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A New Specific Property of Schweizer-Sklard Operators

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Abstract: A new specific property of Schweizer-Sklard operators is discovered that Schweizer-Sklard operations are continued fuzzy operators. The relation between Schweizer-Sklard operators and other Fuzzy operators is analysis. The conclusion that Schweizer-Sklard operations could partly replace other fuzzy operations most in use is given. At the same time the relation between Schweizer-Sklard operations and Zadeh operations as well as probability operators is given.
Keywords: Operator; Fuzzy operator; Schweizer-Sklard operator; Continued fuzzy operator

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INTRODUCTION

Since Zadeh established Fuzzy set theorem in 1965, he introduced a pair of Fuzzy operators. They are named Zadeh operators[1]. There is a lot of discussion about the fuzzy operators[2,3,4,5,6,7,8,9,10], but there is little discussion about the Schweizer-Sklard operators. This paper discussed membership relation in Schweizer-Sklard, Zadeh operator and other generalized operator in common use.

1. PRIMARY DEFINITIONS OF SCHWEIZER-SKLARD OPERATORS

We introduce a pair of important operators, Schweizer-Sklard operators, whose primary definition are as follows[4].

Definition 1.1 $\forall \tilde{A}, \tilde{B}, \tilde{C} \in P(U), \forall u \in U, p \in (£\infty, +\infty)$. Schweizer-Sklard product $\tilde{C}(u)$ of $\tilde{A}(u)$ and $\tilde{B}(u)$, denoted $\tilde{C}(u) = \tilde{A}(u) \overset{\bullet}{s}s\, \tilde{B}(u)$, is defined by

$$\tilde{A}(u) \overset{\bullet}{s}s\, \tilde{B}(u) = \begin{cases} 0 & if\ \ p > 0,\ \ \tilde{A}(u) = 0\ or\ \tilde{B}(u) = 0 \\ [\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]^{-1/p} & if\ \ p > 0, \tilde{A}(u) \bullet \tilde{B}(u) \neq 0 \\ \tilde{A}(u) \bullet \tilde{B}(u) & if\ \ p = 0 \\ [\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]^{-1/p} & if\ \ p < 0, \tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} > 1 \\ 0 & if\ \ p < 0, \tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} \leq 1 \end{cases} \tag{1.1}$$

Definition 1.2 $\forall \tilde{A}, \tilde{B}, \tilde{C} \in P(U), \forall u \in U, p \in (-\infty,+\infty)$. Schweizer-Sklard sum $\tilde{C}(u)$ of $\tilde{A}(u)$ and $\tilde{B}(u)$, denoted $\tilde{C}(u) = \tilde{A}(u) \overset{+}{ss} \tilde{B}(u)$, is defined by

$$\tilde{A}(u) \overset{+}{ss} \tilde{B}(u) = \begin{cases} 1 & if\ p > 0,\ \tilde{A}(u) = 1\ or\ \tilde{B}(u) = 1 \\ 1 - \{[1-\tilde{A}(u)]^{-p} + [1-\tilde{B}(u)]^{-p} - 1\}^{-1/p} & if\ p > 0, \tilde{A}(u) \bullet \tilde{B}(u) \neq 1 \\ \tilde{A}(u) + \tilde{B}(u) - \tilde{A}(u) \bullet \tilde{B}(u) & if\ p = 0 \\ 1 - \{[1-\tilde{A}(u)]^{-p} + [1-\tilde{B}(u)]^{-p} - 1\}^{-1/p} & if\ p < 0, [1-\tilde{A}(u)]^{-p} + [1-\tilde{B}(u)]^{-p} > 1 \\ 0 & if\ p < 0,\ [1-\tilde{A}(u)]^{-p} + [1-\tilde{B}(u)]^{-p} \leq 1 \end{cases} \tag{1.2}$$

The pair of operators $(\overset{\bullet}{ss}, \overset{+}{ss})$ is called Schweizer-Sklard operators.

2. A NEW IMPORTANT SPECIFIC PROPERTY OF SCHWEIZER-SKLARD OPERATORS

On property of Schweizer-Sklard operators, the paper[4] has proved that formula (1.1) and (1.2) are monotone functions for variables $\tilde{A}(u)$ or $\tilde{B}(u)$.

But a new important specific property of Schweizer-Sklard operators has not mentioned up to now.

We give a new conclusion as follows:

Theorem 2.1 When $p > 0$, formula (1.1) is monotone increasing function for the parameter p.

Proof. $\forall u \in U$, if $p > 0$, $\tilde{A}(u) \cdot \tilde{B}(u) \neq 0$, let

$$F(p) = [\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]^{-1/p} = \exp\{\frac{\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}{-p}\}$$

then

$$\frac{\partial}{\partial p} F(p) = F(p) \bullet \frac{\partial}{\partial p}\{\frac{\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}{-p}\} = F(p) \bullet G(p)$$

$$G(p) = \frac{[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1] - \tilde{A}(u)^{-p}\ln\tilde{A}(u)^{-p} - \tilde{B}(u)^{-p}\ln\tilde{B}(u)^{-p}}{p^2[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}.$$

Let $x = \tilde{A}(u)^{-p}$, $y = \tilde{B}(u)^{-p}$ then

$$G(p) = \frac{(x+y-1)\ln(x+y-1) - x\ln x - y\ln y}{p^2(x+y-1)} = \frac{1}{p^2}\ln[(x+y-1)x^{1-\frac{1}{x}+\frac{y}{x}}y^{1-\frac{1}{y}+\frac{x}{y}}].$$

Because $\tilde{A}(u) \in (0,1]$ and $\tilde{B}(u) \in (0,1]$,then $x = \tilde{A}(u)^{-p} \in [1,+\infty)$ and $y = \tilde{B}(u)^{-p} \in [1,+\infty)$.this means $x + y - 1 \geq 1$, $y/x > 0$, $x/y > 0$, $1-1/x \geq 0$, $1-1/y \geq 0$. We have

$$1-1/x + y/x > 0, \quad 1-1/y + x/y > 0$$

then

$$x^{1-\frac{1}{x}+\frac{y}{x}} \geq 1, y^{1-\frac{1}{y}+\frac{x}{y}} \geq 1$$

This means $G(P) \geq 0$. Because $F(p) > 0$,we have

$$\frac{\partial}{\partial p} F(p) = F(p) \bullet \frac{\partial}{\partial p}\{\frac{\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}{-p}\} = F(p) \bullet G(p) \geq 0$$

This means that when $p > 0$, formula (1.1) is monotone increasing function for the parameter p.

Theorem 2.2 When $p<0$, formula (1.1) is monotone decreasing function for the parameter p.

Proof. $\forall u \in U$, if $p<0$, $\tilde{A}(u)^{-p}+\tilde{B}(u)^{-p}>1$, let

$$F(p)=[\tilde{A}(u)^{-p}+\tilde{B}(u)^{-p}-1]^{-1/p}=\exp\{\frac{\ln[\tilde{A}(u)^{-p}+\tilde{B}(u)^{-p}-1]}{-p}\}$$

then

$$\frac{\partial}{\partial p}F(p)=F(p)\bullet\frac{\partial}{\partial p}\{\frac{\ln[\tilde{A}(u)^{-p}+\tilde{B}(u)^{-p}-1]}{-p}\}=F(p)\bullet G(p)$$

$$G(p)=\frac{[\tilde{A}(u)^{-p}+\tilde{B}(u)^{-p}-1]\ln[\tilde{A}(u)^{-p}+\tilde{B}(u)^{-p}-1]-\tilde{A}(u)^{-p}\ln\tilde{A}(u)^{-p}-\tilde{B}(u)^{-p}\ln\tilde{B}(u)^{-p}}{p^2[\tilde{A}(u)^{-p}+\tilde{B}(u)^{-p}-1]}.$$

Let $x=\tilde{A}(u)^{-p}$,$y=\tilde{B}(u)^{-p}$ then

$$G(p)=\frac{(x+y-1)\ln(x+y-1)-x\ln x-y\ln y}{p^2(x+y-1)}=\frac{1}{p^2}\ln[(x+y-1)x^{1-\frac{1}{x}+\frac{y}{x}}y^{1-\frac{1}{y}+\frac{x}{y}}].$$

Because $\tilde{A}(u)\in(0,1]$ and $\tilde{B}(u)\in(0,1]$,then $x=\tilde{A}(u)^{-p}\in(0,1]$ and $y=\tilde{B}(u)^{-p}\in(0,1]$.We know $x+y>1$, then $0<x+y-1<1$.

Because $y\leq 1$ then $y-1\leq 0$, that is $0<x+y-1<x$.

This means$0<\frac{x+y-1}{x}\leq 1$, that is$0<x^{1-\frac{1}{x}+\frac{y}{x}}\leq 1$.

Similarly we have$0<\frac{x+y-1}{y}\leq 1$.that is$0<y^{1-\frac{1}{y}+\frac{x}{y}}\leq 1$.

This means $G(P)\leq 0$. Because $F(p)>0$, we can introduce

$$\frac{\partial}{\partial p}F(p)=F(p)\bullet\frac{\partial}{\partial p}\{\frac{\ln[\tilde{A}(u)^{-p}+\tilde{B}(u)^{-p}-1]}{-p}\}=F(p)\bullet G(p)\leq 0$$

This means that when $p<0$, formula (1.1) is monotone decreasing function for the parameter p.

Theorem 2.3 When $p>0$, formula (1.2) is monotone decreasing function for the parameter p.

Proof. Omitted.

This means that when $p>0$, formula (1.2) is monotone decreasing function for the parameter p.

Theorem 2.4 When $p<0$, formula (1.2) is monotone increasing function for the parameter p.

Proof. Omitted.

This means that when $p<0$, formula (1.2) is monotone increasing function for the parameter p.

The formula (1.1) and the formula (1.2) have defined infinite fuzzy operators. It is quite evident that from the theorem 2.1 to the theorem 2.4 have given a sort of continued Fuzzy operators[7,8]. This is a new important property of Schweizer-Sklard operators.

3. THE RELATION OF THE SCHWEIZER-SKLARD OPERATIONS AND OTHER FUZZY OPERATIONS MOST IN USE

To any fuzzy set$\tilde{A},\tilde{B}\in P(U)$, $\forall u\in U$, Zadeh operator and three other common generalized operators (probability operators, boundary operators and infinite operators) have relation[9,10]:

$$\tilde{A}(u)\hat{\infty}\tilde{B}(u)\subseteq\tilde{A}(u)\otimes\tilde{B}(u)\subseteq\tilde{A}(u)\hat{\bullet}\tilde{B}(u)\subseteq\tilde{A}(u)\wedge\tilde{B}(u)$$

$$\subseteq \tilde{A}(u) \vee \tilde{B}(u) \subseteq \tilde{A}(u)\hat{+}\tilde{B}(u) \subseteq \tilde{A}(u) \oplus \tilde{B}(u) \subseteq \tilde{A}(u)\breve{\infty}\tilde{B}(u)$$

For the Schweizer-Sklard operators($\overset{\bullet}{ss}$, $\overset{+}{ss}$), we give a new conclusion as follows:

Theorem 3.1 $\forall\ \tilde{A}, \tilde{B} \in P(U)$,$\forall u \in U$, For the parameter $p \to +\infty$, the continued Fuzzy operators, that is, Schweizer-Sklard operators ($\overset{\bullet}{ss}$, $\overset{+}{ss}$) and Zadeh operators ($\wedge, \vee$) have relation as follows:

$$[\tilde{A}(u)\ \overset{\bullet}{ss}\ \tilde{B}(u)]_{+\infty} = \tilde{A}(u) \wedge \tilde{B}(u) \subseteq \tilde{A}(u) \vee \tilde{B}(u) = [\tilde{A}(u)\ \overset{+}{ss}\ \tilde{B}(u)]_{+\infty} \tag{3.1}$$

Proof. If the parameter $p \to +\infty$, then $\forall u \in U$, the $\tilde{C}(u)$ in definition 2.1 is denoted $[\tilde{A}(u)\ \overset{\bullet}{ss}\ \tilde{B}(u)]_{+\infty}$ and $[\tilde{A}(u)\ \overset{+}{ss}\ \tilde{B}(u)]_{+\infty}$ as follows:

$$\lim_{p\to+\infty}[\tilde{A}(u)\ \overset{\bullet}{ss}\ \tilde{B}(u)] = [\tilde{A}(u)\ \overset{\bullet}{ss}\ \tilde{B}(u)]_{+\infty},\ \lim_{p\to+\infty}[\tilde{A}(u)\ \overset{+}{ss}\ \tilde{B}(u)] = [\tilde{A}(u)\ \overset{+}{ss}\ \tilde{B}(u)]_{+\infty}.$$

Zadeh operators ($\wedge$, $\vee$Å) are denoted by

$$\tilde{A}(u) \wedge \tilde{B}(u) = \min[\tilde{A}(u), \tilde{B}(u)], \tilde{A}(u) \vee \tilde{B}(u) = \max[\tilde{A}(u), \tilde{B}(u)].$$

We subdivide proof into two steps:

(a) $\forall\ \tilde{A}, \tilde{B} \in P(U)$, $\forall u \in U$, when the parameter $p > 0$,if $\tilde{A}(u)\tilde{B}(u)$=0, then

$$\lim_{p\to+\infty}[\tilde{A}(u)\ \overset{\bullet}{ss}\ \tilde{B}(u)] = 0 = \min[\tilde{A}(u), \tilde{B}(u)] = \tilde{A}(u) \wedge \tilde{B}(u).$$

If $\tilde{A}(u)\tilde{B}(u) \neq 0$, let $\tilde{A}(u) \geq \tilde{B}(u)$ then

$$\begin{aligned}\lim_{p\to+\infty}[\tilde{A}(u)\ \overset{\bullet}{ss}\ \tilde{B}(u)] &= \lim_{p\to+\infty}[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]^{-1/p} \\ &= \lim_{p\to+\infty} \exp\frac{\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}{-p} \\ &= \lim_{p\to+\infty} \exp\frac{\tilde{A}(u)^{-p}\ln\tilde{A}(u) + \tilde{B}(u)^{-p}\ln\tilde{B}(u)}{\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1} \\ &= \tilde{B}(u) = \tilde{A}(u) \wedge \tilde{B}(u)\end{aligned}$$

(b) $\forall\ \tilde{A}, \tilde{B} \in P(U)$, $\forall u \in U$, when the parameter $p > 0$, if $\tilde{A}(u)$=1 or $\tilde{B}(u)$=1, then

$$\lim_{p\to+\infty}[\tilde{A}(u)\ \overset{+}{ss}\ \tilde{B}(u)] = 1 = \max[\tilde{A}(u), \tilde{B}(u)] = \tilde{A}(u) \vee \tilde{B}(u)$$

If $\tilde{A}(u)\tilde{B}(u) \neq 1$, let $\tilde{A}(u) \geq \tilde{B}(u)$ then

$$\begin{aligned}\lim_{p\to+\infty}[\tilde{A}(u)\ \overset{+}{ss}\ \tilde{B}(u)] &= \lim_{p\to+\infty}\{1 - \{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1\}^{-1/p}\} \\ &= 1 - \lim_{p\to+\infty} \exp\frac{\ln\{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1\}}{-p} \\ &= 1 - \lim_{p\to+\infty} \exp\frac{[1 - \tilde{A}(u)]^{-p}\ln[1 - \tilde{A}(u)] + [1 - \tilde{B}(u)]^{-p}\ln[1 - \tilde{B}(u)]}{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1} \\ &= \tilde{A}(u) = \tilde{A}(u) \vee \tilde{B}(u)\end{aligned}$$

It is quite evident that (a) and (b) implies (3.1).

Theorem 3.2 $\forall\ \tilde{A}, \tilde{B} \in P(U)$, $\forall u \in U$, for the parameter $p \to +0$, the continued Fuzzy operators, that is, Schweizer-Sklard operators ($\overset{\bullet}{ss}$, $\overset{+}{ss}$) and probability operators ($\hat{\bullet}$,$\hat{+}$) have relation as follows:

$$\tilde{A}(u)\hat{\bullet}\tilde{B}(u) = [\tilde{A}(u)\ \overset{\bullet}{ss}\ \tilde{B}(u)]_{+0} \subseteq [\tilde{A}(u)\ \overset{+}{ss}\ \tilde{B}(u)]_{+0} = \tilde{A}(u)\hat{+}\tilde{B}(u) \tag{3.2}$$

Proof. If the parameter $p \to +0$, then $\forall u \in U$, the $\tilde{C}(u)$ in definition 1.1 is denoted $[\tilde{A}(u) \overset{\bullet}{ss} \tilde{B}(u)]_{+0}$ and $[\tilde{A}(u) \overset{+}{ss} \tilde{B}(u)]_{+0}$ as follows:

$$\lim_{p\to+0}[\tilde{A}(u) \overset{\bullet}{ss} \tilde{B}(u)] = [\tilde{A}(u) \overset{\bullet}{ss} \tilde{B}(u)]_{+0}, \ \lim_{p\to+0}[\tilde{A}(u) \overset{+}{ss} \tilde{B}(u)] = [\tilde{A}(u) \overset{+}{ss} \tilde{B}(u)]_{+0}.$$

Zadeh operators $(\hat{\bullet}, \hat{+})$ are denoted by

$$\tilde{A}(u)\hat{\bullet}\tilde{B}(u) = \tilde{A}(u)\tilde{B}(u), \tilde{A}(u)\hat{+}\tilde{B}(u) = \tilde{A}(u) + \tilde{B}(u) - \tilde{A}(u)\tilde{B}(u).$$

We subdivide proof into two steps:

(a) $\forall\ \tilde{A}, \tilde{B} \in P(U)$, $\forall u \in U$, when the parameter $p > 0$,if $\tilde{A}(u)\tilde{B}(u)=0$, then

$$\lim_{p\to+\infty}[\tilde{A}(u) \overset{\bullet}{ss} \tilde{B}(u)] = 0 = \tilde{A}(u)\tilde{B}(u) = \tilde{A}(u)\hat{\bullet}\tilde{B}(u)$$

If $\tilde{A}(u)\tilde{B}(u) \neq 0$, let $\tilde{A}(u) \geq \tilde{B}(u)$ then

$$\begin{aligned}\lim_{p\to+0}[\tilde{A}(u) \overset{\bullet}{ss} \tilde{B}(u)] &= \lim_{p\to+0}[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]^{-1/p} \\ &= \lim_{p\to+0} \exp \frac{\ln[\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1]}{-p} \\ &= \lim_{p\to+0} \exp \frac{\tilde{A}(u)^{-p} \ln \tilde{A}(u) + \tilde{B}(u)^{-p} \ln \tilde{B}(u)}{\tilde{A}(u)^{-p} + \tilde{B}(u)^{-p} - 1} = \tilde{A}(u)\tilde{B}(u) = \tilde{A}(u)\hat{\bullet}\tilde{B}(u)\end{aligned}$$

(b) $\forall\ \tilde{A}, \tilde{B} \in P(U)$, $\forall u \in U$, when the parameter $p > 0$, if $\tilde{A}(u)=1$ or $\tilde{B}(u)=1$, let $\tilde{A}(u)=1$ then

$$\lim_{p\to+0}[\tilde{A}(u) \overset{+}{ss} \tilde{B}(u)] = 1 = 1 + \tilde{B}(u) - 1 \bullet \tilde{B}(u) = \tilde{A}(u) + \tilde{B}(u) - \tilde{A}(u)\tilde{B}(u) = \tilde{A}(u)\hat{+}\tilde{B}(u)$$

If $\tilde{A}(u)\tilde{B}(u) \neq 1$, let $\tilde{A}(u) \geq \tilde{B}(u)$ then

$$\begin{aligned}\lim_{p\to+0}[\tilde{A}(u) \overset{+}{ss} \tilde{B}(u)] &= \lim_{p\to+0}\{1 - \{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1\}^{-1/p}\} \\ &= 1 - \lim_{p\to+0} \exp \frac{\ln\{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1\}}{-p} \\ &= 1 - \lim_{p\to+0} \exp \frac{[1 - \tilde{A}(u)]^{-p} \ln[1 - \tilde{A}(u)] + [1 - \tilde{B}(u)]^{-p} \ln[1 - \tilde{B}(u)]}{[1 - \tilde{A}(u)]^{-p} + [1 - \tilde{B}(u)]^{-p} - 1} \\ &= \tilde{A}(u) + \tilde{B}(u) - \tilde{A}(u)\tilde{B}(u) \\ &= \tilde{A}(u)\hat{+}\tilde{B}(u)\end{aligned}$$

It is quite evident that (a) and (b) implies (3.2).

From theorem 2.1 to theorem 3.2 implies that continued Fuzzy operators, Schweizer-Sklard operations, could replace other fuzzy operations most in use. At the same time, when the parameter $p > 0$, from theorem 2.1 to theorem 3.2 implies that continued Fuzzy operators, Schweizer-Sklard operations, and Zadeh operations as well as probability operators have relation as follows:

$$\tilde{A}(u)\hat{\bullet}\tilde{B}(u) \subseteq \tilde{A}(u) \overset{\bullet}{ss} \tilde{B}(u) \subseteq \tilde{A}(u) \wedge \tilde{B}(u) \subseteq \tilde{A}(u) \vee \tilde{B}(u) \subseteq \tilde{A}(u) \overset{+}{ss} \tilde{B}(u) \subseteq \tilde{A}(u)\hat{+}\tilde{B}(u) \tag{3.3}$$

CONCLUSION

From theorem 2.1 to theorem 3.2 implies that Schweizer-Sklard operations are continued Fuzzy operators and that Schweizer-Sklard operations could partly replace other fuzzy operations most in use.

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