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High Pressure Hydraulic Supply System Model

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HIGH PRESSURE HYDRAULIC SUPPLY SYSTEM MODEL

By

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ABSTRACT

A mathematical model is derived to provide quasi-steady state predictions of the performance of a high pressure hydraulic supply system, using equations which govern the physical processes as opposed to equations which match input-output characteristics. Model equations are developed to describe the operation of the power source, control valves, energy source, gas side of the system, hydraulic accumulator, the motorpump, and hydraulic side of the system.

The accuracy of the model is then checked by inserting known parameters from a previously developed control system and comparing model predictions with performance data from this system.

Richard C. Rapson, Jr.
Director of Research Report

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List of Symbols

a	Gas generator constant coefficient, lbsm/s/psi
A_g	Area of gas side of accumulator piston, sq in.
A_O	Area of oil side of accumulator piston, sq in.
A_{Pg}	Area of piston in warm gas motor, sq in.
A_R	Ratio of area of gas side of accumulator piston to area of oil side
D_P	Pump volumetric displacement, cu. in/rad
I	Inertia of rotating mass in pump, in/lb/s^2
K_{FL}	Flow limiter flow constant, cu. in/s/psid
K_M	Volumetric displacement of motor, cu. in/rad
K_{PRV}	Relief valve flow constant, cu. in/s/psid
K_{RV}	Regulator valve flow constant, lbsm/s/psid
M_{AP}	Accumulator piston mass, lbsm
n	Gas generator burn rate exponent
P_{DP}	Pump differential pressure, psid
P_g	Gas generator pressure, psi
$\dot{P}_g$	Rate of change of P_g, psi/s
P_{RV}	Regulator valve cracking pressure, psi
P_{RVD}	Relief valve differential pressure, psi
P_S	System differential pressure, psi
Q_A	Rate of oil flow from accumulator, cu. in/s
Q_P	Pump flow, cu. in/s
R	Gas constant = (1545 x 12)/molecular weight of gas, in lbsf/lbsm - °R

T_M	Motor torque due to gas pressure, in/lbsf
V_{AI}	Instantaneous volume of oil in accumulator, cu. in.
V_{FA}	Volume of oil in accumulator when full, cu. in.
V_g	Gas system volume, cu. in.
$\dot{V}_{gg}$	Volumetric burn rate of gas generator, cu. in/s
V_{gi}	Initial gas system volume, cu. in.
W_g	Mass in gas side of system, lbsm
$\dot{W}gg$	Gas generator mass flowrate, lbsm/s
$\dot{W}_M$	Motor gas mass flowrate, lbsm/s
$\dot{W}_{RV}$	Regulator valve flowrate, lbsm/s
α	Angle of inclination of bent-axis motorpump, degrees
$\dot{\theta}$	Angular velocity of motorpump, rad/s
ρ_M	Density of gas in motor side of motorpump, lbsm/cu. in.

I. INTRODUCTION

High pressure (1000-4000 psi) hydraulic control systems have been used extensively throughout industry to provide accurate positional control. In this report, a general mathematical model will be generated for a hydraulic supply system which contains typical component parts (i.e., actuator(s), pressure supply, control valves, hydraulic accumulator, hydraulic power source). The actuator in this model is used only as a means of generating a hydraulic demand on the system, and is not modeled as a dynamic second-order device. Developing a dynamic simulation of an actuator could be the subject of a research paper in itself.

The model developed herein will be based on equations that govern the physical processes as opposed to equations which match input-output characteristics. The model should be general enough to be readily adaptable to study and/or analysis of similar control systems by modifying the general equations to incorporate the characteristics of the particular system being studied.

As with any model of a physical process, simplifying assumptions will be made where appropriate. The number of assumptions made depends on the degree of complexity and detail desired in the model. Since the intent of this report is to present a relatively simple model which is easy to use and understand, the following assumptions, which should have little effect on the model of the overall system, will be made:

1 Constant Temperature - Although it is recognized that the temperature of a hydraulic control system may vary by several degrees during operation, the temperatures considered in this report (<200°F, 93.3°C) are not severe enough to significantly affect the structural properties of the materials used - typically stainless steel and aluminum. However, the thermal expansion of the hydraulic fluid will be allowed for in sizing reservoirs for quiescent storage in order to preclude the occurrence of damaging pressures which could result from temperature cycling during non-operating periods. For extremely precise models, it might be desirable to include thermal effects during system operation.

2 Rigid Walls - It is assumed that the volumetric expansion which can occur in tubing, accumulators, etc. upon application of pressure is insignificant in its effect on the system model.

3 Constant Bulk Modulus - Although the bulk modulus in reality varies with temperature, pressure, and the amount of entrained air, it is assumed that for the conditions considered herein, the effect is negligible.

II. MODEL DESCRIPTION

The system considered in this model operates in the following manner:

The hydraulic servoactuators are powered by a closed, recirculating hydraulic system with the energy being supplied by a gas-driven motor pump. An accumulator provides the extra hydraulic power capacity to meet the transient flow requirements of the system. Gas power to drive the system is derived from the products of combustion of a solid propellant gas generator. (Although this is a typical aerospace system, the model should be readily adaptable to include equations describing the power source for the particular system to be modeled.) Gas pressure is maintained by pressure relief valves. Transient pressure peaks in the hydraulic system which are due to the accumulator reaching its full position and the pump speed changing are limited by the hydraulic relief valve.

Fluid is supplied to the pump from a reservoir, which also contains a spring to maintain positive pressure during quiescent storage. Even though this will not enter into the model equations, it was mentioned as being typical for systems similar in nature to the system which will be used to check the model developed herein.

III. DERIVATION OF MODEL EQUATIONS

A. POWER SOURCE

As mentioned earlier, the power source included in this model is a solid-propellant warm gas generator. In order to use the model with a different power source, a new set of equations describing the power source characteristics would have to be developed at this point. The equation which describes the output (i.e., mass flowrate) of the gas generator as a function of gas pressure is derived in the following manner:

The rate of propellant consumption ($\dot{W}_{gg}$) is related to the linear burning rate (r_b) for the propellant. It can be assumed that the linear burning rate is given by Baumeister [1]

$$r_b = c\, P_g^{\,n} \qquad (1)$$

where

c and n are experimentally determined constants.

If ρ_P represents the specific weight of the solid propellant (lbs/cu. ft.) and A_P the area of the burning surface (assumed constant), then the propellant consumption rate can be expressed as follows:

$$\dot{W}_{gg} = a\, P_g^{\,n} \qquad (2)$$

where

$a = \rho_P\, A_P\, C.$

The values of a and n depend on the composition of propellant as well as the propellant geometry involved.

B. PRESSURE REGULATOR VALVE

The pressure regulator valve is normally designed such that no flow occurs until the system pressure meets or exceeds some predetermined level, hereinafter called the cracking pressure (P_{RV}). Thus the operation of the valve is illustrated in Figure 1, and can be described as follows:

$$\begin{aligned} \dot{W}_{RV} &= K_{DRV}\,(P_g - P_{RV}) \\ K_{DRV} &= K_{RV} \qquad P_g \geq P_{RV} \\ K_{DRV} &= 0 \qquad P_g < P_{RV} \end{aligned} \tag{3}$$

K_{RV} is a constant which is dependent on the design of the particular valve used and must be determined for each system to which this model is applied.

C. ENERGY SOURCE

Flow of gas through the motor is proportional to the specific weight of the gas and the volumetric flowrate. The volumetric flowrate is equal to the motor angular velocity (rad/s) multiplied times the motor pump geometric constant (cu. in./rad), which can vary depending on the particular unit being used. Thus

$$\dot{W}_M = \frac{P_g}{RT} K_M \dot{\theta} = \rho_M K_M \dot{\theta} \quad \left(\dot{\theta} = \frac{Q_P}{D_P}\right) \tag{4}$$

The geometric constant K_M is determined based on the number of pistons in the pump, the piston stroke per gas intake cycle, piston area, and pump revolution during intake cycle. This constant is usually available from the supplier of the unit used.

D. GAS PRESSURE FUNCTION

The conservation of mass equation for the gas side of the system can then be expressed as follows:

$$\dot{W}_{gg} = \dot{W}_{RV} + \dot{W}_{M} + \frac{d}{dt} W_g \tag{5}$$

$\dot{W}gg$, $\dot{W}_{RV}$, and $\dot{W}_M$ were developed in equations 2, 3, and 4 respectively. The last term in equation (5) represents the rate of change of mass contained within the gas side of the system. This can be expressed as follows:

$$\frac{d}{dt} W_g = \frac{d}{dt} \frac{P_g V_g}{RT} \tag{6}$$

Having assumed the system operates with essentially a constant temperature, this can be expressed as

$$\frac{V_g}{RT} \frac{d}{dt} P_g + \frac{P_g}{RT} \frac{d}{dt} V_g = \frac{\dot{P}_g V_g}{RT} + \frac{P_g \dot{V}_g}{RT} \tag{7}$$

For this model, the variation in the gas system volume ($\dot{V}_g$) is created by an increasing gas generator volume due to propellant consumption, and a transient variation introduced as hydraulic accumulator piston motion occurs due to transient hydraulic demands which exceed the instantaneous capability of the motor-pump. Gas system volume can then be expressed as

$$V_g = V_{gi} + \dot{V}_{gg} t + A_R \left(V_{AF} - V_{AI}\right) \tag{8}$$

Since V_{gi}, $\dot{V}_{gg}$, A_R, and V_{AF} are constants, equation (8) can be differentiated to yield

$$\dot{V}_g = \dot{V}_{gg} + A_R \frac{d}{dt} (-V_{AI}) \qquad (9)$$

But d/dt $(-V_{AI})$ is simply the rate of flow of oil out of the hydraulic accumulator. Equation (9) then becomes

$$\dot{V}_g = \dot{V}_{gg} + A_R Q_A \qquad (10)$$

Combining equations 2, 3, 4, 7, and 10 and solving for $\dot{P}g$ results in:

$$\dot{P}_g = \frac{RT}{V_g} \left[a\ P_g^{\ n} - K_{DRV}\ (P_g - P_{RV})\right] - \frac{P_g}{V_g}\left[\frac{K_M Q_P}{D_P} + \dot{V}g\right] \qquad (11)$$

$$\dot{V}_g = \dot{V}_{gg} + A_R Q_A$$

This expresses the time variation in system pressure as a function of the performance of the power source, the pressure regulator valve(s), and the energy source.

E. ACCUMULATOR FLOW

An expression will now be developed for the flow from the hydraulic accumulator. It will be assumed that relatively low flow velocities are produced in the accumulator. This permits the assumption that the fluid in the accumulator is at hydraulic system pressure. The net force on the accumulator piston, neglecting any frictional effects (which under dynamic conditions should be minimal in a well-designed system), becomes

$$P_g A_P - P_S A_O = \text{net force} \qquad (12)$$

The flow from the accumulator can be expressed as

$$A_o \frac{dx}{dt},$$

where

$\frac{dx}{dt}$ is the accumulator piston velocity.

Thus the rate of change of accumulator flow can be expressed as

$$\dot{Q}_A = A_0 \frac{d^2x}{dt^2}$$

$$\dot{Q}_A = \frac{A_0 F}{M_{AP}} = \frac{A_0}{M_{AP}} (P_g A_g - P_S A_0) \tag{13}$$

This applies only when the instantaneous accumulator volume (V_{AI}) lies between 0 (accumulator empty) and V_{FA}.

The instantaneous volume is determined by

$$V_{AI} = V_{FA} + \int_0^{t} Q_A \, dt \tag{14}$$

$$Q_A = 0 \qquad 0 \geq V_{AI} \text{ or } V_{AI} \geq V_{FA}.$$

F. EQUATION DESCRIBING MOTOR PUMP TORQUE

Due to the transient response characteristics of the motor-pump, an equation must be developed to express the net torque acting on the rotating mechanism. Applying the conservation of energy principle to the hydraulic side of the motorpump yields

$$\left[\left(\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1} \right) + \frac{V_2^{\,2} - V_1^{\,2}}{2} + (U_2 - U_1) \right] \frac{W}{g} = \dot{H} + T_P \dot{\theta} \tag{15}$$

where P_i, ρ_i, V_i, and U_i; are the pressure, density, velocity, and intrinsic internal energy at point i (i = 1 at entrance and i = 2 at exit), W is the weight flow through the pump, $\dot{H}$ is the rate of heat flow to the control volume, and T_P is the torque applied to the pump shaft. Under the assumption that the pumping process is a uniform, constant temperature, adiabatic process (i.e., $U_2 = U_1$) and noting that $V_1 = 0$, $V_2^2 << P_2/\rho_2$, $\rho_1 \sim \rho_2$ (incompressible fluid), and $P_2 - P_1 = P_{DP}$, equation (15) reduces to

$$\frac{P_{DP}}{\rho} \frac{W}{g} = T_P \dot{\theta} \tag{16}$$

However, mass flow through pump divided by density is equal to pump volumetric displacement per radian multiplied by pump rad/s, i.e.,

$$\frac{W}{g\rho} = D_P \dot{\theta} \tag{17}$$

Therefore equation (16) reduces to

$$T_P = P_{DP} D_P \tag{18}$$

which is a standard equation used in determining torque output of hydraulic pumps.

Deriving a comparable expression for the gas side of the pump (i.e., the motor) is somewhat more difficult. Since the flowing medium in the motor is a warm gas, the constant temperature and density assumptions made in the pump analysis are not valid for the motor.

Therefore, in order to develop an expression for the motor torque, the motorpump cross-section and timing diagram in Figures 2 and 3, respectively will be used. From the motor timing diagram it can be seen that the motor pistons are exposed to gas system pressure for approximately 93 percent of a power stroke (considering the power stroke as being inlet open to exhaust open). It is therefore assumed that the useful work extracted from the warm gas is done at nominal gas system pressure Pg. Therefore, the force acting on each piston is just

$$F = A_{Pg} P_g \tag{19}$$

The component of this force normal to the shaft, which produces the useful torque, can be derived by considering the geometry depicted in the cross section of Figure 2. The motorpump considered in this model is a bent-axis unit. The angle α depicted in this figure is the angle between the pump axis and the motor axis. This normal force can be expressed as

$$F_N = P_g A_{Pg} \text{ SIN } \alpha \tag{20}$$

where

F_N is the normal force. The forque generated by F_N is

$$F_N \frac{d_M}{2} \text{ SIN } \beta \tag{21}$$

where

d_M is the working diameter of the motor (see cross-sectional view) and β is the angle of rotation from top dead center, as defined in Figure 3.

The motorpump of this model contains nine (9) pistons on both the gas and oil sides. Therefore, since the pistons are 40 degrees apart (360°/9 pistons), there are typically three pistons being subjected to system pressure, with one possibly being in the expansion region. It will be assumed that since the expansion region is such a small portion of a total power stroke and the average pressure there is probably approximately one-half total system pressure, the torque generated in this region can be neglected. Thus, the total torque acting on the motor shaft due to the hot gas is expressed as

$$T_M = P_g \frac{A_{Pg}\ d_M}{2} \text{ SIN } \alpha \ [\text{SIN } \beta + \text{SIN } (\beta+40°) + \text{SIN } (\beta+80)] \quad (22)$$

From the timing diagram, it can be seen that β varies from 8 degrees to approximately 125 degrees during a power stroke.

For pumps of the type included in this mode, the theoretical torque typically oscillates at a relatively high frequency. The particular pump model considered in checking this simulation displayed 150 Hz oscillation at approximately 1000 rpm. Therefore, it is assumed that the torque is constant at its mean value, i.e.,

$$T_M = P_g \frac{A_{Pg}\ d_M}{2} \text{ K SIN } \alpha \int_8^{45} [\text{SIN } \beta + \text{SIN } (\beta+40°) + \text{SIN } (\beta+80°)]\ d\beta \quad (23)$$

where

K is a constant which converts the distance between adjacent pistons to radians ($K = 57.3/40 = 1.433 \text{ rad}^{-1}$). The theoretical torque can therefore be expressed as

$$T_M = P_g \ D_g \tag{24}$$

where

$$D_g = 1.15 \ A_{Pg} \ d_M \ \text{SIN} \ \alpha \tag{25}$$

(For the unit considered in the model checkout, this value becomes 0.123 cubic inches.)

Using equations (18) and (23), which express the theoretical torque available from the pump and motor respectively, the net theoretical torque available is

$$T_{NT} = D_g \ P_g - D_P \ P_{DP} \tag{26}$$

However, to determine the torque actually available to drive the motorpump, the torque losses encountered (i.e., frictional losses) in operation must be subtracted from the total net theoretical torque available.

In order to derive an expression for the sum of all the losses encountered, the steady-state operating characteristics of a motorpump typical of the unit considered in this model are used. An example of the operating curve is shown in Figure 4. Letting T_L be the sum of all the losses, an expression for steady-state operation would be

$$T_{NT} - T_L = D_g \ P_g - D_P \ P_{DP} - T_L = 0 \tag{27}$$

or

$$T_L = D_g \ P_g - D_P \ P_{DP}. \tag{28}$$

Given a constant supply pressure, the functional relationship between pump differential pressure (P_{DP}) and demand flow from the pump is as shown in Figure 4. Thus a functional relationship between T_L and pump flow can be developed, using this figure and equation (27). The typical shape of this relationship is shown in Figure 5. Since torque is equal to the product of inertia of the rotating mass and the angular acceleration as stated by Sears and Zemansky [2], and angular acceleration is the rate of change of pump flow divided by the pump displacement per radian, the relationship depicted in the referenced figure can be developed as follows:

Steady State $\quad D_g\, P_g - D_P\, P_{DP} - T_L = 0$

Transient $\quad D_g\, P_g - D_P\, P_{DP} - T_L\,(Q_P) = I\,\frac{d^2\beta}{dt^2} \qquad (29)$

where

$T_L\,(Q_P)$ means T_L is a function of Q_P and I is inertia of rotating mass in pump.

But

$$\frac{d^2\beta}{dt^2} = \frac{dQ_P}{dt}/D_P$$

therefore

$$\frac{dQ_P}{dt} = \frac{D_P}{I}\,[D_g\, P_g - D_P\, P_{DP} - T_L\,(Q_P)]. \qquad (30)$$

The pressure drop across the pump (P_{DP}) is equal to system differential pressure (P_S) plus the pressure drop across the flow limiter (P_{FL}). P_{FL} can be expressed as

$$P_{FL} = K'_{FL} \; (Q_P - Q_{PL}) \tag{31}$$

where

Q_{PL} = pump flow limit

$K'_{FL} = K_{FL} \qquad Q_P \geq Q_{PL}$

$K'_{FL} = 0 \qquad Q_P < Q_{PL}$

K_{FL} = flow limiter constant ∿ cu. in/s/psi.

The pump flow expression [equation (30)] can then be written as follows:

$$\frac{dQ_P}{dt} = \frac{D_P}{I} \; [D_g \; P_g - D_P \; (P_S + P_{FL}) - T_L \; (Q_P)]$$

$$= \frac{D_P}{I} \; [D_g \; P_g - D_P \; P_S - D_P \; K'_{FL} \; (Q_P - Q_{PL}) - T_L \; (Q_P)] \tag{32}$$

G. SYSTEM PRESSURE FUNCTION

All that is lacking in the model equations is an expression of system pressure in terms of known constants and variables. This will be developed using a property of hydraulic fluids known as bulk modulus, which expresses the unit change in volume producted by a unit change in pressure and is defined by Harrison and Bollinger [3] as

$$\beta = \frac{\Delta P}{\Delta v/v} \tag{33}$$

where

β = bulk modulus

P = pressure

v = volume.

Equation (33) can also be written [1]

$$\beta = \rho \frac{\Delta P}{\Delta \rho}$$

or

$$\frac{1}{\rho} \Delta\rho = \frac{1}{\beta} \Delta P. \tag{34}$$

Introducing a Δt term on each side of equation (34) and taking the limit as Δt approaches zero yields

$$\frac{1}{\rho} \frac{d\rho}{dt} = \frac{1}{\beta} \frac{dP}{dt} \tag{35}$$

By definition, $\rho = M/V$ where M is mass of fluid contained in volume V. Therefore,

$$\frac{d\rho}{dt} = \left(V \frac{dM}{dt} - M \frac{dV}{dt} \right) / V^2 \tag{36}$$

However, $M = \rho V$ and $dM/dt = \rho Q$, therefore

$$\frac{d\rho}{dt} = \frac{1}{V^2} \left[V \ (\rho Q) - \rho V \frac{dV}{dt} \right]$$

$$\frac{d\rho}{dt} = \frac{\rho}{V} \left(Q - \frac{dV}{dt} \right) \tag{37}$$

where

Q is the volumetric flowrate.

For this model, dV/dt is just the opposite of the accumulator flow (i.e., positive accumulator flow decreases overall hydraulic system volume as accumulator piston displaces hydraulic fluid).

Using equations (35) and (37), the time variation in system pressure can be expressed as follows:

$$\frac{dP}{dt} = \frac{\beta}{V} \left(Q - \frac{dV}{dt} \right) \tag{38}$$

where Q is comprised of pump flow (Q_P), pressure relief valve flow (Q_{RV}), and demand flow (Q_D). For this model, Q_D is the flow required to drive the control actuators. Thus equation (38) can be rewritten as

$$\frac{dP}{dt} = \frac{\beta}{V} (Q_P + Q_A - Q_{RV} - Q_D) \qquad (39)$$

where $Q_{RV} = K'_{PRV} (P_S - P_{RV})$

$$K'_{PRV} = K_{PRV} \qquad P_S \geq P_{RV}$$

$$K'_{PRV} = 0 \qquad P_S < P_{RV}$$

(K_{PRV} is a relief valve flow constant which must be determined for each system studied, where applicable) and

$$V = V_I + V_A$$

where

V_I = hydraulic system volume with full accumulator

V_A = accumulator volume.

This now completes the series of equations required to generate a computerized model of a hydraulic control system. The equations are summarized below, and a summary of the applicable constants and their values as used in the checkout program are also presented. Prior to using this model for any system, the rationale behind the equations derivations should be checked to verify consistency with the system being considered.

H. SUMMARY OF EQUATIONS

(For notation, refer to equations in body of report by number in parentheses.)

Power Source

$$\dot{W}_{gg} = a\ P_g^{\ n} \tag{2}$$

Pressure Regulator Valve

$$\dot{W}_{RV} = K_{DRV}\ (P_g - P_{RV}) \tag{3}$$

$$K_{DRV} = K_{RV} \qquad P_g \geq P_{RV}$$

$$K_{DRV} = 0 \qquad P_g < P_{RV}$$

Energy Source

$$\dot{W}_M = \rho_M\ K_M\ \dot{\theta} \tag{4}$$

$$\dot{\theta} = \frac{Q_P}{D_P}$$

Gas Pressure

$$\dot{P}_g = \frac{RT}{V_g}\left[a\ P_g^{\ n} - K_{DRV}\ (Pg - P_{RV})\right] - \frac{P_g}{V_g}\left[\frac{K_M\ Q_P}{D_P} + V_g\right] \tag{11}$$

$$\dot{V}_g = \dot{V}_{gg} + A_R\ Q_A$$

Accumulator Flow

$$A_Q = \frac{A_O}{M_{AP}}\ (P_g\ A_g - P_S\ A_O) \tag{13}$$

$$Q_A = 0 \qquad 0 \geq V_{AI} \text{ or } V_{AI} \geq V_{FA}$$

$$V_{AI} = V_{FA} + \int_0^{+} Q_A\ dt \tag{14}$$

Motorpump Flow

$$\frac{dQ_P}{dt} = \frac{D_P}{I}\ [D_g\ P_g - D_P\ P_S - D_P\ K'_{FL}\ (Q_P - Q_{PL}) - T_L\ (Q_P) \tag{32}$$

System Pressure Function

$$\frac{dP}{dt} = \frac{\beta}{V} (Q_P + Q_A - Q_{RV} - Q_D) \qquad (39)$$

[Note: As can be seen in the glossary of terms included in the model listing in Appendix I, various terms are included in the model which are not discussed in the body of the report. These are terms which were used to facilitate programming the specific model and are not considered germane to the basic descriptive equations for a general model. This was done in an attempt to present a simplified approach which could be used for various applications, and to avoid prejudicing the model toward a particular system.]

IV. MODEL CHECKOUT/CONCLUSIONS

In order to verify that the mathematical model developed in this report provides a viable means of making at least a preliminary assessment of hydraulic supply system performance, a checkout run was made using input parameters from an existing system. Typical printout data from the checkout runs are shown in the appendix.

The input data were obtained from nominal design values from the components in the selected system. Since a tolerance exists on each of these nominal values, the data obtained are not presented as exact solutions for the various output functions. It is not considered likely that a single system would be manufactured with each component meeting the nominal specified design value. Rather, these data are presented to indicate the trend the output data display.

For each condition checked, the program was structured to iterate for 500 milliseconds, which was sufficient time for the system studied to achieve steady-state operating condition. In each case the last three iterations were printed to verify that a steady state solution had been achieved, which was indicated by the identical values being printed in each output position.

The output included as Appendix II is presented as being typical data for the system which was picked for model verification.

The nominal operating pressures varied from 1900-2100 psi on the gas side of the system and from 2800 to approximately 3700 psi on the oil side in actual system operation; model predictions compare favorably with these values. Also included are data which indicate that hydraulic system pressure decayed to zero. This indicates the capabilities of the modeled system have been exceeded (i.e., hydraulic demand was too high for system to maintain pressure). These data were generated as a self-check by inserting input values which were known to exceed the system design limits, thereby verifying model capability to predict anomalies in system operation.

Based on the data generated when design parameters of an existing hydraulic control system were input into the model and the model was exercised using various demand levels, it is concluded that the basic equations used to describe the performance of the various system components are correct and can be used as the core around which a specific model can be developed. As was done in the checkout runs, additional equations which are not germane to the basic model may be developed for particular components and combined with the basic equations to form a complete system model.

A schematic of the system used in checking the model developed herein is included as Figure 6.

V. MODEL LIMITATIONS

As was indicated earlier, this particular model was developed for a hydraulic system which might typically be encountered in an aerospace/missile system application. In order to be used for other (i.e., commercial) applications, the derivations of the model equations must be reviewed in light of the system being considered. As an example, the model does not consider hydraulic line length, since this is not typically a driving design constraint for the relatively short tubing lengths used in missile applications. However, in commercial/industrial control systems, hydraulic line length can become a significant factor in system design. Relatively long operating times can result in elevated hydraulic fluid temperatures in industrial systems, whereas in the short durations of typical missile systems, temperature effects are miniscule, and were neglected in the model equations.

Therefore, although it is felt the model is a viable tool for predicting control system performance, the basis for the model equations must be considered prior to applying the model.

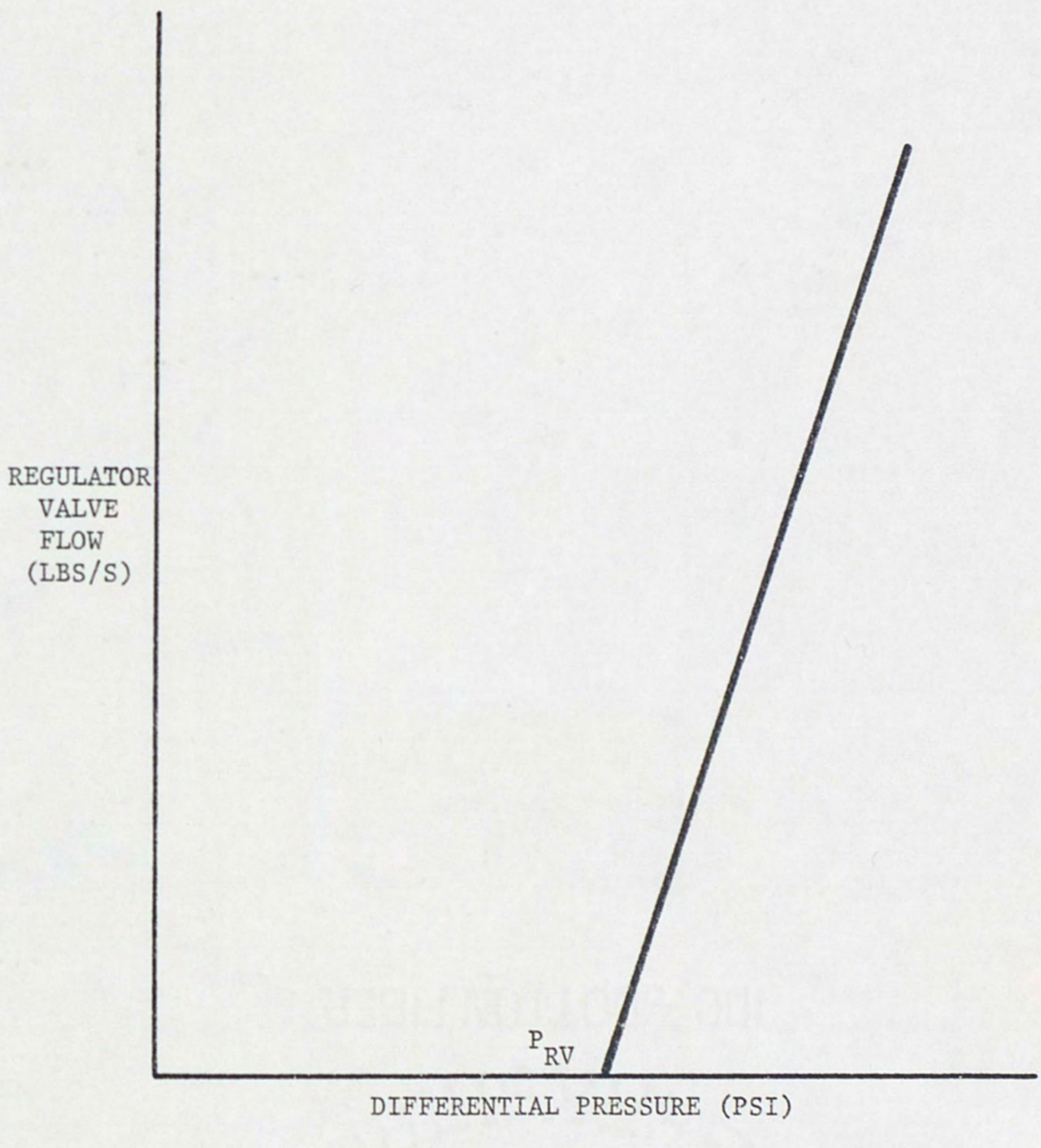

Figure 1. Typical Operating Characteristic of Pressure Regulator Valve

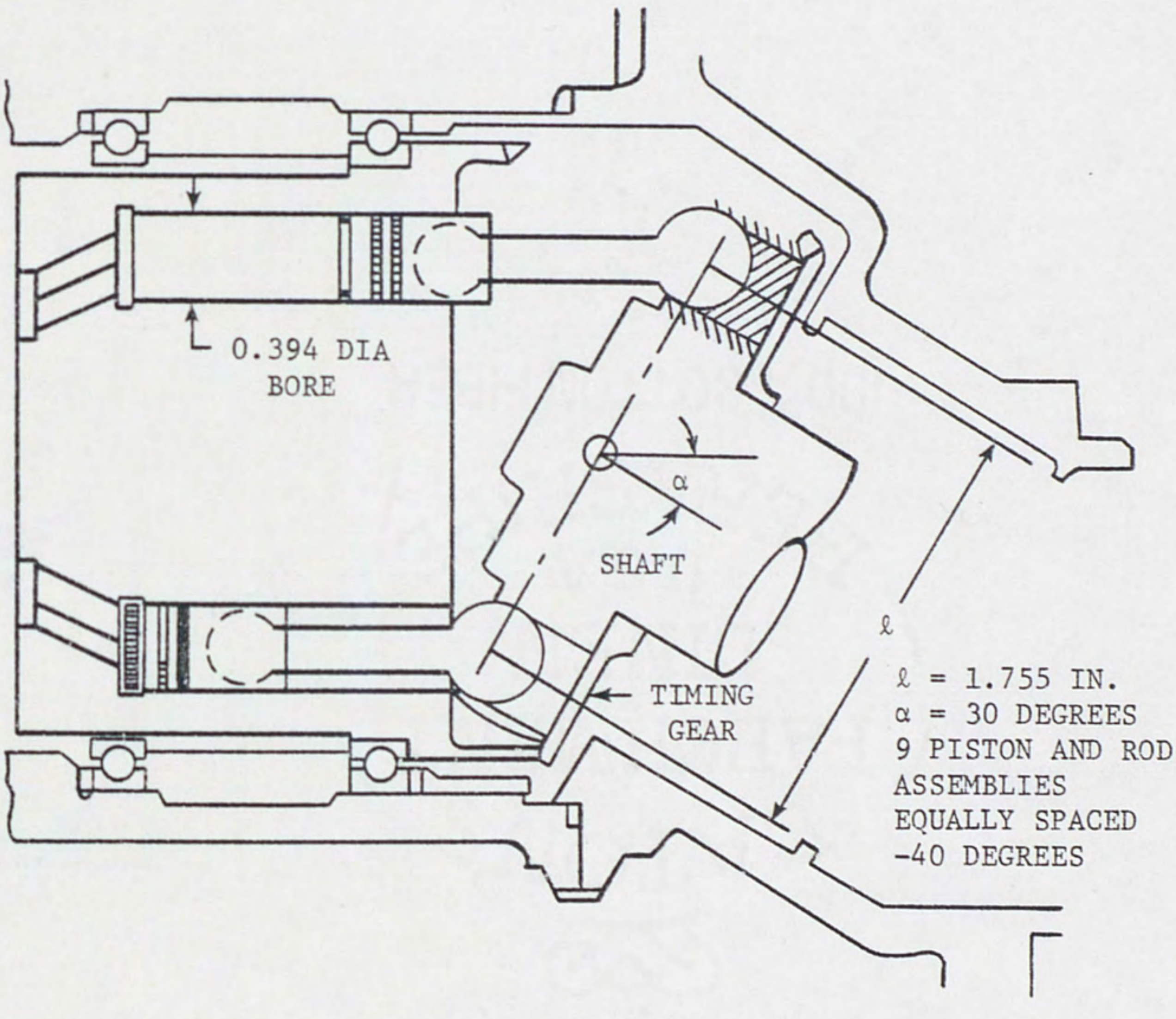

Figure 2. Cross-Section of Warm Gas Motorpump

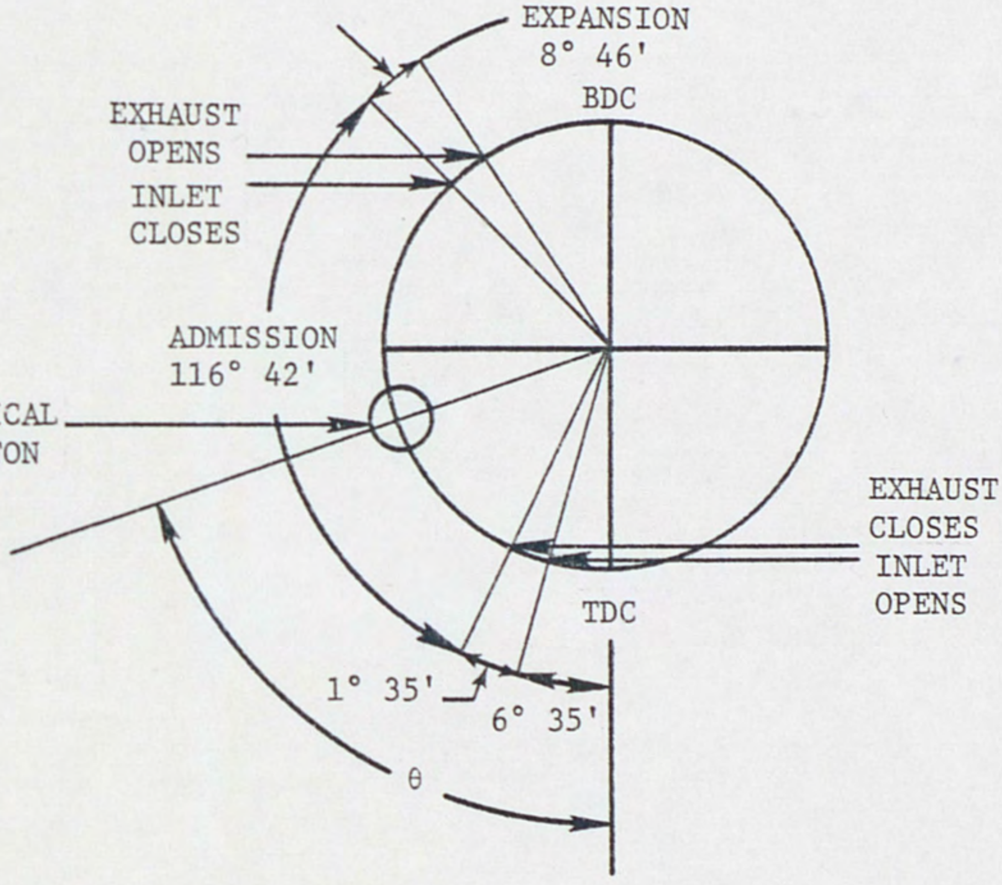

Figure 3. Hot Gas Motor Timing Diagram

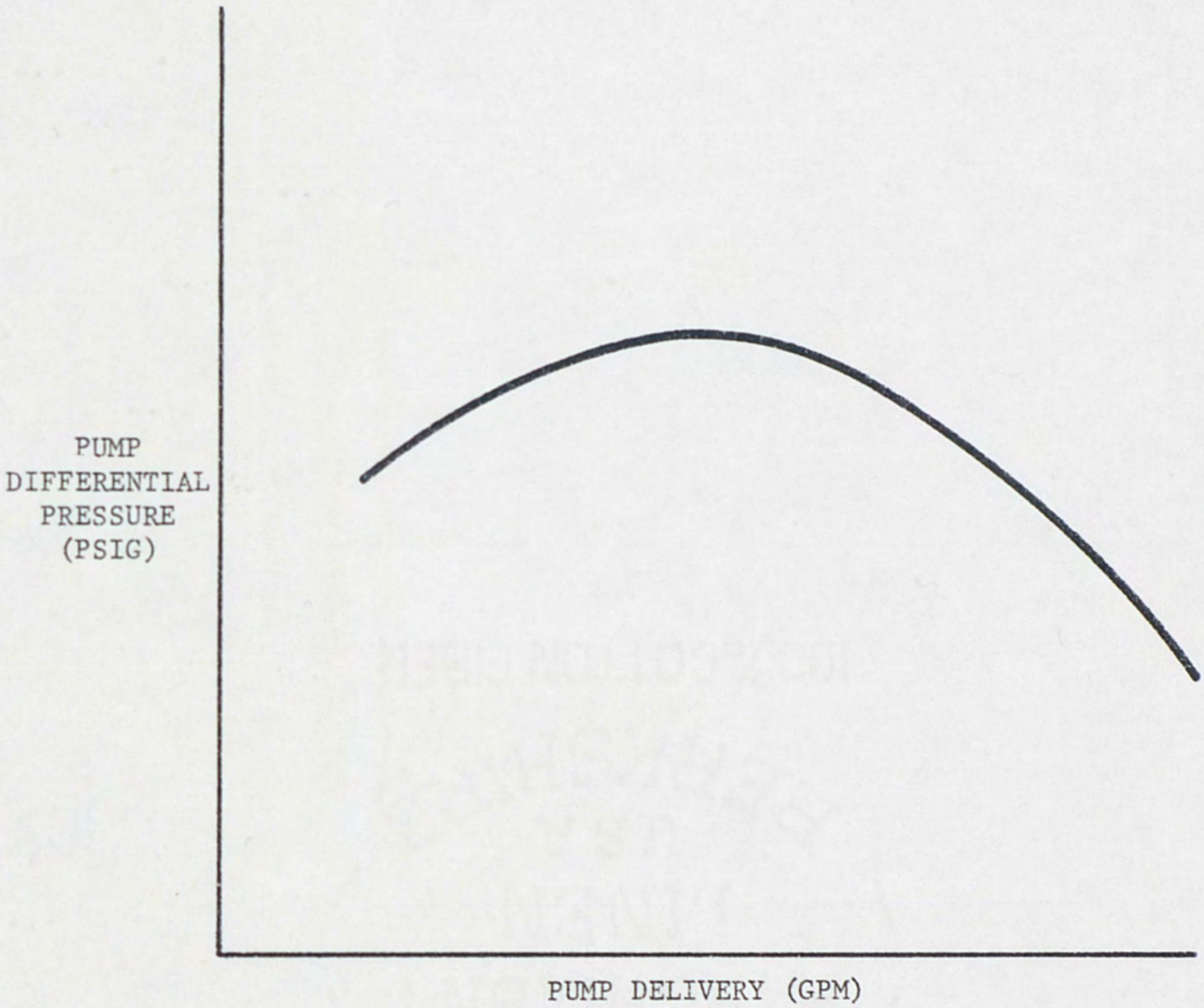

Figure 4. Steady State Operating Characteristics of a Typical Pump

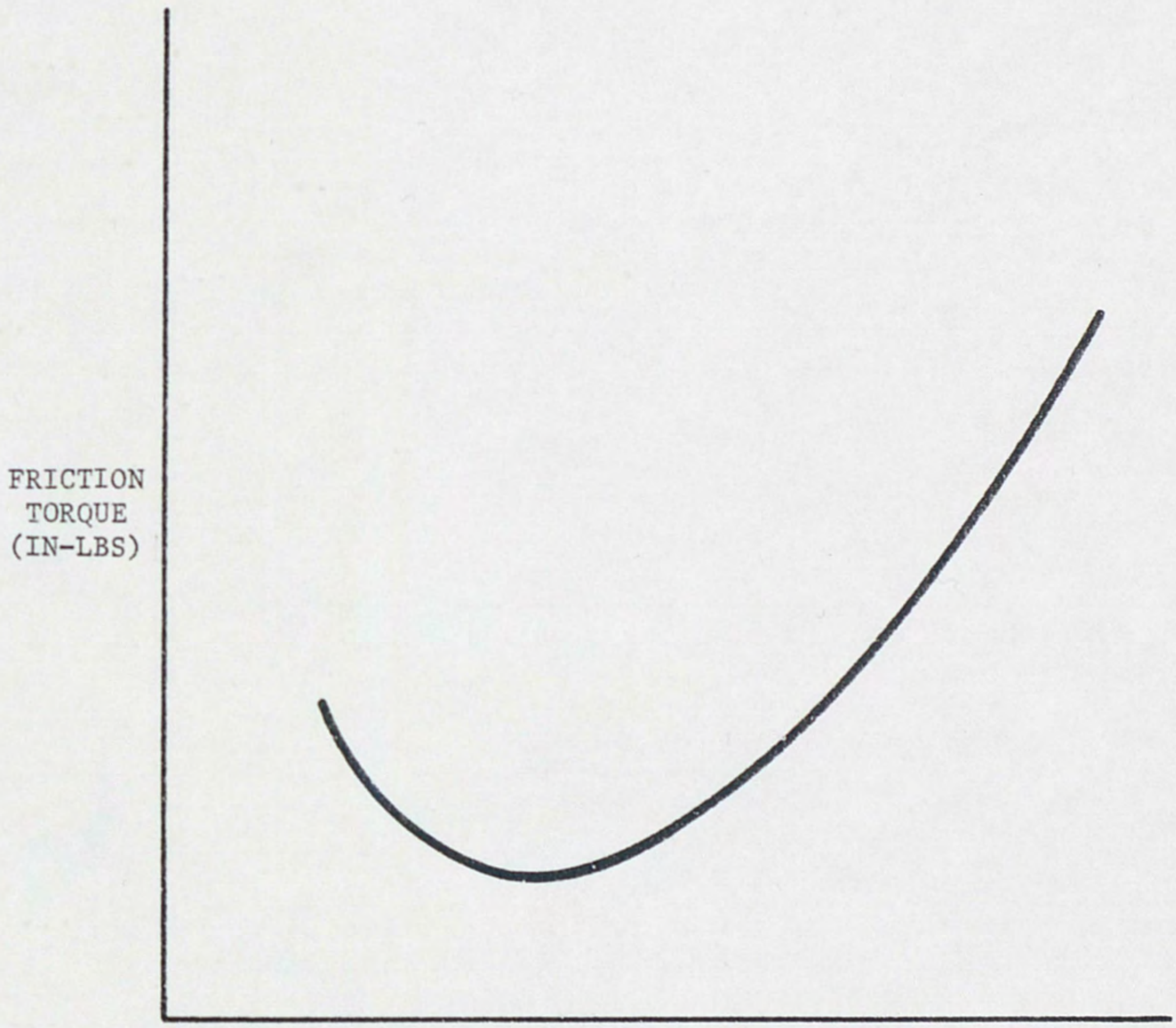

Figure 5. Typical Friction Torque versus Pump Flow Characteristic

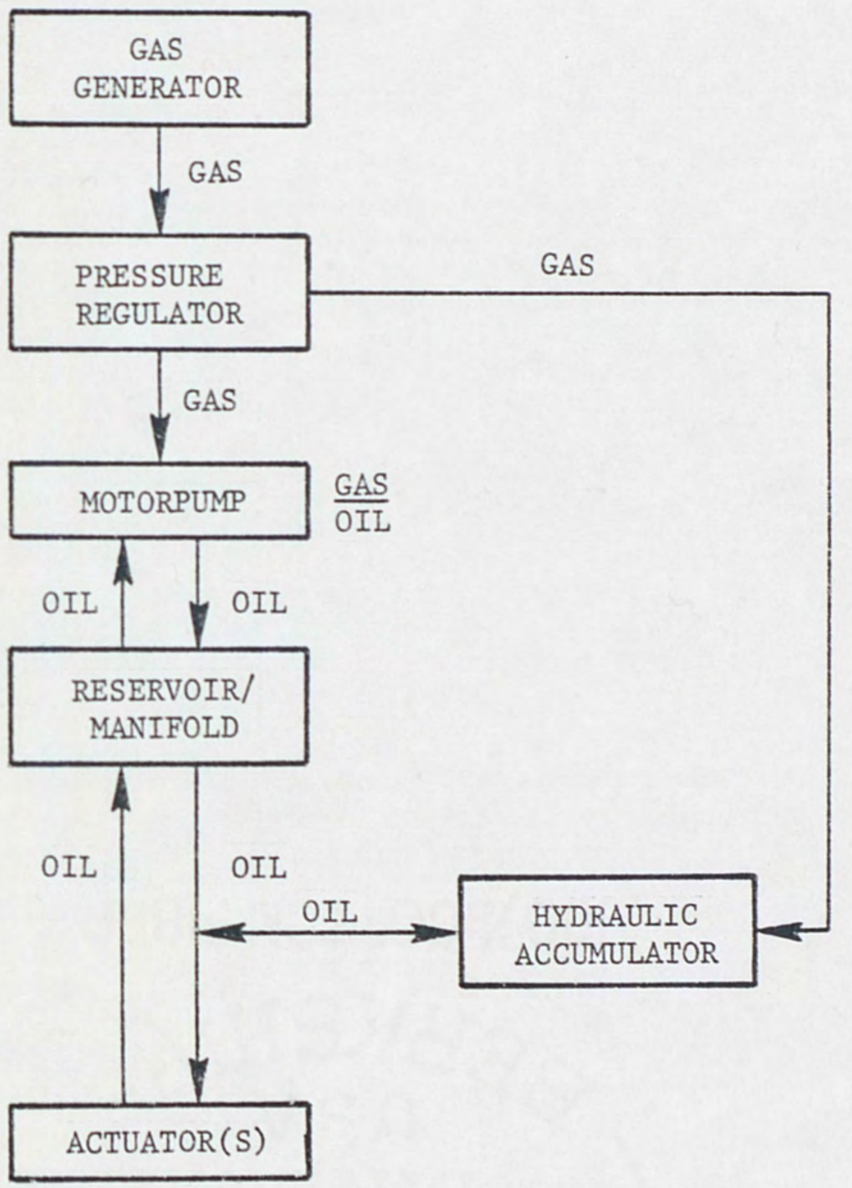

Figure 6. Schematic of System Used to Check Model

APPENDIX I

Listing of Model Program

```
C   THE FOLLOWING IS A GLOSSARY OF TERMS USED IN THIS PROGRAM.
C     A
C          AG= AREA OF GAS SIDE OF ACCUMULATOR PISTON ~ SQ.IN.
C         AKM= DISPLACEMENT PER RADIAN OF MOTOR~CU. IN. / RAD
C        AKRV= REGULATOR VALVE FLOW CONSTANT ~ LBS/SEC/PSI
C          AO= AREA OF OIL SIDE OF ACCUMULATOR PISTON ~ SQ. IN.
C         APA= ACTUATOR PISTON AREA ~ SQ. IN.
C         APM= ACCUMULATOR PISTON MASS ~ LBS/IN/SEC/SEC
C          AR= AREA RATIO (GAS/OIL) OF ACCUMULATOR PISTON
C     B
C       BMOIL= BULK MODULUS OF OIL IN SYSTEM ~ PSI
C          BR= BURN RATE OF GAS GENERATOR
C     D
C          DP= DISPLACEMENT OF PUMP ~ CU.IN./RAD.
C       DPGDT= VARIATION IN SYSTEM PRESSURE WITH TIME(DPG/DT) ~ PSI/SEC
C         DPR= PRESSURE ABOVE REGULATOR VALVE CRACKING PRESSURE ~ PSI
C       DPRES= VARIATION IN RESERVOIR PRESSURE WITH TIME(DP/DT) ~PSI/SEC
C        DPRV= PRESSURE ABOVE RELIEF VALVE CRACKING PRESSURE ~ PSI
C       DPSDT= CHANGE IN SYSTEM PRESSURE WITH TIME ~ PSI/SEC
C       DQADT= RATE OF CHANGE OF ACCUMULATOR FLOW ~ CU.IN./SEC/SEC
C        DQFL= FLOW EXCEEDING FLOW LIMITER LIMIT ~CU.IN./SEC
C       DQPDT= RATE OF CHANGE OF PUMP FLOW ~ CU.IN./SEC/SEC
C          DT= TIME DURATION OF MOTION ~ SEC
C       DVGDT= RATE OF CHANGE OF GAS VOLUME ~ CU.IN./SEC
C       DVRDT= RATE OF CHANGE OF RESERVOIR VOLUME ~ CU.IN./SEC
C        DXDT= RESERVOIR PISTON VELOCITY ~IN./SEC
C     E
C         ELM= EQUIVALENT LINEAR MOTION ~ IN.
C     F
C           F= NET FORCE ON ACCUMULATOR PISTON ~ LBS
C         FBS= TOTAL DAMPING AND FRICTION FORCE ~ LBS
C          FD= ACCUMULATOR PISTON DAMPING
C          FF= TOTAL FORWARD FORCE ON ACCUMULATOR PISTON ~ LBS
C          FG= FORCE ON ACCUMULATOR PISTON DUE TO GAS PRESSURE ~ LBS
C         FLC= FLOW LIMITER FLOW CONSTANT ~ PSI/CU.IN./SEC
C        FLDP= FLOW LIMITER DIFFERENTIAL PRESSURE ~ PSI
C        FRES= NET FORCE ON RESERVOIR PISTON ~ LBS
C       FROIL= TOTAL FORWARD FORCE ON RESERVOIR PISTON ~ LBS
C         FRT= COULOUMB FRICTION FORCE ON ACCUMULATOR PISTON
C          FS= FORCE ON ACCUMULATOR PISTON DUE TO OIL ~ LBS
C         FXD= RESERVOIR PISTON FORCE DUE TO SPRING RATE ~ LBS
C     O
C     P
C        PACC= PUMP ACCELERATION ~RAD/SEC/SEC
C         PCV= PRESSURE DROP ACROSS CHECK VALVE ~ PSI
C         PDP= PUMP DIFFERENTIAL PRESSURE ~ PSI
C         PFE= PRESSURE DROP ACROSS FILTER ELEMENT ~ PSI
C          PG= SYSTEM GAS PRESSURE ~ PSI
C          PJ= INHERENT INERTIA OF PUMP ~IN-LBS-SEC
C         PMR= RESERVOIR PISTON MASS ~ LBSM
C          PR= REGULATOR VALVE CRACKING PRESSURE ~ PSI
C        PRES= PRESSURE IN RESERVOIR ~ PSI
C        PRPM= PUMP SPEED IN REV/MIN
C        PRPS= PUMP SPEED IN RAD/SEC
C         PRV= HYDRAULIC RELIEF CRACKING PRESSURE ~ PSI
C          PS= SYSTEM DIFFERENTIAL PRESSURE ~ PSI
```

```
C      Q
C          QA= ACCUMULATOR FLOW ~ CU.IN./SEC
C         QAL= PREVIOUS VALUE OF QA
C          QD= DEMAND FLOW ~ CU.IN.
C         QFL= FLOW LIMITER LIMIT ~ CU.IN./SEC
C        QNET= NET SYSTEM FLOW ~ CU.IN./SEC
C          QP= PUMP FLOW ~ CU.IN./SEC
C       QPGPM= PUMP FLOW IN GPM
C         QRV= RELIEF VALVE FLOW ~ CU.IN./SEC
C         QSL= SERVOVALVE LEAKAGE FLOW ~ CU.IN./SEC
C        QSUM= SUMMATION OF RESERVOIR FLOWS ~ CU.IN./SEC
C      R
C        RESD= RESERVOIR DAMPING(ASSUMED=0.0 IN MODEL)
C        RGAS= GAS CONSTANT (1545/MOL.WT.)
C         RPA= RESERVOIR PISTON AREA ~ SQ.IN.
C        RPAR= RESERVOIR PISTON AREA ON RETURN SIDE ~ SQ.IN.
C         RSR= RESERVOIR SPRING SPRING RATE ~ LBS/IN
C         RTG= GAS CONSTANT * TEMPERATURE OF GAS
C        RVFC= RELIEF VALVE FLOW CONSTANT ~ CU.IN./SEC/PSI
C      S
C        SFOP= SPRING FORCE ON PISTON ~ LBS
C      T
C         TCM= TORQUE CONSTANT OF MOTOR ~CU.IN.
C         TFL= TORQUE DUE TO FLOW LIMITER ~ IN-LBS
C         TFR= CONSTANT FRICTION TORQUE IN SYSTEM ~ IN-LBS
C          TG= TORQUE DUE TO GAS SUPPLY ~ IN-LBS
C         TGG= GAS TEMPERATURE ~ DEGREES R
C          TN= NET TORQUE ~ IN-LBS
C         TNA= TOTAL NUMBER OF ACTUATORS
C          TP= TOTAL BACK TORQUE ON PUMP ~ IN-LBS
C          TS= BACK TORQUE DUE TO SYSTEM OIL PRESSURE ~ IN-LBS
C      V
C        VACC= ACCUMULATOR OIL VOLUME ~ CU.IN.
C          VG= GAS VOLUME IN SYSTEM ~ CU.IN.
C        VOIL= SYSTEM OIL VOLUME WITH ACCUMULATOR EMPTY ~ CU.IN.
C        VRES= RESERVOIR VOLUME ~ CU.IN.
C       VRESI= INITIAL RESERVOIR VOLUME ~ CU.IN.
C        VSYS= TOTAL SYSTEM OIL VOLUME ~ CU.IN.
C      W
C       WDOT1= RTG*(GAS GENERATOR FLOW - PRESSURE REGULATOR FLOW)
C       WDOT2= A CHECK CALCULATION ON REQUIRED GAS FLOW
C         WPG= REQUIRED GAS GENERATOR FLOW RATE~ LBSM / SEC
C         WPR= PRESSURE REGULATOR FLOW ~LBSM / SEC
C      X
C           X= RESERVOIR PISTON LINEAR MOTION ~ IN.
C        XMAX= MAXIMUM X
C        XMIN= MINIMUM X
C      Z
C          ZD= ACCUMULATOR PISTON DAMPING CONSTANT
C
C THIS CONCLUDES THE LIST OF VARIABLES.THE PROGRAM FOLLOWS.
```

```
   10 READ(5,*)DT,ELM,TNA,VACC,VG,PG,PS
      READ(5,*)VOIL,VRES,VRESI,VSYS,ZD,PR,PRV
      READ(5,*)AG,AKM,AKRV,AO,APA,BMOIL,BR,DP,DXDT
      READ(5,*)FLC,FRT,FCV,FFE,FJ,FMR,FRES,QA,QFL
      READ(5,*)QP,QSL,RESD,RGAS,RPA,RPAR,RSR,RVFC
      READ(5,*)TCM,TGG,WPG,X,XMAX,XMIN,APM,DVRDT
     WRITE(6,60)
      AR=AG/AO
     J=0
     DO 80 J=1,15
     J=J+1
      QSL1 = QSL + 0.15
     I = 0
     DO 50 I=1,250
     I=I+1
      QD=TNA*APA*ELM/DT+QSL
      RTG=RGAS*TGG
      DVGDT=QA*AR+BR
      VG=DVGDT*DT+VG
      DPR=PG-PR
      WPR=AKRV*DPR
      IF(DPR.LT.0.0)WPR=0.0
      WDOT1=RTG*(WPG-WPR)
      WDOT2=DVGDT*PG+((AKM*QP*PG)/DP)
      DPGDT= (WDOT1-WDOT2)/VG
      PG=DPGDT*DT+PG
      FD=QA*ZD
      FG=AG*PG
      FS=AO*PS
      FF=FG-FS
      FBS=FD+FRT
      F=FF-FBS
      DQADT=AO*F/APM
      QA=DQADT*DT+QA
      IF(ABS(FBS).LT.ABS(FF))GO TO 20
      IF(QA*QAL .GT. 0.0) GO TO 20
      QA=0.0
      QAL=0.0
      GO TO 21
   20 CONTINUE
      QAL=QA
      IF(QA.EQ.0.0)FRT=0.0
      IF(DQADT.EQ.0.0)FRT=0.0
   21 CONTINUE
      VSYS= (-1.0*QA*DT)+VSYS
      IF(VSYS.GE.VOIL.AND.VSYS.LE.VOIL+VACC)GO TO 25
      IF(VSYS.LE.VOIL)VSYS=VOIL
      IF(VSYS.GE.VOIL+VACC)VSYS=VOIL+VACC
      QA=0.
      DQADT=0.
   25 CONTINUE
      DPRV=PS-PRV
      QRV=RVFC*DPRV
      IF(DPRV.LT.0.)QRV=0.
      QSUM=QP-QD-QRV
      QNET=QA+QSUM
```

```
         DPSDT=QNET*BMOIL/VSYS
         PS=DPSDT*DT+PS
         IF(PS.LE.0.)PS=0.
         DQFL=QP-QFL
          FLDP=FLC*DQFL
         IF(DQFL.LE.0.)FLDP=0.
         PDP=PS+FLDP+PCV+PFE
         TP=PDP*DP
         TS=PS*DP
         TFL=FLDP*DP
         TG=PG*TCM
         QPGPM=QP/3.85
         TN=TG-TP-TFR
         PACC=TN/PJ
         DQPDT=DP*PACC
         QP=DQPDT*DT+QP
         IF(QP.GT.0.)GO TO 26
         QP=0.0
         TN=0.0
   26    CONTINUE
         DPRES=BMOIL*DVRDT/VRES
         PRES=DPRES*DT+PRES
         FROIL=PDP*RPA+PRES*RPA-PRES*RPAR
         SFOP=X*RSR
         FXD=DXDT*RESD
         FRES=FROIL-SFOP-FXD
         DXDT=DXDT+FRES*DT/PMR
         X=DXDT*DT+X
         IF(X.GT.XMIN)GO TO 30
         X=XMIN
         DXDT=0.
   30    CONTINUE
         IF(X.LT.XMAX)GO TO 35
         X=XMAX
         DXDT=0.
   35    CONTINUE
         VRES=VRESI-X*RPA
         DVRDT=DXDT*RPA
         IF(VRES.GT.VRESI)VRES=VRESI
         PRPS=QP/DP
         PRPM=PRPS*9.5493
        DO 90 I=248,250
   90    WRITE(6,70)QA,QP,PRPM,PG,PS
   70    FORMAT(F10.3,F14.3,F10.3,F9.1,F13.1)
   50   CONTINUE
         WRITE(6,60)
   60    FORMAT(65H  ACCUM. FLOW  PUMP FLOW  PUMP RPM  GAS PRESS.  SYS. PR
        1ESS.        )
   80   CONTINUE
         GO TO 10
         END
```

APPENDIX II

Typical Printout of Sample Run

ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
0.0	11.409	1650.764	2038.3	3079.9
0.0	11.409	1650.764	2038.3	3079.9
0.0	11.409	1650.764	2038.3	3079.9
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
0.0	11.787	1705.460	2038.8	3098.0
0.0	11.787	1705.460	2038.8	3098.0
0.0	11.787	1705.460	2038.8	3098.0
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
0.0	12.155	1758.669	2039.2	3116.8
0.0	12.155	1758.669	2039.2	3116.8
0.0	12.155	1758.669	2039.2	3116.8
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
0.0	12.512	1810.334	2039.6	3136.2
0.0	12.512	1810.334	2039.6	3136.2
0.0	12.512	1810.334	2039.6	3136.2
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
0.0	12.858	1860.401	2040.0	3156.1
0.0	12.858	1860.401	2040.0	3156.1
0.0	12.858	1860.401	2040.0	3156.1
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
0.0	13.193	1908.819	2040.3	3176.6
0.0	13.193	1908.819	2040.3	3176.6
0.0	13.193	1908.819	2040.3	3176.6
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
0.0	13.516	1955.538	2040.6	3197.6
0.0	13.516	1955.538	2040.6	3197.6
0.0	13.516	1955.538	2040.6	3197.6
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
0.0	13.827	2000.510	2040.9	3219.2
0.0	13.827	2000.510	2040.9	3219.2
0.0	13.827	2000.510	2040.9	3219.2
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.

ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
0.0	14.678	2123.649	2041.1	2277.1
0.0	14.678	2123.649	2041.1	2277.1
0.0	14.678	2123.649	2041.1	2277.1
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
19.162	16.050	2322.268	2041.3	1367.2
19.162	16.050	2322.268	2041.3	1367.2
19.162	16.050	2322.268	2041.3	1367.2
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
65.462	17.900	2589.886	2040.8	534.2
65.462	17.900	2589.886	2040.8	534.2
65.462	17.900	2589.886	2040.8	534.2
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
135.795	20.054	2901.486	2038.7	0.0
135.795	20.054	2901.486	2038.7	0.0
135.795	20.054	2901.486	2038.7	0.0
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
220.359	22.203	3212.412	2034.4	0.0
220.359	22.203	3212.412	2034.4	0.0
220.359	22.203	3212.412	2034.4	0.0
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
302.436	24.344	3522.265	2027.4	0.0
302.436	24.344	3522.265	2027.4	0.0
302.436	24.344	3522.265	2027.4	0.0
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
381.992	26.476	3830.691	2018.2	0.0
381.992	26.476	3830.691	2018.2	0.0
381.992	26.476	3830.691	2018.2	0.0
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.
459.004	28.595	4137.375	2006.9	0.0
459.004	28.595	4137.375	2006.9	0.0
459.004	28.595	4137.375	2006.9	0.0
ACCUM. FLOW	PUMP FLOW	PUMP RPM	GAS PRESS.	SYS. PRESS.

APPENDIX III

Summary of Values of Constants Used in
Checkout Runs

Summary of Values of Constants Used in Checkout

The following list is a repeat of pages vi and vii, except that the values assigned the various constants in making a checkout run on the model developed in this report are delineated. These values were derived using the known operating characteristics of an existing hydraulic control system. Refer to the aforementioned pages for definitions of the symbols.

Symbol	Value
α	0.158
Ag	16.75 sq in.
A_O	11.76 sq in.
A_{Pg}	*
A_R	Calculated in model
D_P	0.066 cu. in/rad
I	0.0076 in. lb/s^2
K_{FL}	819.7 psi/cu. in/s
K_M	0.119 cu. in/rad
K_{PRV}	0.20 cu. in/s/psi
K_{RV}	0.002 lbsm/s/psi
M_{AP}	4.55 lbsm
n	*
P_{DP}	Calculated in model
Pg	2000 psi (initialized)
$\dot{P}g$	Calculated in model

P_{RV}	3450 psi
P_{RVD}	Calculated in model
P_S	0.0 psi (initialized)
Q_A	0.0 (initialized)
Q_P	0.0 (initialized)
R	996.0 in lbsf/lbsm-°R
T_M	Calculated in model
V_{AI}	Calculated in model
V_{FA}	40.0 cu. in.
Vg	88.0 cu. in.
$\dot{V}gg$	4.6 cu. in/s
Vgi	88.0 cu. in.
Wg	*
$\dot{W}gg$	0.225 lbsm/s
$\dot{W}_M$	*
$\dot{W}_{RV}$	Calculated in model
α	*
$\dot{\theta}$	*
ρ_M	*

*Symbols marked with an asterisk were not used in the actual model, but were used in the body of the report in deriving equations necessary to the model.

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