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Arthur R. Weeks
University of Central Florida

Lloyd J. Sartor
University of Central Florida

Samuel S. Richie
University of Central Florida

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Arthur R. Weeks
University of Central Florida
School of Electrical and Computer Science
P.O. Box 25000
Orlando, Florida 32816

Lloyd J. Sartor
Raytheon Company
3801 University Drive
McKinney, Texas 75071

Samuel S. Richie
University of Central Florida
School of Electrical and Computer Science
P.O. Box 25000
Orlando, Florida 32816

Abstract. *The morphological binary hit-miss operator has been used extensively to locate features within a binary image. We propose a grayscale hit-miss operator that detects signal shapes and is applicable to scalar-valued functions on one, two, or more dimensions. The hit and miss structuring elements define the lower and upper bounds of the signal: If a signal lies between the hit and miss templates, then the hit-miss operator will produce a one output; otherwise, it will respond with zero. We incorporate a fuzzy logic element to the hit-miss operator to indicate how strongly the signal matches the hit-miss templates.*
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1 Introduction

Morphological techniques for image processing are a powerful tool for feature extraction, edge detection, noise reduction, and compression.[1,2] The term ''morphological'' indicates that some aspect of shape in the image is being operated on. Although based on geometrical concepts, morphological operators can be defined in purely algebraic and set theoretic terms. Binary morphology was originally developed, primarily by Matheron[3] and Serra[4], as an experimental technique for texture analysis. The hit-miss transformation has played a central role in mathematical morphology since that early work.[5] Applications include iron ore analysis, fingerprint recognition, parts inspection, and traffic sign recognition.[5–7] These initial binary operators, such as dilation and erosion, have now been applied to scale images.[1,3,5]

Uncertainty in binary images can result from noise, object orientation, or object scaling (e.g., due to distance from the camera). To compensate for this uncertainty, fuzzy logic techniques have been incorporated into binary morphological operators, including the hit-miss transformation.[8–12]

We propose a grayscale hit-miss transformation. To deal with the uncertainty in grayscale images, the grayscale hit-miss transformation has a fuzzy logic component that indicates the degree of fit between the signal and hit-miss templates.

2 Binary Hit-Miss Transformation

The binary hit-miss transformation locates features or objects within a binary image. It does so by simultaneously identifying a pattern of ''on'' pixels and a pattern of ''off'' pixels. The pattern of ''on'' pixels is represented by the hit structuring element or hit template, and the pattern of ''off'' pixels is represented by the miss structuring element or miss template. The hit-miss transformation is based on the binary erosion operator. Binary erosion can be described as the set of points z at which B_z (the structuring element B translated to location z) completely fits within the binary image:

$$X \ominus B = \cup \{z : X \cap B_z = B_z\}, \tag{1}$$

where X is the binary image, B is the structuring element, and B_z is the translate of B by z. (The translate of set B by point z is $\{b+z | b \in B\}$.) The binary hit-miss transformation is defined as

$$\begin{aligned} X \circledast (C,D) &= (X \ominus C) \cap (X^c \ominus D) \\ &= \{z \in R^2 | C_z \subset X \quad \text{and} \quad D_z \subset X^C\}. \end{aligned} \tag{2}$$

When C_z is entirely contained within X, the hit pattern has been found at location z. When D_z is entirely contained within X^C, the miss pattern has been found at location z. When both the hit and miss patterns are found at location z,

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then the desired feature is detected and the hit-miss operator has a true response at location z. When either the hit or miss pattern match fails at a location, the hit-miss operator produces a false or negative response. For example, the following sets will search for horizontal rising edges (with respect to the downward direction) at least five pixels long in the image X, and the output image will have a one wherever such an edge was found:

$$C=\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ & & \cdot & & \\ 1 & 1 & 1 & 1 & 1 \end{matrix} \qquad D=\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ & & \cdot & & \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

(The dotted element is the center of the mask.) The sets C and D must not intersect, because a pixel may not be on and off simultaneously.

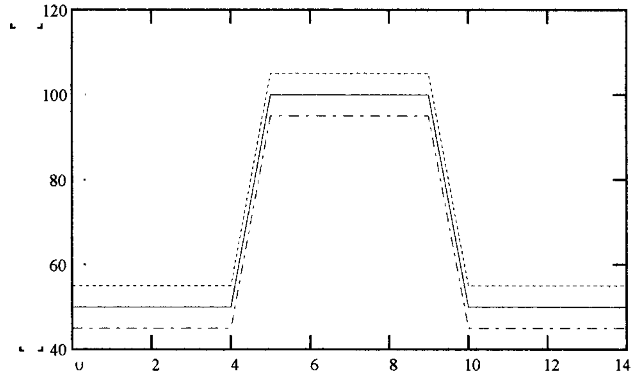


Fig. 1 A signal and a pair of hit-miss templates.

3 Grayscale Hit-Miss Transformation

The extension from binary to grayscale morphology adds a dimension: A pixel value is no longer simply on or off, but takes a scalar value (either on a continuous or discrete range). The proposed grayscale hit-miss transformation is based on grayscale erosion, which is defined by[1]

$$[f\ominus g](x)=\min_{\substack{z\in D\\ z-x\in G}}\{f(z)-g(z-x)\}, \tag{3}$$

where f is the signal, g is the structuring function, D is the domain of f, and G is the structuring function domain [i.e., points at which $g(x)>-\infty$]. The domains D and G and coordinates x and z can be of arbitrary dimension; for images, they are 2-D. Notice that the erosion value is nonnegative if the structuring function lies entirely below the signal: $f(z)\geqslant g(z-x)$. The following equation produces a true response when the structuring function is below the signal:

$$u[f\ominus g](x)=\begin{cases}1 & \text{if } f(z)\geqslant g(z-x)\\ 0 & \text{otherwise}\end{cases}, \tag{4}$$

where $u[x]$ is the unit step function. Likewise, the following indicates if the structuring function $h(x)$ lies below the complement of the signal:

$$u[f^c\ominus h](x)=\begin{cases}1 & \text{if } h(z-x)\leqslant f^c(z)\\ 0 & \text{otherwise}\end{cases}. \tag{5}$$

The complement can be the negative $-f$ when the signal range is the real number line, or the value $M-f$ when the signal range is limited to $[0,M]$. By duality, if the structuring function $h(x)$ is below the signal complement $f^c(x)$, then the signal $f(x)$ is below the structuring function complement $h^c(x)$. That is, $h^c(z-x)-f(z)\geqslant 0$ is equivalent to $f^c(z)-h(z-x)\geqslant 0$.

When both erosions $(f\ominus g)$ and $(f^c\ominus h)$ are positive, then the signal is between $g(x)$ and $h^c(x)$. We define the grayscale hit-miss transformation as an operator that identifies regions of a signal that fit between two templates:

$$\begin{aligned} f\circledast(g,h)&=r\cdot u[(f\ominus g)]\cdot u[(f^c\ominus h)]\\ &=r\cdot\{z|f(x)\geqslant g(z-x)\quad\text{and}\\ &\qquad f(x)\leqslant h^c(z-x)\forall x\}, \end{aligned} \tag{6}$$

where f is the signal or image, g is the hit template, h is the miss template, and r is a function that indicates the strength of the match between the signal and templates. g and h can also be called the "below" and "above" templates, since they are the bounding functions for the signal f. r is a fuzzy membership function that takes a value in the range [0,1]. The condition $g(x)\leqslant h^c(x)$ must hold; if $g(x_0)>h^c(x_0)$ for some x_0, the hit-miss operator would always be zero because the signal could never be simultaneously above $g(x_0)$ and below $h^c(x_0)$. This is analogous to the binary hit-miss operator, where it is required that the intersection of the hit and miss templates be null.

The following example illustrates basic operation of the grayscale hit-miss transformation in 1-D. Let r be unity so that the hit-miss transformation output is binary: any signal fitting between the templates yields a true response. The signal $f(x)$ is a pulse of width 5, the signal's background level is 50, and the pulse level is 100. The below template $g(x)$ is also a 5-wide pulse with a background level of 45 and a pulse level of 95. The complement template $h^c(x)$ is a 5-wide pulse with background level 55 and pulse level 105. When the templates are centered on the signal pulse, the signal fits between the templates, as shown in Fig. 1. Since $f(x)-g(x)\geqslant 0$ and $h^c(x)-f(x)\geqslant 0$, the hit-miss operator has unity response at this location.

Figure 2 shows situations using the same templates where the hit-miss operator has zero response. In Fig. 2(a), the templates are not centered on the signal pulse. In Fig. 2(b), the signal pulse is too narrow, therefore the hit template does not fit below the signal. Likewise, in Fig. 2(c), the signal pulse is too wide and the signal does not fit below the miss template. In Fig. 2(d), the pulse level is too high, so that it does not fit below the miss template. And in Fig. 2(e), noise causes the signal to cross the hit and miss template boundaries.

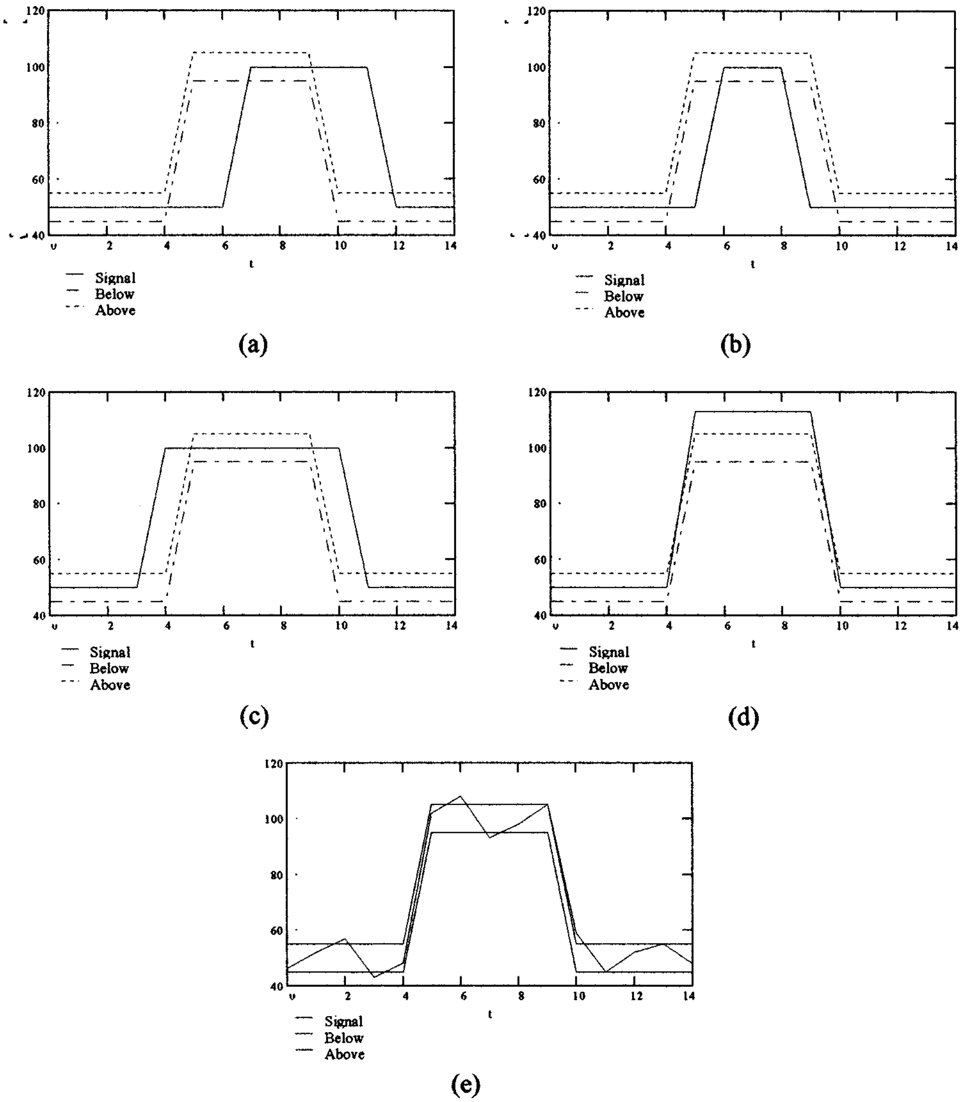


Fig. 2 Examples of conditions that produce zero response from the hit-miss transformation.

To accommodate variations in the signal due to noise or other random processes, the classification margin in the grayscale hit-miss transformation can be increased by increasing the separation between the hit and miss templates. The templates in Fig. 3 are used to detect a signal pulse. The separations in the templates allow for the background signal level to vary between 45 and 55, and the pulse level to vary between 95 and 105. Additionally, because of the horizontal separation of the edges of the templates, they will detect pulses with widths varying between 3 and 7 units.

Separating the hit and miss templates allows for variations in the signal while still requiring it to match some basic shape. It may be desired to detect not only if the signal fits between the hit and miss templates, but also how well the signal fits the desired shape. To achieve this, a "response strength" term has been added to the grayscale hit-miss operator. The response strength is a fuzzy membership value (in the range [0,1]) that indicates to what degree the detected signal matches the ideal desired signal. There is not a single fuzzy membership function for the response strength—its definition may vary depending on the application. The following example illustrates a fuzzy response strength function.

Given the hit and miss templates $g(x)$ and $h(x)$, let the ideal signal be exactly midway between the templates. The

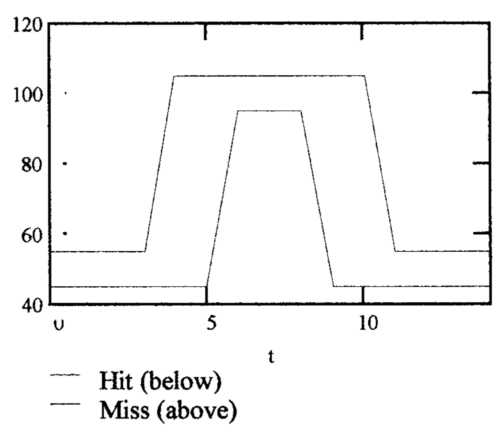


Fig. 3 Hit-miss templates with vertical and horizontal margin.

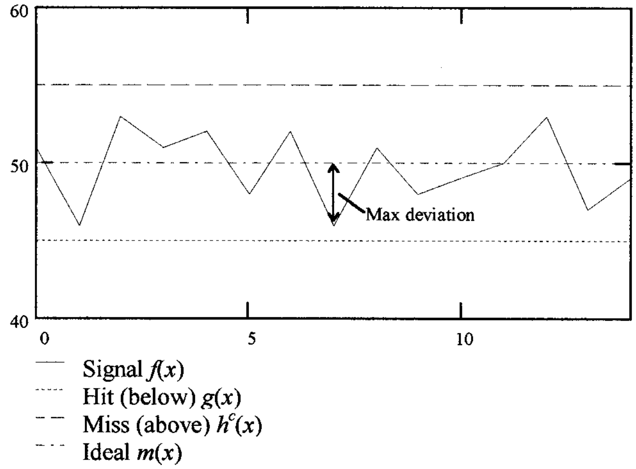


Fig. 4 Fuzzy response of grayscale hit-miss transformation.

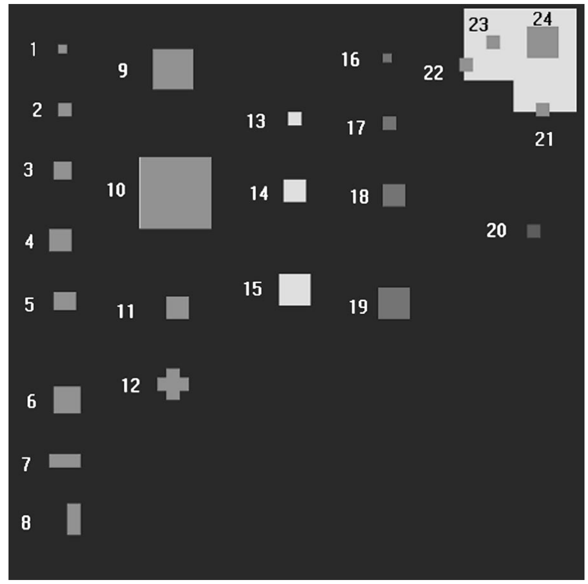


Fig. 5 Boxes test image.

hit-miss operator responds to the ideal signal with a strength of 1. As the signal deviates from the midway point, the strength decreases. There are numerous ways in which the signal can deviate from the ideal: the signal may be offset vertically, may deviate in horizontal dimensions, or may fluctuate due to noise. In this example, the response strength is determined by the maximum deviation from the ideal signal, normalized to a value in the range [0,1]:

$$r(t)=\min\left\{1-2\cdot\left|\frac{f(x)-m(x)}{h^c(x)-g(x)}\right|\right\}$$
$$m(x)=\frac{h^c(x)+g(x)}{2}, \qquad (7)$$

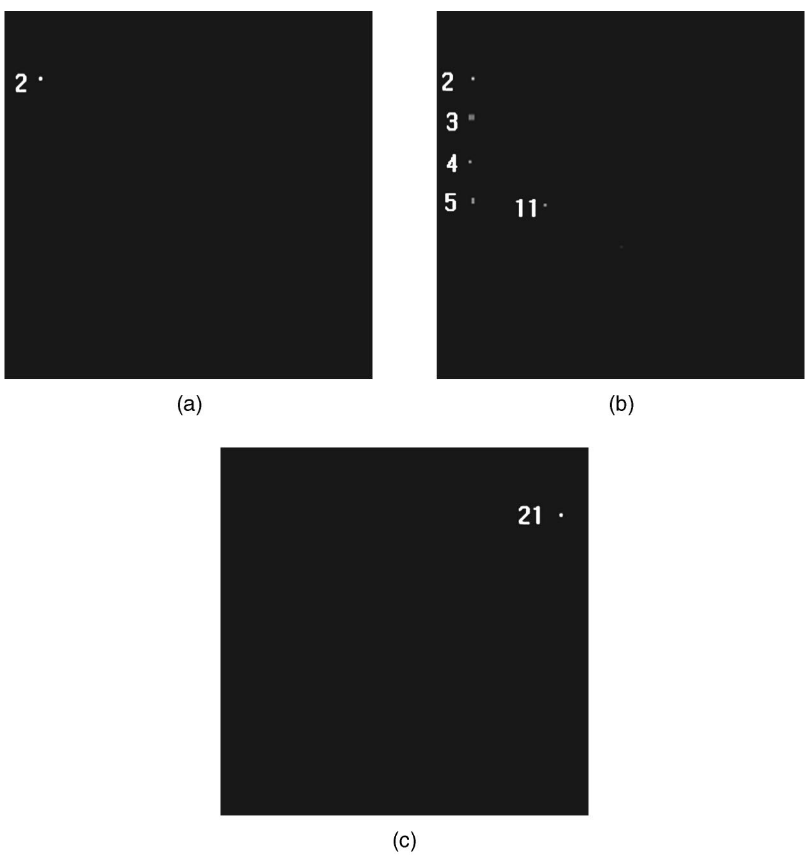
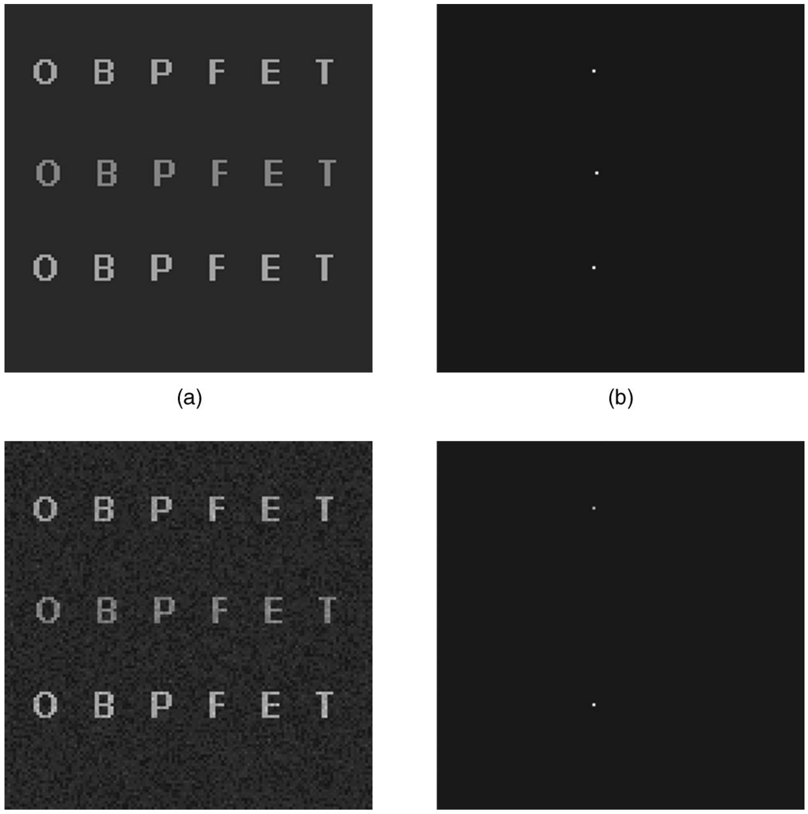


Fig. 6 Results of applying the hit-miss transformation to the image in Fig. 5 for various hit-miss templates.



(a) (b) (c) (d)

Fig. 7 Character recognition of (a) a noiseless image and (c) a noisy image using the grayscale hit-miss transformation.

where $f(x)$ is the signal, $g(x)$ is the hit (below) template, $h^c(x)$ is the complement of the miss (above) template, and $m(x)$ is the ideal signal midway between $g(x)$ and $h^c(x)$. Figure 4 shows a noisy signal along with hit and miss templates set to detect a constant signal at 50 with a noise tolerance of $\pm 10\%$. The ideal signal, with a constant value of 50, is midway between the hit and miss templates. The signal sample shown does not cross the hit or miss boundaries, so the hit-miss operator will have a positive response (i.e., both erosions are nonnegative). The maximum deviation is 80% of the distance between the ideal signal and the hit template. Therefore, the response strength at this location is 0.2.

If the signal exceeds the boundaries set by the hit and miss templates, then the response strength in Eq. (7) may exceed 1, which is not a valid fuzzy membership value. However, the unit step response of the hit erosion or the miss erosion (or both) will be zero, yielding zero response for the hit-miss operator. Therefore, the grayscale hit-miss operator always has a value in the range [0,1]. Even though these examples were presented using a 1-D function, the concepts of the grayscale hit-miss operator are easily extended for a 2-D image by changing scalar variables of x and z in Eq. (6) to 2-D vectors of $\mathbf{x}$ and $\mathbf{z}$.

4 Results

To demonstrate, the grayscale hit-miss transformation is applied to two test images. The first test image is a series of boxes of various intensities and sizes on a uniform background (Fig. 5). A hit-miss template pair is used that searches for a 3×3 square with an intensity in the range [112,144] and background intensity range [0,46]. The "ideal" signal therefore has intensity 128 and background intensity 23. Figure 6(a) shows the result: box 2 responds 100% because it matches the ideal signal exactly. No other feature in the image fits between the hit-miss templates, so there are no other positive responses. Another hit-miss template is then used. This one is similar to the first, but with more separation between the templates, both vertically and horizontally. This increased separation allows a wider range of signals to fit between the templates. Figure 6(b) shows that features 2, 3, 4, 5, and 11 all have positive responses. Because none of these features match the ideal signal, all responses are less than 1. A third pair of hit-miss templates searches for a more complex signal: a gray box that lies on a horizontal boundary between black and white. Figure 6(c) shows that feature 21 was detected.

A second test image consists of a group of alphabetic characters. The hit-miss operator was then used to locate the character P. The hit-miss operator was first applied to a noise-free image with characters of differing intensities [Fig. 7(a)]. The hit-miss templates had enough spacing between them to locate all three P's but with differing strengths [Fig. 7(b)]. The darkest letter P and the brightest letter P yielded the lower strengths. The test image was then corrupted with Gaussian noise [Fig. 7(c)]. The hit-miss operator located two of the P's but was unable to locate the darkest P's because the noise there exceeded the boundaries set by the hit-miss templates [Fig. 7(d)].

5 Conclusions

The proposed grayscale hit-miss operator is effective in detecting features in scalar-valued images. The separation between the hit and miss templates determines the amount the actual feature is allowed to deviate from an ideal feature. The grayscale hit-miss operator can produce a binary output, indicating locations where the feature was detected. More generally, the hit-miss operator can produce a fuzzy output that indicates the degree of fit between the actual and ideal features.

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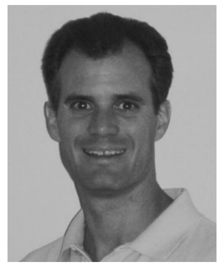
Arthur R. Weeks received his PhD degree in electrical engineering from the University of Central Florida in Orlando in 1987. After completion of his PhD, he spent one year at the Royal Signals and Radar Establishment in Malvern, England, studying laser beam propagation. He recently left his vice president of corporate technology position at Invivo Research Inc. to join the Electrical Engineering Department at the University of Central Florida as a visiting associate professor. His interests include color image processing techniques, the reduction of noise within images using adaptive nonlinear filters, and the use of artificial neural networks in pattern recognition. He is a member of the IEEE, SPIE, and Tau Beta Pi.



Lloyd J. Sartor received his BSEE and MSEE degrees from Southern Methodist University in 1986 and 1988, respectively. He finished his PhD in electrical engineering at the University of Central Florida in 2000 while working at Siemens Telecom Networks, Lake Mary, Florida. He is presently at the Raytheon Company in McKinney, Texas, working in the radar group. His current research interests are color image processing and color morphological image processing.

Samuel S. Richie received the BSE, MSE, and PhD degrees in electrical engineering from the University of Central Florida in 1980, 1983, and 1989, respectively. He is currently an associate professor in the School of Electrical Engineering and Computer Science at the University of Central Florida, and serves as the assistant program director for the electrical engineering program and undergraduate coordinator for the electrical and computer engineering programs. While at the university, he has worked on computer-aided design and analysis of both unidirectional and bidirectional surface acoustic wave (SAW) technology. He is a member of IEEE, Eta Kappa Nu, and Tau Beta Pi.