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Simultaneity in the Scientific Enterprise

Nikias Sarafoglou^{[a],*}; Menas Kafatos^[a]; James H. Beall^[b]

^[a]Chapman University, Orange, CA, USA.

^[b]St. John's College, Annapolis, MD, USA.

* Corresponding author.

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Abstract

In this article, we explore the concept of simultaneity in the scientific enterprise, defined herein as the near-coincident discovery of significant advances in the development of our scientific understanding of the world. We do this by examining two case studies of such coincident or near-coincident discoveries: the development of the so-called Lorentz transformation by H.A. Lorentz (1904) and A. Einstein (1905); and the Aharonov-Bohm effect discovered independently in chronological order by Franz (1939), Ehrenberg and Siday (1949) and Aharonov and Bohm (1959). It is now generally acknowledged that the Lorentz transformations were independently developed by both Lorentz and Einstein as they worked on different approaches to solve a similar problem – i.e., the preservation of the *form* of Maxwell's equations in coordinate systems moving relative to one another, while the relationship between the Ehrenberg-Siday and Aharonov-Bohm works is still controversial. In our view, these independent discoveries allow some speculation about the nature of human discovery and understanding of scientific truths as they progress through time.

Key words: Gauge field theory; Quantum electrodynamics; Aharonov-Bohm effect; Sociology of knowledge; Multiple scientific discoveries

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“We are inclined to think that, with respect to every great addition which has been made to the stock of human knowledge, the case has been similar: that without Copernicus we should have been Copernicans - that without Columbus America would have been discovered - that without Locke we should have possessed a just theory of the origin of human ideas.”

--Lord Macaulay, 1880 (cited in Merton, 1961)

1. MODELS OF SCIENTIFIC DISCOVERIES

It is remarkable to note that ideas which advance the state of human scientific knowledge seem to not to be limited to one individual at a particular moment in history. Thus, Newton and Leibniz both invented the calculus, thought in stunningly different ways. Newton's method for the calculus, which is certainly understood today to be equivalent to Leibniz's, uses a geometric conception, while Leibniz's work is more purely analytical, albeit with geometric analogs. Neither man admitted the other's contribution as original.

This bears on one of the most important subjects in the sociology of knowledge, and one might add of our conception of history: the theory of “multiples”. Merton (1973) comments on these phenomena, by noting that

Sometimes the discoveries are simultaneous or almost so; sometimes a scientist will make a new discovery which, unknown to him, somebody else has made years before.

Merton develops this “multiples' hypothesis” in his paper, opining that multiple discoveries are the most common pattern in science, and that unique discoveries are rarer. Zuckerman (1977) and Lamp and Easton (1984) have brought forward similar ideas on this theory of multiples, and the reader can certainly bring to mind other examples, including the works of Descartes and Bacon, who many consider the actual authors of modernity.

If we look at the history of the Nobel Prize in the sciences, a general useful indicator of the most important discoveries in science, we can find that the majority of the Nobel prizes are occasions where “multiples” have occurred: That is, there are similar threads of investigation which lead to independent discoveries of similar import. Cole (2004) suggests that “great men or women of science might speed the rate of intellectual advance, but they are not necessary for that advance.” The question of priority would seem with this gloss to be one that is at least in some small part arbitrary.

There are certainly what should be called “single” discoveries in the Nobel Pantheon. For example, the winners of the Nobel in Physics 2010, which was awarded to two scientists from Manchester University working together to produce a new material named “graphene”.

The concept of truly new ideas being brought forth has indeed been questioned. Stigler (1980) in characterizing his “law of eponymy,” suggests that “science accepts ideas only when they fit into the then-current state of the science.” But this is clearly not the case for radically new conceptions.

New conceptions actually seem to change the paradigm of scientific thought. In Kuhn’s (1962) now-classic work, “*The Structure of Scientific Revolutions*,” Kuhn posits what might be called an Hegelian reformation of thought. It has not escaped our notice that supposing such a “revolution” does in fact have as one of its underlying assumptions a particular theory of history that is essentially Hegelian.

One can hypothesize different modes for the origination of genuinely new ideas. Following the classification system of Brannigan and Wanner (1983), the scientific discoveries can be classified into three types of models:

The genius models (de Sola Price, 1961)

The cultural maturation models or zeitgeist (Merton, 1973; Hegel, 1979)

The chance models (Simonton, 1979)

In the cultural maturation models, the evolution of research programs is more important than the input of individual workers. For models which suppose chance, the “serendipity pattern” of a scientist is crucial. And of course the model of genius is similar to the “great man” theory of history.

The question of the presence of “multiples” is clearly relevant to these considerations. If multiple scientific discoveries are commonplace, where by “multiples” we mean the independent or nearly independent discovery of genuinely new concepts, then it lends credence to option 2 above. But in many cases (and we consider two later in this

article), the new concept is a natural consequence of either the failure of earlier interpretations or is set upon the stage of scientific advancement by immediately prior work.

The case is less clear in mathematics, where it would seem at first glance that it is a particular moment in history that conditions the advance. For example, we can consider Farkas Bolyai’s warning to his son Janos to publish as soon as possible his non-Euclidian geometry (see, e.g., Merton, 1961). To quote from Merton’s text,

because ideas pass easily from one to another.....that many things have an epoch, in which they are found at the same time in several places, just as the violets appear on every side in spring.....for the advantage is always to the first comer.

In the field of medical research, there is for example, case of multiples in HIV/AIDS research. Two separate teams from the United States and France published in *Science* from 1983-1984, separate articles suggesting that the HIV virus was the cause of AIDS. The American team was under the leadership of Robert Gallo (Institute of Human Virology, Baltimore) and the French team was under the leadership of Luc Montagnier (Pasteur Institute, Paris).

The discovery resulted in controversy between the two groups, and that fight between these two teams for the HIV-patent was ended by a “co-discovery” agreement of the French and the US governments to split the property rights from the patent in 1987.

Gallo and Montagnier accepted their respective discoveries as multiples and started to publish together after 1987.

Interestingly, in 2008, two members of the French team (Montagnier and Barre-Sinoussi) received the Nobel Prize in medicine for the discovery of HIV, but Gallo was not nominated.¹

While at first glance it does appear that most of the great scientific inventions and ideas are associated with certain individuals, a careful consideration shows this not to be the case. Darwin is credited with the theory of evolution, but it is clear from his extensive introduction even in the first (1859 edition) of *Origin of Species* that there was significant concern on his part to delineate his contribution of the concept of natural selection from other authors with very similar arguments. From Darwin’s own comments, it is clear that Wallace had major parts of evolution worked out before Darwin’s publication of the “outline” of his argument.

Other examples can certainly be cited. The natural tendency of history to simplify or obscure leaves for the general public an abridged version of the development of these ideas – especially stories of great discoveries.

¹ This HIV-AIDS-controversy was regenerated by the host institute Karolinska of the Nobel Prize and articles like “Do not cry for me Karolinska” appeared in the journals, news media and the blogs.

Roentgen and Stark independently discovered X-rays, but Roentgen conducted extensive research on their properties and worked out many applications, to the point that Stokes in presenting him in a public lecture in England rightly said that while Stark may have discovered X rays, it is Roentgen who put them in our bones (Riesz, 1995).

At first gloss, the fate of great discoveries would seem to depend very much on the background, outlook, and original motivation of the researchers. But the situation is far more complex than this. In the case of accidental or serendipitous discovers, it would seem to depend on the role of an astute observer such as Becquerel and Curie for natural radioactivity, or Fleming for penicillin. But in some instances, the specific initial goals of potential discoverers prevent them from recognizing the importance of what they stumbled upon.

Two of Merton's (1979) definitions for the reward-system of scientific discoveries are perhaps relevant here: The "Matthew effect" (ME) and "obliteration by incorporation," The Matthew effect describes the pattern of the misallocation of the scientific work based somewhat ironically on the Gospel according to St. Matthew:

For unto every one that hath shall be given, and he shall have abundance: but from him that hath not shall be taken away even that which he hath.

In other words, the more eminent of the collaborating or competing scientists will get the lion's share.

Obliteration by incorporation occurs when the concept becomes so popularized that inventor is not cited in the references. It goes without saying that this tends to ignore the subtle of scientific works. For instance, Einstein's Theory of Relativity is mentioned in papers and books without any reference to the 1905 paper in *Annalen der Physik*.

2. SIMULTANEITY OF SCIENTIFIC DISCOVERIES

Although an optimistic philosopher of science Polanyi (1962) explained science as an "efficient market of ideas," the market of ideas nevertheless does not seem to always be so efficient.

As discussed by many authors – see, e.g., such works as the *Economics of Science* (Stigler), the *Sociology of Science* (Merton), the *Psychology of Science* (Simonton), and the *Philosophy of Science* (Polanyi, Kuhn), an analysis from many different perspectives shows both the circumstances of this efficiency and the possible distortions of the process.

Stigler (1982) pointed out the current state of the science is a critical factor for the acceptance of the new theories:

If an earlier, valid statement of a theory falls on deaf ears, and a later restatement is accepted by the science, this is surely proof that the science accepts ideas only when they fit into the then-current state of the science.

Scientific research can certainly be considered as a public good. The well-known definition by Samuelson states that a public good occurs when "one man's consumption does not reduce some other man's consumption". In the history of economic thought, the lighthouse is the best exemplification of the public good. But research has an idiosyncrasy: it is not only a public good, but a convolution of public and private realms. Merton emphasized the importance of reward system in science, and that this system is based on priority of discovery.

Stephan (2004) wrote of Merton's analysis, that

The quickest way for a scientist to establish a reputation among peers is the "share" knowledge by placing in the public domain, preferably in print. Through this sharing, the idea is established as the private property of scientist.....and leads the scientists to share information rapidly in order to build reputation and hence capture the financial resources bestowed on the eminent.

Fights for deserved priority, intellectual property and property rights can be observed in any process related to scientific discovery.

In this paper, we consider two case studies of simultaneity in scientific discoveries: the development of the Lorentz transformations by both Lorentz and independently by Einstein, and the controversy surrounding the Franz/Ehrenberg-Siday/Aharonov-Bohm effect. In the case of the latter effect, we use an interdisciplinary approach based on physics, bibliometrics and the sociology of science.

3. LORENTZ TRANSFORMATIONS AS OBTAINED BY LORENTZ AND EINSTEIN

We remark briefly on the so-called Lorentz transformation as an example of a kind of simultaneous development of scientific concepts by two authors. The paper by Lorentz (1905 or 1904) from whence the transformations derive their name was an attempt to explain the negative results of Michelson and Morley's (1887) paper to detect the movement of the Earth through the "luminiferous aether." Lorentz's hypothesis was that the motion of the earth through the aether caused a change in the length of the measuring arms of Michelson and Morley's apparatus to just such an extent that it canceled out the expected phase shift that should have been produced by the 90 degree rotation of the apparatus from the supposed direction of motion of the apparatus through the aether. Thus, Lorentz's work questioned neither the simultaneity of events nor the disparate lengths of rods oriented parallel to the direction of motion.

Of course, the well-known derivation of the Lorentz transformation by Einstein (1905) in his "Electrodynamics of Moving Bodies" has a completely different derivation of the Lorentz transformations which is based on a careful analysis of what it means to measure a moving rod in a stationary system. That this analysis revealed a change in length in the direction of motion of the moving rod

as measured in the stationary system is still remarkable. Further, the effect this analysis has on our ability to synchronize clocks in a moving versus a resting system is even more remarkable, and leads to (among other things) the infamous “twin paradox”.

It is now reasonably well established that these two derivations were done independently. This is corroborated not only by considerable scholarship (despite recent popularizations to the contrary), but also by the radically different methods by which Lorentz and Einstein developed their results.

A perusal of the Lorentz (1885 or 1904) paper shows that Lorentz regards the Michelson and Morley (1887) as rather definitive in showing that there is no detectable motion of the Earth with respect to the luminiferous aether. Based on this

the negative result of which has led Fitz Gearld and myself to the conclusion that the dimensions of solid bodies are slightly altered by their motion through the aether.

Lorentz proceeds with a detailed derivation of the consequences of this hypothesis. That is, he derives the now-famous Lorentz transformations by assuming that there is a physical pressure on the electric field surrounding charged particles which tends to essentially compress the field along the direction of motion. The idea appears to be of a momentum exerted upon the electric field moving through the aether. Lorentz explicitly notes that it may be shown that in every electrostatic system, moving with a velocity \mathbf{v} , there is a certain amount of *electromagnetic momentum*. If we represent this, in direction \mathcal{G} and magnitude, by a vector \mathbf{G} , the couple in question will be determined by the vector product $[\mathcal{G}, \mathbf{m}]$.

Where, as Lorentz notes, a vector will be denoted by a German letter, its magnitude by the corresponding Latin letter.

Now, if the axis of z is chosen perpendicular to the condenser plates, the velocity \mathbf{v} having any direction we like, and if U is the energy of the condenser, calculated in the ordinary way, the components are given by the following formulae, which are exact up to the first order:

$$\mathcal{G}_x = \frac{2U}{c^2} m_x, \mathcal{G}_y = \frac{2U}{c^2} m_y, \mathcal{G}_z = 0$$

Lorentz then notes that this tends to exert a physical pressure on the material subjected to this motion through the aether.

To the “modern” ear, this sounds perhaps a bit odd, and it certainly has an ad hoc quality, since it’s purpose is to “save the appearance” of a lack of detectable motion through the luminiferous aether. But a look at Maxwell’s original three papers which developed his electromagnetic theory shows that his use of analogies is quite similar to Lorentz’s. For example, Maxwell uses the motion of an incompressible fluid (whose force on a charge is proportional to its velocity \mathbf{v}), to give us an understanding

of the inverse square force law as the fluid expands and slows by the inverse square of the distance from a positive charge.

This method of analogy is, to say the least of it, *radically* different that the method Einstein used in his 1905 paper.

In the first place, he begins his paper with an acknowledgement of the asymmetries inherent in phenomena

It is known that Maxwell’s electrodynamics...when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena (Einstein, 1905).

He uses the physically straightforward example of moving a magnet relative to a conductor as opposed to moving the conductor with respect to the magnet, to illustrate the essential assymetry of this description in terms of Maxwell’s theory. On the one hand, there is an electric field generated in when the magnet moves. On the other hand, when the coil moves, there is no time-dependent magnetic field, and therefore no electric field. The motion of the charges in the coil wire are produced by the motion of a charge in a magnetic field which produces a force perpendicular to the motion of that charge.

To quote,

Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. (Einstein, 1905)

In order to get around this *asymmetry* in the explanation for the effect, which arises only from the relative motion of the conductor and the magnet, along with the inability of the Michelson and Morley experiment to detect any motion of the Earth with respect to the luminiferous aether, Einstein postulates that the speed of light is the same to all observers moving with uniform velocity with respect to one another.

Einstein raises this assumption to a postulate, by noting that

the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. We will raise this conjecture (the purport of which will hereafter be called the “Principle of Relativity”) to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body.

For the purpose of this discussion, it is important to note that he follows this with a careful analysis of what it means to measure time and extension in the moving versus the stationary systems. It is by this method that he derives his version of the “Lorentz Transformations” of time and extension. That is, his versions of the Lorentz transformations is derived from an insistence of the symmetry between the descriptions of nature in to systems

moving relatively with respect to one another.

It is quite interesting to speculate on why he chose the constancy of the velocity of light as the basis for his derivation. A reader engaged in a contemplation of Maxwell's wave equation for electromagnetic phenomena (from Maxwell's third paper) will note that the velocity of the wave is derived from two laboratory constants, the electric susceptibility, ϵ_0 , of free space, and the magnetic permeability, μ_0 , of free space. These must, by the Principle of Relativity, be constant for all observers. It is remarkable to the authors of this paper that this demand likely led him to consider that the velocity of light must therefore be constant for all observers. This in turn led to his derivation of the so-called Lorentz transformations. Most importantly, it leads to the demonstration in the 1905 paper that using those transformations of length and time leads to a preservation of the *form* of Maxwell's equations, even though it abandons the distinction between electric and magnetic phenomena.

In Einstein's later development of the equations of general relativity, he used an equivalent symmetry, the principle of strong equivalence, to derive the transformations of length and time for uniformly accelerated observers. These derivations were for Hilbert a short foray into the differential geometry of curved spaces. For Einstein, the culmination of the presentation of General Relativity in the 1916 paper, represent the culmination of his efforts to naturally understand the physical principle of equivalence, and led to his use of formalisms from differential geometry to explicate the trajectories of bodies in non-Euclidean space-times.

Such examples of simultaneity are not limited to explicitly technical discoveries like Lorentz and Einstein's. They include famous historical examples from the works of Bacon and Descartes, who are widely regarded as the "authors" of the modern world-view. This suggests that the nearly simultaneous developments of concepts in science are commonplace. We pursue an additional example from quantum mechanics in the next section of this paper.

4. ELECTROMAGNETIC POTENTIAL AND ITS EFFECTS ON PARTICLE WAVE FUNCTIONS

In their widely known paper on the "Significance of Electromagnetic Potentials in the Quantum Theory," Aharonov and Bohm (1959, hereafter called AB59) made the observation that it is well known that in classical

mechanics, the motion of a particle can be affected only by forces acting at the particle at its well-defined location. AB59 noted that this is not the case in quantum mechanics where two different paths or histories of a single particle can interfere and potentials can have measurable effect outside of classically allowed regions.

They illustrate this effect for electromagnetic potentials by two examples: the "electric" and the "magnetic" effects upon particles moving in regions where no force exists, but where differences of the scalar or vector potentials along two possible paths of the particle can physically manifest themselves by affecting the resulting interference pattern of the particle.

They further note that in the magnetic case for an infinitely long and narrow solenoid, and for which they provide an exact solution of the scattering problem, that such a scenario can be experimentally tested by using thin magnetic whiskers.

There is of course some controversy about the priority of their discovery, given the earlier work by Ehrenberg, W. and Siday, R. E. (1949, hereafter ES49) published in 1949 on "The Refractive Index in Electron Optics and the Principles of Dynamics."

In a crucial passage in the conclusion of their paper, they remark that,

One might therefore expect wave-optical phenomena to arise which are due to the presence of a magnetic field, but not due to the magnetic field itself, i.e., which arise whilst the rays are in field-free regions only (ES49)

This remarkable comment is not pursued in any detail in their original paper. Neither is it mentioned in the abstract or introduction to their paper. It has the character of an interesting speculation that is meant for consideration in a subsequent work. Unfortunately, Ehrenberg and Siday seem not to have pursued it any further.

Yet the presence of this comment in the Ehrenberg and Siday paper has caused some to question the priority of the fuller development of the effect of these potentials by Aharonov and Bohm (1959), which occurred 10 years later.

In fairness, Aharonov and Bohm (during his lifetime) stated explicitly that they were unaware of the Ehrenberg and Siday paper. Such a statement is credible, especially when one notes that Einstein was apparently not aware of the paper by Lorentz, published a year before Einstein's paper.

Moreover, it is known (Olariu & Popescu, 1985; Washburn & Webb, 1992; Chirkov & Ageev, 2001; Bakke & Furtado, 2010) that Franz (1939 – hereafter F39), who was a student of Arnold Sommerfeld had first predicted

the “unorthodox nature of interference processes” associated with the ES49 and AB59 effect.²

The controversy is therefore an excellent case study for the possibility of simultaneity (or very nearly so) in scientific discoveries, and would seem to support our discussion in Section 2 of this paper, entitled “Models of Scientific Discoveries.”

4a. Tracing the Origins of Electromagnetic Potentials in Quantum Theory

Preskill, Penrose, Peshkin and Boyer have given interesting surveys of the most important theoretical papers for the development of the effect of electromagnetic potentials in quantum theory, and the Aharonov-Bohm effect.

Table 1
Papers Relevant to the Effect of Electromagnetic Potentials in Quantum Mechanics

Name	Year
J.C. Maxwell	1887
H. Weyl	1918
A. Einstein’s Comment	1919
F. London	1927
V. Fock	1927
H. Weyl	1929
P. Dirac	1931
W. Franz	1939
Ehrenberg and Siday	1949
F. London	1950
Yang and Mills	1954
Aharonov and Bohm	

Source: Preskill (1993), Penrose (2004), Peshkin (2010) and Boyer (1973)

The Aharonov-Bohm effect represents a critical and central juncture in the evolution of the ideas about gauge fields and topological phases. Its conception can be traced back to several important previous developments. Conversely, it directly or indirectly motivated several important later discoveries. As such, it is in the main stream of developments in physical thought.

A consideration of the history can start with the development of the vector potential A . The vector potential is a key to all gauge theories, but was introduced already by Maxwell invariance in the sense invariance under scaling transformations was suggested by H. Weyl in

1918 as a way to generalize Einstein’s General Relativity to include not just gravity but also electromagnetism. The flaw in this idea was pointed out by Einstein (1918).

The idea was reincarnated by the works of F. London (who was mainly interested in superconductivity), Fock and, again, Weyl, but with the crucial difference that the gauge transformation changed the phase of the Schrodinger wave-function of the charged particle by some particular value when the particle of charge e moved along a space-time trajectory. It is this phase effect which underlies the Aharonov-Bohm effect.

Another important previous development which was known to AB but did not play any direct role in motivating their ideas was the remarkable generalization of the gauge notion and gauge theories to the non-Abelian case by Yang and Mills (1954).

The AB59 paper produced widely different reactions in the scientific community. Many immediately espoused the new effect. Thus Furry and Ramsey (1960) suggested a different, complementary point of view, and Chambers at Bristol University embarked on an experimental effort to verify the Aharonov-Bohm Effect. Others, including some prominent physicists did not believe it and pointed possible inconsistencies. Victor Weisskopf (1961) wrote in his “Bolder lectures in theoretical physics” that “The first reaction to this work is that it is wrong; the second is that it is obvious”. A lively debate ensued in which Aharonov and Bohm took an active part. The accumulating evidence, especially from the precise experiments of Akira Tonomura *et al.* (1982) clearly confirmed the effect. After this confirmation, the number of citations of AB59 increased substantially, as might be expected.

Over the years, the Aharonov-Bohm effect has gained increasing prominence and relevance in many areas of physics. It underlies flux quantization, the quantum Hall effect and many aspects of mesoscopic physics. Non-Abelian, “Wilson loop” analogs of the Aharonov and Bohm phase are relevant in gauge /string theories and may underlie the phenomenon of confinement. The Aharonov and Bohm effect is mentioned in thousands of abstracts and titles and appears in most textbooks on quantum mechanics.

As mentioned earlier, it is now widely known that the magnetic Aharonov-Bohm effect had been anticipated

² Franz Walter wrote an article 1939 published in *Verhandlungen der Deutschen Physikalischen Gessellschaft*. In this article the subsequent A-B effect was preceded by 20 years, and this article can, therefore, be considered as the first publication of the phenomenon, before the ES49. Both articles of F39 and ES49 dealt with interference caused by magnetic flux in electron optics.

This article was noticed in the German speaking scientific community, but it was unknown in the English speaking world. The normal procedure for a moving linguistically from German to English for a scientific work is A) to achieve an English translation, and B) to have the article republished in an English-journal or book. Unfortunately, the significant Franz’s work was not appreciated by the English language science community as it was not translated into English. Franz was a theoretical physicist from Munster University known for the Franz-Keldysh effect. Arnold Sommerfeld was Franz’s teacher and an interesting point in the history of A-B effect is that the teacher Sommerfeld was cited in the ES49 article but not his student Franz. The first article using as references all three articles, the F39, ES49 and AB59 is an article by Franz(1965) himself with the title:“Uber zwei unorthodoxe Interferenzversuch”. The works of Franz were made known to some extent in the English speaking world by the articles of Boyer(1973) and Olariu and Popescu (1985). Today, the F39-article has a total of ca. 25 co-citations with ES49 and AB59 in theoretical physics.

and could have been discovered decades earlier. In 1949, Ehrenberg and Siday published their paper in the Proceedings of the Physical Society. In the ES49 paper, they calculated the index of refraction for the de-Broglie waves for the electrons. As with ordinary optics, this index controls the trajectories of the electrons. In particular, appropriate external electromagnetic fields can generate a spatially varying index of refraction which mimics convex (or concave) lenses, thus yielding convergent (or divergent) beams.

These effects of course are technologically important issues in the field of electron microscopy. In their 1949 paper, ES49 claim that some of the previous treatments of these issues were not fully gauge invariant and were therefore incorrect. An elaborate, 12 page discussion then followed in which an appropriate variational action principle yields trajectories which, as in the Fermat principle for light, minimize the travel time determined by the index of refraction along the trajectories. ES49 then illustrated their method by presenting three examples of electron rays. The third example (contained in the last 12 lines and the attendant figure 3) are recognizable by most physics graduates today as the semi-classical magnetic Aharonov-Bohm effect.

This remarkable part of the ES49 paper and its resemblance to AB59 was not recognized (and in fact was completely missed) by the entire physics community until sometime after AB59 was published.

It is also true that the genesis of the ES49 paper is dramatically different from that of the AB59 paper. This stems not so much from the earlier date but mainly from the very different background of the ES and AB researchers.

First, Schrodinger's equation is entirely absent from ES49. Those authors rely instead on an analysis of de Broglie's electron waves. Interestingly, ES49 inspired others to think about focusing beams of electrons by electromagnetic devices, e.g. the electron microscope as guiding waves in lenses with varying indices of refraction for which they sought, using classical Hamiltonian approaches, a gauge invariant formulation.

Berry (2010) and Peshkin (2010) provided helpful illuminations of this history, and build the case conclude that the Aharonov-Bohm effect should indeed be named after those two scientists. Their comments are in response to Sturrock and Groves article (2010) that the Aharonov-Bohm effect should be renamed the Ehrenberg-Siday effect.

Aharonov and Bohm ended their collaboration in 1965 and followed different directions of research. Bohm continued working at Birkbeck College with few collaborators on his own Bohmian interpretation of quantum mechanics. His later references to the Aharonov-Bohm effect were mainly in this context and in a general philosophical context.

Aharonov continued working in more of mainstream physics in connection with AB59 and other effects. Later, Aharonov and Casher (1984) found an effect which is

in a way the dual of the Aharonov-Bohm effect. In the Aharonov-Casher effect the line of magnetic dipoles (namely the original magnetic flux in the Aharonov-Bohm set-up) is replaced by a line of charges and the circulating charged particle. In Aharonov and Bohm, this is replaced by a circulating neutral magnetic dipole (e.g., the neutron).

In 1987, Aharonov and Anandan found a generalization of the Berry phase to non-adiabatic processes. The Berry phase itself is an all important phase effect in many branches of physics, and it is related to the Aharonov-Bohm phase. Indeed in one of its earlier incomplete versions (due to Alden Mead, 1980) is referred to as the molecular Aharonov-Bohm phase.

Furthermore, the Berry phase is clearly related to adiabatic process for classical mechanics, and unlike the Aharonov-Bohm effect is often not a genuinely unique quantum mechanical effect.

Preskill (1993) wrote for the ES49:

In a crucial passage (accompanied by a diagram) in the conclusion of their paper, they remarked, "One might therefore expect wave-optical phenomena to arise which are due to the presence of a magnetic field, but not due to the magnetic field itself, i.e., which arise whilst the rays are in field-free regions only." Yet Ehrenberg and Siday did not make much of a fuss about this effect. (It is not mentioned in the abstract or introduction of the paper.)".

Furthermore, Preskill wrote for the AB59:

In spite of all the anticipations, their paper is justly hailed as a great classic. Much more clearly and comprehensively than previous authors, they stressed the special role of the electromagnetic potentials in quantum theory, and that non-local gauge invariant quantities can have observable effects. They also emphasized the experimental implications,....".

As we noted earlier, it is apparent that Aharonov and Bohm were unaware of the ES49 paper when they published the 1959 paper. Shortly after the 1959 paper was published, Bohm was informed of the Ehrenberg and Siday paper. Subsequently, Aharonov and Bohm referred to it in their 1961 (hereafter, AB61) paper. Interestingly, as noted earlier, the ES49 paper's focus was on the effects of magnetic potentials on particle dynamics. Fortunately, the 1961 paper by Aharonov and Bohm brought proper attention to the ES49 paper, and it now has over 200 citations.

It is also interesting to note that even the authors who cited the ES49 paper during the interval from 1959 to 1961 referred to it for reasons other than the Aharonov-Bohm effect, and completely failed to notice the last part of the paper.

Finding these references is rather non-trivial. The ES49 paper was published in the *Proceeding of the London Physical Society (PLPS)*. Unlike the famous Proceeding of the Royal Society, the PLPS is not widely read - so much so that when the citation index, started around 1960, was extended by scanning to previous years, it did not include articles published in PLPS during 1949.

While the paucity of citations (altogether, the authors

of this work found three) in the 10 year period from 1949 to 1959 can in part be due to the small readership of the PLPS, the fact that the citing authors did not refer to this last part of the ES49 paper cannot be attributed simply to the relatively small readership. All three citations by P.A. Sturrock (1951), by Glaser (1951), and by Ehrenberg and Siday (1951) responding to Glaser, refer only to the general variational method in the ES49 paper. No one seems to have noticed the all-important last example.

4b. Sociology of Science as Applied to Aharonov, Bohm, Ehrenberg, and Siday

We know from Merton's *Sociology of Science* that an early recognition of a scientific result is often a function of the author's perceived eminence. With this in mind, let us compare AB 59 and ES49.

Bohm (1917-1992) was a well-known physicist before the publication of AB59. He was a student of Oppenheimer's, a friend of Einstein's, and a colleague of Feynman's, with whom he studied. His interpretation of quantum mechanics, in part based on his early work, including AB59 has had an impact in the world of quantum physicists. Eminence in a field does not of course make one's life proof against trouble. He was a victim of the McCarthy anti-communist era, and he left the US in 1951.

His current citation profile is (almost 20 years after his death) for the period 1949-1959:

Table 2
Bohm's Citations During the Period 1949-1959

Name	Year	Citations
Aharonov and Bohm	1959	3100
Bohm (a)	1952	1822
Bohm and D. Pines	1953	986
Bohm (b)	1952	931
D. Pines and Bohm	1952	537
Bohm and E.P. Gross	1949	522

His citation profile was clearly grown in the intervening years, but in 1959, he was nevertheless a highly regarded physicist, using citation statistics during the 1950's.

The same is true of Aharonov, although he was a graduate student in 1959 at Bristol University. His publications with Bohm, his thesis advisor, during the period of 1957-1959 were well-received, and although a young man, he was becoming better known to the physics community.

Let us now consider the perceived eminence of Ehrenberg and Siday at Birkbeck College. Siday was a young mathematician who died early in 1956. He was ill in his last years and his publication career effectively ended in 1953.

Ehrenberg was more established than Siday during this period. Hiley (1997) wrote for the Ehrenberg's interests "Ehrenberg's main interest was in experimental solid-state physics and he was responsible for the design of the fine

focus X-ray tube which helped Wilkins in his Nobel-prize winning work on DNA".

Ehrenberg and Siday did not continue with their papers on electron optics, however, and this lack of activity in no small part contributed to the relative obscurity of ES49 until the AB61 paper. The majority of Ehrenberg's citations are from the ES49 paper.

It is fascinating to note that despite the fact that the Aharonov-Bohm Effect was immediately recognized and intensively researched after the publication of the 1959 paper, the prescient contribution of the effect of the magnetic potential on quantum mechanical wave functions was completely and thoroughly missed even after the ES49 paper. The citation count of the AB59 during 1959-1969 was 86 citations, as opposed to three citations for ES49 in the ten years after its publication (op. cit.).

Part of the reason for this may be the format of the presentation: whereas AB59 used the conventional Schrodinger equation, ES49 employed de Broglie waves. Note that the "psi" symbol in their paper refers to the gauge rather than the wave-function.

This apparently trivial and superficial detail is part of a much deeper and more relevant difference. AB59 realized immediately that they had discovered an extremely important, novel, and rather puzzling aspect of quantum mechanics to which their paper (and some which followed soon after) were fully dedicated. Indeed, quantum mechanics was the principal area of research for A&B, both prior to the AB59 paper, and thereafter.

Later, Aharonov and Bohm's scientific paths diverged, with Bohm focusing on his version of quantum mechanics, and his now-famous work in the philosophy of physics, including Bohm's cooperation with Krisnamurti; while Aharonov's work focused on "ordinary" quantum mechanics.

The background and focus of Ehrenberg and Siday was completely different. Following the tradition of electron optics, they were interested in how electron waves and their ensuing classical rays are influenced by electromagnetic fields. Their interesting paper was dedicated to a correct gauge invariant treatment of this subject.

The third example of this treatment (which is indeed correct!) is the concise description of the magnetic effect, presented as an afterthought, to which they ES49 ascribed no particular importance. Indeed Siday in the seven years before his untimely death in 1956, and Ehrenberg, in his long scientific career, never came back to this issue. It was after all just a small part of their main mission: the correct treatment of the de-Broglie electron waves which they indeed achieved. Having it naturally follow from the formalism, they saw nothing surprising and special in this and did not advertise it as such.

This brings us to one of the main conclusions that can be drawn from this remarkable case.

4c. Citations of the Aharonov and Bohm During Their Collaboration Period

In our view, the discovery of a great idea or concept is certainly necessary for scientific renown, but it is perhaps not sufficient, as an equally important key ingredient may be missing. By this we do not refer to the advertising and “selling” of the idea in a putative intellectual market place, though that does not hurt, either.

It is first and foremost *crucial* that the discoverers themselves realize the novelty, importance and potential impact of their discovery. Only with this strong internal conviction will they be able to broadly advertise it in good faith to the rest of the community. And this should be the case irrespective of what their original, and often rather different motivations, were. Time and again, in science and other human endeavors, our initial specific goal limits our horizon and prevents us from seeing the discovery we have made!

Aharonov got his Ph.D in 1960, and he continued his cooperation on quantum mechanics and the Aharonov-Bohm effect with Bohm during the period of 1956-1957 at Technion, and from 1958-1960 at Bristol. Remarkably, it was Ehrenberg who helped Bohm obtain a tenured position at at Birkbeck College, and accounted for Bohm’s tenure there from 1961 onward. If the AB59 is not considered, the citation profile of their collaboration during the period 1957-1964 is:

Table 3
The Co-Citation Profile of Bohm and Aharonov

	Citations
AB-1957	251
AB-1961	187
AB-1961	159
AB-1962	54
AB-1963	54
AB-1964	29
BA-1960	18

Although their papers after 1959 did not have as high a “score” for references, their relatively robust citation rate could keep the Aharonov-Bohm effect alive in the proverbial marketplace of ideas.

Finally, in 1964 Aharonov’s collaboration with Bohm came to an end.

5. THE IMPORTANCE OF AHARONOV-BOHM EFFECT TODAY

There is no question that AB59 has had a significant effect on the development of physics after WWII.

Peshkin (2010) wrote that “the 1959 Aharonov-Bohm paper profoundly changed the way we think about electromagnetic fields in quantum mechanics.” The Aharonov-Bohm effect is the common expression of authors when referring to AB59 paper from the 1960s on. In fact, many researchers have used the Aharonov-Bohm paper without mentioning the paradoxical nature of the

quantum mechanical effects. The diversity of the AB-impact on the sciences can be depicted in the next Table 4, by using citation counts for the AB59 paper.

Table 4
The Diversity of the AB-Impact on Sciences

Subject areas	Aharonov-Bohm	AB59 paper
	Effect	
Physics:		
Multidisciplinary	728	1,247
Condensed matter	546	1,071
Mathematical	183	337
Applied	193	312
Particles and fields	134	229
Nanoscience	86	151
Optics	87	128
Astronomy	54	110
Molecular physics	65	104
Materials science	39	62
Engineering	36	57
Nuclear physics	26	46
Physical chemistry	23	33
Multidisciplinary sciences	21	28
Education	22	27
Other		
Total	1723	3060

Source: Web of Science

We observe from Table 4 that multidisciplinary physics and condensed matter physics have the most citation accounts for both cited items.

The Aharonov-Bohm effect has been the object of a number of philosophical works, besides the works in physics, and we also mention some of them:

Table 5
Some Contributions of the Philosophy of Science for the AB59-Debate

Name	Year
R. Healey	1977
G. Belot	1998
T. Maudlin	1998
S. Leeds	1999
H. Lyre	2001
A.M. Nounou	2003
A. Afriat	2011

Aharonov continued his research on quantum mechanics while Bohm changed his focus from quantum mechanics to the philosophical foundations of physics.

In terms of the number of citations after the 1970’s, Aharonov’s research was more important than Bohm’s, in part because of Bohm’s focus on philosophical issues. Aharonov has continued his research on the Aharonov-Bohm effect and on quantum physics. Aharonov and his co-workers produced new results related to the Aharonov-Bohm effect. This has a significant positive effect on the acceptance of this approach.

For example, Aharonov and Casher’s (1984) work on

the duality of the Aharonov-Bohm effect (i.e., AB59) has more than 500 citations. Aharonov and Anandan (1987) had reformulated and generalized Berry's phase. Their article has more than 1,000 citations. And the research network (Anandan, Casher, Popescu, Tonomura, Nussinov, Vaidman, and Susskind among others) which has been generated by Aharonov, has confirmed and continued the interest in the research on the Aharonov-Bohm effect.

Well-known physicists like the Nobel Prize winner C.N. Yang and Berry have written extensively subjects related to the Aharonov-Bohm effect, and cite AB59 explicitly.

As noted earlier, AB59 has around 3100 citations today. The so-called secondary citations, which show the influence of authors citing AB59, exceeds 65.000 citations at this writing. This suggests to us the remarkable and pervasive influence of Aharonov and Bohm's original 1959 paper.

Finally, Boyer (2000) asked the following question related to the controversy of the Aharonov-Bohm phase shift: "Does the Aharonov-Bohm effect exist?" In his view, classical electromagnetic theory can account for the phase shift.

CONCLUDING REMARKS

Today, it is commonly acknowledged that Einstein and Lorentz developed the so-called "Lorentz transformations" independently.

The priority for the Aharonov-Bohm/Ehrenberg-Siday effect is not so well-determined. One might ask whether the expression, the "Aharonov-Bohm effect," is justified?

Peat (1977) wrote of the Aharonov-Bohm effect:

The work of Aharonov and Bohm was considered by many physicists to be of Nobel quality, and over the years rumors surfaced that they were short-listed for the prize. But no award was ever made-possibly, physicists speculated among themselves, because of the ambiguity over who exactly had discovered the effect.

Although there are well-regarded physicists like Sturrock and Groves who have argued that the effect should be renamed the Ehrenberg-Siday effect, it is apparent that the vast majority of physicists like Berry and Peshkin believe that the use of the term, the Aharonov-Bohm effect, is justified.^{3,4}

If we analyze the difference between a search on the "Aharonov-Bohm effect" in the "Abstracts" of the database *Web of Science* during the years 1959-2010, and the number of direct "Citations" of the Aharonov-Bohm effect (that is, AB59, among others) in the same database during the same period, we find ~ 100 papers more in abstracts than citations. This is an example of "obliteration by incorporation," where the Aharonov-Bohm effect has

become a common phrase. In other words, the "Aharonov-Bohm effect" is so well-known in the community of scientists that it apparently does not need to be included in the references in a particular paper.

This leads brings us to a principal point of our paper that this remarkable case illustrates.

As we have noted earlier, discovering a great idea or concept is necessary for a scientific discovery, but it is apparently not sufficient. Equally important is a key ingredient that can still be missing. By this we do not refer to the advertising and "selling" of the idea. It appears to be crucial that the discoverers themselves realize the novelty, importance, and impact of their discovery. Only with this strong internal conviction will they be able to broadly promulgate it in good faith to the rest of the community. And this should be the case irrespective of what their original and often rather different motivations were.

REFERENCES

- Afriat, A. (2011). *Ex Nihilo*. Retrieved from hal.archives-ouvertes.fr/docs/00/61/12/65/PS/Stokes1.ps
- Aharonov, Y., & Bohm, D. (1959). Significance of Electromagnetic Potentials in the Quantum Theory. *Physical Review*, 115, 485-491.
- Aharonov, Y., & Bohm, D. (1961). Further Considerations on Electromagnetic Potentials in the Quantum Theory. *Physical Review*, 123, 1511-1524.
- Aharonov, Y., & Casher, A. (1984). Topological Quantum Effects for Neutral Particles. *Physical Review Letters*, 53, 319-321.
- Aharonov, Y., & Anandan, J. (1987). Phase-Change During a Cyclic Quantum Evolution. *Physical Review Letters*, 58, 1593-1596.
- Bakke, K., & Furtado, C. (2010). Scalar A-B Effect in the Presence of Topological Defect. *Annalen der Physik*, 522, 447-455.
- Belot, G. (1998). Understaing Electromagnetism. *The British Journal of Philosophy of Science*, 49, 531-555.
- Berry, M.V. (1984). Quantal Phase Factors Accompanying Adiabatic Changes. *Proceedings Royal Society, London A*, 392, 45-57.
- Berry, M. (2010). Aptly Named A-B Effect Has a Classical Analogue, Long History. *Physics Today*, 63(8), 8-8.
- Boyer, T.H. (1973). Classical Electromagnetic Deflections and Lag Effects....Related to A-B Effect. *Physical Review*, 8, 1679-1693.
- Boyer, T.H. (2000). Does the A-B Effect Exists? *Foundations of Physics*, 30, 893-905.
- Brannigan, A., & Wanner, R.A. (1983). Multiple Discoveries in Science. *Canadian Journal of Sociology*, 8, 135-151.
- Chirkov, A.G., & Ageev, A.N. (2001). On the Nature of the A-B Effect. *Technical Physics*, 46, 147-153.
- Cole, S. (2004). Merton's Contribution to the Sociology of

³ Peshkin (1961) gave the name Aharonov-Bohm effect.

⁴ "It would be more revolutionary for this effect to be wrong than for it to be right!" David Bohm, in an USC-Conference (1989).

- Science. *Social Studies of Science*, 34, 829-844.
- Darwin, C. (1859). *On the Origin of Species*. Murray.
- Dirac, P.A.M. (1931). Quantised Singularities in the Electromagnetic Field. *Proceedings Royal Society, London, Series A*, 131, 60.
- Ehrenberg, W., & Siday, R.E. (1949). The Refractive Index in Electron Optics and the Principles of Dynamics. *Proceedings of the Physical Society B*, 62, 8-21.
- Ehrenberg, W., & Siday, R.E. (1951). The Refractive Index in Electron Optics. *Proceedings of Physical Society, London Section B*, 64, 1088-1089.
- Einstein, A. (1905). Zur Elektrodynamik Bewegter Korpen. *Annalen der Physik*, 17, 891-921.
- Einstein, A. (1905). On the Electrodynamics of Moving Bodies. *The Principle of Relativity* (pp. 35). Montreal and London: Dover Publications, Inc..
- Einstein, A. (1918). *Letter of Einstein to Weyl*, 15 April.
- Franz, W. (1939). *Verhandlungen der Deutschen Physikalischen Gessellschaft* (pp. 65).
- Franz, W. (1965). Uber Zwei Unorthodoxe Interferenzversuche. *Zeitschrift fur Physik*, 184, 85-91.
- Fock, V. (1930). Approximation Method for the Solution of the Quantum Mechanical Multibody Problems. *Zeitschrift fur Physik*, 61, 126-148.
- Furry, W. H., & Ramsey, N.F. (1960). Significance of Potentials in Quantum Theory. *Physical Review*, 118, 623-626.
- Glaser, W. (1951). The Refractive Index in Electron Optics and Its Connection with the Routhian Function. *Proceedings Physical Society, London Section B*, 64, 114-118.
- Healey, R. (1997). Nonlocality and the A-B Effect. *Philosophy of Science*, 64, 18-40.
- Hegel, Q. W. F. (1979). *Phenomenology of Spirit* (Translation). Oxford Univ. Press.
- Hiley, B.J., & Bohn, D. (1997). *Biographical Memoirs of Fellows of the Royal. Society*, 43, 107-131.
- Kuhn, T. (1962). *The Structure of Scientific Revolutions*. Chicago Univ. Press.
- Lamb, D., & Easton, S.M. (1984). *Multiple Discovery*. Avebury.
- Leeds, S. (1999). Gauges. Aharonov, Bohm. Yang, Healey. *Philosophy of Science*, 66, 606-627.
- London, F. (1927). Quantenmechanische Deutung der Theorie von Weyl. *Zeitschrift fur Physik*, 42, 375-389.
- London, F. (1948). On the Problem of the Molecular Theory of Superconductivity. *Physical Review*, 74, 562-573.
- Lorentz, H. A. (1895). Elektromagnetische Vorgänge, Beiblätter zu. *Den Annalen der Physik*, 29(4), 168-170
- Lorentz, H. A. (1904). Electromagnetic Phenomena in a System Moving with Any Velocity Less than that of Light. *Proceedings of the Academy of Sciences of Amsterdam*, 6, 809-831
- Lyre, H. (2001). The Principles of Gauging. *Philosophy of Science*, 68(Supplement).
- Maudlin, T. (1998). Discussion. Healey on the A-B Effect. *Philosophy of Science*, 65, 361-368.
- Maxwell, J. C. (1877). *Matter and Motion*, Dover.
- Mead, C. A. (1980). The Molecular A-B Effect in Bound-States. *Chemical Physics*, 49, 23-32.
- Merton, R. K. (1969). Behavior Pattern of Scientists. *American Scientist*, 57, 1-23.
- Merton, R. K. (1961). Singletons and Multiples in Scientific Discoveries. *American Philosophical Society Proceedings*, 105, 470-486.
- Merton, R. K. (1973). *The Sociology of Science. Theoretical and Empirical Investigations*. Univ. Chicago Press.
- Michelson, A., & Morley, E. (1887). On the Relative Motion of the Earth and the Luminiferous Ether. *American Journal of Science*, 34.
- Nounou, A. M. (2003). A Fourth Way to the A-B Effect. Retrieved from doi.10.1007/51070-011-9559-Y
- Olariu, S., & Popescu, I. I. (1985). The Quantum Effects of Electromagnetic Fluxes. *Review of Modern Physics*, 57, 339-436.
- Peat, D. F. (1977). *Infinite Potential, the Life and Times of David Bohm*. Addison-Wesley.
- Penrose, R. (2004). *The Road to Reality*. Vintage Books.
- Peshkin, M. (2010). Aptly Named A-B Effect Has a Classical Analogue, Long History. *Physics Today*, 63(8), 8-8.
- Peshkin, M., & Tonomura, A. (1989). *The Aharonov-Bohm Effect*. Springer.
- Polanyi, M. (1962). The Republic of Science. *Minerva*, 1, 54-73.
- Preskill, J. (1993). Chromatic Aberrations. Yang and Mills Meet Aharonov and Bohm. *Physics Colloquium About the Non-Abelian Aharonov-Bohm Effect* (Cal Tech, Lecture).
- Price, D. De Sola (1961). *Little Science, Big Science*. Columbia Univ. Press.
- Riesz, P. B. (1995). The Life of Roentgen. *American Journal of Roentgenology*, 165, 1533-1537.
- Simonton, D.K. (1986). Multiple Discovery. *Scientometrics*, 9, 269-280.
- Stephan, P. E. (2004). R. K. Merton's Perspective on Priority and Provision of the Public Knowledge. *Scientometrics*, 60, 81-87.
- Stigler, G. J. (1982). *The Economist as a Preacher and Other Essays*. Chicago Univ. Press.
- Sturrock, P. A., & Groves, T. R. (2010). More Variations on A-B. *Physics Today*, 63(4), 8-8.
- Sturrock, P. A. (1951). The Aberrations of Magnetic Electron Lenses Due to Asymmetries. *Philosophical Transactions Royal Society, London Series A*, 243, 387-429.
- Tonomura, A. et al. (1986). Evidence for A-B Effect with Magnetic Field Completely Shield from Electron Wave. *Physical Review Letters*, 56, 792-795.
- Washburn, S., & Webb, R.A. (1992). Quantum Transport in Small Disordered Samples from the Diffusive to the Ballistic Regime. *Reports on Progress in Physics*, 55, 1311-1383.
- Weisskopf, V. F. (1961). *Bolder Lectures in Theoretical Physics*, In W.E. Brittin, B.W. Downs & J. Downs (Eds.). Interscience.
- Weyl, H. (1918). Gravitation Und Elektrizitaet. *Preuss. Akad Wissen Berlin*, 465.
- Weyl, H. (1929). Electron and Gravitation. *Zeitschrift Fur Physik Z Phys*, 56, 330-352.
- Yang, C. N., & Mills, R. (1954). Conservation of Isotopic Spin and Isotopic Invariance. *Physical Review*, 96, 191-195.
- Zuckerman, H. (1977). *Scientific Elite. Nobel Laureates in the United States*. Free Press.