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Nonoscillation Theorems for a Class of Fourth Order Quasilinear Dynamic Equations on Time Scales

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Abstract: In this paper,some sufficient and necessary conditions for nonoscillation of the fourth order quasilinear dynamic equations on time scales $\mathbb{T}$ are established.Our results as special case when $\mathbb{T} = R$ and $\mathbb{T} = N$, involve and improve some known results.
Keywords: Nonoscillation; Quasilinear; Time Scales

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INTRODUCTION

The theory of time scales, which has recently received a lot of attention, was introduced by Stefan Hilger in his Ph.D.Thesis in 1988 in order to unify continuous and discrete analysis[1].A time scale $\mathbb{T}$,is an arbitrary nonempty closed subset of the reals, and the cases when this time scale is equal to the reals or to the integers represent the classical theories of differential and of difference equations. Many other interesting time scales exist, and they give rise to many applications[9].

On any time scale $\mathbb{T}$, we define the forward and backward jump operators by

$$\sigma(t) := \inf\{s > t : s \in \mathbb{T}\}, \quad \rho(t) := \sup\{s < t : s \in \mathbb{T}\}.$$

A point $t \in \mathbb{T}, t > \inf \mathbb{T}$,is said to be left-dense if $\rho(t) = t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, left-scattered if $\rho(t) < t$ and right-scattered if$\sigma(t) > t$. The graininess function μ for a time scale $\mathbb{T}$ is defined by $\mu(t) := \sigma(t) - t$.

A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous function provided it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist (finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous functions f :$\mathbb{T} \to \mathbb{R}$ is denoted by $C_{rd} = C_{rd}(\mathbb{T}) = C_{rd}(\mathbb{T}, \mathbb{R})$.

Let f be a differentiable function on$[a, b]$. Then f is increasing, decreasing, nondecreasing, and nonincreasing on$[a, b]$, if $f^{\Delta}(t) > 0, f^{\Delta}(t) < 0, f^{\Delta}(t) \geq 0$, and $f^{\Delta}(t) \leq 0$ for all $t \in [a, b)$, respectively.

For a function $f : \mathbb{T} \to \mathbb{R}$(the range $\mathbb{R}$ of f may be actually replaced by any Banach space) the delta derivative is defined by

$$f^{\Delta}(t) = \frac{f(\sigma(t)) - f(t)}{\sigma(t) - t}, \tag{0.1}$$

if f is continuous at t and t is right-scattered. We will make use of the following product and quotient rules for the derivative of the product fg and the quotient $\frac{f}{g}$ (where $gg^{\sigma} \neq 0$) of two differentiable functions f and g

$$(fg)^{\Delta} = f^{\Delta}g + f^{\sigma}g^{\Delta} = fg^{\Delta} + f^{\Delta}g^{\sigma} \tag{0.2}$$

$$\left(\frac{f}{g}\right)^{\Delta} = \frac{f^{\Delta}g - fg^{\Delta}}{gg^{\sigma}} \tag{0.3}$$

For $t_0, b \in \mathbb{T}$, and a differentiable function f , the Cauchy integral of f^{Δ} is defined by

$$\int_{t_0}^{b} f^{\Delta}(t)\Delta t = f(b) - f(t_0).$$

An integration by parts formula reads

$$\int_{t_0}^{b} f(t)g^{\Delta}(t)\Delta t = [f(t)g(t)]_{t_0}^{b} - \int_{t_0}^{b} f^{\Delta}(t)g^{\sigma}(t)\Delta t. \tag{0.4}$$

and infinite integral is defined as

$$\int_{t_0}^{\infty} f(t)\Delta t = \lim_{b\to\infty} \int_{t_0}^{b} f(t)\Delta t \tag{0.5}$$

In recent years, there has been much research activity concerning the oscillation and nonoscillation of solutions of various equations on time scales. We refer the reader to the papers[2–8]and the reference cited therein.

In this paper, we consider a fourth order quasilinear dynamic equation

$$(| y^{\Delta^2}(t) |^{\alpha-1} y^{\Delta^2}(t))^{\Delta^2} + q(t) | y(g(t)) |^{\beta-1} y(g(t)) = 0. \tag{0.6}$$

on time scale interval $[t_0, \infty) \subset \mathbb{T}$. ($t_0 \geq 0$) Where
(a) $\alpha, \beta$are positive constants;
(b) $q(t) : [t_0, \infty) \to (0, \infty)$is a rd-continuous function;
(c) $g(t) : [t_0, \infty) \to (0, \infty)$is a rd-continuously differentiable function such that $g(t) \leq t$, and $\lim_{t\to\infty} g(t) = \infty$.

Our purpose here is to make a detailed analysis of the structure of the set of all possible nonoscillatory solutions of the equation (0.6),which can be expressed as

$$((y^{\Delta^2}(t))^{\alpha_*})^{\Delta^2} + q(t)(y(g(t)))^{\beta_*} = 0, \tag{0.7}$$

in terms of the asterisk notation

$$\xi^{\gamma_*} =| \xi |^{\gamma} sgn\xi =| \xi |^{\gamma-1} \xi, \quad \xi \in R, \ \gamma > 0.$$

It is easy to see if $y(t)$ is a nonoscillatory positive solution of (0.7), then so is $-y(t)$.
A) Classification of nonoscillatory solution.
Suppose that $y(t)$ be an eventually positive solution of (0.7). then $y(t)$ satisfies either

$$\text{I}: \ y^{\Delta}(t) > 0, \quad y^{\Delta^2}(t) > 0, \quad ((y^{\Delta^2}(t))^{\alpha_*})^{\Delta} > 0,$$

for all large t or

$$\text{II}: \ y^{\Delta}(t) > 0, \quad y^{\Delta^2}(t) < 0, \quad ((y^{\Delta^2}(t))^{\alpha_*})^{\Delta} > 0.$$

for all large t. It follows that $y^{\Delta}(t), y^{\Delta^2}(t)$, $((y^{\Delta^2}(t))^{\alpha_*})^{\Delta}$ are eventually monotone, so that they tend to finite or infinite limits as $t \to \infty$. Let

$$\lim_{t\to\infty} y^{\Delta^i}(t) = \omega_i, \quad i = 0, 1, 2, \text{ and } \lim_{t\to\infty}((y^{\Delta^2}(t))^{\alpha_*})^{\Delta} = \omega_3.$$

It is clear that ω_3 is a finite nonnegative number. One can easily show that :
(i) If $y(t)$ satisfies I,then the set of its asymptotic values ω_i falls into one of the following three cases:
$\mathrm{I}_1 : \omega_0 = \omega_1 = \omega_2 = \infty, \omega_3 \in (0, \infty)$;
$\mathrm{I}_2 : \omega_0 = \omega_1 = \omega_2 = \infty, \omega_3 = 0$;
$\mathrm{I}_3 : \omega_0 = \omega_1 = \infty, \omega_2 \in (0, \infty), \omega_3 = 0$.
(ii) If $y(t)$ satisfies II,then the set of its asymptotic values ω_i falls into one of the following three cases:
$\mathrm{II}_1 : \omega_0 = \infty, \omega_1 \in (0, \infty), \omega_2 = \omega_3 = 0$
$\mathrm{II}_2 : \omega_0 = \infty, \omega_1 = \omega_2 = \omega_3 = 0$
$\mathrm{II}_3 : \omega_0 \in (0, \infty), \omega_1 = \omega_2 = \omega_3 = 0$.
Equivalent expressions for these six classes of positive solutions of (0.7) are as follows:
$\mathrm{I}_1 : \lim_{t\to\infty} \frac{y(t)}{t^{2+\frac{1}{\alpha}}} = const > 0$;
$\mathrm{I}_2 : \lim_{t\to\infty} \frac{y(t)}{t^{2+\frac{1}{\alpha}}} = 0, \lim_{t\to\infty} \frac{y(t)}{t^2} = \infty$;
$\mathrm{I}_3 : \lim_{t\to\infty} \frac{y(t)}{t^2} = const > 0$;
$\mathrm{II}_1 : \lim_{t\to\infty} \frac{y(t)}{t} = const > 0$;
$\mathrm{II}_2 : \lim_{t\to\infty} \frac{y(t)}{t} = 0, \lim_{t\to\infty} y(t) = \infty$;
$\mathrm{II}_3 : \lim_{t\to\infty} y(t) = const > 0$.
B) Integral representations for nonoscillatory solutions.
We shall establish the existence of positive solutions for each of the above six cases. Let $y(t)$ be a positive solution of (0.7), such that $y(t) > 0$, $y(g(t)) > 0$ for $t \geq t_0 > 0$. Integrating (0.7) from t to ∞ gives

$$((y^{\Delta^2}(t))^{\alpha_*})^{\Delta} = \omega_3 + \int_t^{\infty} q(s)(y(g(s)))^{\beta}\Delta s, \quad t \geq t_0. \tag{0.8}$$

If $y(t)$ is a solution of $I_i (i = 1, 2, 3)$,then we integrate (0.8) three times over $[t_0, t]$ to obtain

$$y(t) = k_0 + k_1(t - t_0) + \int_{t_0}^{t} (t - \sigma(s))[k_2{}^{\alpha} + \int_r^s (\omega_3 + \int_u^{\infty} q(u)(y(g(u)))^{\beta}\Delta u)\Delta r]^{\frac{1}{\alpha}} \Delta s, \tag{0.9}$$

for $t \geq t_0$,where $k_0 = y(t_0)$, $k_1 = y^{\Delta}(t_0)$, $k_2 = y^{\Delta^2}(t_0)$ are nonnegative constant,the equality (0.9) gives an integral representation for a solution $y(t)$ of type I_1. A type I_2 solution $y(t)$ of (0.7) is expressed by (0.9), with $\omega_3 = 0$. If $y(t)$ is a solution of type I_3, then first integrating (0.8) from t to ∞ and then integrating the resulting equation twice from t_0 to t,we have

$$y(t) = k_0 + k_1(t - t_0) + \int_{t_0}^{t} (t - \sigma(s))[\omega_2{}^{\alpha} - \int_s^{\infty} (\sigma(r) - s)q(r)(y(g(r)))^{\beta})\Delta r]^{\frac{1}{\alpha}} \Delta s, t > t_0 \tag{0.10}$$

An integral representation for a solution $y(t)$ of type II_1 is derived by integrating (0.8) with $\omega_3 = 0$ twice from t to ∞. and then once from t_0 to t,we have

$$y(t) = k_0 + \int_{t_0}^{t} (\omega_1 + \int_s^{\infty} [\int_r^{\infty} (\sigma(u) - r)q(u)(y(g(u)))^{\beta}\Delta u]^{\frac{1}{\alpha}}\Delta r)\Delta s, \quad t > t_0 \tag{0.11}$$

An expression for a of type II_2 solution is given by (0.11) with $\omega_1 = 0$, If $y(t)$ is a solution of type II_3,then integrations of (0.9) with $\omega_3 = 0$ three times yield

$$y(t) = \omega_0 - \int_t^{\infty} (\sigma(s) - t)[\int_s^{\infty} (\sigma(r) - s)q(r)(y(g(r)))^{\beta}\Delta r]^{\frac{1}{\alpha}})\Delta s, \quad t > t_0 \tag{0.12}$$

1. NONOSCILLATION THEOREMS

The set of nonoscillatory solution of (0.7) is decomposed into six disjoint classes according to their asymptotic behavior at ∞. It will be shown that necessary and sufficient conditions can be established for the existence of positive solutions of the four type I_1, I_3, II_1 and II_3. and sufficient conditions can also be established for the existence of positive solutions of types I_2, II_2.

Theorem 1.1. The equation (0.7) has a positive solution of type I_1 if and only if

$$\int_0^\infty q(s)(g(s))^{(2+\frac{1}{\alpha})\beta}\Delta s < \infty. \tag{1.1}$$

Proof. Necessary. Suppose that (0.7) has a positive solution of type I_1.then, it satisfies (0.9) for $t \geq t_0$,which implies that

$$\int_{t_0}^\infty q(s)(y(g(s)))^\beta \Delta s < \infty$$

This together with the asymptotic relation $\lim_{t\to\infty} \frac{y(t)}{t^{2+\frac{1}{\alpha}}} = const > 0$; shows that (1.1) is satisfied.

Sufficiency. Suppose that (1.1) holds. Let $k > 0$ be any given constant. choose $t_1 > t_0$ large enough so that

$$(\frac{\alpha^2}{(\alpha+1)(2\alpha+1)})^\beta \int_{t_0}^\infty q(s)(g(s))^{(2+\frac{1}{\alpha})\beta}\Delta s \leq \frac{(2k)^\alpha - k^\alpha}{(2k)^\beta} \tag{1.2}$$

Let $t_* = min\{t_0, \inf_{t>t_0} g(t)\}$, and defined

$$G(t,t_0) = \int_{t_0}^t (t-\sigma(s))(s-t_0)^{\frac{1}{\alpha}}\Delta s = \frac{\alpha^2}{(\alpha+1)(2\alpha+1)}(t-t_0)^{2+\frac{1}{\alpha}} \qquad t \geq t_0$$

$$G(t,t_0) = 0 \qquad t < t_0$$

Let $B(t)$ denote a Banach space of all real-value function, $Y \subset C_{rd}(t_*, R)$ with the norm $\| Y \| = \sup_{t>t_0} | y(t) | < \infty$ Defined a set Ω as follows:

$$\Omega = \{Y = \{y(t)\} \in B(t) \quad kG(t,t_0) \leq y(t) \leq 2kG(t,t_0), t \geq t^*\}$$

Define the operator $F : \Omega \to B(t)$:

$$\begin{cases} Fy(t) = \int_{t_0}^t (t-\sigma(s))[\int_{t_0}^s (k^\alpha + \int_r^\infty (q(u)(y(g(u)))^\beta \Delta u)\Delta r]^{\frac{1}{\alpha}}\Delta s, & t \geq t_0 \\ Fy(t) = Fy(t_0), & t^* \leq t \leq t_0 \end{cases} \tag{1.3}$$

$I)F$ maps Ω into Ω.
Let $y(t) \in \Omega$, for $t \geq t_0$, then

$$Fy(t) \geq k\int_{t_0}^t (t-\sigma(s))(s-t_0)^{\frac{1}{\alpha}}\Delta s = kG(t,t_0)$$

and

$$Fy(t) \leq \int_{t_0}^t (t-\sigma(s))[\int_{t_0}^s (k^\alpha + \int_r^\infty (q(u)(2kG(g(u),t_0)^\beta \Delta u)\Delta r]^{\frac{1}{\alpha}}\Delta s$$

$$\leq \int_{t_0}^t (t-\sigma(s))[\int_{t_0}^s (k^\alpha + (\frac{2k\alpha^2}{(\alpha+1)(2\alpha+1)})^\beta \int_r^\infty q(u)(g(u))^{(2+\frac{1}{\alpha})\beta}\Delta u)\Delta r]^{\frac{1}{\alpha}}\Delta s$$

$$\leq 2k\int_{t_0}^{t}(t-\sigma(s))(s-t_0)^{\frac{1}{\alpha}}\Delta s = 2kG(t,t_0)$$

II) F is rd-continuous . let $y^{(k)}\in\Omega$, $\lim_{k\to\infty}\parallel y^{(k)}-y\parallel=0$

$$\begin{aligned} |(Fy^{(k)})(t)-(Fy)(t)| &= \int_{t_0}^{t}(t-\sigma(s))[\int_{t_0}^{s}(k^{\alpha}+\int_{r}^{\infty}q(u)(y^{(k)}(g(u)))^{\beta}\Delta u)\Delta r]^{\frac{1}{\alpha}}\Delta s \\ &\quad -\int_{t_0}^{t}(t-\sigma(s))[\int_{t_0}^{s}(k^{\alpha}+\int_{r}^{\infty}q(u)(y(g(u)))^{\beta}\Delta u)\Delta r]^{\frac{1}{\alpha}}\Delta s \end{aligned}$$

By using *Lebesgue*'s dominated convergence theorem,we can prove that

$$\lim_{t\to\infty}\parallel Fy^{(k)}-Fy\parallel=0$$

III) F is equicauchy , for all $t_1,t_2>t^*$

$$\begin{aligned} |Fy(t_1)-Fy(t_2)| &= \int_{t_0}^{t_2}(t_2-\sigma(s))[\int_{t_0}^{s}(k^{\alpha}+\int_{r}^{\infty}q(u)(y(g(u)))^{\beta}\Delta u)\Delta r]^{\frac{1}{\alpha}}\Delta s \\ &\quad -\int_{t_0}^{t_1}(t_1-\sigma(s))[\int_{t_0}^{s}(k^{\alpha}+\int_{r}^{\infty}q(u)(y(g(u)))^{\beta}\Delta u)\Delta r]^{\frac{1}{\alpha}}\Delta s \\ &= \int_{t_1}^{t_2}(t_2-t_1)[\int_{t_0}^{s}(k^{\alpha}+\int_{r}^{\infty}q(u)(y(g(u)))^{\beta}\Delta u)\Delta r]^{\frac{1}{\alpha}}\Delta s<\varepsilon \end{aligned}$$

Therefor, by the Schauder fixed point theorem,there exist a fixed point y,such that $Fy=y$,which satisfies (0.7). This completes the proof.

Theorem 1.2. The equation (0.7) has a positive solution of type I_3 if and only if

$$\int_{0}^{\infty}\sigma(t)q(t)(g(t))^{2\beta}\Delta t<\infty \tag{1.4}$$

Proof. Necessity. Suppose that (0.7) has a positive solution of type I_3, then, it satisfies (0.10) for $t\geq t_0$, which implies that

$$\int_{t_0}^{\infty}(\sigma(t)-t_0)q(t)(y(g(t)))^{\beta}\Delta t<\infty$$

This together with the asymptotic relation $\lim_{t\to\infty}\frac{y(t)}{t^2}=const>0$, shows that (1.4) is satisfied.

Sufficiency. Suppose that (1.4) holds. Let $k>0$ be any given constant. choose $t_0>0$ large enough so that

$$\int_{t_0}^{\infty}\sigma(t)q(t)(g(t))^{2\beta}\Delta t\leq\frac{(2k)^{\alpha}-k^{\alpha}}{(k)^{\beta}} \tag{1.5}$$

Let $t^*=min\{t_0,\inf_{t>t_0}g(t)\}$, Let $B(t)$ denote a Banach space of all real-value function, $Y\subset C_{rd}(t_*,R)$ with the norm $\parallel Y\parallel=\sup_{t>t_0}|y(t)|<\infty$ Defined a set Ω as follows:

$$\Omega=\{Y=\{y(t)\}\in B(t)\quad \frac{k}{2}(t-t_0)_+^2\leq y(t)\leq k(t-t_0)_+^2, t\geq t^*\}$$

When $t\geq t_0$, $(t-t_0)_+=t-t_0$; $t\leq t_0$, $(t-t_0)_+=0$, Define the operator $F:\Omega\to B(t)$ as follows:

$$\begin{cases} Fy(t)=\int_{t_0}^{t}(t-\sigma(s))[(2k)^{\alpha}-\int_{s}^{\infty}(\sigma(r)-s)q(r)(y(g(r)))^{\beta}\Delta r]^{\frac{1}{\alpha}}\Delta s, & t\geq t_0 \\ Fy(t)=Fy(t_0) & t^*\leq t\leq t_0 \end{cases}$$

The remainer is similar to theorem 2.1. we omit here. there exists a fixed point y, such that $Fy = y$, which satisfies equation (0.7) and with the propertis $\lim_{t\to\infty} y^{\Delta^2}(t) = 2k > 0$; This completes the proof.

Theorem 1.3. The equation (0.7) has a positive solution of type II_1 if and only if

$$\int_{t_0}^{\infty} [\int_{t}^{\infty} (\sigma(s) - t)q(s)(g(s))^{\beta}\Delta s]^{\frac{1}{\alpha}} \Delta t < \infty \tag{1.6}$$

Proof. Necessity. Suppose that (0.7) has a positive solution of type II_1. then, it satisfies (1.6) for $t \geq t_0$, which implies that

$$\int_{t_0}^{\infty} (\sigma(t) - t_0)q(t)(y(g(t)))^{\beta}\Delta t < \infty$$

This together with the asymptotic relation $\lim_{t\to\infty} \frac{y(t)}{t} = const > 0$; shows that (1.6) is satisfied.

Sufficiency. Suppose that (1.6) holds. Let $k > 0$ be any given constant. choose $t_0 > 0$ large enough so that

$$\int_{t_0}^{\infty} [\int_{t}^{\infty} (\sigma(s) - t)q(s)(y(g(s)))^{\beta}\Delta s]^{\frac{1}{\alpha}} \Delta t < 2^{\frac{-\beta}{\alpha}} k^{1-\frac{\beta}{\alpha}}$$

Let $t^* = min\{t_0, \inf_{t>t_0} g(t)\}$, Let $B(t)$ denote a Banach space of all real-value function, $Y \subset C_{rd}(t_*, R)$ with the norm $\| Y \| = \sup_{t>t_0} | y(t) | < \infty$ Defined a set Ω as follows:

$$\Omega = \{Y = \{y(t)\} \in B(t) \quad kt \leq y(t) \leq 2kt, t \geq t^*\}$$

Define the operator $F : \Omega \to B(t)$:

$$\begin{cases} Fy(t) = kt + \int_{t_0}^{t} \int_{s}^{\infty} [\int_{r}^{\infty} (\sigma(u) - r)q(u)(y(g(u)))^{\beta}\Delta u)\Delta r]^{\frac{1}{\alpha}} \Delta s, & t \geq t_0 \\ Fy(t) = kt & t^* \leq t \leq t_0 \end{cases} \tag{1.7}$$

The remainer is similar to theorem 2.1. we omit here. there exist a fixed point y, such that $Ty = y$, which satisfies equation (0.7) and with the propertis

$$\lim_{t\to\infty} y^{\Delta}(t) = k > 0;$$

This completes the proof.

Theorem 1.4. The equation (0.7) has a positive solution of type II_3 if and only if

$$\int_{t_0}^{\infty} \sigma(t)[\int_{t}^{\infty} (\sigma(s) - t)q(s)\Delta s]^{\frac{1}{\alpha}} \Delta t < \infty \tag{1.8}$$

Proof. Necessity. Suppose that (0.7) has a positive solution of type II_3. then, it satisfies (1.8) for $t \geq t_0$, which implies that

$$\int_{t_0}^{\infty} \sigma(t)[\int_{t}^{\infty} (\sigma(s) - t)q(s)(y(g(s)))^{\beta}\Delta s)]^{\frac{1}{\alpha}} \Delta t < \infty \tag{1.9}$$

This together with the asymptotic relation $\lim_{t\to\infty} y(t) = const > 0$; shows that (1.8) is satisfied.

Sufficiency. Suppose that (1.8) holds. Let $k > 0$ be any given constant. choose $t_0 > 0$ large enough so that

$$\int_{t_0}^{\infty} \sigma(t)[\int_{t}^{\infty} (\sigma(s) - t)q(s)(y(g(s)))^{\beta}]^{\frac{1}{\alpha}} < \frac{1}{2} k^{1-\frac{\beta}{\alpha}} \tag{1.10}$$

Let $t^* = min\{t_0, \inf_{t>t_0} g(t)\}$, let $B(t)$ denote a Banach space of all real-value function, $Y \subset C_{rd}(t_*, R)$ with the norm $\| Y \| = \sup_{t>t_0} | y(t) | < \infty$ Defined a set :

$$\Omega = \{Y = \{y(t)\} \in B(t) \quad \frac{k}{2} \leq y(t) \leq k, t \geq t^*\}$$

Define the operator $F : \Omega \to B(t)$ as follows:

$$\begin{cases} Fy(t) = k - \int_(t)^\infty (\sigma(s) - t)[\int_s^\infty (\sigma(r) - s)q(r)(y(g(r)))^\beta \Delta r)]^{\frac{1}{\alpha}} \Delta s, \ t \geq t_0 \\ Fy(t) = Fy(t_0) \qquad\qquad t^* \leq t \leq t_0 \end{cases} \tag{1.11}$$

The remainer is similar to theorem 2.1. there exists a fixed point y, such that $Fy = y$, which satisfies equation (0.7) and with the propertis

$$\lim_{t\to\infty} y(t) = k > 0;$$

Theorem 1.5. Suppose that

$$\int_{t_0}^\infty q(t)(g(t))^{(2+\frac{1}{\alpha})\beta} \Delta t \leq \infty \tag{1.12}$$

and

$$\int_{t_0}^\infty \sigma(t)q(t)(g(t))^{2\beta} \Delta t = \infty \tag{1.13}$$

then equation (0.7) has a positive solution of type I_2.

Proof. Suppose that (1.12) holds. Let $k > 0$ be any given constant. choose $t_0 > 0$ large enough so that.

$$\int_{t_0}^\infty q(t)(g(t))^{(2+\frac{1}{\alpha})\beta} \Delta t \leq \frac{1}{2^{\alpha+1}} (\frac{(\alpha + 1)(2\alpha + 1)}{\alpha^2})^\alpha \tag{1.14}$$

Let $t^* = min\{t_0, \inf_{t>t_0} g(t)\}$, let $B(t)$ denote a Banach space of all real-value function, $Y \subset C_{rd}(t_*, R)$ with the norm $\| Y \| = \sup_{t>t_0} | y(t) | < \infty$ Defined a set :

$$\Omega = \{Y = \{y(t)\} \in B(t) \quad \frac{1}{2^{1+\frac{1}{\alpha}}} (t - t_0)_+^2 \leq y(t) \leq t^{2+\frac{1}{\alpha}} \qquad t \geq t^*\}$$

Define the operator $F : \Omega \to B(t)$:

$$\begin{cases} Fy(t) = \int_{t_0}^t (t - \sigma(s))[\frac{1}{2} + \int_{t_0}^s \int_r^\infty q(u)(y(g(u)))^\beta \Delta u \Delta r]^{\frac{1}{\alpha}}, \qquad t \geq t_0 \\ Fy(t) = 0 \qquad\qquad t^* \leq t \leq t_0 \end{cases} \tag{1.15}$$

The remainer is similar to theorem 2.1. there exists a fixed point y,such that $Ty = y$, which satisfies equation (0.7) and with the properties

$$\lim_{t\to\infty} y^{\Delta^2}(t) = \infty;$$

Theorem 1.6. Suppose that

$$\int_{t_0}^\infty [\int_{t_0}^\infty (\sigma(s) - t)q(s)(g(s))^\beta \Delta s]^{\frac{1}{\alpha}} \Delta t < \infty \tag{1.16}$$

and

$$\int_{t_0}^\infty \sigma(t)[\int_t^\infty (\sigma(s) - t)q(s)\Delta s]^{\frac{1}{\alpha}} \Delta t = \infty \tag{1.17}$$

then equation (0.7) has a positive solution of type II_2.
Proof. Let $k > 0$ be any given constant.choose $t_0 > 0$ large enough so that.

$$\int_{t_0}^{\infty} [\int_{t_0}^{\infty} (\sigma(s) - t)q(s)(g(s))^{\beta} \Delta s]^{\frac{1}{\alpha}} \Delta t \leq 2^{\frac{-\beta}{\alpha}} k^{1-\frac{\beta}{\alpha}} \tag{1.18}$$

Let $t^* = min\{t_0, \inf_{t>t_0} g(t)\}$, $B(t)$ is defined as Theorem 1.1. Defined a set :

$$\Omega = \{Y = \{y(t)\} \in B(t) \ \ k \leq y(t) \leq 2kt, t \geq t^*\}$$

Define the mapping $F : \Omega \to B(t)$ as follows:

$$\begin{cases} Fy(t) = k + \int_{t_0}^{t} \int_{s}^{\infty} [\int_{r}^{\infty} (\sigma(u) - r)q(u)(y(g(u)))^{\beta} \Delta u)\Delta r]^{\frac{1}{\alpha}} \Delta s, & t \geq t_0 \\ Fy(t) = k & t^* \leq t \leq t_0 \end{cases} \tag{1.19}$$

The remainer is similar to theorem 2.1. there exists a fixed point y,such that $y(t)$ is a positive solution of type II_2.

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