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Dynamic and Stability Characteristics of an Articulated Frame Railway Passenger Truck

David Kenneth Platner
University of Central Florida

DYNAMIC AND STABILITY CHARACTERISTICS OF AN ARTICULATED FRAME RAILWAY PASSENGER TRUCK

BY

DAVID KENNETH PLATNER
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THESIS

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ABSTRACT

Mass transit vehicles in normal rail service frequently attain speeds which can excite carbody oscillations (primary hunting), as well as sustained lateral oscillations of the trucks (secondary hunting). The carbody motions have been shown to generate passenger discomfort and sustained truck hunting can lead to derailment. This thesis developes approximate equations which predict the carbody hunting frequencies, as well as the hunting speed of an articulated frame truck. The linear equations of motion are derived from a simplified model of a railway vehicle. A comparison indicates the the results obtained using the approximate truck hunting equation presented here are within ten percent of the results obtained from more rigorous approaches reported by others.

Director of Thesis

CONTENTS

NOTATION

β = Car Roll Angle (rad)

λ = Wheel Coning Ratio (in/in)

θ = Truck Yaw Angle (rad)

2a = Track Gauge (in)

A = Total Transverse Damping Parameter (lb)

2b = Truck Wheel Base (in)

B = Total Yaw Damping Parameter (lb-in^2/rad)

d = Wheel Diameter (in)

D_θ = Truck Yaw Damping Const. (lb-in-sec^2/rad)

D_L = Truck Longitudinal Damping Const. (lb-sec/in)

D_v = Truck Vertical Damping Const. (lb-sec/in)

D_y = Truck Transverse Damping Const. (lb-sec/in)

f = Vertical Natural Frequency (Hertz)

f_d = Damped Vertical Natural Frequency (Hertz)

f_r = Roll Natural Frequency (Hertz)

F = Friction Force Coefficient (lb)

g = Acceleration of Gravity (in/sec^2)

h = Distance from Lateral Spring to Car C.G. (in)

$\bar{i}$ = Unit Vector in Longitudinal Direction

I_c = Car Moment of Inertia in Roll (lb-in-sec^2/rad)

I_t = Truck Moment of Inertia in Yaw (lb-in-sec^2/rad)

$\bar{j}$ = Unit Vector in Transverse Direction

K_θ = Truck Yaw Stiffness (lb-in/rad)

K_L = Truck Longitudinal Stiffness (lb/in)

K_V = Truck Vertical Stiffness (lb/in)

K_y = Truck Transverse Stiffness (lb/in)

$2L_s$ = Truck Vertical Spring Spacing (in)

M_c = Car Mass ($lb\text{-}sec^2/in$)

M_t = Truck Mass ($lb\text{-}sec^2/in$)

$\bar{P}_1$, $\bar{P}_2$, $\bar{P}_3$, $\bar{P}_4$ = Force Vectors at Wheel-Rail (lb)

r = Wheel Radius (in)

r_a = Distance from Roll Center to Lateral Spring (in)

R = Distance from Roll Center to Car C.G. (in)

V = Truck Forward Velocity (mph)

W_a = Weight on Truck Axle (lb)

W_c = Car Weight (lb)

W_t = Truck Weight(lb)

y = Truck Transverse Displacement (in)

z = Car Vertical Displacement (in)

INTRODUCTION

Mass transit vehicles such as those in service on the New York subway system encounter a wide range of operating conditions. The speed can vary between fifteen miles per hour in curves to eighty miles per hour on straight track. Acceleration, braking, varying passenger load, and varying track bed flexibility all contribute to the complex rail vehicle environment. This paper will be restricted to the analysis of the rail vehicle suspension of an articulated frame truck which is considered to be running under steady state conditions on a straight, level, and rigid track. Additional assumptions will be interjected as they pertain to the topic being discussed. A concise list of assumptions is presented later.

The rail vehicle suspension is defined as the complete assembly, as shown in Photograph 1, whose components are the wheels, axles, motors, truck frame, bolster, primary suspension, and the secondary suspension. The primary suspension is defined as the elastic components connecting the axles to the truck frame, and the secondary suspension is defined as the elastic components connecting the carbody to the truck.

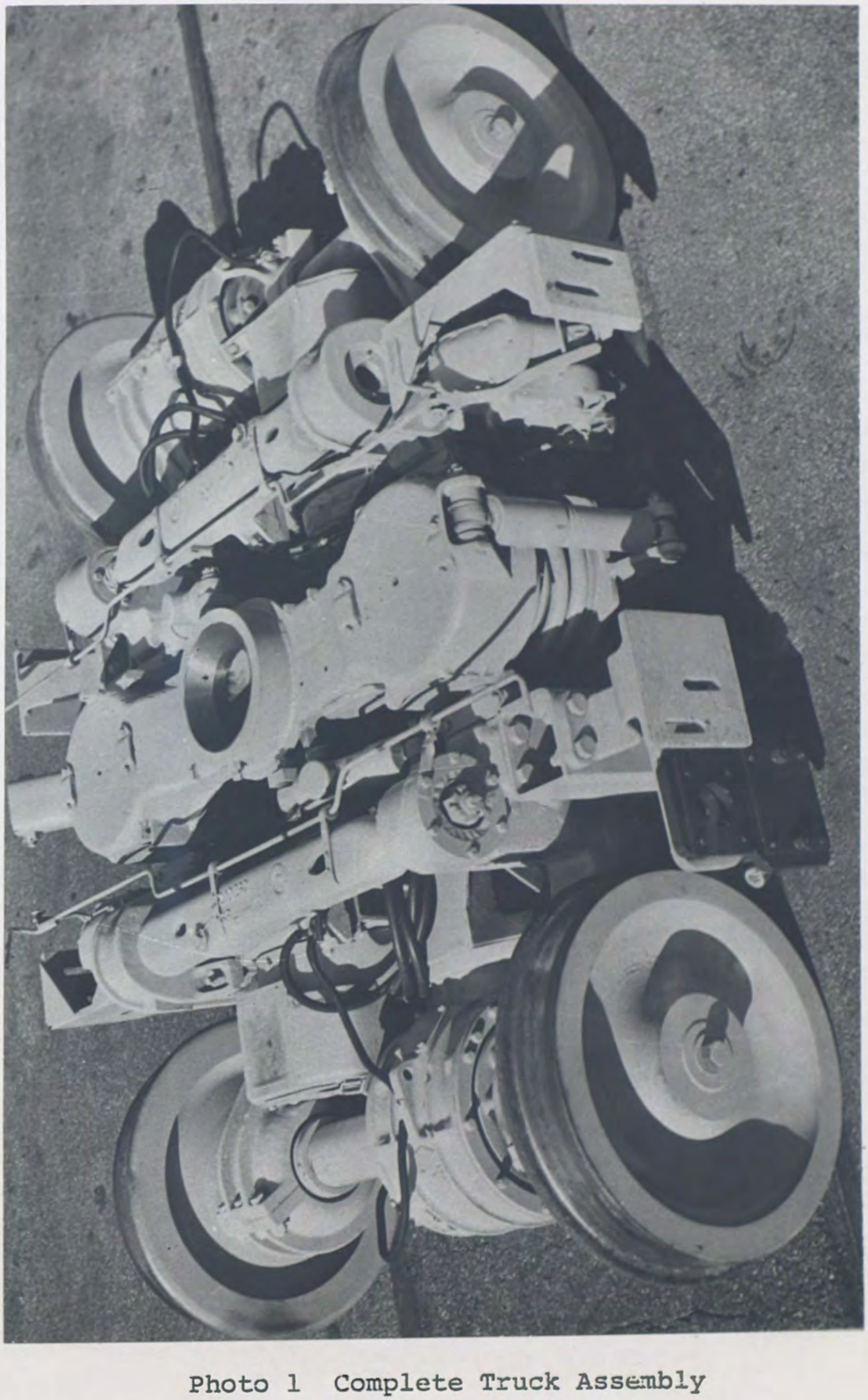


Photo 1 Complete Truck Assembly

An articulated frame truck, as shown in Photograph #2, has two distinct sideframes. Each sideframe is attached to the other through a hinging mechanism which allows sideframe rotation about the hinge line. This characteristic allows each wheel to move vertically over track irregularities independent of the primary suspension rate. Articulation thus allows the components at the primary to be relatively stiff (about 100,000 pounds per inch) in comparison to the secondary suspension (about 2,000 pounds per inch). In this paper, the primary suspension will be considered rigid.

Another major type of truck in mass transit use in the United States is the rigid frame truck. The sideframes of this truck are a single unit and do not articulate. The rigid frame truck permits vertical wheel motion through a soft primary suspension (approximately 15,000 pounds per inch). The difference in the stiffness of the primary suspension is the major item that distinguishes an articulated frame from a rigid frame truck. A rigid primary can only be accommodated by an articulated frame truck and thus consideration of the rigid frame truck is omitted from this analysis.

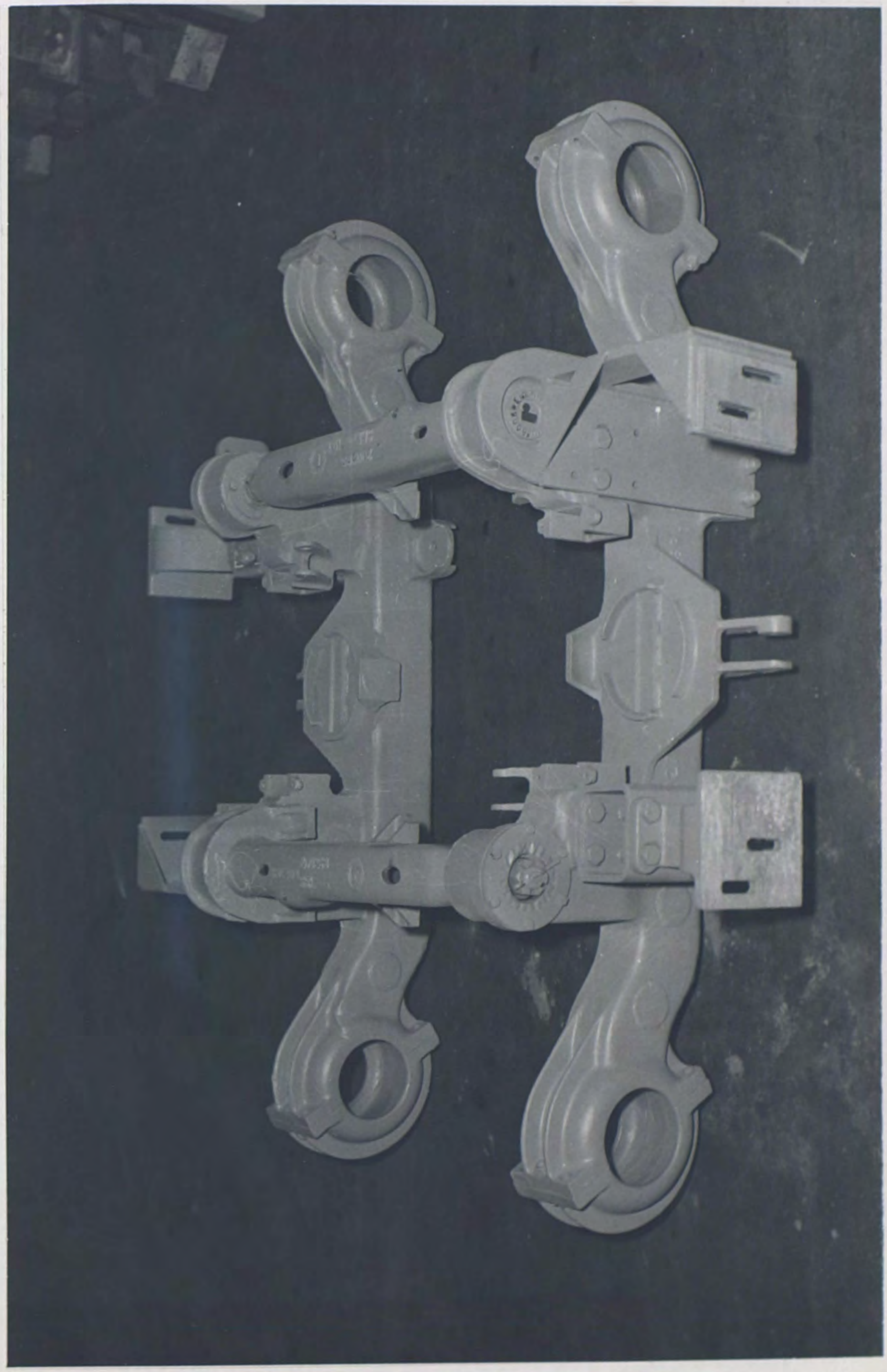


Photo 2 Sideframe Assembly

A survey by Law and Cooperrider (1974) has established the basic criteria for investigations of the rail vehicle system. Of primary concern are the oscillations of the transit vehicle allowed by the suspension elements. The survey mentioned above divides the harmonic oscillations (also called hunting) into two categories: primary hunting and secondary hunting. Primary hunting refers to the harmonic motions of the carbody and secondary hunting to the truck motions. Of the possible motions defined for the total system, only the following will be investigated here. The first is the vertical bounce of the carbody. The second is the combined roll and lateral oscillation of the carbody. The third and final motion considered here describes the lateral truck hunting.

The carbody bounce and roll are defined as resonant conditions (Cooperrider 1968). The dynamic equations describing these two motions will be developed in a later section. These resonant frequencies are usually found to be less than two Hertz and are low speed (less than twenty miles per hour) characteristics. The amplitudes of the bounce and roll oscillations have been found to be adequately controlled by damping (Diboll and Bieniecki 1968).

The lateral truck hunting, however, is found to be an instability phenomenon. The truck hunting is initiated only above a certain critical speed and is characterized by violent truck lateral motions. Unlike a resonant condition which is critical at distinct frequencies, the amplitude of truck hunting oscillations will continue to build as the velocity is increased above the critical speed. The hunting amplitude will usually be limited by the flanges of the wheels, but can lead to derailment.

Truck hunting is generated by the tapered wheel profile commonly in use to aid in curve negotiation. A cylindrical tread theoretically does not hunt, but it also generates excessive flange wear. Wickens (1966) reports that even a cylindrical tread profile will quickly become worn to a measurable taper, which will then generate hunting. Wickens (1965) has also investigated the wheel profile characteristics. His article and an article by Law and Brand (1973) indicate that the true nature of the wheel rail interface is a nonlinear one which can significantly influence the hunting speed of a wheel set. However, Wickens (1966) states that the nonlinear effects are minimal for paired wheel sets as used in a truck. For practical use in this thesis, only tapered wheels and linear forces will be used.

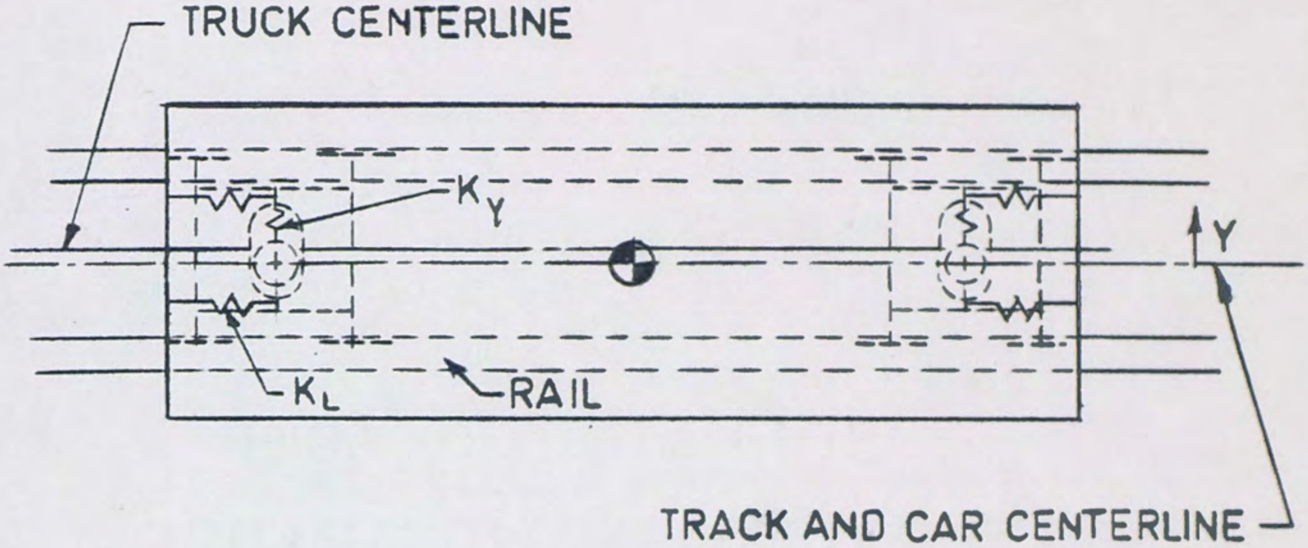
A few articles have presented techniques to determine the critical speed. Cooperrider (1968) emloys a digital computer technique, which plots the root loci of the characteristic equations. Law and Brand (1973) achieve their solution of nonlinear equations with the aid of CSMP (Continuous Systems Modeling Program). Clark and Law (1967) and also Vernon (1967) have presented approximate methods which can be used to determine the hunting speed. The approximation presented by Clark and Law (1967) is used in this thesis and further discussed in the next section. A comparison of the approximate results with those using a more rigorous approach is also presented later.

This thesis incorporates the approximate equations into a computer program which is designed for a timesharing or interactive system. The natural frequencies and critical speed are computed after entering data pertaining to truck geometry, spring rates, and damping constants. A table is also included to indicate how changes made to the input data will influence the calculated data.

THE MATH MODEL

Figure 1 depicts the model of the general railway vehicle. It consists of a rigid carbody supported elastically by the secondary suspension. The trucks are rigidly restrained to move longitudinally with the carbody and elastically restrained vertically and laterally to allow relative motion between the carbody and the trucks. The vertical springs of the secondary suspension are either air bellows or coil springs as shown in Photograph 1. The vertical spring rate K_V and the lateral spring rate K_Y are defined by these secondary suspension elements. The damping D_V and D_Y are supplied by vertical and lateral shock absorbers, which can also be seen in Photograph 1. Rotation of the trucks beneath the carbody is elastically restrained by sidebearers, which provide the values for K_L and D_L.

The following assumptions have also been made in order to develop the equations of motion. The vehicle is operating on straight and level track (tangent track). All vehicle and truck components are considered rigid. The truck frame is articulated and mounted to the axle through a rigid primary suspension. The axles are free running, and no tractive, braking, or frictional forces



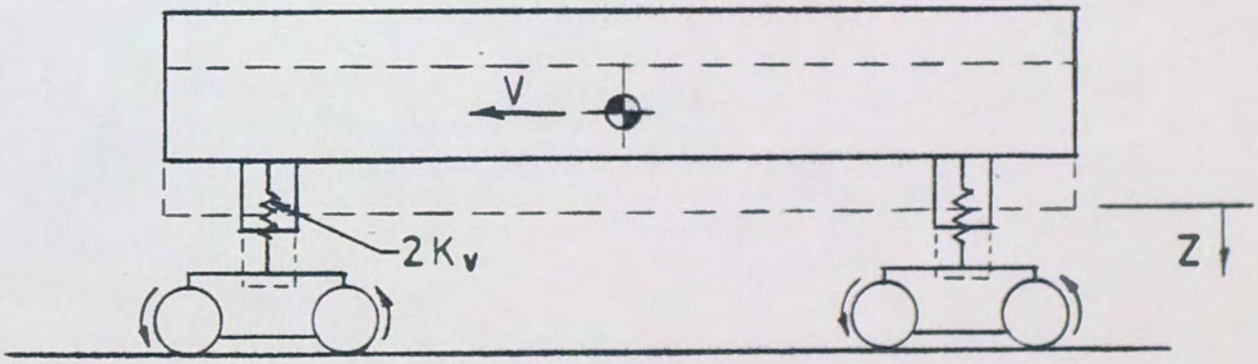


Fig. 1 Entire Vehicle Model

are considered. Aerodynamic forces are also excluded. External forcing functions due to rail joints and wheel flats are considered only as possible exciters of resonant frequencies. The equations are linear, small deflections and rotations are assumed, and only the steady state solution is considered. The wheel tread profile is approximated by a conical taper.

Cooperrider (1968) has presented the equations of motion for a seven degree of freedom vehicle. This thesis, however, is only concerned with the vertical bounce of the carbody, the lateral roll of the carbody, and the truck lateral hunting. Cooperrider (1968) does not include the carbody vertical bounce in his seven degree of freedom model. However, Diboll and Bieniecki (1968) state that no coupling exists between the vertical bounce and the lateral motions of the carbody or the trucks. Also, the roll natural frequency is not influenced by the truck hunting. Cooperrider (1968) and Clark and Law (1967) uncouple the carbody motions from the truck hunting equation. The influence of coupling on the hunting speed will be discussed later. The following sections will develop the uncoupled equations of motion individually.

Vertical Natural Frequency

The vertical natural frequency can be determined with the help of Figure 2. The motion is pure vertical bounce. The truck is assumed to act as a rigid support, which eliminates the influence of the rail deflections. Applying Newton's Law, the equation of motion is

$$M_C \ddot{z} + 4 D_V \dot{z} + 4 K_V z = 0. \tag{1}$$

Here, Mc is the total car mass, K_V is the vertical spring rate, and D_V is the vertical damping constant. It can be shown from Equation 1 that the damped natural frequency, f_d, is

$$f_d = \frac{1}{\pi} \sqrt{\frac{K_V}{M_C} - \left(\frac{D_V}{M_C}\right)^2} . \tag{2}$$

Diboll and Bieniecki (1968) indicate that the damping for optimum passenger comfort is about thirty percent of critical. For the system above, setting D_V equal to $.3\sqrt{K_V M_C}$ results in the following equation for f_d:

$$f_d = .3036 \sqrt{\frac{K_V}{M_C}} . \tag{3}$$

Since this is only a five percent reduction from the undamped natural frequency, f, it is permissible in practical design applications to use

$$f = \frac{1}{\pi} \sqrt{\frac{K_V}{M_C}} . \tag{4}$$

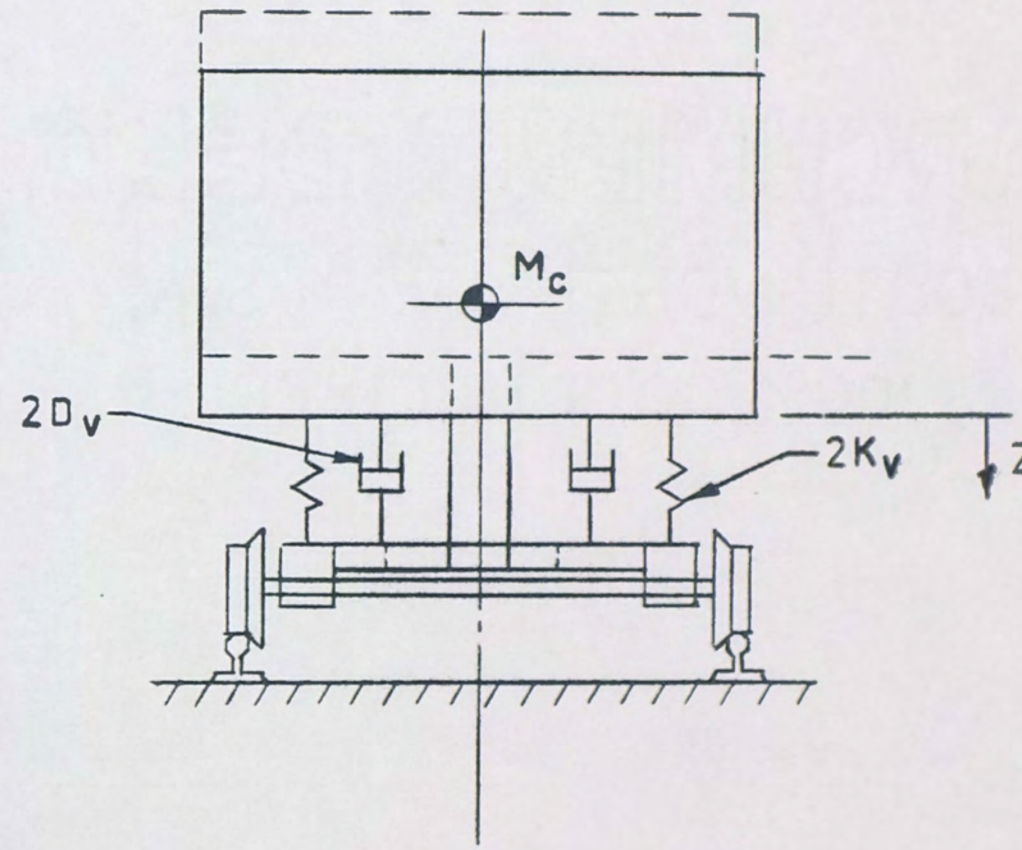


Fig. 2 Vertical Model

The assumption that damping can be neglected in calculating the vertical natural frequency has just been demonstrated. The following development for roll natural frequency will also assume no damping at the outset in order to simplify the equations. However, since optimum roll damping is also about 30 percent of critical, neglecting damping in the roll frquency calculation is a valid simplification (Diboll and Bieniecki 1968)

Roll Natural Frequency

The lateral movement of the carbody is found to be coupled with a rolling motion (Diboll and Bieniecki 1968). As in the vertical natural frequency, the truck is assumed to act as a rigid support, and damping is neglected. Figure 3 shows that the roll motion can be visualized as the carbody mass pivoting about an imaginary point "A". Applying Newton's Law, the equation of motion becomes

$$(M_c R^2 + I_c)\,\ddot{\beta} + (4\,K_v L_s^2 + 2\,K_y r_a^2 - M_c g R)\sin\beta = 0, \qquad (5)$$

and for small β the equation reduces to

$$(M_c R^2 + I_c)\,\ddot{\beta} + (4\,K_v L_s^2 + 2\,K_y r_a^2 - M_c g R)\,\beta = 0. \qquad (6)$$

Here, I_c is the moment of inertia of the carbody in roll, K_y is the lateral spring rate of the secondary suspension, and L_s, R, r_a, and β are as defined in the figure.

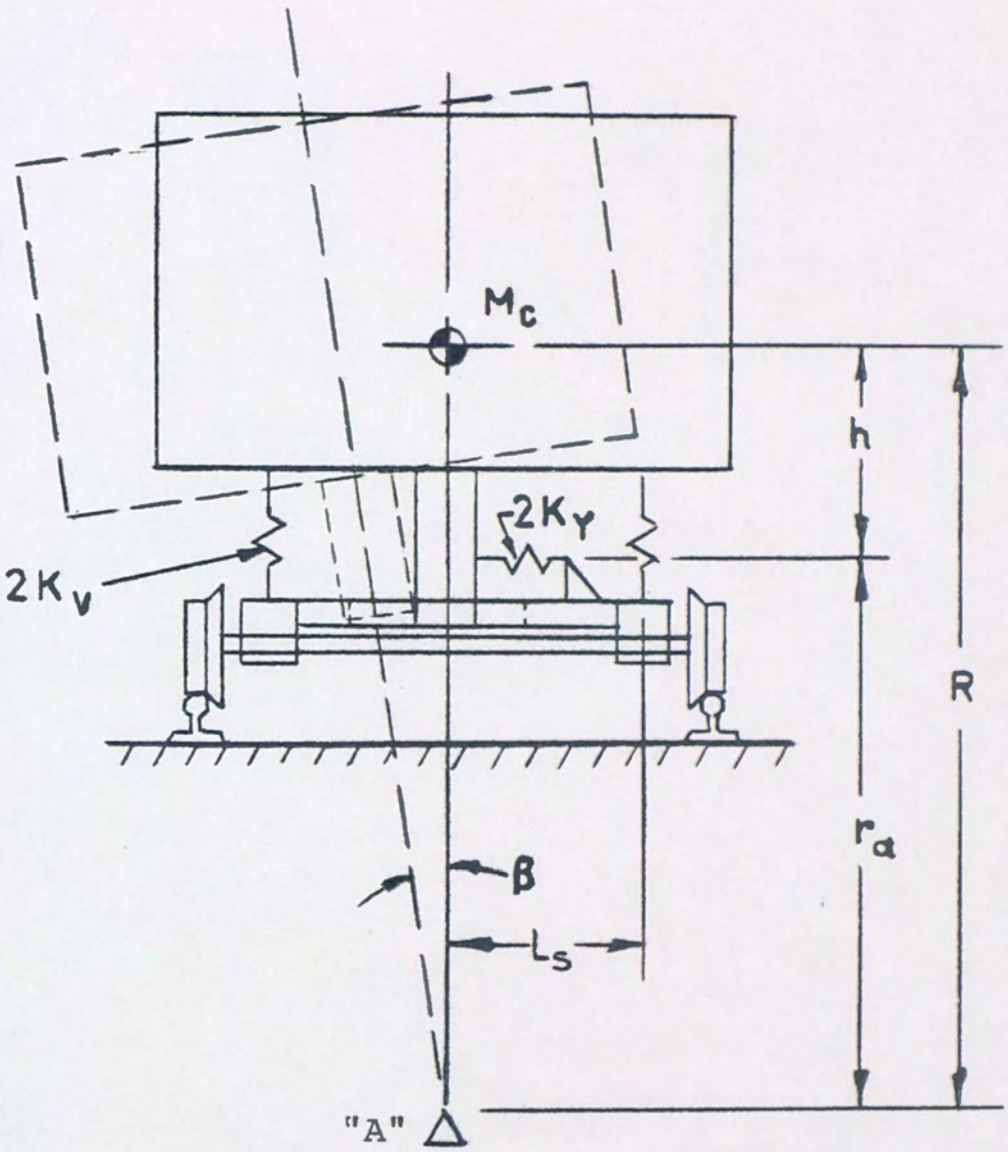


Fig. 3 Roll Model

From Equation 6, the roll natural frequency can be shown to be

$$f_r = \frac{1}{2\pi}\sqrt{\frac{4\, K_v\, L_s^2 + 2\, K_y\, r_a^2 - M_c\, g\, R}{M_c\, R^2 + I_c}} \qquad (7)$$

In order to determine the distance to the roll center, R, consider the static condition generated by a force, F, applied laterally at the carbody C.G. The summation of forces can be shown to be

$$2\, K_y\, r_a \boldsymbol{\beta} - F - M_c\, g\, \boldsymbol{\beta} = 0, \qquad (8)$$

and the moment about "A" can be shown to be

$$2\, K_y\, r_a^2 \boldsymbol{\beta} + 4\, K_v\, L_s^2 \boldsymbol{\beta} - F\, R - M_c\, g\, R \boldsymbol{\beta} = 0. \qquad (9)$$

Combining Equations 8 and 9, the distance to the roll center becomes

$$R = \frac{2\, K_v\, L_s^2 + K_y\, h^2}{K_y\, h}\;, \qquad (10)$$

where, h is given by

$$h = R - r_a\;. \qquad (11)$$

The roll frequency equation is descriptive of a secondary system with independent springs providing the vertical rate K_v. However, the secondary suspension of most new rapid transit trucks is an air spring. The height of the air spring is controlled by a level valve which maintains the floor of the car at a constant height, independent of the number of passengers being carried.

Most transit cars do not have a separate level valve for each air spring. If only one level valve is used on a truck, the air springs are interconnected and allow air to flow freely from one spring to the other as the carbody rotates. The rotation compresses one spring and extends the other. The resistance that remains will be called the roll vertical spring rate.

The two level valve car has one level valve centered on each truck, and the vertical spring rate is replaced by the roll vertical spring rate for roll frequency calculations. A three level valve system has a centered level valve on one truck and independent level valves on the other. On a three level valve system, an average of the roll rate and the vertical spring rate is used for K_V. A four level valve system has independent level valves and the roll equations use the vertical spring rate. The independent air spring is assumed to generate the same rate in roll as it does in vertical bounce. The equation for hunting speed developed in the following section will be shown to be independent of the vertical spring rate.

Hunting Speed

The development that follows is based on the article by Clark and Law (1967). The carbody is assumed to move at the same velocity as the truck and the carbody vertical and roll motions are neglected. Figure 4 will be used to develop the equations of motion. The forces $\bar{P}_1$, $\bar{P}_2$, $\bar{P}_3$, and $\bar{P}_4$ due to friction at the wheel-rail interface have been shown in the above mentioned article to be:

$$\bar{P}_1 = F\left[\frac{a}{V}\dot{\theta} + \frac{\lambda}{r}(y + b\,\theta)\right]\bar{i} + F\left[-\frac{1}{V}(\dot{y} + b\,\dot{\theta}) + \theta\right]\bar{j}\,, \qquad (12)$$

$$\bar{P}_2 = F\left[\frac{a}{V}\dot{\theta} + \frac{\lambda}{r}(y - b\,\theta)\right]\bar{i} + F\left[-\frac{1}{V}(\dot{y} - b\,\dot{\theta}) + \theta\right]\bar{j}\,, \qquad (13)$$

$$\bar{P}_3 = -F\left[\frac{a}{V}\dot{\theta} + \frac{\lambda}{r}(y - b\,\theta)\right]\bar{i} + F\left[-\frac{1}{V}(\dot{y} - b\,\dot{\theta}) + \theta\right]\bar{j}\,, \qquad (14)$$

and

$$\bar{P}_4 = -F\left[\frac{a}{r}\dot{\theta} + \frac{\lambda}{r}(y + b\,\theta)\right]\bar{i} + F\left[-\frac{1}{V}(\dot{y} + b\,\dot{\theta}) + \theta\right]\bar{j}\,. \qquad (15)$$

In these equations, F, the frictional force coefficient is given by

$$F = 3500\sqrt{2\,r\,W_a}\,. \qquad (16)$$

W_a is the weight on the axle, $\bar{i}$ and $\bar{j}$ are unit vectors in the longitudinal and lateral directions, and V is the velocity of the truck. The terms a, λ, y, b, θ, and r are as defined in Figure 4.

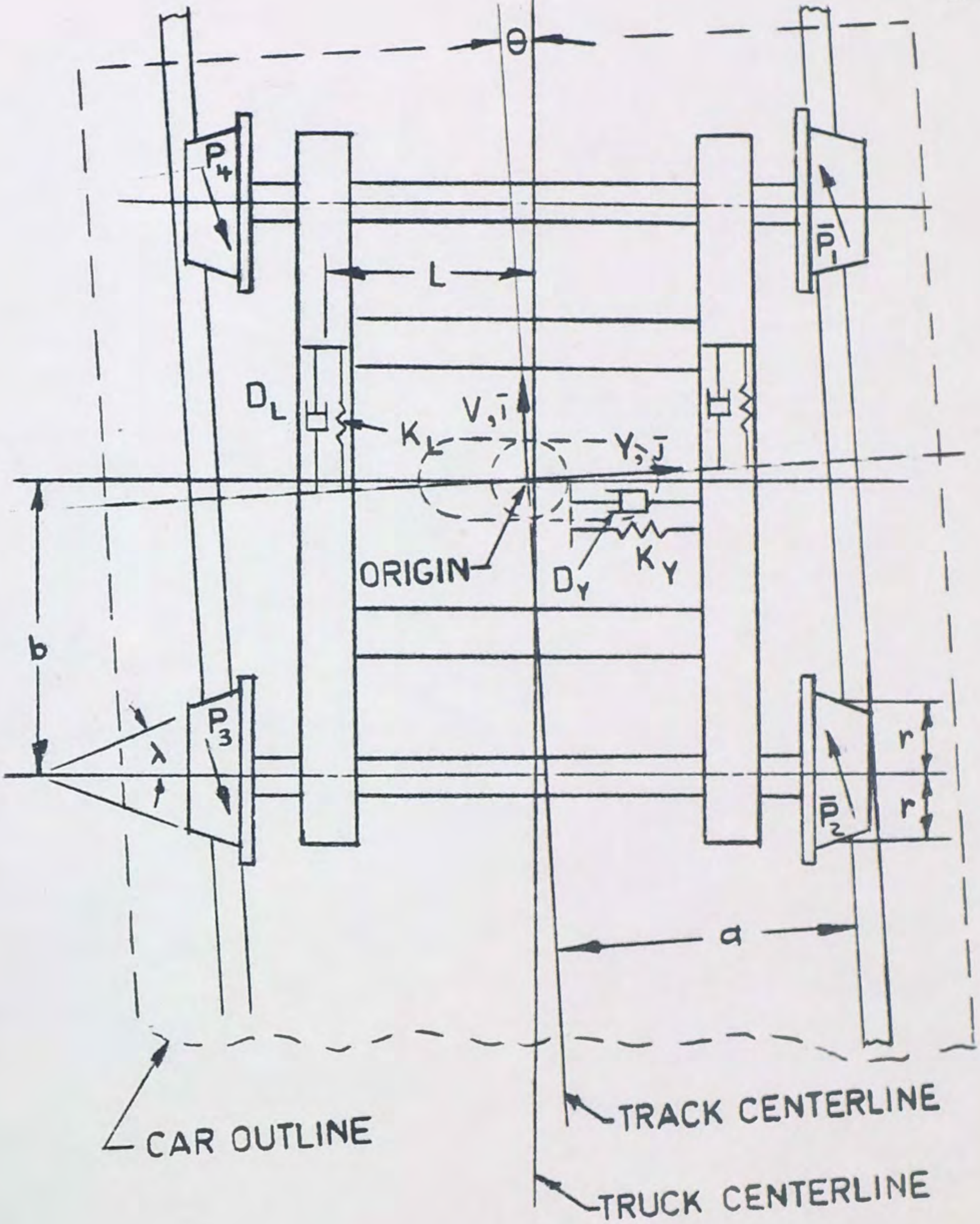


Fig. 4 Hunting Model

The summation of lateral forces at the wheel-rail interface is given by

$$F_y = -4\,F\left[\frac{1}{V}\dot{y} - \theta\right] , \tag{17}$$

and the moment about the origin is

$$-M_o = \frac{4\,F}{V}\,(a^2 + b^2)\,\dot{\theta} + 4\,F\,\frac{a\lambda}{r}\,y \quad . \tag{18}$$

Applying Newton's Law, the equations of motion are

$$M_t\,\ddot{y} + \frac{1}{V}\,(4\,F + V\,D_y)\,\dot{y} + K_y\,y - 4\,F\,\theta = 0, \tag{19}$$

and

$$I_t\,\ddot{\theta} + \frac{1}{V}\left[4\,F\,(a^2 + b^2) + V\,D_\theta\right]\dot{\theta} + K_\theta\theta + 4\,F\,\frac{a\lambda}{r}y = 0. \tag{20}$$

Where, M_t is the truck mass, I_t is the truck moment of inertia in yaw, K_y is the lateral spring rate, D_y is the lateral damping constant, K_θ is the yaw spring rate, and D_θ is the yaw damping constant. It can be noted that K_v, the vertical spring rate, does not appear in the above equations. K_θ and D_θ are defined by

$$K_\theta = 2\,K_L\,L^2 \ , \tag{21}$$

and

$$D_\theta = 2\,D_L\,L^2 \ . \tag{22}$$

After a Laplace transformation, the characteristic equation takes the form

$$s^4 + \left(\frac{A\,I_t + B\,M_t}{M_t\,I_t\,V}\right) s^3 + \left(\frac{K_\theta\,M_t + K_y\,I_t}{M_t\,I_t} + \frac{A\,B}{M_t\,I_t\,V^2}\right) s^2$$

$$+ \left(\frac{A\,K_\theta + B\,K_y}{M_t\,I_t\,V}\right) s + \frac{1}{M_t\,I_t}\left(K_y\,K_\theta + 16\,F^2\,\frac{a\lambda}{r}\right) = 0, \qquad (23)$$

where s is the Laplace transform variable. After applying a neutral stability criteria to the above equation (Clark and Law 1967), the equation for the critical hunting speed can be shown to be

$$V_c^{\,2} = \frac{A\,B\,(A\,I_t + B\,M_t)\,(A\,K_\theta + B\,K_y)\,r}{(A\,I_t + B\,M_t)\,16\,a\,\lambda\,F^2 - A\,B\,r\,(M_t\,K_\theta - I_t\,K_y)^2}, \qquad (24)$$

where the total transverse damping parameter, A, is

$$A = 4\,F + V\,D_y, \qquad (25)$$

and the total yaw damping parameter, B, is

$$B = 4\,F\,(a^2 + b^2) + V\,D_\theta\,. \qquad (26)$$

The critical speed, V_c, is the speed at which truck hunting can start. Further increase above this speed results in sustained lateral oscillations which will eventually cause distruction of the wheel flanges and derailment of the truck. Thus, the critical speed represents an upper limit to the operating speed of the vehicle. Equation 24, for the critical velocity, is nonlinear since both the A and B terms contain velocity. The following section will present the method used to solve for the critical hunting speed.

THE NUMERIC MODEL AND PARAMETRIC STUDY

The equations developed in the previous section have been assembled into a computer program. The program is written in Basic language and designed for a time-sharing or interactive computing system. The program listing is presented in the Appendix. The ability to use the time sharing mode allows the program to be conversational. In other words, the computer prompts the user for a response with questions. In the program for this thesis, the user enters data pertaining to truck geometry, spring rates, and damping constants. The computer then prints out a table of all the input parameters followed by the calculated values of the vertical natural frequency, the roll natural frequency, and the hunting speed. The program also allows the user to change all or any portion of the input data. A typical design example is presented later.

Equations 4 and 7 for vertical frequency and roll frequency generate specific values. However, as stated earlier, the hunting equation is nonlinear and must be solved through an iterative process. Only when the estimated critical velocity used on the right hand side of Equation 24 matches the calculated velocity is the iterative process complete.

A form of the Newton-Raphson method, which automatically iterates to a solution of the critical velocity equation, is incorporated in the program for this thesis. To use this method, Equation 24 for V_c is written as

$$X = (\text{R.H.S.}) - V_c^2 \ , \qquad (27)$$

where R.H.S. is the right hand side of Equation 24. The object then is to find V_c such that X is approximately zero. The Newton-Raphson method uses the initial value chosen for V_c and calculates X. If X is not within the region defined by

$$|X| \leqslant 0.01 \ , \qquad (28)$$

the slope is calculated at the estimated V_c from

$$X' = \frac{X(V_c + 1) - X(V_c - 1)}{2} \ . \qquad (29)$$

A new value of V_c is then calculated from the old value of V_c by using

$$V_c(\text{new}) = V_c(\text{old}) - \frac{X}{X'} \ . \qquad (30)$$

The process then returns to Equation 27 and if X does not satisfy Equation 28, the iteration continues. When Equation 28 is satisfied, the iterating stops and the last value calculated by Equation 30 is used for the critical hunting speed. A plot of X was made for the data presented in Table 1 and found to only have one root for positive values of velocity. In order to assure that only the positive value of velocity results from the method described above, Clark and Law (1967) recommend that the initial value of the critical velocity for Equation 27 be estimated from Equation 24 with zero damping. The assumption of zero damping eliminates the effects of velocity from the right hand side of Equation 24 and yields an acceptable first estimate for V_c. The Newton-Raphson method as used in this thesis has been found to converge on a solution within five iterations.

An example of a typical lightweight truck is shown in Table 1. The first part of the table, which is labeled Design Data, presents the design parameters required to calculate the values of vertical natural frequency, roll natural frequency, and the hunting speed. In general, these are parameters which can be measured by physical means. This data will be used later in the parametric study and in the design example. The last part of the table, which is labeled Calculated Values, presents the numeric output from the computer program.

TABLE 1

DESIGN PARAMETERS AND PROGRAM CALCULATIONS
FOR A TYPICAL LIGHTWEIGHT TRUCK

Design Data	Value
1. Total car weight (lb)	71820.00
2. Total truck weight (lb/trk)	12000.00
3. Truck unsprung weight (lb/trk)	4000.00
4. Radius - car mass moment (in)	50.00
5. Radius - truck mass moment (in)	30.00
6. Vertical spring spacing (in)	71.00
7. Dist. - car C.G. to lateral spg. (in)	29.00
8. Wheel diameter (in)	28.00
9. Track gauge (in)	59.00
10. Wheel base (in)	82.00
11. Coning ratio (in/in)	0.05
12. No. level valves per car (#/car)	3
13. Vertical spring rate (lb/in/spg)	2550.00
14. Roll vert. spring rate (lb/in/spg)	1693.90
15. Lateral spring rate (lb/in/trk)	2066.00
16. Yaw spring rate (in-lb/rad/trk)	529000.00
17. Lat. damper constant (lb-sec/in/trk)	413.00
18. Yaw damper constant (in-lb-sec/rad/trk)	761000.00

Program Calculations	Value
1. Vertical natural frequency (Hz)	1.178
2. Roll natural frequency (Hz)	0.539
3. Hunting speed (mph)	67.437

DISCUSSION OF RESULTS

The parameters which have an influence on the vertical frequency, the roll frequency, and the hunting speed can be determined from Equation 4, Equation 7, and Equation 23, respectively. The effect of changing the vertical or roll parameters can easily be determined. However, the influences of the eleven parameters which make up the truck hunting equation are not so easily determined. The program developed for this theses has the ability to easily change the input data, and thus lends itself to a parametric study. Table 2, which follows, indicates the influence that each of the input parameters has on the calculated values of vertical frequency, roll frequency, and hunting speed. Table 2 provides valuable design information which can be used to maximize the hunting speed of a transit truck. The basic truck data used to initialize the input parameters is found in Table 1.

TABLE 2

INFLUENCE OF DESIGN PARAMETERS
ON CALCULATED FREQUENCIES
AND HUNTING SPEED

Design Parameter Increased	Effect On		
	Vertical Frequency	Roll Frequency	Hunting Speed
Car weight	Decrease	Decrease	Decrease
Truck weight	None	None	Decrease*
Car mass moment	None	Decrease	None
Truck mass moment	None	None	Decrease
Spring spacing	None	Increase	None
Raise car C.G.	None	Lower	None
Wheel diameter	None	None	Increase*
Track gauge	None	None	Decrease
Wheel base	None	None	Increase*
Wheel taper	None	None	Decrease*
Vertical spring rate	Increase	Increase	None
Roll spring rate	None	Increase	None
Lateral spring rate	None	Increase	Increase*
Yaw spring rate	None	None	Increase
Lateral damping	None	None	Increase
Yaw damping	None	None	Increase

* Major hunting parameters

Table 2 indicates the general effects of changing the design parameters, but omits the magnitude of the effect. For example, of the eleven hunting parameters only five have a significant influence on the hunting speed. Also, many of the parameters have maximum and/or minimum limitations dictated by the particular railroad or transist authority's specifications. Each of the parameters will be examined in the following paragraphs with emphasis on the degree of effectiveness and the practical limitations. The data in Table 1 is used as the base for comparisons.

The maximum car weight will always be limited by the authority's specification. However, if the authority would increase the car weight by 10 percent, a 5 percent decrease in vertical and roll frequency would result, but less than a .2 percent decrease in hunting speed would result. Thus car weight has little influence on the hunting speed. The vertical and roll frequencies are the result of the car weight, and as such, they are not used to determine the car weight.

The maximum truck weight is very seldom limited by the authority. The car builder, however, is restricted to a maximum weight on rail, which means the lighter the trucks the greater the number of passengers that can be carried. An increase of 10 percent in truck weight will decrease the hunting speed by over 5 percent. Thus, truck weight has a major influence on the hunting speed, and a reduction in truck weight is desirable. The vertical and roll frequencies are not influenced by the truck weight.

The car mass moment of inertia only influences the roll frequency. A 10 percent decrease produces a 2 percent decrease in the roll frequency. The car moment of inertia is not a parameter which can be easily changed and is usually dictated by the design of the car builder.

The truck mass moment of inertia is defined by the radius of gyration. An increase of 10 percent in the radius of gyration will decrease the hunting speed by about 3 percent. In general it is not practical to reduce the radius of gyration after a truck design is established. However, in the early layout stage of a new truck proposal the large mass items such as the motors and sideframes can be located as close as practical to the center of the truck. The vertical and roll frequencies are not influenced by the truck moment of inertia.

The lateral spring spacing and the height of the car C.G. only influence the roll frequency. A 10 percent increase in spring spacing will generate a 6 percent increase in the roll frequency. Some transit systems will specify a range for the roll frequency.

The vertical springs can be positioned on a new car to provide the desired roll characteristics. If the distance from the lateral spring to the car C.G. is increased by 10 percent, almost a 5 percent decrease in roll frequency will result. The distance to the car C.G. is not a variable and will be determined by the design of the car.

The wheel diameter will be specified by the particular transit authority. If the diameter is allowed to increase by 10 percent, the hunting speed will increase by about 5 percent. However, the larger wheel will weigh more and the negative influence of the heavier truck would have to be considered. Even though the wheel diameter has a significant influence on the hunting speed, restrictions by the transit authority eliminate increasing the wheel diameter as a means to increase the hunting speed. The vertical and roll frequencies are not a function of the wheel diameter.

The track gauge has a very small influence on the hunting speed. If the track gauge is increased from the 59 inch standard gauge to the 69 inch wide gauge used on the BART system in San Francisco, less than a one mile per hour decrease in hunting speed would result. The track gauge does not influence the vertical and roll frequencies.

The maximum wheel base will be specified by the transit authority. It is desirable to design up to the specified limit since an increase of 10 percent in wheel base will result in an increase of nearly 8 percent in hunting speed. However, the sideframe weight will increase with an increased wheel base, and the negative effect on hunting would have to be considered.

The wheel taper is the parameter which generates the hunting phenomenon. A zero taper or cylindrical tread will not generate hunting. However, as stated previously, the cylindrical tread will quickly wear to a tapered tread. A typical wheel will be machined to a one in forty taper. After a period of use, the wheel can become worn to a one in twenty taper. In practice the wheels will be re-machined if the taper exceeds one in twenty. An increase in taper of 10 percent (from one in twenty to one in eighteen) will result in almost a 5 percent decrease in hunting speed. The one in twenty taper is used in this paper since it represents the maximum taper normally permitted. Wheel taper does not influence the vertical and roll frequencies.

The vertical spring rate has a direct influence on the vertical and roll frequencies, but has no influence on the hunting speed. A 10 percent increase in vertical spring rate will increase the vertical frequency by about 5 percent and increase the roll frequency by about 2 percent. Transit authorities will occasionally give ranges for the vertical and roll frequencies. It is common practice in the transit industry to design for a vertical frequency between 1.0 to 1.2 and a roll frequency between .45 to .65.

The roll spring rate is the vertical rate of the air springs when the air supply interconnects the two air springs. This rate is a function of the air springs design and as such is not easily changed. The vertical frequency and the hunting speed are not effected by this characteristic.

The lateral spring rate is another parameter which has a major influence on the hunting speed. A 10 percent increase in the lateral spring rate will increase the hunting speed by almost 5 percent. Unfortunately, the increased lateral rate also increases the roll frequency. Some transit specifications will limit the roll natural frequency which will thereby limit the lateral spring rate. The vertical frequency is not influenced by the lateral spring rate.

The yaw spring rate has a stabilizing influence on the hunting speed, but no influence on the vertical or roll frequencies. At the magnitude shown in Table 1, a 10 percent increase in the yaw spring rate only produces a .5 percent increase in the hunting speed. However, at lower values a 5 percent increase in hunting speed can result from a 10 percent increase in the yaw spring rate. The yaw spring rate, however, restricts the truck's ability to negotiate curves and must be limited to a safe value. The determination of this safe value is not within the scope of this paper.

The lateral and yaw damping have little influence on the hunting speed, and are normally defined by other requirements. The optimum lateral damping has been given by Diboll and Bieniecki (1968) as 30 percent of critical. The yaw damping is determined from the physical characteristics of the side bearing component. The effect of lateral damping is neglected in the roll equation and has no influence on the vertical frequency. Yaw damping has no influence on the vertical or roll frequencies.

To summarize the above, it was determined that, of the eleven design parameters influencing the hunting speed, only five have a significant effect. An increase of 10 percent in the truck weight, wheel diameter, wheel base, wheel taper, or lateral spring rate produced about a 5 percent change in the hunting speed. The above conclusions are based on the initial data found in Table 1. The ability to modify the data leads naturally into a practical design example.

Starting with the data found in Table 1, one is required to increased the hunting speed from 67 miles per hour to at least 90 miles per hour. Of the five parameters which significantly influence the hunting speed, the transit authority's specification is found to define the wheel diameter, the wheel base, and the wheel taper. A weight study has also indicated that the truck weight cannot be reduced more than five percent. The lateral spring rate can be increased, but the specification also limits the roll frequency to .65 Hertz.

Allowing .65 Hertz for the roll frequency, it is determined that the lateral spring rate can be increased from 2066 to 3600 pounds per inch. Also, allowing a five hundred pound decrease in the truck weight, the calculated hunting speed is found to increase to 91 miles per hour. Thus the design goal has been achieved.

The entire design process described above took only a few minutes at a computer terminal. Thus if the equation for hunting speed can be proven reliable, the program developed in this thesis provides a valuable tool. In order to develop confidence in the numeric model, the results of other authors' methods were researched. Table 3 shows a comparison between the hunting speed as reported by others and as calculated by the program presented in this thesis.

TABLE 3

A COMPARISON OF REPORTED HUNTING SPEED TO CALCULATED HUNTING SPEED

Car	Author	Reported	Calculated
Lightweight	Clark & Law (1967)	142	143
Tokaido	Clark & Law (1967)	137	138
High Speed	Cooperrider (1968)	153	157
Lightweight	Wickens (1965)	118	117
Covered Van	Wickens (1966)	83	80
New Pallet Van	Wickens (1966)	34	37
Double Bolster	Wickens (1966)	50	54

The first two entries in Table 3 are from the article by Clark and Law (1967). The hunting equation used in this thesis is based on the equation developed in the above article and therefore should yield the same hunting speed. The agreement shown by the first two entries verifies the numeric model and the Newton-Raphson technique used to solve the critical hunting equation. The 137 mph Tokaido truck was observed to have a critical speed ranging from 110 mph to 152 mph.

The entry by Cooperrider (1968) used a similar math model to the one presented in this thesis, except for the following differences. A profiled wheel tread was assumed, and the wheel rail forces were not as defined in Equation 12 through 15. Yaw damping was also neglected. The reported 153 mph hunting speed was obtained from an approximation derived by using Routh's criteria. Cooperrider (1968) also used a technique which plotted the roots of the characteristic equations. This method gave a critical velocity of 138 mph for the simple truck model and 136 and also 150 mph for the entire vehicle model with coupling effects. Cooperrider (1968) did not consider the effects of coupling between truck hunting and the car rolling motion to be significant.

The lightweight entry by Wickens (1965) followed a similar development to the one presented by Cooperrider (1968). The reported 118 mph hunting speed was obtained after using Routh's discriminant and assuming a tapered wheel tread. The profiled tread was reported to generate a hunting speed of 109 mph.

The last three entries by Wickens (1966) are obtained by a similar approximate technique, but these trucks have substantiating test data. The Covered Van was observed to hunt at speeds between 40 and 50 mph, the New Pallet Van between 25 and 30 mph, and the Double Bolster truck between 55 and 65 mph.

It can be concluded that there is good agreement with other reported methods and that, for design purposes, the program presented here provides a reliable method for calculating the hunting speed.

CONCLUSIONS

A program to calculate the vertical natural frequency, roll natural frequency, and the critical hunting speed has been written and made operational. The previous section has demonstrated the ability of the program to calculate the hunting speed, and earlier sections have demonstrated the ability of the program to accept changes for parametric studies or specific design applications. The program has already proven to be a useful tool for improving trucks presently under design. The program is presently being modified to incorporate additional functions which will calculate the equilization rate, the curving coefficient, and other critical design indices from the data shown in Table 1.

Computer Program Listing

```
      DIMENSION A(29),A1(3),B1(3),Y(3),V(3)
      REAL KT,MC,MT,MMT,KV1,KV2
    1 PRINT, " (1) ENTER TOTAL CAR WEIGHT (LBS)"
      READ, A(1)
      PRINT, " (2) ENTER TOTAL TRUCK WEIGHT (LBS/TRK)"
      READ, A(2)
      PRINT, " (3) ENTER TRUCK UNSPRUNG WEIGHT (LBS/TRK)"
      READ, A(3)
      PRINT, " (4) ENTER RADIUS FOR CAR MASS MOMENT (IN)"
      READ, A(4)
      PRINT, " (5) ENTER RADIUS FOR TRUCK MASS MOMENT (IN)"
      READ, A(5)
      PRINT, " (6) ENTER VERTICAL SPRING SPACING (IN)"
      READ, A(6)
      PRINT, " (7) ENTER VERT. DIST. CAR CG TO LAT. SPRING
     & (IN)"
      READ, A(7)
      PRINT, " (8) ENTER WHEEL DIAMETER (IN)"
      READ, A(8)
      PRINT, " (9) ENTER TRACK GAUGE (IN)"
      READ, A(9)
      PRINT, " (10) ENTER WHEEL BASE (IN)"
      READ, A(10)
      PRINT, " (11) ENTER CONING RATIO (IN/IN)"
      READ, A(11)
      PRINT, " (12) ENTER NUMBER OF LEVEL VALVES PER CAR
     & (#/CAR)"
      READ, A(12)
      PRINT, " (13) ENTER VERTICAL SPRING RATE (LB/IN/SPG)"
      READ, A(13)
      PRINT, " (14) ENTER ROLL VERTICAL SPRING RATE
     & (LB/IN/SPG)"
      READ, A(14)
      PRINT, " (15) ENTER LATERAL SPRING RATE (LB/IN/TRK)"
      READ, A(15)
      PRINT, " (16) ENTER YAW SPRING RATE (IN-LB/RAD/TRK)"
      READ, A(16)
      PRINT, " (17) ENTER LAT. DAMPER CONSTANT (LB/SEC/IN/
     & TRK)"
      READ, A(17)
      PRINT, " (18) ENTER YAW DAMPER CONSTANT (IN-LB-SEC/
     & RAD/TRK)"
      READ, A(18)
    5 PRINT, "CORRECTIONS? ENTER - ITEM NO., NEW VALUE"
      READ, I, VAL
      IF (I.EQ.0) GO TO 10
      A(I)=VAL
      GO TO 5
```

```
 10 CONTINUE
    A(6)=A(6)/2.
    A(20)=A(8)/2.
    A(9)=A(9)/2.
    A(10)=A(10)/2.
    PI=3.1415926
C-----VERTICAL NATURAL FREQUENCY-----
    MC=A(1)/386.
    FV=SQRT(A(13)/MC)/PI
C-----ROLL NATURAL FREQUENCY-----
    IF (A(12)-3) 15,20,25
 15 KV1=A(14)
    KV2=A(14)
    GO TO 30
 20 KV1=A(14)
    KV2=A(13)
    GO TO 30
 25 KV1=A(13)
    KV2=A(13)
 30 CONTINUE
    R=((KV1+KV2)*A(6)**2+A(15)*A(7)**2)/(A(15)*A(7))
    RA=R-A(7)
    FR=SQRT((2*(KV1-KV2)*A(6)**2+2*A(15)*RA**2-A(1)*R)/
&   MC*(R**2+A(4)**2)))/(2.*PI)
C-----HUNTING SPEED-----
    WA=(A(1)+2.*(A(2)-A(3)))/4.
    MT=A(2)/386.
    MMT=MT*A(5)**2
    F=3500.*SQRT(A(8)*WA)
    T3=MT*A(16)-MMT*A(15)
    CNT=0
    V(1)=1200.
 35 V(2)=V(1)+1
    V(3)=V(1)-1
    DO 40 J=1,3
    A1(J)=4.*F+V(J)*A(17)
    B1(J)=4.*F(A(9)**2+A(10)**2)+V(J)*A(18)
    T1=A1(J)*MMT+B1(J)*MT
    T2=A1(J)*A(16)+B1(J)*A(15)
    Y(J)=A1(J)*B1(J)*T1*T2/(T1**2*A(9)*A(11)/A(20)*
&   (4*F)**2-A1(J)*B1(J)*T3**2)-V(J)**2
 40 CONTINUE
    Y1=(Y(2)-Y(3))/2
    V1=V(1)-Y(1)/Y1
    IF (ABS(V1-V(1)).LE..01) GO TO 45
    CNT=CNT+1
    IF (CNT.LT.50) GO TO 35
    PRINT 100
 45 VC=V1*60./(88.*12.)
```

```
      A(6)=2.*A(6)
      A(9)=2.*A(9)
      A(10)=2.*A(10)
      PRINT 126
      PRINT, "                         INPUT"
      PRINT 101, A(1)
      PRINT 102, A(2)
      PRINT 103, A(3)
      PRINT 104, A(4)
      PRINT 105, A(5)
      PRINT 106, A(6)
      PRINT 107, A(7)
      PRINT 108, A(8)
      PRINT 109, A(9)
      PRINT 110, A(10)
      PRINT 111, A(11)
      PRINT 112, A(12)
      PRINT 113, A(13)
      PRINT 114, A(14)
      PRINT 115, A(15)
      PRINT 116, A(16)
      PRINT 117, A(17)
      PRINT 118, A(18)
      PRINT, "                         OUTPUT"
      PRINT 120, FV
      PRINT 121, FR
      PRINT 122, R
      PRINT 123, VC
      PRINT 126
  100 FORMAT("0","THE HUNTING METHOD HAS NOT CONVERGED IN 50
  &   ITERATIONS, RE-CHECK THE INPUT DATA.")
  101 FORMAT("0","(1) TOTAL CAR WEIGHT (LBS)",24X,F11.3)
  102 FORMAT("0","(2) TOTAL TRUCK WEIGHT (LBS/TRK)",18X,
  &   F11.3)
  103 FORMAT("0","(3) TRUCK UNSPRUNG WEIGHT (LBS/TRK)",15X,
  &   F11.3)
  104 FORMAT("0","(4) RADIUS - CAR MASS MOMENT (IN)",17X,
  &   F11.3)
  105 FORMAT("0","(5) RADIUS - TRUCK MASS MOMENT (IN)",
  &   15X,F11.3)
  106 FORMAT("0","(6) VERTICAL SPRING SPACING (IN)",18X,
  &   F11.3)
  107 FORMAT("0","(7) VERTICAL DIST - CAR CG TO LAT SPG (IN)
  &   ",9X,F11.3)
  108 FORMAT("0","(8) WHEEL DIAMETER (IN)", 27X,F11.3)
  109 FORMAT("0","(9) TRACK GAUGE (IN)", 30X,F11.3)
  110 FORMAT("0","(10) WHEEL BASE (IN)", 30X,F11.3)
  111 FORMAT("0","(11) CONING RATIO (IN/IN)",25X,F11.3)
```

```
112 FORMAT("0","(12) # LEVEL VALVES PER CAR (#/CAR)",15X,
&   F11.3)
113 FORMAT("0","(13) VERTICAL SPRING RATE (LB/IN/CAR)",
&   13X,F11.3)
114 FORMAT("0","(14) ROLL VERT. SPRING RATE (LB/IN/SPG)",
&   11X,F11.3)
115 FORMAT("0","(15) LATERAL SPRING RATE (LB/IN/TRK)",
&   14X,F11.3)
116 FORMAT("0","(16) YAW SPRING RATE (IN-LB/RAD/TRK)",
&   14X,F11.3)
117 FORMAT("0","(17) LAT DAMPER CONSTANT (LB-SEC/IN/TRK)"
&   ,10X,F11.3)
118 FORMAT("0","(18) YAW DAMPER CONSTANT (LN-IN-SEC/RAD/
&   TRK)",6X,F11.3)
120 FORMAT("0","(1) VERTICAL NATURAL FREQUENCY (HZ)",15X
&   F11.3)
121 FORMAT("0","(2) ROLL NATURAL FREQUENCY (HZ)",19X,
&   F11.3)
122 FORMAT("0","(3) DIST. TO ROLL CENTER (IN)",21X,F11.3)
123 FORMAT("0","(4) HUNTING SPEED (MPH)",27X,F11.3)
126 FORMAT("1")
    PRINT, "ENTER 1 TO END PROGRAM"
    PRINT, "ENTER 2 TO MODIFY DATA"
    PRINT, "ENTER 3 TO INPUT NEW DATA"
    READ,I
    IF(I-2)100,5,1
100 END
```

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