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We propose a versatile approach for generating multipartite W states in predesigned on-chip multiport photonic lattices. It is shown that is possible to produce photon-encoded W states where exactly one photon is coherently "shared" among N optical modes by judiciously adjusting the coupling coefficients involved in one-dimensional arrays of evanescently coupled single-mode waveguides. Two-dimensional waveguide configurations are also investigated as possible avenues to produce W states with equal probability amplitudes and equal relative phases.

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Today, the generation and manipulation of quantum entanglement is among the most important topics within the framework of quantum computing and information processing [1]. Among the various types of entangled states are the so-called W states that play an important role in quantum-information protocols because their entanglement is known to be robust against losses [2]. These particular states have found diverse applications, such as quantum key distribution, quantum teleportation, and in optimal universal quantum cloning machines [3–5] to mention a few.

In order to generate W states, several physical architectures have been envisioned. Among them, one may mention coupled-cavity QED arrangements, spin systems, trapped ions, superconductors, and quantum dots [6–10]. In this regard, quantum optics can provide a suitable platform for the effective realization of such quantum states [11]. Photons are quite insusceptible to decoherence and allow for high experimental repetition rates with high precision on single qubit operations [12]. In fact, photon-encoded W states can be generated entirely linearly by only using passive optical elements, such as beam splitters, phase shifters, and mirrors along with single-photon sources and standard photodetectors [13]. Indeed, most of the experiments suggested in the quantum optics literature involved photons propagating in free space and interacting with bulk optical elements [14–16]. Such optical settings end up being large in size, and as a result, they are intrinsically sensitive to environmental factors. This seriously affects their performance in terms of quantum coherence, thus, making them unsuitable for photonic quantum technologies. Alternatively, in this same optical realm, integrated photonic structures have recently been utilized in several papers in order to generate maximally entangled states as well as arbitrary one-qubit mixed states, etc. [17–21]. In principle, such integrated optical arrangements can be miniaturized and can be effectively interfaced with quantum photon sources and detectors.

In this paper, we explore the possibility of generating photon-encoded W states using judiciously engineered arrays of evanescently coupled single-mode waveguides. The basic idea is to appropriately design multiport waveguide systems in which the incident photons encounter an auspicious environment that forces the initial wave function to evolve (after a distance z) into a well-defined superposition of states, the so-called W states,

$$|W_N(z)\rangle = \frac{1}{\sqrt{N}}(e^{i\phi_1}|1000\cdots 0\rangle + e^{i\phi_2}|0100\cdots 0\rangle + \cdots + e^{i\phi_N}|0000\cdots 1\rangle). \tag{1}$$

Equation (1) implies that a single photon traversing an array of N identical waveguides can be found "shared" among the N optical modes with equal probability and possibly different relative phases at a distance z.

In general, the quantum dynamics of single photons propagating through planar one-dimensional (1D) photonic lattices (see, for example, Fig. 1) is described by a set of Heisenberg equations for the modal creation operators $i\, d\mathbf{A}^\dagger/d\xi = \mathbf{M}\mathbf{A}^\dagger$, where ξ represents the normalized propagation distance, $\xi = \kappa_0 z$ (z being the actual propagation distance, κ_0 is a characteristic coupling strength), $\mathbf{A}^\dagger = [a_1^\dagger(\xi), a_2^\dagger(\xi), a_3^\dagger(\xi), \ldots, a_N^\dagger(\xi)]^T$, and $\mathbf{M}$ represents the coupling matrix, which is symmetric and tridiagonal with elements $(\mathbf{M})_{m,n} = \kappa_{m,n}$ if $|m-n| \leqslant 1$ and $(\mathbf{M})_{m,n} = 0$, otherwise [22–24]. Throughout our paper, subscripts in the coupling coefficients denote the waveguide pair where the coupling process is taking place. Note that, since we are considering identical waveguides, the individual propagation constants are omitted from the analysis because they just introduce a global phase to the final states. Clearly, given that $\mathbf{M}$ is ξ independent (or time independent), the input-output states are related through the evolution matrix: $\mathbf{A}^\dagger(\xi) = \exp(-i\xi\mathbf{M})\mathbf{A}^\dagger(0)$.

In order to elucidate our proposal, we first consider the case of a single photon propagating through a waveguide array having three identical waveguides with coupling coefficients $\kappa_{1,2} = \kappa_{2,3} = \kappa_0$. As a result, the normalized evolution matrix is given by

$$U(\xi) = \begin{pmatrix} \cos^2\left(\frac{\xi}{\sqrt{2}}\right) & -\frac{i}{\sqrt{2}}\sin(\sqrt{2}\xi) & -\sin^2\left(\frac{\xi}{\sqrt{2}}\right) \\ -\frac{i}{\sqrt{2}}\sin(\sqrt{2}\xi) & \cos(\sqrt{2}\xi) & -\frac{i}{\sqrt{2}}\sin(\sqrt{2}\xi) \\ -\sin^2\left(\frac{\xi}{\sqrt{2}}\right) & -\frac{i}{\sqrt{2}}\sin(\sqrt{2}\xi) & \cos^2\left(\frac{\xi}{\sqrt{2}}\right) \end{pmatrix}. \tag{2}$$

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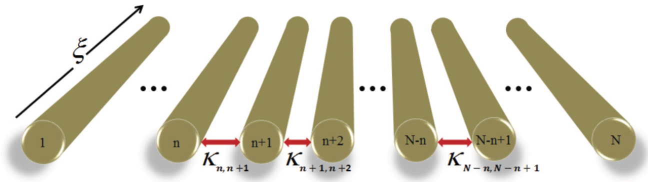

FIG. 1. (Color online) Planar 1D waveguide array of N identical waveguides, ξ is the normalized propagation distance, and $\kappa_{m,n}$ represents the coupling coefficients between elements m and n.

Hence, once a photon is launched into the central waveguide $|\Psi(0)\rangle = a_2^\dagger|0\rangle$, it evolves to the superposition $|W_3\rangle = \frac{i}{\sqrt{2}}\sin(\sqrt{2}\xi)(|100\rangle + |001\rangle) + \cos(\sqrt{2}\xi)|010\rangle$, which is a tripartite W state with different probability amplitudes and different relative phases. It is obvious that, at a distance given by $\xi = \tan^{-1}(\sqrt{2})/\sqrt{2}$ [or at the actual propagation distance $z = \tan^{-1}(\sqrt{2})/\kappa_0\sqrt{2}$], all the probabilities are the same and become equal to $1/3$. Therefore, at this particular distance, we have produced a three-partite W state with equal probability amplitudes but different relative phases. Since the normalized propagation distance depends on κ_0, this example clearly shows the feasibility of using integrated optical multiport waveguide systems to generate multipartite photon-encoded W states by simply choosing the propagation distance. In what follows, we examine the case of producing five-partite W states by propagating a single photon through waveguide arrays of five elements with preengineered coupling coefficients $\kappa_{1,2} = \kappa_{4,5}$ and $\kappa_{2,3} = \kappa_{3,4} = \kappa_0$. In this case, the normalized equations of motion become

$$i\frac{d}{d\xi}\begin{pmatrix} a_1^\dagger \\ a_2^\dagger \\ a_3^\dagger \\ a_4^\dagger \\ a_5^\dagger \end{pmatrix} = \begin{pmatrix} 0 & \kappa_1 & 0 & 0 & 0 \\ \kappa_1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & \kappa_1 \\ 0 & 0 & 0 & \kappa_1 & 0 \end{pmatrix}\begin{pmatrix} a_1^\dagger \\ a_2^\dagger \\ a_3^\dagger \\ a_4^\dagger \\ a_5^\dagger \end{pmatrix}, \tag{3}$$

where $\kappa_1 = \kappa_{1,2}/\kappa_0 = \kappa_{4,5}/\kappa_0$. Again, if a single photon is launched into the central waveguide element $|\Psi(0)\rangle = a_3^\dagger|0\rangle$, it evolves to the superposition $|W_5\rangle = A|10\,000\rangle + B|01\,000\rangle + C|00\,100\rangle + B|00\,010\rangle + A|00\,001\rangle$, where $A = k_1[-1 + \cos(\sqrt{2+k_1^2}\xi)](2+k_1^2)^{-1}$, $B = -i\,\sin(\sqrt{2+k_1^2}\xi)(\sqrt{2+k_1^2})^{-1}$, and $C = 1 + 2[-1+\cos(\sqrt{2+k_1^2}\xi)](2+k_1^2)^{-1}$. In order to equalize all the probabilities, one has to select κ_1 such that $|A|^2 = |B|^2 = |C|^2 = 1/5$. This is easily achieved by searching for a simultaneous numerical root for these probabilities. Along these lines, one finds that, for $\kappa_1 \cong 1.618$, all the probabilities attain the same value of $1/5$ at a normalized propagation distance of $\xi \cong 0.861$. This process is illustrated in Fig. 2. Again, this example demonstrates that, by appropriately adjusting the relative coupling coefficient κ_1, a five-partite W state can be effectively generated at a specific propagation distance ξ. Note that, in these previous cases, the input states were single photons coupled into the central channel of arrays having an odd number of waveguide elements. By doing so, we have guaranteed that the photons will symmetrically evolve toward both sides of the array with exactly the same probability. In contrast, for even waveguide arrays, where a central waveguide cannot exist, the initial state has to be balanced so as to allow the photon to evolve toward both sides of the array with the same probability as in the previous odd arrangements. One way to overcome this problem in even arrays ($N = 2m$) is to launch the path-entangled state $|\psi(0)\rangle = (a_{N/2}^\dagger + a_{(N/2)+1}^\dagger)|0\rangle$ where a single photon is coupled into either one of the two neighboring waveguides at the center of the array with the same probability. This state may be generated by propagating a single photon over one half the coupling length of a standard directional coupler [23], which, in turn, can be cascaded to the central pair of waveguides of the main waveguide array. As an example, consider the propagation of the path-entangled state $|\psi(0)\rangle = (a_2^\dagger + a_3^\dagger)|0\rangle$ through an array of four waveguides with normalized coupling coefficients $\kappa_1 = \kappa_{1,2}/\kappa_0 = \kappa_{3,4}/\kappa_0$ and $\kappa_0 = \kappa_{2,3}$. In this scenario, one can show that, by selecting $\kappa_1 = \kappa_0 = 1$, a four-partite W state is generated at the particular distance $\xi = 0.815$ (or $z = c/\kappa_0$).

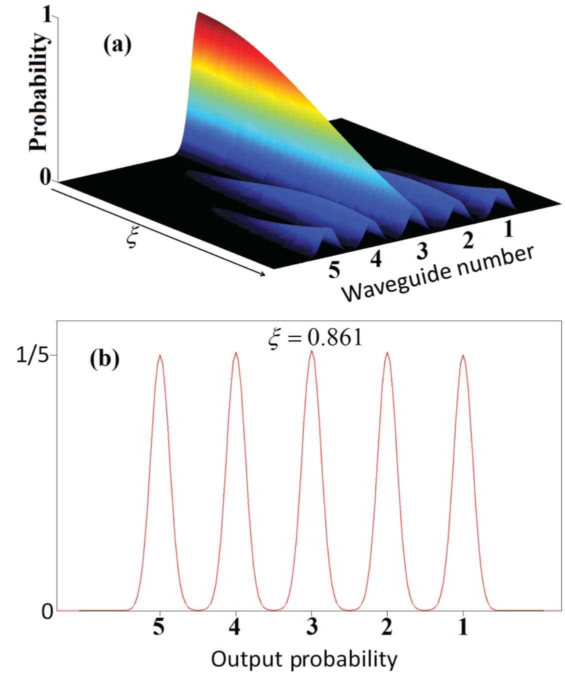

FIG. 2. (Color online) (a) Probability evolution corresponding to a single photon propagating through a planar 1D waveguide array of five elements. The coupling parameter used is $\kappa_1 = 1.618$. (b) Output probability at a normalized propagation distance of $\xi = 0.861$.

Following this process, one can, in principle, generate arbitrary higher N-partite W states using waveguide lattices. In Table I, we show the calculated normalized parameters, coupling coefficients, and propagation distances for the generation of N-partite W states up to $N = 19$ for N being odd, whereas, Table II shows these results up to $N = 10$ for N being even.

As indicated in the previous examples, 1D planar waveguide arrays can, indeed, provide a versatile environment for generating on-chip N-partite W states with equal probabilities and different relative phases. In what follows, we focus our attention on a somewhat different configuration capable of producing large photon-encoded W states that exhibit equal probability amplitudes and equal relative phases. This method

TABLE I. Calculated normalized parameters for generating N-partite W states using odd waveguide arrays. κ_n represents the coupling coefficients corresponding to the nth pair of waveguides starting from the central waveguide toward both sides of the array. In all the cases, the coupling strength between the central guide and its nearest neighbors is considered to be $\kappa_0 = 1$.

N	7	9	11	13	15	17	19
ξ	0.98	1.068	1.137	1.194	1.242	1.283	1.32
κ_1	1.933	2.099	2.204	2.278	2.332	2.373	2.407
κ_2	2.029	2.579	2.906	3.129	3.289	3.408	3.502
κ_3		2.346	3.083	3.555	3.894	4.142	4.326
κ_4			2.607	3.5	4.105	4.555	4.884
κ_5				2.829	3.856	4.584	5.138
κ_6					3.023	4.168	5.012
κ_7						3.195	4.448
κ_8							3.35

is based on two-dimensional (2D) ring waveguide configurations where N waveguide channels are located around a common guide, which serves as the exciting center. In this latter system, all $N+1$ waveguide elements are identical. Figure 3 depicts a transverse view of such an array. In this system, the external waveguides are coupled to each other (their nearest neighbors) through a coupling strength C while, in turn, are coupled to the central guide with the same coupling strength κ. Due to the symmetry of this arrangement, one can recognize the fact that $a_1^\dagger = a_2^\dagger = \cdots = a_N^\dagger = b^\dagger$ such that the Heisenberg equations of motion for a single photon traversing this particular system can be cast in the reduced form

$$i\frac{da_0^\dagger}{d\xi} = N\kappa b^\dagger, \quad i\frac{db^\dagger}{d\xi} = 2Cb^\dagger + \kappa a_0^\dagger. \tag{4}$$

In this case (exciting the center), the input-output states are associated through the following expression,

$$\begin{aligned} a_0^\dagger(\xi) &= \exp(iC\xi)\left[\cos(\sqrt{C^2+N\kappa^2}\xi)\right. \\ &\quad \left. + \frac{iC}{\sqrt{C^2+N\kappa^2}}\sin(\sqrt{C^2+N\kappa^2}\xi)\right]a_0^\dagger(0) \\ &\quad - \exp(iC\xi)\frac{i\kappa N}{\sqrt{C^2+N\kappa^2}}\sin(\sqrt{C^2+N\kappa^2}\xi)b^\dagger(0), \end{aligned} \tag{5}$$

TABLE II. Calculated normalized parameters for generating N-partite W states via even waveguide arrays. κ_n represents the coupling coefficients corresponding to the nth pair of waveguides starting from the central pair toward both sides of the array. In all the cases, $\kappa_0 = 1$.

N	4	6	8	10
ξ	0.815	1.119	1.333	1.502
κ_1	1	1	1	1
κ_2		1.264	1.432	1.485
κ_3			1.483	1.809
κ_4				1.657

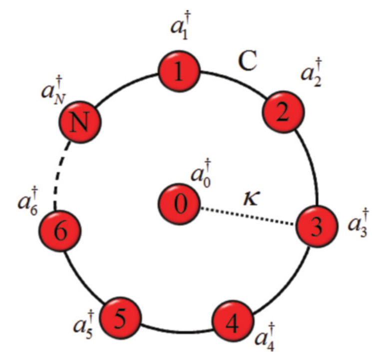

FIG. 3. (Color online) Transverse view of a ring configuration of N identical single-mode waveguides, C is the coupling strength between the external guides, κ is the coupling between the external guides and the central element, and $a_n^\dagger$'s are the creation operators at the nth waveguide.

where $b^\dagger(0)$ represents the creation operators for all the N external terms. For the particular case when $C = 0$, i.e., if the external guides do not interact, Eq. (5) becomes

$$a_0^\dagger(\xi) = \cos(\sqrt{N}\kappa\xi)a_0^\dagger(0) - i\sqrt{N}\sin(\sqrt{N}\kappa\xi)b^\dagger(0). \tag{6}$$

Therefore, under such conditions, if we launch a photon into the central waveguide $|\psi(0)\rangle = a_0^\dagger|0\rangle$, it evolves into the W state,

$$\begin{aligned} |W_N\rangle &= \cos(\sqrt{N}\kappa\xi)|1_0 0_1\cdots 0_N\rangle + \frac{i}{\sqrt{N}}\sin(\sqrt{N}\kappa\xi) \\ &\quad \times(|0_0 1_1\cdots 0_N\rangle + \cdots + |0_0 0_1\cdots 1_N\rangle). \end{aligned} \tag{7}$$

As a result, at a distance of $\xi = s\pi/2k\sqrt{N}$ (with s being an odd integer), the first term on the right-hand side of Eq. (7) vanishes and yields an in-phase state,

$$|W_N\rangle = \frac{i}{\sqrt{N}}(|0_0 1_1\cdots 0_N\rangle + \cdots + |0_0 0_1\cdots 1_N\rangle), \tag{8}$$

which is a N-partite W state where a single photon is found shared among the N external waveguides with exactly the same probability and equal relative phases. In principle, W states also involving the central waveguide can be obtained in this case by choosing the propagation length in Eq. (7) to be $\xi = \tan^{-1}(\sqrt{N})/\kappa\sqrt{N}$ and by simply shifting the phase in the central core by $\pi/2$. On the other hand, if $C = \kappa$, i.e., all the waveguides are located at exactly the same distance from one to each other (polygonal arrangement). This leads to the following version of this solution:

$$\begin{aligned} a_0^\dagger(\xi) &= \exp(iC\xi)\left[\cos(C\sqrt{N+1}\xi)\right. \\ &\quad \left. + \frac{i}{\sqrt{N+1}}\sin(C\sqrt{N+1}\xi)\right]a_0^\dagger(0) \\ &\quad - \frac{iN\,\exp(iC\xi)}{\sqrt{N+1}}\sin(C\sqrt{N+1}\xi)b^\dagger(0). \end{aligned} \tag{9}$$

From Eq. (9), one can clearly deduce that all the elements equally share the photon at a distance of $\xi = s\pi/2C\sqrt{N+1}$ (s being an odd integer) where the term $\cos(C\sqrt{N+1}\xi) = 0$. This implies that, at these specific distances, all the probabilities attain the same value $1/(N+1)$, leading to

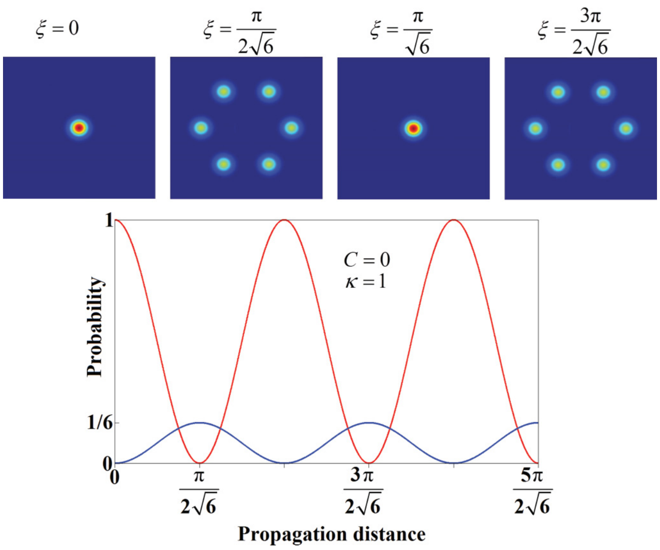

FIG. 4. (Color online) Top: Transverse view of the probability distributions corresponding to a single photon propagating through a ring configuration of waveguides with seven elements. Bottom: Theoretical evolution of the probability of detecting the photon at the blue line (lower): external waveguides and at the red line (upper) central waveguide.

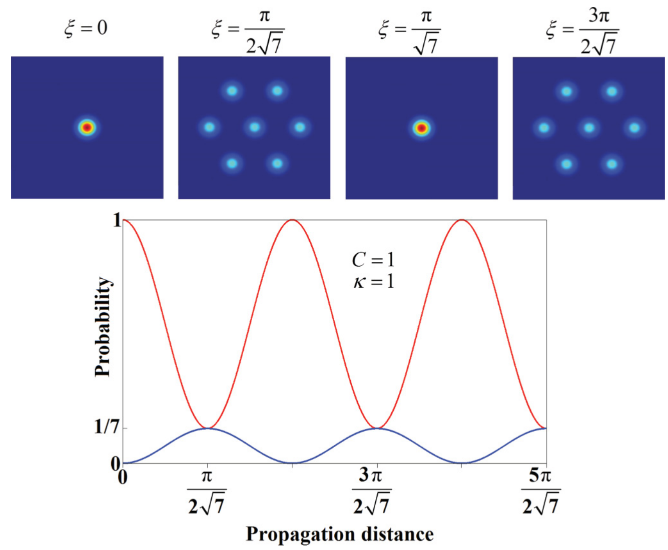

FIG. 5. (Color online) Top: Transverse view of the probability distributions corresponding to a single photon propagating through a ring configuration of waveguides with seven elements. Bottom: Theoretical evolution of the probability of detecting the photon at the blue line (lower): external waveguides and at the red line (upper): central waveguide.

the W state,

$$|W_N\rangle = \exp(-is\pi/2\sqrt{N+1})\left(\frac{i}{\sqrt{N+1}}|1_0 0_1\cdots 0_N\rangle - \frac{i}{\sqrt{N+1}}(|0_0 1_1\cdots 0_N\rangle + \cdots + |0_0 0_1\cdots 1_N\rangle)\right). \tag{10}$$

To elucidate these effects, in Figs. 4 and 5, we show the theoretical evolution of the probability exhibited by a single photon propagating through an array of seven waveguides when launched into the central element. Figure 4 depicts the probability distributions when the external waveguides are uncoupled ($C=0$) but are coupled to the central one with $\kappa=1$. As expected, the probabilities along the external guides, see the blue (lower) line, become the same at regular intervals ($\xi=\pi/2\sqrt{6},3\pi/2\sqrt{6},5\pi/2\sqrt{6},\ldots$), whereas, along the central guide, the probability vanishes. Figure 5 depicts similar results for the case when $C=\kappa=1$. In this latter scenario, all the probability amplitudes and phases become the same at distances of ($\xi=\pi/2\sqrt{7},3\pi/2\sqrt{7},5\pi/2\sqrt{7},\ldots$). In both cases, red (upper) curves describe the probability evolution along the central waveguide, whereas, the blue (lower) curves illustrate the evolution along the external cores.

The waveguide arrays suggested in this paper can be fabricated in bulk fused silica by employing direct femtosecond-laser inscription [25–27]. Since propagation losses in such systems can be as low as 0.05 dB/cm [28], implying an arrival probability at the end of the arrays of approximately 90%, such waveguide configurations are actually suitable for single-photon experiments.

In conclusion, we have shown that 1D and 2D multiport waveguide arrays can be used as a versatile platform to mold the quantum evolution of a single photon into a multipartite photon-encoded W state. Specifically, we have found that single photons, propagating through linear 1D waveguide lattices endowed with properly engineered coupling coefficients, effectively produce a multipartite W state with identical probabilities and different relative phases. On the other hand, by using 2D waveguide systems, it is possible to equalize both the probability amplitudes as well as the relative phases at all sites. The method proposed here may provide a promising avenue in producing on-chip large multipartite W states using miniaturized integrated optical configurations. Such systems could be of relevance in quantum computing and information-processing schemes where scalability and quantum decoherence are issues of importance.

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