# Two Point Resolution of a Defocused Multi-Aperture System Eyelet 

Steven A. Marlow<br>University of Central Florida

Find similar works at: https://stars.library.ucf.edu/rtd
University of Central Florida Libraries http://library.ucf.edu

This Masters Thesis (Open Access) is brought to you for free and open access by STARS. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of STARS. For more information, please contact STARS@ucf.edu.

## STARS Citation

Marlow, Steven A., "Two Point Resolution of a Defocused Multi-Aperture System Eyelet" (1987). Retrospective Theses and Dissertations. 5096.
https://stars.library.ucf.edu/rtd/5096


# TWO POINT RESOLUTION OF A DEFOCUSED MULTI-APERTURE SYSTEM EYELET 

BY
STEVEN ARTHUR MARLOW
B.S., University of West Florida, 1981

THESIS
Submitted in partial fulfillment of the requirements for the degree of Master of Science in the Graduate Studies Program of the College of Engineering University of Central Florida Orlando, Florida

Summer Term

## ABSTRACT

Multi-aperture optical systems based on the insect eye offer an alternative to the common optical system based on the human eye. Some of the advantages of a multi-aperture system include the ability to perform parallel processing, have super resolution and have available large amounts of system redundancy.

An individual eyelet of a multi-aperture system consists of a gradient index lens coupled to optical fibers which transfer the incident light on the lens to individual detectors.

A mathematical model of an individual eyelet was developed. It is a flexible model allowing various system parameters to vary. Computer based algorithms were developed to locate and resolve two points in space. The model was exercised with experimental data and found to have a resolution of $3.1^{\circ}$. The algorithm was also exercised with the computer model and the results compared favorably.

## ACKNOWLEDGMENTS

I have learned, from experience, over the last six months that a graduate thesis is a team effort. The completion of this research would not have been possible without support and I would like to thank:

Dr. Roy Walters for his previous research in multi-aperture optics and for his criticism, support and effort throughout this research.

Dr. Glenn Boreman and Dr. Ronald Phillips for their reviews of this paper and guiding comments.

The staff of Analytics for their support. Special appreciation to Dennis Garbo and John Winterberger for their supportive services.

My very special thanks to Madeline Thompson for all the hours she spent typing and arranging this report. With very few hours to work, she did an excellent and efficient job.

My wife, Jan, who accepted my need to spend nights and weekends study ing.

My parents who instilled the importance of an education.
ACKNOWLEDGMENTS ..... iii
LIST OF TABLES ..... v
LIST OF FIGURES ..... vi
INTRODUCTION ..... 1
Insect Eye ..... 1
Multi-Aperture Optical System ..... 4
THE EYELET MODEL ..... 7
The Source ..... 7
The GRIN Lens ..... 11
The Optical Fiber ..... 21
DETECTION ALGORITHMS ..... 26
Single-Point Detection Algorithm ..... 28
Two-Point Detection Algorithm ..... 31
EXPERIMENTAL PROCEDURES AND RESULTS ..... 34
Experimental Setup ..... 34
Experimental Results ..... 35
CONCLUSION ..... 44
APPENDICES ..... 46
A. Power Calculation ..... 46
B. Software for the Eyelet Model ..... 49
C. Software for Single-Point Detection ..... 54
D. Software for Two-Point Detection ..... 58
REFERENCES ..... 64

## LIST OF TABLES

1. Experimental Hardware Parameters ..... 35
2. Single Point Location Data ..... 36
3. Single Point Location Eyelet Model To Measured Comparison ..... 38
4. Two--Point Resolution Measurements ..... 39
5. Two-. Point Resolution Eyelet Model Predictions ..... 40
6. Two-Point Eyelet Model and Measured Comparison ..... 41

## LIST OF FIGURES

1. Single Eyelet (Bas ic Omatidia) ..... 3
2. Basic Multi-Aperture Eyelet ..... 6
3. Geometry of Source and GRIN Lens ..... 8
4. Radial Refractive Index Profile ..... 12
5. GRIN Lens Input and Output Notation ..... 14
6. Diffraction Geometry ..... 16
7. Square Fiber Array Arrangement ..... 22
8. Hexagonal Fiber Array Arrangement ..... 23
9. Determination of Illuminated Fibers ..... 24
10. Maximum Entrance Angle Definition of Numerical Aperture for an Optical Fiber ..... 25
11. Image Intensity of Two Point Sources Separated by the Rayleigh Criterion ..... 27
12. Determination of Single Point Location ..... 29
13. Experimental Hardware Setup ..... 34
14. Geometry of Point Source and Collecting Aperture ..... 47

## CHAPTER ONE

INTRODUCTION
Insect Eye
Almost all conventional imaging systems are patterned after a human eye; a single large aperture optical system coupled to a large number of detectors to obtain an image. Since each detector is addressed individually, a large amount of time and processing is required for image formation or analysis. In contrast, consider an optical system modeled after the insect eye, a multi-aperture optical system. Insect eyes have an extremely wide field-of-view, as much as 270 degrees. It has been shown that most insect eyes cannot image an object.(1) However, the insect eye is optimized for performing certain tasks such as locating and tracking a target (searching for food, mating, normal flight and defense). The insect eye also processes in parallel; therefore, a short amount of time is required to perform certain tasks.

Most entomologists subscribe to the theory that there are two types of insect eyes, the apposition eye and the superposition eye. In the apposition eye, each eyelet has a small field of view that overlaps with the neighboring eyelets. The sensed information is reconstructed in the brain from tiny segments of information obtained by each eyelet. The superposition eye is based on the superposition principle; the actual image is formed from a layer of images that are superimposed.(2)

The single eyelet or omatidia is shown in Figure 1.(2) The omatidia
consists of 3 distinct structures: (1) lens system (corneal lens and the crystalline cone); (2) a rhabdom which acts like a detector and converts the light to an electric signal; and (3) a transparent, hoselike medium to transfer the light. The omatidia is surrounded by pigment cells which serve as optical insulation between neighboring eyelets.(2) Some insects have up to 20,000 of such eyelets. The most important items learned from insect physiology are: (1) an insect does not need to form an image to perform complex tasks; and (2) multiaperture systems are perfect platforms for application of parallel processing. (3)


Figure 1. Single Eyelet (Basic Omatidia)

## Multi-Aperture Optical System

To develop a multi-aperture optical system it is not necessary to emulate the insect eye, but use it as a guide to assemble a system to perform certain tasks. It is important to remember that an image is not required to perform many tasks.

Some typical characteristics of a multi-aperture optical system are:

1) super resolution capability
2) parallel processing
3) built-in redundancy

There are three general configurations for multi-aperture optical systems. (4)

1) Those that superimpose the image from each lens; i.e., one generates an image from overlapping images from each lens.
2) Those that place each lens in apposition; i.e., each lens has a unique field of view, where their fields of view overlap by a fixed prescription.
3) Those that discard the unified optical image and spacially sample the field of view. Reconstruction of the image information is done by calculational techniques and no alignment is necessary.

The third configuration is the only system considered, since it requires no alignment and is extremely simple to construct.

A random apposition multi-aperture optical system consists of a set of lenses focused onto pixel dividers. This system was first described by

Walters.(4) The lenses are gradient index lenses and the pixel dividers are step index optical fibers. A characteristic of this system is that any point of a concentric circle in space, viewed by a pixel, has a unique detector response.(3)

Kellog(5) compared the resolution and detection characteristics of multi-aperture vs. single aperture systems, and concluded that a multiaperture system resolution improves by the square root of the number of overlapping pixels. Mathews (6) has shown that overlap can be controlled with excellent statistical results and need not be carefully aligned into place. Walters(4) has described the data path in mathematical nomenclature as a set of array operators.

The conventional optical system utilized for point source detection and location usually employs a quadrant detector placed at the focal plane of the system. The quadrant detector divides the focal plane into four quadrants. The object point location is determined by differencing the output signals from the detectors on each side of the axis of interest.

Consider utilizing a single eyelet of a multi-aperture optical system defocused to provide the field of view redundancy. A single Gradient Index (GRIN) lens is used with 16 optical fibers arranged in a hexagonal array and placed far enough out of the focal plane to allow at least three fibers to be illuminated by a single object point. The output from the three fibers can be utilized to determine the point source location in azimuth and elevation with respect to the GRIN lens optical axis. The single eyelet is shown in Figure 2.


Figure 2. Basic Multi-Aperture Eyelet

The purpose of this study was to mathematically model a GRIN eyelet similar to the experimental hardware developed at the University of Central Florida by Dr. R. Walters. The configuration shown in Figure 2 was utilized to characterize the eyelet. A source model, GRIN model and fiber model were developed to simulate the eyelet.

## CHAPTER TWO

THE EYELET MODEL

A mathematical eyelet model allows the system designer to change certain aspects of his system to determine their effects in a fast and efficient manner without changing actual hardware.

The eyelet model is composed of three submodels, a point source, a gradient index lens, and optical fibers.

Ray matrix optics is used to propagate the point source through the system to the fiber plane, locate the centroid and define the spot size. Separate intensity models have been developed to determine the intensity distribution due to the point source. The ray matrix models and intensity models are discussed in detail in the following sections.

## The Source

The point source model is allowed to vary in divergence, wavelength, position (azimuth, elevation and distance from the GRIN lens).

Consider a point source located in the field of view of the GRIN lens. To determine the photon flux collected by the GRIN lens, the geometry of Figure 3 is used.


Figure 3. Geometry of Source and GRIN Lens

The total power received by the entrance aperture of the GRIN lens is given by: (7)

$$
\begin{equation*}
P=\operatorname{EACOS}_{\theta}{ }^{\operatorname{COS}}{ }_{\theta}{ }_{E L} \tag{1}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{P}=\text { power collected by GRIN lens (watts) } \\
& \mathrm{E}=\text { irradiance of the source (watts/cm }{ }^{2} \text { ) } \\
& \mathrm{A}=\text { area of GRIN lens aperture }\left(\mathrm{cm}^{2}\right) \\
& { }^{{ }^{2}} \text { AZ }
\end{aligned}
$$

The derivation of equation 1 is given in Appendix $A$.

The irradiance of the source can be determined by the intensity of the source and the range from source to GRIN lens.

$$
\begin{equation*}
E=\frac{I}{R^{2}} \tag{2}
\end{equation*}
$$

where

$$
\begin{aligned}
& I=\text { intensity of source (watts/sr) } \\
& R=\text { range from source to GRIN lens }\left(\mathrm{cm}^{2}\right)
\end{aligned}
$$

Consider the geometry of Figure 3 to calculate the intensity of the source.(8) Intensity of a source is given by

$$
\begin{equation*}
I=\frac{P_{L}}{\Omega} \tag{3}
\end{equation*}
$$

where

$$
\begin{aligned}
\mathrm{P}_{\mathrm{L}}= & \text { power of laser in watts } \\
\Omega= & \text { solid angle of laser beam with diverging lens } \\
& \text { in steradians }
\end{aligned}
$$

The assumptions made in the model are that the laser is placed far enough from the GRIN lens that a diverging lens placed in front of the laser fills the GRIN lens with a uniform plane wave.

The total power received by the GRIN lens aperture is found by substituting equations 2 and 3 into equation 1.

$$
\begin{equation*}
P=\frac{P_{L} A \operatorname{Cos} \theta_{A Z}}{{ }_{\Omega} R^{2}} \cos \theta_{E L} \tag{4}
\end{equation*}
$$

where

$$
P=\text { watts collected by the GRIN lens, }
$$

Substituting in the equations for the lens area and solid angle of the laser, the power, $P$, becomes

$$
\begin{equation*}
P=\frac{P_{L \pi} r^{2} \cos \theta_{A} A Z^{\cos \theta} E L}{4 \pi \operatorname{Sin}^{2}\left(\theta_{D} / 2\right) R^{2}} \tau \tag{5}
\end{equation*}
$$

where

$$
\begin{aligned}
A= & \text { has been replaced by } \pi r^{2} \text { and } r \text { is the lens } \\
& \text { radius } \\
\Omega= & 4 \pi \operatorname{Sin}^{2}\left(\theta_{0} / 2\right) \text { where } \theta_{D} \text { is the laser beam and } \\
& \text { diverging lens divergence angle } \\
& \text { and } \\
\tau= & \text { transmission of diverging lens }
\end{aligned}
$$

Coupled into $\tau$, are the estimates of fresnel losses in the optical train.

In review, $P$ is the power transferred from a point laser and diverging lens source to a lens with collecting area $A$ with ${ }^{\theta_{A Z}},{ }^{\circ} \mathrm{EL}$ being the position of the point source with respect to the optical axis of the GRIN lens.

The ray matrix model to propagate the rays from the point source to the front of the GRIN lens is a simple transfer matrix.(9)

$$
\left[\begin{array}{l}
r_{1}  \tag{6}\\
\theta_{1}
\end{array}\right]=\left[\begin{array}{ll}
1 & d \\
0 & 1
\end{array}\right]\left[\begin{array}{l}
r_{s} \\
\theta_{s}
\end{array}\right]
$$

where

$$
\begin{aligned}
r_{1} & =r a y \text { height at lens } \\
\theta_{1} & =\text { ray angle at lens } \\
d & =\text { distance from source to lens } \\
r_{s} & =r a y \text { height at source } \\
\theta_{s} & =\text { ray angle at source }
\end{aligned}
$$

Rays are traced from the point source to the optical fibers to locate and define the center of the spot image on the front face of the fibers.

## The GRIN Lens

The gradient index lens is a glass rod whose refractive index decreases quasi-quadratically from the axis to the periphery along the
radius.(10)

In the model the lens is allowed to vary in index of refraction, length, diameter, pitch, and numerical aperture (NA) and also have a flat or spherical radius on one end.

The profile of the refractive index can be expressed by:

$$
\begin{equation*}
n(r)=n_{0}\left(1-\frac{A r^{2}}{2}\right) \tag{7}
\end{equation*}
$$

where

$$
\begin{aligned}
A & =\text { quadratic constant } \\
r & =\text { radial variable } \\
n_{0} & =\text { on axis index of refraction }
\end{aligned}
$$

A typical refractive profile is shown in Figure 4.


Figure 4. Radial Refractive Index Profile

The ray matrix equations that characterize a spherical GRIN lens are:

$$
\left[\begin{array}{l}
r_{2}  \tag{8}\\
\theta_{2}
\end{array}\right]=\left[\begin{array}{ccc}
\operatorname{COS}(\sqrt{\mathrm{A} Z})-\frac{Q_{1}}{N O^{\prime}} & \operatorname{SIN}(\sqrt{A Z}) & \frac{1}{N o} \operatorname{SIN}(\sqrt{A Z}) \\
-\left(Q_{1} \operatorname{CoS}(\sqrt{\mathrm{~A} Z})+\operatorname{No} \sqrt{\mathrm{A} S I N}(\sqrt{\mathrm{~A} Z})\right) & \cos (\sqrt{\mathrm{A} Z)}
\end{array}\right]\left[\begin{array}{l}
r_{1} \\
\\
\theta_{1}
\end{array}\right]
$$

where

$$
\begin{aligned}
& r_{1}=\text { distance between incident ray and optical axis } \\
& \theta_{1}=\text { incident angle in radians } \\
& A=\text { quadratic constant } \\
& Z=\text { lens length } \\
& N_{0}=\text { on-axis index of refraction } \\
& r_{2}=\text { distance between exiting ray and optical axis } \\
& \theta_{2}=\text { exiting ray angle i radians } \\
& R
\end{aligned}
$$

Figure 5 shows this relationship graphically.


Figure 5. GRIN Lens Input And Output Notation

If a flat lens is used, $Q_{1}=0$, the equations reduce to the more familiar

$$
\left[\begin{array}{l}
r_{2} \\
\theta_{2}
\end{array}\right]=\left[\begin{array}{ll}
\cos (\sqrt{A Z}) & \frac{1}{\operatorname{No} \sqrt{A}} \sin (\sqrt{A Z}) \\
-\operatorname{Nov} \sqrt{A S I N}(\sqrt{A Z}) & \cos (\sqrt{A Z})
\end{array}\right]\left[\begin{array}{l}
r_{1} \\
\theta_{1}
\end{array}\right] \text { (9) }
$$

In the experimental device a 3 mm diameter, quarter-pitch flat GRIN lens was used. For a quarter-pitch GRIN lens, $\sqrt{ }$ AZ is equal to $\pi / 2$ and the ray matrix equations reduce to equations 10 and 11 .

$$
\begin{equation*}
r_{2}=\frac{\theta}{N_{0}} \frac{1}{A} \tag{10}
\end{equation*}
$$

$$
\begin{equation*}
\theta_{2}=-N o \sqrt{\mathrm{Ar}}{ }_{1} \tag{11}
\end{equation*}
$$

From equation 10 it is evident that $r_{2}$ is not dependent on $r_{1}$, and all parallel rays entering the quarter pitch GRIN lens focus to one point on the rear surface of the lens. Therefore, the focal plane is located at the rear surface of the lens, with a focal length defined by

$$
\begin{equation*}
f=\frac{1}{\operatorname{No} \sqrt{A S I N}(\sqrt{A Z})} \tag{12}
\end{equation*}
$$

To determine the intensity distribution of the monochromatic light beam either at the focal plane of the GRIN lens, or out of the focal plane, the following diffraction integral for intensity and phase is utilized.(11)

$$
\begin{align*}
U\left(x_{0}, y_{0}\right)= & \frac{\exp (j k z)}{j z} \exp \left[j k / 2 z\left(x_{0}^{2}+y_{0}^{2}\right) \iint_{-\infty}^{\infty}\left\{U\left(x_{1}, y_{1}\right)\right.\right.  \tag{13}\\
& \left.\exp \left[\frac{j k}{2 z}\left(x_{1}^{2}+y_{1}^{2}\right)\right]\right\} \exp \left[-j \frac{2 \pi}{\lambda z}\left(x_{0} x_{1}+y_{0} y_{1}\right)\right] d x_{1} d y_{1}
\end{align*}
$$

where

$$
\begin{aligned}
& k=2 \pi / \lambda \\
& z=\text { distance from aperture to observation region } \\
& \lambda=\text { wavelength }
\end{aligned}
$$

The coordinates for this equation are defined in Figure 6.(11)


Figure 6. Diffraction Geometry

Since we are only concerned with intensity, which is given by UU*, we can ignore the phase components outs ide the integrand and re-write equation 13.

$$
\begin{gather*}
U\left(x_{0}, y_{0}\right)=\frac{A}{j z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} t\left(x_{1}, y_{1}\right) \exp \left[-j k / 2 f\left(x_{1}{ }^{2}+y_{1}{ }^{2}\right)\right] \exp \left[j k / 2 z\left(x_{1}{ }^{2}+y_{1}{ }^{2}\right)\right] \\
\left.\quad \exp \left[-j^{2} \pi / \lambda z\left(x_{0} x_{1},+y_{0} y_{1}\right)\right] d x_{1}, d y_{1}\right] \tag{14}
\end{gather*}
$$

or rearranging terms

$$
\begin{align*}
U\left(x_{0}, y_{1}\right)= & \frac{A}{j \lambda z} \iint_{-\infty}^{\infty}\left[t\left(x_{1}, y_{1}\right) \exp \left[j k / 2\left(x_{1}^{2}+y_{1}{ }^{2}\right)\left(\frac{1}{Z}-\frac{1}{f}\right)\right]\right. \\
& \left.\exp \left[-j 2 \pi / \lambda z\left(x_{0} x_{1}+y_{0} y_{1}\right)\right] d x_{1} d y_{1}\right] \tag{15}
\end{align*}
$$

$$
\text { let } \varepsilon=\frac{1}{Z}-\frac{1}{f}
$$

where $f=$ focal length of the lens
$A=$ electric field amplitude
(and equation 14 becomes)

$$
\begin{array}{r}
U\left(x_{0}, y_{0}\right)=\frac{A}{j \lambda z} \int_{-\infty}^{\infty}\left[t\left(x_{1}, y_{1}\right) \exp \left[(j k / 2) \varepsilon\left(x_{1}^{2}+y_{1}^{2}\right)\right]\right.  \tag{16}\\
\left.\quad \exp \left[-j^{2} \pi / \lambda_{z}\left(x_{0} x_{1}+y_{0} y_{1}\right)\right] d x_{1} d y_{1}\right]
\end{array}
$$

From equation $16, \mathrm{U}\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)$ is found as a Fourier transform of $\left.t\left(x_{1}, y_{1}\right) \exp [k / 2)_{\varepsilon}\left(x_{1}{ }^{2}+y_{1}^{2}\right)\right]$ where the transform must be evaluated at frequencies $\left(f_{x}=\frac{x_{0}}{\lambda z}, f y=\frac{y_{0}}{\lambda z}\right.$ ) to assure correct space scaling in the observation plane.

When $\varepsilon$ goes to zero, i.e. the observation plane is the focal plane, equation 16 gives the Fraunhofer diffraction integral and when $\varepsilon$ is finite, equation 16 gives the Fresnel integral.

Now letting

$$
\begin{aligned}
& \exp \left[(j k / 2) \varepsilon\left(x_{1}^{2}+y_{1}^{2}\right)\right]=g(x, y) \\
& f x=\frac{x_{0}}{\lambda Z} \quad \& f y=\frac{y_{0}}{\lambda Z} \quad x_{1}=x \& y_{1}=y
\end{aligned}
$$

Equation 16 becomes

$$
\begin{equation*}
U\left(x_{0}, y_{0}\right)=\frac{A}{j \lambda \bar{z}} \iint_{-\infty}^{\infty} g(x, y) \exp [-j 2 \pi(x f x+y f y)] d x d y \tag{17}
\end{equation*}
$$

To exploit the circular symmetry of the GRIN lens, the transform to polar coordinates is accomplished by

$$
\begin{array}{ll}
r=\sqrt{x^{2}+y^{2}} & x=r \cos \theta \\
\theta=\tan ^{-1}(y / x) & y=r \sin \theta \\
\rho=\sqrt{f x^{2}+f y^{2}} & f x=\rho \cos \theta \\
\phi=\tan ^{-1}(f y / f x) & f y=\rho \operatorname{SIN} \phi
\end{array}
$$

Applying the coordinate transformations to Equation 17;

$$
\begin{align*}
& U(r)= \frac{A}{j \lambda Z} \int_{0}^{2 \pi} d \theta \int_{0}^{\infty} d r \cdot r g(r) \exp [-j 2 \pi r \rho \operatorname{Cos} \theta \operatorname{CoS} \phi+\operatorname{SIN} \operatorname{Sin} \theta \phi] \\
& \text { or }  \tag{18}\\
& \frac{A}{j \lambda z} \int_{0}^{2 \pi} d r \cdot r g(r) \int_{0}^{2 \pi} d \theta \exp \left[-j 2 \pi r_{\rho} \cos (\theta-\phi)\right]
\end{align*}
$$

where

$$
\begin{aligned}
& g(r)=\exp \left[j k \varepsilon r^{2} / 2\right] \text { and } \\
& \text { Jo }(a)=\frac{1}{2 \pi} \int_{0}^{2 \pi} \exp [-j a \cos (\theta-\phi)] d \theta
\end{aligned}
$$

So rewriting equation 18

$$
U(r)=\frac{A 2 \pi}{j \lambda z} \int_{0}^{\infty} r g(r) J o(2 \pi r \rho) d r
$$

For a circular aperture $t(r)=\operatorname{circ}(r)$
where

$$
\operatorname{circ}(r)=1\left\{\begin{array}{l}
\mathrm{R} \leq 1 \\
0 \text { Otherwise }
\end{array}\right.
$$

Equation 18 becomes

$$
\begin{equation*}
U(r)=\frac{A 2 \pi}{j \lambda z} \int_{0}^{R} r \exp \left(j k \varepsilon r^{2} / 2\right) J o(2 \pi r \rho) d r \tag{19}
\end{equation*}
$$

To simplify the integral let

$$
\alpha=\frac{k \varepsilon r^{2}}{2} \& \quad \beta=2 \pi \rho
$$

Substituting into Equation 19: $A=p^{1 / 2}$
where $P$ is the power injected into the lens (equation 5)

$$
\begin{equation*}
\text { and } \exp \left[j k \varepsilon r^{2} / 2\right]=\operatorname{Cos}\left(k \varepsilon r^{2} / 2\right)+j \operatorname{SIN}\left(k \varepsilon r^{2} / 2\right) \tag{20}
\end{equation*}
$$

$$
U(r)=p^{1 / 2} \frac{2 \pi}{j \lambda z} \int{ }_{0}^{R_{r}(\operatorname{COS} \alpha+j \operatorname{Sin} \alpha) J o(\beta r) d r}
$$

using a change of variables

$$
\begin{gather*}
r_{1}=\beta r \text { and } d r_{1}=\beta d r  \tag{21}\\
U\left(r_{1} / \beta\right)=\frac{p^{1 / 2}}{j \lambda z \beta^{2}} 2 \pi-\int_{0}^{\beta R} r_{1}(\operatorname{COS}+\operatorname{SIN}) \mathrm{Jo}\left(r_{1}\right) d r_{1}
\end{gather*}
$$

so

In the simplifying case where the observation plane is the focal plane, (where $\varepsilon=0$, now contained in $\alpha$ ) and equation 22 reduces to equation 23

$$
\begin{equation*}
U\left(r_{1} / \beta\right)=\frac{p^{1 / 2} 2 \pi}{j \lambda z \beta^{2}} \int_{0}^{\beta R} r_{1} \mathrm{Jo}\left(r_{1}\right) d r_{1} \tag{23}
\end{equation*}
$$

which reduces upon integration to the familiar Fraunhofer intensity distribution referred to as the Airy Pattern

$$
\begin{equation*}
I=P \frac{(K R)^{2}}{Z}\left(\frac{J \frac{1}{2} \frac{(2 \pi \rho R)^{2}}{\pi \rho}}{}\right) \tag{24}
\end{equation*}
$$

When the observation plane is not the focal plane numerical integration is performed on equation 22 using the trapezoid method to determine the intensity at the input plane of the fibers.

## The Optical Fiber

The optical fiber unit that was modeled is multimode. The fibers can be placed either at the focal plane of the GRIN lens or out of the focal plane at another observation plane.

The assumptions made in the fiber model are:

1) The fibers are parallel to optical axis
2) The illuminated single fiber is uniform in core index

The model does not constrain the numbers of fibers that can be used. The fibers are stored in an ( $x, y$ ) grid in the image space with $(0,0)$ being the optical axis. Two different alignments containing 16 fibers placed in square and hexagonal array patterns are shown in figures 7 and 8 respectfully. It should be noted that a symmetrical array is not required, random placement is acceptable.

The ray matrix equation used to transfer the rays from the rear of the lens to the fiber plane is

$$
\left[\begin{array}{l}
r_{f}  \tag{25}\\
\theta_{f}
\end{array}\right]=\left[\begin{array}{ll}
1 & L \\
0 & 1
\end{array}\right]\left[\begin{array}{l}
r_{2} \\
\theta_{2}
\end{array}\right]
$$

where

$$
\begin{aligned}
r_{f} & =r a y \text { height at fiber plane } \\
\theta_{f} & =\text { ray angle at fiber plane } \\
L & =\text { distance from rear of lens to fiber plane } \\
r_{2} & =\text { ray height at rear surface of GRIN lens } \\
\theta_{2} & =\text { exit ray angle at rear of GRIN lens }
\end{aligned}
$$

Rays are traced to define the centroid of the spot in the observation plane (front of fibers).

A simple search algorithm is utilized in the fiber model to determine which fibers are illuminated from the point source. It determines the separation of each fiber centroid from the spot centroid and compares that value with the radius of the spot plus fiber radius to ascertain whether or not that fiber is illuminated.


Figure 7. Square Fiber Array Arrangement


Figure 8. Hexagonal Fiber Array Arrangement

Separation of centers $=\sqrt{\left(x_{f}-x_{s}\right)^{2}+\left(y_{f}-y_{s}\right)^{2}}$

$$
\begin{aligned}
& x_{f}=\text { azimuth coordinate of fiber center } \\
& x_{s}=\text { azimuth coordinate of spot center } \\
& y_{f}=\text { elevation coordinate of fiber center } \\
& y_{s}=\text { elevation coordinate of spot center }
\end{aligned}
$$

If the separation of centers is less than RS+Rf, where RS = radius of spot and Rf = fiber radius, then that fiber is illuminated. This is shown in Figure 9.


Figure 9. Determination of Illuminated Fibers

An area calculation is then performed to determine the proportion of the spot that falls on each illuminated fiber.

The amount of energy that is coupled into each fiber is also dependent on the fibers numerical aperture (12-14).

For a step index fiber the NA is defined as follows, refer to Figure 10. The ray shown transversing the fiber strikes the core-cladding interface at the critical angle, $\theta^{\circ} . \theta$ is the largest external angle for which a mode will propagate in this fiber. The quantity NoSINe is the NA of the fiber. When the medium is air, no=1 and NA $=$ SINo or

$$
\begin{equation*}
N A=\left(n_{1}^{2}-n_{2}^{2}\right)^{1 / 2} \tag{27}
\end{equation*}
$$

Since the fiber accepts only rays contained within the cone defined by $\theta$, an input coupling loss occurs if some fraction of the incident light strikes the fiber at angles greater than $\theta$. Similarly, if the detector at the output of the fibers cannot receive all angles of light
up to $\theta$, power is lost. In this study the detector is not modeled since relative intensity values are used in the detection algorithms, and thus the power emitted at the output of the fibers is the desired quantity to be measured.

To calculate the power transferred into a fiber from the point source (1), the intensity calculated via equation 22 is multiplied by the illuminated area of the front surface of the fiber for rays that have $\theta=\theta$ external critical. For rays outside this angle, no modal excitation is assumed. Further improvements in this model would include a scaling of modal excitation due to the diverging lens input rays from the defocused spot.


Figure 10. Maximum Entrance Angle Definition of Numerical Aperture For An Optical Fiber

This chapter discussed the power transferred from a point source through an eyelet of a multi-aperture optical system. Physical optics and geometrical optics were combined to develop a flexible model useful in predicting the performance of an eyelet. The implementation of this model in software is listed in Appendix A.

## CHAPTER 3

## DETECTION ALGORITHMS

One of the purposes of this research was to develop algorithms to determine the absolute position of two point sources located in the field-of-view of the eyelet. This was to be based on input data from either a real eyelet system or the model described in Chapter 3. This permits direct comparisons between simulation and real data. This chapter discusses the development of the detection algorithms.

Traditionally, the Rayleigh criterion has been used as a measure of optical system resolution or resolving power. According to the Rayleigh criterion (15), two images are just resolved when the principal maximum of one coincides with the first minimum of the other. This is shown in Figure 11.

Using this criterion, for a single aperture optical imaging system of focal length, $f$, and clear aperture diameter of $D$, two point sources are resolved when separated by distance $r$ equal to the distance to the first zero of the Airy pattern ( $J_{1}$ Bessel function), this is

$$
\begin{equation*}
r=\lambda(1.22 \mathrm{f} / \mathrm{D}) \tag{28}
\end{equation*}
$$

where

$$
\begin{aligned}
& r=\text { radius of diffraction spot } \\
& \lambda=\text { wavelength } \\
& f=\text { focal length } \\
& D=\text { aperture diameter }
\end{aligned}
$$



Figure 11. Image Intensity of Two Points Separated By The Rayleigh Criterion

From equation 28 , one can see that as the aperture diameter increases the minimum resolvable distance between the two point sources decreases. Two objects can be brought closer together and still be resolved.

But, unless the detector size approaches the Airy disc size, detector size is the limiting factor for resolution. Two objects whose images fall on the same detector cannot be resolved.

An object's position will not be resolved to a new location until it moves from one detector to another. An object which moves within the field-of-view of a detector can not be further resolved by that detector within its field-of-view.

Due to this fact, the fibers in the eyelet system are moved out of the focal plane to a position where at least three fibers are illuminated by a single point. This allows the equivalent pixel to be smaller than the images of the point source. Therefore, the fibers
(detector) are not the limiting factor for resolution. The resolution is now limited by the accuracy of the algorithms developed utilizing the output from the three illuminated fibers.

## Single-Point Detection Algorithm

The single-point detection algorithm, "LASER3", modified in this research for 2 point detection, is a scaling model using the three highest intensity values from a fiber array. This algorithm and its associated hardware were developed by Dr. Roy Walters at the University of Central Florida. The software listing is given in Appendix C.

The algorithm first sorts the fibers based on intensity; the three largest intensity values are used to determine the position of the spot centroid. The algorithm determines a line both in azimuth and elevation which is based on the separations of the centroids of the three fibers. Once the line is determined, for example in elevation, the relative normalized intensities of the fibers, are used as scaling factors to determine the position of the centroid along the elevation line. This is illustrated in Figure 12.


Figure 12. Determination of Single Point Location

The equations used to determine the azimuth and elevation of the point source are:

$$
\begin{align*}
\Delta A & =\theta E L I-\theta E L 0  \tag{29}\\
\Delta B & =\theta E L 0-\theta E L 2 \\
\Delta Z A & =\theta A Z 1-\theta A Z 0 \\
\Delta Z B & =\theta A Z 2-\theta A Z 0
\end{align*}
$$

where
$\theta$ ELO $=$ elevation position of highest intensity fiber $\theta E L 1=$ elevation position of second highest intensity fiber
$\theta E L 2=$ elevation position of third highest intensity fiber
$\theta A Z O=$ azimuth position of highest intensity fiber
$\theta A Z 1=$ azimuth position of second highest intensity fiber

$$
\begin{aligned}
\Theta A Z 2= & \text { azimuth position of third highest intensity } \\
& \text { fiber }
\end{aligned}
$$

Linear mode excitation scale factors (first approximation) are given by:

$$
\begin{align*}
& \mathrm{F} 1=\mathrm{I} 1 / 2  \tag{30}\\
& \mathrm{~F} 2=\mathrm{I} 2 / 2
\end{align*}
$$

where

$$
\begin{aligned}
& \text { F1 = first scale factor } \\
& \text { F2 = second scale factor } \\
& \text { I1 = normalized intensity of second highest intensity fiber } \\
& \text { I2 = normalized intensity of third highest intensity fiber }
\end{aligned}
$$

Equations 29 and 30 are then combined to determine the centroid of the spot.

$$
\begin{align*}
& \theta E L=[(\theta E L 0+F 1 \Delta A)+(\theta E L 0+F 2 \Delta B)] / 2  \tag{31}\\
& \theta A Z=[(\theta A Z 0+F 1 \Delta Z A)+(\theta A Z 0+F 2 \Delta Z B)] / 2
\end{align*}
$$

where
$\theta E L=$ elevation position of source
$\theta A Z=$ azimuth position of source

In the case of a single point the position can be resolved better than the fiber field-of-view. A position change in the source will cause an intensity change in the three illuminated fibers resulting in a new scale factor and a new object position location prediction.

## Two-Point Detection Algorithm

The two-point detection algorithm developed in this research is in three logical parts. TDA assumes that one of the following three conditions could exist.

1) a single point (or two unresolvable points) is present,
2) two well resolved points are present
3) two close but resolvable points are present (the marginal case)

If three fibers are considered as a "single detector," i.e., when a single point source is in the field-of-view of the system three fibers are illuminated, the second point must be separated a defined resolved distance from the first point in order to cause a fourth fiber to be illuminated. The intensity of the spot in the fiber plane is not the airy pattern and therefore the Rayleigh criterion cannot be invoked.

If only three fibers are illuminated, (condition \#1) the single-point detection algorithm is used to determine the location of the point source.

If six or more fibers are illuminated then condition 2 exists and two well resolved points are present. The algorithm uses the six fibers with the highest intensity values. It sorts the fibers according to position in the image plane. The fiber optic object space centroid positions are known in azimuth and elevation. These values are converted to image space coordinates by the following expression derived from equation 10 .

$$
\begin{equation*}
X=\frac{\Theta A Z F}{N O \sqrt{A}} \tag{32}
\end{equation*}
$$

$$
Y=\frac{\theta E L F}{N O \sqrt{A}}
$$

where

$$
\begin{aligned}
X, Y= & \text { image plane coordinates in mm } \\
\Theta A Z F= & \text { azimuth angle of fiber in object space (radians) } \\
\Theta E L F= & \text { elevation angle of fiber in object space } \\
& \text { (radians) } \\
N o= & \text { on-axis index of refraction of GRIN lens } \\
A= & \text { quadratic gradient constant }\left(\mathrm{mm}^{-1}\right)
\end{aligned}
$$

The separation between illuminated fibers is calculated by:

$$
\begin{equation*}
S=\left[\left(X_{1}-X_{2}\right)^{2}+\left(Y_{1}-Y_{2}\right)^{2}\right]^{1 / 2} \tag{33}
\end{equation*}
$$

where

$$
\begin{aligned}
& S=\text { separation between fiber } 1 \text { and fiber } 2 \\
& X_{1}, Y_{1}=\text { centroid of fiber } 1 \\
& X_{2}, Y_{2}=\text { centroid of fiber } 2
\end{aligned}
$$

The separation between all six fibers is calculated and the two fibers that are farthest apart are then separated into two groups. The remaining four fibers are then compared to the two separated fibers, fiber 1 and fiber 2. The fibers closest to fiber 1 are placed in group 1 and the fibers closest to fiber 2 are placed in group 2. The singlepoint detection algorithm is first used on group 1 , then on group 2 to determine the location of the two points.

If five fibers are illuminated, the same logic is utilized. The two fibers farthest apart are found and separated into two groups, the remaining three fibers are then placed into group 1 and 2 depending on the centroid locations. One fiber will be shared by both groups introducing a slight inaccuracy.

When four fibers are illuminated, condition (3) exists and two close but resolvable points exist. The four illuminated fibers are sorted according to intensity. The separation between the fibers is calculated using equation 33 and the two fibers with the greatest separation are placed into two separate groups. The remaining two fibers are placed in both groups. Thus, two fibers are shared by the two groups for object point location. The single-point detection algorithm is then applied to the two groups to determine the point source locations. This again introduces a small amount of error. The software listing for the twopoint detection algorithms in Appendix D.

This chapter discussed the criterion for single-point and two-point detection. The algorithms developed can be utilized with either the actual hardware or the model based computer simulation.

## CHAPTER 4

## EXPERIMENTAL PROCEDURES AND RESULTS

This chapter explains the experimental procedures and methods used to validate the eyelet model and the two-point detection algorithms.

## Experimental Setup

The eyelet system consists of a quarter pitch, SLW series, three millimeter diameter, NSG America SELFOC MICRO lens, on 16 step-index multimode optical fibers. The eyelet system is shown in Figure 13. The important parameters for those components are listed in Table 1.


Figure 13. Experimental Hardware Setup

TABLE 1.
EXPERIMENTAL HARDWARE PARAMETERS

| GRIN Lens | 3 mm | diameter |
| :--- | :--- | :--- |
|  | 0.25 | pitch |
|  | 1.6075 | No |
|  | 0.206 | A |
|  | 0.46 | NA |
| Fiber | 0.4 mm | core diameter |
|  | 0.37 | NA |
| Point Source | $0.6 \mu \mathrm{~m}$ | wavelength |

An LED was used as a point source in the experimental system.

The experimental configuration was assembled to allow variation of azimuth and elevation of the point source and location of the fibers with respect to the focal plane of the GRIN lens. To achieve angular offset of the point source, the GRIN lens and fiber assembly were mounted on a two-axis rotational stage. The point source remains stationary while the rotational stages provide angular point source offset variation in both azimuth and elevation.

## Experimental Results

Utilizing the experimental configuration described above, various measurements were made to verify the detection algorithms. The detection algorithms were also exercised with eyelet model simulation data.

Various azimuth and elevation angles were used to verify the singlepoint detection algorithm. The measured data and the single-point location prediction are listed in Table 2. The mean error is 1.2 degrees. The eyelet simulation was then exercised using the component characteristic values listed in Table 1. The azimuth and elevation

TABLE 2

## SINGLE POINT LOCATION DATA

$A Z=10$ DEGREES

## ELEVATION =

-2
-1
0
1
2
3
4
5
6
7
8
9
10
11
$\overline{E L}=5$ DEGREES
AZ IMUTH

MEASUREMENT AND ERROR (DEGREES)

LASER3
-0.6
-3.2
-0.2
0.0
0.5
3.9
4.3
5.0
7.9
8.7
9.0
9.3
9.5
10.0

ERROR (ABSOLUTE)
1.4
2.2
0.2
1.0
1.5
0.9
0.3
0.0
1.9
1.7
1.0
0.3
0.5
1.0

| 3 | 1.6 | 1.4 |
| ---: | ---: | ---: |
| 4 | 2.1 | 1.9 |
| 5 | 4.1 | 0.9 |
| 6 | 4.6 | 1.4 |
| 7 | 5.2 | 1.8 |
| 8 | 6.3 | 1.7 |
| 9 | 10.4 | 1.4 |
| 10 | 11.0 | 1.0 |
| 11 | 11.7 | 0.7 |
| 12 | 12.0 | 0.0 |
| 13 | 12.0 | 1.0 |
| 14 | 12.6 | 1.4 |
| 15 | 13.0 | 2.0 |
| 16 | 12.9 | 3.1 |
| 17 | 15.8 | 2.1 |
| 18 | 17.0 | 1.0 |

angles of Table 2 were used for input with detection algorithm results listed in Table 3. (Using the two-point detection algorithm, TDA, with a single point target i.e., case \#1.)

The mean error of the eyelet model input was 0.71 degrees. The mean error between eyelet model inputs and measured data inputs into TDA was 0.88 degrees. It should be noted that the two - point detection algorithm contains LASER3 as the method to locate the point source. For single points, TDA and LASER3 predict the same object coordinates.

A second point source was placed in the field-of-view of the eyelet and the coordinates were determined utilizing the single-point detection algorithm contained in TDA. With both diodes at various azimuth and elevation positions, the two-point detection algorithm was exercised. The results are listed in Table 4. The TDA had the capability to resolve two-points located within 3.1 degrees. Table 5 lists the eyelet model predictions for the same coordinates. Table 6 provides an eyelet model to measured comparison.

The model predicted that two-point sources located at (20.0, 5.0) and $(22.2,3.4)$ respectively could be resolved, a 2.7 degree resolution capability. The actual hardware could not resolve the two sources located at $(20.0,5.0)$ and $(22.2,3.4)$. The model also predicted that two points located at $(20.0,5.0)$ and $(22.2,3.5)$ could not be resolved. The eyelot model and actual measured 2 point resolution differed by 0.4 degrees.

TABLE 3
SINGLE POINT LOCATION EYELET MODEL TO MEASURED COMPARISON
MEASUREMENTS AND ERROR (DEGREES)

| $\begin{gathered} \mathrm{AZ}=10 \text { DEGREES } \\ \text { ELEVATION }= \end{gathered}$ | LASER3 ACTUAL DATA | TDA <br> MODEL DATA | ERROR (ABSOLUTE) |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | MEASURED | MODEL |
| -2 | -0.6 | -1.2 | 1.4 | 0.8 |
| -1 | -3.2 | -0.3 | 2.2 | 0.7 |
| 0 | -0.2 | -0.3 | 0.2 | 0.3 |
| 1 | 0.0 | 0.4 | 1.0 | 0.6 |
| 2 | 0.5 | 1.0 | 1.5 | 1.0 |
| 3 | 3.9 | 2.8 | 0.9 | 0.2 |
| 4 | 4.3 | 3.7 | 0.3 | 0.3 |
| 5 | 5.0 | 4.6 | 1.0 | 0.4 |
| 6 | 7.9 | 5.8 | 1.9 | 0.2 |
| 7 | 8.7 | 8.1 | 1.7 | 1.1 |
| 8 | 9.0 | 8.6 | 1.0 | 0.6 |
| 9 | 9.3 | 10.0 | 0.3 | 1.0 |
| 10 | 9.5 | 10.1 | 0.5 | 0.1 |
| 11 | 10.0 | 10.2 | 1.0 | 0.8 |
| $\overline{E L}=5 \bar{D} \bar{G} \overline{R E E S}$ AZIMUTH $=$ |  |  |  |  |
|  |  |  |  |  |  |  |
| 3 | 1.6 | 2.4 | 1.4 | 0.6 |
| 4 | 2.1 | 3.9 | 1.9 | 0.1 |
| 5 | 4.1 | 4.2 | 0.9 | 0.8 |
| 6 | 4.6 | 4.5 | 1.4 | 1.5 |
| 7 | 5.2 | 6.2 | 1.8 | 0.8 |
| 8 | 6.3 | 6.9 | 1.7 | 1.1 |
| 9 | 10.4 | 10.1 | 1.4 | 1.1 |
| 10 | 11.0 | 10.9 | 1.0 | 0.9 |
| 11 | 11.7 | 11.5 | 0.7 | 0.5 |
| 12 | 12.0 | 11.5 | 0.0 | 0.5 |
| 13 | 12.0 | 11.8 | 1.0 | 1.2 |
| 14 | 12.6 | 12.7 | 1.4 | 1.3 |
| 15 | 13.0 | 16.2 | 2.0 | 1.2 |
| 16 | 12.9 | 16.9 | 3.1 | 0.9 |
| 17 | 15.8 | 17.4 | 1.2 | 0.4 |
| 18 | 17.0 | 17.6 | 1.0 | 0.4 |
|  | MEAN ERROR |  | 1.2 | 0.71 |

TABLE 4
TWO-POINT RESOLUTION MEASUREMENTS

MEASUREMENTS AND ERROR (DEGREES)

| PRIMARY DIODE |  | $\begin{gathered} \text { SECONDARY } \\ \text { DIODE } \end{gathered}$ | $\begin{gathered} \text { TDA } \\ \text { PRIMARY } \end{gathered}$ |  | TDA SECONDARY |  | $\begin{aligned} & \text { ERROR } \\ & \text { PRIMARY } \end{aligned}$ |  | $\begin{aligned} & \text { ERROR } \\ & \text { SECONDARY } \end{aligned}$ |  | SEPARATION |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AZ | EL | AZ EL | AZ | EL | AZ | EL |  |  | AZ | EL | ANGLE |
| 10.0 | 5.0 | 12.43 .0 | 14.8 | 3.1 | 18.7 | 1.3 | -4.8 | 2.0 | -6.3 | 1.7 | 3.1 |
| 10.0 | 5.0 | 15.7-3.9 | 15.0 | 5.8 | 11.4 | -4.4 | -1.7 | -0.8 | 4.3 | 0.5 | 10.6 |
| 10.0 | 5.0 | $20.5 \quad 5.5$ | 15.0 | 3.0 | 18.4 | 1.7 | -5.0 | -0.5 | 2.1 | 3.8 | 10.5 |
| 20.5 | 5.0 | 18.8-6.6 | 20.6 | 5.3 | 19.5 | -6.8 | -0.6 | -0.3 | -0.7 | 0.2 | 11.6 |
| 20.0 | 5.0 | 15.9-5.3 | 20.6 | 5.3 | 15.2 | -5.1 | 0.6 | -0.3 | 0.7 | -0.2 | 11.1 |
| 20.0 | 5.0 | 22.22 .4 | 20.6 | 5.3 | 22.3 | 2.8 | -0.6 | -0.3 | -0.1 | -0.4 | 3.4 |
| 20.0 | 5.0 | 22.23 .4 |  |  |  | OULD NOT | RESOL | VE-- |  |  | ( 2.7) |
| 20.0 | 5.0 | 16.0-5.4 | 20.6 | 5.3 | 15.2 | -5.1 | -0.6 | -0.3 | 0.8 | -0.3 | 11.1 |
| 20.0 | 5.0 | 14.2-2.9 | 20.5 | 5.2 | 15.0 | -2.2 | -0.5 | -0.2 | -0.8 | -0.7 | 9.8 |
| 20.0 | 5.0 | 21.0-3.6 | 20.5 | 5.3 | 21.8 | 1.7 | -0.5 | -0.3 | -0.8 | 5.3 | 8.7 |

MINIMUM RESOLUTION ANGLE
MEAN $\quad 1.7$. $6 \quad 1.8 \quad 1.5 \quad 3.1$
ERROR

TABLE 5
TWO-POINT RESOLUTION EYELET MODEL PREDICTIONS

MODEL AND ERROR (DEGREES)

| PRIMARY DIODE |  | $\begin{gathered} \text { SECONDARY } \\ \text { DIODE } \end{gathered}$ |  | $\begin{gathered} \text { TDA } \\ \text { PRIMARY } \end{gathered}$ |  | TDA SECONDARY |  | ERROR <br> PRIMARY |  | $\begin{aligned} & \text { ERROR } \\ & \text { SECONDARY } \end{aligned}$ |  | SEPARATION ANGLE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AZ | EL | AZ | EL | AZ | EL | AZ | EL | AZ | EL | AZ | EL |  |
| 10.0 | 5.0 | 12.4 | 3.0 | 10.9 | 4.5 | 14.5 | 6.1 | 0.9 | 0.5 | 2.1 | 3.1 | 3.1 |
| 10.0 | 5.0 | 15.7 | -3.9 | 11.7 | 2.4 | 14.2 | -3.8 | 1.7 | 2.6 | 1.5 | 0.1 | 10.6 |
| 10.0 | 5.0 | 20.5 | 5.5 | 12.8 | 4.4 | 19.3 | 4.8 | 2.8 | 0.6 | 1.2 | 0.7 | 10.5 |
| 20.0 | 5.0 | 18.8 | -6.6 | 20.2 | 4.9 | 20.4 | -6.5 | 0.2 | 0.1 | 1.6 | 0.1 | 11.6 |
| 20.0 | 5.0 | 15.9 | -5.3 | 19.3 | 4.5 | 16.2 | -3.2 | 0.7 | 0.5 | 0.3 | 2.1 | 11.1 |
| 20.0 | 5.0 | 22.2 | 2.4 | 19.9 | 5.0 | 21.3 | 3.0 | 0.1 | 0.0 | 0.9 | 0.6 | 3.4 |
| 20.0 | 5.0 | 22.2 | 3.4 | 19.8 | 4.9 | 21.5 | 3.0 | 0.2 | 0.1 | 0.7 | 0.4 | 2.7 |
| 20.0 | 5.0 | 16.0 | -5.4 | 19.0 | 2.6 | 14.9 | -2.6 | 1.0 | 2.4 | 1.1 | 2.8 | 11.1 |
| 20.0 | 5.0 | 14.2 | -2.9 | 17.8 | 1.6 | 16.4 | -1.0 | 2.2 | 3.4 | 2.2 | 1.9 | 9.8 |
| 20.0 | 5.0 | 21.0 | -3.6 | 19.8 | 2.3 | 21.9 | -3.5 | 0.2 | 2.7 | 0.9 | 0.1 | 8.7 |

TABLE 6
TWO-POINT EYELET MODEL AND MEASURED COMPARISON

*Model values resolved these two points
Model predicted two points located at $(20.0,5.0)$ and $(22.2,3.5)$
Could not be resolved

The major causes of error in the experimental results can be attributed to:

1. Inaccurate centroid measurement.
2. Nonsymmetrical modal excitation in fibers.
3. Use of a linear excitation form factor.

When the experimental hardware is used, the object space coordinates of each fiber must be determined. To determine the object space centroid in azimuth and elevation the position of the source is changed with respect to the GRIN lens. The location that produces the highest intensity output for a fiber is used as the object space centroid location for that fiber. Inaccuracies in this measurement account for the majority of errors in the experimental results.

The intensity scale factors that are used in LASER3 assume uniform modal excitation in the fibers and does not account for the variation with angle that exists.

A cause of error in the eyelet model is the arbitrary angular extinction of propagation in the fiber. The fiber model assumes all rays with an angle less than the external critical angle propagate thru the fiber and does not take into account the losses due to the incidence angle.

This chapter discussed the experimental setup that was used to validate the computer simulation model and verify the detection algorithms. The detection algorithms were exercised with experimental data and simulation data, the results were compared to validate the
computer eyelet mode1. The mean error of detection was 1.4 degrees overall for the eyelet model. The detection algorithm locations were compared to actual measured locations and the mean error was 1.2 degrees. The resolution of the hardware system was 3.1 degrees and that of the eyelet model 2.7 degrees.

## CONCLUSIONS

The purpose of this research was to develop a two-point detection algorithm for an eyelet of a multi-aperture optical system. The algorithm was based on allowing three fibers to be illuminated by a single-point source. A computer simulation model of a multi-aperture eyelet was developed and validated with experimental data producing an overall mean error of 1.4 degrees.

The eyelet model was used to develop a two-point detection algorithm. The two-point detection algorithm provided a 3.1 degree resolution capability. If the system had been focused the resolution would have been limited by the size of the fiber and the system resolution point detection algorithm gives a resolution improvement of 40 percent.

The computer eyelet model is composed of three sub-models, a source model, GRIN lens model, and a fiber model. The model combines geometrical and physical optics to predict the output of the eyelet under various conditions. Ray matrices are used to propagate the rays from the point source to the fiber plane and determine the size and position of the spot at the fiber plane. Radiometric principles are applied to the point source to determine the power collected by the GRIN lens. A diffraction integral is utilized to calculate the intensity of the spot at the fiber plane. The fiber plane may be located either at the focal plane of the GRIN lens or removed from the focal plane. The fiber
egergy transfer model is a simple area calculation to determine the amount of power coupled into the fiber. The eyelet model is a flexible model giving multi-aperture researches a valuable tool in developing various algorithms. The results between model and measured values compared favorably.

There are many areas of research in multi-aperture optical systems that could utilize the results of this research. The eyelet model developed could be modified to predict the performance of many eyelets instead of a single eyelet. The detection algorithms could be exercised with 32 inputs instead of 16 to provide a larger field-of-view system. This research demonstrated the ability of an inexpensive, easily assembled system to resolve two-points within 3.1 degrees. Further research can provide a simple, low cost system that could resolve and track targets.

APPENDIX A
POWER CALCULATION

Figure 14 shows the geometry of a collection aperture and emitting source. The power collected by an optical system with a circular aperture of radius, $r$, is given by (7):

$$
\begin{equation*}
P=E A_{p} \tag{34}
\end{equation*}
$$

where

$$
\begin{aligned}
& P=\text { Power in watts } \\
& E=\text { Irradiance of the source }\left(\mathrm{w} / \mathrm{cm}^{2}\right) \\
& A_{p}=\text { Projected area }\left(\mathrm{cm}^{2}\right)
\end{aligned}
$$



Figure 14. Geometry of Point Source and Collecting Aperture

The projected area is given by the area of the aperture projected through the angle between the normal to the aperture and the line of sight.

$$
\begin{align*}
& A_{p}=A \operatorname{COS} \theta  \tag{35}\\
& A=\pi r^{2} \\
& \theta=\text { Angle between normal and line of sight }
\end{align*}
$$

This projected area can also be expressed in terms of the elevation and azimuth angles. Referring to Figure 14 for nomenclature, COS $\theta$ is given by

$$
\begin{align*}
& \cos \theta=\frac{R}{\bar{B}}  \tag{36}\\
& \cos ^{\theta} \theta_{A Z}=\frac{A}{\bar{B}}  \tag{37}\\
& \cos \theta_{E L}=\frac{R}{A} \tag{38}
\end{align*}
$$

rearranging equations 37 \& 38:

$$
\begin{align*}
& \mathrm{B}=\mathrm{A} / \cos \theta_{\mathrm{A}} \mathrm{AZ} \\
& \mathrm{R}=\mathrm{A} \cos \theta_{\mathrm{EL}}
\end{align*}
$$

substituting 39 into 36

$$
\begin{align*}
\cos \theta= & \frac{\mathrm{ACOS} \theta \mathrm{EL}}{\mathrm{~A} / \mathrm{C} \overline{\mathrm{O}} \theta_{\mathrm{AZ}}} \\
& \text { or } \\
\cos \theta= & \cos ^{-2} \mathrm{EL}^{\cos \theta} \mathrm{AZ}
\end{align*}
$$

So equation 34 becomes

$$
P=E A C 0 S \theta_{A Z} \operatorname{CoS} \theta_{E L}
$$

APPENDIX B
SOFTWARE LISTING FOR THE EYELET MODEL


```
THIS PROGRAM CALCULATES THE INTENSITY FOR A SINGLE APERTURE
OF A MULTIAPERTURE OPTICAL SYSTEM
SOURCE WITH INPUT POWER (WATTS) IS PROPAGATED FROM THE SOURCE
O A GRADIENT-INDEX LENS (PLANO-CONVEX OR FLAT) AND FROM THE 
AT ANY POINT BEHIND THE LENS) AND THE POWER IS COUPLED INTO
    THE FIBERS AND PROPAGATED TO. THE OUTPUT, PLANE OF THE FIBERS
```



```
    LENS INPUT PARAMETERS
    A=QUADRATIC GRADIENT CONSTANT (1/MM)
    N=REFRACTIVE INDEX ON AXIS
    RD=RADIUS OF CURVATURE OF CONVEX SURFACE (mm)
    P=LENS PIICH
    PHI=LENS DIAMETER (mm)
    FIBER INPUT PARAMETERS 
    NA=NUMERICAL APERTUURE
    SOURCE INPUT PARAMETERS
    WL=WAVELENGTH IN MICRONS
    TS=SOURCE DIVERGENCE ANGLE (DEGREES)
    TDL =SORCE DIVERGING IENS TRANSMISSION
    AZ =AZIMUTH ANGLE
    EL=ELEVATION ANGLE
    GEOMETRICAL INPUT PARAMETERS
        L=DISTANCE FIBERS ARE FROMM BACK OF LENS (MM)
        THE PROGRAM CALCULATES THE FOILOWING VALUES
        S=DISTANCE FROM EACK OF LENS TO FOCAL POINT (MM)
        EFL=FOCAL LENGTH (MM)
        COORDINATES
        LMNS COORDINATESS ENIRANCE RAY HEIGHIS (mn) INN THE X-PLANE
        *)
        {=ARRAY OF EXIT RAY ANGLES FROM LENS ALIMMUIH
        =ARRAY OF EXITIRA
    M,
        HFX(I)=ARRAY OF RAY HEIGHTS AT FIEER (mm| IN THE X-PLANE 
        *)
            THE SEPERAMIONI BETWEEN THE CENIROID
        PIX(I)=IF PIX IS I THEN FIBER IS ILLULIINATED IF PIX IS O
        THEN FIEER HAS NO ILLUMINATION
    PIXL(I)= IS THE LENGIH OF OVERLAP OF THE SPOT ON THE I(TH) FIBER
    INTENSITY PARAMETERS
    GL=INTENSITY FROM POINT SOURCE INCIDENT AT GRIN LENS (WATTS)
    IFP(r) =INTENSITY AI THE FIBER P: ANE
    PF(I)=ARRAY OF POWER LEAVING FIEERS CALCULATED BY UU*
        WHERE U IS GIVEN BY THE DIFFRACTION INTEGRAL
        SINT = SIN PORTION OF THE DIFFRACTION INTEGRAL
```



```
        EIA(10).OIY (10), HFYDIMENSION STATEMENTS**.*****)
        MIY(10). HFY(10). HIY(10), PF(2O
```



```
        MCR16E,2)AREA(16),SEP(16), PIX(16), PIXL(16) PIXV(16)
```

        L(25), KF (25), PQ(25), AZ (16), AZR(16), EL(16), ELR (16)
    
.....................................
THIS PORTION OF THE PROGRAM COLLECTS THE BASIC PARAMETERS
M NEEDED TO PERFORM THE CALCULATIONS

"WHAT IS NUMERICAL APERTURE OF LENS? "~NAL
"WHAT IS BACK OF LENS TO FIBER DISTTANCE (mm)? ".
"WHAT IS NUMERICAL APERTURE (NA) OF FIBER? "NA
"WHAT IS BACK OF LENS TO FIBER DISTAN
"WHAT IS NUMERICAL APERTURE (NA) OF F
"WHAT IS FIBER DIAMETER (mm) ?
"WHAT IS FIBER DIAMETER (mm)? "FPHI
"WHAT IS SOURCE WOWER SNGTH (TICrons)? "WLM
"WHAT IS SIVERGENCE ANGLE OF SOURCE (DEGREES)?", TS
"WHAT IS SOURC WAVE ANGLE OF SOURCE (D.
"WHAT IS DRIMUTH ANGLE (DEGREES)? "AZ
"WHAT IS
TRANSMISSION OF SOURCE DIVERGING LENS? ",TDL




APPENDIX C
SOFTWARE LISTING FOR SINGLE-POINT DETECTION

```
SIXTEEN CHANNEL INPUT MULTIAPERTURE ARRAY LASER SENSOR
```

SIXTEEN CHANNEL INPUT MULTIAPERTURE ARRAY LASER SENSOR
CLS
CLS
CLEAR 49152!
CLEAR 49152!
SIM TRR(15)
SIM TRR(15)
SNG
SNG
D "DASH16.BIN", O
D "DASH16.BIN", O
DIO% (8)
DIO% (8)
IMP(16
IMP(16
M, by Roy A. Walters. Ph.D.. LASER SENSOR":PRINT, (UCF, Orlando, F1.)"
M, by Roy A. Walters. Ph.D.. LASER SENSOR":PRINT, (UCF, Orlando, F1.)"
FLAG%
FLAG%
O
O
ASH16 (MD%, DIO%(O),FLAG%)
ASH16 (MD%, DIO%(O),FLAG%)
channel scan limits
channel scan limits
MD%=1(0)
MD%=1(0)
=15
=15
4:DIO%(N)=O:NEXT N
4:DIO%(N)=O:NEXT N
ASH16 (MD%, DIO%(O), FLAG%)
ASH16 (MD%, DIO%(O), FLAG%)
N)=0
N)=0
N
N
LAG%=O
LAG%=O
PRINT"This programgives the azimuth and elevation of a laser source Data is "
PRINT"This programgives the azimuth and elevation of a laser source Data is "
PRINT "based on COORDINATE inputs to the CALIE program.
PRINT "based on COORDINATE inputs to the CALIE program.
uUse only STEADY STATE Tight sources.

```
            uUse only STEADY STATE Tight sources.
```




```
            "If you have not changed fiber optic connections, this step is not necessary"
```

            "If you have not changed fiber optic connections, this step is not necessary"
    CLS
CLS
AR(16)
AR(16)
F NORM\$ = "N" OR NORM\$ = "n" THEN GOTO 85O
F NORM\$ = "N" OR NORM\$ = "n" THEN GOTO 85O
PRINT "Place diffusion sheet in front of lens and light it up from the frent"
PRINT "Place diffusion sheet in front of lens and light it up from the frent"
"Adjust, intensity unt il all channels fall below saturation (about 3500),
"Adjust, intensity unt il all channels fall below saturation (about 3500),
"When all 16 channels are within bounas. enter S"
"When all 16 channels are within bounas. enter S"
"Channels below 50 will be deleted from the file because they are defective"
"Channels below 50 will be deleted from the file because they are defective"
%=3
%=3
= O TO 15
= O TO 15
CALL DASH16 (MD%,DIO%(O), FLAG%)
CALL DASH16 (MD%,DIO%(O), FLAG%)
8+S.1:PRINT USING"channel \#\# data = \#\#\#\#\#\#\#":S.DIO%(O)
8+S.1:PRINT USING"channel \#\# data = \#\#\#\#\#\#\#":S.DIO%(O)
NEXTTS
NEXTTS
INKEY\$
INKEY\$
THEN GOTO 620
THEN GOTO 620
PRINT,"I am busy doing the normalization. DO NOT TOUCH THE LIGHT !!!!"
PRINT,"I am busy doing the normalization. DO NOT TOUCH THE LIGHT !!!!"
3:010%(O) =0 : AV =100
3:010%(O) =0 : AV =100
L=1 TO AV
L=1 TO AV
15
15
l
l
\
\
CAK=0
CAK=0
FOR I= \& TO AV
FOR I= \& TO AV
F
F
IO" 1. "NORM". 2048
IO" 1. "NORM". 2048
OPEN "O"\1."NORM".204
OPEN "O"\1."NORM".204
OPEN "O" 1'."NORM". 2048
OPEN "O" 1'."NORM". 2048
P(IN=O "FIBER " ";I;" REMOVED FROM LIST"
P(IN=O "FIBER " ";I;" REMOVED FROM LIST"
N GOTO 82O
N GOTO 82O
NORMALIZATION MULTIPLIER
NORMALIZATION MULTIPLIER
lil SO THEN GO
lil SO THEN GO
DIO%(O),FLAG%)
DIO%(O),FLAG%)
NERATINGGTH
NERATINGGTH
P(I)=PEAK P(I)
P(I)=PEAK P(I)
MRINTH1
MRINTH1
SE
SE
INPUT "NUMBER OF SAMPLES TO AVERAGE ? ".NS
INPUT "NUMBER OF SAMPLES TO AVERAGE ? ".NS
"I""·员"NORM"

```
            "I""·员"NORM"
```

```
\N\vec{Na}
DIM AZ{ 16
    OR K=O TO \5
INPUT A2 AZ(K):INPUT #2,EL(K)
REM PRINT "normalization operator":ND(K);
    NEXT K
    CLOSE
INPUT "DEFINE SATURATION LEVEL ( (-4OOO) " = "SSAT
GOSUB 1960
    FOR ing data
    AR(L)
    NEXT
    MD%=3
    DIO%(O)=0
    FOR R=O=O TOS 15
    CARLLDASH16 (MD% DIO%(O), FLAG%)
    NEXT ?
    FOR I =0 TO 15
    AR (I )=
    PRINT AR(15)
    T=O (1)
        \MEROINGG OUT DEFECTIVE F
        REMPRINT "valid input data":AR(I);
        NEXT I
        CLS
        I
        CRINT "SCAN CHANNELSS=":T
        Z$=INKEY$ ENNOUGH
```



```
        'NORMALIZING THE AR(I) SENSOR DATA
    FOR K=0 TO 15
    AR(K)=AR(K)*ND(K)
    REMPRINT "normalized data":AR(K);
    NEXT K
    ifind the largest three locator in AR(K) array TOP3(O) is largest
    TO1ndothe targestt three locat 
    MAX=0
    FOR= J=0 iO 15
    FOR J)=0 TO 1
    REM PRINT TR(J):
    NEXTJ
    FOR V=0 TO 15
    IF TR(V)>MAX THEN TOP3(O) = V
    MAX = TR(TOP3(O))
    NEXT V
43O NEX(TOP3(O))=0
1440 MAX=0
1450 FOR V=0 TO 15
460 IF TR(y)>MAX THEN TOP3(1) = V
1470 MAX=TR(TOP3(1))
4490 NEXTTVV (1)) =0
1500 MAX = = O TO 15
520 IF TR(V)>MAX THEN TOP3(2)=V
1530 MAX = TR(TOP3(2))
1540 REM PRINT TR(V):
550 NEXT V
1560 :PRINT AR(TOP3(0)),AR(TOP3(1)),AR(TOP3(2))
lol
1600 AR TOP3, (TOP3(0)), AR(TOP3(1)), AR(TOP3(2))
1620 'algoritnm for az 'el of point spurce
670 PO= OOP3(O):P1=TOP3(1):P2=TOP3(2)
lol
```



APPENDIX D
SOFTWARE LISTING FOR TWO-POINT DETECTION



```
880 MD%=3:DIO%(O)=0:AV = 100
890 FOR =1 TO AV
M CALL DASH16 (MD% 
00%6%
```



```
NNN
```

        S
    ```
        S
    EAK S
    EAK S
        O
        O
        ={ TO AV 15
            (I)=PO\IOTOAV
                V
                    OPEN "O" 1. "NORM", 2048
            FOR I=0 to 15
                            #
                    GI <SO THEN GOTO 1070
                THE NORMALIZATION MULTIPLIER
                    REM GENERATING
                    PRINTA
                    NEXT I
            "NUMBER OF SAMPLES to AVERAGE ?",NS
                    PRINT
                    OPEN
                    OPN "II",1."NOR
                    IN INOK
```



```
                    NEMTPRI
            INPUT "DEFINE SATURATIONNLEVEL (4OOO) =".SAT
            REM GET CALIBRATION AND SENSITIVITY DATA. SAT
                    GOSUB A2OO DATA RETURN TO THIS POINT
                    AR(L)=0
                    NEXTLL
                    MD%=3
                    DIO%(0)=0
                    NOR R=0 TO NS
                    CALL DASH16 (MDODIO%(O),FLAG%)
                    AR(z)=AS(Z)+DIO%(0)
                    NRXT Z
                    NERTR R TO 15
                    AR(I)=AR(I)/NS
                    NEXT
        I AR(15)
PRIN
IF AR(I)>SAT THEN AR(I)=O
REM ZEROING OUT DEFECTIVE FIBERS
        ZEROING OUT DEFECTIV
IF AR (I )>1 THEN T =T+1
REM PRINT "VALID INPUT DATA ",AR(I)
NEXT I
NEXT
PRINT "SCAN CHANNELS = ":T
PRINT "SCAN CHANNELS
        T>2 "THEN GOTO 15.4NTS. YOU ARE EITHER TOO WEAK OR SATURATED."
PRINT "NOT ENOUGHY POIN
        Z$="S" THEN END
GOTO 129MO 
GOTO 129O 
```



```
RRMM PRI
* (1)
    I={ TO (15 /NG*A)*PI/ 180
        I
GOTO 1830
lol
AZR(K
ELR(K)=EL{K
ELKK}=\mp@code{ELR(K
FOR K=O TO 15
```





## REFERENCES

(1) Baradar, A. 1983. "Spacially Sampled Multi-Aperture Optical System for Robot Vision. Masters Thesis. University of Central Florida.
(2) Schneider, R. 1983. "Energy Transfer Technology: Wide Field Optics Technique". AFATL-TR-83-23.
(3) Walters, R. and G. Boreman. 1986. "White Paper on Multi-Aperture ptical Systems". Department of Electrical Engineering and Communication Sciences. University of Central Florida.
(4) Walters, R. and B. Mathews. 1983. "Aposition Multi-Aperture Optical Systems Operating in Signature and Pseudo Space". Proceedings of the National Aerospace and Electronics Conference. Dayton, Ohio (May 17-19).
(5) Kellog, S. 1982. "Theoretical Modeling for Detectivity on Resolution Comparison of Single Aperture and Multiple Aperture Imaging Systems". Research Report. University of Central Florida.
(6) Mathews, B.E. and R.A. Walters. 1983. "Random Effects on MultiAperture Element Density:". Proceedings of the National Aerospace and Electronics Conference. Dayton, Ohio (May 17-19).
(7) Seyrofi, K. 1973. Electro-Optical Systems Analysis. ElectroOptical Research Company.
(8) Boyd, R. 1983. Radiometry and the Detection of Optical Radiation. John Wiley and Sons.
(9) Yariv, A. 1976. Introduction to Optical Electronics. Holt, Rhinehart and Winston.
(10) SELFOC Handbook. 1979. New York: Nippon Sheet Glass America.
(11) Goodman, J. 1968. Fourier Optics. McGraw Hill Book Company. San Francisco, CA.
(12) Wendland, P. and L. Wendland. 1987. "Using Far Field Scanning as a Diagnostic Tool". Fiber Optic Product News (May).
(13) Cherin, Allen. 1983. An Introduction to Optical Fibers. New York: McGraw Hill.

## REFERENCES - Continued

(14) Barnoski, M. 1981. Fundamentals of Optical Fiber Communications. Academic Press.
(15) Hecht, E. and A. Zajac. 1979. Optics. Addison-Wesley Publishing Company, Inc., Reading, MA.

