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Factors in Modeling Floridan Aquifer Recharge

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FACTORS IN MODELING
FLORIDAN AQUIFER RECHARGE

BY

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RESEARCH REPORT

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I INTRODUCTION

During the preparation of earlier papers, it has become apparent to the writer that recharge of the Floridan Aquifer is a little understood phenomenon. Of particular interest are the conflicting statements concerning location and volumetric significance of recharge areas.

An example of the conflicting information available is a statement in an abstract of a paper by F. N. Visher and W. S. Wetterhall (1967) that "Most of the piezometric highs indicate low permeability and low or rejected recharge." This concept is in conflict with the generally accepted concept that a piezometric high is indicative of an area of significant recharge. Another indication of the lack of understanding of recharge of the Floridan Aquifer was the study initiated in 1974 by the Florida Geological Survey to determine the significance of the Green Swamp area of Polk County, Florida. Previously, Pride, et al (1966) had indicated that this area was one of the most productive recharge areas in the state.

One possible approach to clarification of the recharge problem might be use of a simulation model of the Floridan Aquifer. The model, through use of such data as permeabilities, storage coefficients, piezometric contours, water withdrawal, and natural discharge, might be used to estimate location and rate of recharge to the aquifer. Alternately, through use of information on rainfall and runoff, soil permeability, aquitard permeability, and evapotranspiration losses, a ground water budget might be formulated and used to estimate aquifer performance.

William James (1972) has formulated a basic method of systematic development of a simulation model which may be applied to either of the above approaches for the Floridan Aquifer. Figure 1 presents a simplified version of his flow chart. In this version, intermediate steps in formulation on the first and second order models have been lumped into one step.

It is the purpose of this paper to pursue that part of the model which has been labeled problem statement. It is hoped that this paper may serve as the basis for possible development of a simulation model of the recharge process of the Floridan Aquifer. It is, however, not the intent of this paper to make the decision as to whether to simulate as indicated in figure 1.

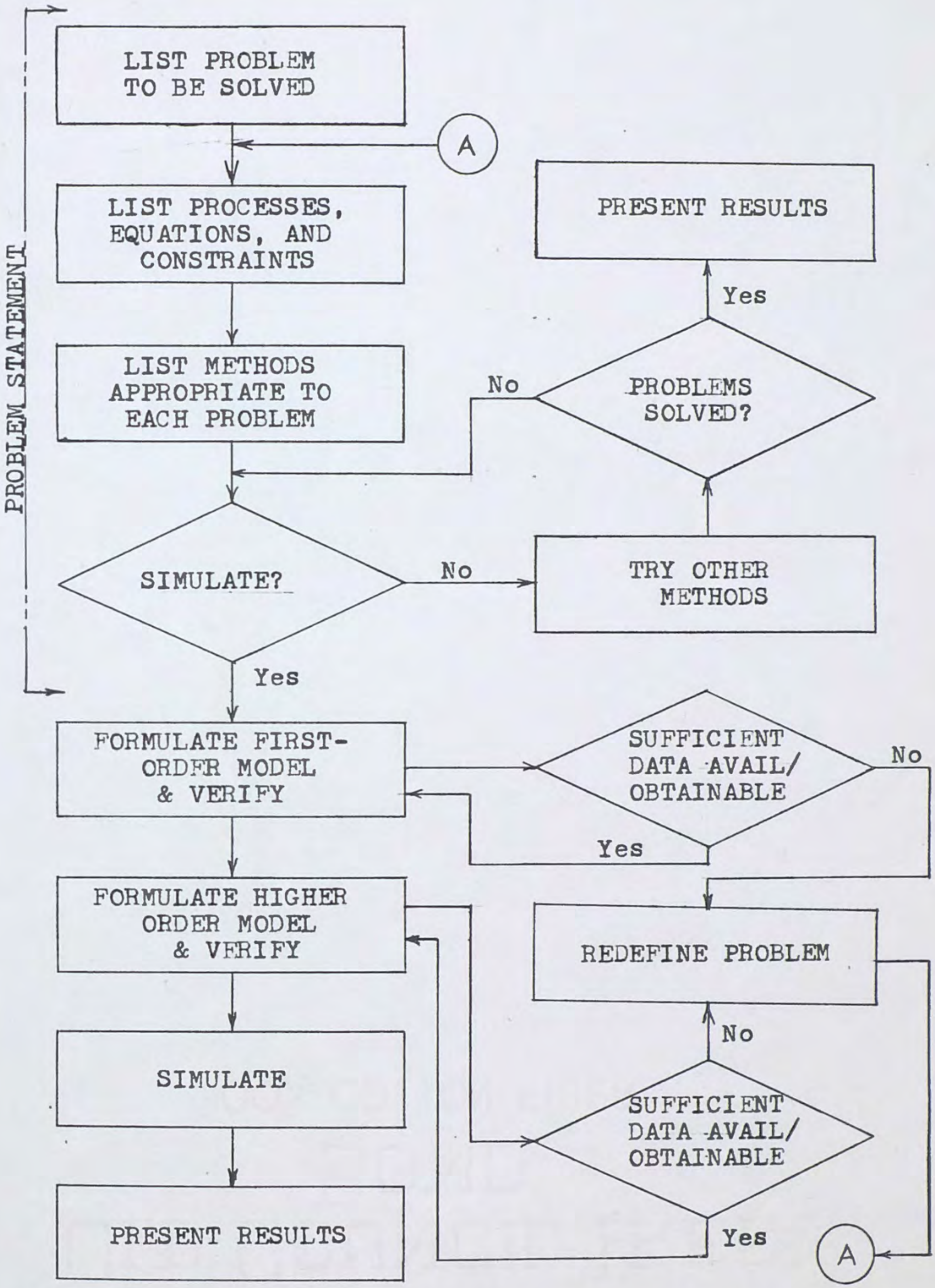


Figure 1. Development of a Simulation Model

II PROBLEMS TO BE ADDRESSED

The basic problem to be solved was stated in the introduction: where, and in what quantities does recharge of the Floridan Aquifer occur? Can modeling of the aquifer clarify the preceding question? Consideration of these questions immediately leads to another set of only sightly less complex questions.

The most immediate secondary question is which of the following is the simplest approach? (1) Simulate the aquifer and vary recharge until a good approximation of the known piezometric surface and estimated drafts on the aquifer are obtained. Or, alternately, (2) model the recharge process on a state wide scale and use the results to prepare a map of recharge rates. If the aquifer is to be simulated, which would be preferred: analog or digital computation? If digital computation is used for solution of the differential equations of the simulation model, is finite difference or finite element analysis more appropriate? Since it is preferred that development of the model not include gathering of new aquifer data, what method would be least sensitive to errors and extrapolation of local data for such parameters as transmissivity, storage coefficients, and evapotranspiration rates to large regions? How can boundary conditions associated with the Floridan Aquifer be simulated?

III CHARACTERISTICS OF THE FLORIDAN AQUIFER

The Floridan Aquifer is a large confined body of groundwater underlying southern South Carolina, southern Georgia, and all of Florida with the exception of the western panhandle. Geologically, the aquifer is composed of a series of saturated limestone and dolomite formations. It is the source of 38% of all water used by the principal communities in Florida (Healy 1972).[1]

As figure 2 illustrates, the Floridan Aquifer is composed of a series of limestone formations which range from a few feet below the surface to as much as 2000 ft. (Pride, et al 1966) (Klein 1971). The base of the aquifer is usually taken to be the Lake City formation, or in some areas, the base of the Avon Park formation. The lower limit of the aquifer is indicated by the occurrence of gypsum which is indicative of a lack of groundwater movement. Overlying the aquifer in most areas is the Hawthorne formation which forms its aquitard. The Hawthorne formation consists of various clays mixed with deposits of sand and interbedded limestone (Pride, et al 1966).

The intervening limestone formations are riddled by solution passages and cavities. The most productive of these formations, the Avon Park, is also highly faulted. This faulting offers a path for vertical movement of water.

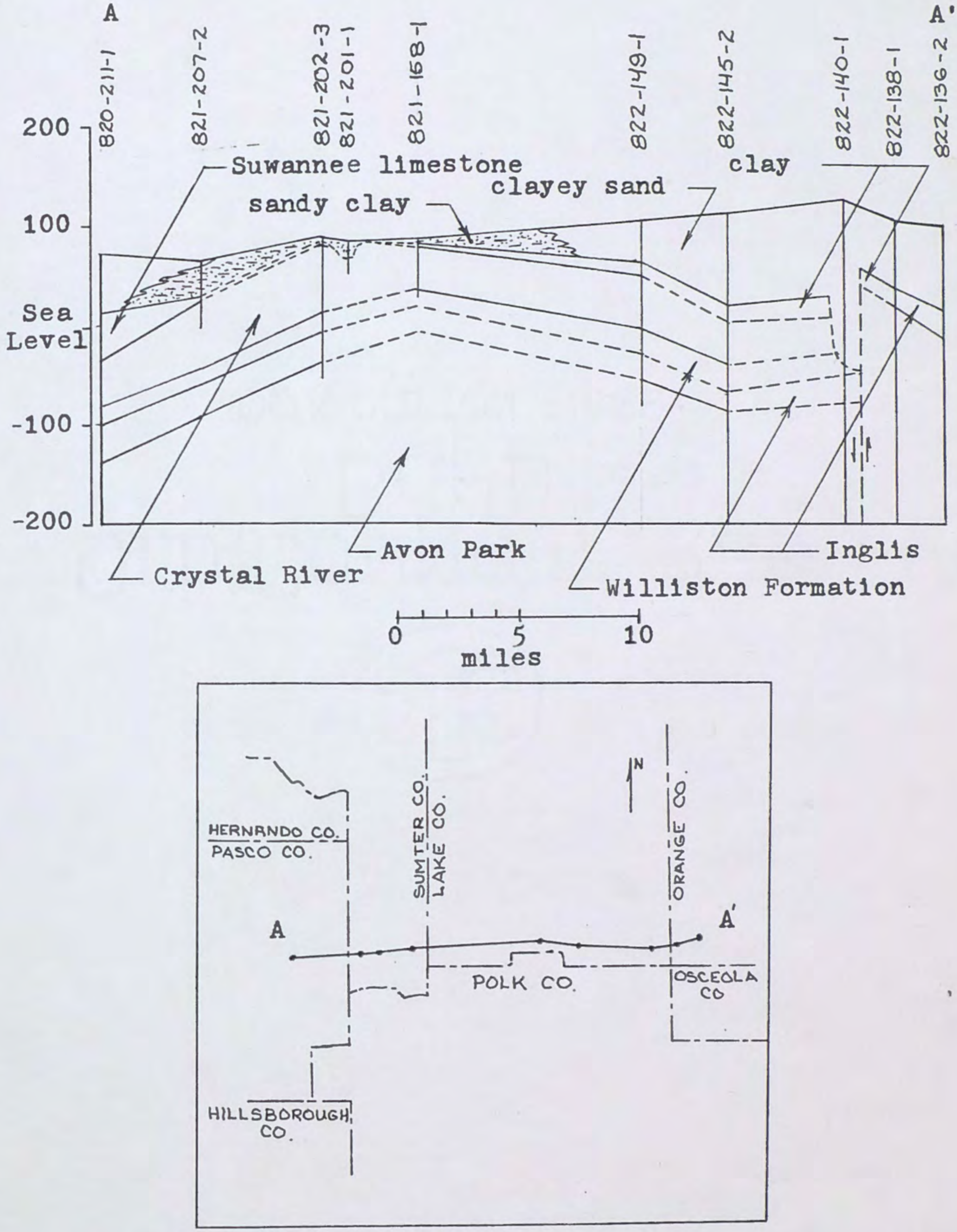


Figure 2. Typical Geological Cross-Section of the Florida Aquifer (after Pride 1966).

Secondly, solution passages along the faults, along with other solution passages and cavities, greatly increase the effective transmissivity of the aquifer. Alternately, some of the solution passages have filled with clastic materials and act as barriers to flow. Also, in some areas, faulting has also placed formations with differing lithology adjacent to each other disrupting hydraulic continuity and causing abruptly changing permeability (Pride, et al 1971).

Water is lost or withdrawn from the aquifer in a number of ways. A major component is 6000 cfs of water that flows from 65 major springs in Florida (Cooper, et al 1953). In the central gulf coast an estimated 230 cfs is lost in leakage to streams (Cherry, et al 1970). Additional leakage occurs to streams in north central Florida. Municipal draft from 65 major cities is approximately 350 cfs (Healy 1972). Drafts on the aquifer from industrial and agricultural users must also be significant. It is assumed that the greatest proportion of the aquifer's flow is lost in submarine discharges.

Recharge appears to occur in four major areas of central and west Florida. These areas are located in Polk, Pasco and Hernando, and Volusia Counties, and in the Keystone Heights area of Clay, Bradford, Alachua, and Putnam Counties. Three major mechanisms are responsible for recharge to the Floridan Aquifer: inflow through the aquitard, direct inflow where the aquifer is exposed, and inflow through sinks.

The Polk County recharge area has been called the most important in Florida. Pride, et al (1966) indicates that the principal recharge mechanism is leakage from the nonartesian aquifer. It has been demonstrated that a good connection exists between the two aquifers (Pride, et al 1961). Also of some significance in the western section of this area is direct infiltration where the water bearing formations of the aquifer outcrops along the Ocala uplift (Pride, et al 1966). Infiltration through sink holes is also thought to occur along the eastern edge of the region, but Stewart (1966) indicates it is not a significant factor.

A piezometric high indicating another recharge area is located in Pasco and Hernando Counties. Wetterhall (1964) indicates that recharge takes place by infiltration from sink holes and leakage through the aquitard. The sink hole infiltration seems to be particularly significant. It is reported that Bear sink, for instance, accepts 41 cfs flow, and that dye tests indicate that there is no connection between the sink and the various nearby streams (Wetterhall 1965).

Recharge also occurs in western Volusia County. This is, however, one of those regions of controversy mentioned in the introduction. The principal mechanism in this region is infiltration through sink holes (Wyrick 1960). Inter-aquifer leakage is also believed to occur in the central area of Volusia County along the eastern edge of the Talbot marine terrance (Wyrick 1960, and Knochennus, et al 1971).

Centered on the intersections of Alachua, Bradford, Clay, and Putnam Counties is another recharge area. It is indicated that the principal mechanism at work here is infiltration through the large number of sinks in the area (Bermes, et al 1963 and Clark, et al 1964).

Though poorly documented, other recharge areas exist in Florida such as one in Orange and Lake Counties hypothesized by Lichtler (1972) and one in southwestern Alachua County. As will be discussed in the following section of this paper, a model using either a water budget or by simulation of the aquifer internal processes would be useful for determining the significance of these areas. If the model is sufficiently detailed, it would also be possible to discover previously unidentified recharge areas.

IV MODELING

Either of two types of models may be used to simulate an aquifer: deterministic and stochastic. Deterministic models are those whose response is equivalent to the physical system of interest. The deterministic model may either be a black box or may use equations which are descriptive of the actual internal physics of the system (Dawdy 1969).

In stochastic models, statistical parameters are determined which describe the response of the system. The statistical parameters are used to generate a record which would be statistically indistinguishable from an actual record (Dawdy 1969).

Deterministic models are of use where transient responses are of interest. Conversely, stochastic models average transients and, therefore, are useful for predications for planning purposes (Dawdy 1969).

If we are interested in the details of recharge locations and quantities, not prediction of future aquifer characteristics, a deterministic model would be appropriate. This deterministic model would be based on a generalized equation for flow and upon a set of boundary equations which would describe the physical characteristics of the aquifer. These boundary conditions could be based upon widespread aquifer data available in various publications of the State of Florida. Initial conditions for the aquifer simulation could be based upon estimates of recharge rates contained in these publications. Aquifer flows could then be varied until the observed piezometric surface matched that generated by the model. In addition to varying recharge flows, it is reasonable to expect to perturb some of the outflows, such as leakage to streams, to locally obtain good agreement between the model and observed piezometric surfaces. Where the data are available, the response of the aquifer to rainfall might also be simulated in an attempt to improve the model's calibration.

In Section II, the alternate possibility of modeling recharge processes directly was suggested. R. A. Freeze (1969) has discussed a model for unsteady, unsaturated flow recharging and discharging a phreatic groundwater system. Using Freeze's model, variation of the phreatic surface could be calculated for a large number of locations distributed over pennisular Florida; and in turn the rate of leakage through the aquitard to the Floridan Aquifer in response to the variation of head of the water table aquifer could also be calculated.

A significant problem is associated with this method. First, a great deal of data would be required for such parameters as soil permeability, soil moisture, rainfall, and evaporation rates. Lack of accuracy of this data would gravely affect the accuracy of the model. Since there is no observed recharge data, it would be, therefore, impossible to calibrate the model. For this reason, direct modeling of recharge processes does not seem a workable alternate.

V MATHEMATICAL BACKGROUND FOR GROUNDWATER MODELS

To prepare a deterministic model, it is necessary to formulate mathematical equations which describe the processes that occur in the system being modeled. In the case of recharge of the Floridan Aquifer, we are interested in the flow of water which is described by general partial differential equations and various boundary conditions. The general equation will be developed below.

The basic equation which is derived from Darcy's equation and continuity is as follows:

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = s_s \frac{\partial h}{\partial t} \qquad (1)$$

where h = head

k_x, k_y, & k_z = coefficient of permeability in x, y, z directors

s_s = specific storage = storage coefficient/ aquifer thickness

For the case of an homogeneous isotropic aquifer equation (1) simplifies in two-dimensions

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{s}{T} \frac{\partial h}{\partial t} \qquad (2)$$

where s = storage coefficient

T = transmissivity coefficient

For the steady state case, the right side of equation (2) equals zero and the equation is of the form of Laplace's equation (Walton 1970).

The solution of these equations either by numerical or analog techniques, require the major effort in construction of a simulation model. In particular, a flow distribution must be found which satisfies the above generalized equation and the boundary conditions of the Floridan Aquifer. By using the flow distribution obtained and known or estimated loss of water from the aquifer, a distribution of recharge flows might be obtained. The solution of the generalized equation and boundary condition equation may be obtained using either an electrical analog or numerical techniques and a digital computer.

VI ANALOG MODELS

The equation for flow of electrical current in a homogeneous media is identical in form to equation (2). As a result, electrical analogs such as conductive sheets or electrolytic tanks have been used to simulate steady state two or three dimensional flow of groundwater in a homogeneous media (Cole 1970). These analogs are of little interest in solution of the problems associated with a large nonhomogeneous aquifer.

Fortunately, for the steady state case, a finite difference approximation may be formulated for Laplace's equation. This approximation may be modeled by a discrete mesh of conductors. It is, therefore, possible to model a homogeneous steady state aquifer with an array of resistive and capacitive elements (Cole 1970).

To continue the analogy for the non-steady state case, the finite difference approximation becomes a series of quasi-steady state approximations varing progressively with a series of time increments. In the analog model, a series of current pulses analogous to the non-steady state variation in flow is supplied to the model.

As will be discussed later, it is also possible to find approximate solutions to Laplace's equation using numerical methods. Analog simulations have several advantages over the alternate numerical solution. Primary of these is that non-homogeneous aquifers with time varying flows can be easily modeled using an analog as compared to using numerical techniques (Sternberg 1971). In general, analog modeling can handle problems of much greater complexity than practical using a digital computer. Other advantages claimed for analog models include providing a visual representation of the aquifer (Lawson et al 1970), and allowing rapid testing of developmental schemes and appraisal of alternate schemes (Walton 1969).

J. A. Cole (1970) has listed inputs and output of an aquifer analog. Data required to produce an analog includes the aquifers properties as a conductor: permeability, storage coefficient, and thickness. Definition of natural boundaries is required. Draft on the aquifer by wells and springs must be known along with estimates of leakage to rivers or other aquifers. Although base flows, natural recharge, and artificial recharge are usually considered inputs and piezometric surfaces outputs, this writer believes that their roles may be reversed.

The output of analog models has been dependent upon the purpose of the model but traditionally has been related to measurement of the potential field of the model (Cole 1970). Electrical current measurements could, however, be made along the boundaries of the model and at its upper surface to determine recharge rates. Input voltages would require regulation to values representative of the observed piezometric surface. Finally after the correct base flows and recharge rates were determined, they might be fixed and the model used to predict the effect of development of this aquifer in the usual manner of measuring the potential field.

As briefly discussed earlier, the analog model's mesh of conductive elements represents a discretized finite difference approximation of the aquifer. A simple case may illustrate the relationship between aquifer and electrical parameters (Walton 1969). For a non-steady, two-dimensional homogeneous, isotropic aquifer with nodes defined as in figure 3a:

$$T(\sum_i h_i - 4h_1) = b^2 S \frac{\partial h}{\partial t} \quad (3)$$

where h_i = head at node i

T = transmissibility

b = grid dimension

S = storage coefficient

Similarly for the conductive element shown in figure 3b:

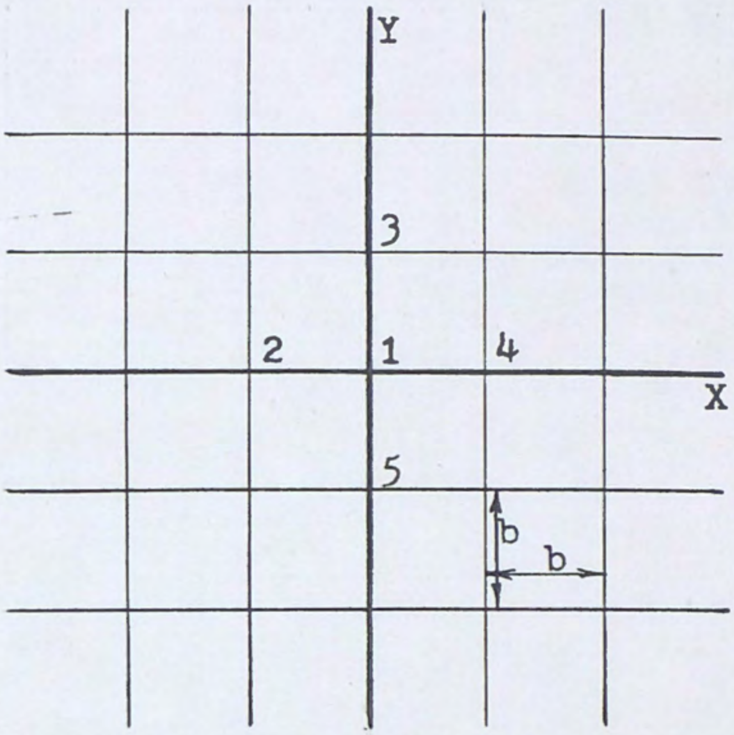
$$1/R(\sum_i V_i - 4V_1) = C \frac{\partial V}{\partial t} \quad (4)$$

where V_i = voltage at node i

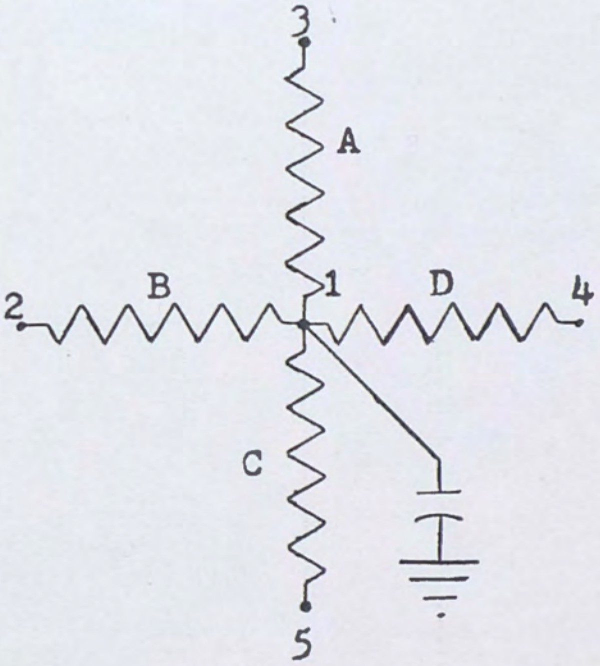
R = resistance for the element

C = capacitance for the element

Note the one to one relationship between the terms of equations (3) and (4). The electrical analogs of the



FINITE DIFFERNECE GRID
3a



ELECTRICAL ANALOG
3b

Figure 3. Finite difference - resistor - capacitor net analogy

aquifers parameters are then as follows (Walton 1969):

Aquifer		Analog	
head	h	electrical potential	V
transmissibility	T	1/ resistance	$1/R$
storage coefficient	S	capacitance	C
volume of fluid	q	charge	θ
flow rate	Q	current	I
time	t	time	t_s

These electrical and hydraulic parameters are related by four scaling factors (Walton 1969). These scaling factors are defined as follows:

$$\begin{aligned} q &= K_1\theta \\ h &= K_2V \\ Q &= K_3I \\ t &= K_4t_s \end{aligned} \tag{5}$$

In addition, K_1, K_3, and K_4 are related by the following equation:

$$\frac{K_3 \times K_4}{K_1} = 1 \tag{6}$$

The relationship between the properties of an element of the aquifer and the values of resistive and capacitive components of the equivalent electrical analog element are a function of these scale factors. For a three-dimensional, anisotropic aquifer, they are as follows (Cole 1970):

$$R_x = R_y = (K_3/K_2)\ T \tag{7}$$

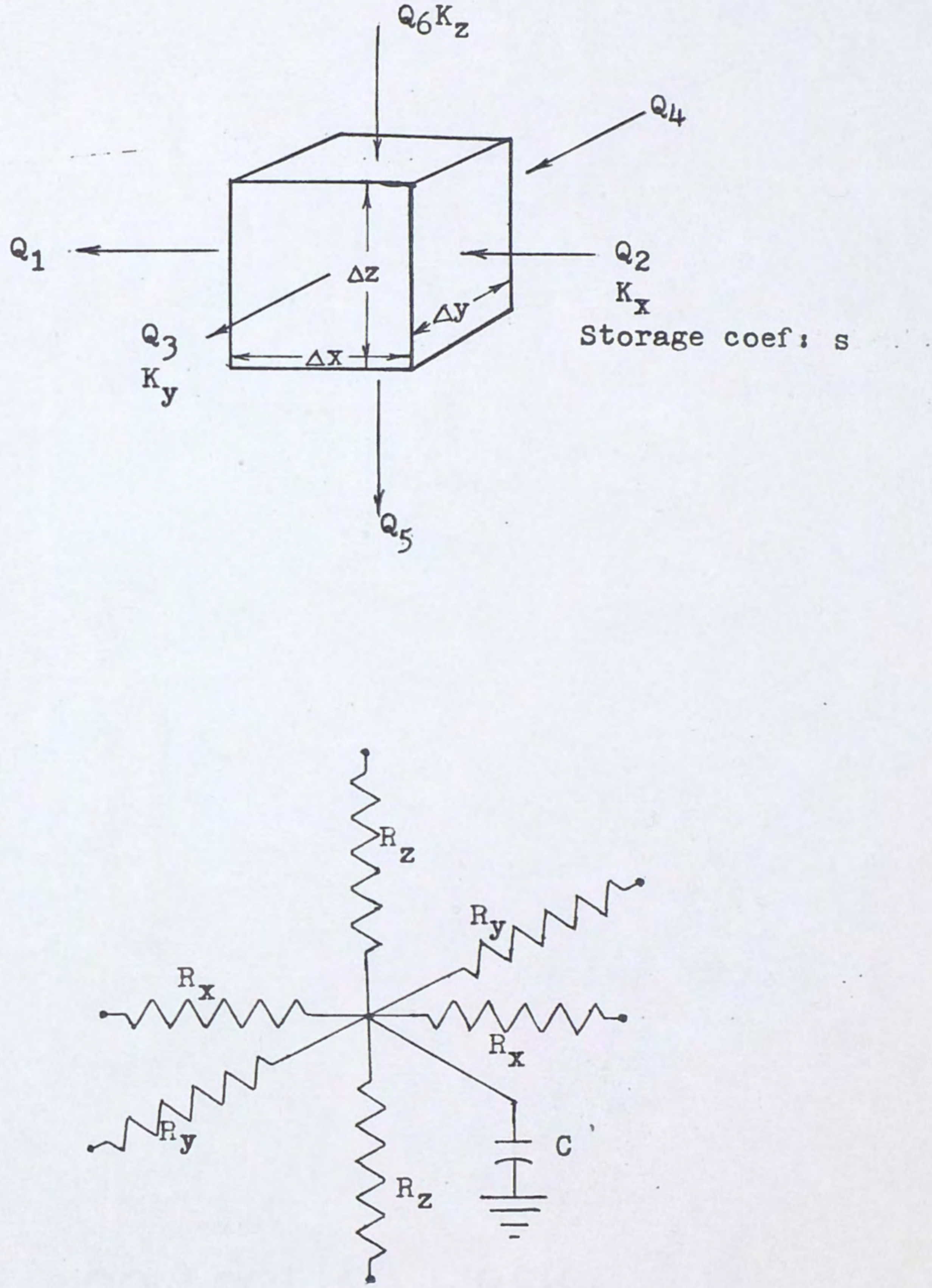


Figure 4. Identification of Analog Elements (after Cole 1970).

where

$$T = k_x \Delta y \Delta z = k_y \Delta x \Delta z$$

and

$$R_z = (K_3/K_2)D/k_z \tag{8}$$

where

D = thickness of the aquifer

For the capacitive component

$$C = 7.48\,\Delta x^2 S(K_2/K_1) \tag{9}$$

The actual value of the scale factors are selected by trial and error (Walton 1969). Their values are perturbated until convenient values of R_x, R_y, and R_z are obtained, and in the case of K_4, to use available model excitation and response measuring equipment.

Leakage through an aquitard may be simulated by the addition of a vertical resistor to the nodes of the analog (Walton 1969). The value of this resistor, R_g, is calculated as:

$$R_g = K_3/(K_2(P'/m)a^2) \tag{10}$$

where

P'/m = aquitard leakage coefficient

a = linear separation of the node points in the aquifer

Walton indicates these resistors should be connected to ground because the hydraulic head of the overlying aquifer remains constant. If the resistors representing the aquitard were connected to ground, electrical current would flow from the model to ground. Keeping in mind the analogy between electrical current and water flow, this flow of current would be analogous to leakage of water out of the aquifer through an aquitard not in as is the case for the Floridan Aquifer. Connecting these resistors to a regulated voltage supply equivalent to the head of the overlying aquifer would seem a better solution.

Walton (1969) also indicates that leakage to a stream may also be simulated with a resistor connected to ground. The value of these resistors may be computed as follows:

$$R_s = K_3/(K_2 I_h A_s) \qquad (11)$$

where I_h = average infiltration rate per unit area of river bottom

A_s = area of streambed

Resistors at aquifer boundaries may be calculated as follows (Walton 1969):

$$R_x = R \frac{\Delta x}{\Delta y} \qquad \text{and } R_y = R \frac{\Delta y}{\Delta x} \qquad (12)$$

where R_x, R_y = values of resistor at the boundary

x, y = proportion of grid spacing represented by the boundary resistors.

Capacitive elements located along the boundary are represented by:

$$C = 7.48\ A_c\ S\ K_2/K_3 \qquad (13)$$

where A_c = the aquifer area represented by the capacitor

Grid spacing may be varied within the model to minimize the number of nodes in the model. The value of resistors at the boundary between the areas of differing grid size may be handled similar to resistors at an aquifer boundary (Walton 1969).

VII DIGITAL MODELING

When flow conditions and the resulting highly complex boundary conditions for the Floridan Aquifer are considered, a closed mathematical solution for the generalized differential equation is not expected. In recent years, the advent of large digital computers has led to the development of methods for the solution of complex partial differential equations.

The use of digital models has some advantages over analog models. D. H. Pilgrim (1970) lists a number of these advantages as follows. The programming of a digital model requires less time than construction of an analog model. Special equipment is needed for an analog model and special skills are required to operate the equipment. Conversely, digital programming is a common skill and large digital computers are readily available to research, engineering, and educational organizations. Most importantly, a digital model is much more flexible, more easily modified. Numerical methods are more versatile than analog modeling when nonlinear boundary conditions exist as would be expected for a large aquifer model. Data readout is more convenient with a digital computer, and more importantly, readout cannot perturb the model (France 1974).

In general, a digital model is comprised of the generalized flow equation and a set of boundary condition equations which describe conditions prevailing in the aquifer. Simplifications may be made to these equations as a result of assumptions made concerning the physical conditions in the aquifer. A numerical solution for these equations may be obtained by use of either finite difference, backward difference or finite element techniques. Finite difference and finite element techniques will be examined in detail in following sections.

The aquifer is physically represented by a finite array of points as with analog models. The properties of the aquifer in the vicinity of each of these points is considered concentrated at each of the points. The process of localizing the aquifer properties at points is called discretization. This process differs slightly with the numerical technique being used to analyze the aquifer. The array of points, or nodes, may be two or three dimensional. The values of transmissibility and storage coefficient may vary in direction for an anisotropic aquifer, and from node to node for a heterogeneous aquifer. Boundaries of the aquifer define the extent of the array (Pilgrim 1970).

In addition, the continuum of time is also discretized as finite time intervals (Pilgrim 1970). A solution is obtained for the differential equation for each time increment (France 1974). Each solution serves as initial conditions for each succeeding time increment. The computation process generally is continued for a specified time period or until steady state conditions are reached.

The computational procedure for a digital model may be divided into two phases: identification of model parameters and simulation (Pilgrim 1970). In the identification phase records of head, inflows, and discharges are used to adjust parametric aquifer data to obtain the fit of calculated to observed data. Generally in the simulation phase, the conditions of the aquifer are calculated for a specified pattern of inputs and withdrawals for the aquifer. For the proposed model of the Floridan Aquifer, the simulation phase would be an extension of the identification phase. An attempt could be made to determine what natural recharge conditions would give aquifer conditions in agreement with those observed.

VIII FINITE DIFFERENCE METHODS

Finite difference methods have been primarily for analysis of homogeneous porous media. The technique is difficult to apply to hetrogeneous and anisotropic regions. Several complex models have been reported in the literature, however, including two in which three-dimensional, unsteady flow in a non-homogeneous, anisotropic aquifer with vertical leakage was simulated.[2] Finite difference analysis often requires a great deal of computer time and storage capacity (France 1974).

Finite difference methods entail writing N algebraic difference equations for each of N node points (Pilgrim 1970). The difference equations are obtained by formulating discrete analogs for the first and second derivations as required and then substituting these analogs into the partial differential equation. As a simple example, for a linear one-dimensional differential equation, the analog of the first and second derivatives of some function u are:

$$\left(\frac{du}{dx}\right)_i = \frac{u_{(i+1)} - u_{(i-1)}}{2\,\Delta x} - \frac{d^3u}{dx^3}\,\frac{(\Delta x)^2}{3!} - \ldots \qquad (14)$$

$$\left(\frac{d^2u}{dx^2}\right)_i = \frac{u_{(i+1)} - 2u_i + u_{(i-1)}}{(\Delta x)^2} - \frac{d^4u}{dx^4}\,\frac{(\Delta x)^2}{4!} - \ldots \qquad (15)$$

These discrete analogs are generally truncated after the first term since $(\Delta x)^2$ is small. The analogs and resulting difference equation becomes more complex for two or three-dimensional elliptical equations such as those of interest. The result is a set of N simultaneous algebraic equations requiring solution (von Rosenberg 1969).

Two types of grids have been used for discretization of the continuous aquifer for finite difference analysis. Usually a square grid is used with the node points at the intersection of the grid lines. If the aquifer is non-homogeneous, the node points may be further grouped into zones of similar transmissibility (Pilgrim 1970).

Alternately, an assymetric grid has been used in some models. The node point is representative of an irregular polygonal area of the aquifer. Often the node points are located at pumped wells, observation wells, or other control locations (Pilgrim 1970).

Three techniques are available for solution of the finite difference equations. Explicit solutions are simple and economical in computation time, but grid size and time increment must be very small to prevent divergence of the calculations. This results in very long computational times. Implicit solutions are stable but require inversion of a large matrix with the resulting demands on computer storage space. The third technique is the alternating direction method which minimizes storage requirements and computational time. Unfortunately, this technique appears not to be useable for the non-homogeneous, anisotropic case in which we are interested (Pilgrim 1970).

IX FINITE ELEMENT ANALYSIS

A technique newer than finite difference analysis is finite element analysis. This numerical method has several advantages over finite difference analysis (France 1974). Most importantly, non-homogeneous and anisotropic aquifers can be simulated with relative ease. A triangular node array can more easily represent the shape of aquifer boundaries. The results of finite element analysis are more accurate. Finally, since the technique is iterative computer solution is relatively simple compared with the matrix inversions associated with solution of the simultaneous equations of a finite difference analysis.

The principle of finite element analysis is based upon the calculus of variations. A head function is found which minimizes a specified function over the aquifer field. This results in a series of simultaneous equations which when solved results in an approximate solution to the original differential equation (Pilgrim 1970).

Some of the boundary conditions given for free surface seepage problems by France (1974) appear to be generally applicable to artesian aquifers such as the Floridan. At an impervious base, no seepage occurs across the boundary, and Darcy's seepage velocity component perpendicular to the boundary is zero. At a water boundary in which the pressure distribution is taken as hydrostatic (i.e., a vertical boundary perpendicular to the direction of flow) pressure varies linearly with depth and the piezometric head is constant. Where a boundary is a seepage face at which fluid gradually flows out of the aquifer, such as leakage to a river, the piezometric head must equal the elevation head.

For phreatic surfaces, such as those identified in Polk and Alachua Counties, two conditions must be satisfied. First is the obvious condition that the piezometric head must equal the elevation head. Secondly, for steady state problems, there is no velocity component normal to the surface. For the case of recharge, this last condition is not applicable (France 1974).

Unlike finite difference techniques, the node points used for finite element analysis are generally located at the corners of triangular elements. Cubic elements, however, may also be used as was demonstrated by France (1974) in his examples.

To solve the general differential equation and the boundary condition equations, discretized forms of these equations must be obtained. This may be done using a variational formula or the Galerkin method as is more commonly found in the available literature. The Galerkin method is briefly presented here (France 1974).

An approximating function of head must be found that when substituted into the governing equation causes the weighted average of the residual over the domain of the aquifer to vanish. The approximating function of head is given by:

$$\phi = \sum_{i} \phi_i N_i \qquad (16)$$

where

ϕ_i = the nodal value of head

N_i = an approximate interpolation function

The interpolation function is defined piecewise for each nodal point and is used as the weighting function.

The Galerkin method then gives:

$$\int_D RN_m \, dD = 0 \quad \text{for } m = 1, 2, \ldots, n \qquad (17)$$

In this integral, the residual, R, is defined by:

$$R = \left[\frac{\partial}{\partial x}\left(K_x \frac{\partial}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial}{\partial z}\right)\right] \sum_{i}^{n} N_i \phi_i \qquad (18)$$

D in equation (17) refers to the flow domain.

Equation (17) may be further simplified using Green's theorem. The boundary equations are then substituted into the resulting equation and a solution is obtained for the unknown heads.

To use this method, it would appear to this writer that approximate values of recharge could be defined as boundary conditions. The resulting piezometric heads then could be compared with observed values and recharge varied both in amount and location until a good approximation of the observed piezometric head was obtained.

X CONCLUSIONS & RECOMMENDATIONS

In the course of preparation of this paper, several observations have been made. First, models have been prepared for regional aquifer problems, but apparently none as extensive as the Floridan Aquifer. Characteristics of some of the models which have been reported are summarized in the appendix. It has become obvious that a model for the Floridan Aquifer would be a major undertaking requiring the services of several people and a significant amount of facilities or, alternately, digital computer time. In view of William James's warning against undertaking formulation of such a model whose real benefits are trivial, this writer has developed serious doubts as to the value of a model formulated for the purpose of defining recharge areas. It seems to this writer that the only benefit, beyond increased knowledge of the aquifer which would accrue from this model, would be the ability to make more rational land use decisions to protect the water resource of the Floridan Aquifer.

As to the question of best method to use for modeling the aquifer, it seems to the writer to be a matter of the qualifications of available personnel. It is felt that an analog model could be constructed by most engineers at the cost of marginal flexibility of the model. If personnel were available with a high degree of expertness in higher mathematics, and numerical methods in particular, then the flexibility of a digital model would make it the choice. This judgment could be altered by answers to questions raised in the statement of problems to the question of sensitivity of the model to the input data.

The following steps are recommended for further development of a Floridan Aquifer model. First, effort should be expended to collect the data available on formation constants, spring flows, well withdrawals, and estimated recharge rates as published in Florida Bureau of Geology publications. The qualitative information also presented in these publications should not be ignored. Additional data may be available in the form of well logs reported to the State. Plotting of the formation constants in the form of a map may be a useful technique for smoothing this data.

Using a map of the piezometric surface of the Floridan Aquifer, a flow net should be drawn. A streamline roughly traverse to the axis of the Florida pennisula and preferably through the Polk County high should be selected. A vertical two-dimensional model should be formulated for the streamline. This model would demonstrate the significance of vertical flow in the aquifer and would serve as a basis for a decision regarding the necessity of formulating a two vs.

three dimensional regional model of the aquifer. In addition, this model would be useful in learning the problems and techniques of aquifer modeling.

Finally, an analysis of the sensitivity of analog and numerical models should be made. Based upon that analysis, the collected data, and the results of the two-dimensional model, formulation of a regional recharge model for the Floridan Aquifer may proceed.

APPENDIX

SURVEY OF AQUIFER MODELS

Aquifer/Location/Remarks	Aq'fer Type	Model Type	Reference
Unk./Upper White River Basin, Indiana	U	A	Maclay, et al, 1972
Biscayne/Southeast Florida	U	A	Appel, 1973
Unk./Camas Prairie, Idaho	C(2) U(1)	FE	Wallace, 1972
Unk./Upper Wabash River, Indiana	U	A	Heisel, 1973
Ogallala/Northeastern Colorado/1400 sq. miles	U	FE	Luckey, et al, 1974
Unk./Livermore Valley, California/anisotropic & non-homogeneous	U(?)	FE	Witherspoon, 1974
Unk./Odessa-Lind Area Washington/recharged by leakage from shallow aquifer	C	FD	Luzier, 1975
Lower Cretaceous/Franklin Area, Southeastern Virginia/non-homogeneous	C	FD	Casner, 1975

Survey of Aquifer Models - Continued

Aquifer/Location/Remarks	Aq'fer Type	Model Type	Reference
Musquopobout/Nova Scotia/ anisotropic, non-homogeneous, irregular mesh	U&C	FD	Lin, 1970
Unk./Houston, Texas	U&C	A	Jorgensen, 1975
Lincolnshire/England minicomputer input/ output to analog	C	A	Rushton, et al 1975
Unk./Arkansas River Basin Colorado/ 3-dimensional	U	FD	Rovey, 1975
Glacial Outwash/ Dayton, Ohio/stream leakage	U	FD	Fidler, 1976
Gravel Aquifer/Walla Walla River Basin, Washington/time dependent flux used to calibrate model	U&C	FD	Baker, 1976
Chipuxet/Rhode Island	U	A	Kelly, 1976
Unk./Sutter Basin, California/ 3-dimensional	U&C	FE	Gupta, 1976

LEGEND:

Aquifer Type:	Model Type:
U — Unconfined	A — Analog
C — Confined	FD — Finite Difference
	FE — Finite Element

FOOTNOTES

[1] Additional data on industrial and agricultural water use is available in Pride, Estimated Use of Water in Florida, 1970. The source of the water is unfortunately not indicated in this document.

[2] These models by Pinder and Bredehoeft (1968) and Prickett and Lonnquist (1968) were reported by Pilgrim 1970.

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