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Two-mode beat phase noise of actively modelocked lasers

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Abstract: An analytic expression for the phase noise spectrum is estimated when two arbitrary longitudinal modes are selected for beating from the output of an actively modelocked laser. A separate experiment confirmed the theory qualitatively. It was found that two-mode beating possesses more phase noise than the beating involving the entire mode spectrum, especially at low offset frequency, even though two mode beating noise is decoupled from the RF oscillator noise to the first order.

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1. Introduction

With recent advances in ultra low noise laser technology, the new research area of microwave photonics is emerging. Photonic radios [1], photonics digital to analog converters [2], and optical clock distribution [3] are a few examples. One of the key elements of the above applications is the capability of generating a low noise microwave signal by beating two highly coherent optical modes. These two optical modes may be obtained from two synchronized lasers [4] or from a single modelocked laser [5]. The latter is especially intriguing because of the extremely low noise nature of modelocked lasers and their capability of generating many modes with a well defined phase relationship. It has been commonly believed that there must be a strong correlation between the phase noise of two-mode beating and that of full mode spectrum from a modelocked laser. In this paper, an analytic expression for the phase noise spectrum of two-mode beating is estimated for actively modelocked lasers and compared to an experimental measurement. The experiment confirms the theory qualitatively. It was found that the noise characteristics of two-mode beating are very different from the phase noise of the entire mode spectrum and typically much noisier at low offset frequency.

2. Theory

In order to calculate the phase noise spectrum from the two-mode beating, we used the same model that has been used to derive the noise spectrum of an actively modelocked laser [6]. The laser cavity is composed of a gain medium and a loss modulator. Gain dynamics such as saturation and recovery were assumed to be negligible. This assumption is not quite realistic for semiconductor lasers but we believe this approach is still relevant to study the general idea of two-mode beat noise. The model includes three noise sources, namely, spontaneous emission, RF driving oscillator phase noise and cavity length fluctuations. First, coupled mode equations for active modelocking were derived in the frequency domain. Then, by treating the noise contribution as a perturbation, the equation for noise was linearized. The perturbation is done by defining the electric field modulated by the phase noise,

$$E_n = E_n^{(0)} \cdot e^{i\theta_n} \quad , \theta_n \ll 1 \quad (1)$$

where $E_n^{(0)}$ is a steady state solution for mode n without any noise and θ_n is optical phase noise. This phase noise can be expanded using Hermite polynomials,

$$\theta_n(t) \approx \theta(t, x) = \sum_{p=0}^{\infty} A_p(t) \cdot H_p(x) \quad (2)$$

Using this electric field, the intensity of the optical pulse train is given by

$$I(t) = I_o \sum_{k,l} \exp[-(k^2 + l^2)/N^2] \cdot \exp \left[-il\omega_m \cdot \left(t + \frac{1}{\omega_m l} (\theta_n - \theta_m) \right) \right] \quad (3)$$

where N is a parameter proportional to the number of modes, ω_m is the modelocker modulation frequency, k and l are related to indices of longitudinal mode n , m by $k = n + m$ and $l = n - m$. This can be summed over k yielding,

$$I(t) = I_o \sqrt{2\pi} N \sum_l \exp[-l^2/N^2] \cdot \exp[-il\omega_m \cdot \{t + J(t)\}] \quad (4)$$

where the timing jitter $J(t)$ is given by

$$J(t) = \frac{\sqrt{2}}{\omega_m N} \sum_{p=0}^{\infty} A_p(t) \left\langle \frac{\partial}{\partial z} H_p(z) \right\rangle \quad (5)$$

and

$$\left\langle \frac{\partial}{\partial z} H_p(z) \right\rangle = \sum_k [\exp(-k^2/2N^2) \cdot \frac{\partial}{\partial z} H_p(z)] / \sum_k \exp(-k^2/2N^2) \quad (6)$$

with $z = k/\sqrt{2}N$.

It is worth mentioning the interpretation of terms with different index p . From the fact that $H_0(x)=1$, $H_1(x)=2x$, $H_2(x)=4x^2-2$, $H_3(x)=8x^3-12x$ etc., each H_p will add to the pulses a uniform spectral phase, temporal delay, linear chirp, cubic spectral phase etc., respectively. Since the timing jitter is related to the random addition of temporal delay, it is obvious that $p=1$ term contributes most to the jitter, which is also consistent with Eq. (6). On the other hand, for the two-mode case, it is not only the $p=1$ term which adds delay but, in addition, all other p terms. Therefore it is natural to expect an increase of phase noise for the two-mode beating case as compared to full mode spectrum. Theoretically, we can calculate the

contribution from all the other additional terms for a given noise source characteristic and derive an analytic expression for the two-mode case. Alternatively a more useful approach is to estimate the range of noise for the two-mode beating using a simpler expression. Another justification for this attempt is that the current model of the modelocked laser is oversimplified, so that even if a rigorous result is derived from the same frame work, its usefulness is mitigated. The two-mode beating formalism can be examined by dropping the summation over k and l from Eq. (3). For example, if $l=1$, then

$$I(t) = I_o \exp[-(k^2 + 1)/N^2] \cdot \exp\left[-i\omega_m \cdot \left(t + \frac{1}{\omega_m}(\theta_n - \theta_{n-1})\right)\right] \quad (7)$$

The timing jitter $J(t)$ is then given by

$$J(t) = \frac{1}{\omega_m} \sum_{p=0}^{\infty} A_p(t) (H_p(n) - H_p(n-1)) \quad (8)$$

For two-mode beating, the maximum timing jitter can be expected when the two modes are completely uncorrelated as if there is no modelocking. In that case, the linewidth of the mode is a major element in determining the noise. It has been noted that the linewidth of each mode is determined by the $p = 0$ term [7]. In the current study, the $p = 0$ term changes the phase of the modes uniformly so that it doesn't affect the noise of the beating term. Here we make an assumption. In order to simulate two completely uncorrelated modes, we treat one mode as perfectly monochromatic, i.e., $\theta_{n-1} = 0$, while keeping only the $p = 0$ term for the other mode.

The maximum timing jitter will then be

$$J(t) = \frac{1}{\omega_m} A_0(t) \quad (9)$$

Therefore Eq. (5) and Eq. (9) will provide the lower and upper limit of two-mode beat noise, respectively. If we now define a constant C between 0 and 1, then the two-mode beat noise can be given by

$$J(t) = \left[\frac{1}{\omega_m} A_0(t) \right] \cdot C + \left[\frac{\sqrt{2}}{\omega_m N} \sum_{p=0}^{\infty} A_p(t) \left\langle \frac{\partial}{\partial z} H_p(z) \right\rangle \right] \cdot (1 - C) \quad (10)$$

where C is 1 for maximum noise and 0 for minimum noise. This C will be determined later when the above result is compared with the experiment.

From Eq. (5), single sideband phase noise spectrum $L(f)$ at carrier frequency $l \times \omega_m$ has been shown to be [6],

$$L(f) = \frac{\sqrt{2}\Delta\omega_{st} \cdot l^2}{N(\gamma/N^2)^2} \frac{(\gamma/N^2)^2}{4\pi^2 f^2 + (\gamma/N^2)^2} \left\{ 1 + \frac{N^2}{2} \left(\frac{\Delta\omega}{\gamma/N^2} \right)^2 \frac{(2\gamma/N^2)^2}{4\pi^2 f^2 + (2\gamma/N^2)^2} + \frac{\gamma/N^2}{(2\sqrt{2}/N) \left(\frac{\Delta\omega_{st}}{\gamma/N^2} \right)} S_{\phi}(f) \right\} \quad (11)$$

where $\Delta\omega_{st}$ is the Schawlow-Towne's linewidth, $\Delta\omega$ is RF modulation frequency detuning from the cavity fundamental frequency, γ is modulation depth of the loss modulator normalized to the modulation frequency and $S_{\phi}(f)$ is phase noise spectrum of the RF driving oscillator.

For the upper limit of two-mode beating, using Eq. (9) and repeating the same procedure, the $L(f)$ was derived to be

$$L(f) = \frac{\sqrt{2}\Delta\omega_{st}N(\gamma/N^2)^2}{(\gamma/N^2)^2} \frac{1}{4\pi^2 f^2} \left\{ 1 + \frac{N^2\Delta\omega^2}{4\pi^2 f^2 + (\gamma/N^2)^2} \right\} \quad (12)$$

This expression shows that the RF source noise $S_\phi(f)$ is not coupled to the upper limit of two-mode beat noise leaving only the spontaneous emission contribution. It also should be noted that the upper limit of two-mode beat noise has a $1/f^2$ as a more dominant term than the Lorentzian shape, which means a strong increase of the noise at low offset frequencies. Another important feature is the l^2 dependence of the lower limit of two-mode noise. This l^2 dependence means that the phase noise increases as the spacing between two-mode increases. One may find this fact is consistent with D. von der Linde's statement about phase noise dependence on harmonic number [8].

3. Experiment and results

In order to validate the above expression with respect to two-mode beat noise, the phase noise was measured for two cases for the same actively modelocked semiconductor laser at 4.283 GHz. First, the output pulse train of the modelocked laser was directed to a photodetector and phase noise of the photo current was measured. Second, the laser output was sent to an optical bandpass filter with 8GHz of FWHM (Little Optics, Inc.) and only two modes were filtered. Then these two modes were detected by the same photodetector and the phase noise was measured. Figure 1 shows the optical spectrum of the two cases. The measured phase noises are shown in Fig. 2 along with the theoretical calculations.

For the RF driving oscillator, the following functional form for $S_\phi(f)$ was used.

$$S_\phi(f) = S_o \left(1 + \left(\frac{f_0}{f} \right)^\beta \right) \frac{f_1^2}{f^2 + f_1^2} \quad (13)$$

All the parameters used in the calculation are shown in Table 1.

Table 1. Parameters used in the noise calculation.

S_o	5×10^{-12}	[1/Hz]
f_0	5	[Hz]
f_1	3×10^2	[Hz]
β	2	
N	80	
ω_{st}	1	[Hz]
$\Delta\omega$	1×10^3	[Hz]
γ	8×10^9	[Hz]

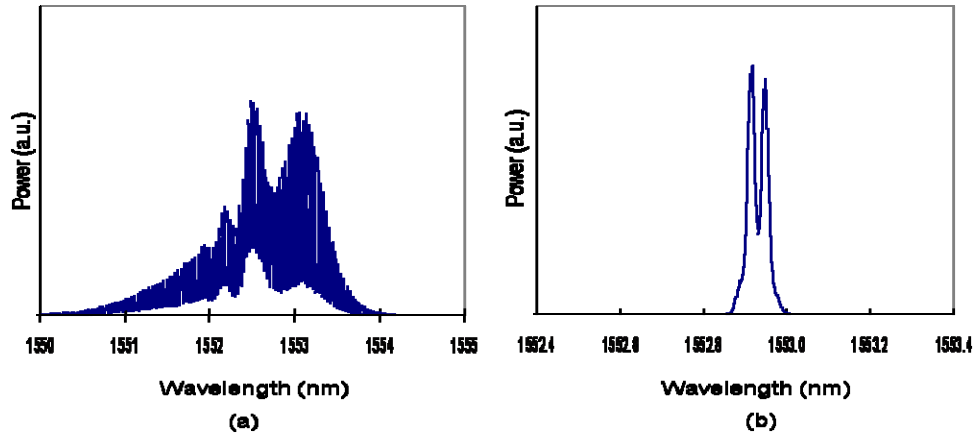


Fig. 1. Optical intensity spectra for the entire modes (a), two selected modes (b).

Figure 2 shows very good agreement between theory and experiment for full mode case. The deviation from the theory arises at the offset frequency higher than 1 MHz, where the measured noise hits the noise floor of our measurement system. At low offset frequency, the noise is mainly caused by the RF driving oscillator phase noise. On the other hand, for the two-mode case, the measured noise fits well when C is 5×10^{-5} showing almost monotone $1/f^2$ behavior at low offset frequency. Again the two-mode noise spectrum meets the measurement noise floor around 10 KHz. (This noise floor is higher than full-mode spectrum case owing to its lower carrier power.). The significance of this work is the fact that for many microwave photonics applications where two-mode beat phase noise is critical, its noise characteristics are very different from the phase noise of the entire mode spectrum and typically much noisier at low offset frequency. Therefore extra caution should be taken when estimating the two-mode beat phase noise from the full-mode phase noise of the modelocked laser.

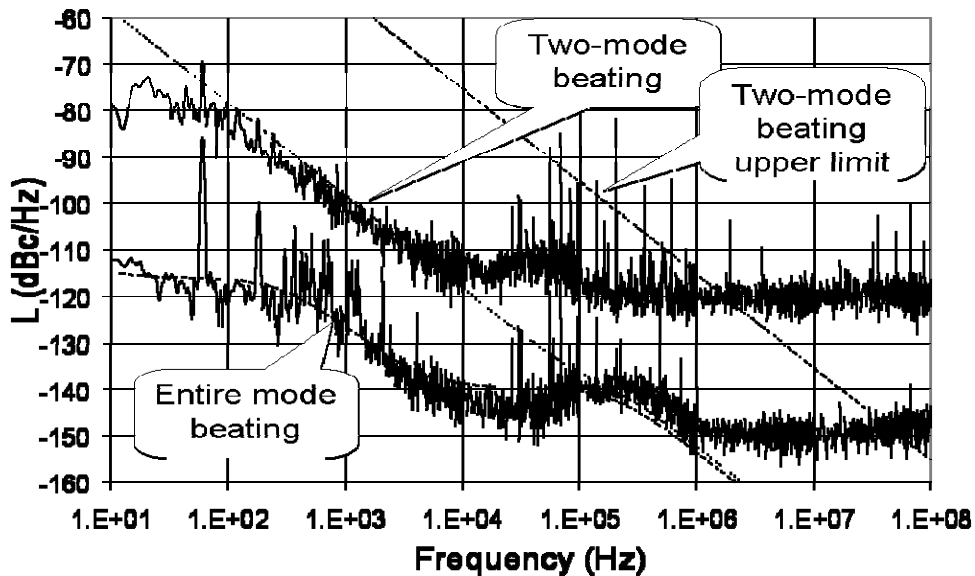


Fig. 2. Single sideband phase noise measurement for both full-mode and two-mode beating cases. (Dotted lines are the theory for lower limit, upper limit and fitted two-mode beat noise with $C = 5 \times 10^{-5}$.)

4. Conclusion

An analytic expression for two-mode beat phase noise spectrum has been estimated using Hjelme's model for the noise of actively modelocked lasers. Two-mode beating suffers more phase noise than the full-mode beating case especially at low offset frequency because of its $1/f^2$ frequency dependence. Separate experiment support the theory qualitatively.

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