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Phase noise analysis and basic measuring techniques

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PHASE NOISE ANALYSIS AND BASIC MEASURING TECHNIQUES

BY

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ABSTRACT

Oscillators are important elements of RF and microwave communications systems. The increased use of the electro-magnetic spectrum requires the use of better and cleaner oscillators. Local oscillators in RF to IF converters play a big role in the overall performance of the receiver systems because phase noise in the local oscillator can drastically reduce the signal to noise ratio at the output of the receiver, especially if the converter is at the front end. Phase noise is the parameter used to describe oscillator stability. The analysis and basic measurement techniques of phase noise are performed in this report.

This research report starts with a description of the ideal oscillator output in both time and frequency domains. Phase noise is then described in the time, frequency, and spectral density domains. The £(f) definition of phase noise is also included, and an example showing the measurement of phase noise.

Finally, an introduction to three different measurement techniques is given.

ACKNOWLEDGEMENTS

I would like to acknowledge and thank the people who have contributed to this research report. First I would like to thank my wife, Colette, for her support and help with this report.

I would also like to express my thanks and appreciation to the members of my committee for their inputs and time spent evaluating the research report. I would like to extend a special thanks to my advisor, Dr. M. Belkerdid, for his help throughout the process of creating this research report.

Finally, I would like to dedicate this research report to my mother, Janet Lyle Kleckner, June 25, 1942 to May 2, 1988.

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CHAPTER ONE

INTRODUCTION

Communications in the electronic industry has experienced rapid growth the past several years. Sine wave oscillators are a critical component in many electronic systems. Associated with oscillators is a problem called phase noise. Typically, phase noise has had little or no exposure in engineering texts and little literature in general is available on this subject. The focus of this research report is twofold.

The first and main focus is to define and describe phase noise. This will be done by first looking at an ideal sine wave in the time and frequency domains. Following this a model of a real sine wave will be introduced and phase noise will be defined. Also at this point sources and problems of phase noise will be introduced. Finally, three different spectral densities involving phase noise will be examined.

The second focus of this research report will be to examine the basic techniques of measuring phase noise. More specifically, this will include direct spectral measurement, the frequency discriminator technique, and finally the phase lock loop technique.

CHAPTER TWO

IDEAL OSCILLATING SIGNAL

Oscillating Signal in Time Domain

An ideal or pure sine wave is shown below in Figure 1.

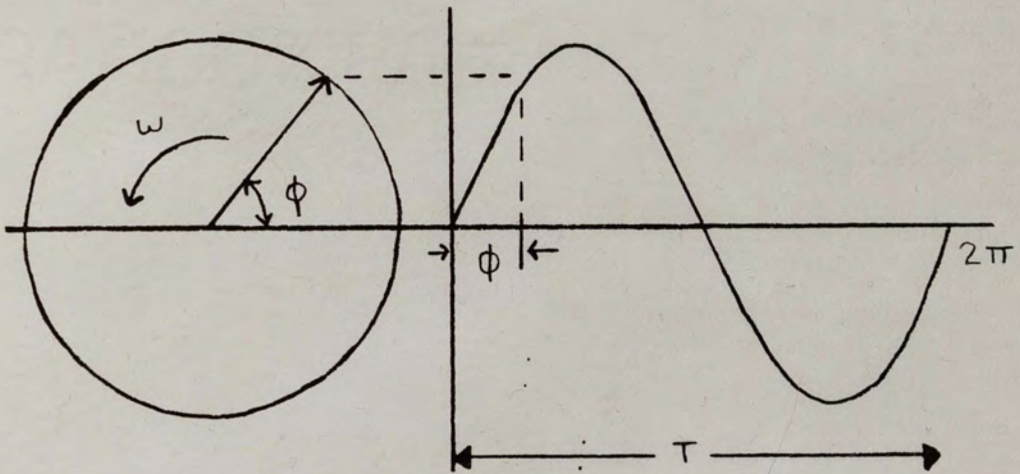

Figure 1. Ideal Oscillating Signal.

The sine wave repeats itself every T seconds. The time T is called the period of the oscillating signal. The reciprocal of the period T is the frequency of the sine wave. The dimension of frequency is cycles/second or Hertz, abbreviated Hz. The phase, ϕ shown in Figure 1 above, is the angle of the sine wave. Phase is measured in radians. One complete sine wave is 2π radians. It also represents one full revolution on the circle in Figure 1 showing the angular representation of phase and angular frequency. The phase angle is related to time in a cycle by

$$\Phi = 2\pi \frac{t}{T}$$

or by

$$\Phi = 2\pi f t$$

The angular frequency, ω has dimensions of radians/second and is related to frequency by

$$\omega = 2\pi f$$

From Figure 1 above it is seen by inspection that a change in phase occurs with time. This is expressed mathematically relating phase to frequency as

$$\frac{\Delta\Phi}{\Delta t} = \omega$$

Taking the limits of this function as $\Delta t \to 0$, yields the equation

$$\frac{d\Phi}{dt} = \omega_i \qquad (2.1)$$

Thus the instantaneous frequency is the derivative of phase with respect to time. The sine wave in Figure 1 can be mathematically represented by

$$V(t) = V_p \sin 2\pi f t \qquad (2.2)$$

Ideal Sine Wave in the Frequency Domain

Another way of representing a sine wave is in the frequency domain. The Fourier transform provides an insightful representation of the sine wave. The Fourier transform will give the distribution of a signal over all frequencies. The Fourier transform and the Inverse Fourier transform are defined as

$$F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

For the pure sine wave

$$F\left[\sin\left(\omega_{o}t\right)\right]=\frac{\pi}{j}\left[\delta\left(\omega-\omega_{o}\right)-\delta\left(\omega+\omega_{o}\right)\right]$$

The magnitude plot of the Fourier transform of the sine function is shown in Figure 2 below.

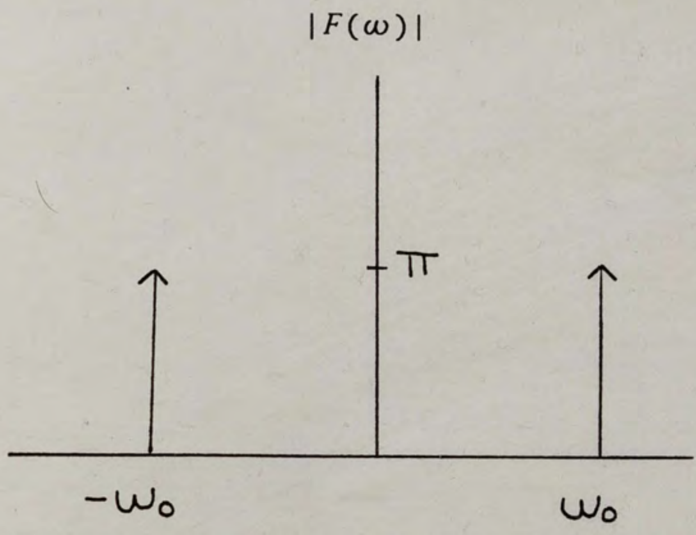

Figure 2. Sine Wave Frequency Components.

Note that the singularity function is present at +/- ω_o. The singularity function has zero width and infinite amplitude. Thus all of the energy is at +/- ω_o.

Power Spectral Density of an Ideal Sine Wave

The final view of the pure sine wave is the power spectral density. The power spectral density function gives the relative power contributions such that if integrated over all frequencies the average power is obtained.

$$P = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_f(\omega)\,d\omega$$

It should be noted that $S_f(\omega)$ does not uniquely describe f(t). This means that there are many different functions that can have the same power spectral density. The spectral density function is very useful in describing random functions.

In the case of periodic power signals, the power spectral density is shown as

$$S_f(\omega) = 2\pi \sum_{n=-\infty}^{\infty} |F_n|^2 \delta(\omega - n\omega_o)$$

The Fn's in the above equation are the coefficients of the exponential Fourier series. For the case of the pure sine wave the power spectral density is given as

$$S_f(\omega) = 2\pi\left[\frac{1}{4}\delta(\omega - \omega_o) + \frac{1}{4}\delta(\omega + \omega_o)\right]$$

This is shown graphically in Figure 3 below. Note that all of the energy is again at +/- ω_o.

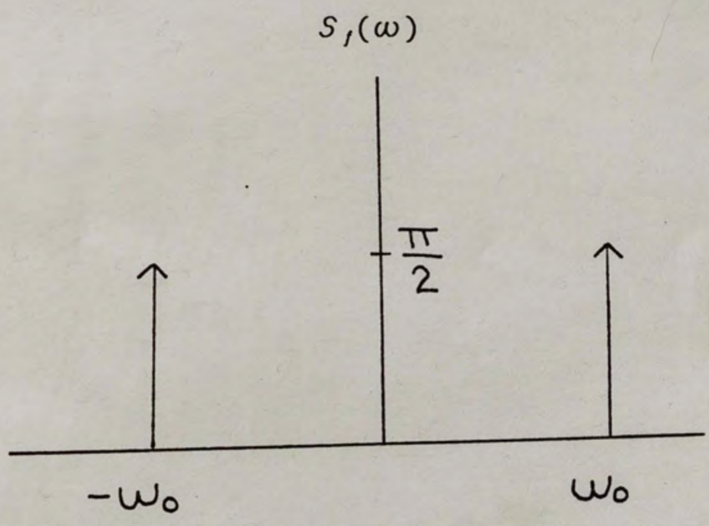

Figure 3. Power Spectral Density.

CHAPTER THREE

PHASE NOISE

Real Signals in the Time Domain

In the real world the ideal oscillator does not exist. Oscillators in practice have frequency, phase, and amplitude errors. To account for these imperfections, real oscillators can be described mathematically by

$$V(t)=\left[V_p+a(t)\right]\sin\left(2\pi f_o t+\phi(t)\right) \qquad (3.1)$$

There are two additional terms in this equation that are not in equation 2.2. The first is a(t), which represents the amplitude noise. It manifests itself in the oscillator as amplitude modulation of the oscillating signal. Usually the amplitude noise is small and can be neglected. In those cases where it does introduce errors or causes problems with phase noise measurements, it can be eliminated. One way to eliminate the effects of amplitude modulation on phase noise is to feed the output of the oscillator into a limiter. The limiter will produce harmonics but the sidebands around the fundamental frequency will be due to phase noise only.

The second additional term in the above equation is $\phi(t)$. This is the term of interest, and it is the mathematical representation of the phase noise. When this term is equal to zero there is no phase noise in the oscillator. Noise here is defined as anything that makes $\Phi(t)$ nonzero.

A nonzero $\Phi(t)$ term manifests itself in the oscillating sinusoid as phase modulation. The effect of the phase noise modulation is to introduce modulation sidebands around the fundamental frequency. Thus the output of a real oscillator may produce the spectrum shown below.

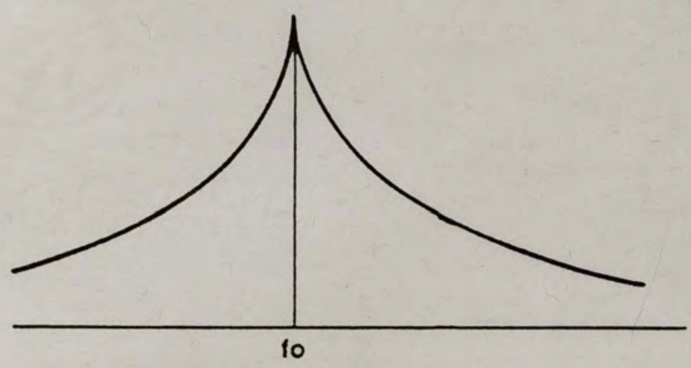

Figure 4. Spectrum Showing Phase Noise.

In contrast to Figure 2, which showed the spectrum of an ideal oscillator that had all of the energy at the fundamental frequency, Figure 4 shows a real oscillator with energy distributed in modulation sidebands around the fundamental frequency. In general, the modulation sidebands may be due to amplitude noise and phase noise. However, as mentioned previously, the amplitude noise can be eliminated by limiting.

Sources of Phase Noise

Oscillators are made from components such as transistors, resistors, capacitors, inductors, and crystals. All of these components are not ideal. Environmental factors can influence the components and thus affect the output of the oscillator. Pressure, gravity, temperature, dust accumulation, and voltage stress can all change the output of an oscillator. Furthermore, some components simply change their characteristics with time. The above mentioned sources of noise typically occur slowly and over a long period of time. This is referred to as long-term stability.

Other types of noise occur randomly and in short bursts. This is called short-term stability. Short-term noise can cause hard to identify errors and problems in systems. A mechanical shock, for example, may cause short term noise. Disturbances on power lines are another example of a way errors can be introduced into electronic systems. Storms and temporary dampness can also be temporary sources of noise. Finally, since long term noise is easier to identify and fix, short term noise is usually the one that causes the most problem. Thus the engineer must take this into consideration when designing oscillators.

Problems Caused by Phase Noise

Why study and measure phase noise? One answer to this question is that increased use of the electromagnetic spectrum requires more frequency stability. Also, signal sources are closer to the carrier as a result of more channels being in a given frequency band. Thus the oscillator has to be clean in order to process these signals. Phase noise is typically the limiting factor in RF and microwave systems. Phase noise is directly related to frequency stability. Furthermore, phase noise can cause errors and reduced performance in electronic systems. As shown below in Figure 5, phase noise side bands can cover up a weak wanted signal and make processing of the wanted signal difficult or impossible.

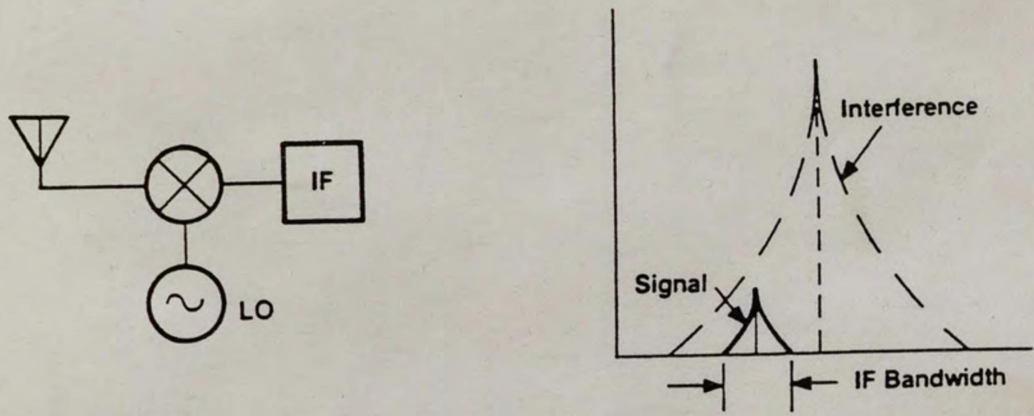

Figure 5. Signal Suppressed by Phase Noise.

In digital communication systems, phase noise can increase bit error rate. Figure 6 below shows an M-ary signaling scheme using QPSK. This scheme is common in the design of modems. Here a phase error of 45 degrees can cause the reception of an incorrect symbol. Furthermore, higher speed modems using an M-ary scheme with bit rates to 19.2K have a phase error margin of only a few degrees.

Finally, phase noise can cause problems in radar systems. In coherent Doppler radar, a strong interfering signal produced by reflections from large stationary objects creates phase noise side bands which can clutter the weak wanted Doppler signal (Sherer 1986).

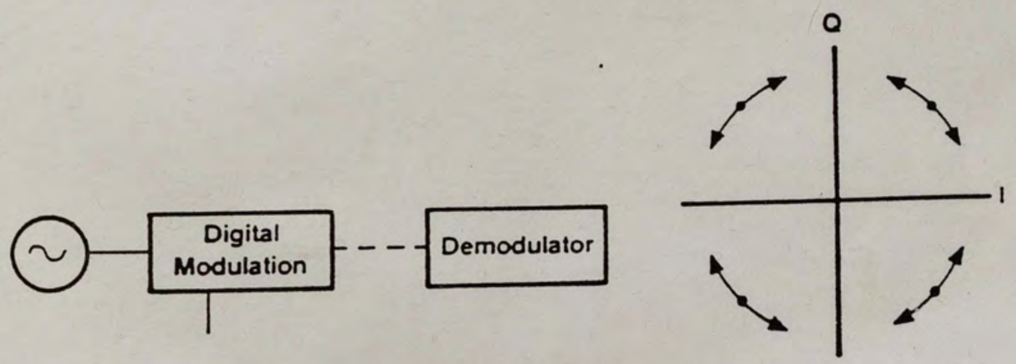

Figure 6. Phase Noise in QPSK System.

CHAPTER FOUR

PHASE NOISE SPECTRAL DENSITIES

Definition of Spectral Density

Earlier in this report, the power spectral density of the pure sine wave was introduced. The power spectral density is a function that gives the power versus frequency for a given waveform in units of watts per units of frequency.

In general a spectral density may be introduced for different types of functions such as phase and frequency. The Fourier transform of a function g(t) corresponds to g(f) in the frequency domain. The spectral density function is defined as

$$S_g(f) = g^2(f)$$

In many cases, as with phase noise, the signal of interest is a random signal. For random signals represented by a sample function x(t) in the time domain, the spectral density is given as

$$S_x(f) = x^2_{\mathrm{RMS}}(f)$$

Note that the RMS value here refers to the RMS value at each frequency and not the entire function. The spectral density function of a random variable is often found using the relation

$$S_x(\omega) = \lim_{T \to \infty} \frac{|F_T(\omega)|^2}{T}$$

where $F_T(\omega)$ represents the Fourier transform of the truncated function f(t). The basic idea is that by increasing the width of truncation T, of f(t), the Fourier transform and thus the spectral density will approach a limit. If such a limit does exist, then the spectral density is given by the above relation. Finally, the above relation provides a method which the spectral density function may be computed with digital computers.

Phase Spectral Density

The phase fluctuations are represented in equation 3.1 in the time domain as $\Phi(t)$. This corresponds to $\Phi(f)$ in the frequency domain. Thus the phase spectral density is given as

$$S_\Phi(f) = \Phi^2_{\mathrm{RMS}}(f) \qquad \left| \frac{\mathrm{rad}^2}{\mathrm{Hz}} \right|$$

Since the units of the phase spectral density function are rad^2 per Hz, the phase spectral density involves no "power" measurement. It turns out, however, that for small phase fluctuations ($\Phi(t)$ << 1 radian) the phase spectral density has approximately the same shape as the power spectral density of the oscillating signal. There are, however, two main differences. The first is that the units are different. The second is that the phase spectral density does not include the carrier, whereas the power spectral density does include the carrier.

Integration of the power spectral density along frequency will yield the power between the limits of integration. In contrast, integration of the phase spectral density over frequency will yield the mean square value of the phase noise over the frequency range integrated. Integration of the phase spectral density over all frequencies gives the total mean square value of the phase noise. This is expressed as

$$\overline{\Phi^2(t)} = \int_{-\infty}^{\infty} S_\Phi(f)\,df \tag{4.1}$$

Equation 4.1 can be derived using the Weiner Khintchine theorem and the definition of a time averaged square signal. The time averaged square value of the phase $\Phi(t)$ is given as

$$\overline{\Phi^2(t)} = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \Phi^2(t)\,dt \tag{4.2}$$

The autocorrelation function is defined as

$$R_\Phi(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \Phi(t)\Phi(t+\tau)\,dt \tag{4.3}$$

When $\tau = 0$ the right side of equation 4.3 can be identified as the average power function, and the following relation can be established.

$$R_{\Phi}(0) = \overline{\Phi^2(t)} \tag{4.4}$$

Furthermore, the Weiner Khintchine theorem relates the autocorrelation function and the spectral density function as Fourier transform pairs. This is given as

$$R_{\Phi}(\tau) = \int_{-\infty}^{\infty} e^{j\omega\tau} S_{\Phi}(f) df \tag{4.5}$$

Again with $\tau=0$ and using equation 4.4, equation 4.1 is realized.

Fractional Frequency Fluctuations

A common point of interest is how frequency is affected by changes in phase. The phase angle, θ, in equation 3.1 is identified as

$$\theta(t) = 2\pi f_o t + \Phi(t)$$

Equation 2.1 gives the angular frequency equal to the time derivative of phase. Thus

$$2\pi f(t) = \frac{d\theta}{dt} = 2\pi f_o + \frac{d\Phi(t)}{dt}$$

This can be rearranged as

$$f(t) - f_o = \frac{1}{2\pi} \frac{d\Phi(t)}{dt}$$

The quantity $f(t)-f_o$ is the change in frequency at time t. This can be denoted as $\Delta f(t)$. Instead of specifying a frequency fluctuation in terms of a shift in frequency, it is more useful to specify a change in frequency with respect to the nominal frequency, f_o. Thus the fractional frequency fluctuation is denoted by

$$y(t)=\frac{\Delta f(t)}{f_o}=\frac{1}{2\pi f_o}\frac{d\Phi(t)}{dt}$$

where y(t) is a dimensionless quantity. The use of y(t) in describing frequency fluctuations becomes very clear with an example comparing two oscillators. Consider Δf(t) = 10 Hz. One oscillaor has frequency, f_{1o} = 100 Hz and another has f_{2o} = 10 MHz. Both oscillators have the same frequency change. However, the frequency change in the first oscillator as a percentage change is much higher than in the second oscillator. Since the quantity y(t) indicates the percentage change, it is a much better quantity showing frequency stability.

Spectral Density of Fractional Frequency Fluctuations

The Fourier transform of y(t) will give y(f). From this the following equation may be written:

$$y(f)=\frac{\Delta(f)}{f_o}=\frac{2\pi f\Phi(f)}{2\pi f_o}$$

The spectral density of fractional frequency fluctuations is given by

$$S_y(f) = y^2_{RMS}(f)$$

The fractional frequency spectral density can be related to the phase spectral density function by

$$S_y(f) = \left(\frac{f \Phi_{RMS}(f)}{f_o} \right)^2 = \frac{f^2}{f_o^2} \Phi^2_{RMS}(f) = \frac{f^2}{f_o^2} S_\phi(f)$$

<u>£(f) Spectral Density</u>

Another common representation of phase noise is £(f). £(f) is defined as the ratio of the single sideband noise power in a 1 Hz bandwidth to the total carrier power at a given offset, f_m, of the carrier. On a logarithmic scale the units of £(f) are dBc/Hz.

$$L(f) = \frac{\text{Pssb (per 1 Hz)}}{\text{Ps}}$$

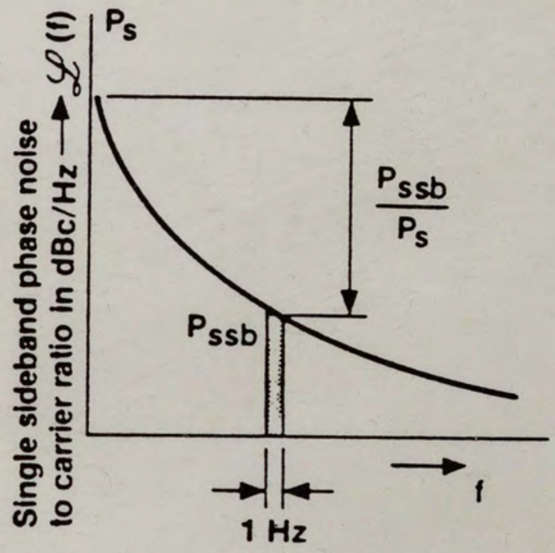

Figure 7. £(f) Definition.

Relationship Between £(f) and Phase Spectral Density

Using phase modulation theory, a phase modulated signal can be mathematically represented in the case of sinusoidal message by

$$X(t) = A\cos\left(\omega_c t + \beta \sin \omega_m t\right) \tag{4.6}$$

where ω_c is the carrier frequency, ω_m is the modulating frequency, and β is the modulation index. Equation 4.1 can be rewritten as

$$X(t) = A\,\mathrm{real}\left(e^{j\omega t} e^{j\beta \sin \omega_m t}\right) \tag{4.7}$$

The function $\exp(j\beta \sin \omega_m t)$ is periodic with frequency ω_m and can therefore be expanded in a Fourier series. Realizing that the coefficients of the Fourier series are given by

Bessel functions, the Fourier series for exp($j\beta \sin\omega_m t$) can be written as

$$e^{j\beta \sin\omega_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn\omega_m t} \qquad (4.8)$$

Substituting equation 4.8 into equation 4.7 and taking the real part allows the phase modulated signal to be written as

$$X(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t \qquad (4.9)$$

In the case of narrow band PM, the Bessel coefficients for $|n| = 2$ and above are approximately zero and the equation for a phase modulated signal can be written as

$$X(t) = AJ_o(\beta)\cos\omega_c t + AJ_1(\beta)cost(\omega_c + \omega_m)t + AJ_{-1}(\beta)\cos(\omega_c - \omega_m)t \qquad (4.10)$$

In order to evaluate the Bessel coefficients the narrow band approximation is used, for small β this approximation

$$e^{j\beta \sin\omega_m t} \sim 1 + j\beta \sin\omega_m t$$

can be substituted into equation 4.7, and the real part is taken to yield

$$X(t) = Acos\omega_c t - \beta A \sin(\omega_c t)\sin(\omega_m t) \qquad (4.11)$$

Using a trigonometric identity equation 4.11 is manipulated to provide a form by which the Bessel coefficient may be recognized.

$$X(t) = Acost\omega_c t + \frac{\beta A}{2}\cos(\omega_c + \omega_m)t - \frac{\beta A}{2}\cos(\omega_c - \omega_m)t \qquad (4.12)$$

The ratios of first order sideband peak amplitude to the carrier peak amplitude in both equation 4.10 and equation 4.12 are equal. This gives

$$\frac{AJ_1(\beta)}{AJ_0(\beta)} = \frac{\frac{A\beta}{2}}{A}$$

Finally, from the definition of £(f) the following relation can be established

$$L(f_m) = \frac{J_1^2(\beta)}{J_0^2(\beta)} = \frac{1}{4}\beta^2 = \frac{1}{4}\left(\sqrt{2}\Phi_{\mathrm{RMS}}(f_m)\right)^2 = \frac{1}{2}\Phi_{\mathrm{RMS}}^2(f_m)$$

The relationship between £(f) and the phase spectral density can now be shown as:

$$L(f) = \frac{1}{2}S_\phi(f) \qquad (4.13)$$

<u>Generating RMS Phase Noise from £(f)</u>

The units of the phase spectral density were previously given as rad^2 per Hz. From equation 4.1, it was seen that integration of the phase spectral density over all frequencies yielded the mean square value of the phase noise. Taking the square root of this value gives the RMS value of the phase noise in radians. This value can also be converted to RMS degrees of phase noise. In the case where the contribution to the phase noise is desired in a certain frequency range, the £(f) spectrum may be integrated to yield this quantity. This can be accomplished via the following example. The mean square value of phase noise can be represented mathematically as

$$\sigma_{\phi}^{2}=2\int_{f_1}^{f_2} L(f)df$$

In this example, Figure 8 provides the £(f) function.

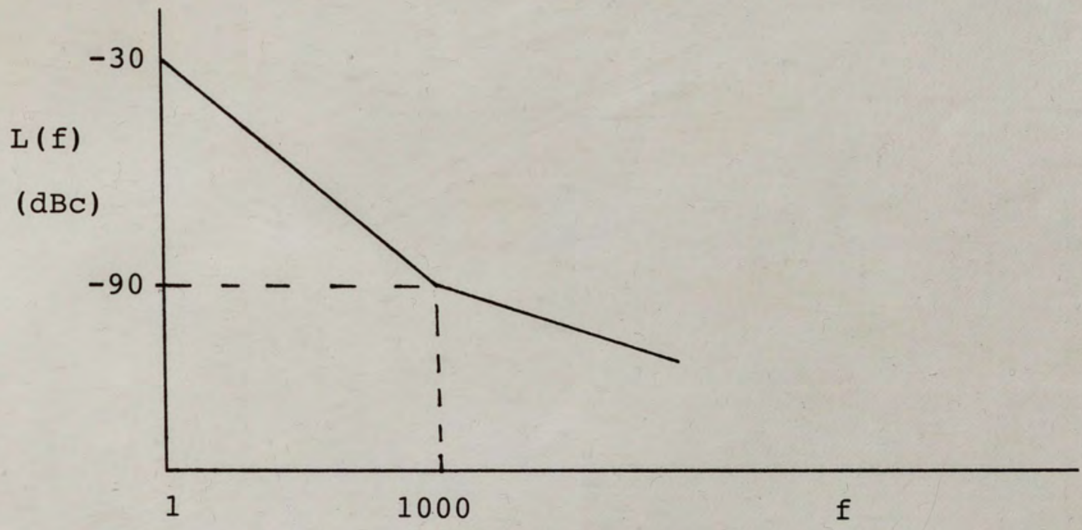

Figure 8. £(f) vs. Frequency.

£(f) is a piecewise linear function which is typical of an oscillators phase noise. A linear function has the form

$$Y=ax+b$$

where

$$a=\frac{Y_2-Y_1}{\mathrm{Log}f_2-\mathrm{Log}f_1}$$

and b is equal to the Y axis intercept. Also note that the X axis is on a logarithmic scale.

In this example the range of integration will be from 1 Hz to 1000 Hz. Within these values of integration b = -30 and

$$a = \frac{-90-(-30)}{\mathrm{Log}\,1000 - \mathrm{Log}\,1} = -20$$

From this the following equation can be written.

$$Y(f) = -20\,\mathrm{Log}\,f - 30 \qquad \left|\frac{\mathrm{dBc}}{\mathrm{Hz}}\right|$$

In order to do the integration, this equation must be converted back to linear. Performing this operation, the linear function g(f) can be written as

$$g(f) = 10^{\frac{Y(f)}{10}} = 10^{-2\,\mathrm{Log}(f)-3}$$

which reduces to

$$g(f) = 10^{-3} f^{-2}$$

At an offset frequency of 1000 Hz from the carrier, the mean square phase noise can now be written as

$$\sigma_{\phi}^{2} = 2\int_{1}^{1000} 10^{-3} f^{-2}\,df = 0.002 \qquad \mathrm{radians}^{2}$$

Finally, taking the square root of this value and converting to degrees σ_{ϕ} is equal to 2.578 degrees RMS of phase noise.

CHAPTER FIVE

PHASE NOISE MEASUREMENT TECHNIQUES

Direct Spectrum Measurement

The most straightforward and probably the easiest technique to obtain $\mathcal{L}(f)$ is directly from the oscillator spectrum using a spectrum analyzer. To generate $\mathcal{L}(f)$, the power in the carrier is read from the spectrum analyzer. Following this, the plot of $\mathcal{L}(f)$ is generated by applying the definition of $\mathcal{L}(f)$. That is, spectrum values in a 1 Hz bandwidth are read at the different frequency offsets from the carrier, the power calculated, and finally divided by the power in the carrier. This final value then can be converted to dBc/Hz and plotted on a graph.

There are some limitations of this method. These limitations become especially apparent when measuring a good oscillator with little phase noise. This arises because the phase fluctuations are out of the dynamic range of the spectrum analyzer.

Frequency Discriminator Technique

The demodulation of a FM signal requires a circuit that yields an output signal proportional to the frequency deviation of the input. A circuit that accomplishes this is called a frequency discriminator. The output of the frequency discriminator is a voltage which represents the frequency fluctuations with time. Thus, the oscillator's output can be fed directly into a frequncy discriminator to obtain the frequency fluctuations. This signal is then operated on by a baseband analyzer to give the spectral density of the frequency fluctuations, $S_{\Delta f}(f)$.

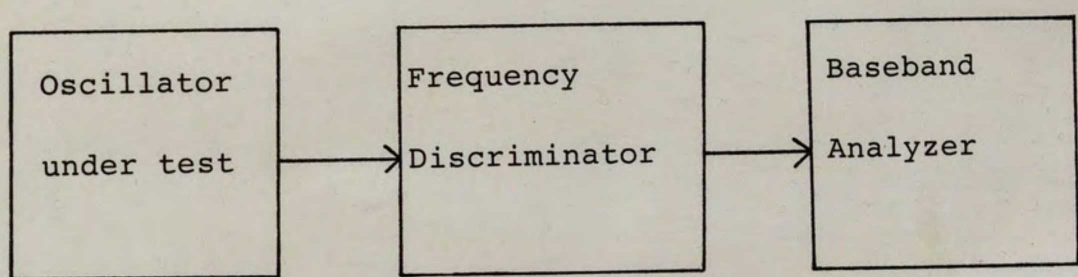

Figure 9. Frequency Discriminator Measurement Technique.

Sometimes the oscillator frequency is very high and in a range difficult to work with. In this case, the frequency can be mixed down with a clean oscillator to a frequency easier to manage. An example of this will be shown in the next section.

Phase Lock Loop Measuring Technique

The phase lock loop measurement technique is the same as the frequency discriminator technique, with one exception. The phase lock loop circuit is substituted for the frequency discriminator. The phase lock loop technique is a good technique to measure phase noise (Jimenez 1986). The phase lock loop provides an adequate performance and is also available in low cost integrated circuits. A phase lock loop provides an output proportional to the frequency fluctuations about a carrier presented to its input.

An example of a phase lock loop measurement is available from Jimenez (1986). The block diagram of the system to measure phase noise is shown below in Figure 10.

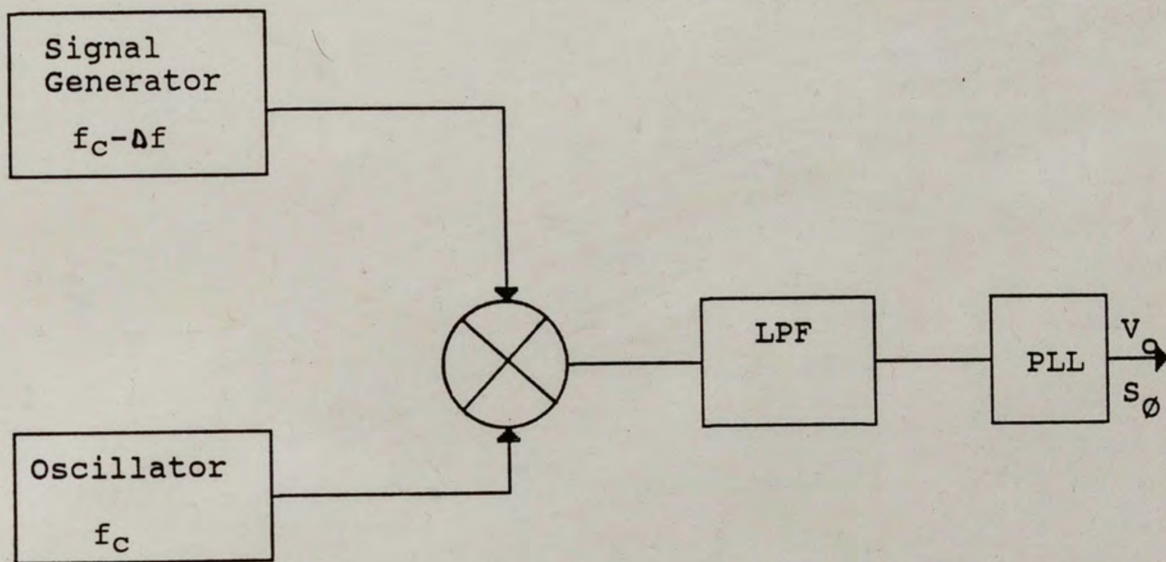

Figure 10. Phase Noise Measurement System.

In this system, a signal generator provided a clean source used to mix with the test oscillator. A low pass filter follows to eliminate the sum component of the mixing process. The frequency of the signal generator is set to $f_c-\Delta f$ and the frequency of the test oscillator is set to f_c. Thus, after the low pass filter a sinusoid with a frequency of Δf is obtained and presented to the phase lock loop. Thus, this is the reason the test oscillator was mixed down to Δf. The output of the phase lock loop was measured with a low frequency spectrum analyzer.

As a comparison to the direct spectrum measurement technique, the £(f) function of the test oscillator was measured using the direct spectrum measurement technique (Jiminez 1986). The limitations of the direct spectrum measurement were attributed to the limited dynamic range of the spectrum analyzer and its high noise floor.

CHAPTER SIX

CONCLUSION

In this report, the ideal oscillator output was presented. It was seen that for the perfect oscillating signal, all of the energy was concentrated at the carrier frequency. With the introduction of noise, energy was introduced into the sidebands around the oscillating frequency. This noise was defined as phase noise and described in the time, frequency, and spectral density domain. In addition to the phase spectral density function, the fractional frequency spectral density, and the £(f) definition were introduced. Also included was a derivation giving the relationship between the phase, fractional frequency, and £(f) spectral densities. The analization of phase noise was concluded with an example showing the procedure to get RMS phase noise from an £(f) graph.

Finally, three different measurement techniques were presented. Of the three presented, the phase lock loop technique was the best.

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