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Rough Fuzzy Distance of the Rough Fuzzy Number

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Abstract
Rough sets theory and fuzzy sets theory research the unperfect problem in information systems. the combination of them formed Rough fuzzy sets and Rough Fuzzy number in this paper, defines the Rough fuzzy distance of the Rough fuzzy number. Then it discusses the nature of Rough fuzzy distance.

Key words
Rough fuzzy number; Distance; Rough fuzzy Distance

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1. FUNDAMENTAL CONCEPT

In this paper, R is the realnumber set, F(R)is all of the fuzzy subsets of the R.

Definition 1.1[2] Assume $\tilde{a} \in F(R)$, we call $\tilde{a}$ a fuzzy number. let

(1) $\tilde{a}$ is regular£that is,there exists a $x_0 \in R$, for which $\tilde{a}(x_0) = 1$

(2) $\forall \lambda \in (0, 1)$, $a_\lambda = \{x|\tilde{a}(x) \geq \lambda\}$ is a bounded and closed interval, noted for $[a^-(\lambda), a^+(\lambda)]$.

Definition 1.2 If $\tilde{b}$is a fuzzy numberdefined by the membership function $b(x)$. Assume $\underline{b}$, $\overline{b}$: R/S($\equiv X_1, \cdots X_n$) $\rightarrow [0, 1]$, Let $\underline{b}(X_i) = \inf_{x \in X_i} b(x)$, $\overline{b}(X_i) = \sup_{x \in X_i} b(x), (1, 2 \ldots n)$, we call $(\underline{b}, \overline{b})$ a Rough fuzzy number and note it for RF number.

Definition 1.3 fuzzyhunmer $\underline{A}, \overline{A} : R \rightarrow [0, 1]$ Suppose $\underline{A}(x) = \underline{b}(X_i)$, $\overline{A}(x) = \overline{b}(X_i)$. $x \in X_i (i = 1, 2 \ldots n)$, we call A=$(\underline{A}, \overline{A})$ a Rough fuzzy number

Definition 1.4 $\underline{A}_a = \{x|\underline{A}(x) \geq a\}$, $\underline{A}_a = \{x|\underline{A}(x) \geq a\}$, $A_a = (\underline{A}_a, \underline{A}_a)$

The properties and calculationrules of the rough fuzzy number refer to the paper [4~6]. Suppose A∈RFN, $\underline{A}_a$ and $\overline{A}_a$ is respectively the confidence interval of $\underline{A}, \overline{A}$ with the speculation degrees of $a(a \in (0, 1))$. We suppose $\underline{A}_a = [a_1^{(a)}, a_2^{(a)}]$, $\underline{A}_a = [a_3^{(a)}, a_4^{(a)}]$. According to thedecomposition theorem [6] of the rough fuzzy number, we have $\underline{A} = \cup_{a \in (0,1]} a.[a_1^{(a)}, a_2^{(a)}]$, $\overline{A} = \cup_{a \in (0,1]} a.[a_3^{(a)}, a_4^{(a)}]$.

We difine the partial order $\leq$ on the RFN as followe: $\forall A, B \in RFN$, $\forall a \in (0, 1] \underline{A}_a = [a_1^{(a)}, a_2^{(a)}]$, $\overline{A}_a = [a_3^{(a)}, a_4^{(a)}]$, $\underline{B}_a = [b_1^{(a)}, b_2^{(a)}]$, $\overline{B}_a = [b_3^{(a)}, b_4^{(a)}]$, if and only if $a_i^{(a)} \leq b_i^{(a)} (i = 1, 2, 3, 4)$, $A < B$ means that $A \leq B$, and there exists a $a_0 \in (0, 1]$, which makes $a_i^{(a)} < b_i^{(a)} (i = 1, 2, 3, 4)$

Definition 1.5 Suppose $\tilde{a}$ is a fuzzy number, we call $\tilde{a}$ a convex fuzzy. let $\forall x, y \in (0, 1]$,

$$\tilde{a}(\lambda x + (1 - \lambda)y) \geq \tilde{a}(x) \wedge \tilde{a}(y)$$

Theorem 1.1 Suppose $\tilde{a}$ is a fuzzynumber, the necessary and sufficient condition of $\tilde{a}$ is a fuzzy convexity is that $\forall\lambda \in (0,1]$, $a_\lambda = \{x|\,\tilde{a}(x) \geq \lambda\}$ is a convex set.
Theorem 1.2 Suppose $\tilde{a}$ is a fuzzy number, then we have the $\tilde{a}$ is a convex fuzzy.

2. ROUGH FUZZYDISTANCE

Definition 2.1 Suppose A, B∈RFN, $RFN_+ = \{A|A \geq 0, A \in RFN\}$, the maping $\tilde{\rho}$: RFNX RFN→ RFN_+ is called the Rough fuzzy distance of the rough fuzzy number.if $\tilde{\rho}$ satisfies the conditions as follow:

(1) $\tilde{\rho}$(A,B)≥ 0, $\tilde{\rho}$(A,B)=0 if and only if A=B;

(2) $\tilde{\rho}$(A,B)=$\tilde{\rho}$(B,A);

(3) $\forall C \in RFN$, we have: $\tilde{\rho}$(A,B)≤ $\tilde{\rho}$(A,C)+$\tilde{\rho}$(C,B).

Theorem 2.1 Assume $\forall A, B \in RFN$, $\tilde{\rho}(A,B) = (\underline{\rho}, \tilde{\rho})$ where:

$$\underline{\rho} = \bigcup_{\lambda\in(0,1]} \lambda[0, \frac{1}{2}\int_\lambda^1 (|a_1(\alpha) - b_1(\alpha)| + |a_2(\alpha) - b_2(\alpha)|)d\alpha]$$

$$\overline{\rho} = \bigcup_{\lambda\in(0,1]} \lambda[|a_3(\lambda) - b_3(\lambda)|, \sup_{\lambda\leq\alpha\leq 1} (|b_3(\alpha) - a_3(\alpha)| \vee |b_4(\alpha) - a_4(\alpha)|)].$$

Then we have $\tilde{\rho}$(A,B)is a Rough fuzzy Distance.

Proof: according to A, B∈RFN, we get the α-confidence interval of speculation level of $\underline{A}, \overline{A}$, $\underline{B}$, $\overline{B}$, $\forall a \in (0,1]$:

$$\underline{A}_a = [a_1^{(a)}, a_2^{(a)}], \quad \overline{A}_a = [a_3^{(a)}, a_4^{(a)}], \quad \underline{B}_a = [b_1^{(a)}, b_2^{(a)}], \quad \underline{B}_a = [b_3^{(a)}, b_4^{(a)}],$$

Noting the definition of $\underline{A}, \overline{A}$, $\underline{B}$, $\overline{B}$. thus, $\underline{A}_a$, $\overline{A}_a$, $\underline{B}_a, \overline{B}_a$ are the convex. and $\underline{A}_a = [a_1^{(a)}, a_2^{(a)}]$, $\overline{A}_a = [a_3^{(a)}, a_4^{(a)}]$, $\underline{B}_a = [b_1^{(a)}, b_2^{(a)}]$, $\overline{B}_a = [b_3^{(a)}, b_4^{(a)}]$ are all bounded,closed interval. so $a_i^{(a)}$, $b_i^{(a)}(i = 1,2,3,4)$ $a \in (0,1]$ are continuous, or there are jump discontinuities, moreover there are bounded. so, the integral: $\int_\lambda^1 (|a_1(\alpha) - b_1(\alpha)| + |a_2(\alpha) - b_2(\alpha)|)d\alpha$ is meaningful.

obviously $\tilde{\rho}$(A,B)≥ 0, (1) let $\tilde{\rho}$(A,B)=0, we have $\forall a \in (0,1]$, $|a_1(\alpha) - b_1(\alpha)| + |a_2(\alpha) - b_2(\alpha)| = 0$, $|a_3(\alpha) - b_3(\alpha)| = 0$. $\sup_{\lambda\leq\alpha\leq 1}\{|a_3(\alpha) - b_3(\alpha)| \vee |a_4(\alpha) - b_4(\alpha)|\} = 0$. thus, $a_i^{(a)} = b_i^{(a)}(i = 1,2,3,4)$

Hence A=B. inverse, letA=B, we have $\forall a \in (0,1]$, $a_i^{(a)} = b_i^{(a)}(i = 1,2,3,4)$, thus $\tilde{\rho}$(A,B)=0.

(2) obviously $\tilde{\rho}$(A,B)=$\tilde{\rho}$(B,A); (∀A, B∈RFN)

(3) let ∀C∈RFN, $\underline{C}_\alpha, \overline{C}_\alpha$ is respectively the confidence interval of $\underline{C}, \overline{C}$ with $a \in (0,1]$ speculation level, Suppose that:

$$\underline{C}_a = [C_1^{(a)}, C_2^{(a)}], \quad \overline{C}_a = [C_3^{(a)}, C_4^{(a)}]$$
$$|a_1(\alpha) - b_1(\alpha)| \leq |a_1(\alpha) - c_1(\alpha)| + |c_1(\alpha) - b_1(\alpha)|.$$
$$|a_2(\alpha) - b_2(\alpha) \leq |a_2(\alpha) - c_2(\alpha)| + |c_2(\alpha) - b_2(\alpha)|,$$

we have $|a_1(\alpha)-b_1(\alpha)|+|a_2(\alpha)-b_2(\alpha)| \leq |a_1(\alpha)-c_1(\alpha)|+|c_1(\alpha)-b_1(\alpha)|+|a_2(\alpha)-c_2(\alpha)|+|c_2(\alpha)-b_2(\alpha)|$, and, $|a_3(\alpha) - b_3(\alpha)| \leq |a_3(\alpha) - c_3(\alpha)| + |c_3(\alpha) - b_3(\alpha)|$, $|a_4(\alpha) - b_4(\alpha)| \leq |a_4(\alpha) - c_4(\alpha)| + |c_4(\alpha) - b_4(\alpha)|$, we have $|a_3(\alpha) - b_3(\alpha)| \leq |a_3(\alpha) - c_3(\alpha)| \vee |a_4(\alpha) - c_4(\alpha)| + |c_3(\alpha) - b_3(\alpha)| \vee |c_4(\alpha) - b_4(\alpha)|$, and, $|a_4(\alpha) - b_4(\alpha)| \leq |a_3(\alpha) - c_3(\alpha)| \vee |a_4(\alpha) - c_4(\alpha)| + |c_3(\alpha) - b_3(\alpha)| \vee |c_4(\alpha) - b_4(\alpha)|$, thus, $\forall\alpha \in [\lambda, 1]$, $(\lambda \in (0,1])$ $|a_3(\alpha) - b_3(\alpha)| \vee |a_4(\alpha) - b_4(\alpha)| \leq |a_3(\alpha) - c_3(\alpha)| \vee |a_4(\alpha) - c_4(\alpha)| + |c_3(\alpha) - b_3(\alpha)| \vee |c_4(\alpha) - b_4(\alpha)| \leq \sup_{\lambda\leq\alpha\leq 1}|a_3(\alpha) - c_3(\alpha)| \vee |a_4(\alpha) - c_4(\alpha)| + \sup_{\lambda\leq\alpha\leq 1}|c_3(\alpha) - b_3(\alpha)| \vee |c_4(\alpha) - b_4(\alpha)|$, Hence, $\forall\lambda \in (0,1]$ $\sup_{\lambda\leq\alpha\leq 1} |a_3(\alpha) - b_3(\alpha)| \vee |a_4(\alpha) - b_4(\alpha)| \leq \sup_{\lambda\leq\alpha\leq 1}|a_3(\alpha) - c_3(\alpha)| \vee |a_4(\alpha) - c_4(\alpha)| + \sup_{\lambda\leq\alpha\leq 1}|c_3(\alpha) - b_3(\alpha)| \vee |c_4(\alpha) - b_4(\alpha)|$, hence, $\tilde{\rho}$(A,B)≤ $\tilde{\rho}$(A,C)+$\tilde{\rho}$(C,B).

Theorem 2.2 Assume $\forall A, B \in RFN\tilde{\rho}$(A,B)=($\tilde{\rho}$, $\underline{\rho}$), where

$$\underline{\rho} = \cup_{\lambda\in(0,1]}\lambda[0, \max(\int_\lambda^1 |a_1(\alpha) - b_1(\alpha)|d\alpha, \int_\lambda^1 |a_2(\alpha) - b_2(\alpha)|d\alpha)]$$

$$\underline{\rho} = \bigcup_{\lambda\in(0,1]} \lambda[\sup_{0\leq\alpha\leq\lambda} |a_3(\alpha) - b_3(\alpha)|, \sup_{\lambda\leq\alpha\leq 1}|a_4(\alpha) - b_4(\alpha)| \vee \sup_{0\leq\alpha\leq 1} |a_3(\alpha) - b_3(\alpha)|]$$

Then we have $\tilde{\rho}$(A,B)is a Rough fuzzyDistance.

Proof: similar to Theorem2.1.

Combining Definition 2.1, Theorem 2.1 or Theorem 2.2. we can obtain

Theorem 2.3 A,B,C,D∈ $RFN, K \in R$. we have:

(1) $\tilde{\rho}$(A+B,A+C)=$\tilde{\rho}$(B,C).

(2) $\tilde{\rho}$(B-A,C-A)=$\tilde{\rho}$(B,C).

(3) $\tilde{\rho}$(A-B,A-C)=$\tilde{\rho}$(-B,-C).

(4) If $K \geq 0$, then we have $\tilde{\rho}(K$A, K B)=$K\tilde{\rho}$(A,B).

If K <0, we have $\tilde{\rho}(K$A, KB)=$|K|\tilde{\rho}(-A, -B)$.

(5) If A≤B≤C, we have: $\tilde{\rho}$(A,B)≤ $\tilde{\rho}$(A,C). $\tilde{\rho}$(B,C)≤ $\tilde{\rho}$(A,C).

(6) If A≤C≤B, A≤D≤B, we have: $\tilde{\rho}$(C,D) ≤ $\tilde{\rho}$(A,B).

Proof: we will proof (4), (6), the others can be proofed similarly.

(4) If $K \in [0, +\infty)$, we have $K\underline{A} = \cup_{a\in(0,1]}a.\ [Ka_1^{(a)}, Ka_2^{(a)}]$, $K\overline{A} = \cup_{a\in(0,1]}a.\ [Ka_3^{(a)}, Ka_4^{(a)}]$. $K\underline{B}=\cup_{a\in(0,1]}a.\ [Kb_1^{(a)}, Kb_2^{(a)}]$ $K\overline{B}=\cup_{a\in(0,1]}a.\ [K\, b_3^{(a)}, Kb_4^{(a)}]$. thus. $\tilde{\rho}(K$A, KB)=$(K\,\underline{\rho}, K\overline{\rho})=K(\underline{\rho},\overline{\rho})=K\tilde{\rho}$(A,B) where

$$\underline{\rho} = \cup_{\lambda\in(0,1]}\lambda[0, \frac{1}{2}\int_{\lambda}^{1}(|a_1(\alpha) - b_1(\alpha)| + |a_2(\alpha) - b_2(\alpha)|)d\alpha].$$

$$\overline{\rho} = \cup_{\lambda\in(0,1]}\lambda[|a_3(\lambda) - b_3(\lambda)|, \sup_{\lambda\leq\alpha\leq 1}(|b_3(\alpha) - a_3(\alpha)| \vee |b_4(\alpha) - a_4(\alpha)|)].$$

In the same way we can proof that: if K <0, we have $\tilde{\rho}(K$A, KB)=$|K|\tilde{\rho}$(-A, -B).

(6) let A≤C≤B, A≤D≤B, thus $\forall a \in (0, 1]$ we have $|c_i^{(a)} - d_i^{(a)}| \leq |b_i^{(a)} - a_i^{(a)}|, (i = 1, 2, 3, 4)$

Hence $\tilde{\rho}$(C,D)≤ $\tilde{\rho}$(A,B).

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