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Reduced Third Order Intermodulation Distortion Utilizing a Push-Pull Class C VHF Transistorized Amplifier

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REDUCED THIRD ORDER INTERMODULATION DISTORTION UTILIZING A PUSH-PULL CLASS C VHF TRANSISTORIZED AMPLIFIER

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ABSTRACT

Reduced Third Order Intermodulation Distortion
Utilizing a Push-Pull Class C VHF Transistorized Amplifier

In a transistorized push-pull amplifier, third order intermodulation distortion was effectively reduced at VHF frequencies with an output power of two watts. The non-linear distortion of the amplifier is modeled using a power series. The resulting expression is used as the basis for choosing the push-pull configuration to reduce the third order intermodulation distortion. The amplifier was built and tested, and the experimental results compare favorably with the theoretical results. The level of the third order intermodulation distortion is found to be at least 30 dB below the interfering signal level.

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LIST OF SYMBOLS

AUT	Amplifier Under Test
A cos a	Wanted Signal
B cos b	Unwanted Signal
IM3	Third Order Intermodulation Distortion
IM5	Fifth Order Intermodulation Distortion
VHF	Very High Frequency (30-300 MHZ)

INTRODUCTION

When it is necessary to place two or more VHF transmitters in close proximity there will be generated certain parasitic frequencies. Transistor amplifiers seem to be more prone to the generation of these parasitic frequencies than their vacuum tube predecessors. Broadband amplifiers are more widely used now and by their very nature do not reject many of these parasitic frequencies. Figure 1 shows how an unwanted signal may become injected into a nearby transmitter operating on a slightly different frequency.

If the two signals represented in Figure 1 are within the passband of the amplifier, the parasitic frequencies generated may also be in the passband. A multi-channel receiver designed to receive the range of frequencies including f_a and f_b will also receive some of the parasitic frequencies. The parasitic frequency does not convey information at a desired frequency so it is considered noise and should be eliminated. This paper will investigate a particular technique for the reduction of a specific class of parasitic frequencies, namely, those due to intermodulation distortion.

Distortion in amplifiers has been represented in a variety of ways, i.e., power series [1], Taylor series [2,3], and Volterra series [4,5]. Of these three, the power series approach is by far the most tractable. By the power series method, the output of an amplifier is represented as a power series of the input signal modulated by an unwanted signal. The relationships between the amplitude and frequency of the distortion components are derived.

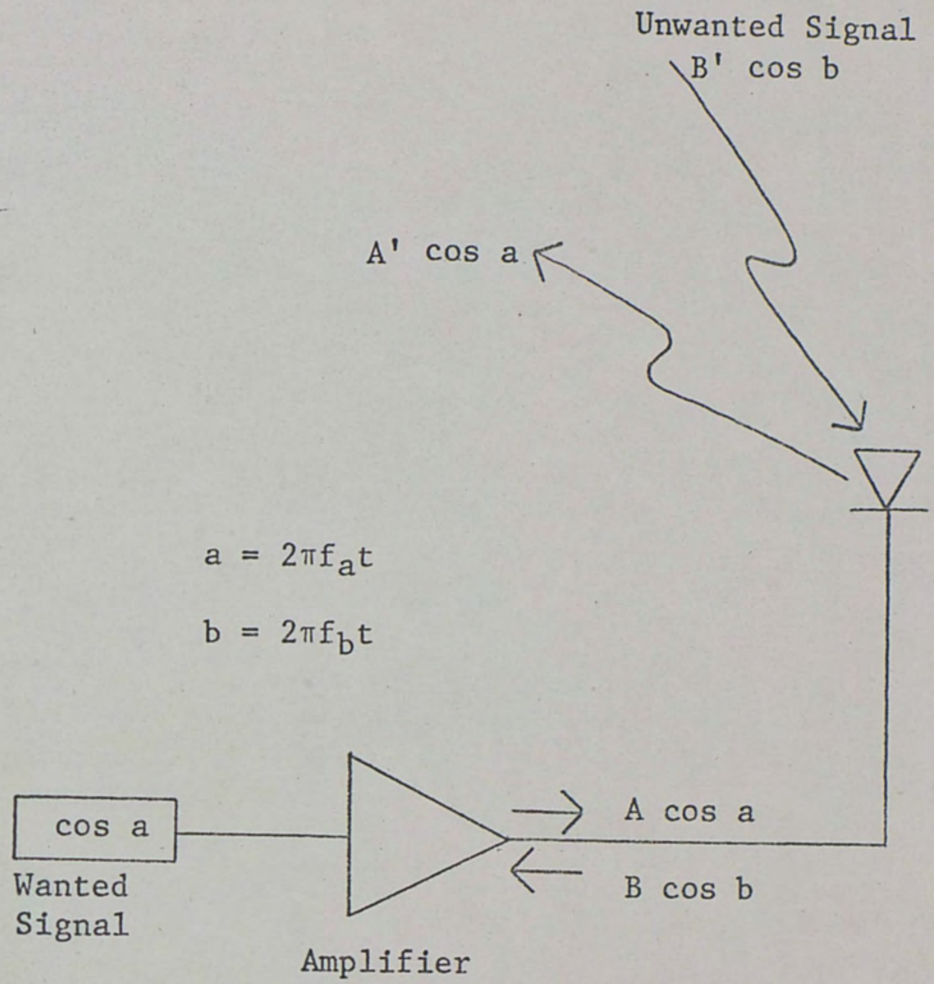


Figure 1. Typical Injection of an Unwanted Signal cos b, into a Nearby Transmitter of the cos a Signal.

This paper will investigate the possibility of reducing the third-order intermodulation distortion (IM3) using a VHF push-pull class C transistor amplifier. The output power of the amplifier will be approximately two watts. The time domain signals of Figure 1, cos a and cos b, correspond to frequencies of 105.6 MHZ and 106.5 MHZ respectively. Futhermore, $a = 2\pi f_a t$ and $b = 2\pi f_b t$. The specific choice of these frequencies was arbitrary but signals in the hundred megahertz range were desired.

CHAPTER I

Power Series Model of a Non-Linear Amplifier

In order to predict the magnitude of the third-order intermodulation distortion (IM3) generated in a given amplifier, a suitable model must be chosen. One of the simpler methods of representing non-linear amplifier characteristics is with a power series. The amplifier output is represented as a power series of the input signal, cos a, modulated by an unwanted signal, (1 + B cos b):

$$y(t) = (1 + B \cos b) A \sum_{0}^{\infty} k_n \cos^n a$$

where

$y(t)$ = amplifier output

$\cos a$ = amplifier input

A = amplifier gain

k_n = coefficient of the n^{th} term

$(1 + B \cos b)$ = modulation caused by the unwanted signal

For analysis purposes, only the terms corresponding to k_1, k_2 and k_3 will be considered. Therefore,

$$y(t) = A(k_1 \cos a + k_2 \cos^2 a + k_3 \cos^3 a)(1 + B \cos b)$$

Since

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

and

$$\cos^3 x = \tfrac{1}{4}(3 \cos x + \cos 3x)$$

$$y(t) = Ak_1 \cos a + \frac{Ak_2}{2} + \frac{Ak_2}{2} \cos 2a + \frac{3Ak_3}{4} \cos a + \frac{Ak_3}{4} \cos 3a +$$

$$ABk_1 (\cos a \cos b) + \frac{ABk_2}{2} (\cos 2a \cos b) + \frac{3ABk_3}{4} (\cos a \cos b) +$$

$$\frac{ABk_3}{4} (\cos 3a \cos b) + \frac{ABk_2}{2} \cos b \qquad (1)$$

The third order intermodulation distortion components are a result of the $\frac{ABk_2}{2}$ (cos 2a cos b) term in Equation 1.

Since

$$\cos x \cos y = \tfrac{1}{2}(\cos (x+y) + \cos (x-y))$$

this term can be expanded to yield:

$$\frac{ABk_2}{4} (\cos (2a+b) + \cos (2a-b)).$$

From the original premise that the two frequencies, f_a and f_b, are neighboring frequencies; $2f_a+f_b \approx 3f_a$ and $2f_a-f_b \approx f_a$.

The $2f_a-f_b$ term results in a frequency within the passband of the amplifier (and likewise a broadband receiver) and may cause interference at the receiver.

The remaining terms of Equation 1 consist of :

(1) the linear amplification of the cos a signal, $k_1A \cos a$, which is the desired output

(2) a d.c. term, $\frac{Ak_2}{2}$, which is blocked by the coupling capacitor

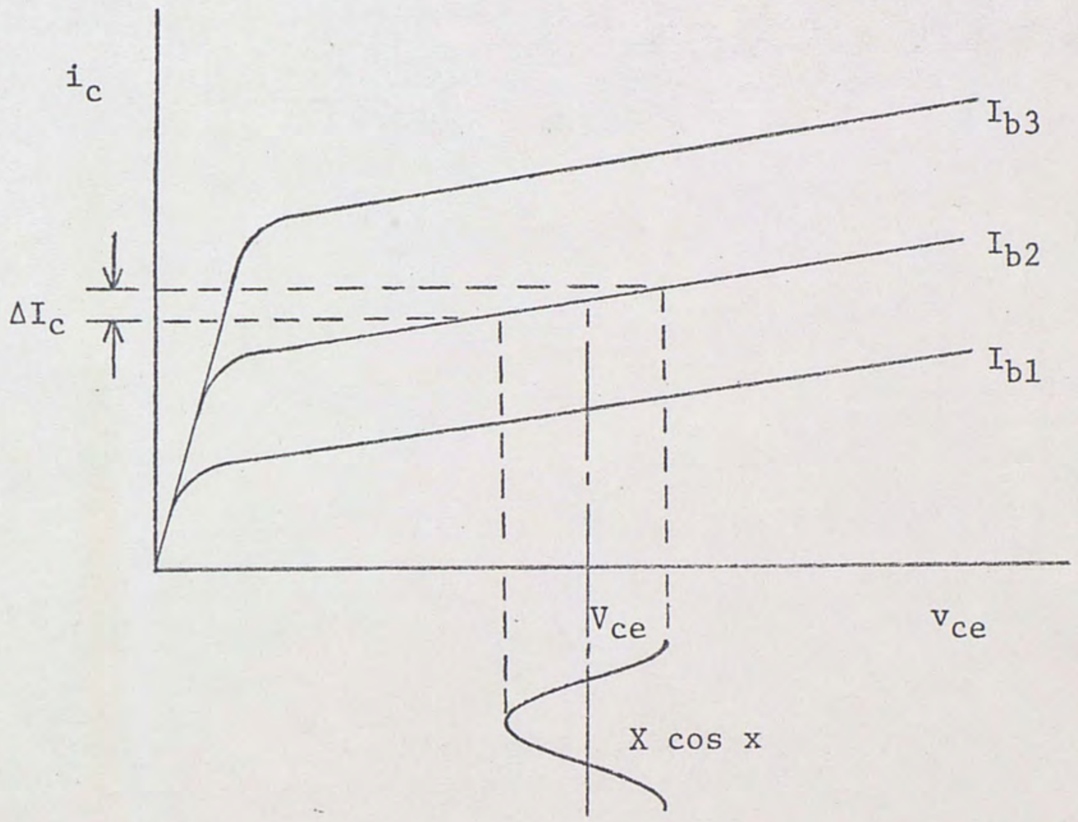
(3) a signal that corresponds to a second harmonic, $\frac{Ak_2}{2} \cos 2a$, which would normally be outside the passband of the amplifier

(4) an original signal component, $\frac{3Ak_3}{4} \cos a$, whose relevance depends on the sign and magnitude of k_3

(5) a signal that corresponds to a third harmonic term, $\frac{Ak_3}{4} \cos 3a$, which would normally be outside the passband of the amplifier

(6) and other intermodulation terms that lie outside the passband of the amplifier.

The modulation of the unwanted signal and the wanted signal is a result of the transistor output characteristics. A typical transistor output characteristic is shown for reference.



As can be seen, the presence of the signal, X cos x, introduced at the collector of a transistor will modulate the output current. Indeed. this is the method used in high level amplitude modulation. The modulation effect can be seen to be greater if the transistor is operated in the saturation region.

CHAPTER II

Parameters of the Power Series Model Effective in Reducing IM3

For a basic amplifier the most straight forward approach to decreasing the effect of the IM3 is to linearize the amplifier. If the amplifier could be linearized so that the first two terms of the power series were sufficient to model the amplifier, the third order components would be non-existent and the IM3 likewise would not present a problem. By operating the transistor below its saturation point, Wollam [6] has shown the marked decrease that occurs in the IM3. This technique is operating the amplifier more linearly and although it is effective in reducing the IM3, it is less efficient than operating in a saturated class C mode.

The use of negative feedback is also an effective method of linearizing the amplifier, but Meyer, Shensa, and Eschenbach [4] have shown that another phenomenon, cross modulation, is reduced significantly but IM3 is not. Feedback also presents difficult problems when applied to VHF broadband transistor amplifiers. The feedback circuit would have to have a constant phase shift over a wide range of frequencies. The difficulty of accomplishing this is compounded by the change in transistor parameters that occurs with changes in frequency and input drive level.

By investigating the nature of the third order intermodulation component, $\frac{ABk_2}{4}\cos(2a-b)$, another approach to reducing IM3 is suggested.

It is suggested that if the second harmonic coefficient is reduced, a likewise reduction in IM3 would occur. It is well known that a push-pull amplifier has reduced even harmonic coefficients and for a perfectly balanced push-pull amplifier, the even harmonic coefficients are zero (see Appendix A). Reducing the second harmonic coefficient would also reduce the second order distortion components (see Equation 1). The effect of reducing certain frequency components can best be seen by defining IM3 quantitatively.

Since A cos a is the wanted signal, defining IM3 in terms of A cos a and the magnitude of the resulting third order distortion frequency would seem justified. However, since IM3 is a result of the unwanted signal B cos b, defining IM3 in terms of B cos b and the magnitude of the resulting third order distortion frequency renders a more accurate picture of the nature of IM3.

From Equation 1 the ratio of the cos 2a cos b and cos b terms is formed, and IM3 is defined:

$$\mathrm{IM3} = \frac{\frac{ABk_2}{2} \cos 2a \cos b}{B \cos b}$$

$$\mathrm{IM3} = \frac{Ak_2}{2} \cos 2a \tag{2}$$

From Equation 2 it can be seen that IM3 is proportional to k_2, the coefficient of the cos 2a term. This term represents the second harmonic of the wanted signal. Reducing the second harmonic of the wanted signal can be accomplished using a push-pull output stage for the amplifier of the wanted signal and is the approach taken in this paper.

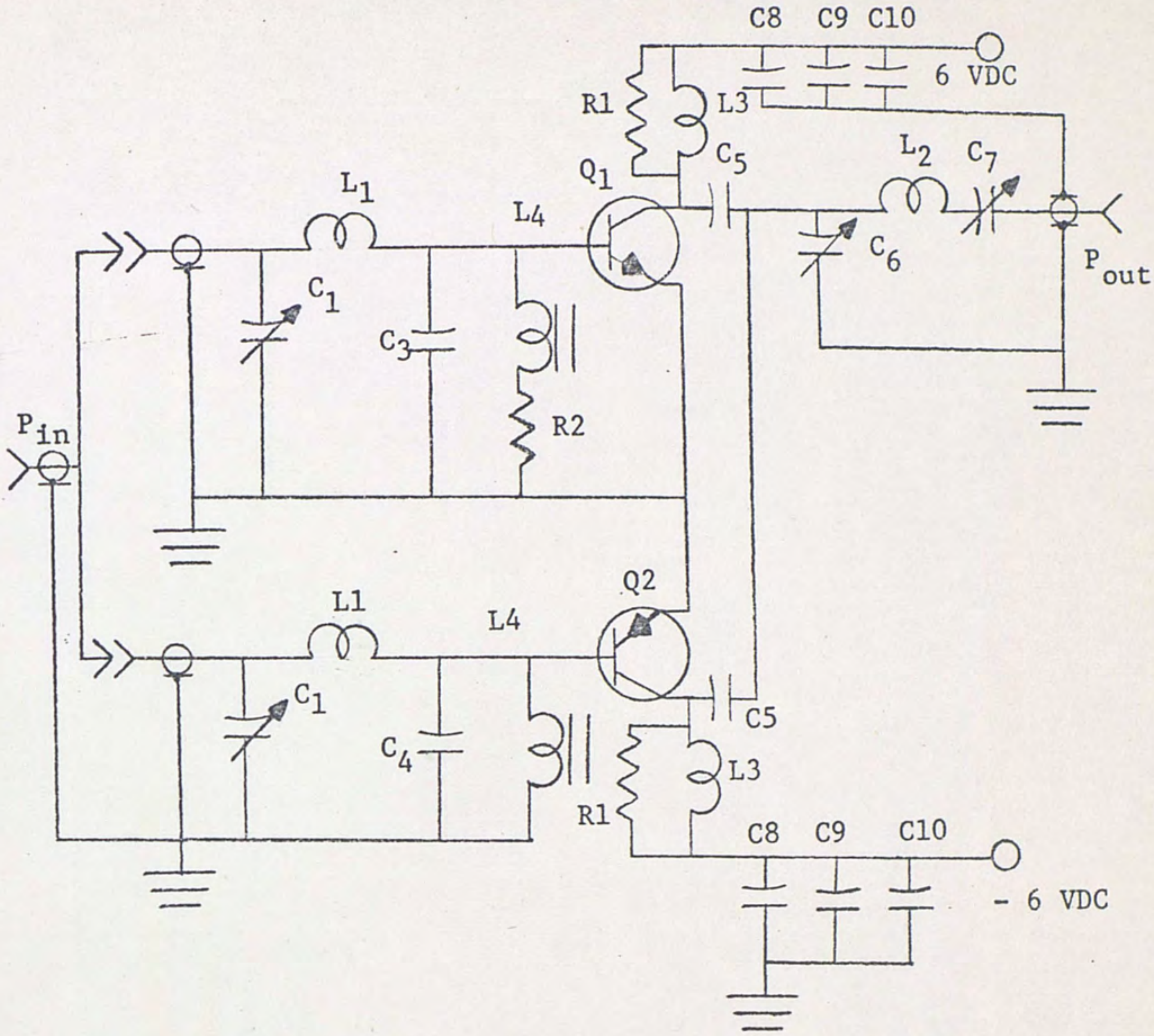
CHAPTER III

Experimental Procedures and Results

The push-pull amplifier as tested is shown in Figure 2. The transistors are Motorola 2N6081 (NPN) and 2N6095 (PNP). No effort was made to match the VHF transistor characteristics except for output power capabilities. Both transistors are rated for 15 watts with a supply voltage of 12.5 VDC. The amplifier was operated well below the output capabilities of the transistors to minimize the heat problem. This was accomplished by reducing the collector supply voltage to 6 VDC for the 2N6081 and to -6 VDC for the 2N6095 and also reducing the input drive power.

The basic configuration of the input circuit for each transistor was taken from the Motorola Semiconductor Handbook. The drive power to the push-pull amplifier was supplied by a four stage transistor amplifier built from a Motorola application note, AN-481.

The testing and measuring configuration is shown in Figure 3. The injection of the unwanted signal, B cos b was accomplished by using a BNC tee connector at the output of the push-pull amplifier, hereafter referred to as the AUT (Amplifier Under Test). The unwanted signal power was obtained from a U.S. Army Radio Transmitter BC-625-A, which is a 1943 surplus tube type transmitter with a high Q output stage. The frequency of the BC-625-A is tunable from 100 MHZ to 150 MHZ, and the output power is variable to approximately 6 watts.



C1: 14-150 pF, ARCO 424
C2: 50-330 pF, ARCO 427
C3: 133 pF
C4: 40.7 pF
C5: 820 pF
C6: 50-330 pF, ARCO 427
C7: 50-330 pF, ARCO 427
C8: 1500 pF
C9: 412 pF
C10: 22 uF, 35V
L1: Copper Strap ¼" wide, 1¼" long, straight
L2: 40 nH, 2 turns #16 AWG ¼" I.D.
L3: 6 turns #16 AWG wrapped on R1
L4: Amidon 43-101 Ferrite bead
Q1: 2N6081
Q2: 2N6095
R1: 100 ohm, 2W
R2: 2.2 ohm, ¼W

Figure 2. VHF Push-Pull Amplifier

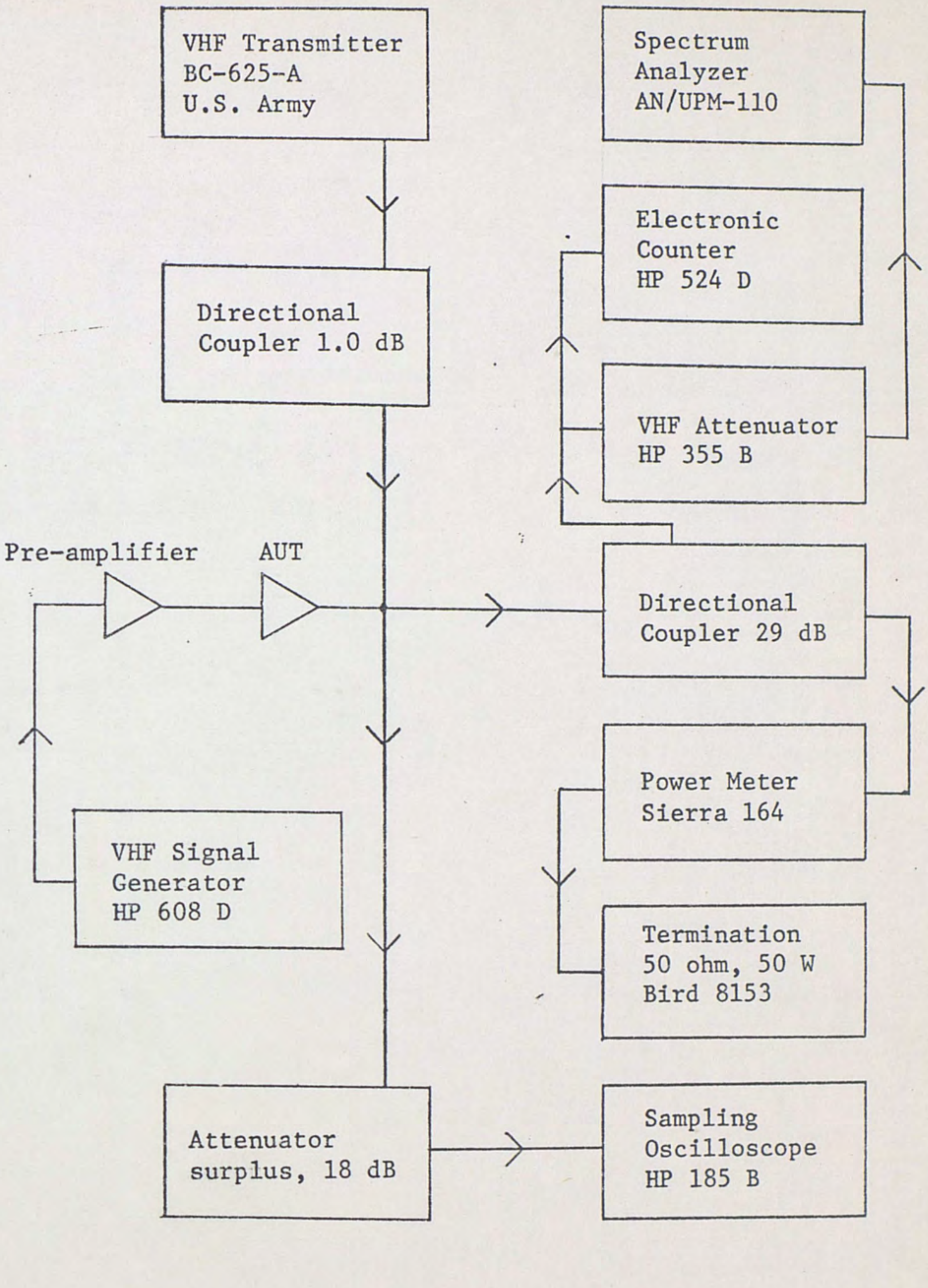


Figure 3. Test Set-up for Measuring Intermodulation Distortion

A two stage transistor amplifier was tried as the source of the unwanted signal but it was found to be too difficult to isolate the output from the AUT. Furthermore, the two stage transistor amplifier for the unwanted signal was found to be far more susceptable to IM3 than was the AUT. This prevented measuring IM3 of the AUT; in fact, it was a good measure of the two stage amplifier IM3.

The directional coupler between the BC-625-A transmitter and the AUT was found to be helpful in reducing the loading effect of the transmitter on the AUT. A Sierra in-line power meter was used for power measurements at the combined output. The power output of the AUT was measured to be 2.0 watts and the output of the BC-625-A transmitter was measured to be 2.5 watts. These power measurements were taken with the AUT and the BC-625-A connected in parallel as they would be during the measurement of the IM3. However, the power measurement of the AUT was taken with the BC-625-A turned off and the power measurement of the BC-625-A was taken with the AUT turned off. Because of the parallel connection of the two power sources the power measurements are relative to 25 ohms rather than the 50 ohms of the single termination. Power measurements made at the input to the AUT with the Sierra power meter were not reliable because of the unknown impedance involved. Separately, the AUT and the pre-amplifier were designed to be matched to 50 ohms. However, when the pre-amplifier was connected to the AUT, both amplifiers were tuned to give a maximum power output from the AUT.

With the cos a signal tuned to 105.6 MHZ and the cos b signal tuned to 106.5 MHZ, IM3 measurements were made. The power spectrum of the two signals with the resulting IM3 is shown in Figure 4. The full scale signal to the right of the centerline is the representation of A cos a. The full scale signal to the left of the centerline represents B cos b. The small signal to the right of A cos a (at +2.4 on the horizontal graticule) represents the IM3 frequency component and is at a frequency of 2a - b i.e., (2(105.6) - 106.5) MHZ or 104.7 MHZ. The small signal to the right of the IM3 frequency is the result of fifth-order intermodulation i.e., 3a - 2b or 103.8 MHZ. It should be noted that the fifth-order intermodulation distortion (IM5) is larger than the IM3. This result should have been expected because the IM5 results from a third harmonic of A cos a which is still present in the output of a push-pull amplifier.

By comparing Figure 4 to Figure 5 it can be seen that the relative amplitude of IM3 and IM5 differ by more than Figure 4 indicates. Figure 5 shows the bandpass characteristics of the output tank circuit and indicates the IM5 signal is being attenuated by approximately 3 dB more than the IM3 signal. Hence, for a flat output tank circuit response, IM5 would be 3 dB larger than shown in Figure 4.

Figures 6, 7, 8, 9, and 10 represent the signals of Figure 4 being attenuated in 6 dB steps using the I.F. attenuator on the spectrum analyzer. This was done to give a calibration reference for Figure 4 and Figure 5. The decibel calibration on the vertical graticule of the spectrum analyzer does not correspond to 6 dB increments.

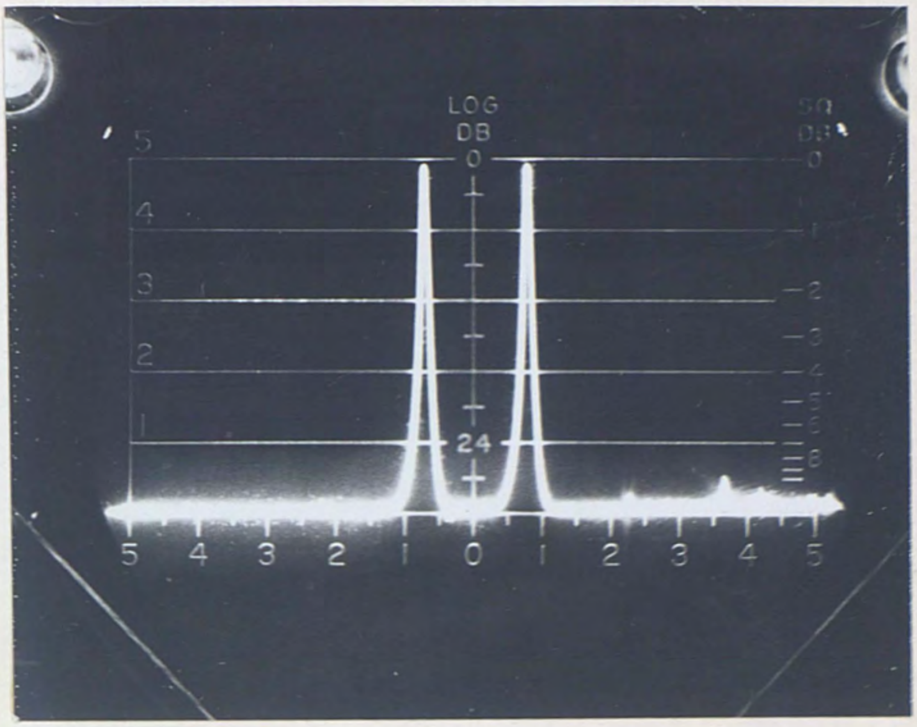


Figure 4. Output Power Spectrum Showing the cos a 105.6 MHZ (Right of Center) and the cos b 106.5 MHZ (Left of Center) Signals and the IM3 cos(2a-b) 104.7 MHZ (at 2.4 on the Horizontal Graticule) and the IM5 cos(3a-2b) 103.8 MHZ (at 3.6 on the Horizontal Graticule. This is from the Push-Pull Amplifier Output.

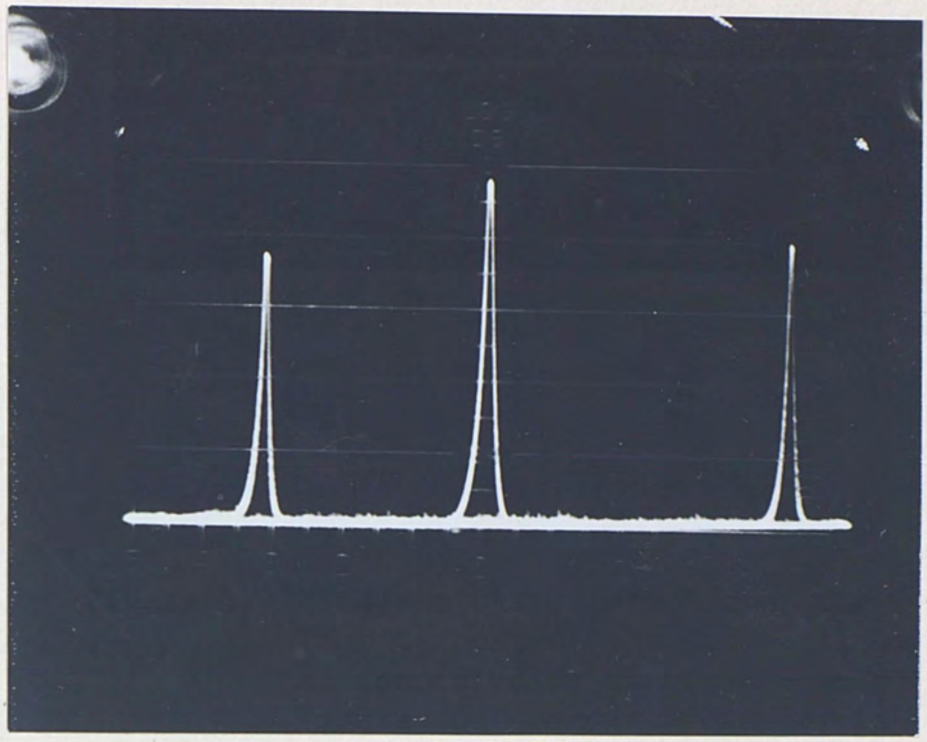


Figure 5. Bandpass Characteristic of the Output Tank Circuit of the Push-Pull Amplifier as Connected in the Test Set-Up of Figure 3. Center Frequency is 105.97 MHZ. The Frequency to the Right of Center is 103.62 MHZ and the Frequency to the Left of Center is 107.43 MHZ.

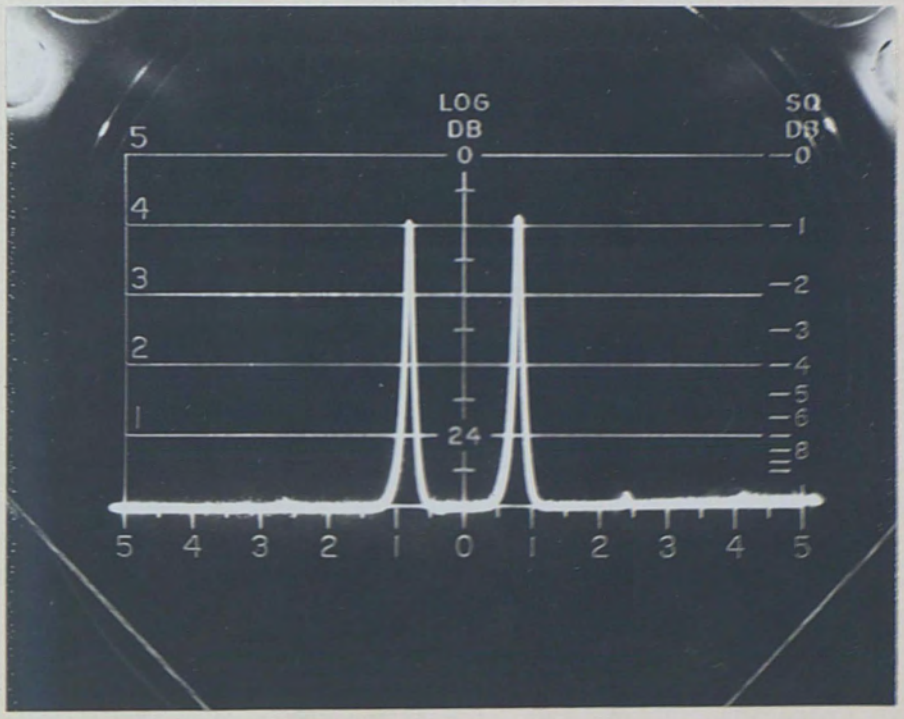


Figure 6. The cos a and cos b Signals Attenuated 6 dB from the Level of Figure 4.

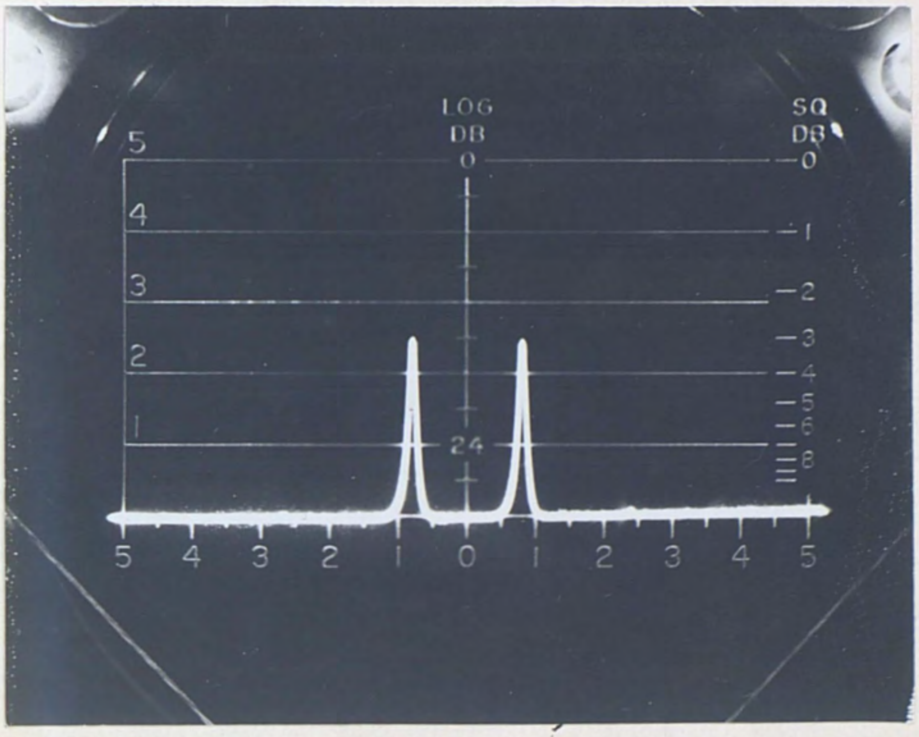


Figure 7. The cos a and cos b Signals Attenuated 12 dB from the Level of Figure 4.

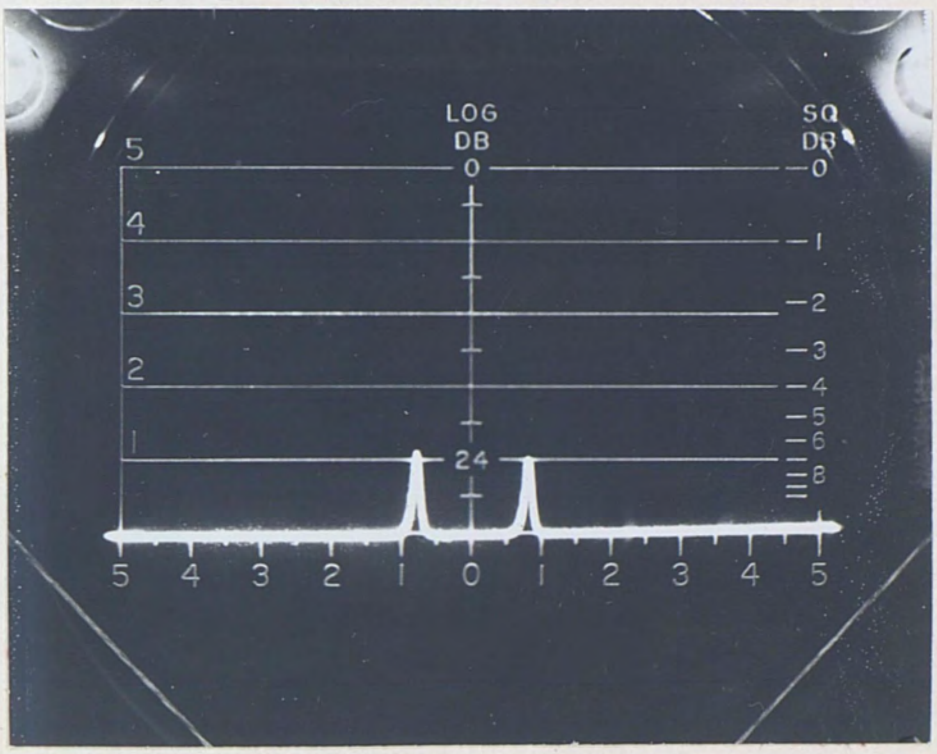


Figure 8. The cos a and cos b Signals Attenuated 18 dB from the Level of Figure 4.

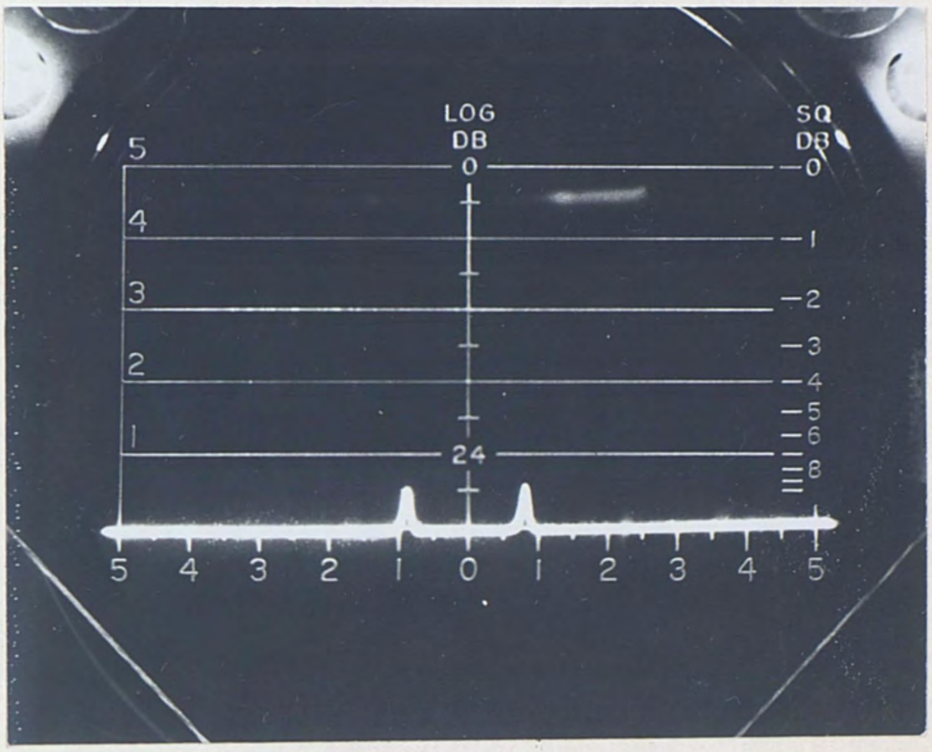


Figure 9. The cos a and cos b Signals Attenuated 24 dB from the Level of Figure 4.

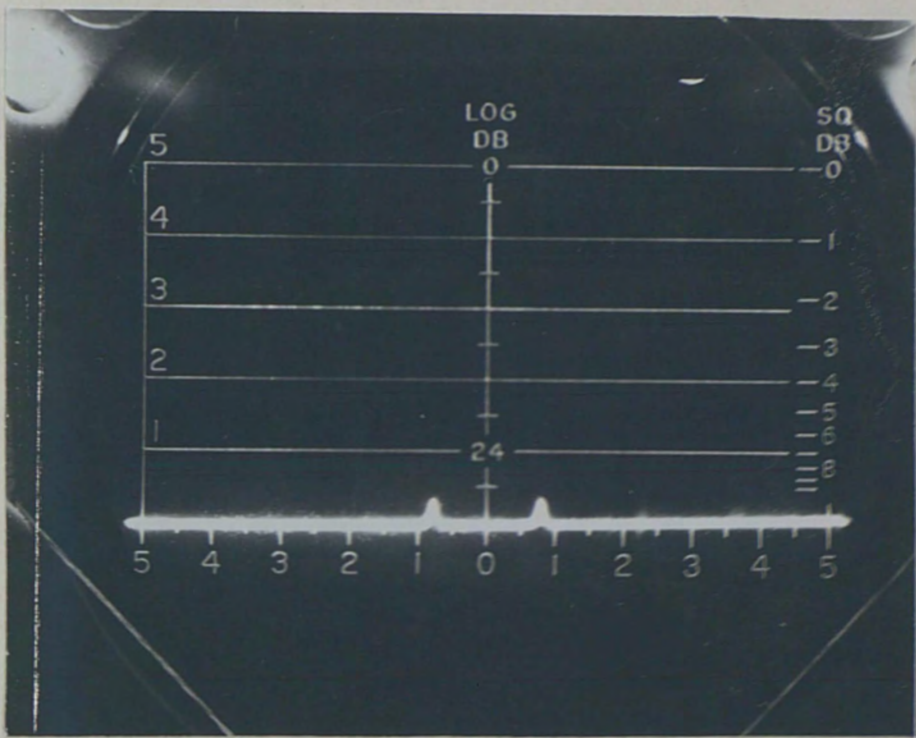


Figure 10. The cos a and cos b Signals Attenuated 30 dB from the Level of Figure 4.

Figure 11 shows the output time waveform of the cos a signal. The picture was taken from the sampling oscilloscope. The graticule is not visible because there is no scale illumination on this oscilloscope; however, the significance of this picture is not its amplitude but rather its shape. The signal shape is close to a sinusoid as is desired.

The results gained from the push-pull amplifier tests agree quite well with the prediction of the theory of Chapters I and II. In order to compare these test results with the IM3 generated in a single-ended VHF transistor amplifier, the pre-amplifier and the BC-625-A were tested, bypassing the push-pull amplifier. The output power level of the BC-625-A was adjusted so the cos a and cos b signals would be of equal amplitude as displayed on the spectrum analyzer. The resulting power spectrum is shown in Figure 12. The IM3 signal component was measured to be 18 dB down from the cos a and cos b signals. This compares to approximately 30 dB or more for the push-pull output stage. Furthermore, the pre-amplifier was designed as a VHF amplitude modulated transmitter using high level modulation. Consequently, the pre-amplifier was likely not being driven into saturation because there was no modulation employed. The level of IM3 obtained in this comparison agrees quite well with work done by Wollam [6] on amplifiers not driven into saturation. Because of the nature of the VHF amplifiers and the limitation of standard laboratory oscilloscopes, it was not possible to observe directly the collector signals.

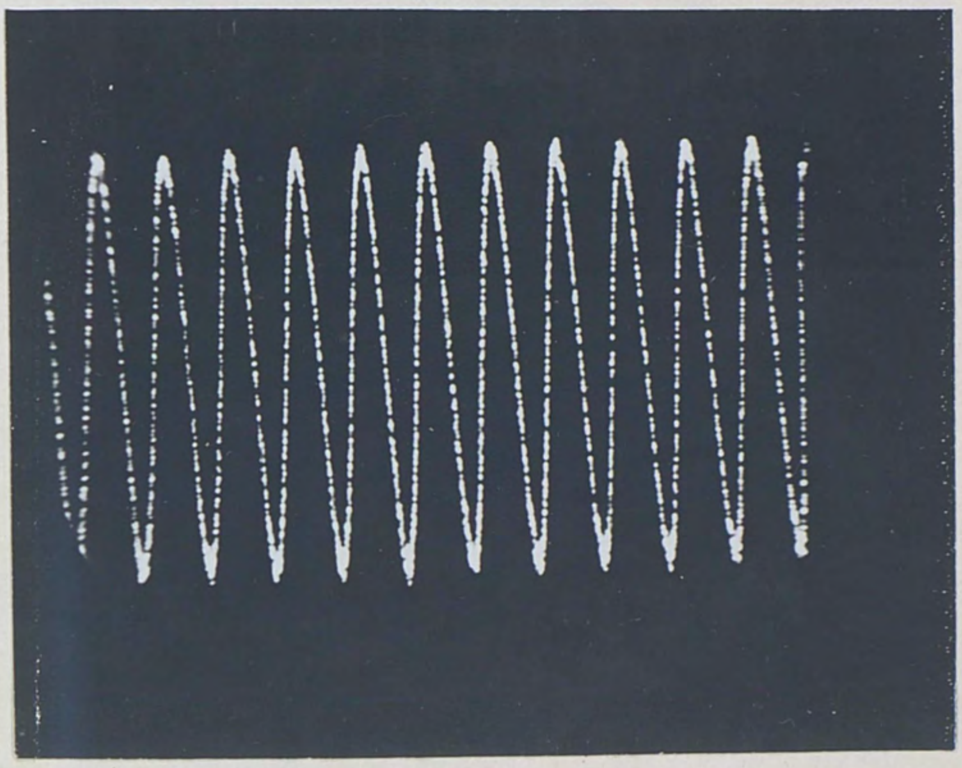


Figure 11. Time Domain Output Signal Showing its Approximate Sinusoidal Shape.

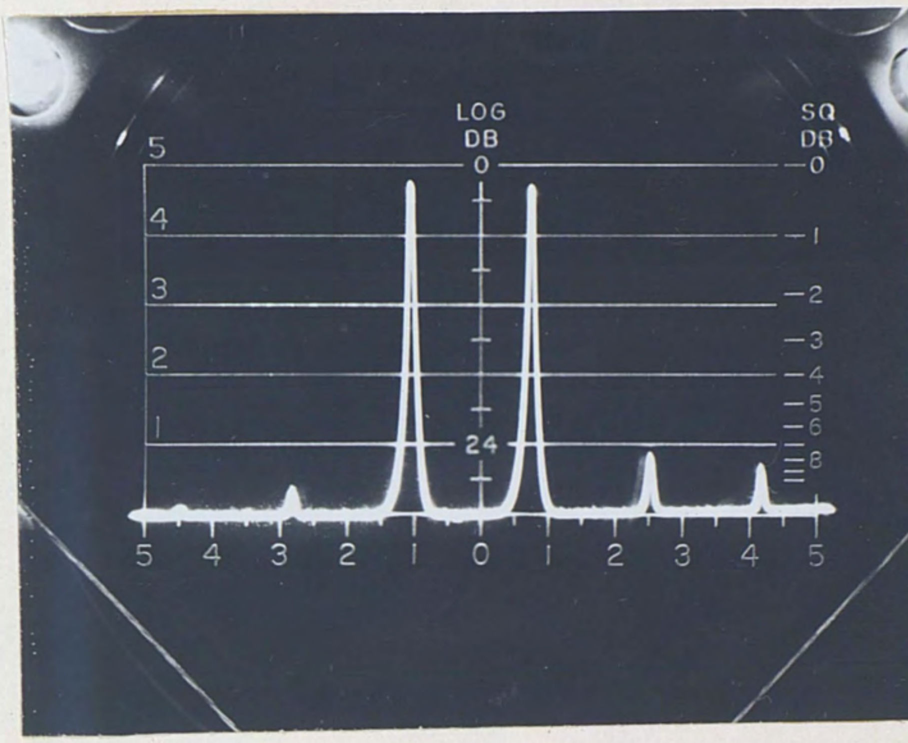


Figure 12. Output Power Spectrum Showing Increased Levels of IM3 and IM5 Present in the Single-Ended Amplifier Output.

As was stated in Chapter II, the reduction in IM3 by operating a single-ended transistor amplifier below saturation is a technique for linearizing the amplifier at the expense of efficiency. The reduction of IM3 in a push-pull output stage is due primarily to the cancellation of the even harmonics. Operating the push-pull transistors into saturation would have little or no effect on the level of IM3 and could be used to increase efficiency.

In order to determine whether or not the AUT was being operated in a saturated mode the output power measurement was used in conjunction with a ratio of the output voltage and approximately the collector voltage. Referring to Figure 13 (a simplified circuit diagram showing the output circuit), P_{out} was measured and the ratio of V_a and V_b used to calculate V_a. The output power was measured to be 1.6 watts into a 50 ohm termination, therefore:

$$V_b = (P_{out}R)^{\frac{1}{2}}$$

$$V_b = [1.6(50)]^{\frac{1}{2}}$$

$$V_b = 8.95 \text{ VRMS}$$

$$V_b = 12.6 \text{ volts peak}$$

The voltage V_b measured 1.6 volts peak (0.8 cm. X 2 volts per cm.), the voltage V_a measured 1.4 volts peak (0.7 cm. X 2 volts per cm.) on a Fairchild oscilloscope that has a 50 MHZ bandwidth.

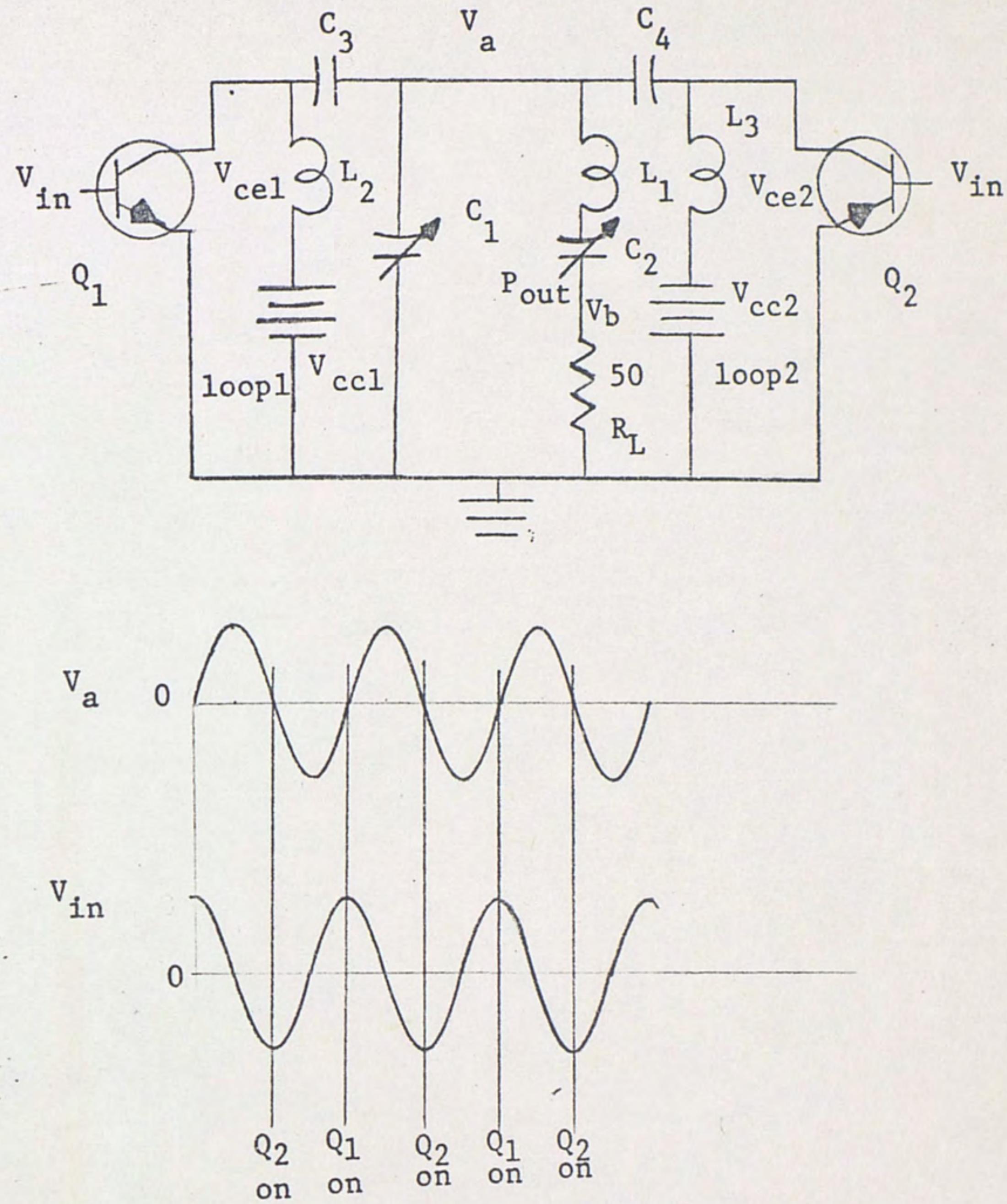


Figure 13. Simplified Output Circuit of the Push-Pull Amplifier Tested Showing Timing of Input and Collector Voltage Signals.

To find the voltage V_a:

$$\frac{V_a}{V_b} = \frac{1.4}{1.6}$$

$$V_a = \frac{1.4}{1.6} V_b$$

from the more accurate power measurement:

$$V_b = 12.6 \text{ volts peak}$$

$$V_a = \frac{1.4}{1.6} (12.6 \text{ volts peak})$$

$$V_a = 11.0 \text{ volts peak}$$

Knowing the value of V_a, the value of V_a under the saturated condition will be calculated.

When Q_1 turns on

$$\sum_{\text{loop 1}} V = 0; \; 6V = V_{L_2} + V_{celsat}$$

for Q_1,

$$V_{celsat} \simeq 0.4 \text{ volts}$$

therefore:

$$V_{L_2} = 6V - 0.4V = 5.6 \text{ volts}$$

When Q_1 turns off, the current that was flowing in L_2 continues to flow in the same direction and charges C_1 to a value of

$$V_{L_2} + V_{cc1} = 5.6V + 6.0V = 11.6V,$$

Capacitor C_1 now discharges thru L_1, C_2, and R_L. At this point, the energy in the tank circuit is stored in the magnetic field of L_1 and $V_a \approx 0$. Now Q_2 turns on and L_3 is "charged" and

$$\Sigma V_{\text{loop 2}} = 0; \; 6V = V_{L_3} + V_{ce2sat}$$

for Q_2,

$$V_{ce2sat} \approx 0.4 \text{ volts}$$

therefore

$$V_{L_2} = 6V - 0.4V = 5.6V$$

When Q_2 turns off, the current in L_2 continues to flow and with V_{cc2} charges C_1 to $V_{cc2} + V_{L_3} = 6V + 5.6V = 11.6V$. The charge on C_1 is in the direction to make V_a minus with respect to ground. Hence, $V_a = -11.6V$.

The preceding analysis is based on Q_1 and Q_2 being saturated at the time each transistor is turned on. The resulting voltage swing, V_a, is $\pm$ 11.6 volts. From the power measurement and comparing the ratios of V_a and V_b it was shown that V_a has a peak value of 11.0 volts. The conclusion is drawn, therefore, that Q_1 and Q_2 are indeed being operated in a saturated condition.

The resulting voltage drop on capacitors C_3 and C_4 was neglected in the analysis because of their large capacitance relative to C_1. Looking at the charge path of C_1 thru C_3 it is seen that the same current flows in each capacitor and since

$$V_{C_1} = \frac{1}{C_1} \int_{-\infty}^{+\infty} i(t)\, dt$$

and

$$V_{C_3} = \frac{1}{C_3}\int_{-\infty}^{+\infty} i(t)\, dt$$

forming the ratio:

$$\frac{V_{C_1}}{V_{C_3}} = \frac{\frac{1}{C_1}\int_{-\infty}^{+\infty} i(t)\, dt}{\frac{1}{C_3}\int_{-\infty}^{+\infty} i(t)\, dt} = \frac{C_3}{C_1}$$

for $C_3 > C_1$, $V_{C_1} > V_{C_3}$

Therefore, the voltage V_{C_3} (and similarly V_{C_4}) was neglected.

CHAPTER IV

Conclusion

The reduction of IM3 at VHF frequencies using a transistorized push-pull output stage has been shown. A comparison made with a single-ended transistor amplifier indicates the push-pull arrangement is at least 12 dB better in IM3 suppression. With the wanted and unwanted signals at the same power level at the output of the AUT, the level of the IM3 was found to be suppressed by at least 30 dB using a push-pull arrangement.

The relationship developed showing the dependence of IM3 on the second harmonic of the wanted signal suggests a possible direction for applying new feedback techniques in the reduction of IM3. Such techniques may be applicable at the receive end of a VHF communications link as well as in the single-ended transmitter stages. However, the results obtained from the push-pull amplifier tests strongly suggest the use of such techniques as a simple, efficient method of reducing the third order intermodulation distortion.

APPENDIX A

Fourier Series Representation of the Output Current Pulse Train in a Push-Pull Amplifier

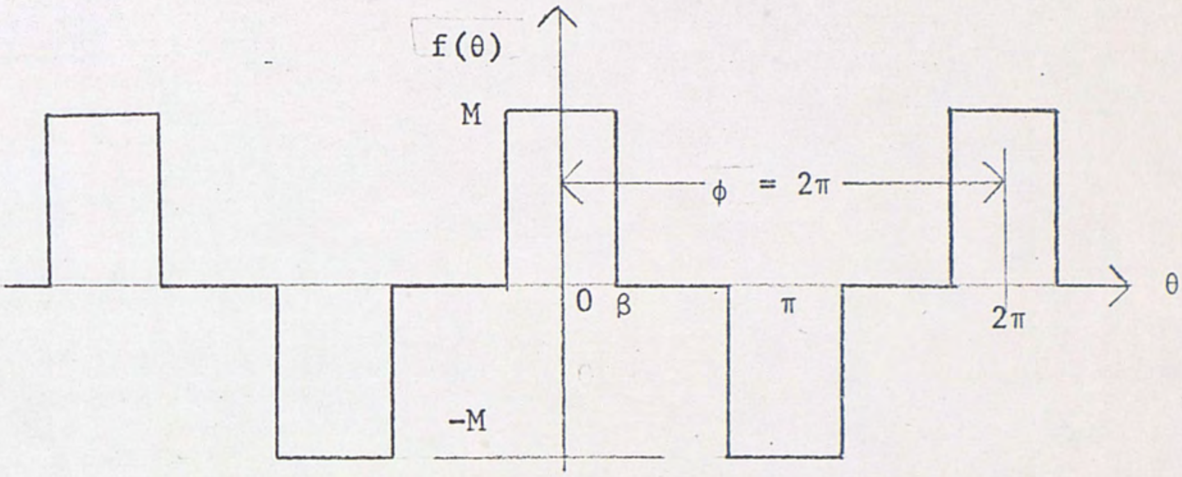


Figure 14. Graphical Representation of a Theoretical Output Current Pulse Train in a Push-Pull Amplifier.

In general, the Fourier series is defined as [1]:

$$f(\theta) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\theta + b_n \sin n\theta)$$

where

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(\theta)\, d\theta$$

$$a_n = \frac{2}{2\pi} \int_0^{2\pi} f(\theta) \cos n\theta\, d\theta$$

$$b_n = \frac{2}{2\pi} \int_0^{2\pi} f(\theta) \sin n\theta\, d\theta$$

Evaluating the a_0, a_n and b_n terms:

$$a_0 = \frac{1}{2\pi}\int_0^{\beta} M d\theta + \frac{1}{2\pi}\int_{\pi-\beta}^{\pi+\beta} (-M) d\theta + \frac{1}{2\pi}\int_{2\pi-\beta}^{2\pi} M d\theta$$

$$a_0 = \frac{M}{2\pi}[\beta - 0 - \pi - \beta + \pi - \beta + 2\pi - 2\pi + \beta]$$

$$a_0 = 0$$

This result was expected because the average value of $f(\theta)$ can be seen to be zero.

$$a_n = \frac{2}{2\pi}\int_0^{\beta} M \cos n\theta \, d\theta + \frac{2}{2\pi}\int_{\pi-\beta}^{\pi+\beta} (-M) \cos n\theta \, d\theta + \int_{2\pi-\beta}^{2\pi} M \cos n\theta d\theta$$

$$a_n = \frac{2M}{n2\pi}\left[\sin n\theta \Big|_0^{\beta} - \sin n\theta \Big|_{\pi-\beta}^{\pi+\beta} + \sin n\theta \Big|_{2\pi-\beta}^{2\pi}\right]$$

$$a_n = 2M [\sin n\beta - \sin 0 - \sin n(\pi + \beta) + \sin n(\pi - \beta) + \sin n2\pi - \sin n(2\pi - \beta)]$$

since

$$\sin 0 = 0, \sin n2\pi = 0$$

and

$$\sin (x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$a_n = \frac{2M}{n2\pi} [\sin n\beta - \sin n\pi \cos n\beta - \cos n\pi \sin n\beta + \sin n\pi \cos n\beta - \cos n\pi \sin n\beta - \sin n2\pi \cos n\beta + \cos n\, 2\pi \sin n\beta]$$

$$a_n = \frac{2M}{n2\pi} [\sin n\beta - \cos n\pi \sin n\beta - \cos n\pi \sin n\beta + \sin n\beta]$$

$$a_n = \frac{4M}{n2\pi} [\sin n\beta - \cos n\pi \sin n\beta]$$

since

$$\cos n\pi = (-1)^n$$

$$a_n = \frac{4M}{n2\pi} [\sin n\beta - (-1)^n \sin n\beta]$$

$$a_n = 0 \qquad \text{for n even}$$

$$a_n = \frac{8M}{n2\pi} \sin n\beta \qquad \text{for n odd}$$

$$b_n = \frac{2}{2\pi}\int_0^{\beta} M \sin n\theta \, d\theta + \frac{2}{2\pi}\int_{\pi-\beta}^{\pi+\beta} (-M) \sin n\theta \, d\theta + \int_{2\pi-\beta}^{2\pi} M \sin n\theta \, d\theta$$

$$b_n = \frac{2M}{n2\pi} \left[-\cos n\theta \Big|_0^{\beta} + \cos n\theta \Big|_{\pi-\beta}^{\pi+\beta} - \cos n\theta \Big|_{2\pi-\beta}^{2\pi} \right]$$

$$b_n = \frac{2M}{n2\pi} [-\cos n\beta + \cos 0 + \cos n(\pi + \beta) - \cos n(\pi - \beta) - \cos n2\pi +$$

$$\cos n(2\pi - \beta)]$$

since

$$\cos 0 = 1, \ \cos n2\pi = 1$$

and

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$b_n = \frac{2M}{n2\pi}[-\cos n\beta + \cos n\pi \cos n\beta - \sin n\pi \sin n\beta - \cos n\pi \cos n\beta -$$

$$\sin n\pi \sin n\beta + \cos n2\pi \cos n\beta + \sin n2\pi \sin n\beta]$$

since

$$\sin n\pi = \sin n2\pi = 0, \cos n2\pi = 1$$

$$b_n = \frac{2M}{n2\pi}[-\cos n\beta + \cos n\beta]$$

$$b_n = 0$$

The final Fourier series representation is given by:

$$f(\theta) = \sum_{n=1}^{\infty} a_n \cos n\theta \qquad \text{for n odd}$$

replacing the index n with (2n-1) insures the index will always be odd.

$$f(\theta) = \sum_{n=1}^{\infty} a_{(2n-1)} \cos(2n-1)\theta$$

where

$$a_{(2n-1)} = \frac{8M}{(2n-1)2\pi} \sin(2n-1)\beta$$

Using the first three terms of the Fourier series:

$$f(\Theta) = \frac{8M}{2\pi} \sin\beta \cos\Theta + \frac{8M}{3(2\pi)} \sin 3\beta \cos 3\Theta + \frac{8M}{5(2\pi)} \sin 5\beta \cos 5\Theta \qquad (3)$$

It can be seen from Equation 3 that only the odd harmonics are present and the even harmonics are zero.

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