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The Membership Relation in Fuzzy Operators

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Abstract: Firstly the membership relation in fuzzy operators located outside Zadeh operators be discussed and given. Secondly the membership relation in fuzzy operators located within Zadeh operators be discussed and given.

Key words: Operator; Fuzzy operator; Membership relation

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1. INTRODUCTION

We since Zadeh established fuzzy set theorem in 1965, he introduced a pair of fuzzy operators. They are named Zadeh operators [1]. There is a lot of discussion about the fuzzy operators [2–10], but there is little discussion membership relation in there operators. This paper discusses membership relation in Zadeh operators and other generalized operators in common use.

2. THE MEMBERSHIP RELATION IN FUZZY OPERATORS LOCATED OUTSIDE ZADEH OPERATORS

In order to discuss the membership relation in fuzzy operators, it is necessary to introduce the concept of generalized operators [4]. Generalized operators are widely-used operators in fuzzy sets. The definition of Zadeh operators and the definitions of generalized operators in common use as well as their membership relation are as follows.

Definition 2.1 Zadeh operators $(\wedge, \vee)$

To any fuzzy set $A, B, C \in P(U)$, fuzzy set $C = A \cap B$ be called intersection of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$C(u) = (A \cap B)(u) = A(u) \wedge B(u);$$

Fuzzy set $C = A \cap B$ be called union of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$C(u) = (A \cup B)(u) = A(u) \vee B(u).$$

Definition 2.2 Probability operators $(\hat{\bullet}, \hat{+})$

To any fuzzy set $A, B, C \in P(U)$, fuzzy set $C = A\hat{\bullet}B$ be called probability product of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$C(u) = (A\hat{\bullet}B)(u) = A(u)B(u);$$

Fuzzy set $C = A\hat{+}B$ be called probability sum of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$C(u) = (A\hat{+}B)(u) = A(u) + B(u) - A(u)B(u).$$

Definition 2.3 Boundary operators $(\otimes, \oplus)$

To any fuzzy set $A, B, C \in P(U)$, fuzzy set $C = A \otimes B$ be called boundary product of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$C(u) = (A \otimes B)(u) = \max[0,\ A(u) + B(u) - 1];$$

Fuzzy set $C = A \oplus B$ be called boundary sum of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$C(u) = (A \oplus B)(u) = \min[1,\ A(u) + B(u)].$$

Definition 2.4 Infinite operators $(\hat{\infty}, \breve{\infty})$

To any fuzzy set $A, B, C \in P(U)$, fuzzy set $C = A\hat{\infty}B$ be called infinite product of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A}\hat{\infty}\tilde{B})(u) = \begin{cases} \tilde{A}(u), & \tilde{B}(u) = 1 \\ \tilde{B}(u), & \tilde{A}(u) = 1 \\ 0, & \text{other} \end{cases}$$

Fuzzy set $C = A \breve{\infty} B$ be called infinite sum of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A} \breve{\infty} \tilde{B})(u) = \begin{cases} \tilde{A}(u), & \tilde{B}(u) = 0 \\ \tilde{B}(u), & \tilde{A}(u) = 0 \\ 1, & \text{other} \end{cases}$$

The paper [7] has proved that Zadeh operations, probability operators and boundary operators as well as infinite operators have membership relation as follows:

$$(\tilde{A} \hat{\infty} \tilde{B})(u) \le (\tilde{A} \otimes \tilde{B})(u) \le (\tilde{A} \hat{\bullet} \tilde{B})(u) \le (\tilde{A} \cap \tilde{B})(u)$$

$$\le (\tilde{A} \cup \tilde{B})(u) \le (\tilde{A} \hat{+} \tilde{B})(u) \le (\tilde{A} \oplus \tilde{B})(u) \le (\tilde{A} \breve{\infty} \tilde{B})(u)$$

Definition 2.5 Schweizer-Sklard operators ($\overset{\bullet}{ss}, \overset{+}{ss}$)[11]

To any fuzzy set $A, B, C \in P(U)$, $p \to (-\infty, +\infty)$, fuzzy set $C = A \overset{\bullet}{ss} B$ be called Schweizer-Sklard product of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$\tilde{A}(u) \overset{\bullet}{ss} \tilde{B}(u) = \begin{cases} 0, & \text{if } p > 0,\ \tilde{A}(u) = 0 \text{ or } \tilde{B}(u) = 0 \\ \left[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1\right]^{-1/p}, & \text{if } p > 0,\ \tilde{A}(u) \bullet \tilde{B}(u) \ne 0 \\ \tilde{A}(u) \bullet \tilde{B}(u), & \text{if } p = 0 \\ \left[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1\right]^{-1/p}, & \text{if } p < 0,\ \tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} > 1 \\ 0, & \text{if } p < 0,\ \tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} \le 1 \end{cases}$$

Fuzzy set $C = A \overset{+}{ss} B$ be called Schweizer-Sklard sum of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$\tilde{A}(u) \overset{+}{ss} \tilde{B}(u) = \begin{cases} 1, & \text{if } p > 0,\ \tilde{A}(u) = 1 \text{ or } \tilde{B}(u) = 1 \\ 1 - \left\{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1\right\}^{-1/p}, & \text{if } p > 0,\ \tilde{A}(u) \bullet \tilde{B}(u) \ne 1 \\ \tilde{A}(u) + \tilde{B}(u) - \tilde{A}(u) \bullet \tilde{B}(u), & \text{if } p = 0 \\ 1 - \left\{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1\right\}^{-1/p}, & \text{if } p < 0,\ [1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} > 1 \\ 0, & \text{if } p < 0,\ [1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} \le 1 \end{cases}$$

The paper [11] has proved that Schweizer-Sklard operators are monotone functions for variables $A(u)$ or $B(u)$, the paper [11] has proved that Schweizer-Sklard operators are monotone functions for the parameter p.

The paper [11] has proved that Schweizer-Sklard operations, and Zadeh operations as well as probability operators have membership relation as follows:

$$\tilde{A}(u)\hat{\bullet}\tilde{B}(u) \subseteq \tilde{A}(u) \overset{\bullet}{ss} \tilde{B}(u) \subseteq \tilde{A}(u) \wedge \tilde{B}(u)$$

$$\subseteq \tilde{A}(u) \vee \tilde{B}(u) \subseteq \tilde{A}(u) \overset{+}{ss} \tilde{B}(u) \subseteq \tilde{A}(u)\hat{+}\tilde{B}(u)$$

Definition 2.6 Dobois-Prade operators $(\overset{\bullet}{d}, \overset{+}{d})$ [13]

To any fuzzy set $A, B, C \in P(U)$, $\lambda \in [0,1]$, fuzzy set $C = A \overset{\bullet}{d} B$ be called Dobois-Prade product of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$(\tilde{A} \overset{\bullet}{d} \tilde{B})(u) = \begin{cases} 0, & \text{if } \lambda = 0 \text{ and } \tilde{A}(u) = \tilde{B}(u) = 0 \\ \dfrac{\tilde{A}(u)\tilde{B}(u)}{\max\{\lambda,\ \tilde{A}(u),\ \tilde{B}(u)\}}, & \text{others} \end{cases}$$

Fuzzy set $C = A \overset{+}{d} B$ be called Dobois-Prade sum of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$(\tilde{A} \overset{+}{d} \tilde{B})(u) = \begin{cases} 1, & \text{if } \lambda = 0 \text{ and } \tilde{A}(u) = \tilde{B}(u) = 1 \\ \dfrac{\tilde{A}(u) + \tilde{B}(u) - \tilde{A}(u)\tilde{B}(u) - \min\{1-\lambda, \tilde{A}(u), \tilde{B}(u)\}}{\max\{\lambda,\ 1-\tilde{A}(u),\ 1-\tilde{B}(u)\}}, & \text{others} \end{cases}$$

The paper [13] has proved that Dobois-Prade operators are monotone functions for the parameter λ and that Dobois-Prade operations, and Zadeh operations as well as probability operators have membership relation as follows:

$$(\tilde{A} \overset{\wedge}{\bullet} \tilde{B})(u) \leq (\tilde{A} \overset{\bullet}{d} \tilde{B})(u) \leq (\tilde{A} \cap \tilde{B})(u) \leq (\tilde{A} \cup \tilde{B})(u) \leq (\tilde{A} \overset{+}{d} \tilde{B})(u)$$

Definition 2.7 Hamacher operators $(\overset{\bullet}{r}, \overset{+}{r})$ [9]

To any fuzzy set $A, B, C \in P(U)$, $r \in [0, +\infty)$, fuzzy set $C = A \overset{\bullet}{r} B$ be called Hamacher product of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$(\tilde{A} \overset{\bullet}{r} \tilde{B})(u) = \begin{cases} 0, & \text{if } r = 0 \text{ and } \tilde{A}(u) = \tilde{B}(u) = 0 \\ \dfrac{\tilde{A}(u)\tilde{B}(u)}{r + [1-r][\tilde{A}(u) + \tilde{B}(u) - \tilde{A}(u)\tilde{B}(u)]}, & \text{others} \end{cases}$$

Fuzzy set $C = A \overset{+}{r} B$ be called Hamacher sum of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$(\tilde{A} \overset{+}{r} \tilde{B})(u) = \begin{cases} 1, & \text{if } r = 0, \text{ and } \tilde{A}(u) = \tilde{B}(u) = 1 \\ \dfrac{\tilde{A}(u) + \tilde{B}(u) + [r-2]\tilde{A}(u)\tilde{B}(u)}{1 - \tilde{A}(u)\tilde{B}(u) + r\tilde{A}(u)\tilde{B}(u)}, & \text{others} \end{cases}$$

The paper [9] has proved that Hamacher operations are monotone functions for the parameter r and that Hamacher operations, and Zadeh operations as well as boundary operators have membership relation as follows:

$$(\tilde{A} \otimes \tilde{B})(u) \le (\tilde{A} \overset{\bullet}{r} \tilde{B})(u) \le (\tilde{A} \cap \tilde{B})(u) \le (\tilde{A} \cup \tilde{B})(u) \le (\tilde{A} \overset{+}{r} \tilde{B})(u) \le (\tilde{A} \oplus \tilde{B})(u)$$

Definition 2.8 Yager operators $(\overset{\bullet}{\Upsilon}, \overset{+}{\Upsilon})$ [10]

To any fuzzy set $A, B, C \in P(U)$, $r \in (0, +\infty)$, fuzzy set $C = A \overset{\bullet}{\Upsilon} B$ be called Yager product of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A} \overset{\bullet}{\Upsilon} \tilde{B})(u) = 1 - 1 \wedge [(1 - \tilde{A}(u))^{\frac{1}{r}} + (1 - \tilde{B}(u))^{\frac{1}{r}}]^r$$

Fuzzy set $C = A \overset{+}{\Upsilon} B$ be called Yager sum of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A} \overset{+}{\Upsilon} \tilde{B})(u) = 1 \wedge [\tilde{A}(u)^{\frac{1}{r}} + \tilde{B}(u)^{\frac{1}{r}}]^r$$

The paper [10] has proved that Yager operations are monotone functions for the parameter r and that Yager operations, and Zadeh operations as well as boundary operators have membership relation as follows:

$$(\tilde{A} \otimes \tilde{B})(u) \le (\tilde{A} \overset{\bullet}{\Upsilon} \tilde{B})(u) \le (\tilde{A} \cap \tilde{B})(u) \le (\tilde{A} \cup \tilde{B})(u) \le (\tilde{A} \overset{+}{\Upsilon} \tilde{B})(u) \le (\tilde{A} \oplus \tilde{B})(u)$$

3. THE MEMBERSHIP RELATION IN FUZZY OPERATORS LOCATED WITHIN ZADEH OPERATORS

The paper [12] introduce new class of fuzzy operators in inner of Zadeh operator. The definitions of the new class of continuous fuzzy operators located within Zadeh operators and their membership membership relation are as follows:

Definition 3.1 Zero operators $(\hat{0}, \breve{0})$

To any fuzzy set $A, B, C \in P(U)$, fuzzy set $C = A \hat{0} B$ be called Zero product of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A} \hat{0} \tilde{B})(u) = \frac{\tilde{A}(u) + \tilde{B}(u)}{2}$$

Fuzzy set $C = A \breve{0} B$ be called Zero sum of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A} \breve{0} \tilde{B})(u) = \frac{\tilde{A}(u) + \tilde{B}(u)}{2}$$

The paper [12] has proved that Zero operations and Zadeh operations have membership relation as follows:

$$(\tilde{A} \cap \tilde{B})(u) < (\tilde{A} \hat{0} \tilde{B})(u) = (\tilde{A} \breve{0} \tilde{B})(u) < (\tilde{A} \cup \tilde{B})(u)$$

Definition 3.2 Q operators $(\hat{\mathrm{Q}}, \breve{\mathrm{Q}})$

To any fuzzy set $A, B, C \in P(U)$, fuzzy set $C = A\hat{\mathrm{Q}}B$ be called Q product of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A}\hat{\mathrm{Q}}\tilde{B})(u) = \frac{2q+1}{4q+1}[\tilde{A}(u) \wedge \tilde{B}(u)] + \frac{2q}{4q+1}[\tilde{A}(u) \vee \tilde{B}(u)]$$

Fuzzy set $C = A\breve{\mathrm{Q}}B$ be called Q sum of fuzzy set A and fuzzy set B if $\forall u \in U$, there is

$$\tilde{C}(u) = (\tilde{A}\breve{\mathrm{Q}}\tilde{B})(u) = \frac{2q}{4q+1}[\tilde{A}(u) \wedge \tilde{B}(u)] + \frac{2q+1}{4q+1}[\tilde{A}(u) \vee \tilde{B}(u)]$$

The Q operators are monotone functions for the parameter q [12]. The paper [12] has proved that Q operations, Zadeh operations as well as Zero operators have membership relation as follows:

$$(\tilde{A} \cap \tilde{B})(u) \le (\tilde{A}\hat{Q}\tilde{B})(u) \le (\tilde{A}\hat{O}\tilde{B})(u) = (\tilde{A}\breve{O}\tilde{B})(u) \le (\tilde{A}\breve{Q}\tilde{B})(u) \le (\tilde{A} \cup \tilde{B})(u)$$

4. CONCLUSIONS

First we have discussed and given the membership relation in fuzzy operators located outside Zadeh operators. Second we have discussed and given the membership relation in fuzzy operators located within Zadeh operators.

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