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METHODS AND TECHNIQUES
USED FOR JOB SHOP SCHEDULING

BY

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RESEARCH REPORT

Submitted in partial fulfillment of the requirements
for the degree of
Master of Science: Operations Research
in the Graduate Studies Program of the College of Engineering
of Florida Technological University

Orlando, Florida
1977

ABSTRACT

The job shop scheduling problem, in which we must determine the order or sequence for processing a set of jobs through several machines in an optimum manner, has received considerable attention. In this paper a number of the methods and techniques are reviewed and an attempt to categorize them according to their appropriateness for effective use in job shop scheduling has been made. Approaches are classified in two categories: a) analytical techniques and b) graphical methods. Also, it should be noticed that this report does not include all the attempts and trials, especially the heuristic approaches.

ACKNOWLEDGEMENTS

I wish to express my sincere thanks to Dr. Yasser Hosni, my committee chairman, for his assistance and guidance throughout the preparation of this research paper. Special thanks go to my committee members, Dr. George Schrader and Dr. Harold Klee, for their assistance to complete this paper. I further wish to express my thanks and appreciation to my wife, Boo Young, and my son, Jung Hoon, for their patience and encouragement.

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CHAPTER I

INTRODUCTION

An important function of job shop scheduling is the coordination and control of complex activities, both optimum resource allocation and sequence in the performance of those activities. The job shop scheduling problem in which we must determine the order or sequence for processing a set of jobs through several machines in an optimum manner, has received considerable attention. A variety of scheduling rules and procedures for certain types of job shops have evolved from these efforts. Network planning and control techniques have found wide application to the scheduling problems associated with project activities. Numerous procedures also have been proposed for determining optimal or near-optimal work station assignments for assembly lines.

The scheduling problem is difficult to standardize due to the variety of criteria involved. On the other hand, the choice of criteria has also been influenced by the prospects of obtaining a solution. In some models it has been possible to find optimal procedures only by departing from what would be considered the most natural and realistic criteria.

Not all job shop scheduling problems for these diverse systems can be efficiently solved, and in several instances heuristic

techniques that yield nonoptimal but relatively good solutions will be employed. In many cases, it may seem that a scheduling algorithm has been developed from an overly simplified model of the real system.

The purpose of this paper is to review a number of the methods and techniques that have been used in job shop scheduling and attempt to categorize them according to their appropriateness for effective use in job shop scheduling. Also, it should be noticed that this report does not include all the attempts and trials which have been developed to solve complicated job shop scheduling problems, especially the heuristic approaches. In this paper, approaches are classified in two categories:

1. Analytical techniques: Mathematical approaches of assigning jobs to machines are derived to optimize certain criteria. Some of these methods can get an exact solution. Others, we apply heuristic routines for computer usage which provide optimal or near optimal solutions.

2. Graphical methods: Graphical techniques do not provide optimal or near optimal solutions directly, but these techniques allow management to forecast problems early enough to take corrective action, which provide means for stimulating alternative plans. Three graphical techniques are reviewed to meet job shop scheduling problems.

CHAPTER II

OBJECTIVE OF JOB SHOP SCHEDULING

Scheduling is the area in production control systems in which the preplanned activities such as aggregate production schedules and aggregate inventory levels are projected on a detailed time scale. The detailed allocation of jobs and materials to human and physical resources, man and machines, take place in scheduling.

Schedules are based on the aggregate planning or the master schedules, the established optimal lot sizes, and the knowledge of available resources. The scheduler tries to find detailed schedules which are optimal with respect to meeting due dates, high machine utilization, low unit cost and other possible goals. These approximate goals are necessary because it is hard to define long-run profit in the short-run situation where it often appears that all costs are fixed. Results of the scheduling activities are fed back to the other planning and control areas to improve their decision making.

To differentiate between schedules and to select the best one, we have to have some measures of effectiveness, as in other areas where we want to "optimize", with which we can compare the different solutions.

In general we want to minimize either the length of operation time such as total processing time, completion time for certain products, average finishing time, total project time, minimize idle time, or we want to minimize certain costs such as the unit cost of production, total cost, etc. The underlying notion for all these objectives is that of profit maximization. Examining the different measure of effectiveness more closely we find that some of them are not applicable for certain problems and that, what is worse, many of them are contradictory. Here we face "the dilemma of scheduling", which is particularly evident in job shop production. The following points are examples of the contradiction in the scheduling operation:

1. We want to decrease the average in-process time of our work orders, thus decreasing in-process inventory and increasing the likelihood of meeting due dates.

2. Also, we want to increase the degree of utilization of equipment, thus increasing the return on our investment in physical facilities.

Achievement of the first goal would lead to the selection of the schedule with the smallest in-process time for all our products, such as the processing time, the transportation time, the waiting time, and the setup time. This objective focuses on the jobs to be done and implies moving them rapidly through the production process.

To achieve the second goal, we need to pick the schedule that maximizes the utilization of existing capacity. This objective focuses on the machines and implies arrangement of jobs to suit the

machines.

It is easily seen that a schedule which is optimal with respect to total in-process time does not have to be optimal with respect to the utilization of existing capacity. It should, however, be noted that the contradiction of these goals exists only for rather short planning horizons or if information concerning future orders is very uncertain. In the long run, the minimum cost goal, including capital cost and inventory cost, includes most of the other sub-objectives of minimum in-process inventory or maximum utilization of machine capacity.

From the two major objectives above, a number of secondary measures of effectiveness can be derived which takes into account some aspects of the overall problem or focuses on important factors which influence the total result:

1. Minimize the time the facilities are occupied.
2. Minimize total idle time.
3. Minimize total waiting time of products.
4. Minimize the total lateness, i.e., the time that it takes to finish products after they were due for delivery. In some cases we might want to provide the lateness with different weight corresponding to different penalties which we have to pay for lateness of different products or corresponding to different degrees of importance of finishing the different products on time.

CHAPTER III

JOB SHOP SCHEDULING PROBLEM

Description of Job Shop Scheduling

Unit production or job shop production involves the manufacture of discrete units. This involves production where the production units are processed either as single entities or in small batches. Scheduling is generally controlled by a routing sheet or short order process rather than by an assembly line system.

Job shop production equipment is usually of a general purpose nature in order to provide the flexibility necessitated by the variation in size, shape, quantity, precision, and type of product. Usually, similar machines are grouped into work centers, and originally each machine can perform a variety of tasks. A work center may also function as an assembly area. The job scheduling problem consists of determining the order or sequence in which the machines will process the jobs so as to optimize some measure of performance.

Characteristics of Jobs Shops

The nature of a wide variety of products and the plants in which they are produced gives certain characteristics common to

virtually all job shops and the following characteristics are stated by Griffin (1):

1. At any time there is a large number of orders at various stages of completion.
2. Orders make conflicting demands on facilities and manpower.
3. Every order differs to some extent. It is difficult, therefore, to predict accurately the time required to complete operations.
4. Work flow is intermittent and orders can be sidetracked.
5. There is usually a queue of work at each machine and it is often difficult to determine which order in the queue should have priority.
6. There are many changes resulting from scrap, rework, machine breakdown, material shortages, engineering changes and rushorders.
7. Considerable effort is expended in determining the status of orders and in expediting orders through various departments. Many orders are marked "rush". Lists of "hot" jobs are developed regularly.
8. Schedules and shop loads are rarely altered due to the very heavy clerical workload required to make the alterations.

In short, the job shop is complex and unpredictable. Close control is rarely established.

In developing a job shop scheduling system it is important to achieve an appropriate balance between control and flexibility to

improve problems rather than to eliminate them completely.

How rigidly should orders be scheduled?

What option should be left to the foreman?

How closely should shop loads be controlled?

How closely should progress be monitored?

The answers to these and similar questions will differ in every circumstance.

Four factors serve to describe and classify a specific job shop scheduling problem.

1. The job arrival pattern. If 'n' jobs arrive simultaneously in a shop that is idle and immediately available for work, then the scheduling problem is said to be static. If jobs arrive intermittently, possibly according to a stochastic process, the scheduling problem is dynamic.

2. It is necessary to specify the number of machines, m , that compose the job shop.

3. The flow process of jobs through the machine must be specified. If all jobs follow the same routing, then the shop is a flow shop. The opposite extreme is the randomly routed job shop, in which jobs do not follow a common sequence of operations.

4. The criterion for evaluating the performance of the shop plays a critical role in the scheduling process.

Outcomes of Scheduling Problems

The outcome of scheduling for a job shop may be stated as follows:

1. To determine the long-term strategic posture of the shop in relation to the market, in particular, to determine the target product mix and the corresponding configuration of shop capacities.

2. To plan and control the timing of production in detail in the shop so as to achieve efficient production, in particular so as to shorten lead times and low setup and in-process inventory costs.

3. To negotiate the timing of deliveries with customers on a realistic basis reflecting the presence of other orders, the capabilities of the shop, and the cost of achieving a certain timing, as well as the value of the timing to the customer.

4. To plan the configuration of the shop, in particular the allocation of man power within the shop, so as to perform the required work efficiently.

5. To negotiate deliveries from suppliers on the basis of a consistent production plan taking account of all the orders to be produced and of shop capabilities.

6. To schedule and control other pre-production activities, such as engineering work; to co-ordinate these activities so as to carry out production as planned.

7. To perform these scheduling and planning operations on the basis of information coming to the form irregularly through time, allowing for the uncertainty about the future and for the occurrence of uncontrolled and unpredictable disturbances in the shop, on the part of suppliers and customers, and indeed in the scheduling system itself.

CHAPTER IV

METHODS AND TECHNIQUES

Introduction

Methods and techniques used for job shop scheduling can be classified into two categories: analytical and graphical methods. Under the analytical method come the mathematical approaches of assigning jobs to machines to optimize certain criteria. Some of these methods are exact seeking sub-optimal solutions. Under the graphical category comes most of the practical methods used in job shop scheduling and control.

Analytical Techniques

The assignment of jobs to machines is a frequently occurring problem in the job shop industry. The assignment of N jobs to up to two machines has an exact solution which is readily obtained. When the problem expanded, the optimal assignment becomes more complex. An exact solution is obtained if all combinations of assignments are made and the elapsed times are determined. The minimum of that set is optimal. However, this can become a large problem very rapidly. There are $6!$ assignments for six jobs. There are heuristic routines for solutions within reasonable computation times.

The following assumptions are made all through this chapter:

1. There are N jobs that require processing on M machines (Job $i = 1, 2, \dots, N$; Machine $j = 1, 2, \dots, M$).
2. Every job requires every machine and no job is processed more than once by any machine.
3. A machine can process only one job at any given time.
4. There is only one machine of each type.
5. All operations, once started, must be completed without interruption.
6. Processing times are assumed to be known without error.
7. Processing times are independent of each other and also of the order in which they are processed.
8. Setup time and the time required to transport jobs between machines is zero.

Exact Methods

N Jobs, One Machine

The criteria to optimize is to minimize the mean flow time¹ by determining an optimal sequence for a set of " N " jobs to be processed by a single machine. So assuming that p_1, p_2, \dots, p_n are the processing times (including any necessary setup time for the machine) of the jobs and it is assumed that the processing times are known with certainty. A schedule will be some permutation of the

¹Mean flow time: mean flow time is equal to the product of the mean number in the system and the mean time between arrivals, or $F = \frac{n}{\lambda}$ where λ is the mean arrival rate.

integers $1, 2, \dots, n$. The procedure is called shortest processing time sequencing (SPT) when it is arranged in order of nondecreasing processing time. The SPT rule results in a minimum mean flow time schedule. The flow time of a job in the k th position of an arbitrary sequence is

$$F_k = \sum_{i=1}^k p_i \quad (4-1)$$

The mean flow time of the entire n job sequence is just

$$\bar{F} = \frac{\sum_{k=1}^n F_k}{n} = \frac{\sum_{k=1}^n \sum_{i=1}^k p_i}{n} = \frac{\sum_{i=1}^n (n-i+1) p_i}{n} \quad (4-2)$$

Now a sum of pairwise products of two sequences of numbers can be minimized by arranging one sequence in nonincreasing order and the other sequence in nondecreasing order. Since the co-efficients $(n-i+1)$ are already in nonincreasing order, \bar{F} would be minimized by sequencing the jobs so that the processing times are in nondecreasing order. Thus, the SPT sequence results in a minimum mean flow time.

In many situations, all jobs are not equally important and the job has an importance weight or value w_i (the larger w_i , the more important the job). We wish to schedule a set of n jobs so as to minimize the mean weighted flow time.

$$\bar{F} = \frac{\sum_{i=1}^n w_i F_i}{n} \quad (4-3)$$

To accomplish the sequencing of the jobs that minimize the mean

flow time, the following condition should be satisfied

$$\frac{p_1}{w_1} \leq \frac{p_2}{w_2} \leq \dots \leq \frac{p_n}{w_n} \quad (4-4)$$

which is a generalization of SPT.

N Jobs, Two Machines

The procedure is to find the job with the shortest time on either machine. If it is on the first machine, it is to be assigned first. If it is on the second machine, assign it last. Then we remove the job just assigned from further consideration. We proceed in the same manner with the remaining jobs until all jobs have been assigned. Symbolically, let p_{i1} be the processing time on machine 1 of the job in the i th position in the sequence. Define p_{i2} similarly for machine 2. It is clear that the last job can not be completed earlier than the time required to process the last job on machine 2. Thus

$$F \geq \sum_{i=1}^n p_{i1} + p_{n2} \quad (4-5)$$

Similarly, the last job cannot be completed in less time than required to process all n jobs on machine 2, plus the delay time before machine 2 can begin, or

$$F \geq \sum_{i=1}^n p_{i2} + p_{11} \quad (4-6)$$

Now the summation of both equations are unaffected by sequence, and so we may only influence these bounds by the choice of p_{n2} and p_{11} . Therefore, we could choose the job with the smallest p_{ij} . If $j=1$,

we would put that job first in sequence so as to minimize p_{11} . If $j=2$, we would put that job last in the sequence so as to minimize p_{n2} . Now with the first job sequenced, one could repeat the same argument for the set of $n-1$ remaining jobs. While not a proof, this does illustrate the logic upon which the procedure is based. This is the most efficient algorithm developed by Johnson (2) which can be expanded to 3 machines under certain circumstances.

Jackson (3) has shown that Johnson's algorithm may be modified to produce a minimum makespan sequence of n jobs in a two-machine job shop (jobs may have different technological orderings). Partition the n jobs into four sets defined as follows:

- {A} = the set of jobs that are processed only on machine 1
- {B} = the set of jobs that are processed only on machine 2
- {AB} = the set of jobs to be processed on machine 1 followed by machine 2
- {BA} = the set of jobs to be processed on machine 2 followed by machine 1

Sequence the jobs in {AB} by Johnson's algorithm. Then sequence the jobs in {BA} by Johnson's algorithm. Now select any arbitrary sequence for the jobs in {A} and {B}. Finally, combine the sets of jobs in the following way, without changing the order within each set:

- Machine 1: jobs in {AB} before jobs in {A} before jobs in {BA}
- Machine 2: jobs in {BA} before jobs in {B} before jobs in {A}

{AB}

N Jobs, Three Machines

Johnson's algorithm for the two-machine case may be extended to the three-machine case under certain circumstances. If either of the following two conditions is true, then Johnson's method is applicable

$$\begin{aligned} &\text{either } \min \{p_{i1}\} \geq \max \{p_{i2}\} \\ &\text{or } \min \{p_{i3}\} \geq \max \{p_{i2}\} \quad i=1,2,3,\dots,n \end{aligned} \quad (4-7)$$

That is, Johnson's method is applicable if machine 2 is completely dominated by either the first or third machine. The working procedure involves defining two dummy machines, say 1' and 2' with processing times

$$\begin{aligned} p_{i1'} &= p_{i1} + p_{i2} \\ p_{i2'} &= p_{i2} + p_{i3} \end{aligned} \quad (4-8)$$

and applying Johnson's algorithm to this new two-machine problem.

Giglio and Wagner (4) applied Johnson's method to 20 general six-jobs, three-machine flow shop problems that did not satisfy either of the required conditions. In 9 out of the 20 cases, an optimal solution was actually generated, and in 8 of the remaining cases, the solution obtained could be made optimal by interchanging two adjacent jobs. Apparently Johnson's method is a useful approximate procedure even if the optimality conditions are not satisfied. At least it would often provide a good starting point for further analysis.

Two Jobs, M Machines

A simple solution to the two-job, m -machine job shop scheduling problem with pre-determined sequencing order can provide a solution to a problem of reasonable dimension case and the problem itself is unrealistic. The approach was first suggested by Akers and Friedman (5), and has been stated more completely by Hardgrave and Nemhauser (6).

The procedure is illustrated by an example, where two jobs are to be processed on four machines. The technological ordering of Job 1 is a, b, c, d and the processing times are $p_{a1}, p_{b1}, p_{c1}, p_{d1}$, for Job 1. The technological ordering of Job 2 is d, b, a, c and the processing times are $p_{d2}, p_{b2}, p_{a2}, p_{c2}$. A graphical representation of the problem is shown in Figure 1.

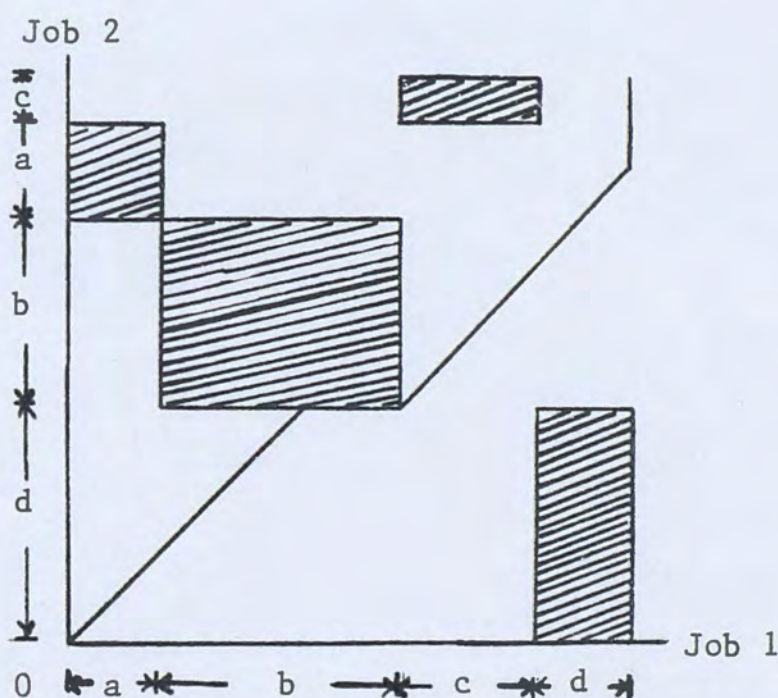


FIGURE 1. GRAPH OF A TWO-JOB, FOUR-MACHINE PROBLEM

A solution to this problem is any line from the point (0,0)

to the point $(\sum_{i=1}^m p_{i1}, \sum_{i=1}^m p_{i2})$, which does not pass through a

shaded region. The line may be composed of horizontal (work on job 1 only), vertical (work on job 2 only), and 45° (simultaneous work on both jobs) segments.

A minimum makespan schedule is a line that minimizes the length vertical (or horizontal) segment, that is, a schedule that maximizes the amount of simultaneous processing. This schedule must be determined by trial and error. Usually, only a few lines must be drawn before the optimal solution is found.

Heuristic Methods

Some of the heuristic methods mentioned in this report are considered sub-optimal if carried to the final solution. The technique can be considered as optimal if it is confined only to certain criteria rather than several or combinations. The heuristic methods are used with the general case of assigning N jobs, to M machines to optimize the elapsed time.

Linear Programming

This problem is considered exact method, since linear programming problems have exact solutions. However, in formulating the problem, it is very similar to the traveling salesman problem which is offered by Little, et al (7). Also, it is the same as the linear programming approach to solve the problem of allocating and assigning limited resources to a number of activities which had to be

performed in a predetermined sequence, i.e., CPM (Critical Path Method).

The main difference between the two models is that one additional set of variables, the sequence of the jobs, are required in the job shop scheduling problem. Rather than allowing only one sequence of operations, we shall include different possible alternatives and leave it to the problem to determine the respective 0 and 1 values for the selected variables².

If we meet a great number of technologically possible sequences, this will increase the number of variables considerably. The linear programming approach is limited to small sequencing problems.

Integer Linear Programming

There are several papers, Bowman (8), Manne (9), Dantzig (10), Wagner (11), in which the n jobs, m machines sequencing problem has been formulated as an integer linear programming problem. In general, the approach is to devise constraints that will satisfy requirements regarding processing times and technological requirements. While mathematically appealing, the application of this approach is severely limited due to hundreds of variables and constraints even for problems of a small size. Some idea of the amount of computational work involved in the n jobs, three machines problem

²Selected variables: an associated variable X_{ij} can have a value of 0 or 1, where i is the initial operation and j the following operation; a 1 indicates that operation j follows operation i ; a zero indicates that i and j occur simultaneously

can be found in Giglio and Wagner (4) and Story and Wagner (12).

Gaglio and Wagner Algorithm

The basic mathematical problem is to select an optimal permutation of n jobs, where the objective function employed is the total amount of processing time elapsing for the completion of all n jobs on three machines. The model can be formulated as follows:

Let $z_{ij} = 1$ if job i is scheduled in order problem j
 0 otherwise

$Z(j) = (z_{1j}, z_{2j}, \dots, z_{nj})$, a column vector

$X_j^k =$ idle-time on machine k before the start of the job
in position j

$Y_j^k =$ idle-time for the job in position j between the end
of the operation on machine k and the beginning on
machine $k+1$

$A, B, D =$ row vector of integer processing times for jobs $1, 2, \dots, n$ on machines $1, 2, 3$, respectively. The model is thus given by

$$\text{Minimize } \sum_{j=1}^n X_j^3 \quad (4-9)$$

Subject to

$$\sum_{i=1}^n z_{ij} = 1 \quad j=1, 2, \dots, n \quad (4-10a)$$

$$\sum_{j=1}^n z_{ij} = 1 \quad j=1, 2, \dots, n \quad (4-10b)$$

$$X_{j+1}^2 + BZ(j+1) + Y_{j+1}^2 - Y_j^2 - DZ(j) - X_{j+1}^3 = 0 \quad j=1, 2, \dots, n-1 \quad (4-10c)$$

$$AZ(j+1)+Y_{j+1}^1 - Y_j^1 - BZ(j) - X_{j+1}^2 = 0 \quad j=1,2,\dots,n-1 \quad (4-10d)$$

They report the number of iterations required for solution using six sample problems, but results do not appear very encouraging. However, they confirm that the exact form in which the problem is stated has an important effect on the efficiency of the algorithm and further work is being directed at the development of more efficient constraints and bounds.

Branch and Bound Techniques

An approach to point the right direction is the branch and bound techniques, which have been used in two ways: first, to make integer programming algorithms more efficient and better suited for solving sequencing problems, and second, to solve sequencing problems directly. The basic idea of branch and bound is that under certain conditions multivariable decision problems can be decomposed into many single-variable subproblems. For the decomposed problem a decision tree can be drawn. In the decision tree each path from beginning to any of the end points represents one complete solution to the original problem.

Ignall and Schrage (13) Algorithm

Ignall and Schrage have proposed a branch-and-bound algorithm for the general three-machine flow shop problem. This procedure requires us to describe the problem as a tree, in which each node represents a partial solution. At each node, a lower bound on make-span is computed for all nodes that emanate from it. It is easy to

see that the flow shop scheduling problem can be expressed as a tree. The first node in the tree structure corresponds to the initial state, with no jobs scheduled. From this node, there are n branches corresponding to the n possible jobs that can be placed first in the sequence. From each of these nodes, there are $n-1$ branches corresponding to the jobs available to be placed second in the sequence. Since there are $n!$ possible sequences, there are $1+n+n(n-1)+\dots+n!$ nodes in the tree.

Each node represents a partial sequence containing from 1 to n jobs. Consider an arbitrary node, say p , with sequence, J_r . That is, J_r is a particular subset of size r ($1 \leq r \leq n$) of the n jobs. Let $\text{TIME } 1(J_r)$, $\text{TIME } 2(J_r)$, and $\text{TIME } 3(J_r)$ be the times at which machine 1, 2, and 3, respectively, complete processing on the jobs in J_r . Then a lower bound on the makespan of all schedules that begin with sequence J_r is

$$LB(P) = LB(J_r) = \max \begin{cases} \text{TIME } 1(J_r) + \sum_{i \in \bar{J}_r} p_{i1} + \min(p_{i2} + p_{i3}) \\ \text{TIME } 2(J_r) + \sum_{i \in \bar{J}_r} p_{i2} + \min(p_{i3}) \\ \text{TIME } 3(J_r) + \sum_{i \in \bar{J}_r} p_{i3} \end{cases} \quad (4-11)$$

where \bar{J}_r is the set of $n-r$ jobs that have been scheduled.

The actual procedure consists of generating the nodes in the tree and computing the lower bounds associated with them. We always branch from the node with the smallest lower bound. To branch from a node, create a new node for every job not yet scheduled by

attaching the unscheduled job to the end of the partial sequence of scheduled jobs. The lower bounds can then be computed from the above equation.

As soon as a node has been found with all n jobs scheduled and a smallest lower bound, the problem is solved and the sequence at that node is optimal. In performing the above steps, dominance can be used to some extent. That is if J_r and I_r are sequences containing the same r jobs, then if $\text{TIME } 2(J_r) \leq \text{TIME } 2(I_r)$ and $\text{TIME } 3(J_r) \leq \text{TIME } 3(I_r)$, the node associated with sequence I_r can be discarded as soon as node J_r is created.

Brooks and White (14) Algorithm

The branch and bound technique has been used by Brooks and White. By using this procedure the criterion of minimizing the total lateness over all orders are examined to determine an optimal sequence of the same numerical example used earlier. For computer applications to large problems, however, it was decided to reduce the number of feasible sequences examined by means of the lower bound decision rule, to yield at least a near-optimal solution. Several sample problems were run using various optimizing criteria, such as minimizing lateness and minimizing total schedule time, and the results are tested. It is observed that the criterion of minimizing lateness produced the shortest machine idle time and total time. Finally, a refinement of this procedure using the minimizing of the weighted combination of machine idle time and lateness as the criterion of optimality, yielded the best first

solution.

Uskup and Smith (15) Algorithm

They determine an optimum solution to a two-stage production sequencing problem with these characteristics: there are n jobs to be sequenced in a two-stage production environment; each production stage is equipped with a single facility: jobs to be sequenced are subject to due-date constraints; facilities in both stages require setup prior to processing each job; setup times in both stages are sequence dependent, and setup cost is assumed to be directly proportional to setup time; and the optimal solution is one that minimizes the total setup cost (of stages I and II) without violating job due-dates. The algorithm employs controlled enumeration through branch-and-bound procedures.

There are basically two important differences between the problem discussed in their paper and the general traveling salesman problem.

1. It concerns sequencing jobs on two facilities in series: the traveling salesman problem is analogous to sequencing jobs on only a single facility.
2. The jobs to be sequenced are subject to due-date constraints in this problem.

The results of this procedure can be applied following industry:

1. Scheduling production in a plastics-manufacturing company.
2. Scheduling plastic fabrication operations.

3. In the textile industry, weaving and dying are two sequential operations with substantial setup costs (times).

Uskup (16) formulated two-stage sequencing problems using a mixed-integer-programming.

The result of their approach is not only a function of the effectiveness of the algorithm but also that of the computer programming approaches that have been employed. The randomly generated data showed that problems involving less than 10 jobs can be solved in a fraction of a second to a few seconds. Problems containing 20 jobs required 45 seconds to three minutes. Two 30-job problems took a little under four minutes and approximately six minutes.

The algorithm is a very powerful procedure currently available and this approach is computationally feasible for a range of problems of practical interest to industry.

Dispatching Methods

Many studies have been performed which focus on the problem of establishing priority decision rules to follow for optimizing the dispatching of orders in a job shop. Many of these studies are based on simulating job shop conditions with set rules for establishing due dates and with a predetermined set of job durations. Early work in the simulation of job shop systems was done by Rowe (17). Since Rowe's study, much research has been done on priority dispatching rules. Numerous others who have searched with the problem include Carroll (18), Conway (19) (20), Legrande (21), and Nanot (22). Texts such as Conway, et al (23), Zimmermann and Sovereign

(24), Greene (25), and Johnson and Montgomery (26) provide discussions in detail.

A number of heuristic methods are applied in the general n -job, m -machine job shop scheduling problem recently and dispatching rules are most widely used. These are simple logical decision rules that enable a decision maker to select the next job for processing at a machine when that machine becomes available. Thus, the scheduling decisions are made sequentially over time instead of all at once. Dispatching procedures always include the concept of job priority.

The results from priority queuing would seem useful in resolving the dynamic job shop scheduling problem, but only limited success has been achieved through this approach. Actual experimentation with various scheduling procedures, such as priority dispatching rules, would be an alternative to the queuing methods in a real job shop scheduling. This is usually not practical, and Monte Carlo simulation is used to overcome this problem.

The general procedure of Monte Carlo simulation consists of developing a computer program that simulates the arrival of jobs and controls the flow of these jobs through the various processing facilities. In addition, the program may assign due dates or other attributes to the jobs and it usually contains lists and files to record the state of the job shop and compute various measures of effectiveness. Through the use of such a computer program, various scheduling procedures, and their impact on shop performance, could be investigated.

The results of these studies seem to indicate that in many situations the SPT priority rule is of dominating importance and it is surprising that it is as a superior rule for many complex dynamic problems as well. Regardless of the measure of performance, the shortest processing time rule is among the best of the many procedures that have been investigated. For a given measure of performance, it is usually possible to produce a rule that performs better than SPT. However, the measures of performance rules are much more complex and require elaborate information systems for implementation. SPT has a smaller variance of flow time and mean flow time than either FCFS (first-come, first-served) and RANDOM (next job chosen at random from among those available) priority rules.

There are many other priority dispatching rules that could be employed in job shop scheduling and discussed in Moore and Wilson (27).

To simulate a dispatching rule, one should recognize that there are three things which have to be considered:

1. The job shop conditions and environment.
2. The dispatching rule to be tested.
3. The criterion by which the rule is evaluated.

Typical shop conditions and environment established for a simulation include ignoring transportation times between jobs, never interrupting jobs, fixing the plant capacity, never permitting machines to break down, etc. The criteria in the studies included mean flow time, flow time variance, meeting due dates, and minimizing

inventory. One criterion may not be the one which is important at all times during any operations.

In many practical cases the testing of alternative dispatching rules by simulating the relevant condition will point to priority rules which yield the best results with respect to certain desirable criteria.

Graphical Techniques

The procedural approach and operating characteristics of the major graphical techniques developed to meet the needs of increasingly complex operating environments are described in this section.

Gantt Charts

The first formal scheduling model used by management was the Gantt chart. This technique provided a powerful tool to management for planning and controlling industrial operations. The Gantt chart has been most successfully applied to highly repetitive production operations.

Generally, a time scale is placed horizontally along the top of a Gantt chart. The rows represent machines, personnel, departments, or whatever resources may be required to accomplish a job. The time scale may be subdivided into calendar time or selected temporal units. An example of Gantt symbols and charts are shown in Figure 2 and Figure 3.

Charts may be prepared for various managerial levels and responsibilities, so that performance may be monitored and responsibility traced throughout the organization.

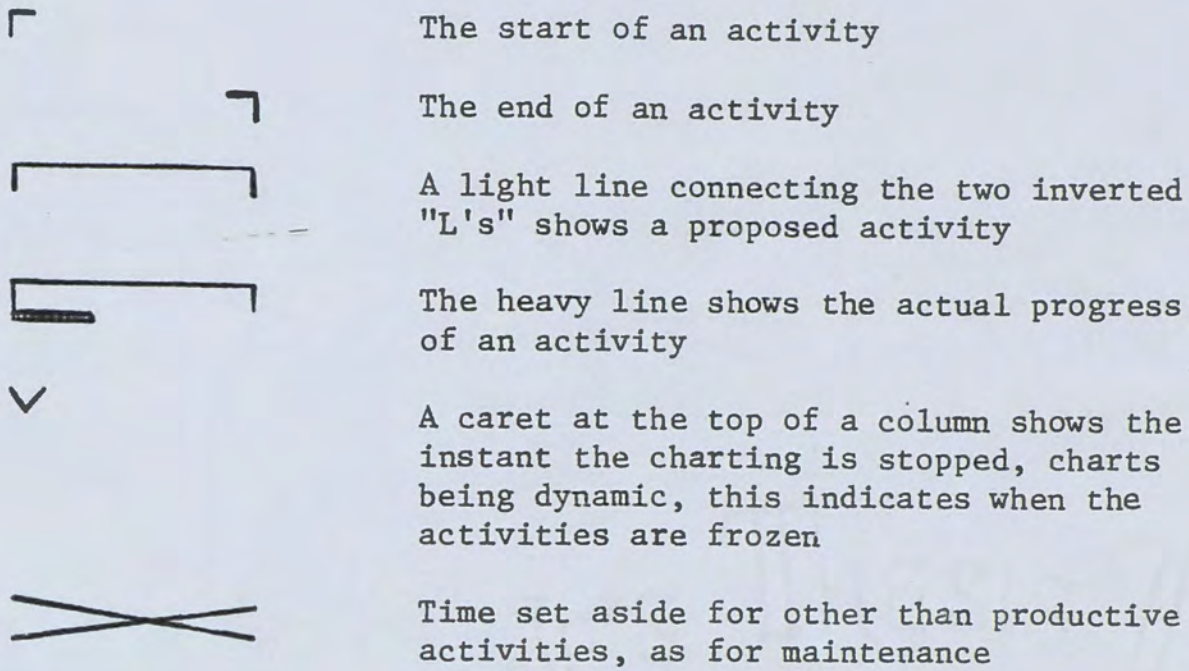


FIGURE 2. GANTT-CHART SYMBOLS

SOURCE: J. H. Greene, Production Control: Systems and Decisions. (Homewood, Illinois: Richard I. Irwin, 1965):426.

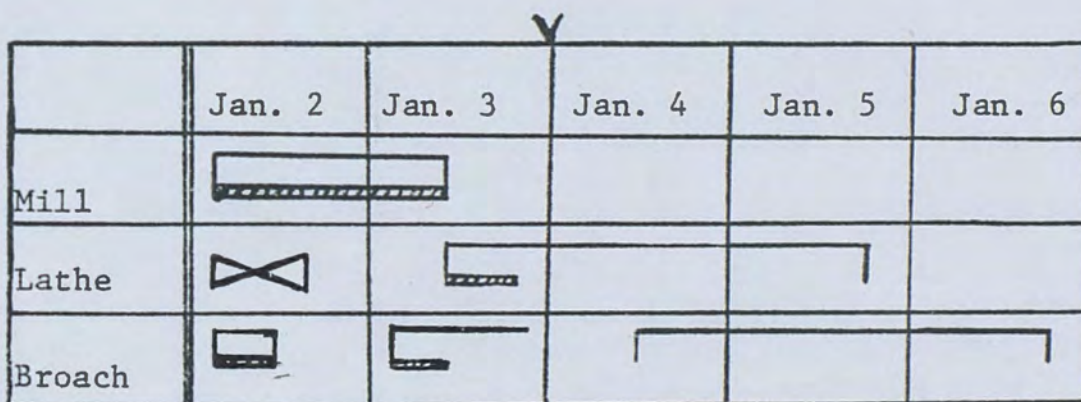


FIGURE 3. MACHINE SCHEDULING CHART

SOURCE: J. H. Greene, Production Control: Systems and Decisions. (Homewood, Illinois: Richard I. Irwin, 1965):427.

Zimmermann and Sovereign (24) summarize the advantages and disadvantages as follows:

The advantage of a Gantt chart are obvious:

1. It is used as a planning tool and as a control device.
2. The progress of work is recorded on a time scale. Therefore the Gantt chart is instructive even for the nonspecialist.

The disadvantages are not visible:

1. The sequence of the planned operations and the time planning have to be performed simultaneously with respect to the same machine. If either the sequence of the operations or the time of the operations change, the entire chart may have to be redrawn.

2. There is no way of optimizing the sequence of operations in a Gantt chart since it does not give any information about other possible sequence.

3. It does not give any information about the impacts of delays of a single operation or the completion time of the entire project.

4. If the number of operations to be planned is high, it is impossible to maintain an overview in a Gantt chart.

5. A Gantt chart is not directly convertible to computerization.

Milestone Charts

An outgrowth of the simple bar technique is the Milestone chart. A Milestone may be described as an important event along the path to project completion. All Milestones are not equally signifi-

cant. The most important are termed "Major Milestones", usually representing the completion of an important group of activities. An example of a Milestone chart is shown in Figure 4.

The method of collecting and organizing data for Milestone charts is similar to the Gantt technique. The primary difference is the graphic display. The Milestone system offers no basic improvement over the Gantt chart except to provide focus on the event to be achieved.

Month	1	2	3	4	5	6
Project No.						
1	●		●			
2		●	●		●	●
3	●		●	●		
4	●	●	●	●		

Milestone ● Schedule * Achievements

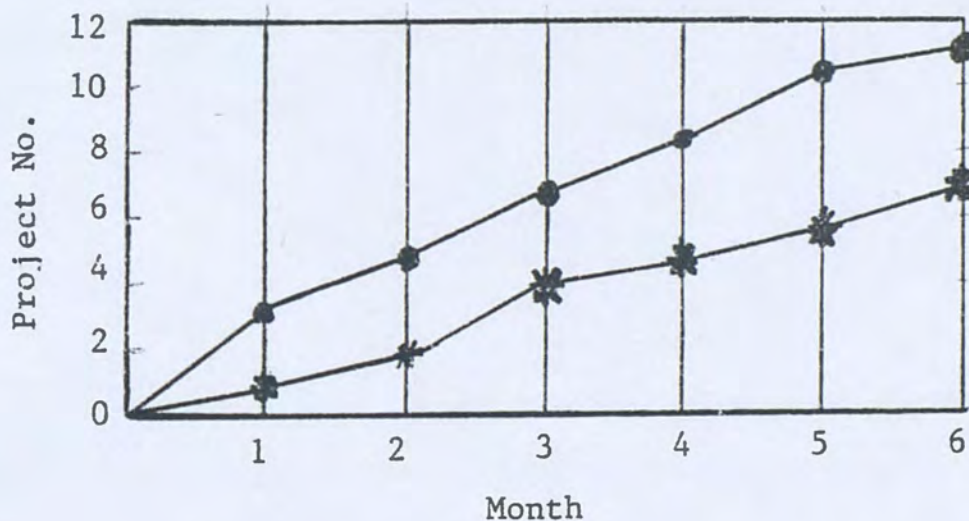


FIGURE 4. MILESTONE SUMMARY

Line of Balance

Line of Balance (LOB) is a production planning system which schedules key events necessary for completing an assembly with respect to the delivery dates for the completed system. The LOB technique is based on the principle of management by exception, wherein management attention is directed to existing or potential problems. The LOB is a useful complement to Gantt charts.

Malcolm and Hill (28) state the following four primary elements in an LOB application:

1. Determination of the objective.
2. Development of a program plan.
3. Measurement of progress.
4. Construction of the line of balance.

The initial step in the application of the LOB method is to graph the cumulative delivery schedule of the end item.

The second step in the implementation of an LOB system is to chart the program.

The third step in the LOB technique is to prepare a progress chart which shows the status of a program at a given point time and a line of balance which represents the number of items that should pass through each control point at a given date if the delivery schedule is to be met. Detailed procedures are discussed in Whitehouse (29) and the LOB technique is widely applied in military, production and manufacturing business. But, in general, the LOB technique affords no simulation capability when management desires

to consider the effects of alternative approaches toward overcoming a problem area.

CHAPTER V

SUMMARY AND CONCLUSIONS

This paper deals with the sequencing problem to schedule n jobs on m machines. The job shop scheduling problem is to determine the order or sequence for processing a set of jobs through machines in an optimum manner. The objective is to minimize the total elapsed time to complete all jobs. We have discussed the analytical methods and graphical methods of the sequencing job shop problem with different approaches to meet the needs of effective algorithms in job shop scheduling.

We come to the following conclusions about the relative merits of the approaches discussed in this paper:

1. The integer programming approach has the obvious advantage that when it succeeds we indeed obtain an optimal solution to the problem. The weak point is that it does not seem to converge fast enough to make it practical. Furthermore, as the problem size increases, the convergence difficulty is likely to become more severe. A measure of dimensional difficulty of an integer programming problem is the sum of the number of variables and constraints in the model as in the linear programming model.

2. However, refraining from the integer values for the

variables might result in formulating the sequencing problem as a linear programming problem which is computationally feasible. This method is not very encouraging.

3. The Johnson approximation appears to be a favorable simple approach, and when it is possibly combined within the condition of processing time dominance it produces excellent results.

4. Branch-and-bound procedure depends heavily on the quality of the lower bounds, particularly, those used in the early stages of branching. It has been used more efficiently and is better suited for solving sequencing problems than integer programming. Brooks and White algorithm is computationally prohibitive for problems of practical dimension. Furthermore, their results suggest some interesting dispatching procedures. Uskup and Smith algorithm proposes a feasible procedure to be applicable to some of the industry. The branch-and-bound procedure necessitates the storage of a large number of partial schedules and the limitation of core storage is more difficult for large problems.

5. Scheduling based upon heuristic methods which give reasonably good suboptimal solutions is getting more and more popular with management in real life problems, but the search of optimal solutions in sequencing problems still remains in a far from satisfactory position.

6. As common job shop scheduling is the dynamic version of problem, analytical techniques explained in this paper have not been practical. Dispatching methods, however, is a practical approach.

Priority dispatching rules in dynamic job shops are developed and tested by many researchers and the results of their studies seem to indicate that the shortest processing time (SPT) priority rule is of dominating importance in many job shop scheduling problems.

7. Graphical techniques are practically applied production systems for scheduling and control, optimizing the sequence of operations by these methods is not supported by any mathematical proof.

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