

Studies in Mathematical Sciences  
Vol. 1, No. 1, 2010, pp. 38-44  
[www.cscanada.org](http://www.cscanada.org)

ISSN 1923-8444 [Print]  
ISSN 1923-8452 [Online]  
[www.cscanada.net](http://www.cscanada.net)

## Soliton Solutions of the Kaup-Kupershmidt and Sawada-Kotera Equations

Sami Shukri<sup>1,\*</sup>

**Abstract:** In this paper I seek soliton solutions of two-component generalizations of the Kaup-Kupershmidt and Sawada-Kotera equations, for this purpose I will apply the extended tanh method. The extended tanh method with a computerized symbolic computation, is used for constructing the travelling wave solutions of coupled nonlinear equations arising in physics. The obtained solutions include soliton, kink and plane periodic solutions.

**Key Words:** Soliton Solutions; Kaup-Kupershmidt Equation; Sawada-Kotera Equation

### 1. INTRODUCTION

Nonlinear wave equations in mathematical physics play a major role in various fields, such as plasma physics, fluid mechanics, optical fibers, solid state physics, chemical kinetics, geochemistry, and so on [1]. The pioneer work of Malfiet in [2] introduced the powerful tanh method for a reliable treatment of the nonlinear wave equations. Later, the extended tanh method, developed by Wazwaz [3], is a direct and effective algebraic method for handling nonlinear equations. Various extensions of the method were developed as well [4].

The equations solvable by the extended tanh method possess moreover a particularly interesting class of solutions; solitons. Recently, Sami Shukri and Kamel Al-Khaled in [5] have solved Generalized coupled Hirota Satsuma KdV equation and obtained the solitons solutions. The solitons are traveling waves that preserve their shape after a collision with other solitons. This property is used in many applications; from hydrodynamics to nonlinear optics, from plasmas to shock waves, from tornados to the Great Red Spot of Jupiter, from traffic flow to internet, and from Tsunamis to turbulence [1]. More recently, solitons are of key importance in the quantum fields and nanotechnology especially in nanohydrodynamics [6].

Many mathematicians wrote papers on the Kaup-Kupershmidt equation and Sawada-Kotera equation , see for example [7–12]. In this paper I solve two-component generalizations of the Kaup-Kupershmidt and Sawada-Kotera equations by the extended tanh method to obtain soliton solutions.

### 2. THE EXTENDED TANH METHOD

---

<sup>1</sup>King Saud University Riyadh 11451, Kingdom of Saudi Arabia.

\*Corresponding author. E-mail address: samishukri@yahoo.com.

†Received 1 November 2010; accepted 23 November 2010.

Wazwaz in [3] has summarized for using extended tanh method. A partial differential equation

$$P(u, u_t, u_x, u_{xx}, \dots) = 0. \tag{1}$$

can be converted to an ordinary differential equation

$$Q(U, U', U''', \dots) = 0. \tag{2}$$

Upon using the wave variable  $\xi = x - \beta t$ . Equation (2) is then integrated as long as all terms contain derivatives where integration constants are considered zeros. Introducing a new independent variable

$$Y = \tanh(\xi), \quad \xi = x - \beta t, \tag{3}$$

leads to change of derivatives:

$$\begin{aligned} \frac{d}{d\xi} &= (1 - Y^2) \frac{d}{dY}, \\ \frac{d^2}{d\xi^2} &= -2Y(1 - Y^2) \frac{d}{dY} + (1 - Y^2)^2 \frac{d^2}{dY^2}, \\ \frac{d^3}{d\xi^3} &= 2(1 - Y^2)(3Y^2 - 1) \frac{d}{dY} - 6Y(1 - Y^2)^2 \frac{d^2}{dY^2} + (1 - Y^2)^3 \frac{d^3}{dY^3}, \\ \frac{d^4}{d\xi^4} &= -8Y(1 - Y^2)(3Y^2 - 2) \frac{d}{dY} + 4(1 - Y^2)^2(9Y^2 - 2) \frac{d^2}{dY^2} \\ &\quad - 12Y(1 - Y^2)^3 \frac{d^3}{dY^3} + (1 - Y^2)^4 \frac{d^4}{dY^4}, \\ \frac{d^5}{d\xi^5} &= -8(-1 + Y^2)(2 - 15Y^2 + 15Y^4) \frac{d}{dY} - 120Y(-1 + Y^2)^2(-1 + 2Y^2) \frac{d^2}{dY^2} \\ &\quad - 20(-1 + Y^2)^3(-1 + 6Y^2) \frac{d^3}{dY^3} - 20Y(Y^2 - 1)^4 \frac{d^4}{dY^4} + (1 - Y^2)^5 \frac{d^5}{dY^5}. \end{aligned} \tag{4}$$

The extended tanh method admits the use of the finite expansion

$$U(\xi) = S(Y) = \sum_{k=0}^M a_k Y^k + \sum_{k=1}^N b_k Y^{-k}, \tag{5}$$

where  $m$  is usually obtained by balancing the linear terms of the highest order in the resulting equation, with the highest order nonlinear terms in equation (2). With  $m$  determined, equate the coefficients of powers of  $Y$  in the resulting of equation (2). This will give a system of algebraic equations involving the unknowns  $a_k, b_k$ , for  $k = 0, \dots, m$ . Determining these parameters, knowing that  $m$  is a positive integer (in most cases) and using equation (5) we obtain an analytic solution in a closed form for equations (5) and (2).

### 3. TWO-COMPONENT GENERALIZATIONS OF THE KAUP-KUPERSHMIDT AND SAWADA-KOTERA EQUATIONS

Consider the two-component generalizations of the Kaup-Kupershmidt and Sawada-Kotera equations [13]

$$\begin{aligned} u_t &= u_{xxxxx} - 10uu_{xxx} + 30v_{xxx} - 25u_x u_{xx} + 45v_x v_{xx} \\ &\quad + 20u^2 u_x - 30v^2 v_x - 60uvv_x, \\ v_t &= -9v_{xxxxx} + 10vu_{xxx} + 30uv_{xxx} + 35v_x u_{xx} + 45u_x v_{xx} \\ &\quad - 20uvu_x - 20u^2 v_x - 30v^2 v_x. \end{aligned} \tag{6}$$

Using the wave variable  $\xi = x - ct$  carries equation (6) into the ordinary differential equation

$$\begin{aligned} -cu' &= u^{(5)} - 10uu''' + 30v''' - 25u'u'' + 45v'v'' \\ &\quad + 20u^2 u' - 30v^2 v' - 60uvv', \\ -cv' &= -9v^{(5)} + 10vu''' + 30uv''' + 35v'u'' + 45u'v'' \\ &\quad - 20uvu' - 20u^2 v' - 30v^2 v'. \end{aligned} \tag{7}$$

Balancing  $u^{(5)}$  with  $uvv'$ ,  $v^{(5)}$  with  $uvu'$  in equation (7) gives

$$\begin{aligned} N + 5 &= N + 2M + 1, \\ M + 5 &= 2N + M + 1. \end{aligned} \tag{8}$$

So that  $N = M = 2$ .

The extended tanh method admits the use of the finite expansion

$$\begin{aligned} u &= a_0 + a_1Y + a_2Y^2 + \frac{b_1}{Y} + \frac{b_2}{Y^2}, \\ v &= c_0 + c_1Y + c_2Y^2 + \frac{d_1}{Y} + \frac{d_2}{Y^2}. \end{aligned} \tag{9}$$

Substituting (9) into (7), and collecting the coefficients of  $Y$  we obtain a system of algebraic equations for  $a_0, a_1, a_2, b_1, b_2, c_0, c_1, c_2, d_1, d_2$  as follows:

$$\begin{aligned} &-16a_1 - ca_1 - 20a_0a_1 - 20a_0^2a_1 + 50a_1a_2 - 16b_1 - cb_1 - 20a_0 \\ &-20a_0^2b_1 - 20a_1^2b_1 - 30a_2b_1 - 40a_0a_2b_1 - 20a_1b_1^2 + 30a_1b_2 \\ &-40a_0a_1b_2 - 40a_1a_2b_2 + 50b_1 - 40a_2b_1b_2 + 30a_1c_0^2 + 30b_1c_0^2 \\ &+ 60c_0c_1 + 60a_0c_0c_1 + 60b_2c_0c_1 + 30b_1c_1^2 + 60b_1c_0c_2 - 90c_1c_2 \\ &+ 60b_2c_1c_2 + 60c_0d_1 + 60a_0c_0d_1 + 60a_2c_0d_1 + 60a_1c_1d_1 \\ &+ 60b_1c_1d_1 + 150c_2d_1 + 60a_0c_2d_1 + 60b_2c_2d_1 + 30a_1d_1^2 \\ &+ 60a_1c_0d_2 + 150c_1d_2 + 60a_0c_1d_2 + 60a_2c_1d_2 + 60a_1c_2d_2 \\ &+ 60b_1c_2d_2 - 90d_1d_2 + 60a_2d_1d_2 = 0, \end{aligned} \tag{10}$$

$$\begin{aligned} &-70a_1^2 - 40a_0a_1^2 - 272a_2 - 2ca_2 - 160a_0a_2 - 40a_0^2a_2 + 100a_2^2 \\ &+ 10a_1b_1 - 80a_1a_2b_1 + 40a_2b_2 - 40a_2^2b_2 + 60a_2c_0^2 + 120a_1c_0c_1 \\ &+ 150c_1^2 + 60a_0c_1^2 + 480c_0c_2 + 120a_0c_0c_2 + 120b_1c_1c_2 - 180c_2^2 \\ &+ 60b_2c_2^2 - 90c_1d_1 + 120a_2c_1d_1 + 120a_1c_2d_1 - 360c_2d_2 + 120a_2c_2d_2 = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} &136a_1 + ca_1 + 80a_0a_1 + 20a_0^2a_1 - 20a_1^3 - 530a_1a_2 - 120a_0a_1a_2 \\ &+ 20a_1^2b_1 + 130a_2b_1 + 40a_0a_2b_1 - 60a_2^2b_1 - 10a_1b_2 + 40a_1a_2b_2 \\ &- 30a_1c_0^2 - 240c_0c_1 - 60a_0c_0c_1 + 180a_2c_0c_1 + 90a_1c_1^2 - 30b_1c_1^2 \\ &+ 180a_1c_0c_2 - 60b_1c_0c_2 + 1170c_1c_2 + 180a_0c_1c_2 - 60b_2c_1c_2 \\ &+ 90b_1c_2^2 - 60a_2c_0d_1 - 60a_1c_1d_1 - 690c_2d_1 - 60a_0c_2d_1 \\ &+ 180a_2c_2d_1 + 90c_1d_2 - 60a_2c_1d_2 - 60a_1c_2d_2 = 0, \end{aligned} \tag{12}$$

$$\begin{aligned} &180a_1^2 + 40a_0a_1^2 + 1232a_2 + 2ca_2 + 400a_0a_2 + 40a_0^2a_2 - 80a_1^2a_2 \\ &- 660a_2^2 - 80a_0a_2^2 - 10a_1b_1 + 80a_1a_2b_1 - 40a_2b_2 + 40a_2^2b_2 \\ &- 60a_2c_0^2 - 120a_1c_0c_1 - 420c_1^2 - 60a_0c_1^2 + 120a_2c_1^2 - 1200c_0c_2 \\ &- 120a_0c_0c_2 + 240a_2c_0c_2 + 240a_1c_1c_2 - 120b_1c_1c_2 + 1380c_2^2 \\ &+ 120a_0c_2^2 - 60b_2c_2^2 + 90c_1d_1 - 120a_2c_1d_1 - 120a_1c_2d_1 \\ &+ 360c_2d_2 - 120a_2c_2d_2 = 0, \end{aligned} \tag{13}$$

$$\begin{aligned} &-240a_1 - 60a_0a_1 + 20a_1^3 + 1030a_1a_2 + 120a_0a_1a_2 - 100a_1a_2^2 \\ &- 90a_2b_1 + 60a_2^2b_1 + 180c_0c_1 - 180a_2c_0c_1 - 90a_1c_1^2 - 180a_1c_0c_2 \\ &- 2430c_1c_2 - 180a_0c_1c_2 + 300a_2c_1c_2 + 150a_1c_2^2 - 90b_1c_2^2 \\ &+ 450c_2d_1 - 180a_2c_2d_1 = 0, \end{aligned} \tag{14}$$

$$\begin{aligned} &-110a_1^2 - 1680a_2 - 240a_0a_2 + 80a_1^2a_2 + 1100a_2^2 + 80a_0a_2^2 \\ &- 40a_2^3 + 270c_1^2 - 120a_2c_1^2 + 720c_0c_2 - 240a_2c_0c_2 \end{aligned}$$

$$-240a_1c_1c_2 - 2460c_2^2 - 120a_0c_2^2 + 180a_2c_2^2 = 0, \quad (15)$$

$$120a_1 - 550a_1a_2 + 100a_1a_2^2 + 1350c_1c_2 - 300a_2c_1c_2 - 150a_1c_2^2 = 0, \quad (16)$$

$$720a_2 - 540a_2^2 + 40a_2^3 + 1260c_2^2 - 180a_2c_2^2 = 0, \quad (17)$$

$$\begin{aligned} &10a_1b_1 - 70b_1^2 - 40a_0b_1^2 - 272b_2 - 2cb_2 - 160a_0b_2 - 40a_0^2b_2 \\ &+ 40a_2b_2 - 80a_1b_1b_2 + 100b_2^2 - 40a_2b_2^2 + 60b_2c_0^2 + 120b_1c_0d_1 \\ &- 90c_1d_1 + 120b_2c_1d_1 + 150d_1^2 + 60a_0d_1^2 + 480c_0d_2 + 120a_0c_0d_2 \\ &+ 120b_1c_1d_2 - 360c_2d_2 + 120b_2c_2d_2 + 120a_1d_1d_2 - 180d_2^2 + 60a_2d_2^2 = 0. \end{aligned} \quad (18)$$

Also in the same manner, we get

$$\begin{aligned} &136b_1 + cb_1 + 80a_0b_1 + 20a_0^2b_1 - 10a_2b_1 + 20a_1b_1^2 - 20b_1^3 \\ &+ 130a_1b_2 + 40a_0a_1b_2 - 530b_1b_2 - 120a_0b_1b_2 + 40a_2b_1b_2 \\ &60a_1b_2^2 - 30b_1c_0^2 - 60b_2c_0c_1 - 240c_0d_1 - 60a_0c_0d_1 \\ &+ 180b_2c_0d_1 - 60b_1c_1d_1 + 90c_2d_1 - 60b_2c_2d_1 - 30a_1d_1^2 \\ &+ 90b_1d_1^2 - 60a_1c_0d_2 + 180b_1c_0d_2 - 690c_1d_2 - 60a_0c_1d_2 \\ &+ 180b_2c_1d_2 - 60b_1c_2d_2 + 1170d_1d_2 + 180a_0d_1d_2 \\ &60a_2d_1d_2 + 90a_1d_2^2 = 0, \end{aligned} \quad (19)$$

$$\begin{aligned} &-10a_1b_1 + 180b_1^2 + 40a_0b_1^2 + 1232b_2 + 2cb_2 + 400a_0b_2 + 40a_0^2b_2 \\ &40a_2b_2 + 80a_1b_1b_2 - 80b_1^2b_2 - 660b_2^2 - 80a_0b_2^2 + 40a_2b_2^2 \\ &60b_2c_0^2 - 120b_1c_0d_1 + 90c_1d_1 - 120b_2c_1d_1 - 420d_1^2 - 60a_0d_1^2 \\ &+ 120b_2d_1^2 - 1200c_0d_2 - 120a_0c_0d_2 + 240b_2c_0d_2 - 120b_1c_1d_2 \\ &+ 360c_2d_2 - 120b_2c_2d_2 - 120a_1d_1d_2 + 240b_1d_1d_2 \\ &+ 1380d_2^2 + 120a_0d_2^2 - 60a_2d_2^2 = 0, \end{aligned} \quad (20)$$

$$\begin{aligned} &-240b_1 - 60a_0b_1 + 20b_1^3 - 90a_1b_2 + 1030b_1b_2 + 120a_0b_1b_2 \\ &+ 60a_1b_2^2 - 100b_1b_2^2 + 180c_0d_1 - 180b_2c_0d_1 - 90b_1d_1^2 \\ &180b_1c_0d_2 + 450c_1d_2 - 180b_2c_1d_2 - 2430d_1d_2 - 180a_0d_1d_2 \\ &+ 300b_2d_1d_2 - 90a_1d_2^2 + 150b_1d_2^2 = 0, \end{aligned} \quad (21)$$

$$\begin{aligned} &-110b_1^2 - 1680b_2 - 240a_0b_2 + 80b_1^2b_2 + 1100b_2^2 + 80a_0b_2^2 \\ &40b_2^3 + 270d_1^2 - 120b_2d_1^2 + 720c_0d_2 - 240b_2c_0d_2 - 240b_1d_1d_2 \\ &2460d_2^2 - 120a_0d_2^2 + 180b_2d_2^2 = 0, \end{aligned} \quad (22)$$

$$120b_1 - 550b_1b_2 + 100b_1b_2^2 + 1350d_1d_2 - 300b_2d_1d_2 - 150b_1d_2^2 = 0, \quad (23)$$

$$720b_2 - 540b_2^2 + 40b_2^3 + 1260d_2^2 - 180b_2d_2^2 = 0, \quad (24)$$

$$\begin{aligned} &20a_1c_0 + 20a_0a_1c_0 + 20b_1c_0 + 20a_0b_1c_0 + 20a_2b_1c_0 \\ &+ 20a_1b_2c_0 + 144c_1 - cc_1 + 60a_0c_1 + 20a_0^2c_1 - 70a_2c_1 + 40a_1b_1c_1 \\ &70b_2c_1 + 40a_2b_2c_1 + 30c_0^2c_1 - 90a_1c_2 + 230b_1c_2 + 60a_0b_1c_2 \\ &+ 60a_1b_2c_2 - 20b_1b_2c_2 + 144d_1 - cd_1 + 60a_0d_1 + 20a_0^2d_1 - 70a_2d_1 \\ &+ 40a_1b_1d_1 - 70b_2d_1 + 40a_2b_2d_1 + 30c_1^2d_1 + 60c_0c_2d_1 - 30c_1d_1^2 \\ &+ 230a_1d_2 + 60a_0a_1d_2 - 20a_1a_2d_2 - 90b_1d_2 + 60a_2b_1d_2 \\ &60c_0c_1d_2 + 60c_1c_2d_2 = 0, \end{aligned} \quad (25)$$

$$\begin{aligned} &20a_1^2c_0 + 160a_2c_0 + 40a_0a_2c_0 + 240a_1c_1 + 60a_0a_1c_1 - 110b_1c_1 \\ &- 20a_0b_1c_1 + 60a_2b_1c_1 - 20a_1b_2c_1 + 60c_0c_1^2 + 2448c_2 - 2cc_2 \\ &+ 480a_0c_2 + 40a_0^2c_2 - 320a_2c_2 + 80a_1b_1c_2 - 20b_1^2c_2 - 480b_2c_2 \end{aligned}$$

$$\begin{aligned}
 &40a_0b_2c_2 + 80a_2b_2c_2 + 60c_0^2c_2 + 30a_1d_1 + 20a_0a_1d_1 + 20a_1a_2d_1 \\
 &+ 20a_2b_1d_1 - 60c_0c_1d_1 + 120c_1c_2d_1 + 20a_1^2d_2 + 160a_2d_2 \\
 &+ 40a_0a_2d_2 - 60c_1^2d_2 + 60c_2^2d_2 = 0,
 \end{aligned} \tag{26}$$

$$\begin{aligned}
 &-80a_1c_0 - 20a_0a_1c_0 + 60a_1a_2c_0 - 20a_2b_1c_0 - 1224c_1 + cc_1 - 240a_0c_1 \\
 &20a_0^2c_1 + 40a_1^2c_1 + 750a_2c_1 + 80a_0a_2c_1 - 40a_1b_1c_1 + 70b_2c_1 \\
 &40a_2b_2c_1 - 30c_0^2c_1 + 30c_1^3 + 1090a_1c_2 + 100a_0a_1c_2 - 690b_1c_2 \\
 &60a_0b_1c_2 + 100a_2b_1c_2 - 60a_1b_2c_2 + 180c_0c_1c_2 - 30a_2d_1 \\
 &+ 20a_2^2d_1 - 60c_1^2d_1 + 90c_2^2d_1 + 10a_1d_2 + 20a_1a_2d_2 - 60c_1c_2d_2 = 0.
 \end{aligned} \tag{27}$$

Also in the same manner, we get

$$\begin{aligned}
 &-20a_1^2c_0 - 400a_2c_0 - 40a_0a_2c_0 + 40a_2^2c_0 - 640a_1c_1 - 60a_0a_1c_1 \\
 &+ 100a_1a_2c_1 + 90b_1c_1 - 60a_2b_1c_1 - 60c_0c_1^2 - 11088c_2 + 2cc_2 \\
 &- 1200a_0c_2 - 40a_0^2c_2 + 60a_1^2c_2 + 2240a_2c_2 + 120a_0a_2c_2 - 80a_1b_1c_2 \\
 &+ 320b_2c_2 - 80a_2b_2c_2 + 120c_1^2c_2 + 120c_0c_2^2 - 10a_1d_1 \\
 &- 20a_1a_2d_1 - 60c_1c_2d_1 = 0,
 \end{aligned} \tag{28}$$

$$\begin{aligned}
 &60a_1c_0 - 60a_1a_2c_0 + 2160c_1 + 180a_0c_1 - 40a_1^2c_1 - 1490a_2c_1 \\
 &- 80a_0a_2c_1 + 60a_2^2c_1 - 30c_1^3 - 2190a_1c_2 - 100a_0a_1c_2 \\
 &+ 140a_1a_2c_2 + 450b_1c_2 - 100a_2b_1c_2 - 60c_0c_1c_2 \\
 &+ 150c_1c_2^2 + 30a_2d_1 - 20a_2^2d_1 = 0,
 \end{aligned} \tag{29}$$

$$\begin{aligned}
 &240a_2c_0 - 40a_2^2c_0 + 400a_1c_1 - 100a_1a_2c_1 + 15120c_2 + 720a_0c_2 - 60a_1^2c_2 \\
 &- 3840a_2c_2 - 120a_0a_2c_2 + 80a_2^2c_2 - 60c_1^2c_2 + 60c_2^3 = 0,
 \end{aligned} \tag{30}$$

$$-1080c_1 + 810a_2c_1 - 60a_2^2c_1 + 1190a_1c_2 - 140a_1a_2c_2 - 30c_1c_2^2 = 0, \tag{31}$$

$$-6480c_2 + 1920a_2c_2 - 80a_2^2c_2 = 0, \tag{32}$$

$$\begin{aligned}
 &20b_1^2c_0 + 160b_2c_0 + 40a_0b_2c_0 + 30b_1c_1 + 20a_0b_1c_1 + 20a_1b_2c_1 \\
 &+ 20b_1b_2c_1 + 20b_1^2c_2 + 160b_2c_2 + 40a_0b_2c_2 - 110a_1d_1 - 20a_0a_1d_1 \\
 &+ 240b_1d_1 + 60a_0b_1d_1 - 20a_2b_1d_1 + 60a_1b_2d_1 + 2448d_2 - 2cd_2 + 480a_0d_2 \\
 &+ 40a_0^2d_2 - 20a_1^2d_2 - 480a_2d_2 - 40a_0a_2d_2 + 80a_1b_1d_2 - 320b_2d_2 \\
 &+ 80a_2b_2d_2 - 60c_1d_1d_2 = 0,
 \end{aligned} \tag{33}$$

$$\begin{aligned}
 &-80b_1c_0 - 20a_0b_1c_0 - 20a_1b_2c_0 + 60b_1b_2c_0 - 30b_2c_1 + 20b_2^2c_1 \\
 &+ 10b_1c_2 + 20b_1b_2c_2 - 1224d_1 + cd_1 - 240a_0d_1 - 20a_0^2d_1 + 70a_2d_1 \\
 &- 40a_1b_1d_1 + 40b_1^2d_1 + 750b_2d_1 + 80a_0b_2d_1 - 40a_2b_2d_1 - 30c_0^2d_1 \\
 &- 30c_1d_1^2 - 690a_1d_2 - 60a_0a_1d_2 + 1090b_1d_2 + 100a_0b_1d_2 - 60a_2b_1d_2 \\
 &+ 100a_1b_2d_2 - 60c_0c_1d_2 - 60c_2d_1d_2 - 30c_1d_2^2 = 0,
 \end{aligned} \tag{34}$$

$$\begin{aligned}
 &-20b_1^2c_0 - 400b_2c_0 - 40a_0b_2c_0 + 40b_2^2c_0 - 10b_1c_1 - 20b_1b_2c_1 \\
 &+ 90a_1d_1 - 640b_1d_1 - 60a_0b_1d_1 - 60a_1b_2d_1 + 100b_1b_2d_1 - 60c_0d_1^2 \\
 &- 11088d_2 + 2cd_2 - 1200a_0d_2 - 40a_0^2d_2 + 320a_2d_2 - 80a_1b_1d_2 + 60b_1^2d_2 \\
 &+ 2240b_2d_2 + 120a_0b_2d_2 - 80a_2b_2d_2 - 60c_0^2d_2 - 120c_1d_1d_2 - 60c_2d_2^2 = 0,
 \end{aligned} \tag{35}$$

$$\begin{aligned}
 &60b_1c_0 - 60b_1b_2c_0 + 30b_2c_1 - 20b_2^2c_1 + 2160d_1 + 180a_0d_1 - 40b_1^2d_1 \\
 &- 1490b_2d_1 - 80a_0b_2d_1 + 60b_2^2d_1 - 30d_1^3 + 450a_1d_2 - 2190b_1d_2 - 100a_0b_1d_2 \\
 &- 100a_1b_2d_2 + 140b_1b_2d_2 - 180c_0d_1d_2 - 90c_1d_2^2 = 0,
 \end{aligned} \tag{36}$$

$$240b_2c_0 - 40b_2^2c_0 + 400b_1d_1 - 100b_1b_2d_1 + 15120d_2 + 720a_0d_2 - 60b_1^2d_2$$

$$-3840b_2d_2 - 120a_0b_2d_2 + 80b_2^2d_2 - 120d_1^2d_2 - 120c_0d_2^2 = 0, \tag{37}$$

$$-1080d_1 + 810b_2d_1 - 60b_2^2d_1 + 1190b_1d_2 - 140b_1b_2d_2 - 150d_1d_2^2 = 0, \tag{38}$$

$$-6480d_2 + 1920b_2d_2 - 80b_2^2d_2 - 60d_2^3 = 0.$$

The above equations are cumbersome to solve. Using a modern computer algebra system, say *Mathematica*, we obtain the two sets of solutions

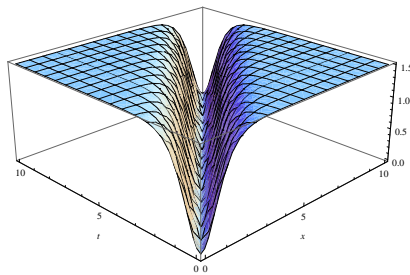
- The first set:  
 $a_0 = 0, a_1 = 0, a_2 = \frac{3}{2}, b_1 = 0, b_2 = 0, c_0 = 0, c_1 = -\sqrt{2}, c_2 = 0, d_1 = 0, d_2 = 0.$
- The second set:  
 $a_0 = 0, a_1 = 0, a_2 = \frac{3}{2}, b_1 = 0, b_2 = 0, c_0 = 0, c_1 = \sqrt{2}, c_2 = 0, d_1 = 0, d_2 = 0.$

In view of this, we obtain the following soliton and kink solutions:

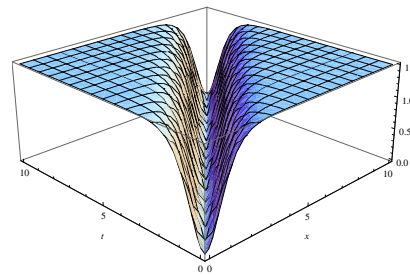
$$u_1(x, t) = \frac{3}{2} \tanh^2(x - ct), \quad v_1(x, t) = -\sqrt{2} \tanh(x - ct), \tag{39}$$

$$u_2(x, t) = \frac{3}{2} \tanh^2(x - ct), \quad v_2(x, t) = \sqrt{2} \tanh(x - ct). \tag{40}$$

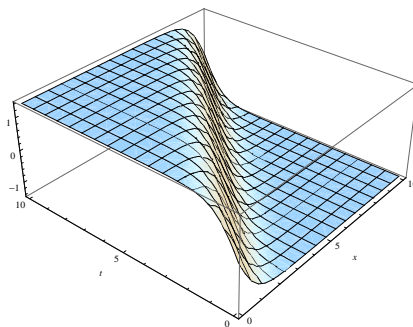
In Figures (1) and (2), the soliton solutions have wings, whereas the kink solutions in Figures (3) and (4) have no wings.



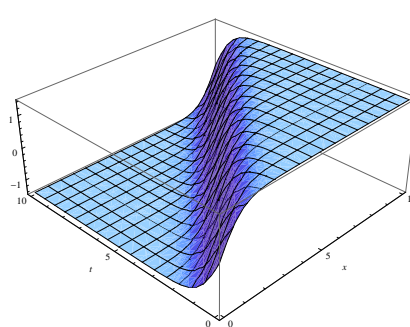
**Figure 1:** The soliton solution  $u_1(x, t)$  of equation (6).



**Figure 2:** The soliton solution  $u_2(x, t)$  of equation (6).



**Figure 3:** The kink solution  $v_1(x, t)$  of equation (6).



**Figure 4:** The kink solution  $v_2(x, t)$  of equation (6).

## 4. CONCLUSIONS

The extended tanh method was successfully used to obtain abundant solitary wave solutions, mostly soliton and kink solutions of two-component generalizations of the Kaup-Kupershmidt and Sawada-Kotera equations.

## ACKNOWLEDGEMENT

The author is grateful for Professor Abdul-Majid Wazwaz for his advices, help and encouragement.

## REFERENCES

- [1] Wazwaz, A. M. (2005). Nonlinear variants of KdV and KP equations with compactons, solitons and periodic solutions. *Commun. Nonl. Sci. Numer. Simul.*, 10, 451-463.
- [2] Malfliet, W., & Hereman, W. (1996). The tanh method, exact solutions of nonlinear evolution and wave equations. *Phys. Scr.*, 54, 563-568.
- [3] Wazwaz, A. M. (2007). The extended tanh method for new soliton solutions for many forms of the fifth-order KdV equations. *Appl. Math. Comput.*, 148, 1002-1014.
- [4] Bekir, A. (2008). Applications of the extended tanh method for coupled nonlinear evolution equations. *Commun. Nonl. Sci. Numer. Simul.*, 13, 1748-1757.
- [5] Shukri, S., & Al-Khaled K. (2010). The extended tanh method for solving systems of nonlinear wave equations. *Appl. Math. Comput.*, 217, 1997-2006.
- [6] Wazwaz, A. M. (2005). Generalized Boussinesq type of equations with compactons, solitons and periodic solutions. *Appl. Math. Comput.*, 167, 1162-1178.
- [7] Parker, A. (2000). On soliton solutions of the Kaup-Kupershmidt equation. I. Direct bilinearisation and solitary wave. *Physica D*, 137, 25-33.
- [8] Musette, M., & Verhoeven C. (2000). Nonlinear superposition formula for the Kaup-Kupershmidt partial differential equation. *Physica D*, 144, 211-220.
- [9] Parker, A. (2000). On soliton solutions of the Kaup-Kupershmidt equation. II. 'Anomalous' N-soliton solutions. *Physica D*, 137, 34-48.
- [10] Wazwaz, A. M. (2010). N-soliton solutions for the integrable bidirectional sixth-order Sawada-Kotera equation. *Appl. Math. Comput.*, 216, 2317-2320.
- [11] Popowicz, Z. (2009). Odd Hamiltonian structure for supersymmetric Sawada-Kotera equation. *Phys. Lett. A*, 373, 3315-3323.
- [12] Zheng, C., Si, Y. Q., & Liu, R. C. (2003). On affine Sawada-Kotera equation. *Chaos, Solitons and Fractals*, 15, 131-139.
- [13] Foursov, M. V., & Maza, M. M. (2002). On computer-assisted classification of coupled integrable equations. *J. Symb. Comput.*, 33, 647-660.