A Network-based Approach to the Repairman Queueing Problem

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A NETWORK-BASED APPROACH TO THE REPAIRMAN QUEUEING PROBLEM

BY

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THESIS

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CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

CHAPTER I

INTRODUCTION

Research Objectives

In today's world almost everyone encounters queueing situations in their daily lives. As the population increases the demand for services increases. This leads, at least temporarily, to longer waiting lines, or queues. In some cases the problem can be solved by simply increasing services to meet the demand. In other cases it may not be physically or economically feasible to increase the services. The growth of queueing theory, as a subject, has been stimulated by the lack of a simple solution to these problems. The most obvious examples of queues formed by people are the checkout counters in supermarkets, banks, ticket counters, and doctors offices.

Queues are not limited to people. In industry we find trucks waiting to be loaded, airplanes waiting to land, and machines waiting to be repaired. In all of these examples time is a common element, and time has value (usually monetary). The time lost by a truck waiting to be unloaded at a warehouse has a definable value to the owner of the truck. Conversly, the time lost by the loading crew when no trucks are there to be unloaded has a definable value to the owner of the warehouse. The proper application of queueing theory can help to minimize this loss of time in the queue and by the server.

The queueing theory that is available today was developed using advanced mathematics. The models are complex and require a good understanding of these advanced mathematical techniques. This complexity points out the need for a simplified approach to queueing problems. The most basic description of a physical system is usually made with a drawing or a diagram. In the field of engineering Gantt charts, PERT charts and logic flow diagrams are used. Graphs and charts of various kinds are used extensively to describe business and industrial systems. The benefits of graphical methods yield a simplified approach to the queueing problem. The use of graphical methods is the primary thrust of this research.

In illustrating a diagram approach, the scope of this research is limited to a specific type of queueing problem, commonly referred to as the "repairman" problem. This problem is characterized by a finite number of repairmen. Once a machine fails it must be repaired before it can be put back into service.

The results of this research are expected to show that a queueing system can be described with a diagram or a graph. After graphically modelling the system, the ease of understanding will be assessed and analytical results obtained. These results will be compared to results obtainable from theoretical models.

Organization of the Thesis

In Chapter 2 the "repairman" problem is physically described and then conceptually presented as a queueing system. The Poisson queueing model is introduced along with the derived formulas which apply to the repairman problem. A numerical example of the repairman problem is solved using the Poisson model.

In Chapter 3 the problem is described as a Markov process which is a special type of stochastic system. The Graphical Evaluation and Review Technique (GERT) is introduced and the repairman problem is modeled as a stochastic network using this technique. An analysis of the GERT model produces steady state results for comparison with the Poisson model, plus additional information about the system which is not available using the Poisson model.

In Chapter 4 the Q-GERT simulation technique is introduced. The repairman problem is modeled as a Q-GERT network and the simulation is performed using the Q-GERTS computer program. The use of a simulation approach makes it possible to remove most of the assumptions which are inherent in the Poisson and the GERT models of the repairman problem. Finally, in Chapter 5, the results of theory, GERT, and Q-GERTS are brought together and analyzed. Advantages and disadvantages of each are discussed.

CHAPTER 2

THEORETICAL MODEL

Definition of Repairman Problem

The most obvious queueing systems are those composed of people waiting for service. Figure 1 shows a barber shop that has three barbers working.

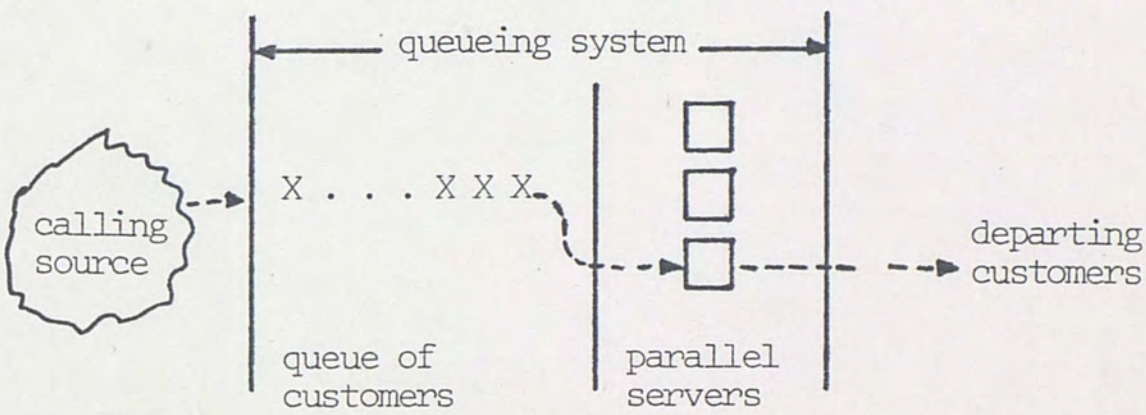


Fig. 1. Barber Shop as a Queueing System

The customers arrive at the barber shop and join the queue if all three barbers are busy. When a barber completes service on one customer, the first customer in line moves to that barber position and his service begins. The queueing system is defined as the queue of customers and the parallel servers. For the barber shop, the calling source of customers is very large, and is assumed to be infinite. For this reason it is not necessary to show the departing customer rejoining the calling source of customers. Furthermore, the characteristics of the calling source will be independent of the number of customers in the barber shop.

The repairman problem is a queueing system that behaves somewhat differently. The customers are machines, which are physically fixed in position. The servers are the repairmen that physically move to the failed machine to repair it. Keeping this physical difference in mind, the repairman queueing system is represented by Figure 2.

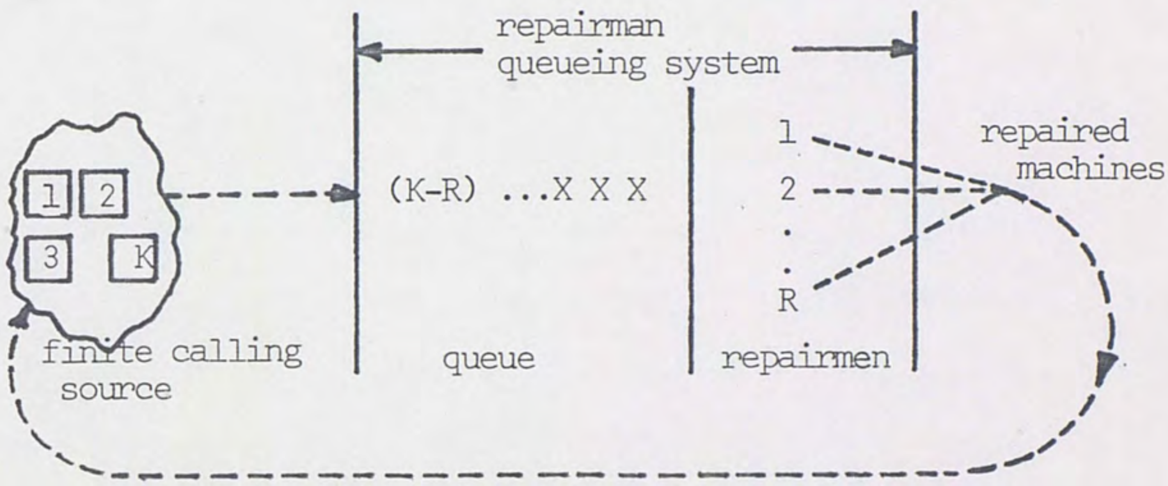


Fig. 2. Repairman Problem as a Queueing System

This representation of the repairman problem resembles the barber shop example in that the customers join a queue, are served, and then depart the queueing system. However, the calling source of machines is now finite in size, and the characteristics of this calling source at any point in time are dependent upon the number of machines in the calling source. If the total number of machines for a specific problem is denoted by (K), then the number of machines in the calling source will vary between zero and K. The number of repairmen is denoted by (R), and the maximum number of machines that can be in the queue is (K-R).

The repairman problem then is represented as a queueing system with a finite calling source and a queue that is limited to a finite size.

Poisson Queueing Model

The Poisson queueing model makes it possible to mathematically analyze queueing systems which meet the following conditions:

(1) The individual units that make up the calling source are identical

(2) All service facilities are identical

(3) The arrivals to the system are Poisson distributed

(4) The service time is exponentially distributed

Real world queueing problems rarely meet all these conditions. However, it has been found that a wide range of queueing systems can be successfully analyzed by assuming the above conditions. The mathematics involved in analyzing models without these assumptions is very laborious. Because the Poisson model reasonably approximates a wide range of queueing systems and the mathematics involved are not extremely difficult, this model has received wide usage in fields other than mathematics. (Panico, 1969)

The Poisson model is further restricted to an analysis of the queueing system in it's "steady state". The steady state condition exists when the behaviour of the system becomes independent of time. This presupposes that the parameters of the system will permit a steady state to be reached and the elapsed time since the start of the operation appraoches infinity. Even though transient state solutions do exist for some models, the mathematics again become extremely difficult.

The following symbols are used in the finite Poisson queueing model and will be continued in the other models in later chapters. (Taha, 1971).

The input parameters for describing the system are:

- K - the total number of machines in the problem
- λ - the mean failure rate of each identical machine
- μ - the mean repair rate of each identical repairman.
- R - number of repairmen
- ρ - defined as λ/μ

The output of the derived formulas for the steady state analysis are:

- λ_{eff} - the mean failure rate of the calling source of machines, once a steady state is reached
- W_s - the expected time that each machine will spend in the queueing system each time the machine fails. This is the same as the expected time the machine is out of operation.
- W_q - the expected time that each machine will spend waiting in the queue after each failure.
- L_s - expected number of machines in the queueing system once steady state is reached. This is the expected number of machines not in operation.
- L_q - expected number of machines in the queue once steady state is reached.
- P_n - the probability that exactly n machines are in the queueing system (out of operation) once steady state is reached. where n is only defined from $n = 0$ to $n = K$.
- $\overline{R}$ - the expected number of idle repairmen

To derive the steady state expressions for P_n, W_s, W_q, L_s, and L_q the system must be examined over an increment of time (h) small enough that only one failure and/or repair can occur during h. In addition, the approximate probability of a failure during h is (K-n) h for $n \leq K$, and the approximate probability of a repair during h is $n\,\mu\,h$ for $n \leq R$ and $R\,\mu\,h$ for $n \geq R$, where n is the number of machines in the queueing system at the beginning of h. The resulting steady state expressions for the system are: (Taha 1971)

$$P_o = \left[\sum_{n=0}^{R} \binom{K}{n} e^n + \sum_{n=R+1}^{K} \binom{K}{n} \frac{n\,\rho^n}{R\,R^{n-R}} \right]^{-1} \qquad (2.1)$$

$$P_n = \begin{cases} \binom{K}{n} \rho^n P_0, & 0 \leq n \leq R \\ \binom{K}{n} \dfrac{n!\,\rho^n}{R!\,R^{n-R}} P_0, & R \leq n \leq K \end{cases} \qquad (2.2)$$

$$L_q = \sum_{n=R+1}^{K} (n-R)\, P_n \qquad (2.3)$$

$$L_s = L_q + \frac{\lambda_{eff}}{\mu} \qquad (2.4)$$

$$W_q = \frac{L_q}{\lambda_{eff}} \qquad (2.5)$$

$$W_s = \frac{L_s}{\lambda_{eff}} \qquad (2.6)$$

$$\overline{R} = \sum_{n=0}^{R} (R-n)\, P_n \qquad (2.7)$$

$$\lambda_{eff} = (R-\overline{R}) \qquad (2.8)$$

To demonstrate the use of the Poisson queueing model, consider a three machine one repairman example. The failures of each machine are Poisson distributed with a mean failure rate of λ=1.0 failure/hour. The repairs by the repairman are also Poisson distributed with a mean repair rate of μ=2.0 repairs/hour. As a result of the special relationship of the Poisson and exponential distributions, the time between failures for each machine is exponential with a mean $^1/_\lambda$= 1 hour. The reapir time is also exponential with a mean $^1/_\mu$= .5 hours.

From the statement of the problem:

λ = 1.0 failure/hour/machine

μ = 2.0 repair/hour/repairman

R = 1 repairman

K = 3 machines

$$\rho = \frac{\lambda}{\mu} = .5$$

From equations (2.1) and (2.2):

P_0 = .2105

P_1 = .3158

P_2 = .3158

P_3 = .1579

From equations (2.3) through (2.8):

L_q = .6316 machines

$\overline{R}$ = .2105 repairmen

λ_{eff} = 1.579 machines/hour

L_s = 1.4211 machines

$W_q = .40$ hours

$W_s = .90$ hours

These results are summarized in Table 1 and discussed in the following paragraphs.

Assuming the machine operation does reach a steady state, the probability of no failed machines is .2105. The probabilities of one and two failed machines are the same and equal to .3158, and the probability of all machines failed is .1579. The expected number of machines out of operation at all time is 1.4211. This means that the machine operation will be operating with 1.579 machines. Once a machine fails it will be out of operation for an average of 0.90 hours, and 0.40 hours of that time will be spent waiting for the repairman. The repairman will be idle 21% of the time, and busy 79% of the time.

Using this information several conclusions can be made about this operation. The most obvious is that over the long run only 1.5 machines will be operating. In most situations that would probably be unacceptable. Machines represent a relatively large investment as compared to repairmen. Machine hours spent in the repair system represent a loss of income, and in this case 4/9 or 44% of those lost machine hours are spent waiting for a repairman. The other 56% are spent actually being repaired.

The Poisson queueing model can be a useful tool for the analysis of actual repairman problems. However, the model is restricted by several assumptions, and the user must be aware of these. Also, the analytical results are limited to the steady state situation.

TABLE 1

Summary of Poisson queueing model analysis of repairman problem

	Steady State Results
λ_{eff} (machines/hour)	1.579
ρ	0.50
W_s (hours)	0.90
L_s (machines)	1.4211
W_q (hours)	0.40
L_q (machines)	0.6316
$\overline{R}$ (repairmen)	0.2105
P_0	0.2105
P_1	0.3158
P_2	0.3158
P_3	0.1579

CHAPTER 3

STOCHASTIC NETWORK APPROACH

In this chapter the repairman problem is studied as a stochastic system which can be graphically represented by a network. The concept of queueing as a stochastic process is certainly not new. The Markov process, which is an important class of stochastic systems, has been used extensively for the study of queueing systems. Unfortunately the Markov process becomes very difficult to analyze once the transitional behavior of the system is expressed as random variables. The Graphical Evaluation and Review Technique (GERT) was developed for the study of stochastic networks. Pritsker (1966) defines stochastic networks as those having activities characterized by a probability of occurrence and a time, where the time is not a constant but a random variable.

The repairman problem is first represented as a Markov process. The concepts and analytical methods of GERT are introduced, and then the repairman problem is represented and analyzed as a GERT stochastic network.

Queueing As A Markov Process

A Markov process is a mathematical model that can be used in the study of complex stochastic systems. In this model the system is described by the states of the system and the state transitions. This definition of a Markov process lends itself to a graphical representation. The graphical representation of the system is called a system state diagram. Figure 3 shows a system state diagram that depicts a three machine, one repairman problem.

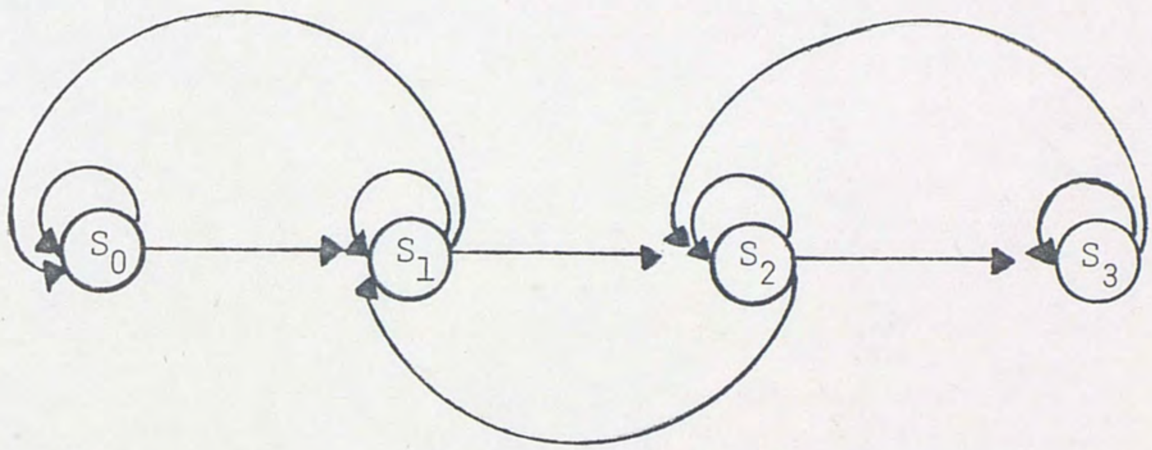


Fig. 3. System State Diagram of Repairman Problem

The states of the system are defined as the number of failed machines (the machines being repaired plus the machines waiting for repairs). At any point in time (t) the system will be in one of the following four states:

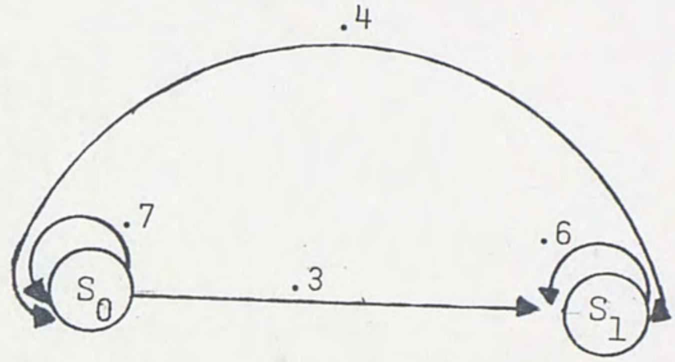
S_0 = 0 Machines failed

S_1 = 1 Machine failed

S_2 = 2 Machines failed

S_3 = 3 Machines failed

If the system is in state S_i at t, then at $(t + \Delta t)$ it will be in state S_i, S_{i+1}, or S_{i-1}. (Δt is defined such that only one state transition can occur during Δt). If (Δt) is taken as a constant finite time, then the system is a discrete-time Markov process. In this case the only element of interest concerning the state transitions is the probability that the transition from S_i to S_j will occur in (Δt). An example is a one machine, one repairman situation as shown below:



If a Δt is chosen for successively examining the system, then there are conditional probabilities associated with each state transition (e.g. .3 is the conditional probability of a transition from state 0 to state 1 given the system is in state 0). This system can be analyzed as a very basic Markov process (Howard, 1960).

In queueing models the time between state transitions must be introduced, and these times are usually state dependent random variables. This results in a continuous-time Markov process, and the mathematics of the analysis becomes more difficult. Examples of this are plentiful in Saaty (1961), Parzen (1962), and Taha (1971). In spite of this difficulty the concept of queueing as a Markov process provides a graphical method for presenting the queueing model.

Graphical Evaluation and Review Technique

The Graphical Evaluation and Review Technique (GERT) was introduced by Pritsker (1966) as a network approach to the analysis of stochastic and logical systems. The network consists of a set of logical nodes which are joined by branches. The nodes and branches of the GERT network are assembled in an analogous fashion to the states and state transitions of the Markov process system state diagram.

Even though several different nodes were introduced, this study is concerned with the Exclusive - Or/Probabilitic node, where Exclusive - Or refers to the input side of the node and Probabilistic refers to the output side of the node. The definition of the logical relations for this node are:

> "The realization of any branch leading into the node causes the node to be realized; however, one and only one of the branches leading into this node can be realized at a given time.
>
> Exactly one branch emanating from the node is taken if the node is realized."
>
> (Pritsker, 1966)

Each branch of the network is characterized by a probability that the branch is taken and a time interval required to traverse that branch. The time interval is usually a random variable.

The concepts described above can be illustrated with a GERT network for a manufacturing and inspection process as shown in Figure 4. Once a part is manufactured, or a rejected part is reworked, node 2 is realized. The realization of node 2 causes one of the branches emanating from node 2 to be taken. The branch selection is based upon the probabilities assigned to the branches. If branch 2-4 is taken, the part goes through a finishing process and departs the system. If branch 2-3 is taken, the part is either scrapped or reworked. Reworked parts are then inspected. It is seen from the diagram that successful inspection is independent of whether a part is first time thru or a reworked part.

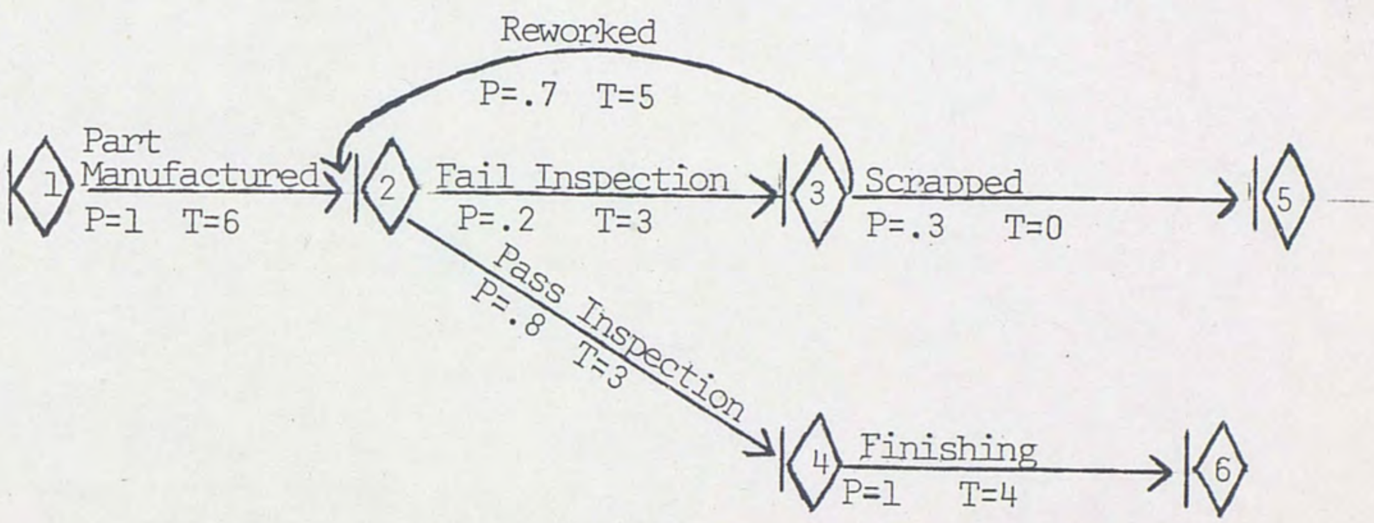
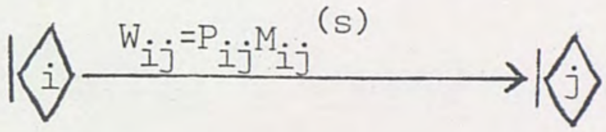


Fig. 4. Gert Network for Manufacturing Inspection Process

Evaluation of the GERT Network

The evaluation of the GERT network is made possible by the use of flowgraph theory and moment generating functions. Flowgraph theory is covered in detail by Lorens (1964) and by Whitehouse (1973). The use of moment generating functions is described by Whitehouse (1973).

Flowgraph theory is a graphical concept which employs nodes and branches. Each branch has a transmittance. The transmittances are a single multiplicative parameter. In the GERT network, however, the branches must have multiple parameters (time and probability) and these parameters are not generally multiplicative. For example, the manufacturing and inspection process of Figure 4 has branches with probability and time parameters. The probability parameters along any path in the network are multiplicative, while the time parameters along that path are additive. To use flowgraph techniques to analyze the network all parameters must be multiplicative. This is accomplished by expressing the time parameter as the moment generating function of the time. The moment generating function of the sum of independent random variables is equal to the product of the moment generating functions of the independent variables. In this way the parameters of the branch can be expressed as a single multiplicative parameter called the "W" function, where the W function is the product of the multiplicative parameters of the branch.



The W function corresponds to the transmittance of flowgraph theory and the network can now be analyzed using Mason's Rule. Mason's Rule as stated by Whitehouse (1973) is: "Write down the product of transmittance along each path from the independent to the dependent variable. Multiply its transmittance by the sum of the nontouching loops to that path. Sum these modified path transmittances and divide by the sum of all the loops in the open flowgraph." Time and space do not permit a detailed description of the use of Mason's Rule to solve flowgraphs.

The definition of the various terms, such as non touching loops, and excellent examples are contained in Lorens (1964) and Whitehouse (1973).

The manufacturing and inspection system is shown in Figure 5 with the W function of each branch assigned.

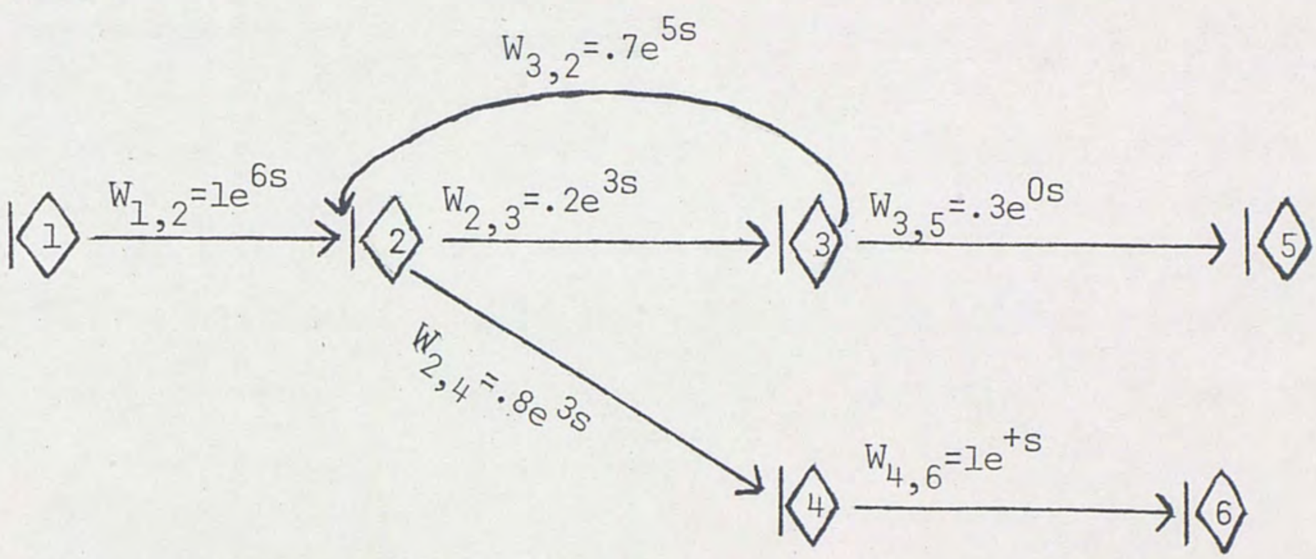


Fig. 5. Manufacturing Inspection Network with W Functions

The times for all branches are constant and the moment generating function for a constant is e^{st} where T is the constant time. Using Mason's Rule an equivalent W function (W_E) can be developed between a start node (dependent variable) and an end node (independent variable). For this example there are two end nodes where the process could terminate with a finished part or a scrapped part. The equivalent W function for a finished part ($W_{1;6}$) will be developed. Only one path exists from node 1 to node 6. The product of the W_{ij}'s along that path is:

$$(W_{1,2})\ (W_{2,4})\ (W_{4,6}) = 0.8e^{13s}$$

There is only one first order loop in the system:

$$(W_{2,3})\ (W_{3,2})\ -0.14e^{8s}\ .$$

There are no higher order loops, and there are no nontouching loops to the path. Therefore:

$$W_{1,6}(s) = \frac{0.8e^{13s}[1-0]}{1-0.14e^{8s}}$$

The equivalent W function for the path contains an equivalent probability (P_E) and an equivalent moment generating function ($M_E(s)$). The equivalent probability is obtained by setting the time to zero and evaluating the W function. Therefore:

$$P_E = W_E\ (0)$$

$$P_{1,6} = W_{1,6}(0) = \frac{0.8}{1\ -0.14}$$

$$P_{1,6} = 0.930$$

The equivalent moment generating function of the time for the path is obtained by removing the probability portion of the W function.

$$M_E(s) = \frac{W_E(s)}{W_E(0)}$$

Successive moments can then be taken where, μ_1 = the first moment, μ_2 = the second moment, etc.

$$\mu_1 = \frac{dM_E(s)}{ds}\Bigg|_{s=0} = \frac{1}{W_E(0)}\frac{dW_E(s)}{ds}\Bigg|_{s=0} \tag{3.1}$$

$$\mu_2 = \frac{d^2M_E(s)}{ds^2}\Bigg|_{s=0} = \frac{1}{W_E(0)}\frac{d^2W_E(s)}{ds^2}\Bigg|_{s=0} \tag{3.2}$$

Higher moments can also be calculated in the same manner, but they are not generally useful. From a study of statistics, it is known that the first moment (μ_1) is the expected value of the random variable and the variance for that variable can be computed using the first and second moments. (Pritsker 1966)

$$\sigma_E^{\ 2} = \mu_{2E} - (\mu_{1E})^2 \tag{3.3}$$

From equation 3.1 the expected time for a part to become a finished product is,

$$\mu_1 = 14.30$$

and the variance of that time is,

$$\sigma_{1,6}^{\ 2} = 12.11$$

where,

$$\mu_2 = 216.60$$

In summary, the manufacturing inspection process has been described by a graphical network. The probability that a finished product will result is 0.930. This also implies that the probability of a scrapped part is 0.070. The expected time to turn out a finished product is 14.30 units time, with a variance of 12.11. The expected time to turn out a scrapped part can be computed for the path leading to that outcome. These are the most basic results of a GERT analysis of a stochastic network. Other useful information can be derived, and will be illustrated in later examples.

GERT Model for 3 Machine/1 Repairman Problem

A GERT Model for the general repairman queueing problem was developed by Whitehouse (1973). A specific example of this general model is the 3 machine/1 repairman example that was analyzed in Chapter 2. The example was also represented as a Markov process earlier in this chapter. The GERT network for this specific example is in Figure 6 and is very similar to the Markov system state diagram of Figure 3.

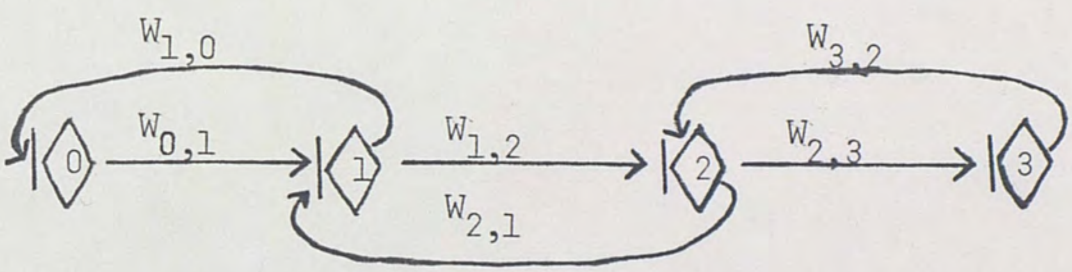


Fig. 6. GERT Network for 3 Machine/1 Repairman Model

The nodes of the GERT diagram represent the four states of the system just as in the system state diagram. The W function of the branches represent the multiplicative parameters of probability and time for the transition from a given state to an adjacent state. It should be noted that in the system state diagram there was a probability that the system would remain in it's present state over the interval of time Δt. This is not true for the GERT network. The interval of time over which the network is examined is a state dependent random variable. It is defined to be an interval of time such that a transition to an adjacent state does occur.

The failures of each identical machine are Poisson distributed with a mean of λ, and the repairs for each identical repairman are Poisson distributed with a mean of μ. Because the number of machines subject to failure and the number of busy repairmen changes, depending upon the state of the system, λ and μ are not adequate descriptors of the system. Therefore the failure rate and repair rate of the system are designated as λ_i and μ_i respectively, where i denotes the state of the system. Each λ_i and μ_i can be defined by the relationship:

$$\lambda_i = (K-i)\lambda$$

$$\mu_i = \begin{cases} i\mu \text{ for } i < R \\ R\mu \text{ for } i \geq R \end{cases}$$

For example, consider the 3 machine/1 repairman problem. If the system is in state 0, all three machines are subject to failure. The failure rate of the system (λ_0) would be 3λ . However, the repair rate of the system (μ_0) would be zero, because the repairman is idle. The state dependent failure and repair rates which completely describe the 3 machine/1 repairman problem are:

State (i)	λ_i	μ_i
0	3λ	0
1	2λ	μ
2	λ	μ
3	0	μ

The probability that each branch of the network is taken can be determined using the λ_i, μ_i. Given that the system is in state i, it can only move to state (i+1) or state (i-1). The probability that the system will move to state (i+1) is the probability that a failure will occur before a repair. Conversely the probability of moving to state (i-1) is the probability that a repair occurs first. These probabilities can be expressed as:

$$P_{i,\, i+1} = \frac{\lambda_i}{\lambda_i + \mu_i}$$

$$P_{i,\, i-1} = \frac{\mu_i}{\lambda_i + \mu_i}$$

and it can be easily seen that:

$$P_{i,\, i+1} + P_{i,\, i-1} = 1$$

Not only are the time distributions state dependent, but they are dependent only upon the current state of the system. The time distribution of interest is the time the system remains in state i. To determine this distribution consider that the actual failures and repairs of the system in state i are independent Poisson distributions with means λ_i, and μ_i respectively. The system will remain in state i until either a failure or a repair occurs. If an occurrence is defined as a failure or a repair, then the occurrence rate will be the sum of the repair rate and the failure rate. The occurrences will also be Poisson distributed with a mean of $(\lambda_i + \mu_i)$. From the relationship between the Poisson and exponential distributions, the occurrence time will be exponentially distributed with a mean of $\left(\frac{1}{\lambda_i+\mu_i} \right)$. The moment generating function for the exponential distribution is $(1 - \frac{s}{a})^{-1}$, where $\frac{1}{a}$ is the mean. The general W function for the branches of the network can now be expressed as,

$$W_{ij} = \frac{\lambda_i}{\lambda_i+\mu_i} \left(1 - \frac{s}{\lambda_i+\mu_i}\right)^{-1} \quad \text{for } j = i + 1$$

$$W_{ij} = \frac{\mu_i}{\lambda_i+\mu_i} \left(1 - \frac{s}{\lambda_i+\mu_i}\right)^{-1} \quad \text{for } j = i - 1$$

where,

$$W_{ij} = P_{ij}M_{ij}(s)$$

For the example problem of Figure 6 with λ=1 failure/hour and μ=2 repair/hour, the state dependent failure and repair rates are:

State (i)	λ_i	μ_i
0	3	0
1	2	2
2	1	2
3	0	2

The W functions for this network are:

$$W_{0,1} = 1(1-\frac{s}{3})^{-1} \qquad W_{2,1} = .67(1-\frac{s}{3})^{-1}$$

$$W_{1,0} = .5(1-\frac{s}{4})^{-1} \qquad W_{2,3} = .33(1-\frac{s}{3})^{-1}$$

$$W_{1,2} = .5(1-\frac{s}{4})^{-1} \qquad W_{3,2} = 1(1-\frac{s}{2})^{-1}$$

The 3 machine/1 repairman queueing problem, where $\lambda = 1$ failure/hour and $\mu = 2$ repairs/hours, is completely described by a GERT network. Before continuing with the analysis of this network, a computer program for the analysis of GERT networks will be introduced. Specific operating instructions for this program are listed in Whitehouse (1973), and are summarized in the next section.

GERT Exclusive - Or Program

A digital computer program is available in FORTRAN for the analysis of GERT networks with Exclusive-Or Probabilistic nodes called GERTXOR. The branches of the network have a probability and a time which can be expressed as a random variable. The input to the program includes a problem identification card and one card for each branch of the network. The input card for each branch in the network includes:

(1) Start Node

(2) End Node

(3) Type of Time Distribution

(4) Probability of Realizing the branch

(5) Parameters defining the time distribution

The output from the program includes:

(1) All paths and loops of the network

(2) The probability of realizing a sink node from any source node.

(3) The mean and the variance of the time to reach a sink node from any source node

To determine the expected value of the time to go from zero machines failed to three machines failed using the GERTXOR program, the network must be opened as shown in Figure 7.

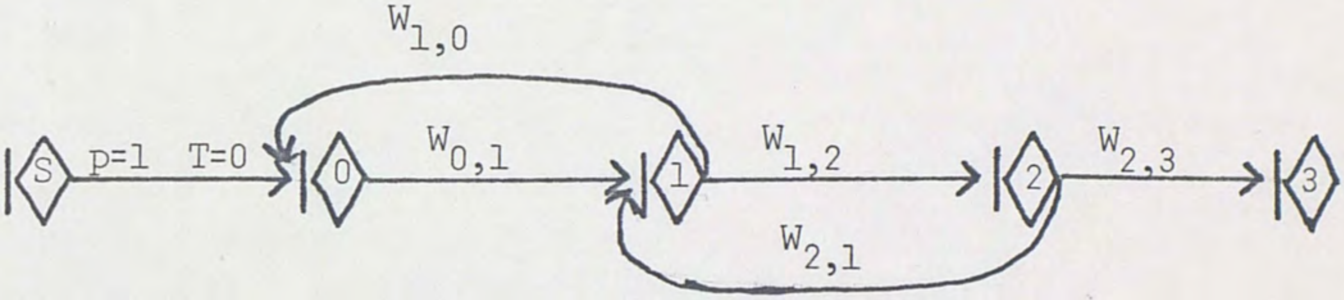


Fig. 7. GERT network for time to go from all machines operating to all machines failed.

The network must have a source node and a sink node. The source node is identified as having no branches leading into it, and the sink node is identified as having no branches leading out of it. For this reason, node s is added to provide a source node, where the probability of branch s-0 is 1 and the time is 0. Branch 3-2 was removed because we are only interested in the time to go from node 0 to node 3 which represents the time to go from all machines operating to all machines failed.

The output is shown in Table 2. The input network section is an echo print of the data. Next is a listing of each branch of the network including it's probability of selection and the mean and variance of the time to travel that branch. The two loops in the network are listed with their W functions evaluated at s=0. The desired path from node s to node 3 is listed with a probability of 1.0, a mean time of 3.8451, and a variance of the time of 11.4652.

TABLE 2

GERTXOR Output For GERT Network of Figure 7

INPUT NETWORK

NODES AND PROBABILITY OF SELECTION WITH
MEAN AND VARIANCE OF TIME FOR EACH LINK

FROM	TO	PROB	MEAN	VAR
s	0	1.00	0.0	0.0
0	1	1.00	0.33	0.109
1	2	0.50	0.25	0.063
1	0	0.50	0.25	0.063
2	1	0.67	0.33	0.109
2	3	0.33	0.33	0.109

LOOP OF ORDER 1 W(0) = 0.5000
W(0) = 0.5000, NODES 0 1
LOOP OF ORDER 1 W(0) = 0.3350
W(0) = 0.335, NODES 1 2

NS	NE	PROB	MT	VT
s	3	1.000	3.8451	11.4652

EQUIVALENT BRANCHES OF THE NETWORK

ENTRY	EXIT	PROBABILITY	MEAN TIME	VARIANCE
s	3	0.10000E 01	0.384515E 01	0.114652E 02

Analysis of GERT Model for 3 Machine/ 1 Repairman

Given this brief introduction to the GERTXOR program, we may now return to the analysis of the GERT model. The initial analysis of the problem will include the development of steady state results for comparison with the Poisson Model of Chapter 2. The remaining analysis will include information about the system that is not available from a steady state analysis, such as the Poisson model.

Steady State Results

The steady state results are developed by first finding the steady state probabilities. The steady state probability for a given state is equal to the expected time in the given state during a regeneration of the system, divided by the expected total time of the regeneration. A regeneration of the system occurs when the system returns to the state from which it started. The GERT network for the expected time of regeneration for node 0 is shown in Figure 8.

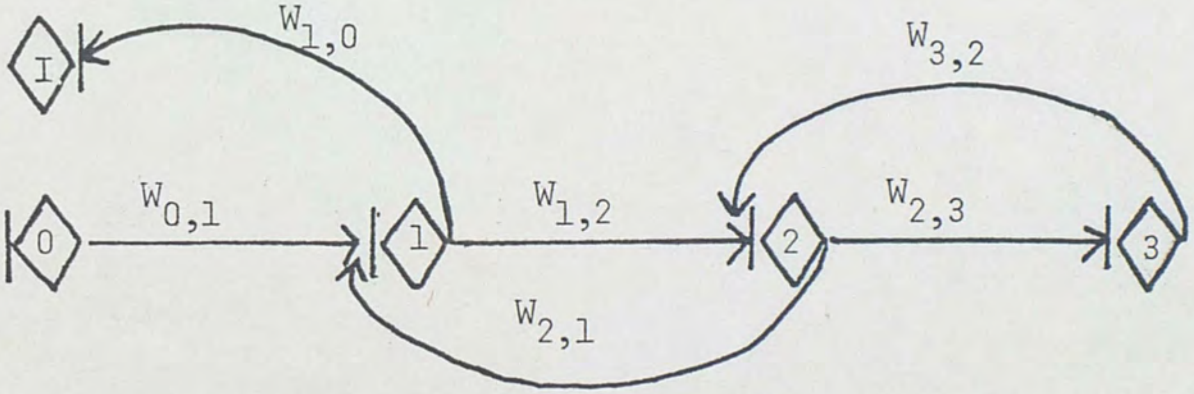


Fig. 8. GERT network for expected time of regeneration.

The network is opened at node 0, and node I represents the input side of node 0. The expected regeneration time of node 0 (representing system regeneration) is the time to go from node 0 to node I. Using the GERTXOR program for the calculation,

$$\mu_1 = \left. \frac{dM_{0,I}(s)}{ds} \right|_{s=0} = 1.5688 \text{ hours}$$

The mean time spent in each state during the system regeneration is represented by the branches starting from that node. For example the time spent in state 1 is represented by branches 1-2 and 1-I. To find the steady state probability for each state:

(1) Set the moment generating functions equal to 1 for all branches not emanating from the node of interest, and again evaluate the network from node 0 to node I.

(2) Divide the mean time spent in each node by the mean regeneration time for node 0.

Figure 9 depicts the network for the mean time spent in node 1. Setting the moment generating function equal to one for a branch effectively reduces the time distribution for that branch to zero.

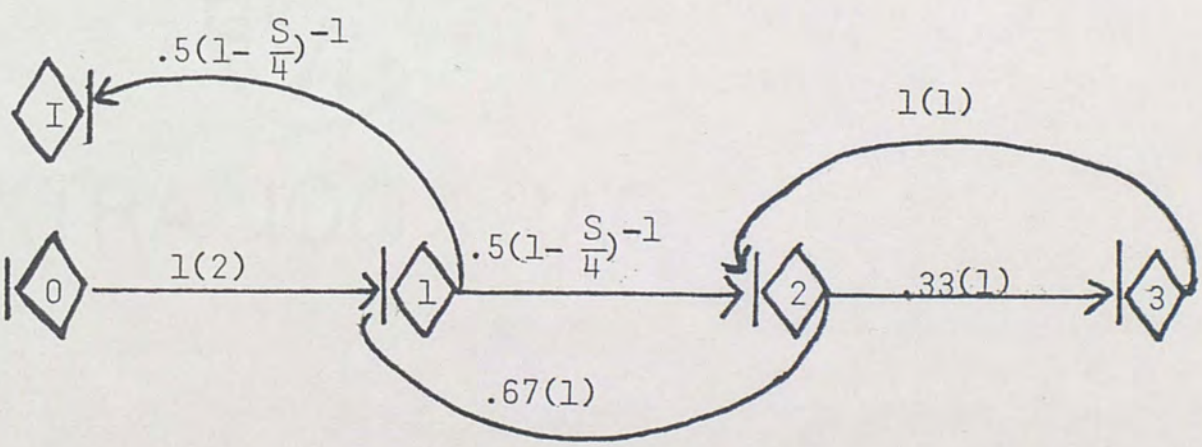


Fig. 9. GERT network for determining P_1

For the network of Figure 9 the mean time spent in node 1 is:

$$\left.\frac{dM_{0,I}(s)}{ds}\right|_{s=0} = .50 \text{ hours}$$

The steady state probability for node 1 (state 1) is:

$$P_1 = \frac{.50}{1.5688} = .319$$

In the same manner all steady state probabilities for the system are found to be:

$$P_0 = .210$$

$$P_1 = .319$$

$$P_2 = .314$$

$$P_3 = .157$$

which closely corresponds with theoretical results as summarized later.

Once the steady state probabilities are determined, the other steady state parameters can be developed. First, the expected number of failed machines is:

$$L_s = \sum_{n=0}^{K} nP_n = (0)(.210)+(1)(.319)+2(.314)+(3)(.157)$$

$$L_s = 1.418 \text{ machines}$$

The effective failure rate for the machines in operation is:

$$\lambda_{eff} = \sum_{n=0}^{K} P_n\lambda_n$$

$$= (.210)(3)+(.319)(2)+(.314)(1)+(.157)(0)$$

$$\lambda_{eff} = 1.582 \text{ machines/hour}$$

The expected number of idle repairmen is:

$$\bar{R} = \sum_{n=0}^{R} (R-n)\, P_n$$

$$= (1-0)(.210) + (0-0)(.319)$$

$$\bar{R} = .210 \text{ repairmen}$$

Therefore, the expected number of busy repairmen is:

$$R - \bar{R} = .790 \text{ repairmen}$$

The expected number of machines being repaired is the same as $(R-\bar{R})$, and the number of failed machines waiting for service is:

$$L_q = L_s - (R - \bar{R})$$

$$L_q = .628 \text{ machines}$$

From the derived formulas of Chapter 2, the expected time each machine is out of operation once it fails is:

$$W_s = \frac{L_s}{\lambda_{eff}}$$

$$W_s = .896 \text{ hours}$$

Also, the expected time waiting for a repairman is:

$$W_q = \frac{L_q}{\lambda_{eff}}$$

$$W_q = .397 \text{ hours}$$

A comparison of these results and the steady state results of the Poisson queueing model is shown in Table 3. Overall, the results are very close. However there are some differences. For the Poisson model P_1 and P_2 are equal. For the GERT model this is not true. This difference can be accounted for in the formulation of the probability portion of the W function for branches 2-1 and 2-3. $P_{2;1}$ was 2/3 and

$P_{2,3}$ was 1/3. They were rounded off to .67 and .33 respectively.

TABLE 3

Comparison of steady state results of
repairman problem using GERT and Poisson model

	GERT Results	Poisson Model Results
P_0	.210	.210
P_1	.319	.316
P_2	.314	.316
P_3	.157	.158
L_s	1.418	1.421
λ_{eff}	1.582	1.579
L_q	.628	.632
W_s	.896	.900
W_q	.397	.400
$\overline{R}$	.210	.210

Other Useful Information

A vast amount of additional information can be developed from the GERT network of the repairman problem. The real difficulty is determining which information is actually useful in the analysis of a particular problem. In this section, the following information will be developed.

(1) Expected time to regeneration

(2) Expected number of machine failures during 1 regeneration

(3) Expected time to complete failure of the system, given the system is in state i.

(4) Expected time to complete recovery of the system, given the system is in state i.

(5) Probability of complete failure occuring before recovery, given the system is in state i.

(6) Probability of recovery occuring before complete failure, given the system is in state i.

The expected time to regeneration can be computed for each state, however it really only has meaning for state 0 (all machines operating). In computing the steady state probabilities earlier, the regeneration time for node 0 was calculated as 1.5688 hours. The expected number of machine failures during one regeneration is determined by using counters. The counter, e^c, corresponds to a moment generating function representing a constant, c, tagged on each branch to physically "count" the number of times a branch is realized.

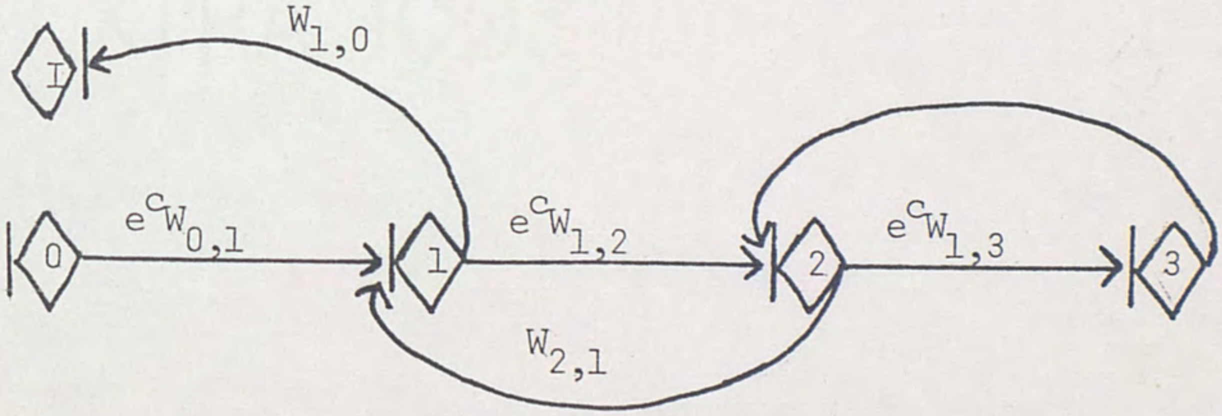


Fig. 10. GERT network with counters.

Figure 10 shows the GERT network with the counters on each branch which represents a machine failure. The equivalent moment generating function from node 0 to node I is now a function of s and c. Setting s equal to zero, the successive moments of the count can be determined. Therefore, the expected number of machine failures during one regeneration of the system to state 0 can be found as follows:

$$M_{0,I}(s,c)\Big|_{s=0} = \frac{.5e^{c} - .165e^{2c}}{1 - .67e^{c}}$$

$$\frac{dM_{0,I}(c)}{dc}\Big|_{c=0} = 2.58 \text{ machine failures}$$

Given that a certain number of machines are not operating, the time distribution to all machines failed or all machines operating can be determined. For example, given that the system is in state 1, Figure 11 shows the GERT network for complete failure of the system (All machines failed).

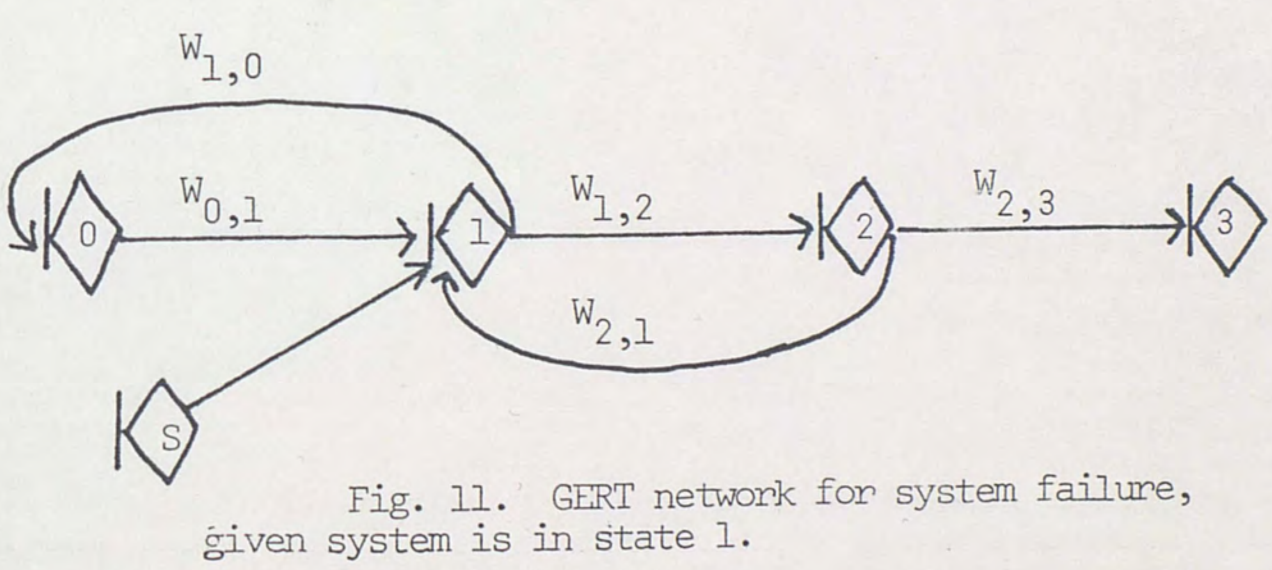


Fig. 11. GERT network for system failure, given system is in state 1.

Node s is added as a dummy node and the branch s-1 has a time of zero. Evaluating the network from node s to node 3 will give the expected time to complete failure given the system is in state 1. Networks can also be constructed for starting in nodes 0 and 2 in a similar manner.

The expected time to complete recovery for state 1 is shown in Figure 12.

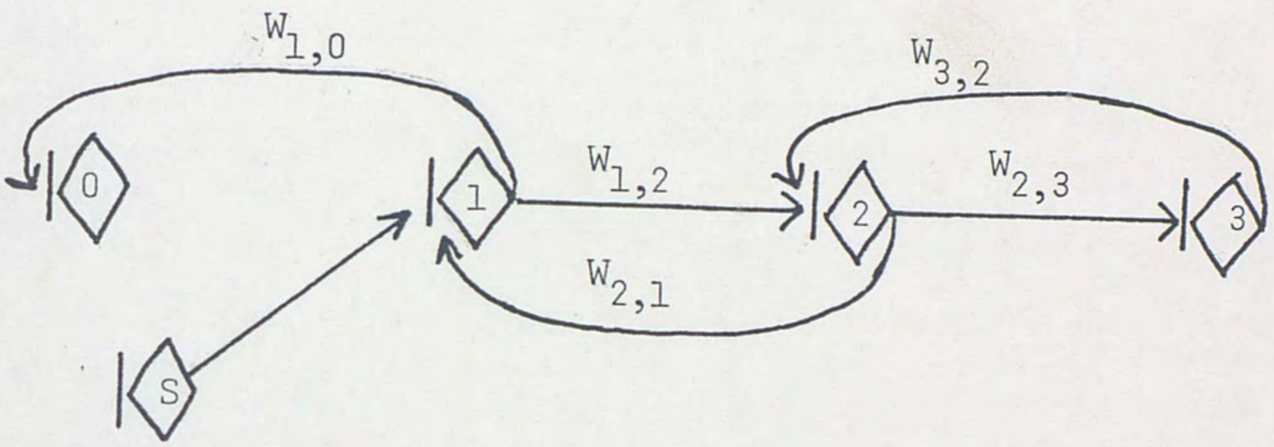


Fig. 12. Expected time to complete recovery from state 1.

Again node s is a dummy start node but node 0 is now the end node.

Given that the system is in an intermediate state (state 1 or state 2), it may be useful to know the probability of recovery occuring before complete failure, and the time to recovery given that complete failure does not occur. This type of conditional information can be developed by constructing the GERT network with an end node for each of the possible outcomes. The network of Figure 13 demonstrates this for the system starting in state 1.

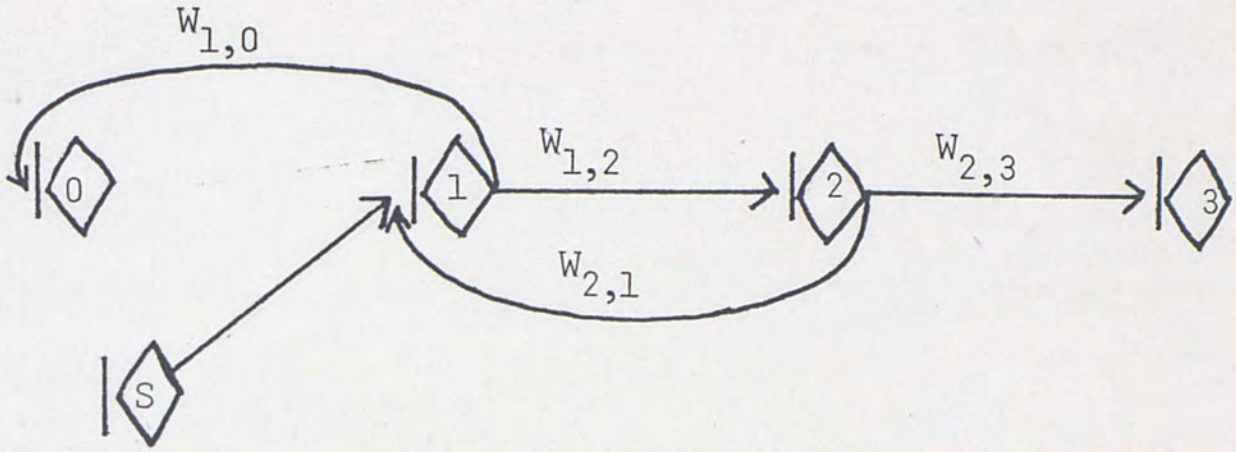


Fig. 13. GERT network for the conditional probabilities of failure and recovery.

Using the GERTXOR program to analyze this network, the output will list the probability, expected time, and variance for moving from node s to node 0 and from node s to node 3.

All of the networks covered in this section were analyzed with the computer program and the results are summarized in Table 4. Whereas the steady state queueing information is more appropriate for planning a machine operation, the information in this table is more appropriate for aiding in day to day operational decisions. For example, once the system is in state 2 (1 machine operating, 2 machines failed) the probability of complete failure is the same as that for recovery. Also, the expected time to recovery is approximately 2 hours. If the system does go to state 3 the expected time to complete recovery will be 2.5 hours. Using this information the work coming into these machines can be rescheduled, or measures can be taken to change the probability or times. Diverting a repairman from other operations would be one way to improve the recovery capability.

TABLE 4

Summary of specific conditional information about the repairman problem as developed using GERT

	0 Machines Failed	1 Machine Failed	2 Machines Failed	3 Machines Failed
Time to Complete Failure	E(t) = 3.845 V(t) = 11.465	E(t) = 3.515 V(t) = 11.356	E(t) = 2.685 V(t) = 10.450	
Time to Complete Recovery		E(t) = 1.239 V(t) = 2.873	E(t) = 1.978 V(t) = 3.665	E(t) = 2.478 V(t) = 3.915
Probability of Complete Failure Before Recovery		.248	.496	
Probability of Recovery Before Complete Failure		.752	.504	
Time to Complete Failure, Given No Recovery		E(t) = .872 V(t) = .513	E(t) = .622 V(t) = .450	
Time to Recovery, Given No Complete Failure		E(t) = .542 V(t) = .404	E(t) = .872 V(t) = .513	

E(t) = mean time in hours
V(t) = variance of the time in hours

In summary, the GERT approach to the repairman queueing problem is a graphical technique for analyzing the stochastic system. It not only gives the traditional steady state analysis of the system, but specific conditional information can be developed for given situations.

CHAPTER 4

A GRAPHICAL SIMULATION MODEL

In the previous chapters the three machine/one repairman problem was analyzed using two different mathematical models. The Poisson queueing model yielded only steady state results. The GERT model yielded steady state results and a limited amount of conditional information. In addition to the limited results these models are restricted by the following assumptions:

(1) The individual units that make up the calling source are identical

(2) All service facilities are identical

(3) The arrival rate to the system is Poisson distributed

(4) The service time is exponentially distributed

Simulation techniques have made it possible to remove these assumptions and study a wider range of complex queueing problems.

In this chapter the repairman example will be modeled using the Q-GERT network language. The statistical information about the model will be developed using the Q-GERTS simulation program. The Q-GERT network language as developed by Pritsker (1974), is designed to simplify the task of constructing the model and to serve as a communications device between the model builder and the decision maker.

The Q-GERT Network

The Q-GERT network is an extension of the GERT network of Chapter 3. It consists of nodes and branches through which transactions flow. These transactions can be either physical units, such as machines, or information units. The transactions can be identified by the assignment of numerical attributes.

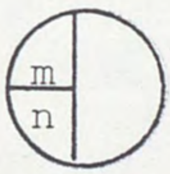
The branches of the Q-GERT network are called activities and the characteristics associated with an activity are:

(1) The probability or conditions under which the activity will process a transaction which has reached the start node of the activity.

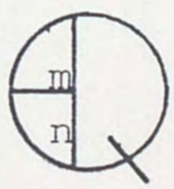
(2) Functions and parameters describing the duration time for the activity.

(3) Activity labels.

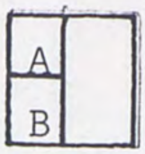
Nodes represent the start and the end of activities. Decisions regarding the flow of transactions are made at the nodes. Activities emanating from a given node cannot be started until that node is realized. The three basic nodes of the network are:



Regular Node - requires m realizations to be released the first time and n realizations to be released thereafter



Queue Node - represents a queue where transactions can wait for further processing. The initial number of transactions in the queue is m and the maximum number allowed is n.



Selector Node - Used to select among parallel queues and/or parallel service activities. A is the queue selection rule and B is the server selection rule.

The output side of the nodes indicate the type of branching that is to be performed from the node. The four possible branching operations are:

DETERMINISTIC - all activities for which this node is the start node will be initiated.

PROBABILISTIC - one and only one activity for which this node is the start node will be initiated. The choice will be on a random basis using the probabilities assigned to the activities leaving the node.

CONDITIONAL, TAKE FIRST - the activities for which this node is the start node are rank ordered. The conditions associated with the activities are tested and as soon as one of the conditions is met that activity is initiated.

CONDITIONAL, TAKE ALL - each activity that has this node as its start node is taken if the condition assigned to the activity is satisfied. The condition for all activities emanating at the node are evaluated.

This brief explanation of nodes and activities is intended to provide a general understanding of the basic components of the Q-GERT network. Discussion of specific networks will show how the nodes and activities can be used to model real systems. For more detailed information on Q-GERT the reader should refer to Pritsker (1974).

Q-GERT Model of the Repairman Problem

The Q-GERT model for the repairman queueing problem is depicted in Figure 14. This is the same problem that was analyzed in Chapters

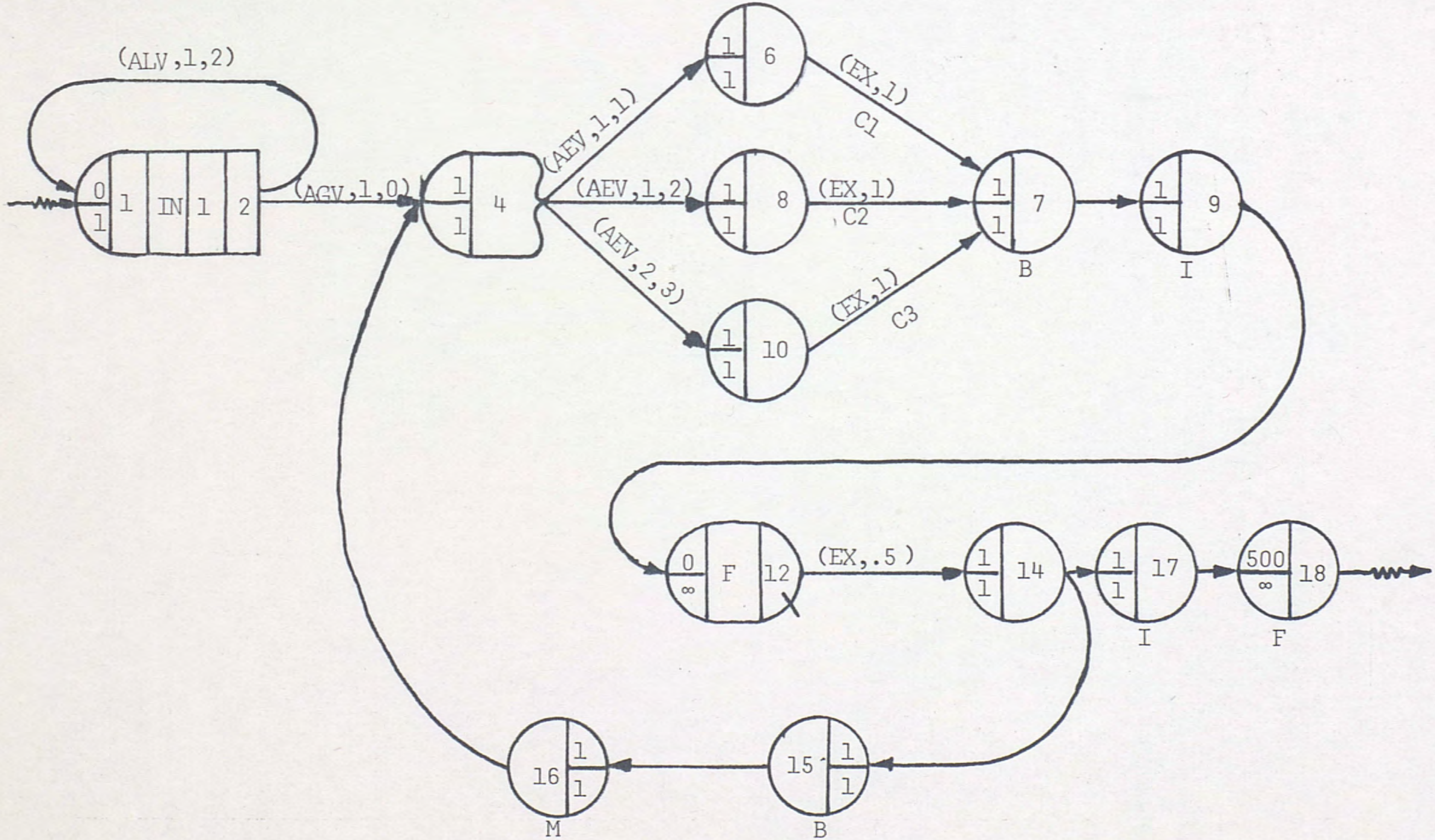


Fig. 14. Q-GERT Model of the 3 machine/1 repairman problem.

2 and 3. It consists of three identical machines and one repairman.

Node 2 is the source node for the network. Three transactions (representing three machines) will be generated from node 2. Attribute 1 of these machines will be assigned a value based on an incremental distribution starting with 1. That is, attribute 1 of each machine (transaction) is assigned the machine number (e.g., 1,2,3). The output of node 2 is Conditional, Take All. The condition for activity 2-2 is that the value of attribute 1 is less than or equal to 2, which stops the generation of transactions after three have been generated. Activity 2-4 is always initiated because the value of attribute 1 is greater than zero. Node 4 is a Conditional Take First node and it identifies the machines by attribute 1. It sends machine 1 to node 6, machine 2 to node 8 and machine 3 to node 10. Activities 6-7, 8-7, and 10-7 represent the time between failures for each machine. For this example they all follow an exponential distribution with a mean of 1 hour. The symbols under the activities of C1, C2, and C3 stand for counter 1, counter 2, and counter 3. They will count the number of transactions each activity processes prior to the release of the statistics nodes. Node 7 is a statistics node for "Between" statistics. It records the time between realizations of the node.

Node 9 is also a statistics node for "Interval" statistics. It records elapsed time since the last mark for every transaction. All transactions are marked with a reference time as they leave a source node. However, they will be marked again with a new reference time if they pass through a mark node. Node 12 is a queue node that initially has zero transactions waiting, and can hold an infinite number of transactions. The "F" in this node designates "First Come, First-Served" service discipline. Activity 12-14 represents the single repairman. The time of repair is exponentially distributed with a mean of 0.5 hours. At node 14 the three machines are returned to operation through nodes 15 and 16. Information transactions also leave node 14 and interval statistics are taken at node 17.

Node 18 is a sink node, and the information transactions which leave it are lost from the system. Node 18 requires 500 transactions for the sink node to be realized the first time. This will constitute one simulation of the system. Node 18 also collects "first" statistics. First statistics are the first time the node is realized during a simulation. Once the actual machines leave node 14 they pass through node 15 which collects between statistics and node 16 which marks the machines with a new reference time for the next failure repair cycle.

Q-GERTS Simulation of the Repairman Problem

Using the Q-GERTS computer program the network model of the repairman problem was simulated ten times. Each simulation consisted of one realization of the sink node, which represents 500 machine repairs. The complete printout for this simulation is contained in the Appendix. The printout is organized into three sections: (1) Echo check of the input date (2) Final Results for First Simulation and (3) Final results for 10 simulations.

The accuracy of the model can be determined by comparing the simulation results to the theoretical results from Chapter 2. In some simulations it is difficult to determine whether the model has actually reached a steady state during the simulation. Even if the model reaches a steady state, the statistics are sometimes distorted by the data collected during transient behavior of the system. For this model it was determined that a good measure of steady state would be a comparison of the effective failure rate of the machines to the effective repair rate of the repairman. The mean effective failure rate of the machines was previously defined as λ_{eff}. Therefore, the mean effective repair rate will be defined as μ_{eff}. Statistically, the system is in a steady state when the distributions of effective failures and effective repairs are identically distributed. For this simulation statistics taken at node 7 represent the time between failures of the machines, and those taken at node 15 represent time between repairs. These in effect measure $1/\lambda_{eff}$ and $1/\mu_{eff}$ respectively. From the computer output the mean and standard deviations are:

	MEAN	STANDARD DEVIATION
$1/\lambda_{eff}$	0.6356 hours	0.5801 hours
$1/\mu_{eff}$	0.6371 hours	0.5744 hours

In addition, the histograms for nodes 7 and 15 are almost identical. Therefore, the statistical results of this simulation should closely approximate the steady state situation.

Using the statistics nodes of the model and the automatic queue and server statistics, the steady state results of the Q-GERT simulation will be developed in the following six measures.

1. L_q - Expected number of machines in the Queue

   The mean of the average number in the queue is 0.6353 machines.

2. P_0 - Probability of 0 Machines in the System

   The mean of the average server utilization is 0.7853. This says that the probability of the repairman being busy is .7853. Therefore, the probability of the repairman being idle is (1-.7853).

   $$P_0 = .2147$$

3. λ_{eff} - Effective Arrival Rate of Machines to the Queue.
   From the between statistics taken at node 7, the expected time between failure of the three machines is 0.6356 hours. Therefore,

   $$\lambda_{eff} = \frac{1}{.6356} = 1.5733 \text{ machines/hour}$$

4. W_s - Expected Time in the Repair System for each Machine.

   The interval statistic at node 17 measures the total time for a complete cycle for every machine. A cycle consists of operation, failure, waiting, and repair. Node 9 measures the time of operations for each machine during one cycle. Therefore:

   $$W_s = \text{Expected cycle time - expected operation time}$$

   $$W_s = 1.9051 - .9973$$

   $$W_s = .9078 \text{ hours}$$

5. L_s = Expected Number of Machines in the Repair System

   The expected number of machines in the repair system is the sum of the expected number waiting for repair and the expected number being repaired. The expected number waiting is L_q, and the expected number being repaired is the same as the average server utilization.

$$L_s = L_q + \text{Repairman Utilization}$$

$$L_s = .6353 + .7853$$

$$L_s = 1.4206 \text{ Machines}$$

6. W_q = Expected Waiting Time in the Queue

Both L_q and λ_{eff} have already been determined. Therefore,

$$W_q = \frac{.6353}{1.5733} = .4079 \text{ hours}$$

These results are compared to the theoretical results in Table 5. The comparison shows that the simulation results are very close to the theoretical. This establishes the Q-GERT network as a reasonable model of the (M/M/1):(GD/3/3) repairman problem.

TABLE 5

Comparison of steady state results by simulation and by derived formulas

Steady State Parameter	By Simulation	By Derived Formula
λ_{eff}	1.5733	1.5789
W_s	0.9078	0.9000
L_s	1.4206	1.4211
W_q	0.4079	0.4000
L_q	0.6353	0.6316
P_0	0.2147	0.2105

The output for the Q-GERTS simulation also contains other useful information. The standard deviation, standard deviation of the mean, and coefficient of variation are given for each statistic node. Also listed are the minimum and maximum value of that statistic. For example, from the interval statistic at node 9 it is found that the minimum and maximum times spent in operation by any machine was 0.0005 and 8.4923 hours respectively.

Counter statistics are computed in relation to each statistic node. In this problem the counters only have meaning in relation to the sink node (node 18). Counters 1, 2, and 3 count the number of times machines 1, 2, and 3 fail during one simulation of the network. The mean number of failures after ten simulations are:

Machine 1 - 171.8 Failures/simulation

Machine 2 - 165.3 Failures/simulation

Machine 3 - 163.5 Failures/simulation

The average number in the queue nodes has a standard deviation and the minimum and maximum values are recorded. The statistics on the servers includes the longest periods idle and busy. For the repairman of this example, the longest idle period was 2.8083 hours and the longest busy period was 12.5053 hours.

Using the same approach that was used in the development of the Q-GERT model for this example, more complex repairman problems can be simulated. Because each individual machine can always be identified, the assumption of identical machines and repairmen is no longer necessary. Also, the Poisson failure rate and exponential repair time assumption is no longer needed. Other time distributions can be used for each machine. The use of spare machines can be introduced, and priorities can be assigned so that more important machines spend less time in the repair system. These capabilities combined with the logical network depiction of the system makes the Q-GERT technique very useful in the analysis of repairman type problems.

CHAPTER 5

SUMMARY AND CONCLUSIONS

Comparison of Models

This study has concentrated on the development of graphical or network approaches to the solution of the repairman type queueing problem. The more traditional approach of using the derived formulas of the Poisson queueing model was presented first. The steady state solution of these formulas was used to verify the results of the graphical models as they were developed. Table 6 summarizes these steady state results using the three different techniques. Using the Poisson model as a standard it can be seen that the GERT stochastic network approach can provide the same steady state measures of the system. In the GERT approach the other measures of the steady state were developed using the steady state probabilities. Therefore, the numerical differences that occured in P_1 and P_2 are also evident in the other measures.

In the Q-GERTS simulation the measures of the steady state were developed using the statistical capabilities of the simulation technique. The values of P_1, P_2, and P_3 are not easily determined. From the comparison with the Poisson model it can be seen that the simulation provides a very good approximation of the theoretical system.

TABLE 6

Comparison of steady state results using Poisson Models, GERT, and Q-GERTS

	Poisson Model Derived Formulas	GERT	Q-GERTS Simulation
λ_{eff} (Machines/hour)	1.5789	1.582	1.5733
W_s (hours)	0.9000	0.896	0.9078
L_s (machines)	1.4211	1.418	1.4206
W_q (hours)	0.4000	0.397	0.4079
L_q (machines)	0.6316	0.628	0.6353
P_0	0.2105	0.210	0.2147
P_1	0.3158	0.319	
P_2	0.3158	0.314	
P_3	0.1579	0.157	

For those problems which do not violate the assumptions of the Poisson model, it is certainly easier to determine the steady state results using the derived formulas. However, the assumptions are quite restrictive and most repairman systems do not meet them all. For example, a system with a spare machine cannot be analyzed using the Poisson queueing model. In terms of understanding the problem, the Poisson model is the least desirable. It does not treat the problem as a system or provide a graphical depiction of the problem.

The GERT model of the repairman problem is constructed as a stochastic network. This provides a graphical representation of the problem as a system, and the analysis of the system develops a clearer understanding of the problem. The GERT network for the problem is very similar to the system state diagram of the Markov process, which is also a stochastic representation. Information about the system is no longer limited to the steady state as shown in Chapter 3. The GERT model as developed in this study is subject to the same assumptions as the Poisson model, however this does not preclude the development of GERT models using distributions other than Poisson and exponential. Whitehouse (1973) has done work in this area.

The GERT analysis makes it possible to develop additional information about the repairman problem. The analysis is no longer restricted to the steady state as with the Poisson model. For every state of the system the probability, expected time, and variance of the time to move to every other state can be predicted. For the example discussed in Chapter 3 it was found that once one machine failed, the probability of getting all machines operating before complete failure of the system was 0.752. The expected time until full operation was 0.542 hours, with a variance of 0.404 hours. This type of information makes it possible to predict the actual performance of the machine system over a short period of time.

The Q-GERT simulation technique is potentially the most useful model of the three. The assumptions of the Poisson and GERT models are no longer necessary. Actual machine operations can be realistically simulated using Q-GERT. Other simulation techniques are available that can simulate the same problems. However, they do not provide a graphical description of how the system operates. The network approach simplifies the formulation of the model. Once the network is constructed it provides a clear, logical picture of the system being simulated. One disadvantage is that the symbology of Q-GERT is not obvious, and it does require some study. Also, an analysis of the network requires the use of the Q-GERTS computer program.

Areas for Further Research

In the use of GERT for analysis of the repairman problem the first area of concern should be a model that does not require the Poisson/exponential relationship between the failure rate and repair time.

The use of the Poisson distribution for the machine failure rate implies randomly distributed failures. In many cases this is an acceptable assumption. The assumption of exponential repair times may not occur as frequently in actual systems. Whitehouse (1973) discusses two possible GERT representations of the repairman problem with other distributions for the repair times. However, he does not attempt to analyze the system using these models.

For the Q-GERT model the most challenging area seems to be the inclusion of regularly scheduled maintenance and in spections in the model. Another area of concern is the statistical output of the Q-GERTS program and how it can best be used. For example, counter statistics are generated in relation to each statistics node. As was demonstrated in Chapter 4, the interpretation of these statistics is not completely understood.

APPENDIX

Computer Output for Q-GERTS Simulation
of Repairman Problem

GERT SIMULATION PROJECT 1 BY NEWTON
DATE 12/16/1974 QGERT VERSION 6/22/74

** NETWORK DESCRIPTION **

NUMBER OF SINK NODES IS 1
STATISTICS COLLECTED ON 5 NODES %INCLUDING SINKS<
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
NUMBER OF SIMULATIONS REQUESTED IS 10
INITIAL RANDOM NUMBER IS 1783
NUMBER OF ATTRIBUTES PER TRANSACTION IS 2
MODIFICATIONS - NO
BEGINNING, ENDING TRACE RUNS - 0, 0
SCALE FACTOR FOR SC DISTRIBUTION IS 1.0000
TIME FROM WHICH STATISTICS ARE KEPT 60.0000
TYPE OF HISTOGRAM DESIRED - PLOTTED
FIRST RUN PRINTOUT OPTION - YES
LIST ALL INPUT OPTION - YES
EXECUTION OPTION - E3

** NODE CHARACTERISTICS **

NODE	NUMBER OF REQUIREMENTS	NO. OF SUBSEQUENT REQUIREMENTS	OUTPUT TYPE	MARK	TYPE OF STATISTICS	ATTRIBUTE CHOICE %CRITERION<	%ATTR. NO.<	HALTING DESIRED AT RELEASE
2	0	1	A			LAST	0	
4	1	1	F			LAST	0	
6	1	1	D			LAST	0	
8	1	1	D			LAST	0	
10	1	1	D			LAST	0	
7	1	1	D		B	LAST	0	
9	1	1	D		I	LAST	0	
15	1	1	D		B	LAST	0	
17	1	1	D		I	LAST	0	
14	1	1	D			LAST	0	
16	1	1	D	M		LAST	0	
18	500	9999	D		F	LAST	0	

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
18

STATISTICS COLLECTED ALSO ON NODES
17 15 9 7

** QUEUE NODES **

I NODE I	INITIAL NO. IN QUEUE	MAXIMUM NO. ALLOWED	OUTPUT TYPE	PRIORITY SCHEME	MAY BLOCK INCIDENT SERVERS	NODE FOR BALKERS	ASSOCIATED SELECTOR I I
12	0	9999	D	FIFO	NO	-1	0

ATTRIBUTE ASSIGNMENT INFORMATION

NODE NUMBER	ATTRIBUTE NUMBER	DISTRIBUTION TYPE	PARAMETER SPECIFICATION
2	1	IN	1

PARAMETER SPECIFICATION

PARAMETER SET	PARAMETERS 1	2	3	4
1	0.0	0.0	0.0	0.0
2	1.0000	0.0	50.0000	0.0
3	0.5000	0.0	50.0000	0.0

** ACTIVITY DESCRIPTION **

START NODE	END NODE	DISTRIBUTION TYPE	PARAM SPEC	ACTIVITY NUMBER	COUNT TYPE	PROBABILITY	CONDITIONAL BRANCHING INFORMATION %CODE<	%FIELD A<	%FIELD B<	%ORDER<
2	2	SC	0	0	0	1.0000	ALV	1.0	2.0	1
2	4	SC	0	0	0	1.0000	AGV	1.0	0.0	2
4	6	SC	0	0	0	1.0000	AEV	1.0	1.0	1
4	8	SC	0	0	0	1.0000	AEV	1.0	2.0	2
4	10	SC	0	0	0	1.0000	AEV	1.0	3.0	3
6	7	EX	2	0	1	1.0000				
7	9	SC	0	0	0	1.0000				
8	7	EX	2	0	2	1.0000				
9	12	SC	0	0	0	1.0000				
10	7	EX	2	0	3	1.0000				
12	14	EX	3	98	0*	1.0000				
14	15	SC	0	0	0	1.0000				
14	17	SC	0	0	0	1.0000				
15	16	SC	0	0	0	1.0000				
16	4	SC	0	0	0	1.0000				
17	18	SC	0	0	0	1.0000				

```
*** INPUT CARDS ***

GEN,NEWTON,1,12,16,1974,1,5,1,10,1783,2,,,,1.,60.,P,F,*
SOU,2,0,1,A*
VAS,2,1,IN,1*
REG,4,1,1,F*
REG,6,1,1,D*
REG,8,1,1,D*
REG,10,1,1,D*
STA,7,1,1,,B,.2,.2*
STA,9,1,1,,I,.2,.2*
STA,15,1,1,,B,.2,.2*
STA,17,1,1,,I,.3,.3*
QUE,12,0,(9).4469,.0108*
REG,14,1,1,D*
REG,16,1,1,D,M*
SIN,18,500,(7)282,3.8*
PAR,2,1.,0.,50.*
PAR,3,.5,0.,50.*
ACT,2,2,,,,,,ALV,1,2*
ACT,2,4,,,,,,AGV,1,0*
ACT,4,6,,,,,,AEV,1,1*
ACT,4,8,,,,,,AEV,1,2*
ACT,4,10,,,,,,AEV,1,3*
ACT,6,7,EX,2,,1*
ACT,8,7,EX,2,,2*
ACT,10,7,EX,2,,3*
ACT,7,9*
ACT,9,12*
ACT,12,14,EX,3*
ACT,14,15*
ACT,15,16*
ACT,16,4*
ACT,14,17*
ACT,17,18*
FIN*

*** NO ERRORS DETECTED IN INPUT DATA ***

*** EXECUTION WILL BE ATTEMPTED ***
```

FINAL RESULTS FOR FIRST SIMULATION

TOTAL ELAPSED TIME # 320.2131

NODE	MEAN	STAT TYPE
18	320.2131	F
17	1.9194	I
15	0.6401	B
9	1.0081	I
7	0.6395	B

NO. OF COMPLETIONS OF ACTIVITIES WITH COUNTERS PRIOR TO REALIZATION OF STATISTICS NODES

NODE	MEAN	COUNTERTYPE
18	166.0000	1
18	174.0000	2
18	160.0000	3
17	82.4540	1
17	85.9040	2
17	82.9520	3
15	82.6192	1
15	86.0761	2
15	83.1162	3
9	82.1500	1
9	85.6320	2
9	82.7180	3
7	82.3146	1
7	85.8036	2
7	82.8818	3

NUMBER IN Q-NODE

NODE NO.	MEAN	MIN.	MAX.
12	0.6364	0.	2.

SERVER UTILIZATION

SERVER NO.	MEAN	LONGEST PERIOD IDLE	LONGEST PERIOD BUSY
98	0.7936	2.0159	11.8879

GERT SIMULATION PROJECT 1 BY NEWTON
DATE 12/ 16/ 1974

FINAL RESULTS FOR 10 SIMULATIONS

NODE	PROBABILITY	MEAN	STD.DEV.	STD.DEV. OF MEAN	COEFF. VAR.	NO OF OBS.	MIN.	MAX.	STAT TYPE
18	1.0000	318.6440	11.9898	3.7915	0.0376	10.	305.7441	346.7178	F
17	1.0000	1.9051	1.2806	0.0181	0.6722	5000.	0.0280	9.8728	I
15	1.0000	0.6371	0.5744	0.0081	0.9016	4990.	0.0000	5.1792	B
9	1.0000	0.9973	1.0001	0.0141	1.0029	5006.	0.0005	8.4923	I
7	1.0000	0.6356	0.5801	0.0082	0.9126	4996.	0.0	4.9637	B

AVERAGE NO. OF COMPLETIONS OF ACTIVITIES WITH COUNTERS PRIOR TO REALIZATION OF STATISTICS NODES

NODE	COUNTERTYPE	MEAN	STD.DEV.	NO OF OBS.	MIN.	MAX.
18	1	171.8000	8.5479	10.	159.0000	188.0000
18	2	165.3000	9.0068	10.	154.0000	184.0000
18	3	163.5000	8.1684	10.	154.0000	182.0000
17	1	84.9492	49.8638	5000.	0.0	188.0000
17	2	84.0222	47.8014	5000.	0.0	184.0000
17	3	82.3272	47.4264	5000.	0.0	182.0000
15	1	85.1182	49.7704	4990.	0.0	188.0000
15	2	84.1898	47.7023	4990.	0.0	184.0000
15	3	82.4908	47.3327	4990.	0.0	182.0000
9	1	84.7849	49.9211	5006.	0.0	188.0000
9	2	83.8508	47.8545	5006.	0.0	184.0000
9	3	82.1652	47.4978	5006.	0.0	182.0000
7	1	84.9538	49.8279	4996.	0.0	188.0000
7	2	84.0180	47.7560	4996.	0.0	184.0000
7	3	82.3291	47.4037	4996.	0.0	182.0000

AVERAGE NUMBER IN Q-NODE

NODE NO.	MEAN	STD.DEV.	NO. OF OBS.	MIN.	MAX.
12	0.6353	0.0628	10.	0.5385	0.7259

NUMBER IN Q-NODE

MIN.	MAX.
0.	2.

AVERAGE SERVER UTILIZATION

SERVER NO.	MEAN	STD.DEV.	NO. OF OBS.	MIN.	MAX.
98	0.7853	0.0149	10.	0.7614	0.8033

TIME PERIODS OF SERVER

LONGEST PERIOD IDLE	LONGEST PERIOD BUSY
2.8083	12.5053

```
STAT HISTOGRAM FOR NODE   18

OBSV     RELA      CUML          UPPER       0         20        40        60        80        100
FREQ     FREQ      FREQ     BOUND OF CELL    I....I....I....I....I....I....I....I....I....I....I
   0     0.0       0.0          282.00       I                                                  I
   0     0.0       0.0          285.80       I                                                  I
   0     0.0       0.0          289.60       I                                                  I
   0     0.0       0.0          293.40       I                                                  I
   0     0.0       0.0          297.20       I                                                  I
   0     0.0       0.0          301.00       I                                                  I
   0     0.0       0.0          304.80       I                                                  I
   2     0.200     0.200        308.60       I**********                                        I
   0     0.0       0.200        312.40       I         C                                        I
   3     0.300     0.500        316.20       I***************             C                     I
   2     0.200     0.700        320.00       I**********                            C           I
   1     0.100     0.800        323.80       I*****                                      C      I
   0     0.0       0.800        327.60       I                                           C      I
   1     0.100     0.900        331.40       I*****                                           C I
   0     0.0       0.900        335.20       I                                                C I
   0     0.0       0.900        339.00       I                                                C I
   0     0.0       0.900        342.80       I                                                C I
   0     0.0       0.900        346.60       I                                                C I
   1     0.100     1.000        350.40       I*****                                            CI
   0     0.0       1.000        354.20       I                                                 CI
                                             I....I....I....I....I....I....I....I....I....I....I
TOTAL     10
```

STAT HISTOGRAM FOR NODE 17

```
 OBSV    RELA    CUML        UPPER        0         20        40        60        80        100
 FREQ    FREQ    FREQ    BOUND OF CELL    I....I....I....I....I....I....I....I....I....I....I
  161   0.032   0.032             0.30    I**                                               I
  366   0.073   0.105             0.60    I****C                                            I
  567   0.113   0.219             0.90    I******    C                                      I
  569   0.114   0.333             1.20    I******          C                                I
  609   0.122   0.454             1.50    I******                C                          I
  494   0.099   0.553             1.80    I*****                      C                     I
  469   0.094   0.647             2.10    I*****                          C                 I
  383   0.077   0.724             2.40    I****                               C             I
  330   0.066   0.790             2.70    I***                                    C         I
  219   0.044   0.833             3.00    I**                                       C       I
  195   0.039   0.872             3.30    I**                                         C     I
  149   0.030   0.902             3.60    I*                                           C    I
  113   0.023   0.925             3.90    I*                                            C   I
   94   0.019   0.944             4.20    I*                                             C  I
   74   0.015   0.958             4.50    I*                                             C  I
   45   0.009   0.967             4.80    I                                               C I
   42   0.008   0.976             5.10    I                                               C I
   25   0.005   0.981             5.40    I                                                CI
   17   0.003   0.984             5.70    I                                                CI
   79   0.016   1.000             6.00    I*                                               CI
                                          I....I....I....I....I....I....I....I....I....I....I
TOTAL  5000
```

STAT HISTOGRAM FOR NODE 15

OBSV FREQ	RELA FREQ	CUML FREQ	UPPER BOUND OF CELL
1174	0.235	0.235	0.20
965	0.193	0.429	0.40
827	0.166	0.594	0.60
568	0.114	0.708	0.80
439	0.088	0.796	1.00
288	0.058	0.854	1.20
218	0.044	0.898	1.40
160	0.032	0.930	1.60
121	0.024	0.954	1.80
71	0.014	0.968	2.00
54	0.011	0.979	2.20
34	0.007	0.986	2.40
22	0.004	0.990	2.60
14	0.003	0.993	2.80
9	0.002	0.995	3.00
9	0.002	0.997	3.20
3	0.001	0.997	3.40
3	0.001	0.998	3.60
7	0.001	0.999	3.80
4	0.001	1.000	4.00

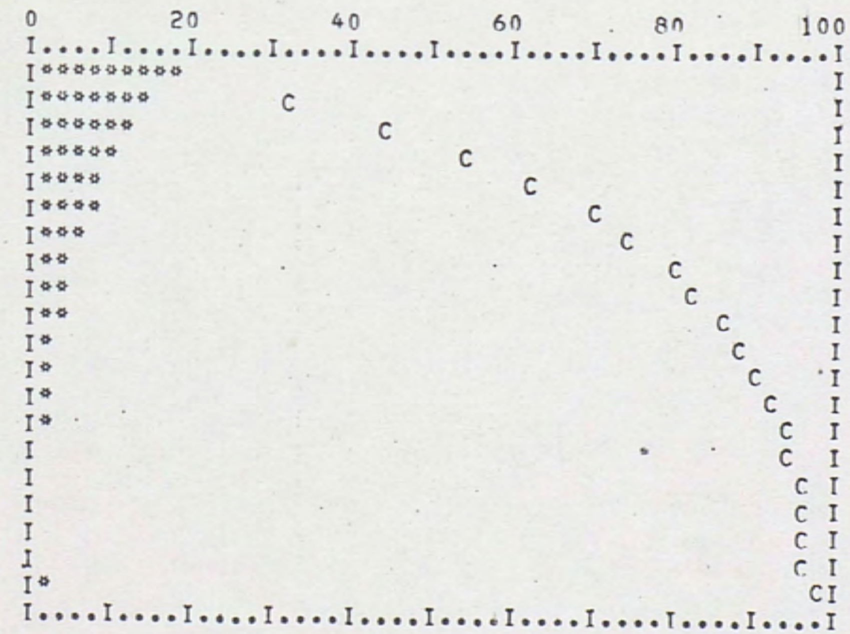
TOTAL 4990

```
0         20        40        60        80        100
I....I....I....I....I....I....I....I....I....I....I
I************                                     I
I**********          C                            I
I********                    C                    I
I******                            C              I
I****                                  C          I
I***                                      C       I
I**                                         C     I
I**                                           C   I
I*                                             C  I
I*                                              C I
I*                                              C I
I                                                CI
I                                                CI
I                                                CI
I                                                CI
I                                                CI
I                                                CI
I                                                CI
I                                                CI
I                                                CI
I....I....I....I....I....I....I....I....I....I....I
```

STAT HISTOGRAM FOR NODE 9

OBSV FREQ	RELA FREQ	CUML FREQ	UPPER BOUND OF CELL
918	0.183	0.183	0.20
712	0.142	0.326	0.40
599	0.120	0.445	0.60
510	0.102	0.547	0.80
412	0.082	0.629	1.00
377	0.075	0.705	1.20
264	0.053	0.757	1.40
230	0.046	0.803	1.60
170	0.034	0.837	1.80
155	0.031	0.868	2.00
120	0.024	0.892	2.20
98	0.020	0.912	2.40
95	0.019	0.931	2.60
64	0.013	0.944	2.80
45	0.009	0.953	3.00
38	0.008	0.960	3.20
37	0.007	0.968	3.40
25	0.005	0.973	3.60
18	0.004	0.976	3.80
119	0.024	1.000	4.00

TOTAL 5006



STĀT HISTOGRAM FOR NODE   T

```
 OBSV    RELA    CUML       UPPER        0        20        40        60        80        100
 FREQ    FREQ    FREQ   BOUND OF CELL    I....I....I....I....I....I....I....I....I....I....I
 1157   0.232   0.232       0.20         I************                                     I
 1017   0.204   0.435       0.40         I**********          C                            I
  792   0.159   0.594       0.60         I********                    C                    I
  600   0.120   0.714       0.80         I******                            C              I
  418   0.084   0.797       1.00         I****                                  C          I
  322   0.064   0.862       1.20         I***                                       C      I
  201   0.040   0.902       1.40         I**                                          C    I
  146   0.029   0.931       1.60         I*                                            C   I
  109   0.022   0.953       1.80         I*                                             C  I
   68   0.014   0.967       2.00         I*                                              C I
   50   0.010   0.977       2.20         I*                                              C I
   28   0.006   0.982       2.40         I                                                CI
   32   0.006   0.989       2.60         I                                                CI
   10   0.002   0.991       2.80         I                                                CI
   12   0.002   0.993       3.00         I                                                CI
   12   0.002   0.996       3.20         I                                                CI
    9   0.002   0.997       3.40         I                                                CI
    5   0.001   0.998       3.60         I                                                CI
    2   0.000   0.999       3.80         I                                                CI
    6   0.001   1.000       4.00         I                                                CI
                                         I....I....I....I....I....I....I....I....I....I....I
TOTAL   4996
```

STAT HISTOGRAM FOR NODE 12

```
OBSV    RELA     CUML       UPPER         0         20        40        60        80       100
FREQ    FREQ     FREQ   BOUND OF CELL     I....I....I....I....I....I....I....I....I....I....I
  0    0.0      0.0             0.45      I                                                  I
  0    0.0      0.0             0.47      I                                                  I
  0    0.0      0.0             0.49      I                                                  I
  0    0.0      0.0             0.51      I                                                  I
  0    0.0      0.0             0.53      I                                                  I
  2    0.200    0.200           0.55      I**********                                        I
  0    0.0      0.200           0.57      I         C                                        I
  0    0.0      0.200           0.59      I         C                                        I
  1    0.100    0.300           0.61      I*****        C                                    I
  0    0.0      0.300           0.63      I             C                                    I
  3    0.300    0.600           0.64      I***************              C                    I
  0    0.0      0.600           0.66      I                             C                    I
  2    0.200    0.800           0.68      I**********                             C          I
  1    0.100    0.900           0.70      I*****                                       C     I
  0    0.0      0.900           0.72      I                                            C     I
  1    0.100    1.000           0.74      I*****                                            CI
  0    0.0      1.000           0.76      I                                                 CI
  0    0.0      1.000           0.78      I                                                 CI
  0    0.0      1.000           0.80      I                                                 CI
  0    0.0      1.000           0.82      I                                                 CI
                                          I....I....I....I....I....I....I....I....I....I....I
TOTAL  10
```

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