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Optimization Analysis of a Simple Position Control System

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OPTIMIZATION ANALYSIS OF A SIMPLE POSITION CONTROL SYSTEM

BY

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LIST OF SYMBOLS

SYMBOL	DESCRIPTION
A	Plant description matrix
B	Driving function matrix
C	Controlled output matrix
D	Viscous friction in newton-meter-seconds per radian
DC	Direct Current voltage
E_e	Error, $(\theta_i - \theta_o)$ between output shaft angle and reference input shaft angle converted to volts
E_f	Voltage applied to motor field in volts
E_i	Required shaft angular displacement in volts
E_o	Motor shaft angular displacement in volts. (See θ_o)
$E_{\dot{\theta}}$	Velocity feedback in volts
G	Plant transfer function
H_{eq}	Equivalent feedback expression
I_a	Armature current in amperes
i_f	Field current in amperes
J	Inertia of the motor shaft including load in kilogram-meters^2 (Kg-m^2)
K	System gain factor, $(K = W_n^{\ 2})$
K_e	Potentiometer error constant in volts per radian
K_m	System gain factor adjusted by feedback, i.e., $K_m = K_v/1 + K_v k_2$
K_s	Motor speed constant in radians per second per volt

LIST OF SYMBOLS (continued)

SYMBOL	DESCRIPTION
K_t	Electrical torque constant in newton-meters per ampere
K_v	Velocity error constant in radians per second at the output shaft per radian error
k_1	Output shaft displacement feedback coefficient in volts per radian displacement
k_2	Output shaft velocity feedback coefficient in volts per radian per second
L	State function of Lagrange operator
L_f	Motor field inductance in henries
N	Worm-gear reduction ratio (30:1)
$\underline{P}$	Positive definite cost function matrix
p	Cost function (scalar)
$\bar{p}$	Cost function weighting factor
$\underline{Q}$	Semi-definite characteristic matrix of n(n+1)/2 elements
R_a	Motor armature resistance in ohms
R_f	Motor field resistance in ohms
s	Frequency domain operator
T	Time constant of model system in seconds
T_e	Electrical torque in newton-meters
t_f	Motor field electrical time constant in seconds
T_m	Mechanical torque in newton-meters
t_m	Motor mechanical time constant in seconds

LIST OF SYMBOLS (continued)

SYMBOL	DESCRIPTION
W_n	Natural frequency of the motor in radians per second
X_1	State variable for shaft angular displacement, θ_o
X_2	State variable for shaft angular velocity, $\dot{\theta}_o$
$\underline{X}$	Underscored variable represents a matrix of the variable
$\dot{X}$	Dotted variable represents derivative with respect to time
Y	State variable for controlled output variable
Z	Zeta, damping factor
γ	Reduced characteristic of n-elements to replace $\underline{Q}$ in performance index
Γ (s)	Synthetic transfer function
λ	Lagrange multiplier [8]
θ_e	Error, $(\theta_i - \theta_o)$ between output shaft angle and reference input shaft angle in radians
θ_m	Motor shaft angular displacement in radians at the armatures
θ_o	Motor shaft angular displacement in radians reduced by gear reduction ratio, N. (i.e., $\theta_o = \theta_m/N$)
θ_r	Input angular error causing one radian per second revolution of the motor shaft
Ø	State Transition Matrix or its elements

1. INTRODUCTION

1.1 Objective and Procedures

The main goal of this thesis is to analyze optimal control of a simple position control system by means of a first order system model and linear-quadratic state regulator theory. The technique used is that of selecting a single, forward system time constant for an (n-1)-order model of an n-order system which allows the use of optimal techniques without becoming involved in the complexities of solving the Hamilton-Jacobi equation or the Matrix Ricatti equation.

In this modeling approach, the physical interpretation of the performance index in terms of the desired first-order system forward time constant is used in conjunction with the Kalman equation to obtain the optimal control feedback coefficients and the closed loop gain.

The parameters selected for the position control system to be analyzed are based on the MS150 Modular Servo System which is a universal teaching aid and demonstration model for automatic control and feedback systems. The MS150 is described in detail in references [1] and [2] and is manufactured by FEEDBACK LTD. of England.

To attain the stated objective, it will first be necessary to evaluate the DC-motor characteristics of the power drive unit. After a discussion of the system in terms of DC-motor analysis, second-order differential equations and state equations in sections 2 and 3, modern control theory is discussed in section 4 as a background to the establishment of a model of the system. The optimal feedback coefficients are then determined by an application of the Kalman equation.

The time response of the model system is determined by classical methods, and the IBM 360 Continuous System Modeling Program, (360/CSMP) for several values of the control weighting factor, $\bar{p}^{2}$.

1.2 Computer Programs

In reference [3], J. L. Melsa has compiled a group of computer programs which may be used for analysis and design of linear control systems represented in state variable form as

$$\dot{\underline{X}}(t) = \underline{A}\underline{X}(t) + \underline{b}U(t) \tag{1-1}$$

$$U(t) = K[\theta_i(t) - \underline{K}^T\underline{X}(t)] \tag{1-2}$$

$$\underline{Y}(t) = \underline{C}^T\underline{X}(t) \tag{1-3}$$

The state variable programs included in reference [3] can be summarized as follows:

BASMAT - Basic matrix computation of determinant, inverse, characteristic equation, eigenvalues and state transition matrix.

RTESP - Rational time response of the linear feedback control system described above.

GTRESP - Graphical time response display for an arbitrary input.

SENSIT - Analysis of sensitivity to parameter variations.

STVARFD - State variable feedback program may be used to find both open and closed loop system transfer functions.

FRESP - Frequency response of a rational transfer function over a specified range of frequencies. Both Bode and Nyquist plots are available.

RTLOC - Root locus plots.

PRFEXP - Partial fraction expansion of rational transfer functions.

These programs were used throughout the background work of this thesis in addition to the IBM 360 Continuous System Modeling Program, (360/CSMP) which is detailed in reference [4]. Selected computer output prints are shown in Appendix 1.

2. SYSTEM DESCRIPTION AND ANALYSIS

2.1 Rationale of Section 2

The MS150 Modular Servo System is defined for purposes of this thesis as a simple position control with a small field-controlled DC motor as the plant driving force. The purpose of this section is to develop the system describing differential equations, transfer function and closed loop parameters based on resistance and inductance values of the MS150 DC-motor field and armature as measured in the laboratory.

The development begins in section 2.2 with fundamental definitions and proceeds to the normal form second-order system transfer function. In section 2.3, the method used to approximate a linear motor is discussed. The effect of velocity feedback on the system gain and transfer function is discussed in section 2.4.

2.2 System Dynamics

In a simple position control system, an error signal is used to energize the field of a DC-motor with constant armature excitation. The motor shaft torque is controlled by the voltage applied to the field terminals as the output controlled variable is moved to the desired position. The analysis then involves the electrical transients in the field circuit and the dynamics of the mechanical load. The following discussion draws extensively from references [1], [5], and [6]. All symbols used are listed and defined on pages vii through ix. A field controlled motor is shown schematically in Figure 2-1.

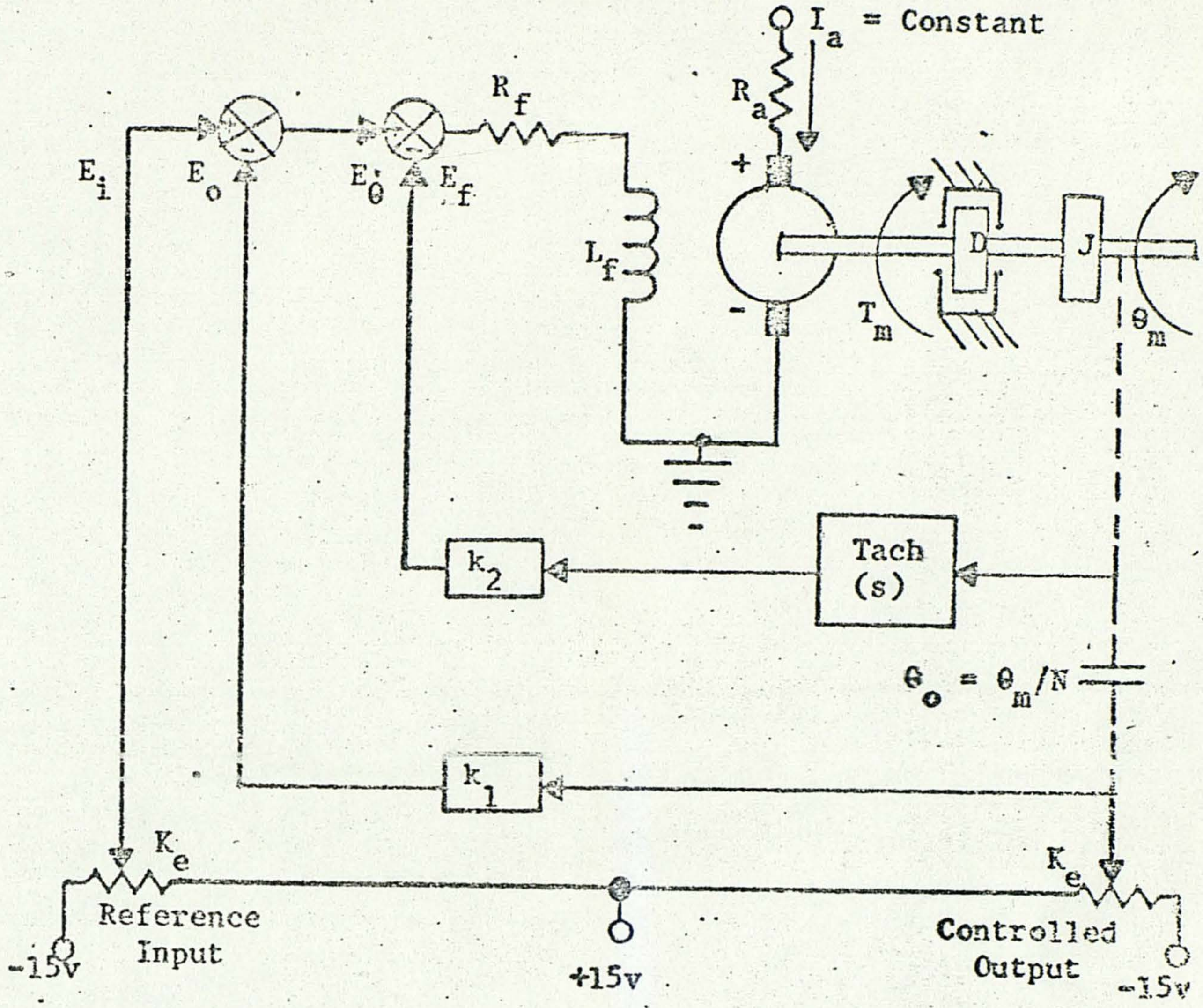


Figure 2-1 Field Controlled DC-Motor Positioning System

With torque as the link between the electrical and mechanical portions, the following equations can be written where a compatible system of units (MKS) is assumed. The electrical motor torque is proportional to the field current,

$$T_e(s) = K_t i_f(s) \qquad (2\text{-}1)$$

where the electrical torque and field current are expressed as frequency domain variables.

The field current can be found in terms of the field control voltage by application of Kirchoff's Voltage Law as,

$$i_f(s) = E_f(s)/(R_f + L_f s) \qquad (2\text{-}2)$$

where s represents differentiation with respect to time. Substitution into equation 2-1 gives,

$$T_e(s) = K_t E_f(s)/R_f + L_f s) \qquad (2\text{-}3)$$

The electrical motor torque (T_e) is used to accelerate the total inertia (J) of the motor armature and load and in overcoming the viscous friction torque (D). This relation is expressed in the frequency domain as,

$$T_e(s) = T_m(s) \qquad (2\text{-}4)$$

where,

$$T_m(s) = (Js^2 + Ds)\theta_m$$

With the above expressions combined, the torque and field current are eliminated from the expression and the system differential equation is,

$$K_t E_f(s)/(L_f s + R_f) = (Js^2 + Ds)\theta_m \qquad (2\text{-}5)$$

Algebraic manipulation and defining,

$$t_f = L_f/R_f \tag{2-6}$$

and,

$$t_m = J/D \tag{2-7}$$

allows the writing of equation (2-5) as,

$$K_t E_f(s)/t_f s + 1)DR_f = (t_m s^2 + s)\theta_m \tag{2-8}$$

To simplify the subsequent development, it is worthwhile to consider the time constants defined in equations (2-6) and (2-7). Manufacturer's data given in reference [1] and supplementary data measured in the laboratory are summarized in Table 2-1. From the data in Table 2-1, the rated torque of the motor is,

$$\text{Torque} = \text{HP}(746\ \text{watts/HP})/\text{Rated Speed(radians/second)} \tag{2-9}$$

$$\text{Torque} = (1/50)(746)/628$$

$$\text{Torque} = 2375.8 \times 10^{-5}\ \text{newton-meters.}$$

If a viscous load is assumed, it will have a torque curve which passes through the origin and the rated torque of the motor. Then the viscous damping constant (D) at the rated torque is,

$$D = \text{Rated Torque/Rated Speed} \tag{2-10}$$

$$D = 2375.8 \times 10^{-5}/628$$

$$D = 3.78 \times 10^{-5}\ \text{(newton-meter-second/radian).}$$

The manufacturer's data states the motor inertia is about 3.1×10^{-5} $Kg\text{-}m^2$ (see Table 2-1). With the load inertia equal to the motor inertia, the inertial time constant is,

$$t_m = J/D \tag{2-11}$$

$t_m = (6.2 \times 10^{-5})/(3.78 \times 10^{-5})$

$t_m = 1.64$ seconds.

Table 2-1

MS150 DC MOTOR DATA

MANUFACTURERS DATA:

Name: FRACMO

Mfg. Fractional HP Motors Lmtd., Enfield, England

Volts: 24DC

Amps: 1.5 A

RPM: 6000

HP: 1/50

Inertia: 3.1×10^{-5} Kg-m/amp

LABORATORY DATA:

ARMATURE:	SYMBOL	VALUE
Resistance, D.C.	R_a	2.94 ohms
Inductance	L_a	11.79 mh
FIELD:		
Resistance, D.C.	R_f	4.4 ohms
Inductance	L_f	25 mh

Also from Table 2-1, the motor field time constant is,

$$t_f = L_f/R_f \qquad (2\text{-}12)$$

$t_f = 0.02483/4.4$

$t_f = 5.64 \times 10^{-3}$ seconds.

From the equations (2-11) and (2-12) it can be observed that the motor field electrical time constant, t_f, is about three orders of magnitude smaller than the inertial time constant, t_m. That is,

$$t_f \lll t_m$$

$$0.00564 \lll 1.64 \tag{2-13}$$

In other words, the effect of the motor field time constant is negligible in comparison to the relatively sluggish mechanical time constant. Therefore, to simplify subsequent system considerations, t_f may be neglected without serious error and equation (2-2) becomes,

$$i_f(s) = E_f(s)/R_f \tag{2-14}$$

To further simplify equation (2-8), it is convenient to define a speed constant, K_s as,

$$K_s = K_t/DR_f \tag{2-15}$$

where K_s indicates how fast the motor will rotate per volt applied at DC. Substitution of these results in equation (2-8) gives,

$$K_s E_f(s) = (t_m s^2 + s)\theta_m \tag{2-16}$$

In a closed loop system, the error signal is used to operate the forward path as shown in Figure 2-1. From the figure, it is possible to define the reference input, $E_i = K_e\theta_i$, the controlled output, $E_o = k_1 K_e \theta_o$ and the resulting error as,

$$E_e = K_e(\theta_i - k_1\theta_o) \tag{2-17}$$

where K_e is the conversion factor to change radians to volts. For simplicity in the discussion to follow, let $k_1 = k_2 = 1.0$. Therefore,

$$E_f(s) = K_e(\theta_i - \theta_o) \tag{2-18}$$

where one radian of error gives K_e volts and, since $\theta_o = \theta_m/N$, equation (2-16) is,

$$K_s[K_e(\theta_i - \theta_o)/N] = (t_m s^2 + s)\theta_o \tag{2-19}$$

By defining a velocity error constant $K_v = (K_s K_e/N)$, we have,

$$K_v(\theta_i - \theta_o) = t_m s^2 + s)\theta_o \tag{2-20}$$

or,

$$K_v \theta_i = (t_m s^2 + s + K_v)\theta_o \tag{2-21}$$

One further rearrangement of equation (2-20) expresses the system transfer function as,

$$\theta_o/\theta_i = K_v/(t_m s^2 + s + K_v) \tag{2-22}$$

Equation (2-22) is a second order differential equation. If this equation is compared to the normalized form of a second order equation,

$$Y/X = W_n^2/[s^2 + (2ZW_n)s + W_n^2] \tag{2-23}$$

it can be seen that the parameter W_n, the undamped natural frequency, and Z, the damping factor are given by

$$W_n = \sqrt{K_v/t_m} \tag{2-24}$$

and,

$$Z = 1/2\sqrt{K_v t_m} \tag{2-25}$$

Some discussion of K_v is called for before proceeding to other considerations. It is referred to as the velocity error constant and gives the output shaft speed per unit error. This factor determines the steady state following accuracy and the magnitude of the peak in the open loop frequency response. Increasing K_v reduces the steady state following error, but increases the tendency to oscillate. Altering any of the factors that make up K_v will modify K_v. It is not necessary to know the individual factors since K_v can be determined directly. It can be measured by,

1. opening the position feedback loop,
2. setting the controller gain to zero,
3. zeroing the preamplifier so that no rotation occurs,
4. setting in an error at the input so that the motor will rotate,
5. advancing the controller gain to 50 percent of full-scale,
6. adjusting the input error until the output rotates at one revolution per second as determined by a strobe light.

The input error required to cause the one revolution per second rotation will be in the range of 10 to 20 degrees. Hence, when the controller gain setting is maximum, $K_v(\max)$ is,

$$K_v(\max) = 4\pi/(\theta^o_r/57.3^o) \qquad (2\text{-}26)$$

The angle, θ^o_r, with the existing amplification, was found to be about 17.5 degrees average so that,

$$K_v(\max) = 4\pi/(17.5/57.3) \qquad (2\text{-}27)$$

$$K_v(\max) = 41.2.$$

The controller gain scale can, therefore, be calibrated directly in values of K_v.

2.3 MS150 System Time Constant

The motor-tachometer unit used in the MS150 consists of a fractional (one-fiftieth) horsepower direct current motor with a 30:1 worm gear and a permanent magnet tachometer as an integral unit. In small machines such as this, the brush friction is the dominant frictional force and is independent of speed. Therefore, the frictional characteristics are not viscous and the motor cannot be considered linear. The manufacturer of the MS150 has included compensating networks and inertia effects in providing what is termed a "defined time constant" to approximate a "linear" motor. The effect of brush friction is characterized as a motor time constant which varies inversely with the actual motor speed and is simulated in the MS150 by a phase advance network in series with a tachometer feedback signal. With this arrangement, the forward path time constant, t_m, is reduced to be about 0.25 seconds substantially independent of frictional effects. Therefore, the system time constant, $t_m = J/D$, in equation (2-11) is reduced to be 0.25 seconds by the above arrangement.

Following procedures outlined in reference [1], section 2.2.1, the MS150 system time constant was measured and found to be 0.25 seconds as expected. The method used consists of applying a square wave input to the system and observing the speed response with an oscilloscope. The motor speed responds exponentially and the time constant is estimated from the time required to rise to 63 percent of the final value.

2.4 Tachometer Feedback

As pointed out before, increasing K_v reduces the steady state following error, but increases the tendency to oscillate. In order to adjust system parameters to improve performance, one method is to introduce a signal from a tachometer in the forward path as mentioned in section 2.3. Since the signal is proportional to output shaft speed, it is termed "velocity feedback" and provides a powerful means of stabilization and improved transient response.

The schematic block diagram for this method is shown in Figure 2-2(a). (The parameters, $K_v = 41.2$ and $t_m = 0.25$ were defined in the previous sections.) The velocity feedback signal from the tachometer is returned to the input with k_2 representing a potentiometer across the tachometer output. The polarity of the returned signal is negative. The position feedback control is shown as k_1, which is across the output of an operational amplifier. The output position signal is returned for comparison to the desired input to generate the error E_e.

The effect of tachometer feedback on the forward gain and time constant can be shown by deriving the transfer function of the inner loop and single integration shown in Figure 2-2(a) where unity position feedback is assumed, (i.e., $k_1 = 1$). Since,

$$G(s) = Kv/s(t_m s + 1) \tag{2-28}$$

and,

$$\theta_o(s)/E_e(s) = G(s)/(1 + G(s)H(s)) \tag{2-29}$$

Then,

$$\theta_o(s)/E_e(s) = K_v/s[t_m s + (1 + K_v k_2)] \tag{2-30}$$

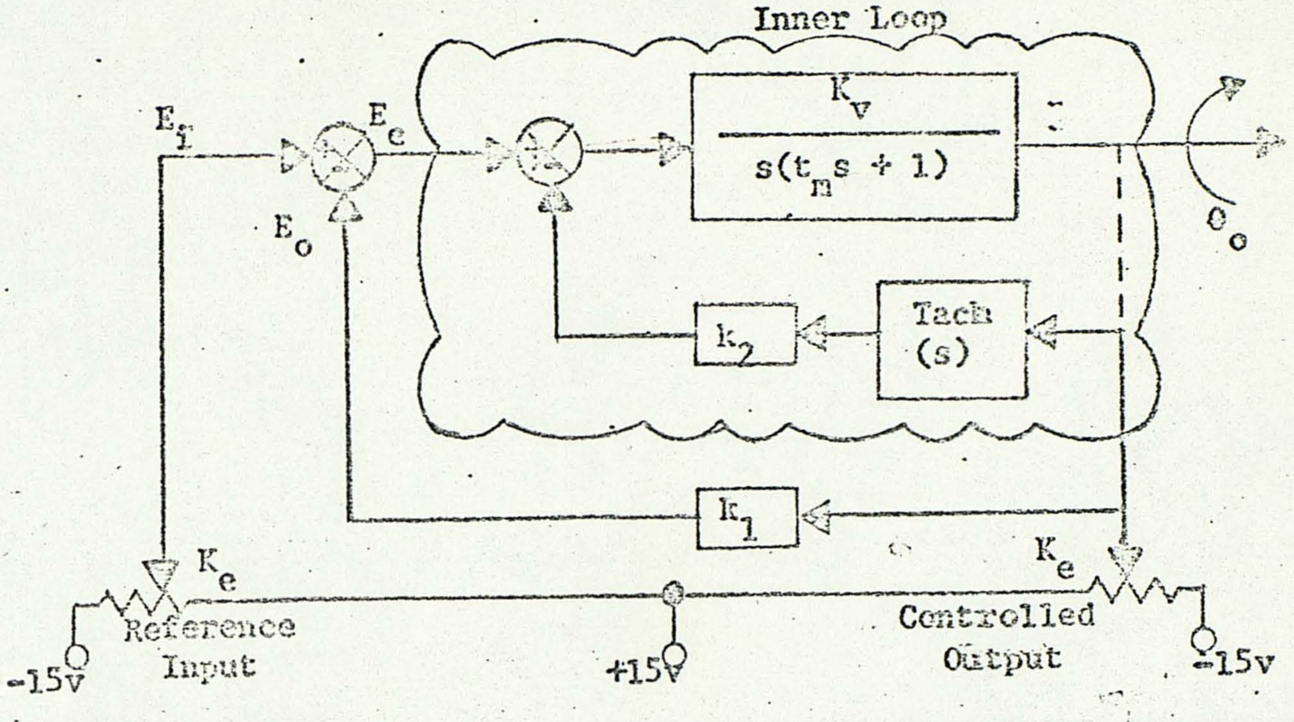
which may be rearranged algebraically by separating the factors,

$$K_m = K_v/(1 + K_v k_2) \tag{2-31}$$

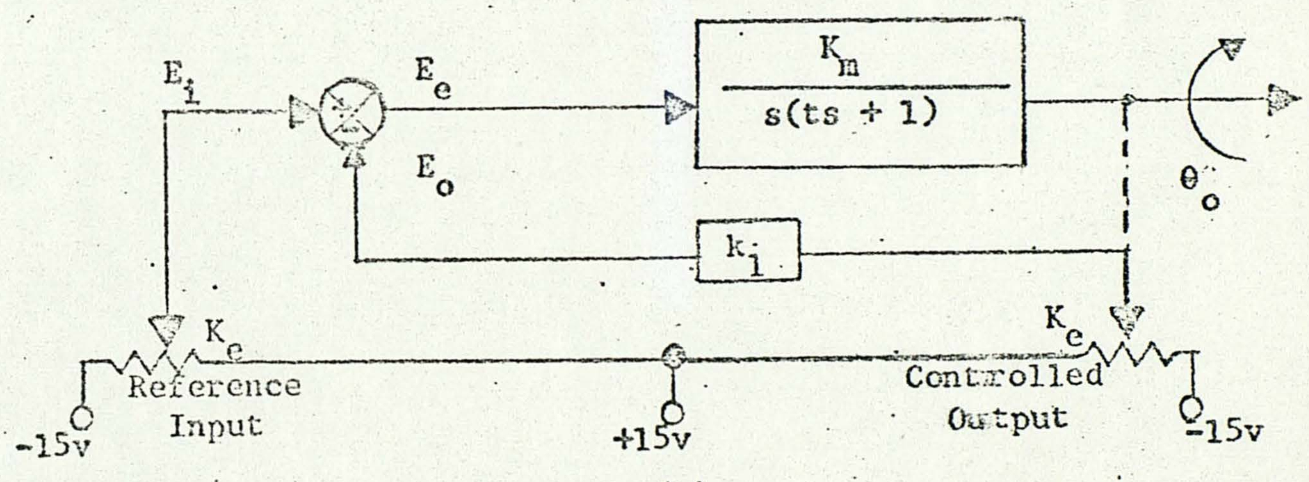
and,

$$t = t_m/(1 + K_v k_2) \tag{2-32}$$

Thus, the tachometer feedback has the effect of making the inner loop transfer function equivalent to a single time constant forward path. Both the time constant, t_m, and the velocity error constant, K_v, are reduced by the factor, $1/(1 + K_v k_2)$. It does not



(a)



(b)

Figure 2-2 Closed Loop Positioning System matter that, as in the case of the MS150, the time constant is not a genuine one but is established by compensation as stated in the previous section. Substitution of the two factors into equation (2-30) simplifies the inner loop transfer function to,

$$\theta_o(s)/E_e(s) = K_m/s(ts + 1) \quad (2\text{-}33)$$

The tachomater feedback coefficient is shown in the denominator of equations (2-31) and therefore tends to reduce forward gain and time constant as k_2 is increased. The effect of tachometer feedback on damping factor (Z) can be determined by equating the coefficient of s in the normalized equation, (2-23), with the same coefficient when tachometer feedback is present. That is,

$$(1 + K_v k_2)/t_m = 2ZW_n \quad (2\text{-}34)$$

$$Z = (1 + K_v k_2)/2\sqrt{K_v t_m} \quad (2\text{-}35)$$

Thus, the damping factor is increased as the tachometer feedback is increased.

The tachometer feedback coefficient, k_2 represents a voltage divider across the output of the tachometer. The writer found in laboratory tests that the tachometer generator output was about 2.69 volts per 1000 RPM, measured at the high speed shaft. Therefore, at the low speed shaft, the output voltage represents,

$$[2.69 \text{ volts}/3.5 \text{ radians/second}] = 0.76 \text{ volts/radian/second} \quad (2\text{-}36)$$

This value of k_2 is the maximum feedback attainable with the existing tachometer in the MS150.

3. STATE FUNCTIONS

3.1 Rationale of Section 3

In section 3.2, the state equations which define the closed-loop positioning system shown in Figure 2-1 are developed. The development is for the second-order system in terms of the field controlled (constant armature current) mode of operation. The effect of gain and feedback on pole position in the complex plane are pointed out. In addition, the state equations are put into the forms convenient for computer programming.

3.2 Equations of State

The closed loop positioning system shown in Figure 2-1 was described in Section 2 by equation (2-16),

$$K_s E_f(s) = (t_m s^2 + s)\theta_m$$

Division by N allows the equation to be written in terms of θ_o,

$$K_s E_f(s)/N = (t_m s^2 + s)\theta_o \qquad (3\text{-}1)$$

From Figure 2-1, using the voltage conversion factor, K_e, the voltage applied to the field may be defined as,

$$E_f(s) = K_e[\theta_i(s) - k_1\theta_o(s) - k_2\dot{\theta}_o(s)] \qquad (3\text{-}2)$$

Incorporation of equation (3-2), in effect, closes the loop and the expression is,

$$(K_s K_e/N)\,[\theta_i(s) - k_1\theta_o(s) - k_2\dot{\theta}_o(s)] = (t_m s^2 + s)\theta_o \qquad (3\text{-}3)$$

Making use of the definition of K_v from equation (2-20), we may rewrite equation (3-3) in the time domain as,

$$K_v[\theta_i(t) - k_1\theta_o(t) - k_2\theta_o(t)] = t_m\ddot{\theta}_o(t) + \dot{\theta}_o(t) \qquad (3\text{-}4)$$

By definition, the phase variables of the system are,

$$X_1(t) = \theta_o(t)$$
$$X_2(t) = \dot{\theta}_o(t) \qquad (3\text{-}5)$$

Therefore, equation (3-4) may be written in phase variables,

$$t_m\dot{X}_2(t) + X_2(t) = K_v\theta_i(t) - K_vk_1X_1(t) - K_vk_2X_2(t) \qquad (3\text{-}6)$$

and the state equations describing the system are,

$$\dot{X}_1(t) = X_2(t)$$
$$\dot{X}_2(t) = -Kk_1X_1(t) - \left[(1/t_m) + Kk_2\right] K_2(t) + K\theta_i(t) \qquad (3\text{-}7)$$

where,

$$K = W_n^{\ 2} = K_v/t_m.$$

In matrix form, the closed-loop system state equations are expressed as,

$$\dot{\underline{X}}(t) = \begin{bmatrix} 0 & 1 \\ -Kk_1 & -(1/t_m) - Kk_2 \end{bmatrix} \underline{X}(t) + \begin{bmatrix} 0 \\ K \end{bmatrix} \theta_i(t) \qquad (3\text{-}8)$$
$$Y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{X}(t).$$

where Y(t) is the state variable representation of the output variable, $\theta_o(t)$. This expression shows the effect of feedback on the system state equations, and consequently, the effect on system performance. Specifically, the poles of the closed-loop transfer function may be set as desired by proper selection of the controller gain, K, and the feedback coefficients, k_1 and k_2.

For convenience of computer programming, equation (3-8) may be broken down into parts which allow the driving function, U(t), to be shown separately as follows,

$$\underline{X}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1/t_m \end{bmatrix} \underline{X}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \underline{U}(t) \qquad (3\text{-}9)$$

$$\underline{U}(t) = K[\theta_i(t) - k_1X_1(t) - k_2X_2(t)]$$

$$\underline{Y}(t) = [\ 1\ 0]\ \underline{X}(t)$$

This form of the state equation is commonly used to facilitate identification of all the system elements involved.

One method of solution of equation (3-7) is by integration. That is,

$$X_1(t) = \int_o^t X_2(t)\ dt$$

$$X_2(t) = \int_o^t [\ Kk_1X_1(t) - [(1/t_m + Kk_2]\ X_2(t) + K\theta_i(t)]\ dt \quad (3\text{-}10)$$

If the initial conditions are taken to be,

$$IC_1 = X_1(0) = 1 \quad (3\text{-}11)$$

$$IC_2 = X_2(0) = 1 \quad (3\text{-}12)$$

then the reference input, θ_i, is assumed to the zero, or desired position. The IBM 360 Continuous System Modeling Program (360/CSMP), of reference [4] provides a convenient method of obtaining a graphical and digital presentation of equations (3-7) and (3-10). The 360/CSMP output plots and amplitude versus time results are included in section 4.5.

3.3 State Transition Matrix and Time Performance

The solution of the first order linear differential equations

$$\dot{\underline{X}}(t) = \underline{A}\underline{X}(t) + \underline{B}\underline{U}(t) \quad (3\text{-}13)$$

involves the determination of the time response of all the state variables. The state transition matrix (STM) provides one vehicle to accomplish this task. A knowledge of the STM and the initial state of the system allows one to determine the state at any later time. Thus, the STM,

$$\phi(t) = e^{\underline{A}t} \quad (3\text{-}14)$$

is used to obtain,

$$X(t) = \phi(t)X(0) \quad (3\text{-}15)$$

where $\underline{X}(0)$ is the initial conditions. The STM is related to a generalized impulse response associated with the system described by the $\underline{A}$ Matrix. The time response of equation (3-13) where the initial state is given at t = 0 can be obtained by application of the convolution integral,

$$X(t) = \phi(t)X(0) + \int_0^t \phi(t - \bar{t})\ \underline{B}\ \underline{U}\ (\bar{t})d\bar{t} \quad (3\text{-}16)$$

However, in the particular application under discussion, the initial displacement error of the system output variables is assumed to be one, that is, $\underline{X}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and the desired position is the zero position, (θ_i) toward which the system tends to move. Therefore, $\underline{U}(\bar{t})$ may be considered to be zero for all $\bar{t}$, and,

$$\int_0^t \phi(t - \bar{t})\ \underline{B}\ \underline{U}\ (\bar{t})d\bar{t} = 0 \quad (3\text{-}17)$$

Hence, the system time response may be written as,

$$\underline{X}(t) = \phi(t)\underline{X}(0) \quad (3\text{-}18)$$

$$\underline{X}(t) = \begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (3\text{-}19)$$

$$\underline{X}(t) = \begin{bmatrix} \phi_{11}(t) + \phi_{12}(t) \\ \phi_{21}(t) + \phi_{22}(t) \end{bmatrix} \quad (3\text{-}20)$$

In other words, the expression for the position, $X_1(t)$ and velocity, $X_2(t)$, respectively are,

$$X_1(t) = \phi_{11}(t) + \phi_{12}(t) \quad (3\text{-}21)$$

$$X_2(t) = \phi_{21}(t) + \phi_{22}(t) \tag{3-22}$$

where $\phi_{11}(t)$, $\phi_{12}(t)$, $\phi_{21}(t)$ and $\phi_{22}(t)$ are the elements of the state transition matrix (STM). The STM for each of the conditions which will be discussed in section 4 was obtained using the Melsa programs [3] on the IBM 370. The complete printed computer output is included in Appendix 1. The computer input data in each case is the elements of the A-matrix of equation (3-13) which are,

$$\underline{A} = \begin{bmatrix} 0 & 1 \\ -Kk_1 & -(1/t_m) - Kk_2 \end{bmatrix} \tag{3-23}$$

from equation (3-8) with the appropriate values of the feedback coefficients, k_1 and k_2 while the gain factor, K = 164.8 and τ_m = 0.25 second.

Time response solution by means of the STM is included to complete the discussion of problem solution by state variables. However, in simple systems such as the one described here, direct analysis by classical methods are adequate. The STM was simplified here by judicious definitions which caused the convolution integral to drop out.

4. MODERN CONTROL THEORY

4.1 Rationale of Section 4

In this section the concepts of modern control theory involving the state function equations and the quadratic performance index are discussed in section 4.2 to set the background for a discussion of the basis for system modeling via performance indices in sections 4.3 and 4.4. In section 4.4, the desired positioning system is modeled by application of techniques involving the Kalman equation. This approach takes advantage of optimal concepts in a greatly simplified manner. Data and results relative to this model are presented in Table 4-1, and Figure 4-2. Comparable results obtained by the relatively simple 360/CSMP simulation are then presented in Figure 4-3 through 4-6. Closing remarks and conclusions are presented in section 4.6.

4.2 Optimal Control

In the general optimal control problem, the plant to be controlled is assumed to be described by a set of differential equations of the form,

$$\underline{\dot{X}} = f(\underline{X}, \underline{U}, t) \tag{4-1}$$

The performance of the system is judged on the basis of an integral performance index of the form,

$$PI = S[X(t_f), t_f] + \int_{t_i}^{t_f} L(\underline{X}, \underline{U}, t)dt \tag{4-2}$$

The time interval of interest is designated to be from the initial time, t_i, to the final time, t_f, of the control period. The integrand of the performance index, $L(\underline{X}, \underline{U}, t)$, is assumed to be a positive definite function of $\underline{X}$, $\underline{U}$, and t, where $\underline{X}$ is an n-dimensional state vector and $\underline{U}$ is an m-dimensional control vector. $S[X(t_f), t_f]$ is the final cost function. It is desired to find the optimal control $\underline{U}^o(X)$ that minimizes the performance index PI, or preferably to find the optimal control law. The optimal control law enables the generation of $\underline{U}(x,t)$ from $\underline{X}(t)$ in the true feedback sense. The references for the above discussion are [7] and [8].

4.3 Modeling and Performance Indices

A basic problem in designing a system by methods of optimal control theory is the selection of the performance index. One choice is to use a quadratic performance index. The selection then is of the positive definite matrix $\underline{P}$ and positive semi-definite matrix $\underline{Q}$ in,

$$PI = \int_o^\infty (\underline{X}^T \underline{Q}\ \underline{X} + \underline{U}^T \underline{P}\ \underline{U})dt \qquad (4-3)$$

For the scalar input case equation (4-3) becomes,

$$PI = \int_o^\infty (\underline{X}^T \underline{Q}\underline{X} + pU^2)dt \qquad (4-4)$$

Minimization of the initial performance index, equation may be equivalently stated as a minimization of a reduced performance index,

$$PI = \int_0^\infty [(\gamma^T X)^2 + pU^2] dt \tag{4-5}$$

Since there are only n unknown elements in $\underline{\gamma}$, of the n(n+1)/2 elements in $\underline{Q}$, only n combinations are critical in determining the optimal system. Now $\underline{\gamma}$ may be chosen directly, rather than selecting $\underline{Q}$. The reduced performance index has a very important feature related to system modeling.

If the term p in equation (4-5) is allowed to vanish, (i.e., p = 0), the performance index of equation (4-5) has an absolute minimum value of zero, if and only if,

$$\gamma^T X = \gamma_1 X_1(t) + \gamma_2 X_2(t) + . . . \gamma_n X_n(t) = 0 \tag{4-6}$$

for all $0 \leq t \leq \infty$.

With the system expressed in phase variables, so that

$$X_i(t) = d^{i-1} X_1(t)/dt^{i-1} \tag{4-7}$$

for all $i \leq n$, equation (4-6) becomes,

$$\gamma_1 X_1(t) + \gamma_2 dX_1(t)/dt + . . . + \gamma_n d^{n-1} X_1(t)/dt^{n-1} = 0 \tag{4-8}$$

Since the system here is represented in phase variables and actual system parameters are not involved, equation (4-8) may then be written,

$$\gamma_1 Y(t) + (\gamma_2 dY(t)/dt) + . . . + \gamma_n [d^{n-1} Y(t)]/dt^{n-1} = 0 \tag{4-9}$$

where Y(t) represents the output variable, $\theta_o(t)$.

Equation (4-9) represents an (n-1) order differential equation which the output Y(t) must satisfy if the performance index is to achieve an absolute minimum of zero. A time response which satisfies this equation is the ideal or model time response. In other words, the describing differential equation (4-9) may be referred to as the system model.

Since the model order is (n-1) while the system order is n, Y(t) cannot satisfy (4-9) exactly. However, if the performance index is to be minimized with p = 0, Y(t) must approximate the model response as closely as possible.

The model time response is determined by the selection of the elements of $\underline{\gamma}$. Therefore, $\underline{\gamma}$ should be chosen so that the model time response meets such specifications as rise time, overshoot, damping ratio, M-peak and phase margin.

Schultz and Melsa, [8], state that if the scalar cost function, p, of equation (4-5), is less than 0.1, the optimal response approximates the model time response specified by $\underline{\gamma}$. If p is greater than 10, the response is approximately equal to the unforced response, $\dot{\underline{X}} = \underline{A}\,\underline{X}$, of the forward system alone. This concept will be investigated in sections 4.4 and 4.5.

4.4 System Model and Optimal Control

Despite the simplicity of the concept presented above, the choice of $\underline{\gamma}$ in most cases still presents a difficult problem. It can be shown, [8], that the choice of $\underline{\gamma}$, and for that matter, choice of a performance index may be side-stepped by specifying a system model of the form,

$$Y(t) + T\dot{Y}(t) = 0 \qquad (4\text{-}10)$$

and by selection of a weighting factor, $\bar{p}$, to be applied to the required control. In other words, for discussion purposes, the performance index is of the form,

$$PI = \int_0^\infty (\bar{Y}^2 + \bar{p}^2 U^2)\, dt \qquad (4\text{-}11)$$

where,

$$\bar{Y} = X_1(t) + TX_2(t) \qquad (4\text{-}12)$$

from the phase variable definitions given in equation (3-5). In terms of the discussion in the last paragraph of the previous section, $\bar{p} = \sqrt{p}$ and the same effects are observed as $\bar{p}$ is varied.

With equation (4-10) as the system model, there are several approaches available to determine the required feedback coefficients. For example, there is the Hamilton-Jacobi approach [8] to the basic optimal control problem where the system is described by equation (4-1) and the quadratic performance index. One technique for solution of the Hamilton-Jacobi equation is the matrix Ricatti method of determining the feedback coefficients which minimize the performance index. However, the Kalman equation was developed to obtain an exact solution for the feedback coefficients without the necessity for solving the matrix Ricatti equation. There are several useful forms of the Kalman equation, but the one which will be used in this thesis is,

$$\left| 1 + G(s)H_{eq}(s) \right|^2 = 1 + \left| \Gamma(s)/\bar{p} \right|^2 \qquad (4\text{-}13)$$

Since both sides of equation (4-13) are complex quantities, the notation, $\left| m(s) \right|^2 = m(s)m(-s)$, can be used to expand the equation even though it is only true for $s = jw$.

This technique begins by defining a synthetic transfer function,

$$\Gamma(s)/\bar{p} = (1 + Ts)\ G(s)/\bar{p} \tag{4-14}$$

where T is the model time constant and G(s) is the system transfer function,

$$G(s) = K_v/s\ (t_m s + 1) \tag{4-15}$$

where $K_v = 41.2$ and $t_m = 0.25$. The value of the model time constant, T, may be selected at the designer's choice. In this case, a value of T = 0.1 second will be used and equation (4-14) becomes,

$$\Gamma(s)/\bar{p} = (1 + 0.1s)\ 41.2/s(0.25s + 1)\ \bar{p} \tag{4-16}$$

The right-hand side of equation (4-13) becomes,

$$1 + \left|(1 + 0.1s)\ 41.2/s\ (0.25s + 1)\bar{p}\ \right|^2 \tag{4-17}$$

In terms of the notation, $m(s)^2 = m(s)\ m(-s)$, equation (4-17) may be expressed,

$$1 + (41.2/\bar{p})^2 \left[(1 - 0.1s)/(-s)(-0.25s + 1)\right]\left[(1 + 0.1s)/s(0.25s + 1)\right] \tag{4-18}$$

or,

$$\left[0.0625\bar{p}^{2}s^4 - s^2(\bar{p}^2 + 16.97) + 1697\right]/\left[0.625s^4 - s^2\right] \tag{4-19}$$

The roots of the numerator of equation (4-19) are,

$$s^2 + \left[(\bar{p}^2 + 16.97) \pm\sqrt{(\bar{p}^2 + 16.97)^2 - 424.25\bar{p}^2}\ \right]/0.125\bar{p}^2 \tag{4-20}$$

In the case where $\bar{p}^2 = 0.1$, the roots are,

$$s^2 = 1365.6 \pm 1256 = 2622 \text{ or } 110$$

$$s = \pm 51.2 \text{ and } \pm 10.5 \tag{4-21}$$

Therefore, the roots in the left-half of the complex plane, designated by $[\ldots]^+$, can be expressed as,

$$\left[1 + \Gamma(s)/\bar{p}\right]^+ = (s + 10.5)(s + 51.2)$$

$$\left[1 + \Gamma(s)/\bar{p}\right]^+ = s^2 + 61.7s + 537.6 \tag{4-22}$$

The left-hand side of equation (4-13) is formed using the equivalent feedback expression,

$$H_{eq}(s) = k_1 + k_2 s \qquad (4\text{-}23)$$

$H_{eq}(s)$ is defined as the equivalent feedback of the system. $H_{eq}(s)$ is an artificial means of displaying the effects of state variable feedback in the closed-loop system. The equivalent feedback is obtained by moving k_2 forward to sum with k_1. Therefore, the left-hand side of equation (4-13) becomes,

$$1 + GH_{eq}(s) = 1 + [41.2/s(0.25s + 1)][k_1 + k_2 s]$$

$$1 + GH_{eq}(s) = [s^2 + 4(1 + 41.2k_2)s + 164.8k_1]/s(s + 4) \qquad (4\text{-}24)$$

By equating the coefficients of s in equations (4-22) and (4-24), the feedback coefficients may be determined,

$$61.7 = 4 + 164.8k_2$$

$$k_2 = 57.7/164.8 = 0.350 \qquad (4\text{-}25)$$

and,

$$537.6 = 164.8k_1$$

$$k_1 = 537.6/164.8 = 3.262 \qquad (4\text{-}26)$$

Similarly, by successively letting $\bar{p}^2$ = 0.1, 1.0, 10.0 and 100.0, Table 4-1 was formed based on the system block diagram shown in Figure 4-1.

The time performance equations obtained by classical methods of differential equations are also tabulated in Table 4-1. The equation for $\bar{p}^2$ = 0.1, for example, was determined using the roots

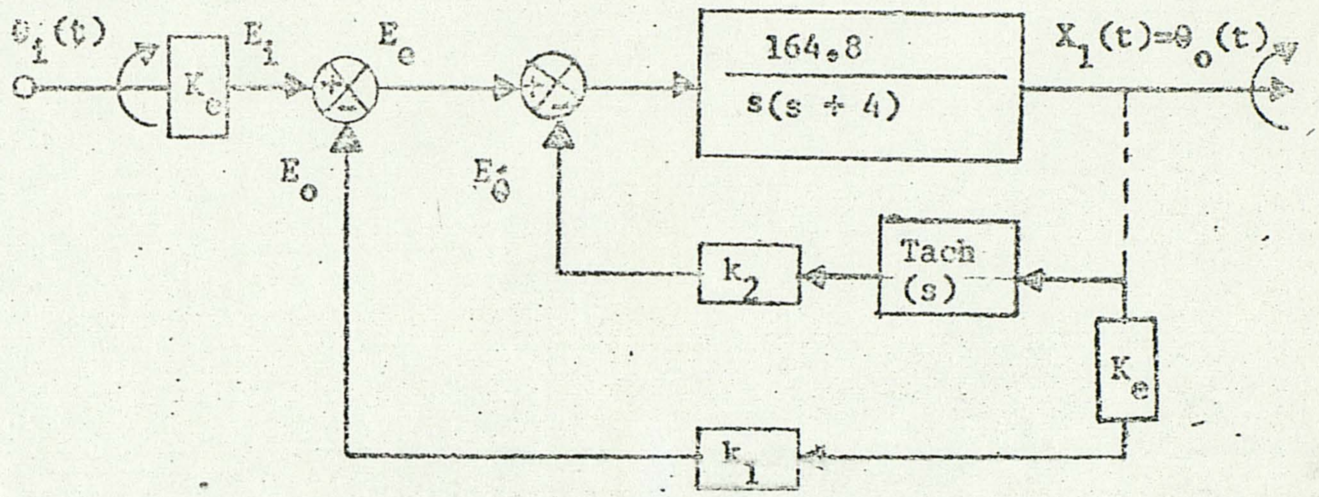


Figure 4-1 Position Control System Block-Diagram

given in equation (4-22) and by writing,

$$X_1(t) = Ae^{-10.5t} + Be^{-51.2t} \qquad (4\text{-}27)$$

The initial conditions were chosen to be $X_1(0) = 1$ and $X_2(0) = 1$. Solution for A and B leads to the time performance equation,

$$X_1(t) = 1.28e^{-10.5t} - 0.28e^{-51.2t} \qquad (4\text{-}28)$$

Table 4-1

Performance vs Weighting Factor, $\bar{p}^2$

$\bar{p}^2$	k_1	k_2	Performance Equations
0.1	3.262	0.350	$X_1(t) = 1.28\ e^{-10.5t} - 0.28\ e^{-51.2t}$
1.0	1.000	0.126	$X_1(t) = e^{-12.4t}(\text{Cos } 3.3t + 4.03 \text{ Sin } 3.3t)$
10.0	0.315	0.049	$X_1(t) = e^{-6.06t}(\text{Cos } 3.9t + 1.81 \text{ Sin } 3.9t)$
100.0	0.099	0.019	$X_1(t) = e^{-3.58t}(\text{Cos } 1.86t + 2.45 \text{ Sin } 1.86t)$
∞	-	-	$X_1(t) = 1.25 - 0.25\ e^{-4t}$

Solutions for the other value of $\bar{p}^2$ were obtained by similar means. These equations are plotted in Figure 4-2. The ascending line shown in Figure 4-2 is the initial slope of the curves due to the initial conditions. In the limit where $\bar{p}^2 = \infty$, it also represents the unforced system response where,

$$G(s) = 41.2/s(0.25s + 1) \qquad (4\text{-}29)$$

or, by multiplying through by 4,

$$G(s) = 164.8/s(s + 4) \qquad (4\text{-}30)$$

The roots of the denominator, (characteristic equation), of the forward system are $s = 0$ and $s = -4$. Therefore,

$$X_1(t) = A + Be^{-4t} \qquad (4\text{-}31)$$

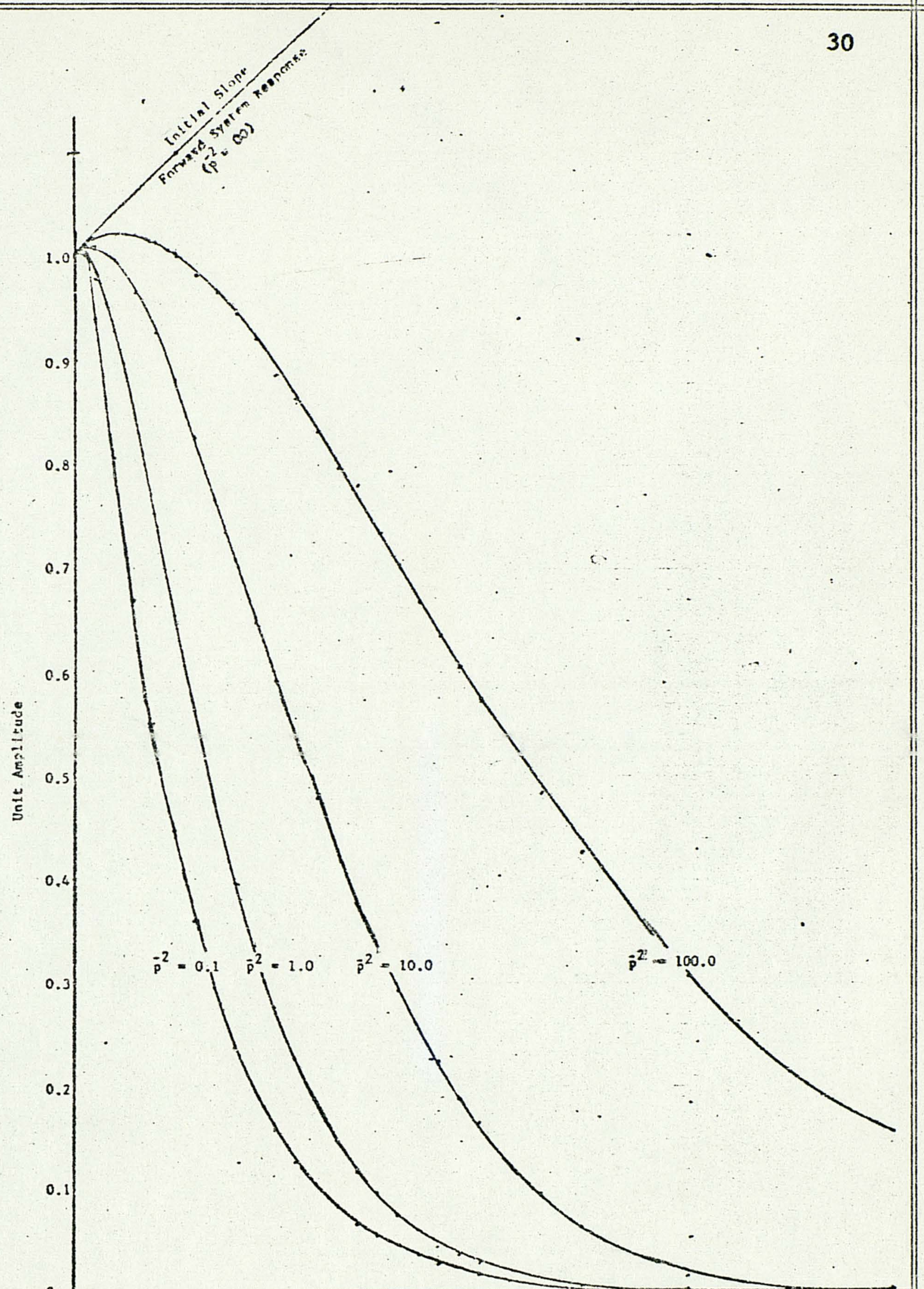


Figure 4-2 Performance Vs Weighting Factor

and with $X_1(0) = 1$ again,

$$X_1(t) = 1.25 - 0.25e^{-4t} \qquad (4\text{-}32)$$

As stated above, the unforced response is equivalent to making $\bar{p}^2$ go to infinity. Therefore, we see in Figure 4-2, the effects of variation of the weighting factor, $\bar{p}^2$, from 0.01 to infinity, or, from the model to the unforced response of the forward system.

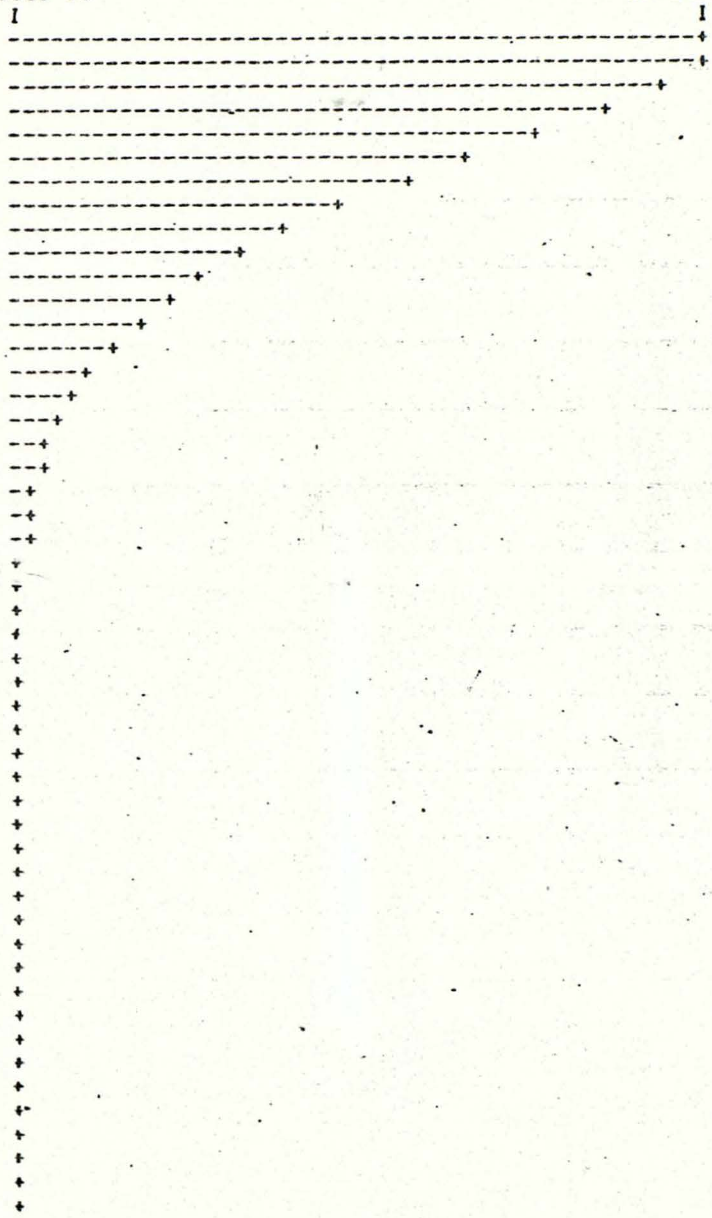
4.5 Simulation in 360/CSMP

Using the form of the state equation developed in equation (3-10), the IBM 360 Continuous System Modeling Program was used on the IBM 370 to obtain the output position plot and data shown in Figure 4-3 through 4-6, corresponding to $\bar{p}^2 = 0.1, 1.0, 10.0$, and 100.0. (The complete programs and output prints are included in Appendix 1.) Figures 4-3 through 4-6 confirm the results obtained by classical methods shown in Figure 4-2. The point by point data printed in Figure 4-3 through 4-6 are in exact agreement with data calculated over the range plotted in Figure 4-2. The response can be seen to be more sluggish as $\bar{p}^2$ advances from 0.1 to 100.0. The 360/CSMP method offers an extremely simple, convenient and informative technique for evaluating the time performance of linear control systems when described as in equation (3-10). Otherwise, very sophisticated techniques involving interfacing of analog and digital systems have been used as in reference [9] to solve complex systems of equations such as these.

```
FIGURE 4-3 MODEL POSITION RESPONSE, P**2=0.1                         PAGE   1
                        MINIMUM              X1     VERSUS TIME           MAXIMUM
                      3.5318E-05                                        1.0000E 00
TIME          X1            I                                                I
0.0           1.0000E 00    -------------------------------------------------+
2.0000E-02    9.3813E-01    ----------------------------------------------+
4.0000E-02    8.0625E-01    ----------------------------------------+
6.0000E-02    6.6998E-01    ---------------------------------+
8.0000E-02    5.4598E-01    ---------------------------+
1.0000E-01    4.4712E-01    ----------------------+
1.2000E-01    3.6319E-01    ------------------+
1.4000E-01    2.9467E-01    --------------+
1.6000E-01    2.3896E-01    -----------+
1.8000E-01    1.9373E-01    ---------+
2.0000E-01    1.5705E-01    -------+
2.2000E-01    1.2730E-01    ------+
2.4000E-01    1.0319E-01    -----+
2.6000E-01    8.3647E-02    ----+
2.8000E-01    6.7803E-02    ---+
3.0000E-01    5.4960E-02    --+
3.2000E-01    4.4550E-02    --+
3.4000E-01    3.6112E-02    -+
3.6000E-01    2.9272E-02    -+
3.8000E-01    2.3727E-02    -+
4.0000E-01    1.9233E-02    +
4.2000E-01    1.5590E-02    +
4.4000E-01    1.2637E-02    +
4.6000E-01    1.0243E-02    +
4.8000E-01    8.3030E-03    +
5.0000E-01    6.7303E-03    +
5.2000E-01    5.4555E-03    +
5.4000E-01    4.4221E-03    +
5.6000E-01    3.5845E-03    +
5.8000E-01    2.9055E-03    +
6.0000E-01    2.3552E-03    +
6.2000E-01    1.9091E-03    +
6.4000E-01    1.5475E-03    +
6.6000E-01    1.2544E-03    +
6.8000E-01    1.0168E-03    +
7.0000E-01    8.2417E-04    +
7.2000E-01    6.6806E-04    +
7.4000E-01    5.4152E-04    +
7.6000E-01    4.3895E-04    +
7.8000E-01    3.5580E-04    +
8.0000E-01    2.8841E-04    +
8.2000E-01    2.3378E-04    +
8.4000E-01    1.8950E-04    +
8.6000E-01    1.5360E-04    +
8.8000E-01    1.2451E-04    +
9.0000E-01    1.0093E-04    +
9.2000E-01    8.1803E-05    +
9.4000E-01    6.6313E-05    +
9.6000E-01    5.3752E-05    +
9.8000E-01    4.3572E-05    +
1.0000E 00    3.5318E-05    +
```

FIGURE 4-4 MODEL POSITION RESPONSE, P**2=1.0 PAGE 1

MINIMUM -8.0681E-06 X1 VERSUS TIME MAXIMUM 1.0025E 00



TIME	X1
0.0	1.0000E 00
2.0000E-02	9.8762E-01
4.0000E-02	9.2903E-01
6.0000E-02	8.4534E-01
8.0000E-02	7.5068E-01
1.0000E-01	6.5423E-01
1.2000E-01	5.6169E-01
1.4000E-01	4.7630E-01
1.6000E-01	3.9968E-01
1.8000E-01	3.3237E-01
2.0000E-01	2.7420E-01
2.2000E-01	2.2460E-01
2.4000E-01	1.8279E-01
2.6000E-01	1.4738E-01
2.8000E-01	1.1893E-01
3.0000E-01	9.5229E-02
3.2000E-01	7.5844E-02
3.4000E-01	6.0120E-02
3.6000E-01	4.7437E-02
3.8000E-01	3.7261E-02
4.0000E-01	2.9139E-02
4.2000E-01	2.2687E-02
4.4000E-01	1.7585E-02
4.6000E-01	1.3559E-02
4.8000E-01	1.0423E-02
5.0000E-01	7.9676E-03
5.2000E-01	6.0608E-03
5.4000E-01	4.5864E-03
5.6000E-01	3.4514E-03
5.8000E-01	2.5819E-03
6.0000E-01	1.9190E-03
6.2000E-01	1.4161E-03
6.4000E-01	1.0366E-03
6.6000E-01	7.5199E-04
6.8000E-01	5.3979E-04
7.0000E-01	3.8266E-04
7.2000E-01	2.6720E-04
7.4000E-01	1.8306E-04
7.6000E-01	1.2239E-04
7.8000E-01	7.9106E-05
8.0000E-01	4.8659E-05
8.2000E-01	2.7603E-05
8.4000E-01	1.3354E-05
8.6000E-01	3.9869E-06
8.8000E-01	-1.9236E-06
9.0000E-01	-5.4234E-06
9.2000E-01	-7.2743E-06
9.4000E-01	-8.0255E-06
9.6000E-01	-8.0651E-06
9.8000E-01	-7.6771E-06
1.0000E 00	-7.0421E-06

```
FIGURE 4-5 MODEL POSITION RESPONSE, P**2=10.0                              PAGE   1

                               MINIMUM              X1      VERSUS TIME             MAXIMUM
                             -7.6043E-03                                          1.0081E 00
  TIME             X1            I                                                    I
0.0             1.0000E 00   ---------------------------------------------------+
2.0000E-02      1.0081E 00   ---------------------------------------------------+
4.0000E-02      9.9595E-01   --------------------------------------------------+
6.0000E-02      9.6806E-01   -------------------------------------------------+
8.0000E-02      9.2636E-01   -----------------------------------------------+
1.0000E-01      8.8013E-01   ---------------------------------------------+
1.2000E-01      8.2607E-01   ------------------------------------------+
1.4000E-01      7.6646E-01   ---------------------------------------+
1.6000E-01      7.0910E-01   ------------------------------------+
1.8000E-01      6.4947E-01   ---------------------------------+
2.0000E-01      5.9072E-01   ------------------------------+
2.2000E-01      5.3374E-01   ---------------------------+
2.4000E-01      4.7921E-01   ------------------------+
2.6000E-01      4.2760E-01   ----------------------+
2.8000E-01      3.7923E-01   -------------------+
3.0000E-01      3.3429E-01   -----------------+
3.2000E-01      2.9287E-01   ---------------+
3.4000E-01      2.5498E-01   -------------+
3.6000E-01      2.2055E-01   -----------+
3.8000E-01      1.8946E-01   ----------+
4.0000E-01      1.6160E-01   --------+
4.2000E-01      1.3675E-01   -------+
4.4000E-01      1.1474E-01   ------+
4.6000E-01      9.5350E-02   -----+
4.8000E-01      7.8381E-02   ----+
5.0000E-01      6.3621E-02   ---+
5.2000E-01      5.0868E-02   --+
5.4000E-01      3.9924E-02   --+
5.6000E-01      3.0601E-02   -+
5.8000E-01      2.2722E-02   -+
6.0000E-01      1.6121E-02   -+
6.2000E-01      1.0644E-02   +
6.4000E-01      6.1495E-03   +
6.6000E-01      2.5098E-03   +
6.8000E-01     -3.9272E-04   +
7.0000E-01     -2.6632E-03   +
7.2000E-01     -4.3959E-03   +
7.4000E-01     -5.6747E-03   +
7.6000E-01     -6.5739E-03   +
7.8000E-01     -7.1585E-03   +
8.0000E-01     -7.4855E-03   +
8.2000E-01     -7.6043E-03   +
8.4000E-01     -7.5577E-03   +
8.6000E-01     -7.3820E-03   +
8.8000E-01     -7.1082E-03   +
9.0000E-01     -6.7626E-03   +
9.2000E-01     -6.3669E-03   +
9.4000E-01     -5.9392E-03   +
9.6000E-01     -5.4943E-03   +
9.8000E-01     -5.0440E-03   +
1.0000E 00     -4.5978E-03   +
```

FIGURE 4-6 MODEL POSITION RESPONSE, P**2=100.0

PAGE 1

MINIMUM 5.7314E-02   X1 VERSUS TIME   MAXIMUM 1.0229E 00

TIME	X1
0.0	1.0000E 00
2.0000E-02	1.0155E 00
4.0000E-02	1.0228E 00
6.0000E-02	1.0229E 00
8.0000E-02	1.0167E 00
1.0000E-01	1.0052E 00
1.2000E-01	9.8919E-01
1.4000E-01	9.6927E-01
1.6000E-01	9.4612E-01
1.8000E-01	9.2032E-01
2.0000E-01	8.9237E-01
2.2000E-01	8.6273E-01
2.4000E-01	8.3181E-01
2.6000E-01	7.9997E-01
2.8000E-01	7.6751E-01
3.0000E-01	7.3473E-01
3.2000E-01	7.0186E-01
3.4000E-01	6.6911E-01
3.6000E-01	6.3667E-01
3.8000E-01	6.0470E-01
4.0000E-01	5.7331E-01
4.2000E-01	5.4264E-01
4.4000E-01	5.1276E-01
4.6000E-01	4.8376E-01
4.8000E-01	4.5568E-01
5.0000E-01	4.2859E-01
5.2000E-01	4.0251E-01
5.4000E-01	3.7747E-01
5.6000E-01	3.5347E-01
5.8000E-01	3.3053E-01
6.0000E-01	3.0865E-01
6.2000E-01	2.8781E-01
6.4000E-01	2.6800E-01
6.6000E-01	2.4921E-01
6.8000E-01	2.3141E-01
7.0000E-01	2.1458E-01
7.2000E-01	1.9870E-01
7.4000E-01	1.8373E-01
7.6000E-01	1.6964E-01
7.8000E-01	1.5640E-01
8.0000E-01	1.4397E-01
8.2000E-01	1.3233E-01
8.4000E-01	1.2144E-01
8.6000E-01	1.1127E-01
8.8000E-01	1.0177E-01
9.0000E-01	9.2930E-02
9.2000E-01	8.4702E-02
9.4000E-01	7.7057E-02
9.6000E-01	6.9964E-02
9.8000E-01	6.3393E-02
1.0000E 00	5.7314E-02

4.6 Conclusion

One of the problem areas of modern control theory still requiring basic research as reported in reference [10] is the area involving plant model accuracy, performance index definition to consolidate design and model inaccuracies, and in general, a systematic approach to optimal system design. The approach used in this thesis utilizes a rational method of establishing quadratic cost functionals based on specification of a desired system model. To be specific, a model with a first order system time constant which gives the desired output variable time response is used. Once the cost functional is established, a method is demonstrated whereby feedback coefficients for each of the state variables are computed based on model and control power considerations for optimization.

An extension to higher order systems and models require additional computation but not added theory. Thus, the principles can be applied to higher order systems since the designer can choose the model time constant to obtain the desired n-order system response, limited only by the control power considerations.

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[10] Athans, M., "On the LQG Problem", IEEE Transactions on Automatic Control, Vol. AC-16, No. 6, Dec. 1971.

APPENDIX 1

COMPUTER OUTPUT DATA

```
****CONTINUOUS SYSTEM MODELING PROGRAM****

***PROBLEM INPUT STATEMENTS***

INITIAL
      IC1=1.0
      IC2=1.0
DYNAMIC
      X1=INTGRL(IC1, X2)
      X2=INTGRL(IC2,-537.6*X1-61.7*X2)
TIMER FINTIM=1., OUTDEL=0.02, PRDEL=0.02
PRTPLOT X1
LABEL FIGURE 4-3 MODEL POSITION RESPONSE, P**2=0.1
PRTPLOT X2
LABEL MODEL VELOCITY RESPONSE, P**2=0.1
END
STOP

OUTPUT VARIABLE SEQUENCE
IC1    IC2    X1     ZZ0003 X2

OUTPUTS    INPUTS   PARAMS  INTEGS + MEM BLKS  FORTRAN  DATA CDS
 9(500)   13(1400)  3(400)    2+  0=  2(300)    6(600)      6
```

```
FIGURE 4-3 MODEL POSITION RESPONSE, P**2=0.1                              PAGE   1

                             MINIMUM             X1    VERSUS TIME            MAXIMUM
                            3.5318E-05                                       1.0008E 00
 TIME              X1        I                                                    I
0.0            1.0000E 00    --------------------------------------------------+
2.0000E-02     9.3813E-01    ----------------------------------------------+
4.0000E-02     8.0625E-01    ----------------------------------------+
6.0000E-02     6.6998E-01    ---------------------------------+
8.0000E-02     5.4398E-01    ---------------------------+
1.0000E-01     4.4712E-01    ----------------------+
1.2000E-01     3.6319E-01    ------------------+
1.4000E-01     2.9467E-01    --------------+
1.6000E-01     2.3696E-01    -----------+
1.8000E-01     1.9373E-01    ---------+
2.0000E-01     1.5705E-01    -------+
2.2000E-01     1.2730E-01    ------+
2.4000E-01     1.0319E-01    -----+
2.6000E-01     8.3647E-02    ----+
2.8000E-01     6.7803E-02    ---+
3.0000E-01     5.4960E-02    --+
3.2000E-01     4.4550E-02    --+
3.4000E-01     3.6112E-02    -+
3.6000E-01     2.9272E-02    -+
3.8000E-01     2.3727E-02    -+
4.0000E-01     1.9233E-02    +
4.2000E-01     1.5590E-02    +
4.4000E-01     1.2637E-02    +
4.6000E-01     1.0243E-02    +
4.8000E-01     8.3030E-03    +
5.0000E-01     6.7303E-03    +
5.2000E-01     5.4555E-03    +
5.4000E-01     4.4221E-03    +
5.6000E-01     3.5845E-03    +
5.8000E-01     2.9055E-03    +
6.0000E-01     2.3552E-03    +
6.2000E-01     1.9091E-03    +
6.4000E-01     1.5475E-03    +
6.6000E-01     1.2544E-03    +
6.8000E-01     1.0168E-03    +
7.0000E-01     8.2417E-04    +
7.2000E-01     6.6806E-04    +
7.4000E-01     5.4152E-04    +
7.6000E-01     4.3895E-04    +
7.8000E-01     3.5580E-04    +
8.0000E-01     2.8841E-04    +
8.2000E-01     2.3378E-04    +
8.4000E-01     1.8950E-04    +
8.6000E-01     1.5360E-04    +
8.8000E-01     1.2451E-04    +
9.0000E-01     1.0093E-04    +
9.2000E-01     8.1800E-05    +
9.4000E-01     6.6313E-05    +
9.6000E-01     5.3752E-05    +
9.8000E-01     4.3571E-05    +
1.0000E 00     3.5318E-05    +
```

MODEL VELOCITY RESPONSE, P**2=0.1 PAGE 1

MINIMUM -6.9820E 00 — X2 VERSUS TIME — MAXIMUM 1.0000E 00

TIME	X2
0.0	1.0000E 00
2.0000E-02	-5.7199E 00
4.0000E-02	-6.9820E 00
6.0000E-02	-6.5014E 00
8.0000E-02	-5.5726E 00
1.0000E-01	-4.6259E 00
1.2000E-01	-3.7888E 00
1.4000E-01	-3.0852E 00
1.6000E-01	-2.5058E 00
1.8000E-01	-2.0330E 00
2.0000E-01	-1.[illegible]66E 00
2.2000E-01	-1.3366E 00
2.4000E-01	-1.0835E 00
2.6000E-01	-8.7828E-01
2.8000E-01	-7.1193E-01
3.0000E-01	-5.7708E-01
3.2000E-01	-4.6777E-01
3.4000E-01	-3.7917E-01
3.6000E-01	-3.0735E-01
3.8000E-01	-2.4913E-01
4.0000E-01	-2.0194E-01
4.2000E-01	-1.6369E-01
4.4000E-01	-1.3269E-01
4.6000E-01	-1.0755E-01
4.8000E-01	-8.7181E-02
5.0000E-01	-7.0668E-02
5.2000E-01	-5.7282E-02
5.4000E-01	-4.6432E-02
5.6000E-01	-3.7637E-02
5.8000E-01	-3.0508E-02
6.0000E-01	-2.4729E-02
6.2000E-01	-2.0045E-02
6.4000E-01	-1.6248E-02
6.6000E-01	-1.3171E-02
6.8000E-01	-1.0676E-02
7.0000E-01	-8.6538E-03
7.2000E-01	-7.0146E-03
7.4000E-01	-5.6859E-03
7.6000E-01	-4.6089E-03
7.8000E-01	-3.7359E-03
8.0000E-01	-3.0283E-03
8.2000E-01	-2.4547E-03
8.4000E-01	-1.9897E-03
8.6000E-01	-1.6128E-03
8.8000E-01	-1.3073E-03
9.0000E-01	-1.0597E-03
9.2000E-01	-8.5899E-04
9.4000E-01	-6.9628E-04
9.6000E-01	-5.6440E-04
9.8000E-01	-4.5749E-04
1.0000E 00	-3.7083E-04

```
                ****CONTINUOUS SYSTEM MODELING PROGRAM****

          ***PROBLEM INPUT STATEMENTS***

      INITIAL
            IC1=1.0
            IC2=1.0
      DYNAMIC
            X1=INTGRL(IC1, X2)
            X2=INTGRL(IC2,-164.8*X1-24.8*X2)
      TIMER FINTIM=1., OUTDEL=0.02, PRDEL=0.02
      PRTPLOT X1
      LABEL FIGURE 4-4 MODEL POSITION RESPONSE, P**2=1.0
      PRTPLOT X2
      LABEL MODEL VELOCITY RESPONSE, P**2=1.0
      END
      STOP

OUTPUT VARIABLE SEQUENCE
IC1    IC2    X1     ZZ0003 X2

 OUTPUTS    INPUTS   PARAMS  INTEGS + MEM BLKS  FORTRAN  DATA CDS
  9(500)  13(1400)   3(400)   2+  0=  2(300)    6(600)       6
```

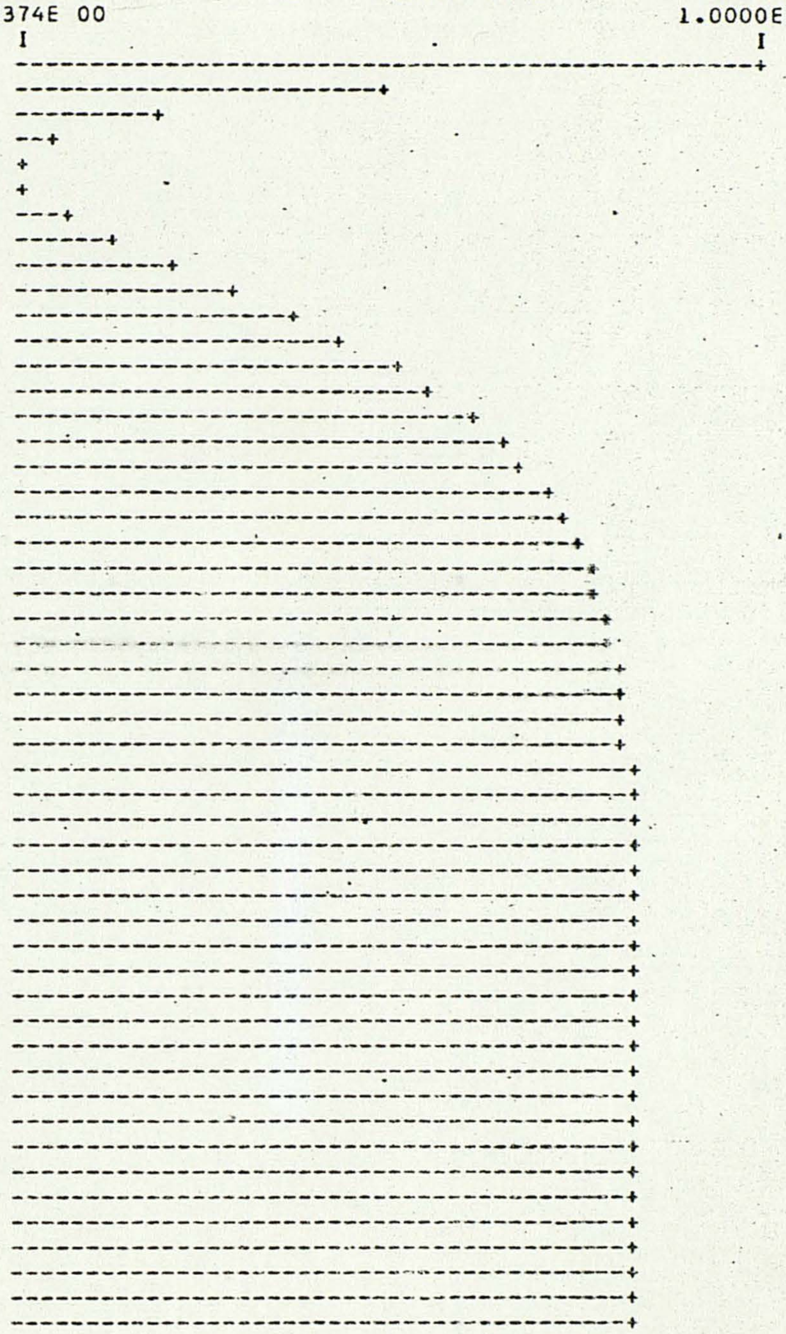
FIGURE 4-4 MODEL POSITION RESPONSE, P**2=1.0 PAGE 1

MINIMUM -8.0681E-06 — X1 VERSUS TIME — MAXIMUM 1.0025E 00

TIME	X1
0.0	1.0000E 00
2.0000E-02	9.8762E-01
4.0000E-02	9.2903E-01
6.0000E-02	8.4534E-01
8.0000E-02	7.5068E-01
1.0000E-01	6.5423E-01
1.2000E-01	5.6169E-01
1.4000E-01	4.7630E-01
1.6000E-01	3.9968E-01
1.8000E-01	3.3237E-01
2.0000E-01	2.7420E-01
2.2000E-01	2.2460E-01
2.4000E-01	1.8279E-01
2.6000E-01	1.4788E-01
2.8000E-01	1.1898E-01
3.0000E-01	9.5229E-02
3.2000E-01	7.5844E-02
3.4000E-01	6.0120E-02
3.6000E-01	4.7437E-02
3.8000E-01	3.7261E-02
4.0000E-01	2.9139E-02
4.2000E-01	2.2687E-02
4.4000E-01	1.7585E-02
4.6000E-01	1.3569E-02
4.8000E-01	1.0423E-02
5.0000E-01	7.9676E-03
5.2000E-01	6.0608E-03
5.4000E-01	4.5864E-03
5.6000E-01	3.4514E-03
5.8000E-01	2.5819E-03
6.0000E-01	1.9190E-03
6.2000E-01	1.4161E-03
6.4000E-01	1.0366E-03
6.6000E-01	7.5199E-04
6.8000E-01	5.3979E-04
7.0000E-01	3.8266E-04
7.2000E-01	2.6720E-04
7.4000E-01	1.8308E-04
7.6000E-01	1.2239E-04
7.8000E-01	7.9106E-05
8.0000E-01	4.8659E-05
8.2000E-01	2.7603E-05
8.4000E-01	1.3354E-05
8.6000E-01	3.9869E-06
8.8000E-01	-1.9236E-06
9.0000E-01	-5.4234E-06
9.2000E-01	-7.2743E-06
9.4000E-01	-8.0255E-06
9.6000E-01	-8.0681E-06
9.8000E-01	-7.6771E-06
1.0000E 00	-7.0421E-06

MODEL VELOCITY RESPONSE, P**2=1.0 PAGE 1

MINIMUM -4.8374E 00 X2 VERSUS TIME MAXIMUM 1.0000E 00



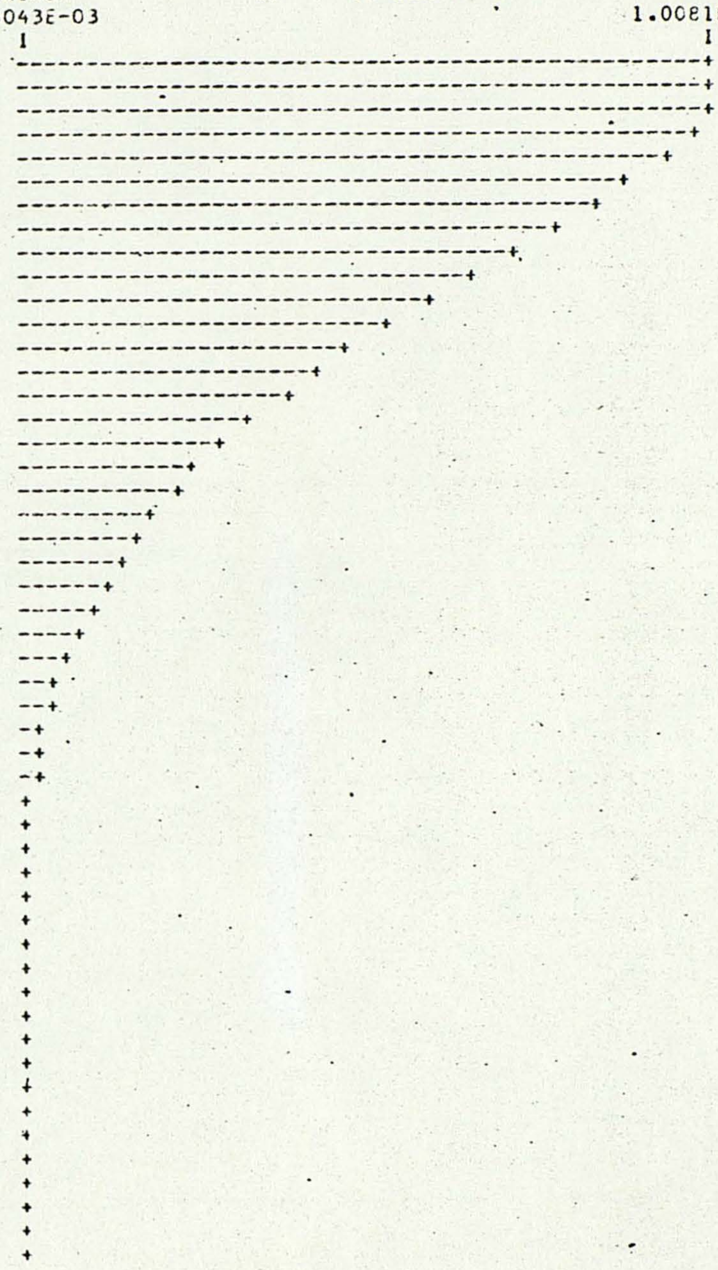
TIME	X2
0.0	1.0000E 00
2.0000E-02	-1.9849E 00
4.0000E-02	-3.7000E 00
6.0000E-02	-4.5532E 00
8.0000E-02	-4.8374E 00
1.0000E-01	-4.7605E 00
1.2000E-01	-4.4675E 00
1.4000E-01	-4.0583E 00
1.6000E-01	-3.5993E 00
1.8000E-01	-3.1337E 00
2.0000E-01	-2.6883E 00
2.2000E-01	-2.2780E 00
2.4000E-01	-1.9105E 00
2.6000E-01	-1.5879E 00
2.8000E-01	-1.3094E 00
3.0000E-01	-1.0721E 00
3.2000E-01	-8.7222E-01
3.4000E-01	-7.0540E-01
3.6000E-01	-5.6735E-01
3.8000E-01	-4.5397E-01
4.0000E-01	-3.6145E-01
4.2000E-01	-2.8643E-01
4.4000E-01	-2.2594E-01
4.6000E-01	-1.7743E-01
4.8000E-01	-1.3872E-01
5.0000E-01	-1.0797E-01
5.2000E-01	-8.3669E-02
5.4000E-01	-6.4544E-02
5.6000E-01	-4.9562E-02
5.8000E-01	-3.7877E-02
6.0000E-01	-2.8804E-02
6.2000E-01	-2.1790E-02
6.4000E-01	-1.6392E-02
6.6000E-01	-1.2258E-02
6.8000E-01	-9.1066E-03
7.0000E-01	-6.7170E-03
7.2000E-01	-4.9146E-03
7.4000E-01	-3.5631E-03
7.6000E-01	-2.5558E-03
7.8000E-01	-1.8104E-03
8.0000E-01	-1.2628E-03
8.2000E-01	-8.6414E-04
8.4000E-01	-5.7669E-04
8.6000E-01	-3.7183E-04
8.8000E-01	-2.2786E-04
9.0000E-01	-1.2841E-04
9.2000E-01	-6.1215E-05
9.4000E-01	-1.7130E-05
9.6000E-01	1.0601E-05
9.8000E-01	2.6941E-05
1.0000E 00	3.5498E-05

```
****CONTINUOUS SYSTEM MODELING PROGRAM****

***PROBLEM INPUT STATEMENTS***

INITIAL
      IC1=1.0
      IC2=1.0
DYNAMIC
      X1=INTGRL(IC1, X2)
      X2=INTGRL(IC2, -51.9*X1-12.12*X2)
TIMER FINTIM=1., OUTDEL=0.02, PRDEL=0.02
PRTPLOT X1
LABEL FIGURE 4-5 MODEL POSITION RESPONSE, P**2=10.0
PRTPLOT X2
LABEL MODEL VELOCITY RESPONSE, P**2=10.0
END
STOP

OUTPUT VARIABLE SEQUENCE
IC1     IC2     X1      ZZ0003 X2

OUTPUTS     INPUTS    PARAMS  INTEGS + MEM BLKS  FORTRAN  DATA CDS
  9(500)   13(1400)   3(400)    2+  0=  2(300)    6(600)       6
```

FIGURE 4-5 MODEL POSITION RESPONSE, P**2=10.0 PAGE 1

MINIMUM -7.6043E-03 X1 VERSUS TIME MAXIMUM 1.0081E 00

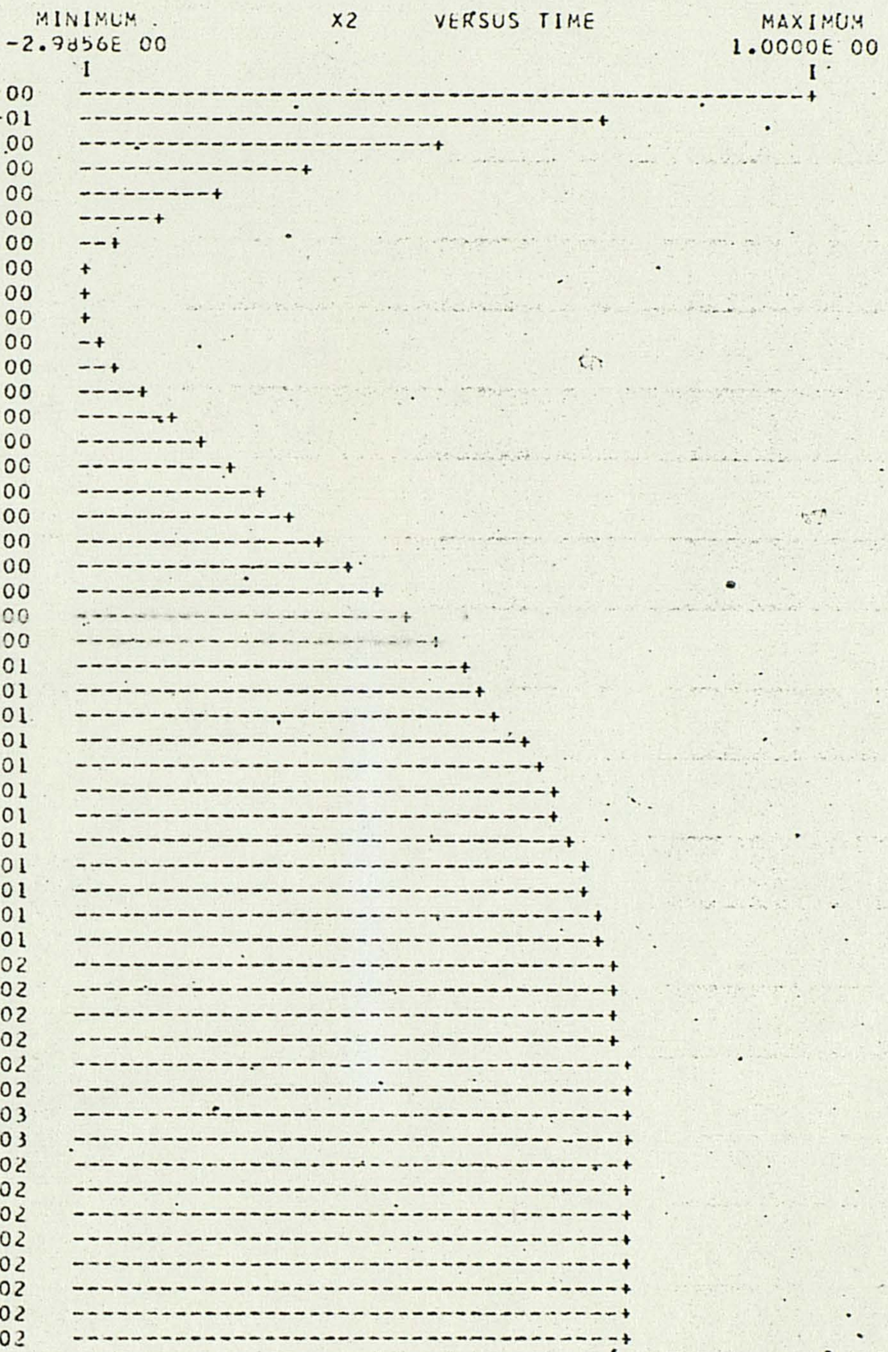
TIME	X1
0.0	1.0000E 00
2.0000E-02	1.0081E 00
4.0000E-02	9.9595E-01
6.0000E-02	9.6806E-01
8.0000E-02	9.2836E-01
1.0000E-01	8.6013E-01
1.2000E-01	8.2607E-01
1.4000E-01	7.6846E-01
1.6000E-01	7.0910E-01
1.8000E-01	6.4947E-01
2.0000E-01	5.9072E-01
2.2000E-01	5.3374E-01
2.4000E-01	4.7921E-01
2.6000E-01	4.2760E-01
2.8000E-01	3.7923E-01
3.0000E-01	3.3429E-01
3.2000E-01	2.9287E-01
3.4000E-01	2.5498E-01
3.6000E-01	2.2055E-01
3.8000E-01	1.8948E-01
4.0000E-01	1.6160E-01
4.2000E-01	1.3675E-01
4.4000E-01	1.1474E-01
4.6000E-01	9.5350E-02
4.8000E-01	7.8381E-02
5.0000E-01	6.3621E-02
5.2000E-01	5.0868E-02
5.4000E-01	3.9924E-02
5.6000E-01	3.0601E-02
5.8000E-01	2.2722E-02
6.0000E-01	1.6121E-02
6.2000E-01	1.0644E-02
6.4000E-01	6.1495E-03
6.6000E-01	2.5098E-03
6.8000E-01	-3.9272E-04
7.0000E-01	-2.6632E-03
7.2000E-01	-4.3959E-03
7.4000E-01	-5.6747E-03
7.6000E-01	-6.5739E-03
7.8000E-01	-7.15[illegible]E-03
8.0000E-01	-7.4855E-03
8.2000E-01	-7.6043E-03
8.4000E-01	-7.5577E-03
8.6000E-01	-7.3820E-03
8.8000E-01	-7.1032E-03
9.0000E-01	-6.7626E-03
9.2000E-01	-6.3659E-03
9.4000E-01	-5.9392E-03
9.6000E-01	-5.4943E-03
9.8000E-01	-5.0440E-03
1.0000E 00	-4.5978E-03



MODEL VELOCITY RESPONSE, P**2=10.0

PAGE 1

TIME	X2
0.0	1.0000E 00
2.0000E-02	-1.4267E-01
4.0000E-02	-1.0368E 00
6.0000E-02	-1.7193E 00
8.0000E-02	-2.2233E 00
1.0000E-01	-2.5778E 00
1.2000E-01	-2.8087E 00
1.4000E-01	-2.9381E 00
1.6000E-01	-2.9856E 00
1.8000E-01	-2.9679E 00
2.0000E-01	-2.8994E 00
2.2000E-01	-2.7923E 00
2.4000E-01	-2.6569E 00
2.6000E-01	-2.5017E 00
2.8000E-01	-2.3339E 00
3.0000E-01	-2.1593E 00
3.2000E-01	-1.9825E 00
3.4000E-01	-1.8072E 00
3.6000E-01	-1.6365E 00
3.8000E-01	-1.4724E 00
4.0000E-01	-1.3165E 00
4.2000E-01	-1.1699E 00
4.4000E-01	-1.0334E 00
4.6000E-01	-9.0720E-01
4.8000E-01	-7.9150E-01
5.0000E-01	-6.8613E-01
5.2000E-01	-5.9082E-01
5.4000E-01	-5.0515E-01
5.6000E-01	-4.2863E-01
5.8000E-01	-3.6069E-01
6.0000E-01	-3.0073E-01
6.2000E-01	-2.4815E-01
6.4000E-01	-2.0232E-01
6.6000E-01	-1.6263E-01
6.8000E-01	-1.2849E-01
7.0000E-01	-9.9339E-02
7.2000E-01	-7.4632E-02
7.4000E-01	-5.3870E-02
7.6000E-01	-3.6585E-02
7.8000E-01	-2.2348E-02
8.0000E-01	-1.0765E-02
8.2000E-01	-1.4764E-03
8.4000E-01	5.8397E-03
8.6000E-01	1.1473E-02
8.8000E-01	1.5683E-02
9.0000E-01	1.8698E-02
9.2000E-01	2.0720E-02
9.4000E-01	2.1926E-02
9.6000E-01	2.2468E-02
9.8000E-01	2.2480E-02
1.0000E 00	2.2076E-02

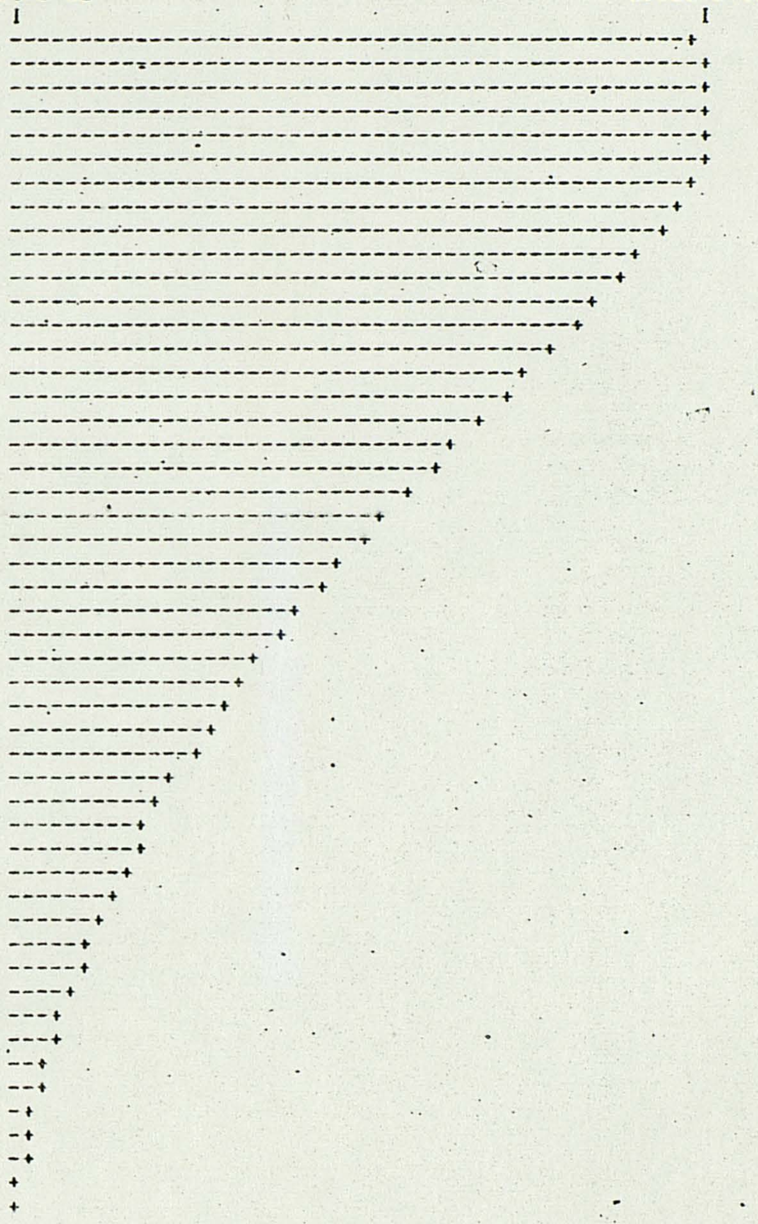


```
                ****CONTINUOUS SYSTEM MODELING PROGRAM****

        ***PROBLEM INPUT STATEMENTS***

      INITIAL
            IC1=1.0
            IC2=1.0
      DYNAMIC
            X1=INTGRL(IC1, X2)
            X2=INTGRL(IC2,-16.3*X1-7.16*X2)
      TIMER FINTIM=1., OUTDEL=0.02, PRDEL=0.02
      PRTPLOT X1
      LABEL FIGURE 4-6 MODEL POSITION RESPONSE, P**2=100.0
      PRTPLOT X2
      LABEL MODEL VELOCITY RESPONSE, P**2=100.0
      END
      STOP

OUTPUT VARIABLE SEQUENCE
IC1    IC2    X1    ZZ0003 X2

 OUTPUTS    INPUTS   PARAMS  INTEGS + MEM BLKS  FORTRAN  DATA CDS
  9(500)  13(1400)   3(400)    2+  0=  2(300)    6(600)     6
```

FIGURE 4-6 MODEL POSITION RESPONSE, P**2=100.0

PAGE 1

MINIMUM 5.7314E-02 — X1 VERSUS TIME — MAXIMUM 1.0229E 00



TIME	X1
0.0	1.0000E 00
2.0000E-02	1.0155E 00
4.0000E-02	1.0228E 00
6.0000E-02	1.0229E 00
8.0000E-02	1.0167E 00
1.0000E-01	1.0052E 00
1.2000E-01	9.8919E-01
1.4000E-01	9.6927E-01
1.6000E-01	9.4612E-01
1.8000E-01	9.2032E-01
2.0000E-01	8.9237E-01
2.2000E-01	8.6273E-01
2.4000E-01	8.3181E-01
2.6000E-01	7.9997E-01
2.8000E-01	7.6751E-01
3.0000E-01	7.3473E-01
3.2000E-01	7.0186E-01
3.4000E-01	6.6911E-01
3.6000E-01	6.3667E-01
3.8000E-01	6.0470E-01
4.0000E-01	5.7331E-01
4.2000E-01	5.4264E-01
4.4000E-01	5.1276E-01
4.6000E-01	4.8376E-01
4.8000E-01	4.5568E-01
5.0000E-01	4.2859E-01
5.2000E-01	4.0251E-01
5.4000E-01	3.7747E-01
5.6000E-01	3.5347E-01
5.8000E-01	3.3053E-01
6.0000E-01	3.0865E-01
6.2000E-01	2.8781E-01
6.4000E-01	2.6800E-01
6.6000E-01	2.4921E-01
6.8000E-01	2.3141E-01
7.0000E-01	2.1456E-01
7.2000E-01	1.9870E-01
7.4000E-01	1.8373E-01
7.6000E-01	1.6964E-01
7.8000E-01	1.5640E-01
8.0000E-01	1.4397E-01
8.2000E-01	1.3233E-01
8.4000E-01	1.2144E-01
8.6000E-01	1.1127E-01
8.8000E-01	1.0177E-01
9.0000E-01	9.2930E-02
9.2000E-01	8.4702E-02
9.4000E-01	7.7057E-02
9.6000E-01	6.9964E-02
9.8000E-01	6.3393E-02
1.0000E 00	5.7314E-02

```
MODEL VELOCITY RESPONSE, P**2=100.0
                                                                                          PAGE    1

                                   MINIMUM                X2      VERSUS TIME               MAXIMUM
                                 -1.6432E 00                                               1.0000E 00
 TIME           X2                 I                                                          I
 0.0            1.0000E 00         -------------------------------------------------+
 2.0000E-02     5.6022E-01         -----------------------------------------+
 4.0000E-02     1.7571E-01         ----------------------------------+
 6.0000E-02    -1.5857E-01         ----------------------------+
 8.0000E-02    -4.4728E-01         ----------------------+
 1.0000E-01    -6.9476E-01         -----------------+
 1.2000E-01    -9.0500E-01         -------------+
 1.4000E-01    -1.0817E 00         ----------+
 1.6000E-01    -1.2283E 00         -------+
 1.8000E-01    -1.3476E 00         -----+
 2.0000E-01    -1.4432E 00         ---+
 2.2000E-01    -1.5171E 00         --+
 2.4000E-01    -1.5720E 00         -+
 2.6000E-01    -1.6100E 00         +
 2.8000E-01    -1.6331E 00         +
 3.0000E-01    -1.6432E 00         +
 3.2000E-01    -1.6420E 00         +
 3.4000E-01    -1.6310E 00         +
 3.6000E-01    -1.6116E 00         +
 3.8000E-01    -1.5850E 00         -+
 4.0000E-01    -1.5523E 00         -+
 4.2000E-01    -1.5145E 00         --+
 4.4000E-01    -1.4726E 00         ---+
 4.6000E-01    -1.4273E 00         ----+
 4.8000E-01    -1.3795E 00         ----+
 5.0000E-01    -1.3296E 00         -----+
 5.2000E-01    -1.2783E 00         ------+
 5.4000E-01    -1.2261E 00         -------+
 5.6000E-01    -1.1734E 00         --------+
 5.8000E-01    -1.1206E 00         ---------+
 6.0000E-01    -1.0680E 00         ----------+
 6.2000E-01    -1.0160E 00         -----------+
 6.4000E-01    -9.6475E-01         ------------+
 6.6000E-01    -9.1451E-01         -------------+
 6.8000E-01    -8.6540E-01         --------------+
 7.0000E-01    -8.1759E-01         ---------------+
 7.2000E-01    -7.7119E-01         ----------------+
 7.4000E-01    -7.2630E-01         -----------------+
 7.6000E-01    -6.8299E-01         ------------------+
 7.8000E-01    -6.4131E-01         ------------------+
 8.0000E-01    -6.0129E-01         -------------------+
 8.2000E-01    -5.6297E-01         --------------------+
 8.4000E-01    -5.2634E-01         ---------------------+
 8.6000E-01    -4.9140E-01         ---------------------+
 8.8000E-01    -4.5814E-01         ----------------------+
 9.0000E-01    -4.2654E-01         -----------------------+
 9.2000E-01    -3.9656E-01         -----------------------+
 9.4000E-01    -3.6818E-01         ------------------------+
 9.6000E-01    -3.4134E-01         ------------------------+
 9.8000E-01    -3.1602E-01         -------------------------+
 1.0000E 00    -2.9215E-01         -------------------------+
```

U.F.C.C.   S Y S T E M   L O G   4 OCT 72

```
$ 16.46.14 JOB 596 -CANNON  -BEG EXEC-110-C M
$ 16.46.14 JOB 596  00 7-TRK, 01 9-TRKS FTU001
$*16.46.16 JOB 596 CANNON .   OCO,FTU001-RING OUT
 *16.46.16 JOB 596 IEF233A M OCO,FTU001,,CANNON,LOAD
 *16.47.37 JOB 596 IEC209I CANNON    FTU001 OCO TR=000,TW=000,EG=000,CL=000,N=000,SIO=00993
 *16.47.37 JOB 596 IEC202E K OCO,FTU001,NL,CANNON,LOAD
$ 16.47.47 JOB 596 END EXEC

//CANNON JOB (3010,0076,9,9),'CANNON A.',CLASS=M,MSGLEVEL=(2,0)          JOB 596
***SETUP        TAPE9,1,FTU001
***MESSAGE *** VOLSER=FTU001 - RINGOUT, KEEP ****
//LOAD EXEC PGM=IEHMOVE
//SYSPRINT DD SYSOUT=A
//SYSUT1 DD UNIT=SYSDA,DISP=OLD,VOL=SER=WORK01
//DD1 DD DSN=SYSCTLG,DISP=SHR,UNIT=3330,VOL=SER=SYSIPL
//LCT DD DSN=FTULCT,DISP=(NEW,KEEP),SPACE=(CYL,(2,2,2)),
//             VOL=SER=WORK01,UNIT=SYSDA
//TAPEIN DD UNIT=TAPE9,VOL=SER=FTU001,LABEL=(1,NL),
//             DCB=(RECFM=FB,LRECL=80,BLKSIZE=800,DEN=3),DISP=OLD
//SYSIN DD *
IEF373I STEP /LOAD    / START 72278.1646
IEF374I STEP /LOAD    / STOP  72278.1647 CPU   0MIN 01.36SEC MAIN  62K LCS   OK
// EXEC  PGM=BASMAT
//STEPLIB DD DSN=FTULCT,DISP=(OLD,KEEP),UNIT=SYSDA,VOL=SER=WORK01
//FT05F001 DD DDNAME=SYSIN
//FT06F001 DD SYSOUT=A
//SYSIN DD *
IEF373I STEP /        / START 72278.1647
IEF374I STEP /        / STOP  72278.1647 CPU   0MIN 00.23SEC MAIN  96K LCS   OK
// EXEC  PGM=FRESP
//STEPLIB DD DSN=FTULCT,DISP=(OLD,KEEP),UNIT=SYSDA,VOL=SER=WORK01
//FT05F001 DD DDNAME=SYSIN
//FT06F001 DD SYSOUT=A
//SYSIN DD *
IEF373I STEP /        / START 72278.1647
IEF374I STEP /        / STOP  72278.1647 CPU   0MIN 02.23SEC MAIN 116K LCS   OK
IEF375I  JOB /CANNON  / START 72278.1646
IEF376I  JOB /CANNON  / STOP  72278.1647 CPU   0MIN 03.82SEC
```

```
SYSTEM SUPPORT UTILITIES ---- IEHMOVE

   COPY PDS=FTULCT,TO=3330=WORK01,FROM=2400=FTU001,FROMDD=TAPEIN
A PREALLOCATED DATA SET IS BEING USED
  MEMBER BASMAT   HAS BEEN MOVED/COPIED.
  MEMBER FRESP    HAS BEEN MOVED/COPIED.
  MEMBER GTRESP   HAS BEEN MOVED/COPIED.
  MEMBER PRFEXP   HAS BEEN MOVED/COPIED.
  MEMBER RTLOC    HAS BEEN MOVED/COPIED.
  MEMBER RTRESP   HAS BEEN MOVED/COPIED.
  MEMBER SENSIT   HAS BEEN MOVED/COPIED.
  MEMBER STVARFD  HAS BEEN MOVED/COPIED.
        DATA SET FTULCT HAS BEEN COPIED TO VOLUME(S)
                  WORK01
```

```
BASIC MATRIX PROGRAM
PROBLEM IDENTIFICATIONS     Model STM p^-2 = 0.10

**********************************************

     THE A MATRIX

      0.0                 1.0000000E 00
     -5.3759985E 02      -6.1699997E 01

     THE DETERMINANT OF THE MATRIX

      5.3759985E 02

     THE INVERSE OF THE MATRIX

IHC208I IBCOM - PROGRAM INTERRUPT-  UNDERFLOW   OLD PSW IS  FF45000DA20D53BA . REGISTER

TRACEBACK  ROUTINE  CALLED FROM ISN   REG. 14    REG. 15    REG.  0    REG.  1

           SIMEQ              0033    420C65E2   000D4CF8   00000000   000C5D74

           MAIN                       00012912   010C5C48   FD000008   00104FF8

ENTRY POINT=  010C5C48

STANDARD FIXUP TAKEN , EXECUTION CONTINUING
     -1.1476934E-01      -1.8601194E-03
      1.0000000E 00       0.0

**********************************************

     THE MATRIX COEFFICIENTS OF THE NUMERATOR OF THE RESOLVENT MATRIX

     THE MATRIX COEFFICIENT OF S**1

      1.0000000E 00       0.0
      0.0                 1.0000000E 00

     THE MATRIX COEFFICIENT OF S**0

      6.1699997E 01       1.0000000E 00
     -5.3759985E 02       0.0

**********************************************

     THE CHARACTERISTIC POLYNOMIAL - IN ASCENDING POWERS OF S

      5.3759985E 02       6.1699997E 01       1.0000000E 00

**********************************************

     THE EIGENVALUES OF THE A MATRIX
        REAL PART           IMAGINARY PART

     -1.0500000E 01       0.0
     -5.1199982E 01       0.0

**********************************************
```

```
THE ELEMENTS OF THE STATETRANSITION MATRIX

THE MATRIX COEFFICIENT OF EXP(-1.050000E 01)T

 1.2579851E 00        2.4570033E-02
-1.3208847E 01       -2.5798571E-01

THE MATRIX COEFFICIENT OF EXP(-5.119998E 01)T

-2.5798535E-01       -2.4570033E-02
 1.3208847E 01        1.2579851E 00
```

BASIC MATRIX PROGRAM
PROBLEM IDENTIFICATIONS   Model STM $\bar{p}^2 = 1.0$

****

THE A MATRIX

0.0	1.0000000E 00
-1.6479999E 02	-2.4799988E 01

THE DETERMINANT OF THE MATRIX

1.6479999E 02

THE INVERSE OF THE MATRIX

-1.5048534E-01	-6.0679615E-03
1.0000000E 00	0.0

****

THE MATRIX COEFFICIENTS OF THE NUMERATOR OF THE RESOLVENT MATRIX

THE MATRIX COEFFICIENT OF S**1

1.0000000E 00	0.0
0.0	1.0000000E 00

THE MATRIX COEFFICIENT OF S**0

2.4799988E 01	1.0000000E 00
-1.6479999E 02	0.0

****

THE CHARACTERISTIC POLYNOMIAL - IN ASCENDING POWERS OF S

1.6479993E 02	2.4799988E 01	1.0000000E 00

****

THE EIGENVALUES OF THE A MATRIX

REAL PART	IMAGINARY PART
-1.2399994E 01	-3.3226624E 00
-1.2399994E 01	3.3226624E 00

****

THE ELEMENTS OF THE STATETRANSITION MATRIX

THE MATRIX COEFFICIENT OF EXP(-1.239999E 01)T*COS(3.322662E 00)T

1.0000000E 00	0.0
0.0	1.0000000E 00

THE MATRIX COEFFICIENT OF EXP(-1.239999E 01)T*SIN(3.322662E 00)T

3.7319450E 00	3.0096340E-01
-4.9593755E 01	-3.7319450E 00

```
BASIC MATRIX PROGRAM            Model STM p^-2 = 10.0
PROBLEM IDENTIFICATIONS

*********************************************

THE A MATRIX

 0.0                     1.0000000E 00
-5.1899994E 01          -1.2120000E 01

THE DETERMINANT OF THE MATRIX

 5.1899994E 01

THE INVERSE OF THE MATRIX

-2.3352599E-01          -1.9267824E-02
 1.0000000E 00           0.0

*********************************************

THE MATRIX COEFFICIENTS OF THE NUMERATOR OF THE RESOLVENT MATRIX

THE MATRIX COEFFICIENT OF S**1

 1.0000000E 00           0.0
 0.0                     1.0000000E 00

THE MATRIX COEFFICIENT OF S**0

 1.2120000E 01           1.0000000E 00
-5.1899994E 01           0.0

*********************************************

THE CHARACTERISTIC POLYNOMIAL - IN ASCENDING POWERS OF S

 5.1899994E 01           1.2120000E 01           1.0000000E 00

*********************************************

THE EIGENVALUES OF THE A MATRIX
   REAL PART              IMAGINARY PART

-6.0599995E 00          -3.8956909E 00
-6.0599995E 00           3.8956909E 00

*********************************************

THE ELEMENTS OF THE STATETRANSITION MATRIX

THE MATRIX COEFFICIENT OF EXP(-6.059999E 00)T*COS( 3.895691E 00)T

 1.0000000E 00           0.0
 0.0                     1.0000000E 00

THE MATRIX COEFFICIENT OF EXP(-6.059999E 00)T*SIN( 3.895691E 00)T

 1.5555639E 00           2.5669384E-01
-1.3322409E 01          -1.5555649E 00
```

```
BASIC MATRIX PROGRAM              Model STM p-2 = 100.0
PROBLEM IDENTIFICATIONS

**********************************************

   THE A MATRIX

    0.0                     1.0000000E 00
   -1.6299988E 01          -7.1599998E 00

   THE DETERMINANT OF THE MATRIX

    1.6299988E 01

   THE INVERSE OF THE MATRIX

   -4.3926412E-01          -6.1349738E-02
    1.0000000E 00           0.0

**********************************************

   THE MATRIX COEFFICIENTS OF THE NUMERATOR OF THE RESOLVENT MATRIX

   THE MATRIX COEFFICIENT OF S**1

    1.0000000E 00           0.0
    0.0                     1.0000000E 00

   THE MATRIX COEFFICIENT OF S**0

    7.1599998E 00           1.0000000E 00
   -1.6299988E 01           0.0

**********************************************

   THE CHARACTERISTIC POLYNOMIAL - IN ASCENDING POWERS OF S

    1.6299988E 01           7.1599998E 00           1.0000000E 00

**********************************************

   THE EIGENVALUES OF THE A MATRIX
      REAL PART               IMAGINARY PART

   -3.5799999E 00          -1.8664379E 00
   -3.5799999E 00           1.8664379E 00

**********************************************

   THE ELEMENTS OF THE STATETRANSITION MATRIX

   THE MATRIX COEFFICIENT OF EXP(-3.5800000E 00)T*COS( 1.8664398E 00)T

    1.0000000E 00           0.0
    0.0                     1.0000000E 00

   THE MATRIX COEFFICIENT OF EXP(-3.5800000E 00)T*SIN( 1.8664398E 00)T

    1.9180918E 00           5.3577983E-01
   -8.7332038E 00          -1.9180918E 00
```

VITA

Arthur G. Cannon, Jr. was born in Wewoka, Oklahoma, on September 2, 1930. He received the Bachelor of Science degree in electrical engineering from the University of Missouri, Columbia, in 1957.

From 1957 to 1966, he was employed by the Sandia Corporation, Albuquerque, New Mexico. In 1966, he moved to Orlando, Florida where he was employed by Martin Co. until 1968. From 1968 to the present time, he has been employed by the U. S. Naval Training Equipment Center which sponsored his return to full-time graduate study at Florida Technological University in September, 1971.

Mr. Cannon is a member of Eta Kappa Nu, Institute of Electrical and Electronic Engineers, Florida Engineering Society and the National Society of Professional Engineers. He is a registered Professional Engineer in the State of Florida.

OPTIMIZATION ANALYSIS OF A SIMPLE

POSITION CONTROL SYSTEM

ABSTRACT

One of the problem areas of modern optimal control theory is the definition of suitable performance indices.

This thesis demonstrates a rational method of establishing a quadratic performance index derived from a desired system model. Specifically, a first order model is used to provide a quadratic performance index for which a second order system is optimized. Extension of the method to higher order systems, while requiring more computations, involves no additional theoretical complexities.