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The Effects of Introducing Skewness into Capital Rationing Decision Models

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THE EFFECTS OF INTRODUCING SKEWNESS INTO CAPITAL RATIONING DECISION MODELS

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ABSTRACT

When investment projects are described by subjective probability distributions, the measure of investment worth becomes a difficult task. One of the basic assumptions underlying investment analysis under risk is that decision makers would base their decisions on only the first two statistical moments of the probability distribution of returns. However, the mean and variance can adequately describe only certain symmetric distributions such as the normal and the uniform distributions. As a result, if probability distributions of investment returns are actually asymmetric, the classic first two moments analysis ignores information (skewness) that is needed to make a better investment decision. Even though the importance of the third moment in project selection has been recognized, nowhere in the literature is there a successful application of the concept to a regular periodic decision process where the decision maker lacks full knowledge of his future as well as present investment opportunities. Therefore, it is the purpose of this research to investigate the effectiveness of utilizing the higher statistical moments in capital rationing situation.

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INTRODUCTION

Capital rationing can be defined as a situation in which an organization does not have and cannot obtain enough capital to make all of the investments that are available to it. The paramount problem that confronts the decision maker is to determine how the available capital should be allocated to the investment proposals that are competing for these funds. Since the decision maker is commonly forced to make decisions in the face of uncertainty about the future, it is this lack of certainty about the future that makes capital allocation decisions one of the most difficult and challenging tasks. The decision problem to be investigated in this research is as follows:

Statement of The Problem

Uncertainty About Future Investment Opportunities

When investment decisions are made on a regular periodic basis, one of the important considerations is the amount of information the decision maker can obtain about the future. One view of this problem is that the decision maker at the time of decision has complete knowledge about the investment opportunities that are to be selected for implementation in both the present and future. Another view of this problem is that the decision maker does not have any knowledge concerning future investment opportunities.

The assumption that a decision maker in most real world situations will have either complete information or no information about the future seems quite improbable. This study utilizes an approach which describes an investment framework that allows the decision maker some expectation as to future investment opportunities without requiring specific knowledge about particular investment proposals. This view describes some middle ground concerning the availability of information regarding the outcomes of future investments.

Uncertainty About Future Cash Flows

In view of the fact that investment decisions frequently require judgemental estimates about future events, complete information regarding future cash flows of the investment proposals is not likely. At each decision period, cash flows commonly are projected at the time the investment is first proposed, and at least implicitly, the future cash flows are considered to be subject to probabilistic deviation from their expected values. That is, while initial outlays in a given project are known with certainty, the future cash flows are only estimates that can be described by subjective probability distributions.

Statistical Moments in Capital Rationing

When investment projects are described by subjective probability distributions, the measure of investment worth becomes a difficult task. One of the basic assumptions underlying investment analysis under risk is that decision makers would base their decisions on only the first two statistical moments of the probability distribution of returns. However, the mean and variance can adequately describe only certain symmetric distributions such as the normal and the uniform distributions. As a result, if probability distributions of investment returns are actually asymmetric, the classic first two moments analysis ignores information (skewness) that is needed to make a better investment decision. Even though the importance of the third moment in project selection has been recognized, nowhere in the literature is there a successful application of the concept to a regular periodic decision process where the decision maker lacks full knowledge of his future as well as present investment opportunities. Therefore, it is the purpose of this research to investigate the effectiveness of utilizing the higher statistical moments in capital rationing situation.

Objectives of the Research

The primary purpose of the research is to develop a decision model useful in hedging uncertainty stemming from an investment decision process. Another purpose of this research is to develop an understanding of a dynamic decision process where the decision maker has neither complete information regarding future investment opportunities nor complete information regarding the cash flows of the investment proposals. These objectives will be accomplished in two ways.

Development of a Decision Model Considering the Higher Statistical Moments

A set of decision rules which incorporate the concept of profitability, variability and skewness of investment proposals' returns will be utilized and incorporated into a single index model. The criterion will be called Expectation-Variance-Skewness (EVS) Criterion.

Simulated Investment Settings

Given the lack of available actual data and the need to examine the performance of the models under a variety of conditions regarding investment settings, the logical alternative is to select a simulated environment in which the important parameters generating investment data could be controlled. By applying the EVS and two other well accepted decision models in the literature to identical groups of projects through computer simulation, the effectiveness of these criteria will be compared. The two decision models are expected value criterion and Mean-Variance Criterion.

The primary contribution of this research is to answer the following specific questions:

1) What improvement in investment proposal selection can be attained with incorporation of the higher statistical moments in capital rationing problem?

2) How does the EVS criterion perform with respect to the other existing investment decision models which do not consider explicitly the higher statistical moments of the probability distribution of investment return?

Plan of Study

Chapter II provides a review of the literature related to the issues raised in the various areas of the research problem. The review of the literature reveals that while there are discussions regarding the desirability of incorporating skewness in measuring investment worth, nowhere in the literature is there a successful application of the concept to capital rationing situations.

Chapter III discusses the role of the third moment in capital rationing problems. A measure of the third moment-skewness is derived based on the principles of expectation and expected utility. This derivation, especially through the later principle, makes it obvious as to what kind of skewness (positive or negative) risk-aversive investors might prefer. Then, the EVS criterion is precisely defined.

Chapter IV describes the features and assumptions of the simulation model which is used to test the effectivness of these criteria in the investment decision process. The input parameters, the shapes of the probability distributions used, the method of generating cash flows, variance, and skewness of the investment proposals are presented. Also, the starting conditions and other elements of the simulation are presented.

Chapter V presents the simulation results and the analyses of the data regarding the objectives of this study. Three types of investment situations are described and their specific investment parameters are defined. Based on these investment settings, the effectiveness of the EVS criterion is compared to the other tested decision criteria. To examine the effects of the risk parameters on the performance of the criteria, the sensitivity of the specific input parameters is analyzed. Chapter VI contains the summary and conclusions of the research.

CHAPTER II

Review of Related Literature on Capital Budgeting

Due to the significance of the investment decision-making process of the firm, extensive effort has been directed at the problem of capital rationing under risk. Results and ideas that stem from this effort have been reported in the literature of a variety of disciplines such as accounting, business, economics, financial management, operations research, and industrial engineering. In particular, the importance of considering skewness have been recognized in recent literature dealing with capital rationing. This chapter reviews the existing capital rationing models which treat skewness explicitly.

Nondeterministic Capital Rationing Models

The future is rarely known with certainty and thus capital rationing decisions are normally based on predictions about the future. Depending on the difficulty in predicting future outcomes, decision outcomes may be divided into two categories, namely, those that involve risk and those that involve uncertainty.

The distinction between these two terms made in this research is that decisions involve <u>risk</u> if the probabilities of the alternative possible outcomes are known while uncertainty implies that the frequency distribution of the possible outcomes is not known.

The Concept of Risk

The concept of risk most widely used in the literature is the variability of return, which is measured by the variance or standard deviation [1,2]. This means that the more an investment's return varies about its expected return, the larger is the investor's risk. When variance is used as a measure of risk, it implies that deviations below expected value are regarded the same as deviations above the expected value. This measure of risk has become popular mainly due to its ease of computation and familiarity.

Recently, there has been considerable interest in using semivariance rather than variance as a measure of risk [3, 4]. Unlike the variance, semivariance is a measure of "downside" risk and does not consider the possibility of a large favorable return to be a risk.

Another measure of risk common in capital rationing literature is the probability of loss. If risk is defined as the chance of experiencing a loss, this measure is the area of a probability distribution which lies below the point of profitability. In this research, the risk of an investment proposal is defined by the variability about its expected value.

Capital Rationing Criteria With Probabilistic Considerations

Most often, investment proposals have been analyzed by using net present value as a criterion function, even though the choice of a criterion for optimization is rather difficult. Numerous decision criteria have been presented in the literature for evaluating risky investment proposals given some budgetary and other constraints [1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Most of these criteria only consider the first two moments (mean and variance) in the evaluation of the economic desirability of investment proposals. However, the introduction of higher moments beyond the mean and variance into the capital rationing decision model would be of importance. Thus, this study examines the effectiveness of incorporating the first three moments in capital rationing decisions.

Capital Rationing Criteria Based on the First Two Moments

Expected Value Maximization

The expected value maximization decision model is formulated based on the assumption that the decision maker is risk indifferent and is only interested in selecting the feasible solution vector having the largest expected net present value without violating the budget constraint. This type of model has been discussed by Weingartner [13].

The linear programming formulation of the expected value maximization criterion is, therefore,

Model I: Maximize $Z \sum_{i=1}^{n} E_i X_i$

Subject to: $\sum_{i=1}^{n} Ci\ Xi \leq B$

where,

$$Xi = \begin{cases} o \text{ if project i is not selected} \\ 1 \text{ if project i is selected} \end{cases}$$

and

E_i = expected present value of proposed i

C_i = first cost or initial outlay of proposal i

B = available budget

Expected Value - Variance Criterion

The expected value - variance (EV) criterion as proposed by Markowitz [14] and reformulated by Weingartner [13,15] consists of successively minimizing a portfolio's variance for each of a number of expected values or expected returns. Weingartner's approach is often referred to as the portfolio approach and is based on 0-1 conditions to reflect proposal's indivisibility.

The EV model requires the stipulation of the rate of trade-off (λ) between the reduction in expected value and reduction in variance, and it also assumes that variance of a return is a measure of risk. The linear programming formulation of the EV criterion assuming statistical independence among proposals is,

Model II: Maximize $Z = E - \lambda\sigma^2$

$$= \sum_{i=1}^{n} E_i\ X_i\ \ -\lambda(\sum_{i=1}^{n} \sigma_i^2 X_i)$$

subject to:

$$\sum_{i=1}^{n} C_i X_i \leq B$$

$$X_i = (0, 1)$$

which can be solved as a zero - one integer programming problem.

Capital Rationing Criteria Based on the First Three Moments

The need to include the third moment in the evaluation of risky investment proposals has been discussed by many authors including [7, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24]. The third moment can be measured by the skewness of the probability density function and this is the approach taken in this study. Skewness can be either positive or negative depending on the direction of the "tail" of the distribution.

Two main reasons have been cited for incorporating skewness in capital rationing decisions. First, use of only the first two moments is restrictive in the sense that only normally distributed asset returns are appropriate, which is not always the case in real world investment settings [24]. Secondly, since positive skewness is associated with a large right tail ('upside potential"), all other things being equal, it can be reasonably assumed that risk aversive investors will prefer right-skewness and dislike left-skewness ("down-side risk") [19, 23, 25].

Stone [26] introduces skewness into the capital rationing model with the extension of the mean-variance criterion (model III) and the resulting criterion is referred to as the expected value-variance-skewness (EVS) criterion. To use the EVS criterion, another trade-off parameter (δ) is desired. Stone's EVS model, which will be used in this study (with some modifications) requires the maximization of the parametric objective function $E_p - \theta\sigma_\rho^2 + \lambda S_\rho^3$ where E_ρ is expected portfolio return, σ_ρ^2 is the variance of portfolio return, S_ρ^3 is portfolio skewness and θ and λ are risk parameters [26]. Since the solution of a general cubic programming model is not feasible, he developed a linear approximation to the cubic programming model. With a single - index model, portfolio skewness can be represented as:

$$S_\rho^3 = B_\rho^3 S_m^3 + \sum_{i=1}^{n} X_i^3 S_i^3$$

where S_m^3 is skewness of return on the market index and S_k^3 is skewness of the independent random component of a security's return. Allowing an investor to have different attitude toward market skewness and indepent skewness, Stone's EVS criterion is:

$$\text{Maximize } Z = E_\rho - \theta_1 \sigma_m B_\rho - \theta_2 \sum_{i=1}^{n} V_i X_i^2 + \lambda_1 S_m B_\rho + \lambda_2 \sum_{i=1}^{n} S_i^3 X_i^3$$

$$\text{Subject to: } \sum_{i=1}^{n} X_i = 1$$

$$0 < X < P \text{ for } i = 1,\ldots,N$$

where

p = maximum fraction of the portfolio that may be held in any one security

B_p = market response of security p

X_i = fraction of the portfolio invested in security i

V_i = variance of return on security i

Even though potential importance of considering higher moments, especially skewness, in capital rationing decisions has been addressed by several authors, nowhere in the literature is there a successful application of the concept to a capital rationing situation under risk. Therefore, it is the purpose of this research to investigate the effectiveness of utilizing the higher statistical moments in capital rationing environments.

CHAPTER III

Decision Criteria Incorporating Skewness

In Chapter II, the literature review revealed the interest of many authors in considering higher moments, especially the third moment, in capital rationing decision models. This chapter discusses skewness specifically in terms of measures, implications and its effects in capital rationing decisions. It also discusses a decision criterion - the expected value - variance - skewness criterion - that incorporates skewness. Then, a zero-one linear programming formulation of this criterion is developed for statistically independent proposals.

Measures of Skewness

Most statistical literature acknowledges the existence of probability distributions which are asymmetrical or skewed; however, a general measure of this characteristic has never been resolved [27]. The family of unimodal asymmetrical distributions is characterized as having one tail longer than the other with the probability function rising more steeply on the short-tail side of the mode, depending on the direction of the skew. An assymmetrical distribution with a long right tail and sharply rising left tail is said to be *positively* skewed. On the other hand, if the distribution has a long left tail and a sharply falling right tail, it is said to be <u>negatively</u> skewed.

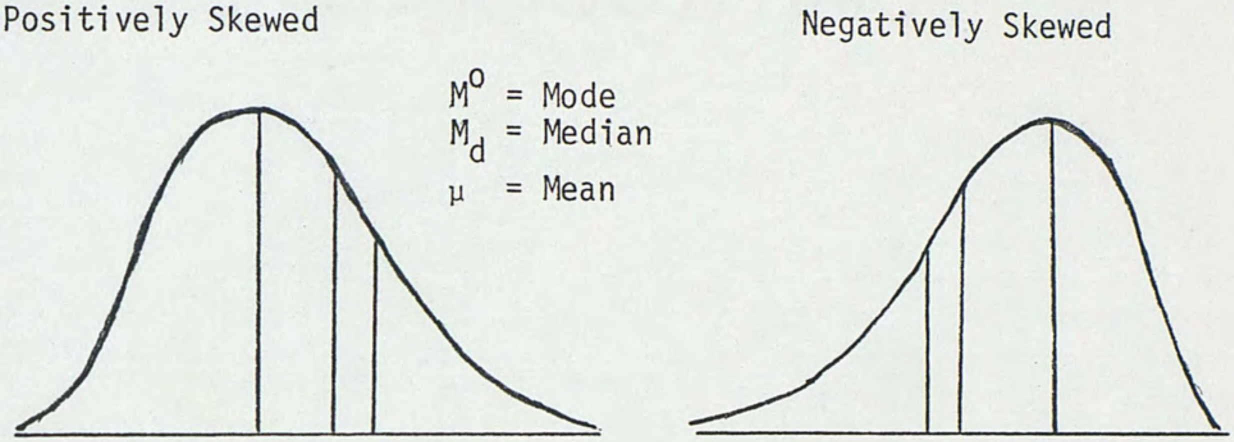

Figure 1. Typical unimodal skewed (continuous) probability distributions

As a measure of skewness, there are three main categories of definitions; momental skewness, Pearsonian Measure of skewness and order-statistic measure of skewness. [28]. However, Becker [27] concludes that the momental measures of skewness provide the best theoretical measure of skewness for a Theory of Parameter-preference security valuation.

Momental measures of skewness are further classified according to how one treats scale dependency, i.e., third central moment and relative skewness. The third central moement is defined to be the expectation of deviations from the mean cubed.

$$M = E[(X-E[X])^3] \tag{3.1}$$

Since M is scale dependent, numerous authors [27, 28] have noted this and have suggested that skewness be measured instead by the third moment relative to its dispersion. Hence, the coefficient of skewness or relative skewness is measured to be

$$\alpha = \frac{\mu}{\sigma^3} .$$

However, M and α may be equal to zero even though the distribution is asymetric. Unfortunately, there is no way that this condition can be detected or corrected from the properties of moments from the above. Thus, M (or α) must always be interpreted with some caution [27]. Even though many authors claim the superiority of α over M as a measure of skewness because of scale independency, it is not always true in the capital rationing area. The reason is that capital rationing decisions are rather sensitive to the actual magnitudes of cash flows so that scale dependance is a more desirable measure [27]. Because of computational simplicity and wide acceptance of the concept, M will be used as a measure of skewness in this research.

Momental Measure of Skewness for Portfolio

Equation (3.1) expressed the third central moment of the return on a single project. In this section, a measure of skewness of a group of independent projects using the third central moment approach is discussed. The approach taken is similar to that adopted by Jean [21], and it will be illustrated using an arbitrary investment situation.

Consider a decision maker faced with three investment proposals each having an estimated return of R_1, R_2, and R_3 respectively. If all three proposals are financed, the estimated total return is $(R_1 + R_2 + R_3)$. If we let:

$$r_1 = R_1 - E(R_1)$$

$$r_2 = R_2 - E(R_2) \quad \ldots\ldots\ldots \quad (3.2)$$

and $$r_3 = R_3 - E(R_3)$$

$M \quad (R_1 + R_2 + R_3)$ = third central moment of $(R_1 + R_2 + R_3)$

by definition,

$$M\,(R_1 + R_2 + R_3) = E(r_1 + r_2 + r_3)^3$$

expansion of $(r_1 + r_2 + r_3)^3$ and taking the expected values of the expanded terms and using the property that the expected value of a sum is equal to the sum of expected values, we obtain:

$$M = E(r_1^{\,3}) + E(r_2^{\,3}) + E(r_3^{\,3}) + 6E(r_1 r_2 r_3) + 4E(r_1^{\,2} r_2) + 4E(r_1 r_2^{\,2}) + 3E(r_1^{\,2} r_3) + 3E(r_1 r_3^{\,2}) + 2E(r_2^{\,2} r_3) + 2E(r_2 r_3^{\,2}) \ldots (3.3)$$

where,

$$E(r_1^{\,3}) = M(r_1)$$

$$E(r_2^{\,3}) = M(r_2)$$

$$E(r_3^{\,3}) = M(r_3) \quad \text{by definition.}$$

Also, since independence of the proposals are assumed,

$$E(r_1 r_2 r_3) = E(r_1)\,E(r_2)\,E(r_3)$$

but $E(r_1) = E(r_2) = E(r_3) = 0$ and this implies that $6E(r_1 r_2 r_3)$
$= 4E(r_1^2\ r_2) = 0$ and so also a all the
terms in equation (3.3) except the first three.

Therefore, for independent projects, the momental measure of skewness of the projects' returns is equal to the sum of the third central moments of the individual returns. Or symbolically,

$$M = E(r_1^3) + E(r_2^3) + E(r_3^3)$$

or

$$M(R_1+R_2+R_3) = M(r_1) + M(r_2) + M(r_3)$$

In general terms,

$M(R_i) = \sum_i M(r_i)$, where $i = 1, \ldots, n$ =total number of investment projects;

R_i = represents the return on project i, and r_i are as defined in equations (3.2). Now that a means of measuring skewness has been established, the next section examines the implications of skewness in project selection.

Implications of Skewness in Project Selection

The principle of expected utility theory can be used to specify and demonstrate the role of skewness in investment decisions. If we let u(v) be a time-invariant utility function for money where v is the amount of money subject to a statistical distribution, u(v) can be expanded around the mean cash flow

E(v) by the Taylor series to obtain:

$$U(V) = U[E(V)] + U^{1}[E(v)][V-E(V)]+U^{11}\frac{[E(V)][V-E(V)]^{2}}{2^{1}_{\cdot}}$$

$$+ U^{111}\frac{[E(V)][V-E(V)]^{3}}{3^{1}_{\cdot}} + \ldots\ldots\ldots\ldots\ldots \quad (3.4)$$

The expected value over V is taken of each side of equation (3.4) to derive the expected utility,

$$E[E(V)] = U[E(V)] + U^{1}[E(V)]E[V-E(V)] + \frac{U^{11}[E(V)]}{2^{1}} = E[V-E(V)]^{2}$$

$$+ U^{111}\frac{[E(V)]}{3^{1}_{\cdot}} E[V-E(V)]^{3} + \ldots\ldots\ldots\ldots\ldots \quad (3.5)$$

The first term on the right of equation (3.5) is U(V) evaluated at the mean cash flow. E[V-E(V)] is zero thus making the second term equal to zero; the third term is a constant $U^{11}\frac{[E(V)]}{2^{1}_{\cdot}}$ times the second moment (variance) of cash flows, while the forth is also a constant times the third moment (skewness) of cash flows.

The constant $U^{111}\frac{[E(V)]}{6}$ is positive if the utility function meets the usual conditions, a positive but decreasing marginal utility of money. Thus the third moment when multipled by this constant can be either negative or positive. If the distribution is positively skewed, the contribution of this fourth product element to utility is positive. For a risk-aversive decision maker who wants to maximize his expected utility, positive skewness will be preferred.

Thus, it can be deduced that given a set of investment proposals, an investor is most likely to select those that have higher positive skewness even though they might have lower expected returns. In otherwords, a decision maker who is risk aversive, under normal circumstances, is reluctant to undertake any investment that presents him with the possibility, however, small, of a large loss and only a limited gain, skewness, therefore, is a measure of this asymmetry factor. The following section discusses a measure of investment worth that incorporates skewness - the mean - variance - skewness criterion.

Mean - Variance - Skewness Criterion

As can be observed on page 15, positive skewness is associated with a large right tail. Thus, all other things being equal, it can be reasonably assumed that decision makers prefer positive skewness and dislike negative skewness. Based on this reasoning, Stone [26] postulates a generalization of the mean-variance criterion to a mean-variance-skewness (EVS) criterion with the following assumptions:

1. Portfolio return is maximized for given levels of variance and skewness.
2. Portfolio variance is minimized for given levels of portfolio return and skewness, and
3. Portfolio skewness is maximized for given levels of portfolio return and variance.

Thus, as a measure of investment worth, the following index is proposed to evaluate an investment proposal under risk:

$$Z_j = E_j - \lambda\ \sigma_j^2 + \delta M_j$$

where,

E_j = expected return on investment j

σ_j^2 = variance on return on investment j

M_j = skewness of the return on project j

and

λ, δ are risk aversion parameters set by the decision maker.

In order to apply the E-V-S criterion to a capital rationing situation, a mathematical programming appraoch is derived. Thus is the subject of the next section.

Capital Rationing With E-V-S Criterion

Stone's index of investment worth, Z_j, stated in the last section can be developed into a zero-one integer linear programming model of the E-V-S criterion. If Z_j is known or can be computed, and assuming statistical independence among the investment proposals, an efficient set of proposals can be obtained using the E-V-S criterion by maximizing Z_j over the range of the proposals. The L.P model can be stated as:

$$\text{Maximize } Z = \sum_{j=1}^{n} (E_j - \lambda\sigma_j^2 + \delta\mu_{3j})X_j$$

Subject to:

$$\sum_{j=1}^{n} C_j X_j \leq B$$

where,

$$X_j = \begin{cases} o & \text{if proposal } j \text{ is not selected} \\ 1 & \text{if proposal } j \text{ is selected} \end{cases}$$

and

C_j = first cost or initial outlay of proposal j

B = available budget for financing selected proposals

for $\delta = o$, the E-V-S criterion reduces to the mean-variance criterion and for $\lambda = \delta = 0$, the E-V-S criterion reduces to the expected value maximization criterion.

The E-V-S criterion developed in this section will thus be tested against mean-variance and the expected value maximization criteria discussed in Chapter II. To accomplish this, a computer simulation model was developed and the details and components of this model are presented in Chapter IV.

CHAPTER IV

The Description of the Simulation Model

In Chapters II and III, the EVS criterion was proposed along with two methods of selecting investment alternatives. The two methods are the expected present worth criterion and the mean-variance criterion. These two decision criteria are to be compared with the EVS criterion. This Chapter contains a detailed description of the simulation model used to test the effectiveness of these criteria. First, the specific assumptions of the simulation model used in this analysis are described. Then follows the description of the simulation process which is used to test the effectiveness of those decision criteria.

Basically, the simulation model consists of two parts. The first part of the simulation model includes the generation of a schedule of investment proposals (SIP) that are submitted at each decision period. In particular, detailed descriptions are given as to how the investment proposals are generated. The second part of the simulation model is the application of the different decision criteria to the schedule of investment proposals generated in the first part. In addition, the second part of the simulation consists of the accumulation and calculation of statistics to evaluate the performance of these decision rules.

Modeling Assumptions

The basic assumptions made in the simulation model are:

1. The firm's primary objective is to make investment decisions that promise to maximize its present worth with the limitation on funds available for investment. Although other goals are also legitimate, in order to keep the analysis manageable, these other goals are not considered in this study. Further, nonmomentary considerations which affect investment decisions are not considered in this study.
2. The size of each proposal's first cost is assumed to be known when it is proposed, but future cash flows are random in magnitude. This assumption seems valid because for many investment proposals cash outlays are known in advance but occur either at the beginning of the proposal's life or at given times during the earlier life of the proposal. Each proposal is also assumed to have a known investment life. Each investment proposal is considered to be an indivisible unit, and it is not possible to undertake "multiples" of any investment proposal.

Description of the Simulation Process

The simulation process can be separated into three phases in this study, and Figure 2 shows the general modeling process.

Phase I: The generation of a schedule of investment proposals containing proposals that are submitted within a specific time interval for the decision maker's consideration.

Phase II: The application of the three different decision criteria described in Chapter III to the SIP generated is Phase I. In addition, the second phase of the simulation consists of the accumulation and calculation of statistics relevant to the stated purposes of this study.

Phase III. Once Phase I and II are completed, the realizations of cash flows of all the projects generated during the study period are preserved. Given these realizations of the proposal's cash flows, the optimal solution to the decision problem is obtained. This result is found by solving a zero-one linear programming formulation of the model.

Generation of a Schedule of Investment Proposals

A schedule of investment proposals (SIP) is a set of

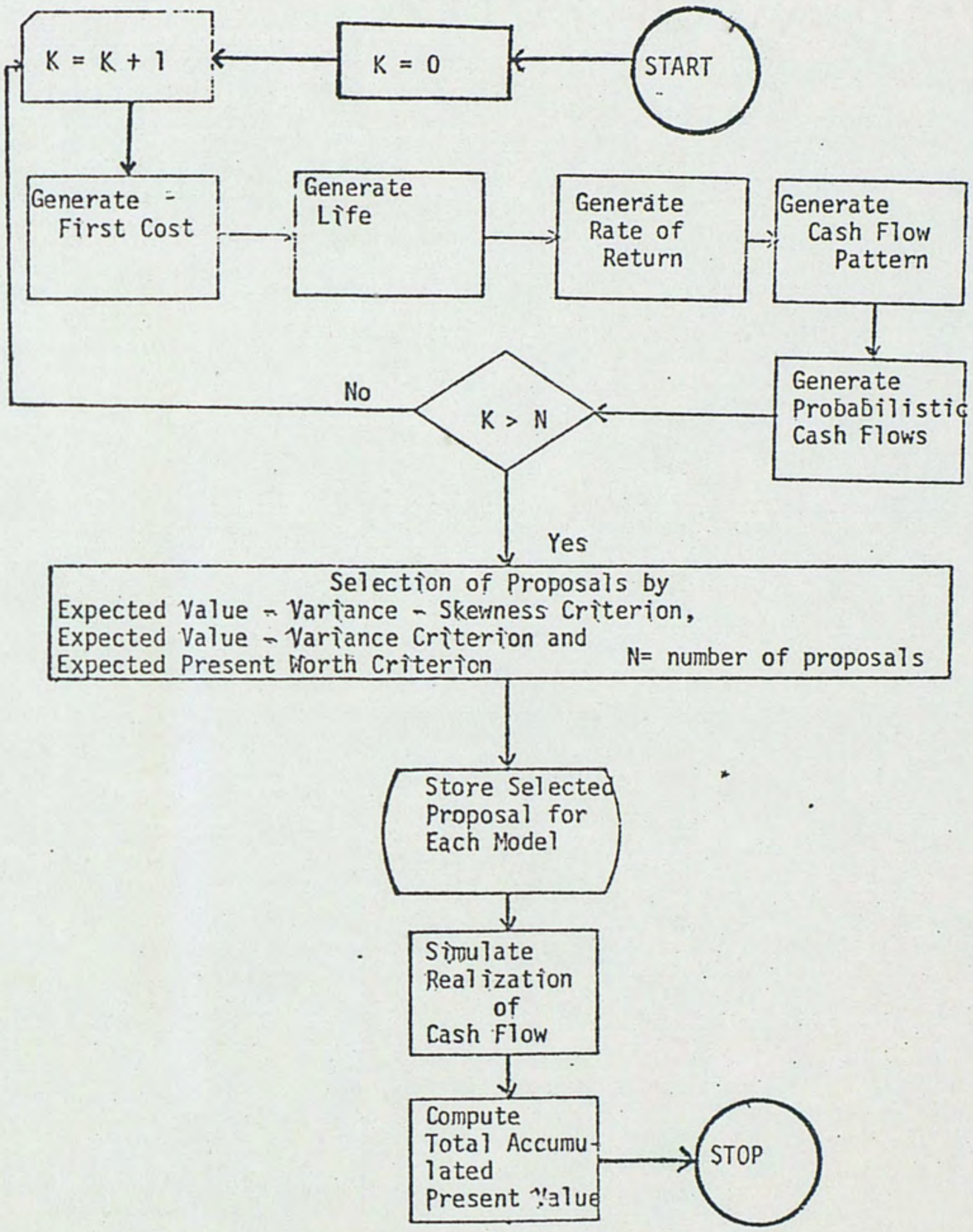

Figure 2. The general modeling process utilized in the study

investment opportunities submitted for consideration during a decision period. Due to the variability among proposals with respect to the size of investment, expected pattern of cash flows, life, and expected rate of return, it is reasonable to visualize a decision maker having schedules consisting of investment proposals drawn from an underlying distribution in which the size of investment, expected pattern of cash flows, life, and expected rate of return are all random variables.

In this section, the general framework for Phase I simulation is described. In order to generate a particular proposal, the following five basic characteristics are defined:

1. The interrelationships among proposals.
2. The initial investment required by the proposal.
3. The proposal life.
4. The rate of return.
5. The cash flow patterns (timing and magnitude)

Assumptions of Independence Among Proposals

The proposals generated in this analysis are all considered to be functionally independent. The adoption of the assumption that the proposals are independent eliminates the necessity of generating the covariances among the proposals.

The Proposal's First Cost

In the simulation, the generation of the proposal's first cost is based on the approach taken by Park [29]. He generates the first cost of the proposal from a C_0 distribution that is described by a mean first cost $\bar{C}_0$ and six other parameters which represent three different exponential distributions. Thus, the codistribution is essentially a combination of three exponential distributions placed so that the mean of the resulting C_0 distribution is $\bar{C}_0$. Graphically, this relationship can be depicted as in Figure 3.

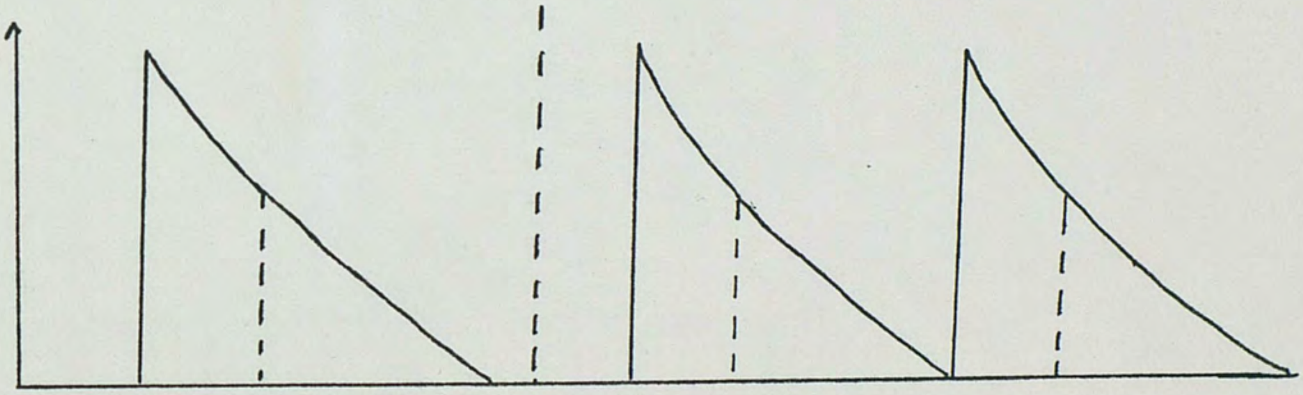

Figure 3. Combination of three exponential distributions

In Figure 3, the three exponential distributions correspond to

$$f(x) = (1/(c_i-a_i))e^{-(1/c_i-a_i))(x-a_i)}$$

where $x > a_i$, $i = 1,2,3$

and the cumulative distribution F(x)

$$F(x) = 1 - e^{-(1/(c_i-a_i))(x-a_i)}$$

Thus, x can be viewed as

$$x = a_i - (c_i - a_i)\ell n(1 - F(x)) \qquad (4\text{-}1)$$

Therefore, by specifying a_i and c_i, an exponential distribution can be placed anywhere on the x axis. By placing three such distributions on one axis, and by sampling from the three distributions an appropriate freaction of the time, it is possible to have the expected value of all the sampling equal to $\bar{C}_o$. To make sure the samples drawn in this way represent those from the C_o distribution, Park provides the following conditions that must be held between those distribution parameters and the fraction of time (f_i) one should sample from distribution i (i = 1,2,3):

$$\bar{C}_o = f_1c_1+f_2c_2+f_3c_3 \qquad (4\text{-}2)$$

$f_1+f_2+f_3 = 1$

$c_1f_1 = c_2f_2$ and $c_2 \leq C_o \leq c_3$

Several reasons are stated in [29] for using this rather complicated scheme in the generation of the proposals first cost. An algorithm for generating the proposal's first cost is shown in Appendix A.

Proposal's Life

In practice, it is common to observe that there are usually more investment proposals with short lives then with long lives. Therefore, the proposal life is generated from a single exponential distribution. Since a large number of proposals are generated throughout the study period, it is necessary to limit the maximum life which a proposal can take at $L_{max.}$ Thus, three parameters are used to define this truncated exponential distribution (L_{min}, $\bar{n}$, L_{max}).

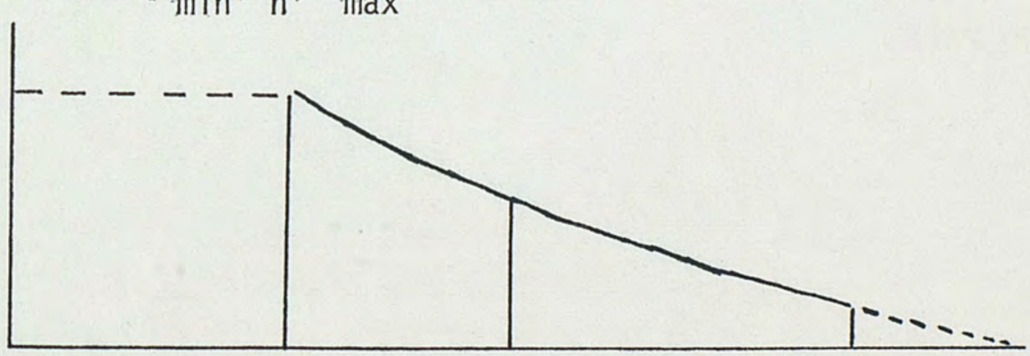

Figure 4. Average life distribution

Let L_{max} = the maximum proposal life specified in the shifted life distribution.

$\bar{n}$ = the average proposal life.

L_{min} = the maximum proposal life allowed in the life distribution.

An algorithm for generating the proposal life n is presented in Appendix A.

The Distribution of Investment Opportunities with Rate of Return g_k

The distribution of investment opportunities with growth rate g_k can be viewed as one which describes the average fraction f_k of dollars worth of proposals with growth g_k. This growth rate represents the internal rate of return of the proposal. That is, it is the rate which sets the receipts equal to the disbursements of the proposal.

The type of distribution of future schedule of investment opportunities utilized in this study is the exponential as shown in Figure 5. The maximum value for any g_k (internal rate of return) is 32% with the lower limit of 6%. The shape of this distribution reflects fact that the firm has a greate proportion of low-return proposals available than it has of high-return proposals. An algorithm for generating the rate of return for the proposals is in Appendix A.

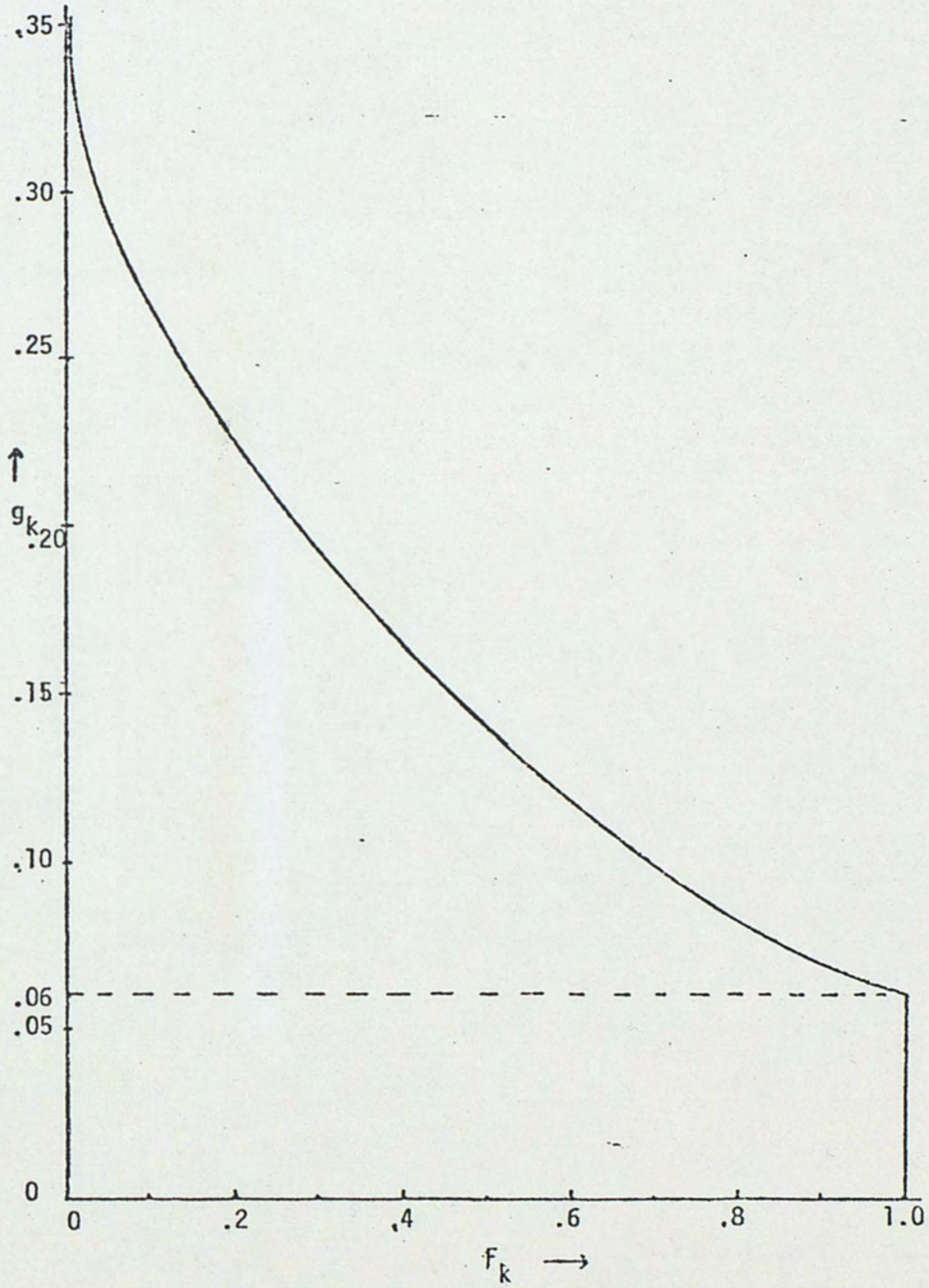

Figure 5. The distribution of investment opportunities (exponential shape)

Expected Cash - Flow Pattern

In order to generate the series of cash flows to be represented by a probability distribution, it is necessary to identify the expected pattern of a proposal's cash-flow receipts series. Once a proposal's first cost C_o, its life n, and its rate of return g_j are known, it is possible to determine the expected amount and timing of its cash flows, provided its cash-flow pattern is known. In this simulation, four basic cash-flow patterns are used to generate the probabilistic cash flows:

1. Single Payment.
2. Uniform Series.
3. Gradient Series (Increasing)
4. Gradient Series (Decreasing)

By using combinations of the gradient series patterns and the uniform series pattern, it is possible to generate an unlimited number of variations of these patterns in the simulations. Following Park's development in [29], a variety of combinations of cash flow patterns can be achieved by defining and controlling the size of R_2 relative to R.

$$R = R_1 + R_2 = C_o(^{A/P}\ g,^{n}) \qquad (4\text{-}3)$$

In the simulation a particular cash-flow pattern is randomly generated for each proposal from a predetermined distribution of cash-flow shapes. If the cash-flow pattern selected is a gradient series cash flow, a random choice is made between the increasing series and the decreasing series. Then the value of R_2 relative to R is determined in the simulation by a fraction f_R that is a random variable such that

$$0 \leq f_R \leq 1$$

and that

$$R_1 = f_R R$$

$$R_2 = (1 - f_R)R$$

Thus, when f_R is selected for either of the two combination series, the following distinctive cash-flow patterns result for the particular values of f_R shown below:

If $f_R = 0.$, the resulting series is strictly a gradient series.

If $f_R = 1.$, the resulting series is strictly a uniform series.

If $0 < f_R < 1.$, the resulting series is a combination of a uniform series and a gradient series.

Symbolically,

Q_1 = the probability of observing a single payment from the cash flow distribution.

$Q_1'=1-Q_1$ = probability of a series payment type cash flow.

Q_2 = the probability of the cash flow being a combination of decreasing series, if the proposal is a series payment type cash flow.

$Q_2'=1-Q_2$ = the probability of the cash flow being a combination of increasing series.

The computations of the expected cash-flow series F_t for a single proposal are given in Equation 4-4 and the logic to generate $E[F_t]$ is presented in Appendix A.

$$E[F_t] = R_1 + F\,(t-1), \text{ for increasing series.}$$

$$E[F_t] = R_1 + (n - 1)G - G(t-1), \text{ for decreasing series} \qquad (4\text{-}4)$$

Now, it is possible to determine the expected present value associated with the particular cash flow pattern generated. That is,

$$E[PW] = \sum_{t=1}^{n} E[F_t]/(1+i)^t - C_0 \qquad (4\text{-}5)$$

where

i = minimum attractive rate of return

Generation of Skewed Present Worth Distribution

In previous sections of this chapter, the expected cash flow patterns and other profiles of the proposals have been discussed. However, since the interest of this study is in skewness of returns and its effects in capital rationing, it is pertinent to generate skewed present worth distributions of the proposals. To do this, the beta probability function will be used. Choice of the beta distribution function is appropriate because of its flexibility in assuming different skewed shapes depending on the values of the parameters that describe it.

The Basic Beta Distribution

The probability density function of the basic beta distribution whose range is between 0 and 4 can be defined as:

$$f(x) = \frac{1}{B(a,b)} x^{a-1} (1 - x)^{b-1} \qquad (4\text{-}6)$$

where

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}, \text{ a and b} > 0$$

The shape of the beta distribution is determined by controlling the values of the two parameters, a and b, respectively. Table 1 summarizes the variety of shapes of Beta distribution for different combinations of a and b, and Figure 6 depicts the corresponding shapes of Beta distributions.

TABLE 1
Shapes of Beta Distribution
For Different Combinations
of a and b

Conditions on a and b	Shape of Beta Beta Distribution
1. $a=b=1$	Rectangular
2. $a=b>1$	Symmetric unimodal
3. $1<a<b$	Unimodal, positively skewed, mode at $(a-1)^*/(N-2)$
4. $1<b<a$	Unimodal, negatively skewed, mode at $(a-1)^*/(N-2)$
5. $a\leq 1$	J-shape, mode at 0 or 1
6. $a+b\leq 2$	U-shape, mode at 0 or 1

*a+b=N

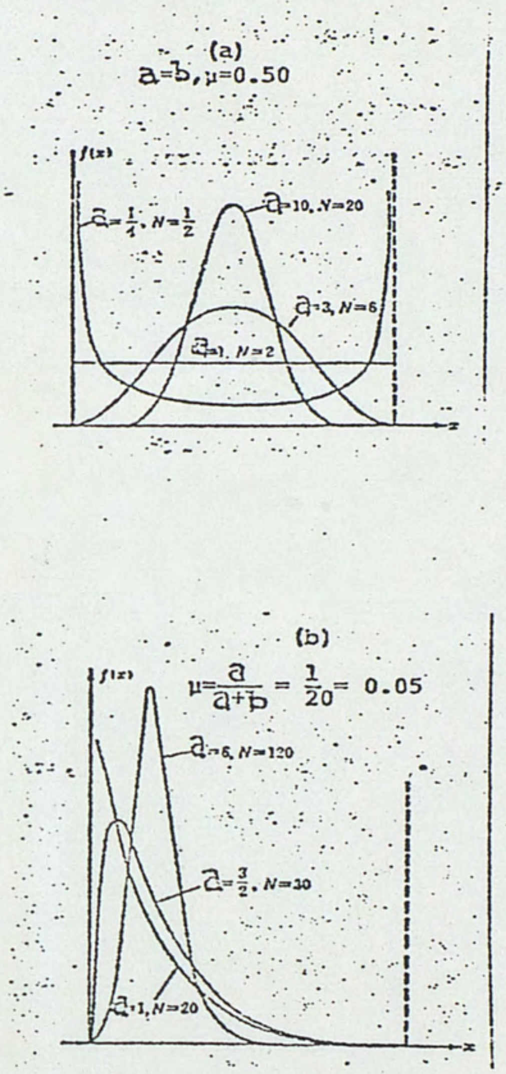

Figure 6. Shapes of Beta distribution for different combination of a and b

The Transformed Beta Distribution

Since investors will not be interested in expected returns that vary between 0 and 1 units, a transformation of the basic beta distribution to one with some practical range of limits are necessary. For the purpose of this study, the range of interest regarding a project's return is defined as (A,B), where A represents the worst or lowest expected return and B the best or highest possible value.

To go from X(0,1) to y(A,B), one can use the following linear relationship.

$$X = \frac{y-A}{B-A}$$

or,

$$y = A + (B-A)\,X \tag{4-7}$$

Moments of X(0,1) and y(A,B)

The moment generating function of X(0,1) is given by:

$$E[X^n] = \frac{(2a+2)\ (2a+4)\dots(2a+2n)}{v(v+2)(v+4)\dots.(v+2n-2)}, \quad \text{where}$$

$$v = 2a + 2b + 4$$

Therefore,

$$E[X] = \frac{a + 1}{a + b + 2} \tag{4-8}$$

$$E[X^2] = \frac{(a+1)(a+2)}{(a+b+2)(a+b+3)} \tag{4-9}$$

$$E[X^3] = \frac{(a+1)(a+2)(a+3)}{(a+b+2)(a+b+3)(a+b+4)} \tag{4-10}$$

and the variance of x is given by:

$$\text{Var}(x) = \frac{(a+1)(b+1)}{(a+b+2)^2(a+b+3)} \qquad (4\text{-}11)$$

From equation (4-7), the moment generating function of y can be represented as:

$E[y] = E[\{A+(B-A)X\}]$, which, after some mathematical manipulations [28] result in the following expression:

$$E[y^n] = \sum_{k=0}^{n} \binom{n}{k} (B-A)^k A^{n-k} E[X^k] \qquad (4\text{-}12)$$

Thus, from (4-12),

$$E[y] = A+(B-A)E[X] \qquad (4\text{-}13)$$

$$E[y^2] = A^2+2(B-A)A.E[X]+(B-A)^2E[X^2] \qquad (4\text{-}14)$$

$$E[y^3] = A^3+3(B-A)A^2.E[X]+3(B-A)^2A.E[X^2]+ (B-A)^3.E[X^3] \qquad (4\text{-}15)$$

The variance and skewness of y are thus given as follows:

$$\text{Var}[y] = E[\{y-E[y]\}^2]$$

or $$\text{Var}[y] = E[y^2] - (E[y])^2 \qquad (4\text{-}16)$$

and $$M[y] = \text{Skewness of } y$$

$$= E[\{y-E[y]\}^3]$$

$$= E[y^3]-3E[y]E[y^2]+2E[y^3] \qquad (4\text{-}17)$$

Thus, if the values of A,B, a, and b were known, the expected value, variance, and skewness of a project's return are readily obtained. The actual generation of the values of the above parameters are obtained through the Monte Carlo sampling.

Generation of Beta Random Deviates

Once the expected present worth of an investment proposal, E[y], is determined from Equation (4.5), the next step is to compute the variance and skewness of the investment proposal. For doing so, it is necessary to generate ranges of parameters of a, b, A and B for the Beta distribution. By sampling the parameter values from the following uniform distributions, if is possible to generate a variety of shapes of the Beta probability distribution.

$$a_1 \leq a \leq a_2$$
$$b_1 \leq b \leq b_2$$
$$A_1 \leq A \leq A_2$$

In order to make sure that the values of a, b, and A generated above give E[y] determined by Equation (4.5), the value of B can be found automatically by solving the following equation. From Equation (4.13), solving for B yields,

$$B = \frac{(a+b+2)E[y]+bA}{b+1}$$

Now, if a, b, A, and B are known, one can easily determine $E[y^2]$ and $E[y^3]$, and generate a Beta random deviate with a minimum value A and a maximum value B. The program logic to generate the Beta deviates is given in [30] and presented in Appendix A.

Measure of Effectiveness

The basis for comparing the effectiveness of different decision criteria is their ability to maximize the present value of the firm's capital over a given decision period. By applying each decision criterion to those proposals generated and keeping track of all realizations of cash flows associated with proposals undertaken, it is possible to determine the total present value of the firm's capital under each decision criterion. Thus, the total present value for applying decision criterion k is:

$$PW[RLZ]_k = \sum_{j=1}^{n} (RLZ_j)\, X_{kj} \tag{4-18}$$

where

(RLZ_j) = Realization of cash flow associated with proposal j

X_{kj} = 0, if proposal j is rejected under decision criterion k

1, if proposal j is accepted under decision criterion k

It is of interest to compare the present value obtained from the three decision criteria with the one obtained with complete knowledge about the future investment opportunities and the realizations of their future cash flows. Therefore, if complete information about the realizations of all the proposals is available at the time of capital allocation, it should be possible to select the better set of proposals. The global optimum with the perfect information is thus obtained from solving the following zero-one integer linear programming problem:

$$\text{Maximize } Z = \sum_{j=1}^{n} (RLZ_j)X_j$$

$$\text{Subject to } \sum_{j=1}^{n} C_j X_j \leq B$$

<u>Replication of Simulation Runs</u>

As discussed above, each of the decision criteria being considered is applied to the same set of proposals generated to compare the effectiveness of one decision criterion with another. A completion of one simulation run produces a single value of total present value, $PW[RLZ]_k$, for each criterion being tested. Since the present value obtained in each simulation run is a random variable, several simulation runs must be made to compute the mean of these present value, E_p, and the variability of the values about their mean, σ_p. The decision of how many runs should be made for a particular set of parameters can be determined by the variability that is observed in the total present value figure after some preliminary runs are made.

CHAPTER V

Simulation Results and Analysis

In this Chapter, the simulation results produced by the decision models described in Chapter III are presented and analyzed. To provide a background regarding the various simulations undertaken, three types of investment situations are described and their specific parameters are defined. The simulation results based on these investment settings are compared, and the effects of critical input parameters on the performance of each decision criterion are examined. In summary, this Chapter is to answer two specific questions;

1) what improvement in proposal selection can be attained with incorporating the skewness into a decision criterion other than the first two statistical moments? 2) what are the effects the choice of risk parameters on the effectivenss of the E-V-S criterion?

Description of The Investment Situations

Three types of investment settings are precisely defined in this section. These situations will be referred to subsequently as Case I , Case II, and Case III. The reason for evaluating each case in different investment settings is to provide a contrast between a SIP with all positively skewed distributions, a SIP with mixed composition of positive and negative skewed distributions, and a SIP with all negatively skewed distributions.

Case I - Parameters

1. Distribution of investment opportunities with rate of return g_k - exponential shape (see P. 32)
2. Discount rate (MARR) used = 10%
3. Size of average investment per proposal (see p. 28)

   $\bar{C}_o$ = \$15,000

   a_1 = \$6,000 c_1 = \$11,000

   a_2 = \$10,000 c_2 = \$15,000

   a_3 = \$14,000 c_3 = \$19,000

4. Proposal life in years (See p. 30)

   $L_{min} = 2$, $L_{AVG} = 5$, $L_{max} = 8$
5. Size of external funds available for allocation (see p. 28)

   B = \$150,000
6. Probability of a particular expected cash flow pattern (see p. 33)

   $Q_1 = 0.2$, $Q_2 = 0.6$, $0 \leq f_R \leq 1$
7. Number of investment proposals generated in the study period

   N = 50

8. The parameters of the Beta distribution (see p. 36).

The probability distributions of a, b and A are uniform distributions with

$1.0 \le a \le 2.0$

$2.0 \le b \le 3.0$

$0 \le A \le 500$

Case II - Parameters

1-7. Same as Case I

8. Fifty percent of investment proposals are positively skewed distributions whereas the other fifth percents negatively skewed ones with the following specific parameters:

Positively Skewed	Negatively Skewed
$2.0 \le a \le 3.0$	$2.0 \le a \le 3.0$
$3.0 \le b \le 4.0$	$1.0 \le b \le 2.0$
$0 \le A \le 500$	$0 \le A \le 500$

Case III - Parameters

1-7. Same as Case I

8. All the investment proposals are negatively skewed with the following parameters:

$2.0 \le a \le 3.0$

$1.0 \le b \le 2.0$

$0 \le A \le 500$

Risk Aversion Parameters

As discussed in Chapter III, a direct comparison of the EVS criterion with other decision criteria calls for specification of a coefficient of risk aversion (λ,δ) in advance. The same argument applies to the utilization of the E-V criterion in which a coefficient of risk aversion (λ) needs to be specified. Since different present values are possible for different values of λ or δ, it is desirable to define the efficient set of investment by varying the coefficient of risk aversion, while holding all other parameters fixed.

In the simulation, for a given value of λ (or δ), twenty runs are performed. Then E_p and σ_p are estimated from these 20 sample runs. For a different value of λ (or σ), another 20 runs are made, using the same parameters to compute E_p and σ_p. This procedure is repeated a number of times and the values of $E[E_p]$ and $E[\sigma_p]$ are plotted with $E[E_p]$ on the horizontal axis and $E[\sigma_p]$ on the vertical axis. In the simulation, the values of σ range from 0 to 0.00005 while the value of λ is fixed at 0.0002.

Simulation Results and Analysis

Case I

The simulation results for this investment situation are shown in Figure 7 and the detailed statistics are presented in Table 5 in Appendix C. In Figure 7, the values of a δ used for the E-V-S criterion range from 0. to .0, 0-05. Thus, the line connecting 0^1 and 0^5 represents the efficient frontier generated by the E-V-S criterion for a fixed λ value of 0.0002. The solid circle (O) represents the statistics for the E-V criterion while a shaded triangle (▲) does for the expected present value-variance criterion. As described in Chapter IV, if the decision maker has complete information about the SIP at the time of decision, the global optimum with this perfect information is denoted by a shaded rectangular (■).

For the E-V-S criterion, the greatest E_p value is obtained at δ = 0.00005, and as the value of δ increases, the value of $E[E_p]$ gradually increases. This increasing trend in the expected present value as δ becomes large can be explained as follows: From the linear relationship of $Z_j=E_j-\lambda\sigma_j^2+\delta M_j$ with M>0, it is evident that the effect of $\lambda\sigma_j^2$ will diminish and one can eventually observe the point where $\lambda\sigma_j^2 = \delta M_j$. When this happens, the project selection can be effected by only the first and third statistical moments.

Assigning a higher value of δ implies that for a proposal j, the requirement to meet the E-V-S criterion becomes less dependent upon the first two statistical moments (recall the fact that the value Z_j must be positive in order to be considered for investment by the E-V-S criterion). Thus, for a higher value of δ, the firm generally select the proposals with higher expected present values while placing a little emphasis on the variability of the investment proposals. Therefore, using a higher value of δ normally results in a larger $E[E_p]$ with a higher variability, $E[\sigma_p]$. When δ becomes extremely large, the E-V-S criterion virtually requires no consideration of the first two statistical moments. Theoretically, it is however possible to place an upper bound on the limit which δ can take for an expected utility maximizer, if his utility function is given. From Equation (3.5) δ could be set equal to $\frac{U^{III}}{3!}$ E(V). However, it is a rather difficult task to find such a utility function in the real world decision environment.

Figure 7 indicates that one can expect an improvement in project selection by incorporation the third statistical moment. Recall the fact that the point (▲) is obtained for the E-V criterion with a fixed sample of 20 runs. Thus, as the sample size changes, the location of ▲ is likely to change. Therefore, it is necessary to place a 95% confidence interval about this point to reflect any statistical significance bearing upon this random variable with respect to E_p and σ_p. The range set by RS is determined by finding an interval giving that ▲ and any value of 0 to not differ statistically at the significance level of 0.05 with regard to their average performance [31, See Page 3-23]. On the other hand, the range given by RT is found by solving for

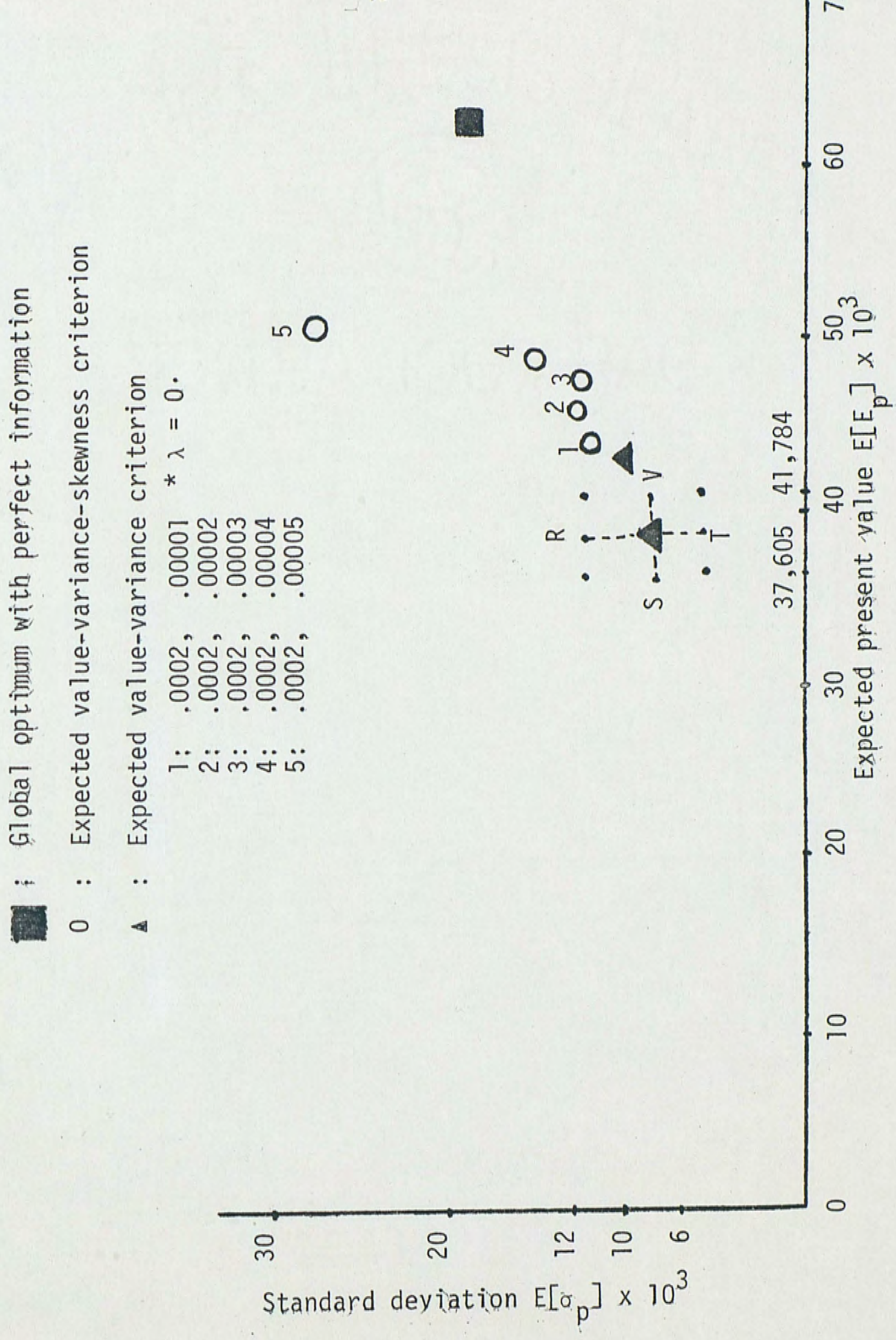

Figure 7. Risk-return chart: Case I

an interval giving that ▲ and any value of 0 do not differ statistically at the significance level of 0.05 with regard to their variability [31, See Page 4-9]. Therefore, the dotted square RSTV around ▲ represents the area in which any value falls assumes to be statistically insignificant with regard to their performance in both E_p and σ_p when compared with the performance of ▲. Following the logic above, it can be shown that the points 0^1, 0^2 and 0^3 obtained from the E-V-S criterion do not differ statistically with regard to their variability when compared with that of ▲, but their average performance (E_p) becomes pronounced statistically.

Finally, it is of interest to answer the following two questions; 1) what if the future cash flow realizations were known with certainty before the decision is to be made? 2) how much improvement would occur in the selection of projects? When the decision maker has complete information about all the future SIP's at the current decision time, a Zero-One integer programming formulation given in Page 42 generates the global optimal solution at the point denoted by a square (■) in Figure 7. It is seen that the value of knowing the project realizations is far more pronounced that in the case of perfect information. This is largely because the perfect knowledge of present and future project realizations allow for the optimization of sources of capital.

Case II

The simulation results for this investment situation are shown in Figure 8 and the detailed statistics are presented in Table 6 in Appendix C. In Figure 8, the values of λ and δ used for the E-V-S criterion are the same as in Case I.

For the E-V-S criterion, the greatest E_p value is obtained at $\delta = 0.00001$. In order to compare the effectiveness of the E-V-S criterion against the E-V criterion at $\lambda = 0.0002$, the two points 0^1 and ▲ in Figure 8. If a 95% confidence interval is constructed for the two points, it is observed that:

$$51{,}309 \leq E_p\ (0^1) \leq 62{,}358$$

and

$$36{,}686 \leq E_p\ (\blacktriangle) \leq 45{,}019$$

using the two-tailed t-test. This means that the probability that the above inequalities are satisfied is 0.95. Hence, given the same level of confidence, the E-V-S criterion is most likely to yield an E_p greater than that obtained for the E-V criterion for this particular case.

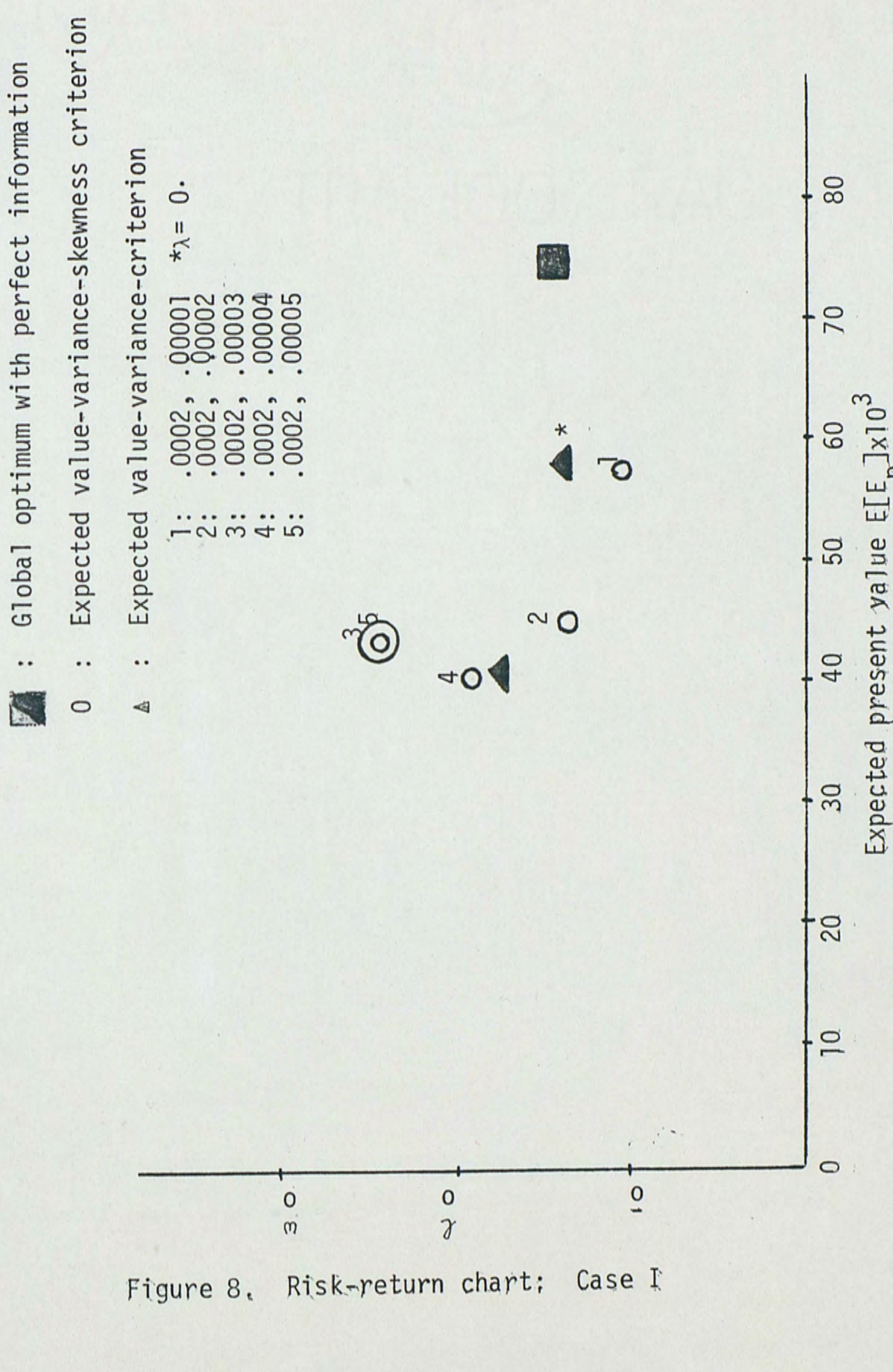

Figure 8. Risk-return chart; Case I

Case III

Figure 9 shows the results of the simulation obtained for the case where all the returns from the proposals are negatively skewed. Detailed statistics are presented in Table 7 in Appendix C.

For this particular case, the E-V-S criterion does not dominate the E-V criterion. A risk averter dislikes regatively skewed returns from a proposal and since the E-V-S model developed in this study assumed the decision maker to be a risk averter, the poor performance of the E-V-S criterion when returns are all negatively skewed is understood.

Consider points 0^1 and ▲ in Figure 9. If a 95% confidence interval is constructed for the two points, it is seen that

$$39{,}352 \leq E_p\ (0^1) \leq 43{,}990$$

and

$$47{,}659 \leq E_p\ (\blacktriangle) \leq 54{,}804$$

Thus, given the same level of confidence, the E-V criterion dominates the E-V-S criterion for this particular case.

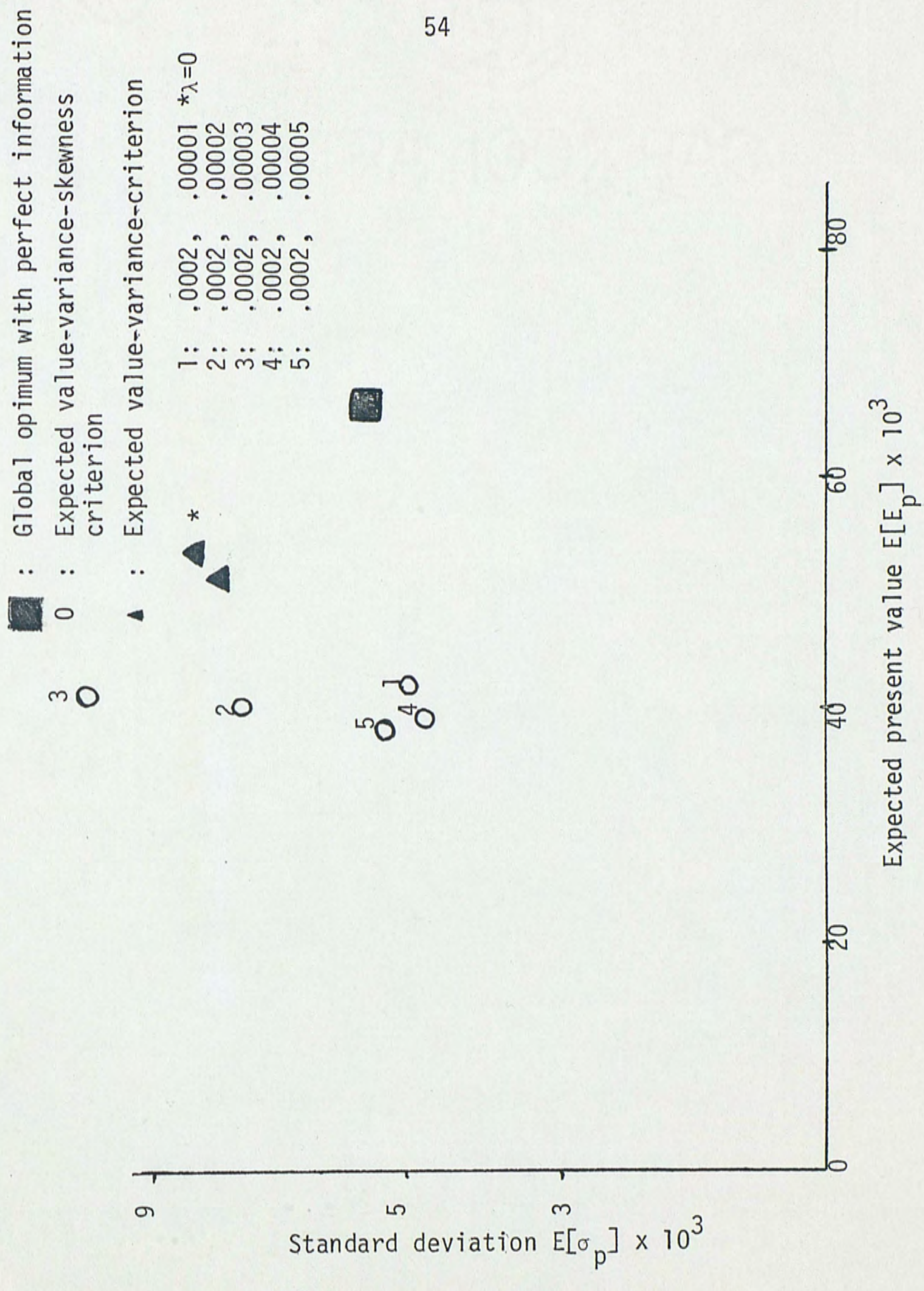

Figure 9. Risk-return chart: Case III

CHAPTER VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The primary purpose of this research was to investigate the significance of the concept of skewness in capital allocation problems. A complete summary of the results of the research is given in this chapter, following by conclusions, and recommendations for future research.

Summary of Results

This research begins with the discussion of the decision criteria considering risk commonly mentioned in the literature. In particular, attention is given to decision criteria that considers skewness. As is revealed by the review of literature, one of the basic assumptions underlying investment analysis under risk is that decision maker's would base their decisions on only the first two statistical moments of the probability distribution of returns. However, the mean and variance can adequately describe only certain symmetric distributions such as the normal and the uniform distributions. As a result, if probability distributions of investment returns are actually asymmetric, the classic first two moments analysis ignores information (skewness) that is needed to make a better investment decision. Even though the importance of the third moment in project selection has been recognized, nowhere in the literature is there a successful application of the concept to a regular periodic decision process where the decision maker lacks full knowledge of his future as well as present investment opportunities.

Skewness can be measured by the third central moment of a probability distribution, and in order to incorporate skewness in capital rationing decision criteria, a zero-one linear programming decision model called the expected value-variance-skewness (EVS) model is developed. This model consists of a single index which seeks a practical trade-off among the three major investment factors: profitability, variability, and skewness.

In order to investigate the effectiveness of incorporating skewness in capital rationing decisions, an investment situation is suggested where investment decisions are made on a one-time basis with the objective of maximizing total expected present values. It is assumed that knowledge of that investment would be available and their associated cash flows are probabilistic.

Given the lack of available actual data and the need to examine the performance of the models under a variety of conditions regarding investment settings, the logical alternative is to select a simulated environment in which the important parameters generating investment data could be controlled. Therefore, a simulation model based on three investment situations (positively skewed negatively skewed, mixed-positively and negatively-skewed and negatively skewed distributions of expected returns) is developed to test the effectiveness of the EVS criterion with other frequently mentioned decision criteria. These criteria are the expected value maximization and the expected value-variance (EV). In addition to comparing these two criteria with the EVS criterion, the value of having complete information about the future project realizations is also introduced to compare the overall effectiveness of the EVS criterion.

Conclusions

The primary contribution of this research is to answer the following specific questions:

1) What improvement in investment proposal selection can be attained with incorporating higher statistical moments in capital rationing problem?
2) How does the EVS criterion perform with respect to other existing investment decision model which do not consider explicitly the higher statistical moments of the probability distribution of investment return?

The analyses of the results obtained through the simulation process indicate that the EVS criterion is generally superior and more effective than the other two criteria under the investment situations considered in the study except when all returns are negatively skewed. It is also observed that the EVS criterion is sensitive to the choice of the risk aversion parameter.

Using a higher value of δ results in a larger average expected value but with a higher variability. When the risk parameter, δ, becomes extremely large, the EVS criterion virtually requires no consideration of the first two statistical moments. However, the predetermination of appropriate value of δ remains unsolved.

Recommendations for Further Research

A logical extension of this study is the incorporation of the fourth statistical moment which measures the peakedness of a distribution in capital rationing problems. This, of course, would be an extension of the EVS criterion and would require the stipulation of another risk coefficient and the development of a similar single index solveable by linear programming techniques.

The assumption of independence among investment proposals, though practical, is limited in actual application. Thus, the consideration of the effects of interrelationships among the proposals can be of special interest.

The decision process considered in the study assumed a single stage "once-for-all" type of decision. The application of the E-V-S model to multi-stage decision process and investigation of its effectiveness as a decision criterion with respect to other decision criteria would be of great interest.

APPENDICES

APPENDIX A

LOGIC OF SIMULATION MODELS USED IN THE RESEARCH

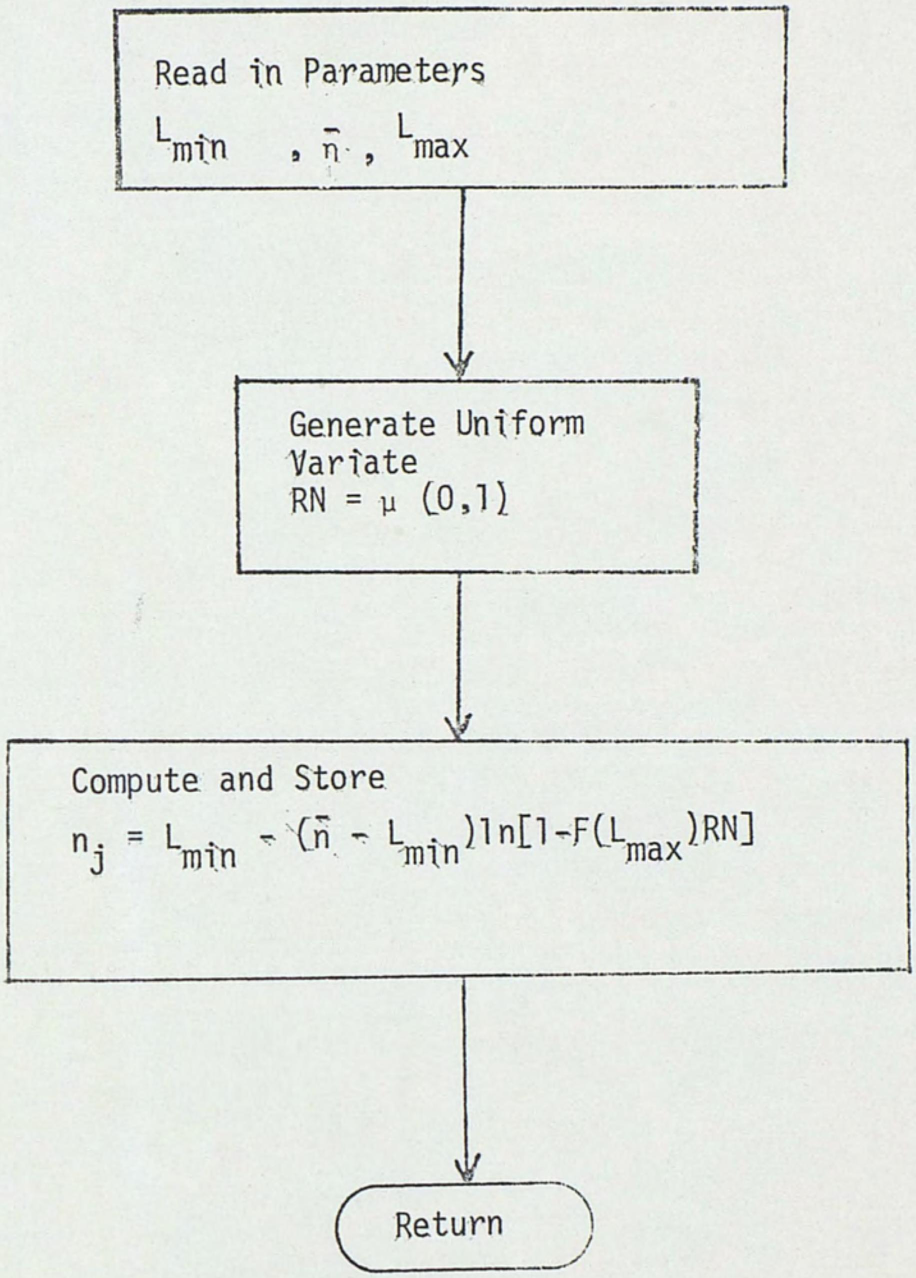

Figure 10. Logic to generate life of proposal

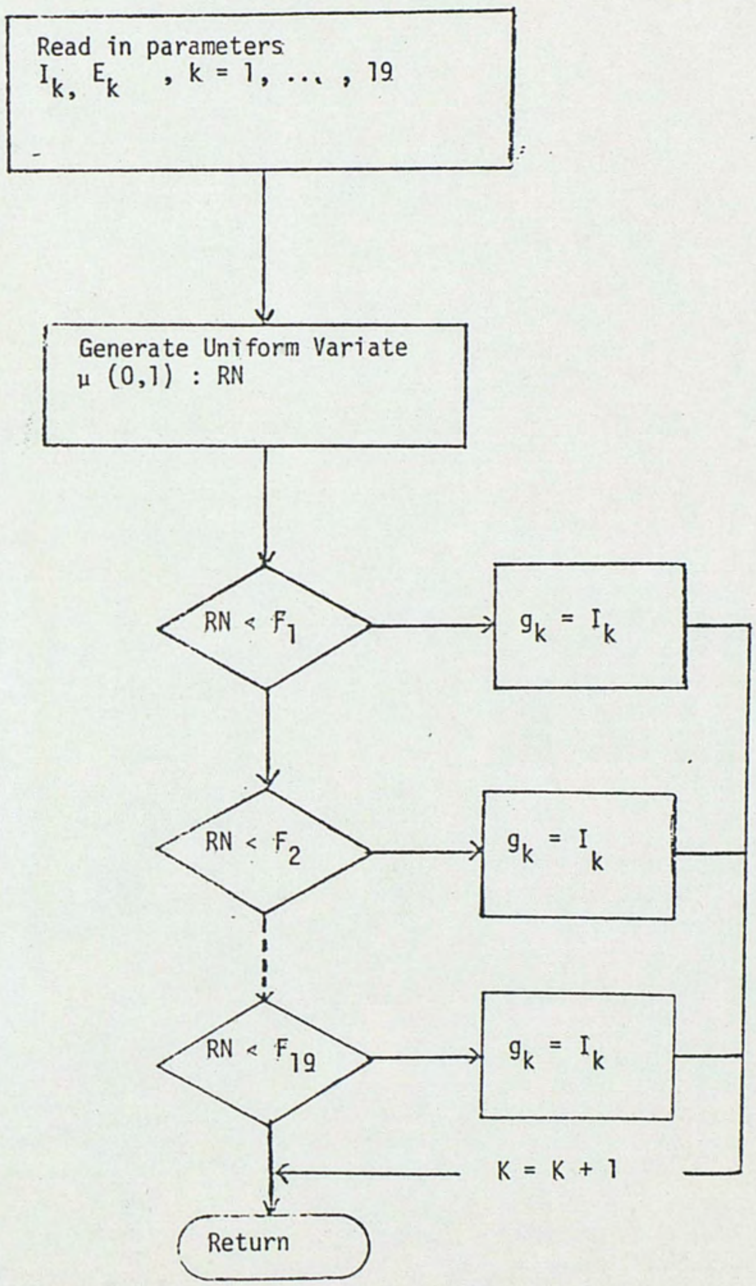

Figure 11. Logic to generate rate of return for proposal

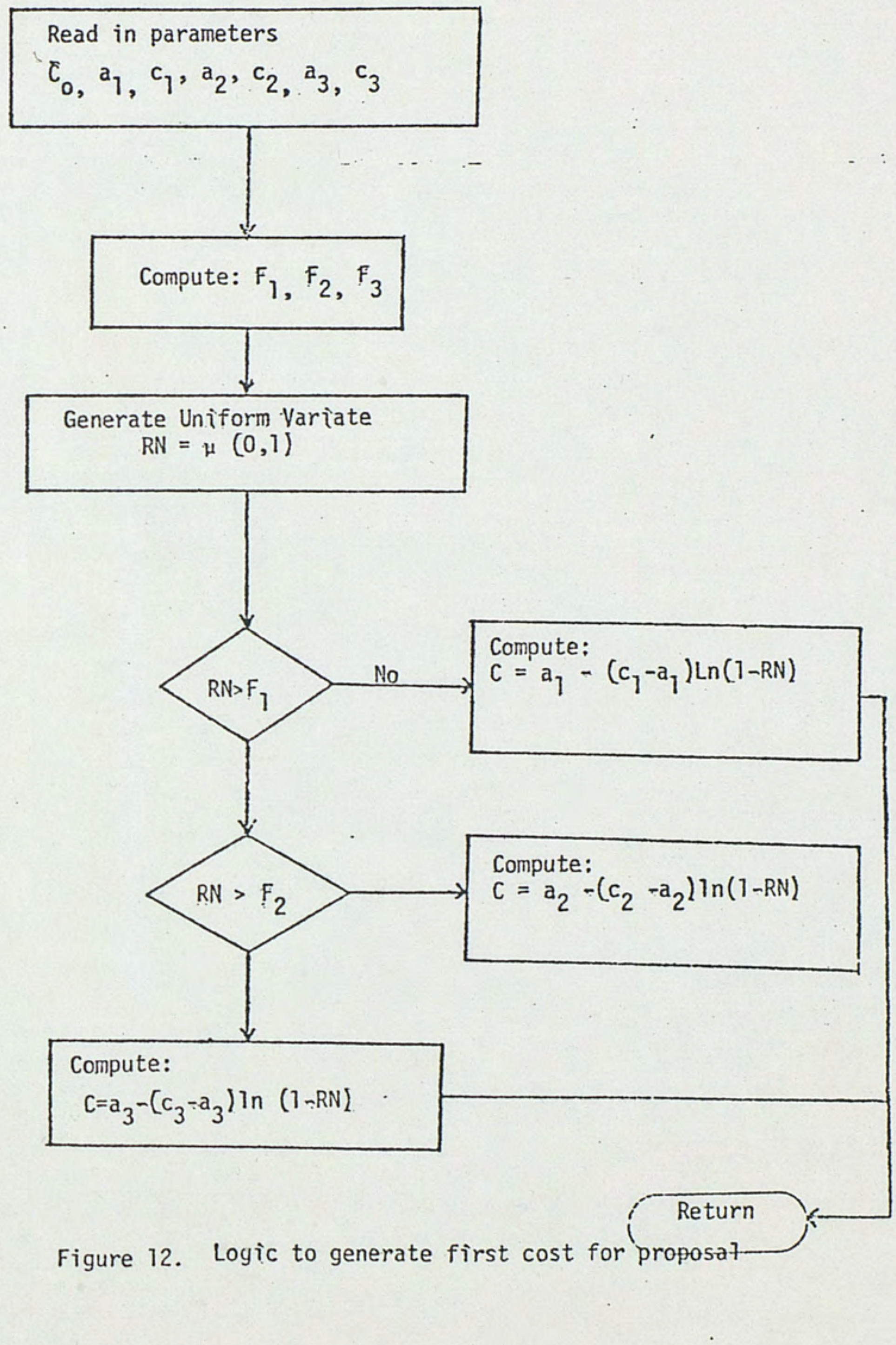

Figure 12. Logic to generate first cost for proposal

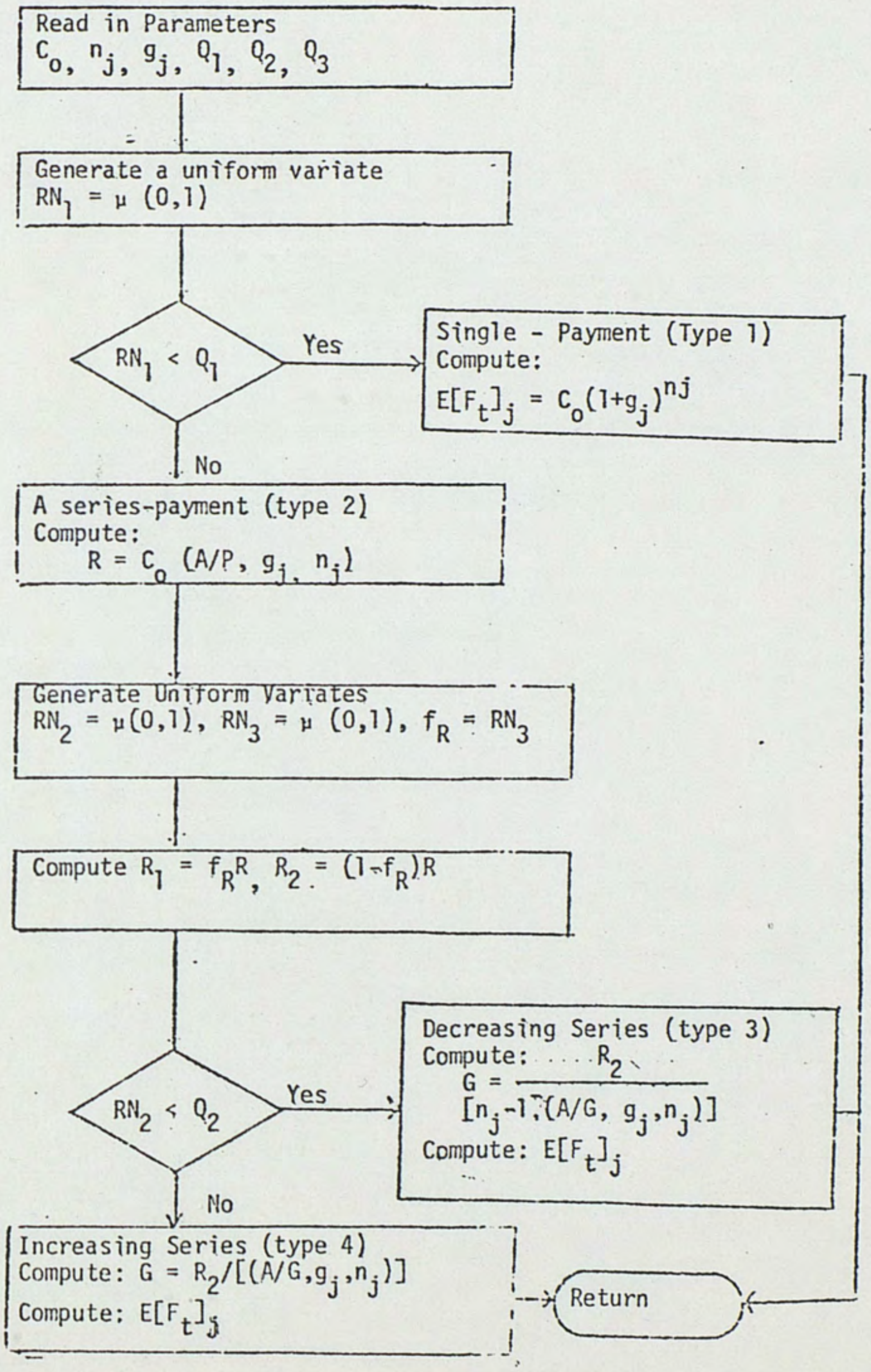

Figure 13. Logic to generate expected cash flows for proposal

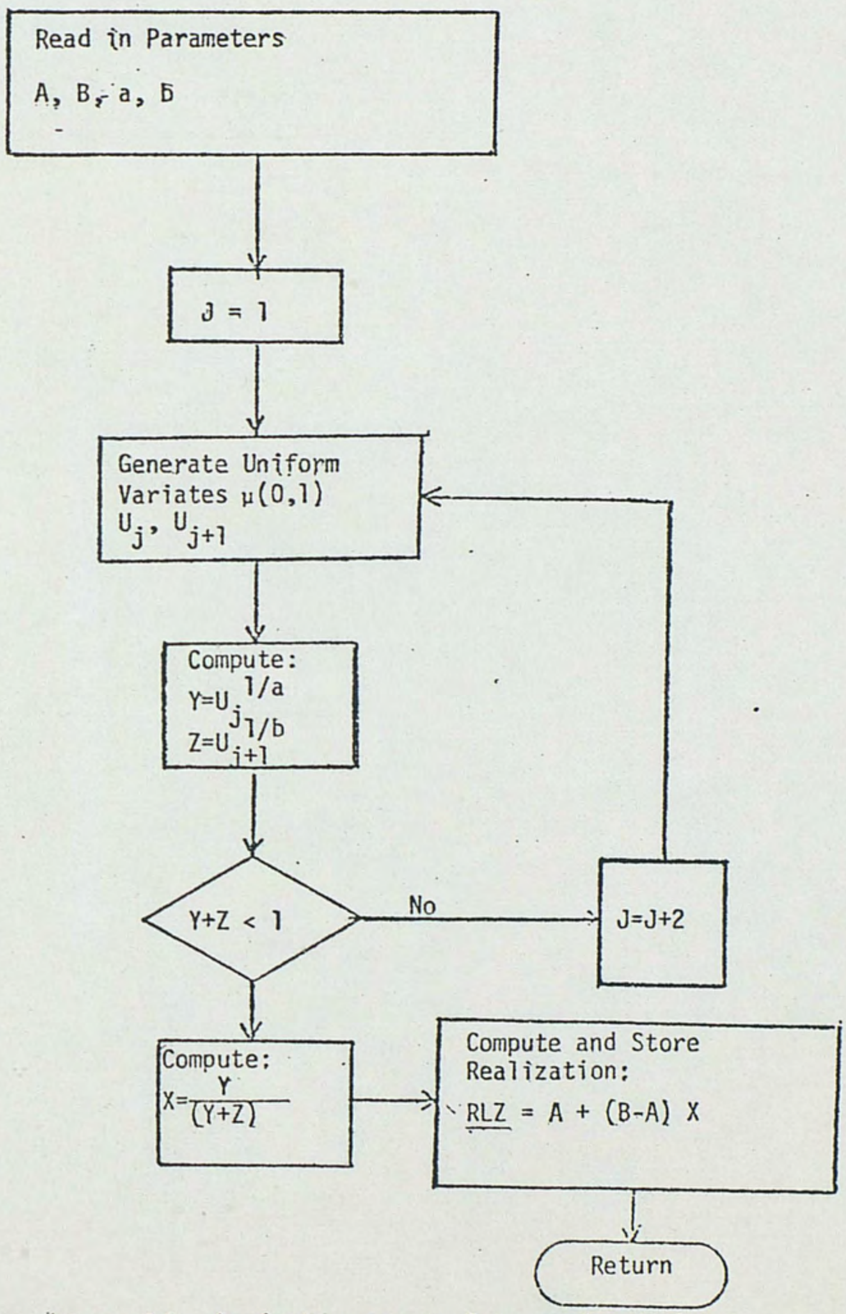

Figure 14. Logic to generate Beta random deviates

APPENDIX B

GENERATED PROJECT PROFILES

(Mean, Variance, Rate of Return, First

Cost, Expected PW, Skewness)

Table 2. Project profiles; Case I

PROJECT	TYPE	LIFE	ALPHA	BETA	ROR	FCOST	EXPV	SDEV	SKEW
1	3	7	1.869	2.685	0.15	7140.77	1680.80	689.353	52825233.73
2	3	4	1.843	2.760	0.16	24948.35	4204.58	1677.256	847805315.93
3	4	6	1.514	2.033	0.21	6038.12	1532.00	637.550	32975190.20
4	1	7	1.573	2.065	0.25	6932.59	10031.03	4185.316	8648880710.30
5	3	4	1.111	2.556	0.21	21269.56	5987.85	2878.843	8474598194.66
6	2	2	1.753	2.450	0.17	16530.35	1567.47	515.585	20263039.93
7	3	6	1.606	2.041	0.19	6001.13	1864.62	743.239	42728670.79
8	3	3	1.219	2.656	0.30	27128.21	13601.35	6563.617	94955774914.30
9	3	4	1.909	2.088	0.23	7136.23	2254.17	741.482	15098237.83
10	1	3	1.639	2.538	0.19	7625.13	2028.93	695.465	64896353.78
11	3	6	1.477	2.834	0.15	6207.79	978.23	432.200	23121297.40
12	3	4	1.621	2.516	0.20	6852.85	2231.35	838.540	114060065.68
13	1	3	1.922	2.364	0.25	7797.64	3644.72	1444.320	275516188.72
14	4	6	1.284	2.208	0.15	6045.28	326.68	51.046	30890.29
15	3	3	1.512	2.634	0.11	7663.49	149.09	27.136	4869.74
16	4	4	1.889	2.606	0.20	6238.88	545.26	144.501	432276.50
17	3	3	1.809	2.81	0.12	35994.0	1553.82	508.16	25779571.87
18	3	2	1.467	2.592	0.21	16357.82	2816.12	508.161	291782551.72
19	3	3	1.518	2.584	0.19	21118.99	3503.84	1054.055	657800983.28
20	2	4	1.266	2.677	0.20	8067.10	1810.88	1412.448	127102712.19
21	3	4	1.262	2.784	0.20	34648.17	8410.85	731.812	20710545570.65
22	3	5	1.345	2.141	0.19	34187.49	11520.01	3923.009	28070846375.73
23	1	7	1.109	2.983	0.11	6843.21	447.54	5203.160	514006.10
24	3	5	1.250	2.826	0.19	7501.46	1881.82	106.510	230031182.65
25	4	6	1.141	2.315	0.21	23206.04	539.07	866.418	264090.98
26	3	4	1.472	2.170	0.12	23592.72	1267.92	95.827	9056101.23
27	2	7	1.852	2.972	0.11	17162.39	568.94	377.674	688900.27
28	3	2	1.159	2.455	0.17	6468.52	819.66	148.374	15785118.29

Table 2. Project profiles: Case I (Cont'd)

29	1	5	1.111	2.029	0.13	17721.19	2551.96	374.182	281169417.35
30	4	3	1.766	2.630	0.30	7557.37	1645.72	1037.155	26574537.58
31	3	2	1.382	2.236	0.25	19882.34	4395.39	531.972	1514070473.88
32	3	3	1.180	2.437	0.16	25199.62	3217.43	1938.496	727166359.08
33	4	3	1.140	2.775	0.25	18107.93	4288.88	1328.475	3775544488.04
34	3	3	1.448	2.581	0.17	8057.99	1361.32	2145.952	45370159.06
35	3	2	1.155	2.225	0.12	6954.09	194.64	568.126	75978.91
36	3	3	1.399	2.477	0.20	6848.94	1640.54	64.921	42879618.07
37	3	4	1.112	2.559	0.20	8106.82	2001.69	556.637	241693738.21
38	3	3	1.953	2.379	0.13	6697.47	401.51	879.374	50361.44
39	3	2	1.408	2.518	0.11	20588.84	362.12	62.299	61196.23
40	4	4	1.323	2.658	0.16	26826.81	2862.35	1203.761	528832420.76
41	2	3	1.188	2.106	0.23	7084.88	1674.78	676.788	75106051.52
42	1	2	1.267	2.472	0.25	17574.88	5119.98	2392.267	3943347106.06
43	2	4	1.613	2.648	0.25	39811.41	13625.22	5850.375	43788574784.11
44	3	2	1.144	2.761	0.19	17343.59	2793.33	1263.602	763850224.04
45	3	2	1.704	2.615	0.20	21449.95	3824.88	1580.734	748312624.55
46	3	2	1.985	2.286	0.17	18184.83	2224.76	804.186	32581972.91
47	1	3	1.142	2.964	0.16	17753.05	3066.40	1494.964	1373338567.06
48	1	3	1.035	2.459	0.14	8111.35	917.43	228.633	4359777.18
49	3	6	1.237	2.123	0.16	17158.66	3225.50	1459.084	713797715.60
50	3	4	1.136	2.559	0.12	17781.93	1014.35	370.664	17669332.02

Table 3. Project profiles: Case II

PROJECT	TYPE	LIFE	ALPHA	BETA	ROR	FCOST	EXPV	SDEV	SKEW
1	2	2	2.37	1.26	0.25	22080.85	4531.50	1282.76	-575366617.31
2	3	4	2.17	1.68	0.16	24948.35	4204.58	1413.70	-311942611.63
3	4	2	2.58	1.29	0.25	22458.37	3646.10	1111.46	-412105862.32
4	4	2	2.51	1.47	0.23	31789.58	3052.85	962.04	-208401315.04
5	2	2	2.20	1.97	0.13	8052.18	325.50	109.58	-63764.74
6	3	2	2.01	1.32	0.15	18666.64	1359.22	412.25	-12554620.91
7	3	3	2.79	1.83	0.25	21006.53	6979.02	2040.78	-1588051142.58
8	4	6	2.86	1.68	0.25	23230.62	4409.75	1249.18	-459923723.43
9	3	2	2.28	1.55	0.12	18668.72	503.69	37.12	-8632.24
10	2	2	2.73	1.42	0.17	6272.31	594.76	66.95	-85600.63
11	3	4	2.93	1.74	0.25	17031.41	5864.91	1745.18	-1225472033.02
12	3	2	2.09	1.84	0.15	16401.18	1200.03	304.37	-1572260.23
13	3	4	2.23	1.80	0.23	7136.23	2254.17	668.73	-27897092.49
14	3	2	2.75	1.26	0.13	7265.98	374.61	13.24	-785.49
15	3	6	2.80	1.45	0.30	7986.81	5711.14	1614.65	-1209010771.35
16	3	2	2.50	1.92	0.19	17049.10	2383.35	795.23	-58841049.98
17	1	3	2.99	1.60	0.30	26576.94	17291.98	5019.49	-34820739887.04
18	3	4	2.70	1.74	0.12	16409.13	966.21	288.52	-4669253.49
19	3	3	2.14	1.15	0.12	35994.01	1553.82	397.46	-16398608.30
20	3	4	2.65	1.54	0.13	18646.95	1517.64	432.09	-19230377.72
21	1	7	2.37	1.36	0.11	6843.21	447.54	66.52	-70856.18
22	3	6	2.79	1.49	0.25	34386.37	22613.79	6696.58	-82840360217.28
23	3	4	2.81	1.44	0.20	8299.20	2148.67	629.12	-72605132.75
24	2	5	2.37	1.23	0.11	16973.69	435.73	61.67	-66135.61
25	3	3	2.00	1.95	0.17	6510.59	928.68	263.40	-217795.50

Table 3. Project profiles: Case II (Cont'd)

26	3	2	2.66	3.42	0.23	21918.41	3968.27	1350.95	278305192.21
27	3	2	2.57	3.59	0.20	18548.89	2761.81	893.16	107464843.07
28	3	4	2.44	3.30	0.15	7748.50	1117.02	352.45	5923334.62
29	3	2	2.81	3.18	0.17	20988.19	2016.14	703.44	19130988.88
30	2	3	2.74	3.83	0.12	20230.79	716.01	110.37	200947.40
31	3	2	2.28	3.20	0.21	16508.37	3156.53	1154.36	233669263.43
32	4	3	2.66	3.16	0.19	18572.13	1389.39	363.05	3739283.92
33	3	2	2.04	3.75	0.16	17745.73	1626.56	591.23	56216674.34
34	3	5	2.11	3.95	0.16	31915.76	7054.25	2886.92	6724692362.68
35	4	3	2.66	3.41	0.25	-18107.93	4288.88	1547.46	410794219.84
36	4	4	2.47	3.16	0.25	7757.49	1232.81	426.64	8552131.55
37	3	4	2.69	3.68	0.25	23118.22	9617.73	3462.26	5836262755.41
38	3	7	2.86	3.05	0.11	19324.12	728.92	246.10	422973.79
39	2	3	2.51	3.62	0.23	7084.88	1674.78	540.37	25804706.71
40	4	7	2.08	3.88	0.23	6563.81	1050.34	291.83	6923217.38
41	3	3	2.53	3.30	0.15	7821.09	840.12	236.90	1568890.50
42	4	4	2.95	3.00	0.25	7544.07	1340.82	314.27	248556.75
43	1	3	2.45	3.69	0.25	6511.69	3043.65	1008.69	187629250.25
44	2	3	2.04	3.99	0.17	8000.50	1003.91	330.67	10862055.07
45	3	3	2.60	3.66	0.17	32492.91	5024.66	1778.49	863920173.12
46	3	3	2.56	3.36	0.23	7161.09	1894.48	691.32	40671340.38
47	3	4	2.15	3.63	0.25	7183.28	2775.46	948.78	199525003.41
48	2	5	2.37	3.58	0.13	38925.44	3027.37	1003.28	186799026.43
49	1	7	2.13	3.53	0.20	6466.06	5423.36	2177.87	2328073050.36
50	1	3	2.96	3.95	0.14	8111.35	917.43	158.71	516675.52

Table 4. Project profiles; Case III

PROJECT	TYPE	LIFE	ALPHA	BETA	ROH	FCOST	EXPV	SDEV	SKEW
1	2	2	2.37	1.26	0.25	22080.85	4531.50	1282.76	-575366617.31
2	3	4	2.17	1.08	0.16	24948.35	4204.58	1413.70	-311942611.63
3	4	2	2.56	1.29	0.25	22458.37	3646.10	1111.46	-412105862.32
4	4	2	2.51	1.47	0.23	31789.58	3052.85	962.04	-208401315.04
5	2	2	2.20	1.97	0.13	8052.18	325.50	109.58	-63764.74
6	3	2	2.01	1.32	0.15	18666.64	1359.22	412.25	-12559620.91
7	3	3	2.79	1.83	0.25	21006.53	6979.02	2040.78	-1588051142.58
8	4	6	2.86	1.68	0.25	23230.62	4409.75	1249.18	-459923723.43
9	3	2	2.25	1.55	0.12	18668.72	503.69	37.12	-8632.24
10	2	2	2.73	1.42	0.17	6272.31	594.76	66.95	-85600.63
11	3	4	2.93	1.74	0.25	17031.41	5864.91	1745.18	-1225472033.02
12	3	2	2.04	1.84	0.15	16401.18	1200.03	304.37	-1572260.23
13	3	4	2.23	1.80	0.23	7136.23	2254.17	668.73	-27897092.49
14	3	2	2.75	1.26	0.13	7265.98	374.61	13.24	-785.49
15	3	6	2.80	1.45	0.30	7986.81	5711.14	1614.65	-1209010771.35
16	3	2	2.50	1.92	0.19	17049.10	2383.35	795.23	-58841049.98
17	1	3	2.99	1.60	0.30	26576.94	17291.98	5019.49	-34820739887.04
18	3	4	2.70	1.74	0.12	16409.13	966.21	288.52	-4669253.49
19	3	3	2.14	1.15	0.12	35994.01	1553.82	397.46	-16398608.30
20	3	4	2.65	1.50	0.13	18646.95	1517.64	432.09	-19230377.72
21	1	7	2.37	1.36	0.11	6843.21	447.54	66.52	-70856.18
22	3	6	2.79	1.49	0.25	34386.37	22613.79	6696.58	-82840360217.28
23	3	4	2.81	1.44	0.20	8299.20	2148.67	629.12	-72605132.75
24	2	5	2.37	1.23	0.11	16973.69	435.73	61.67	-66135.61
25	3	3	2.00	1.95	0.17	6510.59	928.68	263.40	-217795.50

Table 4. Project profiles; Case III (Cont'd)

26	3	2	2.66	1.42	0.23	21918.41	3968.27	1132.40	-397465973.88
27	3	2	2.57	1.59	0.20	18548.89	2761.81	759.16	-91817226.66
28	3	4	2.44	1.30	0.15	7748.50	1117.02	293.55	-6881154.20
29	3	2	2.81	1.18	0.17	20988.19	2016.14	576.19	-71641853.17
30	2	3	2.74	1.83	0.12	20230.79	716.01	94.99	-154277.68
31	3	2	2.28	1.20	0.21	16508.37	3156.53	955.76	-239121325.55
32	4	3	2.66	1.16	0.19	18572.13	1389.39	297.69	-9346154.73
33	3	2	2.04	1.75	0.16	17745.73	1626.56	511.82	-9070011.81
34	3	5	2.11	1.95	0.16	31915.76	7054.25	2524.84	-572668253.86
35	4	3	2.66	1.41	0.25	18107.93	4288.88	1295.65	-604776921.34
36	4	4	2.47	1.16	0.25	7757.49	1232.81	350.82	-13944626.02
37	3	4	2.69	1.68	0.25	23118.22	9617.73	2954.72	-5363654713.70
38	3	7	2.86	1.05	0.11	19324.12	728.92	198.87	-3362118.98
39	2	3	2.51	1.62	0.23	7084.88	1674.78	460.56	-18863519.05
40	4	7	2.08	1.88	0.23	6563.81	1050.34	254.38	-738165.41
41	3	3	2.53	1.30	0.15	7821.09	840.12	196.99	-2212273.43
42	4	4	2.95	1.00	0.25	7544.07	1340.82	252.34	-7363685.12
43	1	3	2.45	1.69	0.25	6511.69	3043.65	864.45	-106068427.22
44	2	3	2.04	1.99	0.17	8000.50	1003.91	290.18	-243217.48
45	3	3	2.60	1.66	0.17	32492.91	5024.66	1518.07	-690041056.11
46	3	3	2.56	1.36	0.23	7161.09	1894.48	577.79	-52651450.68
47	3	4	2.15	1.63	0.25	7183.28	2775.46	813.68	-65775572.31
48	2	5	2.37	1.58	0.13	38925.44	3027.37	854.93	-110396535.59
49	1	7	2.13	1.53	0.20	6466.06	5423.36	1856.41	-920916720.39
50	1	3	2.96	1.95	0.14	8111.35	917.43	137.18	-479351.67

APPENDIX C

DETAILED SIMULATION RESULTS

Expected Value-Variance-Skewness Criterion				
λ	δ	E_p	σ_p	F_Θ
.0002	.00001	44,100	11,664	256
.0002	.00002	44,240	12,069	697
.0002	.00003	45,202	11,914	524
.0002	.00004	46,650	13,995	456
.0002	.00005	50,021	29,592	5,806

Expected Value-Variance Criterion *		
E_p	σ_p	F_Θ
39,695	9,343	1,903

* λ = 0.0002

Expected Value Maximization Criterion		
E_p	σ_p	F_Θ
42,165	11,628	14

With Perfect Information		
E_p	σ_p	F_Θ
63,000	18,200	563

Table 5. Detailed simulation results: Case I

Expected Value-Variance Skewness Criterion				
λ	δ	E_p	σ_p	F_θ
.0002	.00001	56,834	11.805	1,438
.0002	.00001	43,680	15,539	3,237
.0002	.00001	42,091	25,706	5,580
.0002	.00001	40,533	18,311	4,040
.0002	.00001	42,091	25,706	5,580

Expected Value-Variance Criterion*		
E_p	σ_p	F_θ
40,853	18,631	3,959

*λ = 0.0002

Expected Value Maximization Criterion		
E_p	σ_p	F_θ
59,600	11,805	1,286

With Perfect Information		
E_p	σ_p	F_θ
72,520	12,580	735

Table 6. Detailed simulation results: Case II

Expected Value-Variance-Skewness Criterion				
λ	δ	E_p	σ_p	F_θ
.0002	.00001	41,671	4,955	708
.0002	.00002	40,350	7,096	1,048
.0002	.00003	40,719	8,875	1,533
.0002	.0004	39,865	4,952	558
.0002	.00005	38,926	5,302	562

Expected Value-Variance Criterion*		
E_p	σ_p	F_θ
51,232	7,633	532

λ = .0002

Expected Value Maximization Criterion		
E_p	σ_p	F_θ
51,382	7,822	464

With Perfect Information		
E_p	σ_p	F_θ
66,540	5,870	812

Table 7. Detailed simulation results: Case III

APPENDIX D

FORTRAN SIMULATION PROGRAM LISTINGS

```
C
C
C      THIS IS A FORTRAN SIMULATION PROGRAM TO GENERATE DATA
C
C      TO BE USED IN TESTING VARIOUS CAPITAL BUDGETING CRITERIA
C
C      THE UNDERLYING DISTRIBUTION IS THE BETA DISTRIBUTION
C
C
      DIMENSION S(20),Y(20),CAT(20),VAR(20)
      DIMENSION REAL1(50),REAL5(50)
      DIMENSION REAL2(50),REAL3(50),REAL4(50),REAL6(50),REAL7(50),REAL8(
     *50),REAL9(50),REAL10(50)
      DIMENSION REAL11(50),REAL12(50),REAL13(50),REAL14(50),REAL15(50)
      DIMENSION REAL16(50),REAL17(50),REAL18(50),REAL19(50),REAL20(50)
      DIMENSION ROT(50),LIFE(50),COST(50),C(300)
      DIMENSION ROR(300),RET(300),N(300),EOF(300),CR(300)
      DIMENSION GOF(10),TOF(10)
      DIMENSION PA(300),STD(50),PLI(20)
      DIMENSION PV(10),Z(20),MODE(50),FC(300),LL(50)
      DOUBLE PRECISION RA(300),LAMDA,DELTA,CP(300),CPV(50),AL,PI,BI
      DOUBLE PRECISION D3,D4,ALPHA(50),BETA(50),[illegible]
      DOUBLE PRECISION SKEW(50),A(50),B(50),EX(50),EXSQ(50),[illegible]
      REAL MODE
C
C     READ IN VALUES OF PARAMETERS AND ECHO-PRINT
C
C
      READ(5,11) G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,G16,
     *G17,G18,G19
      READ(5,11) FK1,FK2,FK3,FK4,FK5,FK6,FK7,FK8,FK9,FK10,FK11,FK12,FK13
     *,FK14,FK15,FK16,FK17,FK18,FK19
C
      READ(5,33) A1,C1,A2,C2,A3,C3
      READ(5,44) F1,F2,F3
      READ(5,27) NY
      READ(5,35) LMIN,LAVG,LMAX
      WRITE(6,20) LMIN,LAVG,LMAX
      READ(5,55) D1,D2,D3,D4
      WRITE(6,333) D1,D2,D3,D4
      READ(5,88) RATE
      WRITE(6,85) RATE
C
      READ(5,50) AL,UP,ALL,UPP
      WRITE(6,51) AL,UP,ALL,UPP
      READ(5,57) AA,BB
      WRITE(6,58) AA,BB
      READ(5,65) MX
      READ(5,19) SL,SLL,SD,SDD
      WRITE(6,60) SL,SLL,SD,SDD
      READ(5,107) FCOST
      WRITE(6,108) FCOST
C
C     EVM = E - LAMDA*VARIANCE + DELTA*SKEWNESS
C
      EE=-(1./(LAVG-LMIN))*(LMAX-LMIN)
      FLMAX=1.-EXP(EE)


C
      K=1
      JJ=0
      KI=0
      IX=12345
      II=1
      MD=1
      WRITE(6,29)
C
C
C     GENERATION OF RATE OF RETURN FOR THE PROJECTS
C                                                                   C
      DO 25 J=1,NY
      CALL RANDU(IX,IY,YFL)
      RR=YFL
      IF(RR.LE.FK1) GO TO 1
      IF(RR.LE.FK2) GO TO 2
      IF(RR.LE.FK3) GO TO 3
      IF(RR.LE.FK4) GO TO 4
      IF(RR.LE.FK5) GO TO 5
      IF(RR.LE.FK6) GO TO 6
      IF(RR.LE.FK7) GO TO 7
      IF(RR.LE.FK8) GO TO 8
      IF(RR.LE.FK9) GO TO 9
      IF(RR.LE.FK10) GO TO 10
      IF(RR.LE.FK11) GO TO 40
      IF(RR.LE.FK12) GO TO 41
      IF(RR.LE.FK13) GO TO 42
      IF(RR.LE.FK14) GO TO 43
      IF(RR.LE.FK15) GO TO 50
      IF(RR.LE.FK16) GO TO 51
      IF(RR.LE.FK17) GO TO 52
      IF(RR.LE.FK18) GO TO 63
      IF(RR.LE.FK19) GO TO 64
C
    1 ROR(J)=G1
      GO TO 14
    2 ROR(J)=G2
      GO TO 14
    3 ROR(J)=G3
      GO TO 14
    4 ROR(J)=G4
      GO TO 14
    5 ROR(J)=G5
      GO TO 14
    6 ROR(J)=G6
      GO TO 14
    7 ROR(J)=G7
      GO TO 14
    8 ROR(J)=G8
      GO TO 14


    9 ROR(J)=G9
      GO TO 14
   10 ROR(J)=G10
```

```
      GO TO 14
   40 ROR(J)=G11
      GO TO 14
   41 ROR(J)=G12
      GO TO 14
   42 ROR(J)=G13

      GO TO 14
   43 ROR(J)=G14
      GO TO 14
   50 ROR(J)=G15
      GO TO 14
   61 ROR(J)=G16
      GO TO 14
   62 ROR(J)=G17
      GO TO 14
   63 ROR(J)=G18
      GO TO 14
   64 ROR(J)=G19
      GO TO 14
   14 RET(J)=ROR(J)
C
C
C
C     COMPUTATION OF PROJECT FIRST COST
C
C
      CALL RANDU(IX,IY,YFL)
      RN1=YFL
      IF(RN1.GT.F1) GO TO 12
      CC(J)=A1-(C1-A1)*ALOG(1.-RN1)
      GO TO 15
   12 IF(RN1.GT.F2) GO TO 13
      CC(J)=A2-(C2-A2)*ALOG(1.-RN1)
      GO TO 15
   13 CC(J)=A3-(C3-A3)*ALOG(1.-RN1)
      GO TO 15
   15 FC(J)=CC(J)
C
C
C     GENERATION OF LIFE OF PROJECT
C
C     LIFE OF THE PROJECTS ARE GENERATED BASED ON THE ASSUMPTION
C
C     OF A TRUNCATED EXPONENTIAL DISTRIBUTION
C
C
      CALL RANDU(IX,IY,YFL)
      R=YFL
      N(J)=LMIN-(LAVG-LMIN)*ALOG(1.-(FLMAX*R))
C
      CALL RANDU(IX,IY,YFL)
      D3=YFL
      GA(J)=AA+(BB-AA)*D3
C
C     COMPUTING EXPECTED CASH FLOW FROM FOUR CASH FLOW PATTERNS
C
C     THE CASH FLOW PATTERNS ARE, SINGLE PAYMENT, UNIFORM SERIES,
C
C     INCREASING SERIES, AND DECREASING SERIES.
C
C
C     ITYPE=1 REPRESENTS  SINGLE PAYMENT
C
C     ITYPE=2 REPRESENTS SERIES PAYMENT
C
C     ITYPE=3 REPRESENTS INCREASING GRADIENT SERIES

C
C     ITYPE=4 REPRESENTS DECREASING GRADIENT SERIES
C
      CR(J)=(1.+RET(J))**N(J)
      PA(J)=(1.+RATE)**N(J)
      CALL RANDU(IX,IY,YFL)
      RN2=YFL
      IF(RN2.LE.G1) GO TO 16
      IF(RN2.LE.G2) GO TO 17
      IF(RN2.LE.G3) GO TO 31
      GO TO 31
   16 ITYPE=1


      ECF(J)=FC(J)*CR(J)
      EPW(J)=ECF(J)*(1./PA(J))-FC(J)
      IF(EPW(J).LE.0.) GO TO 25
       IF(GA(J).GT.EPW(J)) GO TO 25
      NPW(K)=EPW(J)
      A(K)=GA(J)
      COST(K)=FC(J)
      LIFE(K)=N(J)
      ROT(K)=RET(J)
      GO TO 18
   17 ITYPE=2
      ECF(J)=FC(J)*RET(J)*CR(J)/(CR(J)-1.)
      EPW(J)=(ECF(J)/RATE)*((PA(J)-1.)/PA(J))-FC(J)
      IF(EPW(J).LE.0.) GO TO 25
      IF(GA(J).GT.EPW(J)) GO TO 25
      NPW(K)=EPW(J)
      A(K)=GA(J)
      COST(K)=FC(J)
      LIFE(K)=N(J)
      ROT(K)=RET(J)
      GO TO 18
   31 ECF(J)=FC(J)*RET(J)*CR(J)/(CR(J)-1.)
      CALL RANDU(IX,IY,YFL)
      FR=YFL
      RF=1.-FR
      GRAD=ECF(J)*FR
      UNIF=ECF(J)*RF
      C=GRAD/((1./RET(J))-(N(J)/(CR(J)-1.)))
C
      LIFE(K)=N(J)
      NN=LIFE(K)
```

```
C
      CALL RANDU(IX,IY,YFL)
      IF(YFL.LE.DA) GO TO 32
      ITYPE=3
      DO 59 I=1,NN
      GCF(I)=G*JJ
      TCF(I)=UNIF+GCF(I)
      PW(I)=TCF(I)/((1.+RATE)**I)
      SUM=SUM+PW(I)
      JJ=JJ+1
   59 CONTINUE
      JJ=0
      EPW(J)=SUM-FC(J)
      IF(EPW(J).LE.0.) GO TO 25
      IF(DA(J).GT.EPW(J)) GO TO 25
      NPW(K)=EPW(J)

       A(K)=DA(J)
      COST(K)=FC(J)
      LIFE(K)=N(J)
      ROT(K)=RET(J)
      GO TO 18
   32 ITYPE=4
      DA=GCF(J)
C
      DO 91 I=1,NN
      GCF(I)=G*KI
      TCF(I)=DA-GCF(I)
      PW(I)=TCF(I)/((1.+RATE)**I)
      SUM=SUM+PW(I)
      KI=KI+1
   91 CONTINUE
      KI=0
      EPW(J)=SUM-FC(J)
      IF(EPW(J).LE.0.) GO TO 25
      IF(DA(J).GT.EPW(J)) GO TO 25
      NPW(K)=EPW(J)
       A(K)=DA(J)
      COST(K)=FC(J)
      LIFE(K)=N(J)
      ROT(K)=RET(J)
C
C
C                                                                       C
   18 CALL RANDU(IX,IY,YFL)
      D1=YFL
      ALPHA(K)=AL+(UP-AL)*D1
      CALL RANDU(IX,IY,YFL)
      D4=YFL
      BETA(K)=ALL+(UPP-ALL)*D4
      EX(K)=(ALPHA(K)+1.)/(ALPHA(K)+BETA(K)+2.)
      EXSQ(K)=((ALPHA(K)+1.)*(ALPHA(K)+2.))/((ALPHA(K)+BETA(K)+2.)*(ALPH
     *A(K)+BETA(K)+3.))
      EXCUB(K)=((ALPHA(K)+1.)*(ALPHA(K)+2.)*(ALPHA(K)+3.))/((ALPHA(K)+BE
     *TA(K)+2.)*(ALPHA(K)+BETA(K)+3.)*(ALPHA(K)+BETA(K)+4.))
      B(K)=((NPW(K)-A(K))*(ALPHA(K)+BETA(K)+2.)+A(K)*(ALPHA(K)+1.))/(ALP
     *HA(K)+1.)
C
      EYSQ(K)=A(K)**2+(2.*(B(K)-A(K))*A(K)*EX(K))+((B(K)-A(K))**2*EXSQ(K
     *))
      VAR(K)=(((B(K)-A(K))**2)*(ALPHA(K)+1.)*(BETA(K)+1.))/((ALPHA(K)+BE
     *TA(K)+3.)*((ALPHA(K)+BETA(K)+2.)**2))
      STD(K)=SQRT(VAR(K))
      MODE(K)=(A(K)*BETA(K)+B(K)*ALPHA(K))/(ALPHA(K)+BETA(K))
C
      EYCUB(K)=A(K)**3+(3.*(B(K)-A(K))*(A(K)**2)*EX(K))+(3.*((B(K)-A(K))
     ***2)*A(K)*EXSQ(K))+(((B(K)-A(K))**3)*EXCUB(K))
C
      SKEW(K)=EYCUB(K)-(3.*NPW(K)*EYSQ(K))+(2.*NPW(K)**3)
C
C
      WRITE(6,222) K,ITYPE,LIFE(K),ALPHA(K),BETA(K),ROT(K),COST(K),NPW(K
     *),STD(K),SKEW(K)
C
      IF(K.EQ.50) GO TO 117
C
      K=K+1

C
C
C
   25 CONTINUE
C
C
C
  117 WRITE(6,106)
C
C
      DO 104 NL=1,K
      WRITE(6,105) NL,A(NL),B(NL),MODE(NL)
  104 CONTINUE
C
      WRITE(6,69)
C                                                                       C
C     THIS PORTION OF THE PROGRAM CALCULATES THE REALIZATIONS
C
C     FOR THE PROJECTS
C                                                                       C
C     FOR EACH PROJECT, TEN REALIZATIONS (BETA VARIATES) ARE GENERATED
C
C     THE VARIATES ARE STORED IN ARRAYS REAL1,REAL2,REAL3,REAL4,
C
C     REAL5,REAL6,REAL7,REAL8,REAL9,AND REAL10
C
C
      DO 73 I=1,K
C
      DO 67 J=1,NN
   71 CALL RANDU(IX,IY,YFL)
      R1=YFL
      CALL RANDU(IX,IY,YFL)
```

```
      R2=YFL
C
      Y(J)=R1**(1./ALPHA(I))
      Z(J)=R2**(1./BETA(I))
      S(J)=Y(J)+Z(J)
      IF(S(J).LE.1.0) GO TO 72
      GO TO 71
   72 CAT(J)=Y(J)/S(J)
C
      RLZ(J)=A(I)+(B(I)-A(I))*CAT(J)
   67 CONTINUE
C
      REAL1(I)=RLZ(MD)
      REAL2(I)=RLZ(MD+1)
      REAL3(I)=RLZ(MD+2)
      REAL4(I)=RLZ(MD+3)
      REAL5(I)=RLZ(MD+4)
      REAL6(I)=RLZ(MD+5)
      REAL7(I)=RLZ(MD+6)
      REAL8(I)=RLZ(MD+7)
      REAL9(I)=RLZ(MD+8)
      REAL10(I)=RLZ(MD+9)
      REAL11(I)=RLZ(MD+10)
      REAL12(I)=RLZ(MD+11)
      REAL13(I)=RLZ(MD+12)

      REAL14(I)=RLZ(MD+13)
      REAL15(I)=RLZ(MD+14)

      REAL16(I)=RLZ(MD+15)
      REAL17(I)=RLZ(MD+16)
      REAL18(I)=RLZ(MD+17)
      REAL19(I)=RLZ(MD+18)
      REAL20(I)=RLZ(MD+19)
C
      WRITE(6,103) I,REAL1(I),REAL2(I),REAL3(I),REAL4(I),REAL5(I),REAL6(
     *I),REAL7(I),REAL8(I),REAL9(I),REAL10(I)
C
   73 CONTINUE
C
      WRITE(6,30)
C
      DO 113 I=1,K
      WRITE(6,103) I,REAL11(I),REAL12(I),REAL13(I),REAL14(I),REAL15(I),R
     *EAL16(I),REAL17(I),REAL18(I),REAL19(I),REAL20(I)
  113 CONTINUE
C
C
C
   11 FORMAT(19F4.2)
   19 FORMAT(4F5.2)
   20 FORMAT(5X,'LMIN=',I3,5X,'LAVG=',I3,5X,'LMAX=',I3,///)
   27 FORMAT(I3)
   33 FORMAT(6F10.0)
   44 FORMAT(3F5.2)
   50 FORMAT(4F5.2)
   51 FORMAT(5X,'AL=',F5.2,5X,'UP=',F5.2,5X,'ALL=',F5.2,5X,'UPP=',F5.2,/
     *//)
   55 FORMAT(3I2)
   57 FORMAT(2F10.0)
   58 FORMAT(5X,'AA=',F10.0,5X,'BB=',F10.0,///)
   65 FORMAT(I2)
   66 FORMAT(4F5.2)
   80 FORMAT(5X,'SL=',F5.2,2X,'SLL=',F5.2,2X,'SD=',F5.2,2X,'SDD=',F5.2,/
     */)
   86 FORMAT(F5.2)
   89 FORMAT(5X,'MARR=',F5.2,//)
  107 FORMAT(F10.0)
  108 FORMAT(10X,'FIRST COST=',F10.0,/)
  333 FORMAT(5X,'Q1=',F5.2,5X,'Q2=',F5.2,5X,'Q3=',F5.2,5X,'Q4=',F5.2,//)
   30 FORMAT('1',1X,'PROJ NO',4X,'REAL11',5X,'REAL12',6X,'REAL13',8X,'RE
     *AL14',6X,'REAL15',7X,'REAL16',5X,'REAL17',6X,'REAL18',7X,'REAL19',
     *8X,'REAL20',///)
  103 FORMAT(2X,I3,5X,10(F8.2,4X),///)
   69 FORMAT(1X,'PROJ NO',4X,'REAL1',7X,'REAL2',7X,'REAL3',9X,'REAL4',7X
     *,'REAL5',9X,'REAL6',7X,'REAL7',7X,'REAL8',8X,'REAL9',8X,'REAL10',/
     *//)
  106 FORMAT('1',1X,'PROJECT',6X,'A',17X,'B',15X,'MODE',//)
  105 FORMAT(5X,I3,3(F15.8,4X),//)
  222 FORMAT('1',3X,I3,3X,I2,4X,I2,5X,F10.8,5X,F10.6,5X,F5.2,5X,F10.3,4X
     *,F14.8,3X,F14.8,2X,F22.8,///)
   29 FORMAT('1',1X,'PROJECT',1X,'TYPE',2X,'LIFE',4X,'ALPHA',10X,'BETA',
     *11X,'ROP',7X,'FCOST',11X,'EXPV',13X,'SDEV',19X,'SKEW',//)
C
      STOP
      END
C
C

      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*65539
      IF(IY) 5,6,6
    5 IY=IY+2147483647+1
    6 YFL=IY
      YFL=YFL*0.4656613E-9
      IX=IY
      RETURN
      END
C
C
C
```

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