# STARS

University of Central Florida
STARS

**Retrospective Theses and Dissertations** 

1976

# Torque-speed Linearization of a D.C. Servo System

Nicholas A. Mantello University of Central Florida

Part of the Engineering Commons Find similar works at: https://stars.library.ucf.edu/rtd University of Central Florida Libraries http://library.ucf.edu

This Masters Thesis (Open Access) is brought to you for free and open access by STARS. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of STARS. For more information, please contact STARS@ucf.edu.

#### **STARS Citation**

Mantello, Nicholas A., "Torque-speed Linearization of a D.C. Servo System" (1976). *Retrospective Theses and Dissertations*. 234. https://stars.library.ucf.edu/rtd/234



## TORQUE-SPEED LINEARIZATION OF A D.C. SERVO SYSTEM

BY

NICHOLAS A. MANTELLO B.S., Florida Institute of Technology, 1968

# RESEARCH REPORT

Submitted in partial fulfillment of the requirements for the degree of Master of Science in the Graduate Studies Program of Florida Technological University

> Orlando, Florida 1976

#### PREFACE

In early 1973 Professor Fred O. Simons Jr., of Florida Technological University, suggested the topic for this research report. During control engineering discussions, it was proposed that a D.C. Shunt Motor could be constrained to perform in an idealized sense for the static case. The purpose of this report is to provide a reasonably comprehensive examination of a control which linearizes the torque-speed relationship of a dc servo system.

The fundamental goal is to determine a control which linearizes the torque-speed output relation of a system, in which a dc shunt motor is under basic armature control. Consequently, the emphasis is on a concept and technique rather than the design of the system process. In this regard, the report does not delve into performance analysis, or design of the system to constrain the torque and speed parameters to operate along an idealized curve.

iii

# TABLE OF CONTENTS

PREFAC	CE.	•	•	•	•	•	•	•	•	•	•	•	•	•	•	iii
Chapte I.	er TORQI	UE-SF	PEED	NON	LINE	ARITY	Y IN	THE	SHUN	м ті	IOTOR	•	•		•	1
	In The	trodu e Shu	uctio unt M	on lotoi	r											
II.	EQUA	TIONS	5 FOF	R TH	E LIN	VEAR	ZAT	ION F	ROCE	SS	•	•	•	•	•	20
	The The The	e Par e Lir e Lir	ramet lear leari	Rela Rela	Enve ation ion N	lope nship Mode	os I									
III.	MECH	ANIZA	TION	I FOI	R LIN	VEAR	ZAT	ION		•	•	•	•	•	•	29
	To	rque	and	Spee	ed Se	ensor	rs									
IV.	CONC	USIC	NS	•	•	•	•	•	•	•	•	•	:-	•	•	30
• •	•	•		•	•	•	•	•	•	•	•	•	•	•	•	
LIST O	F REF	EREN	CES	• -	•	•				•	•	•				32

# LIST OF FIGURES

1.	Schematic Diagram of a DC Shunt Motor	•	2
2.	Ideal Motor Torque Characteristics at Zero Speed	•	4
3.	Ideal Motor Speed Characteristics at Free Running Speed	•	5
4.	Ideal Motor Torque-Speed Characteristic	•	6
5.	Shunt Motor Schematic Diagram	•	9
6.	Shunt Motor Characteristic Curve	•	9
7.	Armature Controlled Shunt Motor Schematic Diagram	•	10
8.	Armature Controlled Shunt Motor Characteristic Curves .	•	11
9.	Basic Armature Controlled Motor-Load Schematic Diagram .		13
10.	Armature Controlled Motor-Load Mathematical Model	•	15
11.	Closed-Loop Speed Control System With Armature Controlled Motor	•	17
12.	Basic Qualitative Steps for Speed Control Systems		21
13.	Qualitative Steps for Torque-Speed Parameter Control .		24
14.	Linearization Model Simplified Diagram	•	26
15.	Torque-Speed Linearization Model		27

## I. TORQUE-SPEED NONLINEARITY IN THE SHUNT MOTOR

## Introduction

A servomotor transforms a control signal into a torque which is applied to a load, in accordance with a selected control strategy, to satisfy the control system specification. Various types of dc motors exhibit different torque-speed characteristics. These characteristics, usually shown as torque-speed curves, are factors which severely limit a system's dynamic performance. The concept of an "ideal motor" provides an artifice for considering torque-speed characteristics, having practical use in the design of dc motors and the servo systems which use them [ 6, 12, 16 ].

The ideal motor is one in which there is negligible brush and bearing friction, and the effects of saturation, armature reaction, and armature inductance are negligible. These criteria, when used with the mathematical modeling of a dc shunt motor allow an equation to be developed relating both the torque and speed parameters with the control signal. Consider the ideal dc motor shown in Figure 1:

where  $R_a$  = armature resistance  $R_f$  = field resistance  $L_a$  = armature inductance  $L_f$  = field inductance  $I_a$  = armature current

 $I_f = field current$ 

V <sub>cs</sub>= control signal

 $V_{f}$  = applied field voltage

 $T_d$  = developed torque

N = speed

 $K_T$  = motor torque constant

 $K_q$  = generator constant

 $E_q$  = induced voltage ( counter-emf )



Figure 1. Schematic diagram of a dc shunt motor

The armature-circuit voltage equation, assuming negligible inductance, is

And the coupling relationship

$$E_{g} = K_{g} N I_{f}$$
 (2)

Then,

$$V_{cs} = I_a R_a + K_g N I_f \qquad (3)$$

The torque delivered by a dc motor depends upon the flux and armature current, in a relationship

$$\Gamma_{d} \sim \Phi I_{a} = K I_{f} I_{a} \tag{4}$$

Therefore, with (3) and (4)

$$T_{d} = \frac{KI_{f}}{R_{a}} (V_{cs} - K_{g} N I_{f})$$
 (5)

and the torque equation has an idealized "linear" form of

$$T_{d} = K_{T} V_{cs} - D N \qquad (6)$$

if field current is held constant, and armature voltage is considered the control variable. It remains to be established, that the terminology "linear" used in conjunction with the torque equation (-6 ) is in fact consistent with the accepted definition of linearity [ 6, 14, 16 ].

Of course, since the effects of friction and other factors have been neglected in the model, torque-speed parametric curves derived from equation (6) are idealized. In terms of physical systems, a system is linear if and only if

 $H(\alpha x_{1} + \beta x_{2}) = \alpha H x_{1} + \beta H x_{2}$ (7) where  $x_{1}$  and  $x_{2}$  are input signals

 $\alpha$  and  $\beta$  are constants

and possesses both the property of additivity and homogeneity [ 3, 5, 13 ]. For generality consider the dc motor from the principle of superposition aspect because of its simplicity. If, in equation ( 6 )

the developed torque is plotted for zero speed and with a variable input ( control signal ), the ideal torque-control signal characteristic of Figure 2 is obtained.



Figure 2. Ideal motor torque characteristics at zero speed

With zero speed, the characteristic curve of Figure 2 can be seen to possess both the property of additivity and homogeneity; first from the simplified equation showing proportionality

$$T_{d} = K_{T} V_{cs}$$
 (8)

and secondly, from the straight line ( linear function ) relation shown in the graphical portrayal. Conversely, at the free running speed, where torque is zero,

$$N = \frac{K_T}{D} V_{cs}$$
(9)

and the principle of superposition applies as shown in Figure 3. The end point conditions where speed and torque were taken to be zero have been demonstrated to be linear.



Figure 3. Ideal motor speed characteristic at free running speed

If the ideal torque-speed characteristics of equation (6) is plotted as shown in Figure 4, with the terminal points satisfying both Figures 2 and 3, the physical system demonstrates linearity in both the torque and speed parameters. Torque and speed display the properties of additivity and homogeneity with respect to the control signal in the ideal motor [12, 16].



Figure 4. Ideal motor torque-speed characteristic

Ideally then, the developed torque T  $_{\rm d}$  of a motor should be proportional to the control signal V  $_{\rm cs}$  and speed N in a relation,

 $T_{d} = K_{T} V_{cs} - D N \qquad (10)$ 

where  $K_T$  is the torque constant and D is the electrical damping constant for the motor, in order to have linear operation. This equation is a straight line curve representing ideal torque-speed characteristics, which may be considerably different than actual motor curves [ 16 ]. In practice, linear theory is utilized in analysis and system design, by considering the actual motor curves as linear for small departures in the region of operation. If a nonlinear situation arises, the standard approach of analysis is to limit attention to small perturbations about the operating point ( reference state )

and find a first order or linear approximation of the nonlinear system. With this technique, the control strategy needed to satisfy the system specification, can be obtained from the linearized equations even though the system equations are nonlinear [5].

In servomotor design, great emphasis is placed upon linearizing the relationship between control voltage and torque. The design goal is to have a straight line characteristic from the stall torque to the no-load speed as indicated by equation (10). Since this is not the case for a normal dc shunt motor, one alternative is to provide a method of adaptive control which constrains the system to perform in an idealized sense.

If a technique can be developed which linearizes the shunt motor torque-speed curve; then it is conceivable that one can benefit from the linearization. One aspect of this proposal is the possibility of a cost saving resulting from the use of a less expensive motor rather than a special design motor for servo system use. Several other factors must be considered; the effects upon system performance and dynamic behavior must be ascertained, and the cost must be minimized so that potential users can effectively trade-off costs and other parameters with the system philosophy and performance specification of the design process. Thus as a first step, this report is directed towards examination of a control technique to linearize the torque-speed curve of a system utilizing a dc shunt motor under basic armature control.

#### The Shunt Motor

In the shunt motor, a field is set up by the armature current in quadrature with the main field which results in a net reduction of the main flux. This effect ( armature reaction ) is caused by the load current saturating the magnetic field path, or creation of an armature flux in opposition to the main flux. The creation of an armature flux by the load current causes a displacement shift in the neutral zone and resultant field flux. Accordingly, the flux density is saturated on part of the pole while the remainder is weakened, causing a net reduction of the main flux. The effect of armature reaction is proportional to load current. Figures 5 and 6 show the schematic diagram of a dc shunt motor connection and its characteristic curve, relating torque, speed and current. The characteristic curve shows that as the load increases, and armature current sets up an opposing field, motor speed decreases. Speed decreases as the main field is weakened until there is a drastic change at an operating point where the load torque approaches the stall torque value [ 11, 16 ].



Figure 5. Shunt motor schematic diagram



# Figure 6. Shunt motor characteristic curve

The armature control mode of connection represents one method of utilizing a shunt motor in a system. In this mode, the field current is held constant and the armature voltage is used as the control variable. When this mode is utilized, the torque-speed characteristic curve approaches that of the ideal motor previously shown in Figure 4. Figures 7 and 8 show the schematic diagram of a dc shunt motor under armature control and its characteristic curve. This is the mode of control to be investigated for development of a linearizing scheme. The torque-speed curve of the armature controlled motor is linear over a wide range and to a first approximation is represented by the ideal curves shown in Figure 8. The straight line curves neglect the effects of brush and bearing friction and armature reaction; all of which can be made very small in a well designed motor [ 12, 14, 16 ].



Figure 7. Armature controlled shunt motor schematic diagram



Figure 8. Armature controlled shunt motor characteristic curves

In addition to the nonlinearities which are inherent to the motor, nonlinearities can arise in a servo system due to associated components. Thaler and Wilcox [15] in presenting a basic approach to the analysis and design of speed control systems with dc motors, account for modulator impedance in the analysis of performance. Both the characteristics of the motor and amplifier are considered together since the characteristics of both affect speed regulation. Truxall [16] provides an excellent treatment of nonlinearities arising from the basic torque equations, magnetic saturation, armature reaction, dead zone, and static friction from both the aspects of design techniques and electromechanical actuators.

Both natural and induced environments which a servo system experiences under use conditions can influence system nonlinearity. These environments which can be either internal or external to the system and its components, affect electrical and mechanical portions of the system. As an example; thermal shock and vibration environments can affect friction characteristics by causing a change in mechanical clearances, while electrical parameters can also change in response to system use conditions. Simple wearout ( a condition of repeated system use ), continuous environmental stress or environmental cycling create changes in the original system characteristics, influencing nonlinearity and causing performance degradation. The development of a linearizing control could compensate for both the effects of environment and inherent component nonlinearities in the system.

Consider the basic armature control system shown in Figure 9, where

J<sub>M</sub> = motor moment of inertia f<sub>M</sub> = motor viscous friction J<sub>L</sub> = load moment of inertia f<sub>L</sub> = load viscous friction n = motor to load gear ratio  $\Theta_{M}$  = motor shaft position  $\dot{\Theta}_{M}$  = motor shaft velocity  $\ddot{\Theta}_{M}$  = motor shaft velocity  $\ddot{\Theta}_{M}$  = motor shaft acceleration  $\Theta_{L}$  = load shaft position





Figure 9. Basic armature controlled motor-load schematic diagram For dynamic equilibrium the torque equation at the motor shaft is,

$$T_{d} = (J_{M} + J_{L}) \overset{\circ}{\Theta}_{M} + (f_{M} + \frac{f_{L}}{n^{2}}) \overset{\circ}{\Theta}_{M} + T_{L}$$
(11)

This equation does not include the effects of retarding torque due to shaft stiffness. The basic assumption is that the shaft spring constant ( K <sub>S</sub> ) is comparatively large or alternately that all shafts have infinite stiffness. In instrument servos, the spring constant is considered comparatively large. Equation (11) can be simplified to

$$T_{d} = J \overset{\bullet}{\Theta}_{M} + f \overset{\bullet}{\Theta}_{M} + T_{L}$$
 (12)

with  $J = (J_M + J_L/n^2)$  and  $f = (f_M + f_L/n^2)$ . The field current for the armature control motor is usually constant and equation (4) thus becomes

$$T_{d} = K_{Ta} I_{a}$$
(13)

where K  $_{Ta}$  is the motor torque constant in the armature control mode. The circuit voltage equation for Figure 9 is

$$V_a = R_a I_a + L_a I_a + K_{ga} \Theta_M \qquad (14)$$

where K  $_{ga}$  is the counter-emf constant for the motor. Utilizing equations (12), (13), and (14) to develop the system equation yields

$$L_{a}J\overset{\textcircled{}_{}}{\Theta}_{M} + (R_{a}J + fL_{a})\overset{\textcircled{}_{}}{\Theta}_{M} + (fR_{a} + K_{ga}K_{Ta})\overset{\textcircled{}_{}}{\Theta}_{M}$$
$$= K_{Ta}V_{a} - L_{a}\overset{\textcircled{}_{}}{T}_{L} - R_{a}T_{L}$$
(15)

Applying the Laplace transform,

$$s [L_a J s^2 + (R_a J + f L_a) s + f R_a + K_{ga} K_{Ta}] \Theta_M(s)$$
  
=  $K_{Ta} V_a(s) - (s L_a + R_a) T_L(s)$  (16)

If T  $_{L}(s) = 0$ , the transfer function for the armature controlled motor-load combination is,

$$\frac{s \Theta_{M}}{V_{a}}(s) = \frac{K_{Ta} / L_{a} J}{s^{2} + 2 \zeta \omega_{n} s + \omega_{n}^{2}}$$
(17)

where  $2 \zeta \omega_n = (R_a J + fL_a) / L_a J and$  (18)

$$\omega_n^2 = (fR_a + K_{ga}K_{Ta}) / L_J \qquad (19)$$

The mathematical model of the developed motor-load combination for the armature control connection is shown in Figure 10. The transfer function notation in this figure has been simplified, by representing  $K_{M} = K_{Ta} / L_{a} J, K_{a} = R_{a} / K_{Ta}$ , and  $\tau_{a} = L_{a} / R_{a} [1, 14, 15, 16].$ 



Figure 10. Armature controlled motor-load mathematical model

When it is necessary to provide accurate control of steadystate speed, closed-loop feedback control is established. In closedloop feedback control, the speed measurement is compared to the reference signal and an error is generated to control speed regulation.

The change from the model shown in Figure 10 is very simple, but the steady state accuracy is improved greatly with the regulation reduced to less than 1 percent with tachometer feedback. Figure 11 illust-rates a closed-loop speed control system with an armature controlled motor and tachometer feedback. The model of Figure 10 has been mod-ified by the addition of an amplifier and tachometer [ 14, 15, 16 ].

In this closed-loop model, the amplifier is connected to the motor block with the assumption that no loading effect exists to disturb the transfer functions. The amplifier output circuit impedance must be added to those of the motor circuit being driven by the amplifier. This model shows no poles in the amplifier circuit. As such, it is applicable to vacuum tube and transistor amplifier control. When rotating amplifiers (Ward-Leonard ) or other types such as thyratron control are utilized, the model must be modified to account for the addition of open-loop amplifier poles. The amplifier with gain  $G_A$  in this model is a pure gain component. If it provided rectification; its transfer function would include an open-loop pole [ 14, 15, 16 ].

The system transfer function can be formulated from the block diagram of Eigure 11 and the basic relationship

$$\frac{C}{R} = \frac{G}{1 + G H}$$
(20)

where C represents the system output, R is the reference input, G is the forward transfer function and H is the feedback transfer function. The open-loop speed equation is given by,





18

$$s \Theta_{L} = \frac{K_{A} K_{M} R(s) - K_{a} K_{M} (s + 1/\tau_{a}) T_{L}(s)}{s^{2} + 2 \zeta \omega_{n} s + \omega_{n}^{2}} (21)$$

The closed-loop speed is

$$s \Theta_{L} = \frac{G_{A}G_{M}R(s) - K_{a}(s + 1/\tau_{a})G_{M}T_{L}(s)}{1 + G_{A}G_{M}G_{t}}$$
 (22)

The output-input function at  $T_{1} = 0$  is

$$\frac{\text{speed}}{R} = \frac{K_A K_M}{s^2 + 2 \zeta \omega_n s + \omega_n^2 + K_A K_M K_t}$$
(23)

The output-input function with R a constant is

$$\frac{speed}{T_{L}} = \frac{K_{a}K_{M}(s+1/\tau_{a})}{s^{2}+2\zeta\omega_{n}s+\omega_{n}^{2}+K_{A}K_{M}K_{t}}$$
(24)

The 'closed-loop speed control system model of Figure 11 is described by the equations of motion (11), (13), and (14), the speed transfer functions of equations (21) and (22), and the outputinput functions of equations (23) and (24). This model is a basic uncompensated system; compensation for obtaining stability or acceptable performance is not included. Some significant remarks can be made about this closed-loop system [5, 14, 15, 16].

- 1. The closed-loop system is inherently stable
- The model is incorrect if the motor is driven into nonlinear operation, but it can be used as a first approximation
- 3. The forward gain must be increased when feedback is used
- High amplifier gain may cause system oscillation or instability, compensation may be required

- The system may be underdamped if the amplifier is pure gain with no open-loop poles
- Additional poles may be introduced by the amplifier, increasing the need for compensation
- Introduction of additional amplifier poles causes systems to become unstable
- Load torques may be considered separately for constant, periodic or random application
- 9. The transient response to load change is not the same as for reference speed change
- 10. Load change can be examined by substituting s = 0 into the openloop system function
- 1]. The speed / T \_ closed-loop response for load disturbance has greater bandwidth than speed / R, and possibly larger overshoot

#### II. EQUATIONS FOR THE LINEARIZATION PROCESS

#### The Parameter Envelope

The family of nonlinear torque-speed curves, for the armature controlled motor ( Figure 8 ) is bounded by:

- The torque-speed curve at maximum control voltage determined by motor design
- 2. The loci of stall torque end point conditions ( N = 0 )

3. The loci of no-load speed end point conditions (T = 0) Within these boundaries ( parameter envelope ) for the particular motor it is possible to have infinitely many torque-speed combinations for incremental changes in control voltage. Torque-speed combinations within these boundaries do not impose any restraints upon the motor, except for points lying on or near the boundaries. The dc motor may not be designed for continuous stall operation or operation in the vicinity of stall; in general this is not usually a system requirement except under intermittent or random conditions [14, 16].

In view of the torque-speed characteristic curves, it is appropriate to qualitatively consider the closed-loop speed control system for the static case with reference to Figure 11. To attain accurate speed control several basic steps, as shown in Figure 12, are accomplished. Assume that at a point in time the desired speed  $N_1$  at torque  $T_1$  has changed in response to load conditions to speed  $N_2$  at torque  $T_2$ . The tachometer measures the speed  $N_2$  and generates an electrical signal proportional to it. With this signal and the reference signal representing desired speed, the motor control voltage is either increased or decreased depending upon the speed N<sub>2</sub> relation to speed N<sub>1</sub> to drive the system speed to the desired value at N<sub>3</sub>.



- 2 Determine difference between desired and actual speed  $(N_1 N_2)$
- 3 Adjust control signal to account for speed difference to arrive at desired speed N 3

Figure 12. Basic qualitative steps for speed control systems

#### The Linear Relationships

Equation (10) expressed the developed torque of a motor  $T_d$  in a linear relationship with the control signal V and speed N. This equation was developed from analysis of the motor electrical circuit with appropriate assumptions. Actual output torque of the motor shaft is distinctively different. The motor actual output torque T measured at the shaft of the unloaded motor assuming negligible static or coulomb friction is related to T by,

$$T_{A} = T_{d} - f_{M} N \qquad (25)$$

where  $f_{M}$  is the motor viscous friction constant. Equation (25) can be referred through the gears to the load by the use of the gear ratio n defined in terms of speed as,

$$\frac{\dot{\Theta}_{M}}{\dot{\Theta}_{L}} = n \qquad (26)$$

It may be shown that with the definition of gear ratio n,

$$T_{L} = n T_{A}$$
 (27)

From equations (25) and (27)

(

(

$$T_{A}_{L} = n(T_{A}) = n(T_{d} - f_{M}N)$$
 (28)

where (T  $_{A}$  ) is the load torque delivered by the motor at the load. Substituting for T  $_{d}$  in equation (28) with equation (10),

$$T_{A}_{L} = n [K_{T}_{cs} - (D + f_{M})N]$$
 (29)

and

$$(T_A)_{L} = n (K_T V_{CS} - D_M N)$$
(30)

where D  $_{M}$  is the motor damping constant including both viscous friction and electrical damping [ 14, 15, 16 ].

Without loss of generality, equation ( 30 ) can be written,

T = K V - D N(31) A T cs M (31) Caution must be exercised with the use of equation (31); if load torque is under consideration, the gear ratio n must be a factor in the equation.

Equation (31) allows a model to be defined which can be utilized to adjust the motor control so that torque-speed output of the servo system can be constrained to follow the straight line form of the ideal servo motor. It is more convenient to express equation (31) in terms consistent with the need to adjust the torque of the system. The actual torque delivered by the motor T<sub>A</sub> can be considered the desired torque T<sub>D</sub> needed at the system output, and the output torque T<sub>O</sub> experienced due to load variations are related by

$$T_{D} = T_{O} - m N \qquad (32)$$

where m is the geometrical slope. Figure 13 shows one of the family of curves for variable V<sub>cs</sub>, illustrating the interpretation of equation (32). Consider initially, that torque T<sub>0</sub> = T<sub>1</sub> at speed N<sub>1</sub> and there are system components for measuring torque and speed at the system output. The tachometer measures speed N<sub>1</sub> and generates a voltage signal proportional to it. With N<sub>1</sub> and equation (32), the value of T<sub>D</sub> corresponding to speed N<sub>1</sub> on the straight line curve can be determined. Then, a motor control signal corresponding to T<sub>D</sub> can be provided by adjustment of the motor control voltage e<sub>a</sub>. T<sub>D</sub> can be summed with measured torque T<sub>0</sub> to generate an error signal driving the system to the desired value of torque-speed on the straight line curve. Subsequent changes in speed, torque or motor control signal e operate in the same manner and the torque-speed parameters are constrained to move along the straight line in the static case.



Determine actual speed N<sub>1</sub>, and output torque T<sub>1</sub>
 Determine desired torque T<sub>D</sub> = T<sub>2</sub> from T<sub>D</sub> = T<sub>0</sub> - m N
 Adjust control signal e<sub>a</sub> to drive motor to T<sub>D</sub>
 Determine T<sub>D</sub> - T<sub>0</sub> and adjust T<sub>D</sub> signal to account for error in torque

Figure 13. Oualitative steps for torque-speed parameter control

Figure 14 shows a simplified linearization model block diagram, incorporating the basic steps to control the torque-speed parameter. The block labeled Model Definition at the motor control voltage input e a remains to be defined. Its function is to represent the straight line curve of equation (32) and provide the implementation of basic steps 2 and 3 of Figure 13.

## The Linearization Model

In general, the linearization might be implemented as shown in Figure 15. A tachometer is connected to the motor drive shaft producing a voltage proportional to speed. This feedback signal is used as a point of reference by which the system operation is transferred from the nonlinear to the linear curve. Since it is desired to constrain the torque-speed parameter to move along the straight line curve shown in Figure 13; a voltage divider is used to provide a voltage proportional to T<sub>D</sub> which lies on the straight line curve. The divider has a proportionality factor m, equal to slope m of the straight line, which divides the speed voltage to a value representative of T<sub>D</sub>. An equivalent statement is that for each point on the straight line curve representing speed, the torque at this point is proportional to it by a factor m ( slope ).

A torque sensor provides a voltage proportional to output torque T<sub>0</sub>. The desired torque T<sub>D</sub> is subtracted from T<sub>0</sub> resulting in a torque difference  $\varepsilon$ . Epsilon represents the difference between torques, at the same speed, on the nonlinear and linear curves. This difference can have a positive or negative value. The sign of  $\varepsilon$  dep-







Figure 15. Torque-speed linearization model

ends upon whether T  $_0$  lies above or below the straight line curve at the time of consideration. The torque difference  $\varepsilon$  is used to adjust the drive voltage e  $_a$  to either decrease or increase the motor voltage to drive the torque-speed parameter to operation along the linear curve. If  $\varepsilon$  is positive, e must be decreased and if  $\varepsilon$  is negative, e must be increased for proper operation.

The torque difference  $\varepsilon$  and the drive voltage must be operated upon in a circuit which has the following basic specification:

- 1. With  $\varepsilon$  positive, e must decrease until  $\varepsilon$  = 0 and then hold at constant e
- 2. With  $\varepsilon$  negative, e must increase until  $\varepsilon$  = 0 and then hold at constant e

This circuit is shown in Figure 15 as the basic summation symbol modified by these two constraints.

Basically, by the use of both speed and torque feedback the motor voltage is decreased or increased to a value at which the torque-speed parameter lies on the straight line. With the model, the torque-speed parameter is constrained to operate along the straight line curve in the static sense.

#### III. MECHANIZATION FOR LINEARIZATION

#### Torque and Speed Sensors

Figure 15 shows both torque and speed feedback. The speed feedback utilizes a conventional tachometer to provide a voltage proportional to speed. To measure the output torque T  $_{\mathrm{O}}$  a dynamic-torque transmitter is shown in the linearization model. Miller [8] describes dynamic torque transducers which measure torque while transmitting power. One type of transducer ( or transmission dynamometer ) is called a noncontact type. This transducer measures twist in the drive shaft and translates it to torque without contact with the physical system. A coupling shaft is incorporated between the motor and load. This coupling shaft incorporates two discs which are aligned with optical sensors. As the motor spins up, the twist in the connecting shaft is measured through the determination of phase lag between the optical signals. This phase lag (twist) is converted to a signal representing dynamic torque. Other methods utilizing magnetic, capacitive, or inductive noncontact signal paths have been considered by manufacturers for noncontact type dynamic torque transducers, each operating in essentially the same mode [ 8 ].

# IV. CONCLUSIONS

A system utilizing a dc shunt motor can be constrained, through the use of control engineering concepts so that the motor performs in a linear fashion even though the torque-speed characteristic is nonlinear. The simple linearization model presented constrains the torque-speed characteristic to operate along a linear curve which lies within the motor parameter envelope, for the static case. With the concept presented, the linearization model adjusts for nonlinearities caused by the motor design, and those caused by natural or induced environments. As nonlinearities are introduced by wear or, other factors, the process continuously adjusts for time oriented modes of nonlinearity.

The linearization scheme, simple in nature, controls the torque-speed parameter by adjusting motor control voltage in response to the actual nonlinear and desired characteristics. Conceptually the model performs in a comparator mode of operation constraining the torque-speed to match the desired linear curve. No design mechanization of the model is given, but it is apparent that the model could be designed for variable slope m and axis intercept, parameters of the line to provide generality for system utility.

Perhaps the greatest utility of the closed-loop linearization model would be to use the process with low cost motors for which no special efforts have been taken to control the torque-speed charact-

eristics linearity. If a low cost, and general utility linearization system could be designed, one has the advantage of being able to take a "cheap" motor, add the linearization package and achieve linear operation in the static case.

The linearization model presented is a concept with commercial application if developed and produced as a low cost product with today's technology. The design and performance analysis of the system remains to be accomplished as further development of the concept. Basic mathematical models developed for the armature controlled motor used in a speed control system can be used as a basis for the performance analysis.

#### LIST OF REFERENCES

- Carlstein, Joe. "Systems Designer's Handbook, Servomechanisms." Electromechanical Design (October 1969): 89-105.
  - Chestnutt, Harold and Mayer, Robert W. Servomechanisms and <u>Regulating System Design</u>. 2vols. New York: John Wiley, 1955.
  - 3. Derusso, Paul M.; Roy, Rob J.; and Close, Charles M. <u>State</u> Variables for Engineers. New York: John Wiley, 1967.
  - Edwards, Robert S. "Selecting Electric Servomotors." <u>Machine</u> <u>Design</u> 21 (January 1949): 104-109.
  - 5. Elgerd, Olle I. <u>Control Systems Theory</u>. New York: McGraw-Hill, 1967.
  - 6. Gibson, John E., and Tuteur, Franz B. <u>Control System Components</u>. New York: McGraw-Hill, 1958.
  - 7. Kyser, James G. "Experimental Servo Analysis." <u>Military Systems</u> Design (December 1963): 2-7.
  - Miller, William S., ed. "Torque and Power." <u>Machine Design</u> 46 (November 1974): 88-89.
- 9. Natkin, Hy. "Guide to Selecting Servomotors." Instruments and Control Systems 46 (June 1973): 69-77.
- Newton, George C., Jr. "What Size Motor?" <u>Machine Design</u> 22 (November 1950): 125-131.
- 11. Porter, E. "DC Motors." Machine Design 46 (April 1974): 5-8.
- 12. Savant, C. J., Jr. <u>Basic Feedback Control System Design</u>. New York: McGraw-Hill, 1958.
- 13. Schwarz, Ralph J., and Friedland, Bernard. Linear Systems. New York: McGraw-Hill, 1965.
- 14. Thaler, G. J., and Brown, Robert G. <u>Analysis and Design of</u> <u>Feedback Control Systems</u>. 2nd ed. New York: McGraw-Hill, 1960.
- 15. Thaler, G. J., and Wilcox, M. L. "Adjustable Speed D.C. Drives." Control Engineering 10 (November 1963): 75-99.

- 16. Truxall, John G., ed. <u>Control Engineer's Handbook</u>. New York: McGraw-Hill, 1958.
- 17. Walker, Peter. "Toward the Ideal Servomotor." Machine Design 46 (May 1974): 105-107.