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State Feedback Stabilization of Discrete Singular Large-scale Control Systems

SUN Shuiling[1,*]

CHEN Yuanyuan

Abstract: This paper studies the state feedback stabilization of discrete singular large-scale control systems by using Lyapunov matrix equation, generalized Lyapunov function method and matrix theory. There gives some sufficient conditions for determining the asymptotical stability and instability of the corresponding singular closed-loop large-scale systems while the subsystems are regular, causal and R-controllable. At last, an example is given to show the application of main result.

Key words: Discrete Singular Large-Scale Control System; Asymptotical Stability; Stabilization; Generalized Lyapunov Function; State Feedback

The theory of singular control systems has been applied to our life more and more extensively, such as power systems, economic systems, population systems, etc. But these systems often contain more than one singular system which compose singular large-scale control systems. Singular large-scale control systems have a more practical background. The actual production process can be described preferably by singular large-scale control systems, particularly by discrete singular large-scale control systems. The causality of discrete singular systems makes related results complicated and challenging for us. At present, the research results of the problem above are seldom. The asymptotical stability and stabilization of singular large-scale systems has been considered by Lyapunov function method in [Zhao Jian-chuan,(2008), Zhang Qing-ling,(1997)]. This paper consider the same question by introduce weighted sum Lyapunov function method, and give its interconnecting parameters regions of stability.

1. DEFINITIONS AND PROBLEM FORMULATION

Consider the discrete singular large-scale control systems with m subsystems:

$$E_i x_i(k+1) = A_{ii} x_i(k) + \sum_{j=1, j\neq i}^{m} A_{ij} x_j(k) + B_i U_i(k) \quad (i=1,\cdots,m) \tag{1}$$

[1] Guangdong Polytechnic Normal University, Guangzhou, China Email: sun600103@yahoo.com.cn
* Corresponding Author.
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where $x_i(k) \in R^{n_i}$ is a semi-state vector, $U_i \in R^{m_i}$ is a control input vector; A_{ii} , $E_i \in R^{n_i \times n_i}$, $B_i \in R^{n_i \times m_i}$, they are constant matrices; $\sum_{i=1}^{m} n_i = n$, $\sum_{i=1}^{m} r_i = r$, $\sum_{i=1}^{m} m_i = l$, $rank(E_i) = r_i \le n_i$, $rank(E) = r < n$,

$E = Block-diag(E_1, E_2, \cdots, E_m)$, $B = Block-diag(B_1, B_2, \cdots, B_m)$.

Now we give some concepts about discrete singular system

$$Ex(k+1) = Ax(k) \tag{2}$$

and discrete singular control system

$$Ex(k+1) = Ax(k) + Bu(k) \tag{3}$$

where E and A are $n \times n$ constant matrices, B is a $n \times m$ constant matrix, $rank(E) = r < n$, $x(k) \in R^n$ is a semi-state vector, $u(k) \in R^m$ is a control input vector.

Definition 1[Yang Dong-mei. Zhang Qing-ling, 2004]**:** Discrete singular system (2) is said to be regular if $\det(zE - A) \neq 0$ for some $z \in C$.

Definition 2[Yang Dong-mei, et al., 2004]**:** The zero solution of discrete singular system (2) is said to be stable if for every $\varepsilon > 0$,there exists $\delta > 0$, such that $\|x(k;\ k_0, x_0)\| < \varepsilon$, for all $k \ge k_0$, whenever the arbitrary initial consistency value $x(k_0) = x_0$ which satisfies $\|x_0\| < \delta$.

Definition 3[Yang Dong-mei, et al., 2004]**:** Discrete singular control system (3) is said to be causal if $x(k)$ can be uniquely determined by $x(0)$ and control input vectors $u(0)$, $\cdots$, $u(k)$ for any k . Otherwise, it is said to be non-causal.

Now consider the isolated subsystems of systems:

$$E_i x_i(k+1) = A_{ii} x_i(k) + B_i U_i(k) \quad (i = 1, \cdots, m) \tag{4}$$

Assume that all systems of systems (4) are R-controllable, we choose the linear control law

$$U_i(k) = -K_i x_i(k) (i = 1, \cdots, m) \tag{5}$$

Then singular closed-loop large-scale systems of systems (1) are given by

$$E_i x_i(k+1) = (A_{ii} - B_i K_i) x_i(k) + \sum_{j=1, j \neq i}^{m} A_{ij} x_j(k) \ (i = 1, \cdots, m) \tag{6}$$

The corresponding closed-loop isolated subsystems are

$$E_i x_i(k+1) = (A_{ii} - B_i K_i) x_i(k) (i = 1, \cdots, m) \tag{7}$$

In order to investigate the stabilization of discrete singular large-scale control systems (1), we give the following lemmas:

Lemma 1[Dai Li-yi, 1986]: The system (3) is said to be R-controllable if $rank[zE - A \quad B] = n$ for some $z \in C$.

Lemma2[Dai Li-yi, 1986]: Discrete singular control system (3) is said to be causal if and only if $\deg\{\det(zE - A)\} = rank(E)$.

Lemma 3 [Wo Song-lin, 2004]: Assume that $u, v \in R^n$, $V \in R^{n \times n}$ is a positive semi-definite matrix, then $2u^T V v \le \varepsilon u^T V u + \varepsilon^{-1} v^T V v$ holds for all $\varepsilon > 0$.

Lemma 4[Wo Song-lin, Zou Yun, 2003]: Assume that the system (2) is regular and causal, then it is asymptotically stable if and only if given positive definite matrix W ,there exists a positive semi-definite matrix V which satisfies $A^TVA - E^TVE = -E^TWE$.

Lemma 5[Liang Jia-rong, 2000]: Assume that the system (2) is regular, causal, and there exists a function $v(Ex)$ which satisfies the following conditions:

(a) $v(Ex) = (Ex(k))^T V(Ex(k))$,where V is an positive semi-definite matrix, and $rank(E^TVE) = rank(E) = r$;

(b) $\Delta V(EX) \le -(Ex(k))^T W(Ex(k))$, where W is a positive definite matrix;

then the sub-equilibrium state of system(2) $EX = 0$ is asymptotically stable.

2. MAIN RESULTS

Theorem 1: Assume that all isolated subsystems (4) of systems (1) are R-controllable, all closed-loop isolated subsystems (7) are regular, causal and asymptotically stable, and there exists a real number $\mu > 0$ which satisfies that

$$\left[A_{ij}x_j(k)\right]^T \left[A_{ij}x_j(k)\right] \le \mu\left[E_jx_j(k)\right]^T \left[E_jx_j(k)\right] \ (i, j = 1,\cdots m, j \ne i) \tag{8}$$

then when

$$2W_i - V_i - 2(m-1)^2 \mu\lambda_M I_i > 0 \ (i = 1,\cdots m) \tag{9}$$

the zero solution of the singular closed-loop large-scale systems(6) are asymptotically stable, the discrete singular large-scale control systems(1) are stabilizable. The interconnecting parameter region of stability is given by (9). Here W_i is a positive definite and V_i is a positive semi-definite matrix by Lemma4 given , and $\lambda_M = \max\limits_{1\le i\le m}\{\lambda_M(V_i)\}$ and I_i is a $n_i \times n_i$ identity matrix.

Proof: systems (7) are regular and causal, as they are asymptotically stable, then given positive definite matrix W_i , Lyapunov matrix equation $(A_{ii} - B_iK_i)^T V_i(A_{ii} - B_iK_i) - E_i^TV_iE_i = -E_i^TW_iE_i$ have positive semi-definite solution V_i .

Construct quadratic form $v_i\left[E_ix_i(k)\right] = \left[E_ix_i(k)\right]^T V_i\left[E_ix_i(k)\right]$ as the scalar Lyapunov function of systems (7).

Let $v\left[Ex(k)\right] = \sum\limits_{i=1}^{m} v_i\left[E_ix_i(k)\right]$ as the Lyapunov function of systems (1).

We have

$$\Delta v_i\left[E_ix_i(k)\right]\Big|_{(6)} = \left\{\left[E_ix_i(k+1)\right]^T V_i\left[E_ix_i(k+1)\right] - \left[E_ix_i(k)\right]^T V_i\left[E_ix_i(k)\right]\right\}\Big|_{(6)}$$

$$= \left[(A_{ii} - B_iK_i)x_i(k) + \sum_{j=1, j\ne i}^{m} A_{ij}x_j(k)\right]^T V_i \times \left[(A_{ii} - B_iK_i)x_i(k) + \sum_{j=1, j\ne i}^{m} A_{ij}x_j(k)\right] - \left[E_ix_i(k)\right]^T V_i\left[E_ix_i(k)\right]$$

$$= x_i^T(k)\left[(A_{ii} - B_iK_i)^T V_i(A_{ii} - B_iK_i) - E_i^TV_iE_i\right]x_i(k) + 2\left[\sum_{j=1, j\ne i}^{m} A_{ij}x_j(k)\right]^T V_i\left[(A_{ii} - B_iK_i)x_i(k)\right]$$

$$+\left[\sum_{j=1,j\neq i}^{m} A_{ij}x_j(k)\right]^T V_i \left[\sum_{j=1,j\neq i}^{m} A_{ij}x_j(k)\right]$$

By using Lemma 3，choose $\varepsilon = 1$, we have

$$\Delta v_i\left[E_i x_i(k)\right]\Big|_{(6)} \le x_i^T(k)\left(-E_i^T W_i E_i\right)x_i(k) + 2\left[\sum_{j=1,j\neq i}^{m} A_{ij}x_j(k)\right]^T V_i \left[\sum_{j=1,j\neq i}^{m} A_{ij}x_j(k)\right]$$

$$+\left[\left(A_{ii}-B_iK_i\right)x_i(k)\right]^T V_i\left[\left(A_{ii}-B_iK_i\right)x_i(k)\right]$$

$$= x_i^T(k)\left(-E_i^T W_i E_i\right)x_i(k) + 2\left[\sum_{j=1,j\neq i}^{m} A_{ij}x_j(k)\right]^T V_i \left[\sum_{j=1,j\neq i}^{m} A_{ij}x_j(k)\right] + x_i^T(k)\left[\left(A_{ii}-B_iK_i\right)^T V_i \left(A_{ii}-B_iK_i\right)\right]x_i(k)$$

$$= x_i^T(k)\left[-2E_i^T W_i E_i + E_i^T V_i E_i\right]x_i(k) + 2\left[\sum_{j=1,j\neq i}^{m} A_{ij}x_j(k)\right]^T V_i \left[\sum_{j=1,j\neq i}^{m} A_{ij}x_j(k)\right]$$

Noticing that

$$\left[\sum_{j=1,j\neq i}^{m} A_{ij}x_j(k)\right]^T V_i \left[\sum_{j=1,j\neq i}^{m} A_{ij}x_j(k)\right] = \sum_{j=1,j\neq i}^{m}\left[A_{ij}x_j(k)\right]^T V_i \left[\sum_{j=1,j\neq i}^{m} A_{ij}x_j(k)\right]$$

$$= \sum_{j=1,j\neq i}^{m}\sum_{s=1,s\neq i}^{m}\left(A_{ij}x_j(k)\right)^T V_i A_{is}x_s(k)$$

$$= \frac{1}{2}\sum_{j=1,j\neq i}^{m}\sum_{s=1,s\neq i}^{m} 2\left(A_{ij}x_j(k)\right)^T V_i A_{is}x_s(k)$$

$$\le \frac{1}{2}\sum_{j=1,j\neq i}^{m}\sum_{s=1,s\neq i}^{m}\left[\left(A_{ij}x_j(k)\right)^T V_i A_{ij}x_j(k) + \left(A_{is}x_s(k)\right)^T V_i A_{is}x_s(k)\right]$$

$$\le \frac{1}{2}\lambda_M(V_i)\sum_{j=1,j\neq i}^{m}\sum_{s=1,s\neq i}^{m}\left[\left(A_{ij}x_j(k)\right)^T A_{ij}x_j(k) + \left(A_{is}x_s(k)\right)^T A_{is}x_s(k)\right]$$

$$\le \frac{\mu}{2}\lambda_M(V_i)\sum_{j=1,j\neq i}^{m}\sum_{s=1,s\neq i}^{m}\left[\left(E_jx_j(k)\right)^T E_jx_j(k) + \left(E_sx_s(k)\right)^T E_sx_s(k)\right]$$

$$= \frac{\mu}{2}\lambda_M(V_i)\sum_{j=1,j\neq i}^{m}\left[(m-1)\left(E_jx_j(k)\right)^T\left(E_jx_j(k)\right) + \sum_{s=1,s\neq i}^{m}\left(E_sx_s(k)\right)^T E_sx_s(k)\right]$$

$$= \frac{\mu}{2}\lambda_M(V_i)\left[(m-1)\sum_{j=1,j\neq i}^{m}\left(E_jx_j(k)\right)^T\left(E_jx_j(k)\right) + (m-1)\sum_{s=1,s\neq i}^{m}\left(E_sx_s(k)\right)^T E_sx_s(k)\right]$$

$$= \frac{\mu}{2}\lambda_M(V_i)\left[2(m-1)\sum_{j=1,j\neq i}^{m}\left(E_jx_j(k)\right)^T\left(E_jx_j(k)\right)\right] = (m-1)\mu\lambda_M(V_i)\sum_{j=1,j\neq i}^{m}\left(E_jx_j(k)\right)^T\left(E_jx_j(k)\right)$$

Thus

$$\Delta v_i\left[E_i x_i(k)\right]\Big|_{(6)} \le -2\left[E_ix_i(k)\right]^T W_i\left[E_ix_i(k)\right] + \left[E_ix_i(k)\right]^T V_i\left[E_ix_i(k)\right] + 2(m-1)\mu\lambda_M(V_i)\sum_{j=1,j\neq i}^{m}\left[E_jx_j\right]^T\left[E_jx_j\right]$$

Here $\lambda_M(V_i)$ denotes the maximum eigenvalue of matrix V_i. Thereby，

$$\Delta v[Ex(k)]|_{(6)}=\sum_{i=1}^{m}\Delta v_i[E_ix_i(k)]|_{(6)}\le-\sum_{i=1}^{m}\left((E_ix_i(k)\right)^T\left(2W_i-V_i-2(m-1)^2\mu\lambda_M I_i\right)\left(E_ix_i(k)\right)$$

Noticing that

$$2W_i-V_i-2(m-1)^2\mu\lambda_M I_i>0(i=1,2,\cdots,m)$$

by using Lemma5, we know, $\lim_{k\to\infty}V(Ex)=0$, therefore $\lim_{k\to\infty}E_ix_i(k)=0\ (i=1,\cdots m)$.

To prove $z_i(k)=Q_i^{-1}x_i(k)=\begin{bmatrix}z_i^{(1)}(k)\\ z_i^{(2)}(k)\end{bmatrix}\to 0(k\to\infty)$. By noticing that systems (7) are regular and causal, there exists reversible ，matrices P_i ，$Q_i\ (i=1,\cdots m)$ which satisfy

$$P_iE_iQ_i=\begin{bmatrix}I_i^{(1)}&0\\0&0\end{bmatrix},\ P_i(A_{ii}-B_iK_i)Q_i=\begin{bmatrix}M_i&0\\0&I_i^{(2)}\end{bmatrix}$$

Let

$$P_iA_{ij}Q_j=\begin{bmatrix}A_{ij}^{(1)}&A_{ij}^{(12)}\\A_{ij}^{(21)}&A_{ij}^{(2)}\end{bmatrix}(i,j=1,\cdots,m,i\ne j)$$

$$z_i(k)=Q_i^{-1}x_i(k)=\begin{bmatrix}z_i^{(1)}(k)\\ z_i^{(2)}(k)\end{bmatrix},$$

where $I_i^{(1)}$, $I_i^{(2)}$ is $r_i\times r_i$ and $(n_i-r_i)\times(n_i-r_i)$ identity matrix, respectively. P_i ，Q_i ，M ，$A_{ij}^{(1)}$ ，$A_{ij}^{(12)}$ ，$A_{ij}^{(21)}$ ，$A_{ij}^{(2)}$ are corresponding dimension constant matrices. Thus the singular closed-loop large-scale systems (6) are equivalent to

$$\begin{cases}z_i^{(1)}(k+1)=Mz_i^{(1)}(k)+\sum\limits_{j=1,j\ne i}^{m}\left(A_{ij}^{(1)}z_j^{(1)}(k)+A_{ij}^{(12)}z_j^{(2)}(k)\right)\\ 0=z_i^{(2)}(k)+\sum\limits_{j=1,j\ne i}^{m}\left(A_{ij}^{(21)}z_j^{(1)}(k)+A_{ij}^{(2)}z_j^{(2)}(k)\right)\end{cases}$$

Noticing that

$$P_iE_ix_i=P_iE_iQ_iQ_i^{-1}x_i=P_iE_iQ_iz_i(k)=\begin{pmatrix}z_i^{(1)}(k)\\0\end{pmatrix}$$

we have $z_i^{(1)}(k)=\begin{pmatrix}I_i^{(1)}&0\end{pmatrix}P_iE_ix_i$. $\lim_{k\to\infty}z_i^{(1)}(k)=0$ holds from $\lim_{k\to\infty}E_ix_i(k)=0$.

Noticing that

$$P_iA_{ij}x_j(k)=\begin{pmatrix}A_{ij}^{(1)}&A_{ij}^{(12)}\\A_{ij}^{(21)}&A_{ij}^{(2)}\end{pmatrix}\begin{pmatrix}z_j^{(1)}(k)\\z_j^{(2)}(k)\end{pmatrix}$$

we have

$$\left(A_{ij}^{(21)}z_j^{(1)}(k)+A_{ij}^{(2)}z_j^{(2)}(k)\right)=\begin{pmatrix}0&I_i^{(2)}\end{pmatrix}P_iA_{ij}x_j(k).$$

Noticing that $\lim_{k\to\infty} A_{ij}x_j(k)=0$ holds from (8), that is $\lim_{k\to\infty}\left(A_{ij}^{(21)}z_j^{(1)}(k)+A_{ij}^{(2)}z_j^{(2)}(k)\right)=0$, so $\lim_{k\to\infty} z_i^{(2)}(k)=0$.

Hence, $\lim_{k\to\infty} z_i(k)=0$, that is $\lim_{k\to\infty} x(k)=0$. The theorem 1 is proved.

Theorem 2: Assume that all subsystems (4) of system (1) are R-controllable, all closed-loop isolated systems (7) are regular, causal, and given positive definite matrix W_i, there exists a positive semi-definite matrix V_i which satisfies

$$\left(A_{ii}-B_iK_i\right)^T V\left(A_{ii}-B_iK_i\right)-E_i^TV_iE_i=E_i^TW_iE_i \qquad (i=1,\cdots m),$$

if there exists a real number $\mu>0$ which satisfies

$$\left[A_{ij}x_j(k)\right]^T\left[A_{ij}x_j(k)\right]\le\mu\left[E_jx_j(k)\right]^T\left[E_jx_j(k)\right] \quad (i,j=1,\cdots m, i\ne j) \qquad (10)$$

then when

$$W_i-V_i-4(m-1)^2\mu\lambda_M I_i>0 \;(i=1,\cdots m) \qquad (11)$$

the zero solution of the discrete singular closed-loop large-scale systems(6) are unstable, the discrete singular large-scale control systems(1) are not stabilizable.

Proving is similar with Theorem 1. Here taking $\varepsilon=\frac{1}{2}$ (in Lemma 3).

3. EXAMPLE

Consider the following 5-order discrete singular large-scale control system which consists of two sub-systems

$$E_ix_i(k+1)=A_{ii}x_i(k)+\sum_{j=1,j\ne i}^{2}A_{ij}x_j(k)+B_iU_i(k) \quad (i=1,2) \qquad (12)$$

where

$$E_1=\begin{pmatrix}1&0&0\\0&0&1\\0&0&0\end{pmatrix},\quad E_2=\begin{pmatrix}1&0\\0&0\end{pmatrix},\quad B_1=\begin{pmatrix}0&1\\0&1\\1&0\end{pmatrix},\quad B_2=\begin{pmatrix}0\\ \frac{1}{2}\end{pmatrix},$$

$$A_{11}=\begin{pmatrix}-\frac{1}{2}&0&0\\0&0&\frac{1}{2}\\0&1&0\end{pmatrix},\quad A_{12}=\begin{pmatrix}\frac{1}{4}&0\\ \frac{1}{5}&0\\0&0\end{pmatrix},\quad A_{21}=\begin{pmatrix}\frac{1}{6}&0&\frac{1}{6}\\-\frac{1}{5}&0&\frac{1}{5}\end{pmatrix},\quad A_{22}=\begin{pmatrix}\frac{1}{2}&0\\0&1\end{pmatrix}.$$

We choose the control law

$$U_i(k)=-K_ix_i(k)\;(i=1,2),$$

here $K_1 = \begin{pmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix}$, $K_2 = \begin{pmatrix} \frac{1}{2} & 0 \end{pmatrix}$, and $W_1 = I_3$, $W_2 = I_2$, $\mu = \frac{1}{8}$, then $V_1 = \begin{pmatrix} \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $V_2 = \begin{pmatrix} \frac{4}{3} & 0 \\ 0 & 0 \end{pmatrix}$.

it is easy to test that (8) and (9) are held, then we know this system (12) is stabilizable from Theorem 1.

4. CONCLUSION

In this paper, the state feedback stabilization of discrete singular large-scale control systems is investigated by using generalized Lyapunov function method. According to the bound limit parameter of interconnecting terms, there gives some sufficient conditions for determining the asymptotical stability and unstability of the singular closed-loop large-scale system while the subsystems are regular, causal, and R-controllable.

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