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Simple physics of quadratic spatial solitons

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Abstract: Spatial solitons in quadratically nonlinear media result from the interplay of parametric gain, diffraction and cascading phase shift. Their main features are well understood in mathematical terms, and several experiments have been successfully carried out which demonstrate their observability and most important properties. Here we provide an intuitive interpretation of some of the underlying physics, outlining the processes that govern their excitation, propagation and interaction forces.

OCIS codes: (190.5940) Self-action effects; (190.4410) Nonlinear optics, parametric processes; (190.4180) Multiphoton processes

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Solitons exhibit universal properties that do not depend on the underlying physical mechanisms responsible for their existence. This has been amply demonstrated for spatial solitons observed to occur in Kerr, saturating Kerr, liquid crystalline, photorefractive and quadratically nonlinear media.[1-4] Of these, there are simple intuitive approaches to understanding the physical mechanisms leading to spatial solitons, their properties and their interactions in Kerr and saturating Kerr media, in liquid crystals and in photorefractive crystals.[5-6] A beam of light is known to diffract when propagating without any constraints on its transverse dimensions. It diffracts in one dimension if confined in the other transverse coordinate by a planar waveguide, or in two dimensions when no guidance is provided. It does not diffract when a bi-dimensional guiding geometry counteracts its spreading, as it is the case in optical fibers or channel waveguides. Diffraction can also be arrested by a nonlinear interaction with the material in which the beam propagates, for example, when an intensity-dependent response operates as a focusing mechanism. A nonlinear medium able to counteract diffraction in one dimension is said to support 1D bright spatial solitons, i. e., stable nonlinear eigen-modes of the system. 1D spatial solitons exist in planar waveguides, and are known to be stable in self-focusing materials. Similarly, when diffraction is balanced in both transverse dimensions, the resulting beams are referred to as 2D bright spatial solitons. In both cases, these eigen-solutions have a spatial beam profile that is invariant upon propagation. 2D spatial solitons, however, are known to be unstable in ideal Kerr media, i. e. those described by an intensity-dependent linear increase in refractive index: $n=n_0+n_2I$, with n_2 a constant and I the local intensity.[6] Conversely, they tend to be stable and robust in media such as photorefractives, liquid crystals and $\chi^{(2)}$-active media. All of the afore-mentioned media exhibit a saturable nonlinear response, i. e. the variation in optical path-length upon propagation depends in a sub-linear way on the field intensity at each point in the transverse beam profile. [7] Although the change in path-length can be associated to a change in refractive index in most nonlinear media, such as Kerr, saturating Kerr, liquid crystals or photorefractives, it is linked to a nonlinear phase-shift in $\chi^{(2)}$-active media for multi-frequency solitons: there is no change in the refractive index for the quadratic solitons, because they consist of multiple frequency waves coupled together by the second-order nonlinearity $\chi^{(2)}$. Because of this complexity, they are invariably discussed in terms of the detailed numerical solutions to the pertinent nonlinear wave equations and not in terms of the underlying physics.[8-12] Fortunately, it is possible to explain in a simple way how self-focusing processes occur during parametric interactions. In fact, one can obtain a great deal of insight and construct quite simple pictures for some of the properties of these quadratic solitons and their interaction "forces" without relying on the detailed solutions. It is our goal in this Article to show this. However, it is important to note at the outset that there are properties such as stability, soliton fusion etc. which cannot be handled in a simple way and require detailed mathematical attention.

Let us consider the classical second harmonic generation (SHG) process in which only the fundamental wave is injected into the medium. The simplest case occurs for a 1D, Type I interaction between a single fundamental field (of envelope a_{FF}) and a single harmonic field (of envelope a_{SH}). Diffraction can occur in only one dimension (y), i. e. in the plane (y, z) of a

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slab waveguide, and the nonlinear wave evolution along z is described by the well-known equations for SHG:

$$2ik_{FF}\frac{\partial}{\partial z}a_{FF}(y)-\frac{\partial^2}{\partial y^2}a_{FF}(y)=2k_{FF}\Gamma\left[a_{SH}(y)a_{FF}^{*}(y)\right]e^{i\Delta kz} \quad (1)$$

$$2ik_{SH}\frac{\partial}{\partial z}a_{SH}(y)-\frac{\partial^2}{\partial y^2}a_{SH}(y)=2k_{SH}\Gamma\left[a_{FF}^{2}(y)\right]e^{-i\Delta kz} \quad (2)$$

with $\Gamma=\omega_{FF}\chi_{eff}^{(2)}\left(2n^3c^3\varepsilon_0\right)^{-1/2}$ the nonlinear coupling strength and $\chi_{eff}^{(2)}$ the "effective" nonlinearity (accounting for a specific crystal orientation, set of field polarizations and overlap integral). Here $\Delta k = 2k(\omega) - k(2\omega) = 2k_{FF} - k_{SH}$ is the wavevector mismatch, and near the phase-matching condition $n = n_{FF} \sim n_{SH}$. The other symbols have their usual definitions. When only the fundamental (FF) is input, in the plane-wave case (the second harmonic (SH) wave initially grows $\pi/2$ out of phase with the fundamental. This relative phase changes with propagation distance when $\Delta k \neq 0$ because, in the general case of a wavevector (and phase-velocity) mismatch, $v_\omega \neq v_{2\omega}$. Up-(FF→SH) and down-(SH→FF) conversion occur successively due to this velocity mismatch and the FF develops a nonlinear phase-shift through "cascading", as pictured in Figure 1. When $k_{SH} \neq 2k_{FF}$, the up-converted phase-fronts acquire a time lag/gap with respect to the un-converted FF and, once down-conversion takes place and the energy flows back into the FF, this results in a nonlinear phase-shift in the FF, with size and sign depending on intensity and mismatch, for a given nonlinearity. [13] In fact, energy flows continuously into and out of the fundamental and this process leads to a continuous accumulation of the nonlinear phase shift. Furthermore, as will be discussed later, this exchange also occurs in the steady state (soliton case) and leads to beam narrowing.

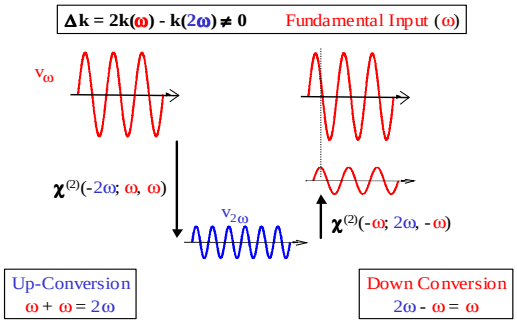

Fig. 1 Intuitive sketch on the origin of the "cascading" phase shift in Type I SHG with plane waves.

The situation becomes somewhat more complex when dealing with beams of finite widths in the diffraction coordinate. In fact, each beam tends to diffract in y but, simultaneously, up- and down-conversions give rise to a phase-front distortion via cascading. At the peak of the fundamental envelope the nonlinear phase shift is the largest, and it decreases to zero in the wings. This phase distortion leads to either self-focusing or self-defocusing of the FF beam.

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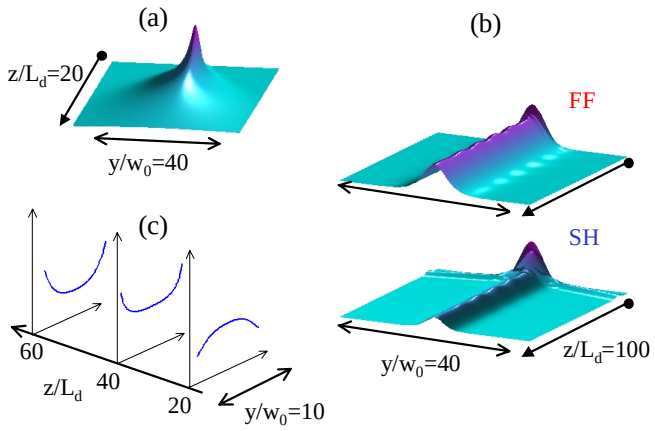

Fig. 2 Simulated evolution of the transverse intensity of (a) a linearly diffracting 1D beam and (b) a quadratic spatial soliton excited by a gaussian FF beam. (c) The FF phase-front evolution for the latter case. Propagation distances are in units of the diffraction length L_D, whereas the transverse coordinate is in units of the input waist w_0. Here $\Delta k \cdot L_D = 2$.

The formation of a stable spatial soliton can now be understood in terms of a focusing-like phase-front curvature due to cascading: such curvature can counteract diffraction in the transverse plane.[14-15] This is shown in Figure 2 above. Notice that, although a cascading phase shift can only originate in the presence of a phase mismatch between plane waves, this is not true when dealing with finite beams, due to their spatial distribution of wavevectors.

There is another approach to understanding this self-focusing process. Parametric gain refers to the process by which one of the beams, fundamental or harmonic, gains energy from the other via the nonlinear polarization on the right hand side of the coupled mode equations (1)-(2). For finite, bell-shaped beams, parametric gain takes place preferentially where the envelopes entering into the field product have the maximum amplitude, i. e. at the peaks. This tends to regenerate photons on axis, thereby counteracting the tendency to spreading. Consistently with the notion that SHG gives rise to a narrower beam at twice the pump frequency, we can simply describe the generation of a quadratic spatial soliton excited by an FF beam (of peak intensity I) as in Figure 3. The input FF ($\propto e^{-y^2/w_o^2}$), undergoing SHG, forms a narrower SH beam ($\propto e^{-2y^2/w_o^2}$). The two waves interact parametrically where both intensities are higher, in such a way that the down-converted photons at FF ($\propto e^{-3y^2/w_o^2}$) occur preferentially on axis leading also to compress the otherwise diffracting FF beam. When the diffraction ($L_D = \pi w_o^2 n/\lambda$) and parametric-gain ($\gamma$) distances ($L_{pg} = 1/\gamma\sqrt{I}$) are comparable to each other, i. e., when this narrowing mechanism is balanced by linear diffraction, a solution with an invariant transverse profile can propagate. The latter is a bright spatial soliton and, by its own nature, contains both frequency components. For this very reason, quadratic solitons are also referred to as *simultons*.[16] It is important to recognize that, for a simulton to exist, both waves must be present and dynamically interact in the presence of parametric gain and diffraction. This imposes specific constraints on their transverse profiles in amplitude and phase, as apparent from the simulton profile in the simplest case of a medium without walk-off.[10-12]

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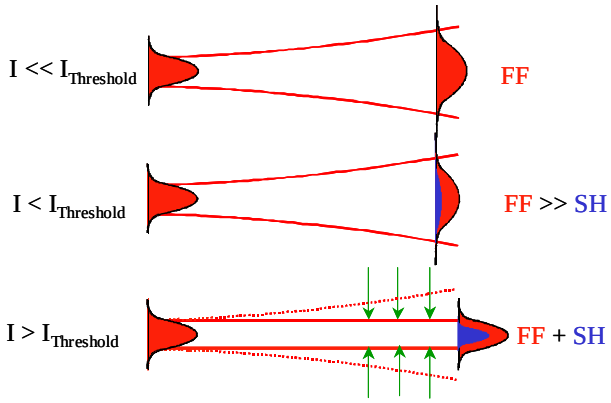

Fig. 3 Beam narrowing through SHG. The input is an FF beam. From top to bottom: Linear diffraction at low powers, weak SHG below threshold for soliton generation, and spatial soliton formation.

One can also utilize some of the universal properties of solitons, in conjunction with the well-known coupled mode equations described above, to argue some of the properties of the quadratic soliton fields, without actually solving for them. For non-diffracting [$\partial^2 a(y)/\partial y^2 = 0$) and stationary ($\partial|a(y)|/\partial z = 0$) solutions, $|a_{FF}(y)|$ and $|a_{SH}(y)|$ must be independent of z. This implies that the field profiles in the plane (y, z) must be constant. More information can be gained by examining the structure of the coupled mode equations, in the phase-matched case. After propagation for a short distance Δz, the evolution of the fields is given by

$$\Delta a_{FF}(y) = -i\Gamma a_{SH}(y) a_{FF}^{*}(y)\Delta z \qquad \Delta a_{SH}(y) = -i\Gamma a_{FF}^{2}(y)\Delta z \tag{3}$$

Because the envelopes are independent of z, both Δa_{FF} and Δa_{SH} must correspond to a pure phase rotation of the fields, i. e. they must be orthogonal to a_{FF} and a_{SH}, respectively. Given the "*i*" pre-factor in the equations (3), the two fields must therefore be parallel to each other. Furthermore, since they must remain parallel to one another in the soliton, they must also rotate together. As indicated in Figure 4, this is in contrast to the standard phase-matched SHG case, for which the fields are orthogonal to one another. Note that this is a nonlinear phase rotation since it is proportional to the product of two fields, and occurs for all solitons by virtue of their nonlinear propagation. Now, equations (1) and (2) apparently predict the amount of rotation to be different for different transverse positions, i. e. a function of y. However, it is

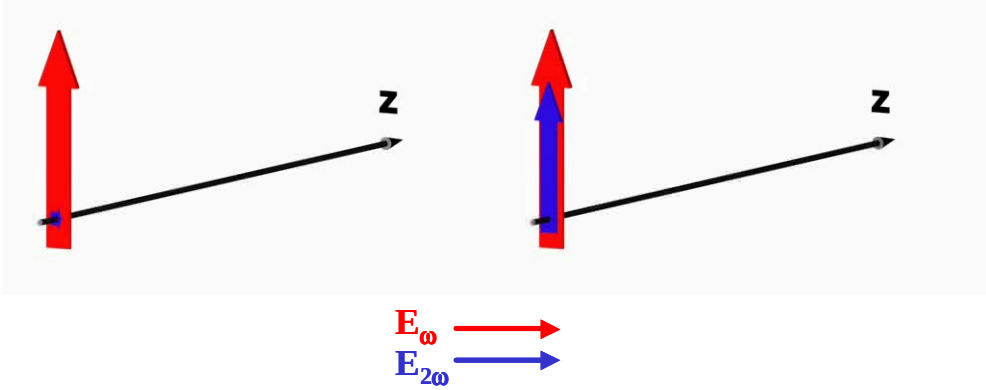

Fig. 4 (205 + 205KB) Animations showing the phase rotation of FF and SH field vectors in standard SHG (left) and quadratic soliton propagation (right).

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again a well-known property of solitons that any changes occur uniformly across the envelope, including phase. Thus the soliton experiences an "averaged" nonlinear phase rotation. Furthermore, the higher the intensity of the soliton, the faster the nonlinear rotation.

Having deduced the conditions for the solutions being stationary, it is also clear what occurs in the case of wavevector mismatch, i. e. when $\Delta k \neq 0$. The field vectors *a* still must remain parallel and rotate together. Thus there is an additional nonlinear contribution to the nonlinear phase rotation for the field which lags in phase due to $\Delta k \neq 0$, so that the envelopes remain in phase.

When an FF beam alone is injected into a crystal (i. e., SHG), a soliton is not created right at the input. Instead, the fundamental and the harmonic generated by it evolve towards the exact simulton by generating the appropriate field components in both amplitude and phase. During this excitation process some energy is radiated before complete trapping takes place. This can be quite efficient provided that the final simulton is not too far away in parameter space, i. e., if the input beam is close enough to the FF component of the resulting spatial soliton. An example of such evolution is shown in Fig. 2 for a Gaussian FF input.

Similar arguments can be applied in a straightforward manner to describe the excitation of a quadratic spatial soliton when injecting an SH input beam. In this case, down-conversion takes place first, generating the FF components needed to support the stable propagation of the simulton.[17] Noise photons or a small seed at FF can initiate the interaction process (from eqn. (2)) via parametric instability or gain (down-conversion), respectively.

Although this simple approach has predicted a self-focusing mechanism, it cannot guarantee that the balance between diffraction and self-focusing leads to a stable soliton. For that, more sophisticated analyses are necessary.[18] It is possible, however, to argue that there is a feedback mechanism that should stabilize a mutually self-trapped structure. For example, an increase in the fundamental amplitude due to some perturbation will increase the nonlinear phase rotation for the harmonic, causing the fields to dephase and hence exchange energy. Energy will flow into the harmonic. However, this will in turn increase the down conversion process, returning energy back to the fundamental. This process will tend to stabilize the relative field amplitudes, hence avoiding catastrophic self-focusing.

Finally, some of the simple concepts outlined above, in conjunction with equations (1) and (2), can be used to describe the *effective forces* between quadratic spatial solitons. To this extent, two simultons are assumed whose fields overlap in space while propagating in essentially the same direction. In the first approximation, this allows the role of relative transverse velocities and additional terms deriving from tilt-induced wavevector-mismatch to be neglected. Identifying by a_{FF} and a_{SH} the field amplitudes in one of the solitons, and by b_{FF} and b_{SH} those in the other one, the nonlinear polarization terms driving the FF will take the form:

$$P_{FF} = \varepsilon_o \chi^{(2)} (a_{SH} a_{FF}^* + b_{SH} b_{FF}^* + a_{SH} b_{FF}^* + b_{SH} a_{FF}^*) \quad (4)$$

whereas the corresponding terms generating the SH will be:

$$P_{SH} = \varepsilon_o \chi^{(2)} (a_{FF} a_{FF} + b_{FF} b_{FF} + 2 a_{FF} b_{FF}) \quad (5)$$

Clearly, the terms containing the cross-products of *a* and *b* are responsible for the soliton-soliton interaction. In Figure 5 below the contributions of each term across the soliton transverse field distributions are identified.

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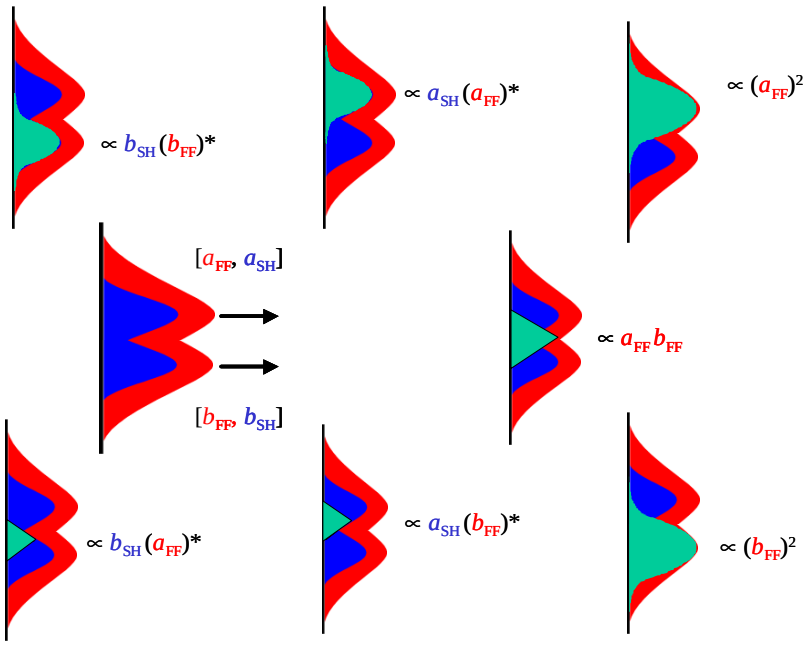

Fig. 5 Soliton collisions: schematic illustration of the interaction terms (in green) due to overlapping envelopes at FF and SH for solitons "a" and "b".

Since coherent fields are involved, it is now convenient to analyze the various cases of soliton-soliton interactions for different relative phases between them. In so doing, we will explicitly write the pertinent coupling terms describing the interaction. These terms will add to the usual ones which couple the individual soliton fields together on the RHS of eqn. (1-2) for both $[a_{FF}, a_{SH}]$ and $[b_{FF}, b_{SH}]$, i. e.,

$$2ik_{FF}\frac{\partial}{\partial z}a_{FF} - \frac{\partial^2}{\partial y^2}a_{FF} = 2k_{FF}\Gamma\left[a_{SH}a_{FF}^* + \delta(a_{FF})\right] \tag{6}$$

$$2ik_{FF}\frac{\partial}{\partial z}b_{FF} - \frac{\partial^2}{\partial y^2}b_{FF} = 2k_{FF}\Gamma\left[b_{SH}b_{FF}^* + \delta(b_{FF})\right] \tag{7}$$

$$2ik_{SH}\frac{\partial}{\partial z}a_{SH} - \frac{\partial^2}{\partial y^2}a_{SH} = 2k_{SH}\Gamma\left[a_{FF}^2 + \delta(a_{SH})\right] \tag{8}$$

$$2ik_{SH}\frac{\partial}{\partial z}b_{SH} - \frac{\partial^2}{\partial y^2}b_{SH} = 2k_{SH}\Gamma\left[b_{FF}^2 + \delta(b_{SH})\right] \tag{9}$$

For in-phase simultons, i. e., $\angle a_{FF} = \angle b_{FF}\,(= \angle a_{SH} = \angle b_{SH})$:

$$\delta(a_{FF}) = a_{SH}b_{FF}^* + b_{SH}a_{FF}^* + b_{SH}b_{FF}^* \tag{10}$$

$$\delta(b_{FF}) = a_{SH}b_{FF}^* + b_{SH}a_{FF}^* + a_{SH}a_{FF}^* \tag{11}$$

$$\delta(a_{SH}) = b_{FF}^2 + 2\,a_{FF}\,b_{FF} \tag{12}$$

$$\delta(b_{SH}) = a_{FF}^2 + 2\,a_{FF}\,b_{FF} \tag{13}$$

Since all the amplitudes are in phase, the cross terms corresponding to the fields' overlap provide constructive contributions to the parametric process. As discussed previously, this will increase the nonlinear phase rotation, corresponding to progressively increasing energy for each soliton. Moreover, the "self-terms" ($b_{SH}b_{FF}^*$, $a_{SH}a_{FF}^*$, a_{FF}^2, b_{FF}^2) due to the adjacent

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soliton tend to shift the effective center-of-mass towards the axis (z). This provides an attractive force which improves the overlap between the soliton fields, favoring the convergence of the two simultons. It is indeed well known that in-phase quadratic solitons can either fuse upon collision or "pass through" each other, depending on the details (transverse velocity) of the initial trajectories.[19-21]

It is worthwhile to stress that, due to the *eigen*-nature of each simulton, when considering the actual spatial overlap of the pertinent field distributions with the nonlinear perturbing polarization (4) at FF or (5) at SH, respectively, the weights of the individual quadratic terms in (10) thru (13) are comparable, i. e., for simultons of similar power and size ($|f|_x$ indicates the peak amplitude):

$$\int_y dy\,|(a_{FF}^*/|a_{FF}|_x)\,a_{SH}b_{FF}^*| \approx \int_y dy\,|(a_{FF}^*/|a_{FF}|_x)\,b_{SH}a_{FF}^*| \approx \int_y dy\,|(b_{FF}^*/|b_{FF}|_x)\,a_{SH}b_{FF}^*| \approx \int_y dy\,|(b_{FF}^*/|b_{FF}|_x)\,b_{SH}a_{FF}^*|$$

$$\int_y dy\,|(a_{SH}^*/|a_{SH}|_x)\,a_{FF}b_{FF}| \approx \int_y dy\,|(a_{SH}^*/|a_{SH}|_x)\,b_{FF}^2| \approx \int_y dy\,|(b_{SH}^*/|b_{SH}|_x)\,a_{FF}b_{FF}| \approx \int_y dy\,|(b_{SH}^*/|b_{SH}|_x)\,a_{FF}^2|$$

as summarized in Figure 6 below.

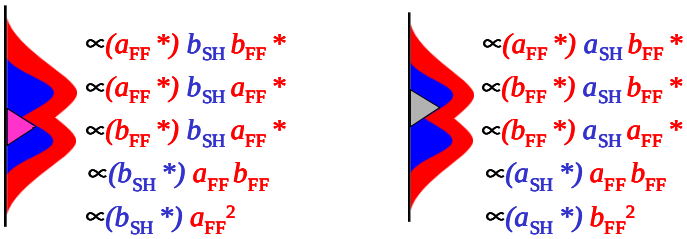

Fig. 6 Soliton collisions: as in Figure 5, but taking into account the spatial overlap with the eigen-distributions.

For out-of-phase simultons, i. e., $\angle a_{FF} = \angle b_{FF} - \pi$ $(= \angle a_{SH} = \angle b_{SH} - \pi)$ and :

$$\delta(a_{FF}) = -\,a_{SH}\,|b_{FF}^*| - |b_{SH}|\,a_{FF}^* + |b_{SH}||b_{FF}^*| \tag{14}$$

$$\delta(b_{FF}) = -|a_{SH}|\,b_{FF}^* - b_{SH}\,|a_{FF}^*| + |a_{SH}||a_{FF}^*| \tag{15}$$

$$\delta(a_{SH}) = b_{FF}^2 - 2\,a_{FF}\,|b_{FF}| \tag{16}$$

$$\delta(b_{SH}) = a_{FF}^2 - 2\,|a_{FF}|\,b_{FF} \tag{17}$$

All of the cross terms subtract from the individual solitons' driving terms, reducing the nonlinear phase rotation and the effective energy of each soliton, despite the additional self-terms from the neighboring simulton. Therefore, the overlap effectively leads to repulsive forces pushing the solitons apart and minimizing their mutual energy.

For in-quadrature simultons, i. e., $\angle a_{FF} = \angle b_{FF} - \pi/2$ $(= \angle a_{SH} = \angle b_{SH} - \pi/2)$:

$$\delta(a_{FF}) = -\,i\,a_{SH}|b_{FF}^*| + i\,|b_{SH}|\,a_{FF}^* + |b_{SH}||b_{FF}^*| \tag{18}$$

$$\delta(b_{FF}) = -\,i\,|a_{SH}|b_{FF}^* + i\,b_{SH}\,|a_{FF}^*| + |a_{SH}||a_{FF}^*| \tag{19}$$

$$\delta(a_{SH}) = -\,|b_{FF}|^2 + 2i\,a_{FF}\,|b_{FF}| \tag{20}$$

$$\delta(b_{SH}) = -\,|a_{FF}|^2 - 2i\,|a_{FF}|\,b_{FF} \tag{21}$$

From (20) in (8) it is apparent that the cross-term increases a_{SH}, whereas (21) in (9) shows a reduction in b_{SH}, despite the rotation slow-down due to the self-terms from the neighboring

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soliton. Thus a_{SH} grows while b_{SH} decreases or, equivalently, the [a_{FF}, a_{SH}] simulton takes energy from the i[b_{FF}, b_{SH}] soliton, undergoing amplification through the interaction.

Similarly, when the solitons are $3\pi/2$ out-of-phase, it is straightforward to show that the -i[b_{FF}, b_{SH}] simulton will be amplified at the expense of [a_{FF}, a_{SH}].

All of these results are in agreement with more sophisticated treatments and experiments. BPM propagation graphs for the various cases are shown in Fig. 7.

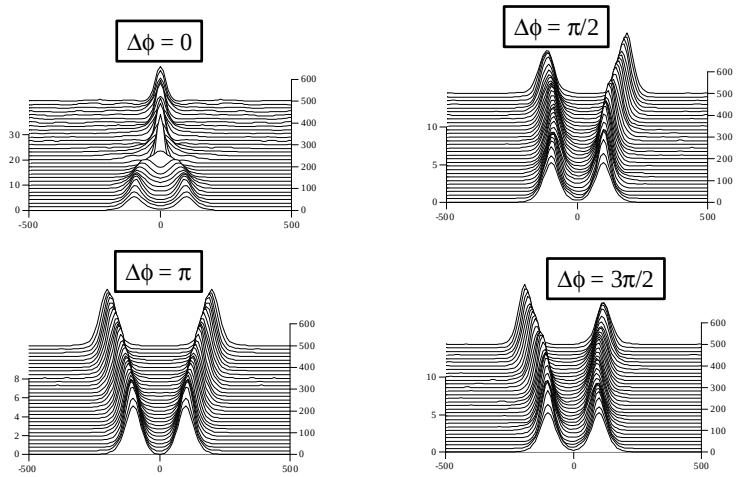

Fig. 7 BPM propagation (z versus y) 3D-graphs for the various cases of interactions mentioned above. Resulting FF intensity for equi-power Gaussian beams launched parallel to z. Units are real (distances and intensity) but should be viewed as arbitrary.

It is important to note that this approach cannot predict the outcome of a collision, just the effective forces that are operative. Effective-particle approaches provide physical-mechanical pictures of collisions, albeit with some additional mathematics, and can actually predict their outcome.[21-23]

In summary, although quadratic spatial solitons do not conform to the usual concepts associated with other spatial solitons that rely on refractive index changes, it is nevertheless feasible to understand some of their properties and interactions in terms of insightful concepts. In particular, after outlining the role of a cascading phase shift and parametric gain, we have revisited quadratic soliton-soliton interactions with the aid of simple coupled equations and spatial overlap considerations, providing an intuitive basis to the understanding of the effective forces between quadratic solitons.

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