative games, at least one coalition will object to any possible allocation. The nucleolus is especially

The following example illustrates a problem of defection that afflicts the Shapley value but not the nucleolus.[99] Suppose that three communities, A, B, and C, agree to build and share a wastewater treatment facility. The characteristic function specifies the benefit to each coalition of communities from building the facility on their own. Suppose the characteristic function gives the following payoffs:

$v(A) = v(B) = v(C) = 2$
$v(A,B) = v(A,C) = 5$
$v(B,C) = 7$
$v(A,B,C) = 9.$

The communities clearly benefit from cooperation since each pair of communities and the coalition of all three communities get benefits that exceed the sum of the benefits when the communities are on their own. The Shapley value allocation for this problem gives a payoff of $2\frac{1}{3}$ to community A, and $3\frac{1}{3}$ to communities B and C. A weakness of the Shapley value in this setting is that communities B and C could defect from the three community project and get a higher payoff on their own: the Shapley value gives the pair $3\frac{1}{3} + 3\frac{1}{3} = 6\frac{2}{3}$ compared to $v(B,C) = 7$. The nucleolus for this cooperative game gives community A a payoff of 2, and communities B and C get payoffs of $3\frac{1}{2}$ each. B and C do as well under the nucleolus as they could do on their own. No coalition has an incentive to defect under the nucleolus allocation.

Defection is obviously relevant to fair division problems in which players voluntarily participate in a joint enterprise.[100] It deserves attention, but is less relevant in fair division problems in which participation is mandated by the government. Voluntary participation is present for many collective decisions like the water treatment project. Other examples include international economic and

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resistant to defection because it is defined in such a way that it is always in the core if the core exists. *See id.* at 201-02.

99. For similar examples *see* MOULIN, *supra* note 11, at 24-25; YOUNG, *supra* note 12, at 82-84.

100. The reader should notice that I am mingling normative and positive theory here. Cooperative games and their solutions concern how should a group divide costs or benefits. Whether members of a group can and will defect is a positive question. The link that I am drawing is based on the notion that defection will not occur if every coalition is doing as well under the solution of the game as they could on their own. Economists use the core to examine this link.

environmental treaties, joint ventures, and partnerships. There is no point embracing an allocation method as *fair* if some players object to their allocation and actually have an incentive to leave the joint enterprise.[101] In most of the fair division problems discussed in this review, the allocation is mandated and players do not have an option of leaving the joint enterprise. Nevertheless, there is a normative argument that the nucleolus is desirable because it is consistent with a goal of autonomy since coalitions receive an allocation that is at least as good as what they can achieve on their own (when this is possible). A sensible rejoinder to the autonomy argument is that sometimes it is just for one coalition to subsidize another. The universal service requirement in public utility regulation is a prominent example of a mandated subsidy policy.[102]

IV. FAIRNESS AND EFFICIENCY

Moulin and Young address two main criticisms of the use of cooperative game theory or other notions of end-state justice to guide legal policy-making. The first criticism holds that fair division of cost or benefit ignores economic efficiency. The second criticism holds that fair division should be assessed with reference to procedural rather than end-state justice. The authors are sensitive to both criticisms and believe that end-state analysis must accommodate concerns about efficiency and process. In this part of the Review I will mention some of the highlights of analysis that combines efficiency with end-state justice. In the next part of the Review I will discuss fair process.

Two efficiency issues frequently arise in fair division problems. One is the problem of shirking. I use that term broadly to cover both the free-rider problem[103] and the moral hazard problem.[104] Generally, the issue is whether parties

101. Notice that defection is not always a problem with the Shapley value. Recall the earlier discussion of the bidder cartel that settled on the Shapley value as a fair way to distribute the gains from collusion. That cartel was remarkably resistant to defection. The important point is that the very construction of the nucleolus assures that coalition dissatisfaction is minimized.

102. *See* ZAJAC, *supra* note 14, at 134, 203-10.

103. Free-riding occurs when people who benefit from a public good avoid paying for it. *See* MOULIN, *supra* note 11, at 27, 340.

104. Moral hazard occurs when a person deviates from an agreed course of action and takes an action that advances personal interests. *See* ZAJAC, *supra* note 14, at 60-61.

will take private actions that optimally advance a joint project. The other efficiency issue concerns strategic misrepresentation of information by parties holding private information.[105]

Self-interested parties usually will not take optimal actions to advance the common good. They tend to shirk their responsibility because they personally bear the cost of effort, resource contribution, or investment in the joint project. The government can eliminate the free-rider problem through coercion, and firms can mitigate or eliminate the problem by building cooperative institutions like joint ventures or trade associations. Moral hazard can also be eliminated, provided that inefficient actions can be observed and verified. Governmental fiat or private contract can assure efficient action given observability and verifiability.[106]

If a public good like a dam is funded by voluntary contributions there is likely to be an inefficiently low level of funding. In this setting, free-riders are users who benefit from the dam, but contribute less than their fair share to the cost of the dam. If the government provides the dam, then governmental fiat eliminates the free rider problem. The government can use general tax revenues or user fees to fund the project. Here, reliance on distributive justice to divide fairly the cost of the dam avoids the shirking problem.

Yet many public goods are provided by the private sector without serious shirking problems. Suppose, for example, three oil companies hire an exploration company to investigate oil deposits at a remote site. The companies agree they will share the resulting information. The companies certainly care about the incentive effects of their contract with the exploration company. But efficiency issues recede when it comes to dividing the cost of this simple joint venture. The companies are likely to use some fairness criterion to guide their decision about how they will share the cost of compensating the exploration company. The companies can handle the free-rider problem with a contract that binds them to sharing the compensation

105. *See* YOUNG, *supra* note 12, at 130-31.

106. Economists apply the term observable to information shared by the parties to a contract. They apply the term verifiable to information shared by the parties and the trier of fact at trial.

cost.[107]

The power of contract is also a key to assuring efficient actions are taken in the presence of moral hazard. Moral hazard is an issue in the nuisance problem in Part II because the polluter has an incentive to cheat on an agreement to abate her polluting activity. In my earlier discussion I used fairness criteria to assign property rights without regard to possible inefficiencies. Assuming the victims can observe the polluting action, the parties will set the optimal level of abatement by contract. Following Coase, if transaction costs are sufficiently low, then contract rights, not property rights, assure efficiency.

Shirking and similar problems occur when actions are unobservable. The government cannot assure compliance with regulations, and private parties cannot assure compliance with contracts if relevant actions cannot be observed. Inefficiency, however, is not inevitable. Optimal actions can still be induced if parties are provided the right incentives.

The sharing rules in a joint project can be chosen to ameliorate undesirable incentives for parties to choose inefficient activities. For example, if a party's share of the benefit from a joint venture depends indirectly on his or her effort, that creates a positive incentive to exert effort. Equal benefit-sharing in partnership does not give partners much of an incentive to exert effort that will contribute to the partnership's profit. If sharing is proportional to an indirect measure of effort like attracting new clients, then a greater and more efficient incentive is provided. The main problem with unobservable actions is that sharing rules that provide proper incentives often clash with sharing rules that assure a fair end-state.[108]

Strategic misrepresentation of private information is the second efficiency problem. I alluded to this problem in the second estate division example.[109] An efficient allocation scheme should give the painting left in the estate to the party who values it the most. I assumed that the priest knew who valued the painting the most, and even the

107. More precisely, the availability of contract law diminishes the impact of free-riding. Although the contract between the three companies deters free-riding after an agreement, there is still a temptation to free ride by refusing to sign an agreement or by demanding a small cost share.

108. *See* MOULIN, *supra* note 11, at 22 and sections 6.4 and 6.5.

109. *See supra* text accompanying notes 46-51.

precise value both Yves and Zack attached to the painting. Often such information will not be available and there is a danger of misallocation. The efficiency loss is measured by the gap between the highest value user and the player who actually receives the item. The inefficiency can be alleviated if the parties can trade items once they are allocated. If trade is possible, then the inefficiency cost is reduced to the cost of the transaction.

Another danger created by the information revelation problem is that the decision whether to build a public project like a dam, and also the size of the project, may be distorted. If users know that their payment for the dam will be proportional to their benefit, then they have an incentive to understate their benefit. If the government believes the understated valuations, then it could mistakenly decide to cancel a project or reduce its scale because costs appear large in comparison to benefits.

Private information does not always create efficiency problems.[110] Some people will report their private information honestly regardless of the consequences to themselves. Other people are deterred from lying because they fear getting caught and the resulting shame or punishment. Deterrence requires some probability that liars are detected and punished. This is not possible with some kinds of information that cannot be verified in court; e.g., how strongly a person desires a public good. Even when detection is not feasible or effective, sharing rules are another source of incentives for truth-telling.[111]

In the following example the sharing rule achieves both the fairness and efficiency goals. Consider a public project that will serve Ann, Bob, and Carol. The expected benefits to each party from an optimally designed project are equal. Suppose that there are two design choices: X or Y. Further suppose that there are two possible states of the world: x or y, and that Ann and Bob observe the true state of the world,

110. The collusive bidders at the used machinery auction dealt with the information revelation problem effectively. They chose a mechanism that induced the high value party to identify themselves. The mechanism also implemented the Shapley value allocation of the gains from collusion. *See* Graham et al., *supra* note 58.

111. There is a vast literature in economics that addresses the goal of getting parties to reveal their (private) preference information concerning a public good. *See e.g.,* JEAN-JACQUES LAFFONT, FUNDAMENTALS OF PUBLIC ECONOMICS 112-131 (1988).

while Carol and the government manager in charge of the project simply know that x and y are equally likely. Assume that choice X is optimal given x is the true state of the world. Similarly, Y is optimal given y. Finally, assume that Ann prefers project X and Bob prefers project Y regardless of the true state of the world. The problem for the manager is to fairly assign the costs of the project and induce Ann and Bob truthfully to reveal their information. There is a simple sharing rule that is fair and induces truthfulness. If Ann and Bob report the same state of the world, then the manager should believe their reports and choose the product design that is optimal for the reported state. The costs of the project are divided equally between Ann, Bob, and Carol. If Ann and Bob make inconsistent reports, then the manager should randomly choose a product design and divide the costs of the project equally between just Ann and Bob. The equilibrium outcome consists of truthful reports, optimal project design, and equal cost sharing, despite the private information and the conflicting interests of Ann and Bob.[112]

There is no consensus about how conflicting fairness and efficiency imperatives should be resolved. Moulin believes there is frequent conflict between the goals of efficiency and fairness.[113] When conflict is inescapable he favors an intuitive balancing between the two interests.[114] Young believes that the degree of conflict is overstated, and that fairly assigning entitlements in the context of competitive markets is often the best approach to public policy.[115] Generally, law and economics scholars shy away from a

112. The following numbers can be used to illustrate the example. The cost of the project is 9. The benefit to Carol when the project design matches the state is 9. The benefit to Carol is 0 otherwise. The benefit to Ann is: 10 if the state is x and the design is X; 9 if the state is y and the design is X; 8 if the state is y and the design is Y; and 6 if the state is x and the design is Y. The benefit to Bob is: 8 if the state is x and the design is X; 6 if the state is y and the design is X; 10 if the state is y and the design is Y; and 9 if the state is x and the design is Y. The equilibrium reports of Ann and Bob will be the true state. All three get an expected payoff of 6.

113. *See* MOULIN, *supra* note 11, at 22 (discussing the tension between efficiency and equity).

114. *See id.* at 163 (endorses an approach to fair division that defines "reasonable tests of equity, selecting as often as possible a small subset of efficient and 'just' outcomes.").

115. *See* YOUNG, *supra* note 12, at 161 (stating the trade-off is "largely chimerical"); *id.* at 19 (justifying competitive markets with fairly defined entitlements as equitable).

serious analysis of fairness issues when efficiency is a concern. The usual justification is that particular policies can be designed on the basis of efficiency. If the result is an unfair distribution of wealth, then society can address that problem via government taxes and transfers.[116] I agree with Moulin and Young that fairness and efficiency issues should be faced as they arise, but that topic is too complex for me to comment on here.[117]

V. FAIR PROCESS

Fair outcomes are the main concern of both authors, but they both recognize that fairness depends on means as well as ends, and they both comment at length on economic analysis of procedural fairness.[118] Most of the economic interest in process relates to implementation of a fair end-state allocation.[119] Courts, legislatures, and arbitrators face

116. *Compare* Louis Kaplow and Steven Shavell, "Why the Legal System is Less Efficient than the Income Tax in Redistributing Income," 23 J. Legal Stud. 667 (1994) (arguing that the income redistribution policies should be done through the tax system not legal rules. The problem with legal rules is that redistributional policies are likely to be directly inefficient in the market affected by the legal rule and indirectly inefficient through their impact on the labor-leisure choice. In contrast, the income tax only distorts the labor-leisure choice.) *with* Chris William Sanchirico, *Taxes Versus Legal Rules as Instruments for Equity: A More Equitable View*, USC Law School, Working Paper No. 98-21 (June 1998) (stating the Kaplow and Shavell result is not robust to various changes in their modeling assumptions. Particular attention is paid to heterogeneity of tortfeasors.)

117. There are three problems with the usual approach to fairness in law and economics. First, fairness is manifest in dimensions other than wealth or income inequality. I doubt that fiscal policy is the appropriate venue to address fairness issues raised by most of the fair division problems discussed earlier in this review. Second, political reality may favor interventions for the sake of fairness at the level of particular policies rather in the general fiscal policy arena. Third, the claim that fairness can be assured more efficiently through fiscal policy is open to challenge.

118. *See* MOULIN, *supra* note 11, at 36-44; YOUNG, *supra* note 12, at 130-45.

119. For an extensive discussion of "cake-cutting" methods that can be used to implement fair outcomes *see generally* BRAMS AND TAYLOR, *supra* note 50 (commenting that cake-cutting requires that one party divides resources that will be shared. The other party or parties get first choice among the allotments of resources. The name comes from the just solution parents use to settle distributional fights among children.); *see also* Moulin, *supra* note 24, at 2-3, 11-12 (Economists study fair procedure by specifying a game that implements a social choice. The "rights" of a player in the game are modeled as the set of actions available to the player under the rules of the game.). The concept of implementation is explained in KEN BINMORE, FUN AND GAMES: A TEXT ON GAME

constraints in implementing fair end-states because they have limited knowledge and power. Economic analysis is used to derive processes that implement desired end-states despite these constraints. Alternatively, economic analysis is used to show what ends are feasible given limited knowledge and power.[120]

In the second estate division problem I skirted the question of implementation by supposing that the priest knew the valuations that Yves and Zack attached to the painting. In fact, there is a simple auction process that implements the end-state in which Yves takes the painting and pays $200 to Zack.[121] The priest simply puts the painting up for public auction and lets Yves and Zack split the proceeds. Yves would submit a winning bid of $400, take the painting, and pay half of the winning bid to Zack.

The preceding example shows that sometimes a fair end-state can be implemented despite private information. Closer attention to the example reveals another potential implementation problem: a wealth constraint. If Yves has wealth less than $200, the supposed transfer is not feasible. The outcome of the public auction would be an inefficient sale to a third party, and Yves and Zack would split the proceeds of $400.

A third implementation difficulty is caused by collusion between the players.[122] Recall the discussion of the allocation of pollution damages between Xerxes, Yves, and Zack. For heuristic purposes I suggested that the Shapley value could be interpreted in terms of a lottery that determined the order in which the players made payments to satisfy the damage claim. For example, whoever draws the shortest straw makes the first payment. Whoever draws the longest straw makes the last payment (if any payment is still required after the first two). One problem with this lottery approach is that Xerxes and Zack have an incentive to collude.[123] In an effective collusive agreement Xerxes

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THEORY 531 (1992).

120. *Cf.*, Roger Myerson & Mark Satterthwaite, *"Efficient Mechanisms for Bilateral Trading,"* 29 J. ECON. THEORY 265 (1983) (impossibility of finding a bargaining mechanism that is voluntary, unsubsidized, and leads to trade if and only if trade is efficient).

121. The task of implementing the end-state in which both Yves and Zack get a payoff of 5 is more difficult.

122. *See* MOULIN, *supra* note 11, at 79.

123. Alternatively, Xerxes and Yves could collude against Zack.

would always take Zack's straw if Zack had a shorter straw. In return, Zack would compensate Xerxes for the extra expense. The result of the collusion is to shift more of the damages to Yves.[124]

In addition to the implementation problem, Moulin and Young use economic methods to analyze particular processes.[125] They choose candidate fair division processes and analyze their efficiency and end-state justice attributes. The fairness of a particular process is assessed by factors like: do players have an individual or group incentive to lie;[126] are players coerced;[127] are players ignored; and can one player dictate the outcome.[128] In some examples an apparently fair process may be inefficient[129] or lead to a patently unfair end-state.[130] In other examples, it is possible to achieve a degree of fairness in both process and outcome as well as efficiency. I will illustrate the combined analysis of process, outcome and efficiency with brief comments on Moulin's treatment of voting rules.[131]

Moulin compares two arguably fair voting schemes that might be used to resolve a social choice problem. The social

124. Yves would have to pay 16(2/3) more because of the collusion. Xerxes and Zack could agree to any split of the cost savings.

125. *See* YOUNG, *supra* note 12, at 42-63 (legislative apportionment), 156-61 (assignment of students to dormitory rooms); MOULIN, *supra* note 11, at 205-13 (divide and choose and auctions).

126. *See* MOULIN, *supra* note 11, at 40, 195.

127. As I explained in Part III when participation in the joint project is voluntary the end-state must be chosen so that no coalition of players has an incentive to defect from the grand coalition. In other words, the end-state must be in the core. Players must be coerced to accept an allocation outside of the core. *See supra* notes 92-93.

128. *See* MOULIN, *supra* note 24, at 7-8.

129. *See infra* text accompanying notes 124-25.

130. *See* MOULIN, *supra* note 24, at 7 (Fair procedures can produce unfair outcomes. In some cases the voluntary provision of a public good results in all costs of provision falling to one party. Also "in the celebrated 'gloves market' with 101 owners of a right glove and 100 owners of a left glove the unique core allocation gives all the surplus to the left glove owners.").

131. Young has an extensive treatment of a market-like process. *See* YOUNG, *supra* note 12, at 151-61. He argues that when there are different kinds of items to allocate and people have diverse preferences over those items then it is important to try to achieve an efficient allocation, and that the market is often an attractive method of achieving fair and efficient allocations. *See id.* at 161. ("Competitive markets allocate property both efficiently and equitably *provided* the goods were equitably allocated to begin with.") *Id.* Assuming rationality, people will trade from their initial endowments in a way that makes everyone better off. We see market-like forces in action when children trade food from their school lunch.

choice concerns the provision of a public good. In one of Moulin's examples he supposes that a group of five neighbors who live in adjoining condominiums decide to share the joint cost of hiring a gardener to care for a garden.[132] The neighbors are assumed to combine equal cost sharing, a notion of end-state justice, with either majority or unanimity voting as a fair process for eliciting preference information. Although unanimity voting respects a voluntary participation constraint, majority voting is more efficient.[133]

Moulin illustrates the problem by supposing that the garden gives the neighbors the following benefits in dollars: $800, $600, $450, $350, and $300 for a total value of $2500. The cost of the gardener is $2000, so the project is efficient if the benefits are commensurable and weighted equally. Equal cost sharing of $400 implies that two of the neighbors will suffer a loss. The project will not be undertaken given a unanimous voting rule. Two of the neighbors will object to the project because they would have to pay more than they get. In contrast, the project will be approved using a majority voting rule with equal cost sharing. The vote would be three in favor and two opposed. So majority voting leads to an efficient outcome while unanimity does not.

Majority voting does not always yield efficiency. If the resident holding the value of $450 is assigned a lower value of $350, then an inefficient result occurs. The total value of a gardener is now $2400 and the garden's value still exceeds its cost, but a majority of residents now oppose the plan. A possible remedy for this inefficiency is to change the cost sharing rule. In the original problem, if costs are shared proportionally to benefits then the costs are allocated: (640, 480, 360, 280, 240). The neighbors will now unanimously vote for to hire the gardener, if and only if it is efficient to do so. Of course, the allure of the proportional rule fades once we recall that the residents have an incentive to understate their benefit to reduce their cost share.[134]

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132. *See* MOULIN, *supra* note 11, at 23-24.

133. *See id.*, at 325. A different approach studied in the same chapter is voluntary contribution to the production of the public good. Like the two voting procedures, voluntary contribution usually leads to inefficiently low levels of production, but it is not systematically lower than the level under the voting procedures. *Id.* at 339-49.

134. With equal cost sharing no one has an incentive to lie. *Cf.* YOUNG,

CONCLUSION

I conclude with brief remarks on a bothersome question: If these books are so valuable for legal scholarship, why aren't they read and cited more often by legal scholars – after all, they were written four and five years ago? There are two plausible answers to that question, but only one that I can accept. First, the methods of cooperative microeconomics are too hard to apply and the results obtained from these methods are too weak to be interesting. Second, this subfield of microeconomic theory is relatively obscure and no one has noticed its potential usefulness in law and economics. Of course, I favor the second answer.

The purpose of this review was to introduce the methods of cooperative microeconomics to an audience of legal scholars. The task was fairly easy because so many of the canonical problems studied by Moulin and Young are directly relevant to legal policy. The examples I have presented in this Review are certainly accessible to noneconomists and at the same time they offer rich insights into problems of fair division. With a nascent interest in fairness appearing in law and economics the time is ripe to apply cooperative game theory and other methods from cooperative microeconomics to the study of legal policy.

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supra note 12, at 23 (proportional rule might create bad incentives).