# Game-Theoretic Analysis for a Supply Chain With Distributional and PeerInduced Fairness Concerned Retailers 

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Supported by the National Natural Science Foundation of China (No.71271199, No.70901067) and the Science Fund for Creative Research Groups of NSFC (No.71121061).

Received 2 December 2013; accepted 6 March 2014


#### Abstract

In this paper, we consider a supply chain system with one supplier and two homogeneous retailers to investigate the influence of distributional and peer-induced fairness concerns on supply chain. The Nash bargaining solution is used as distributional fairness reference and the first retailer's monetary payoff is used as the peer-induced fairness reference. We first analyze the decision-making process of the fairness concerned retailers under a given wholesale price and make a comparison with the fairnessneutral counterparts, then we derive the supplier's optimal wholesale price and the retailers' corresponding optimal order quantity in equilibrium. The results show that the second retailer orders less product and receives a higher wholesale price than the first retailer. The peer-induced fairness concerned retailer is in a worse position than the distributional fairness concerned retailer.


Key words: Supply chain management; Game theory; Distributional fairness; Peer-induced fairness

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## INTRODUCTION

Traditional theories in economics assume that individuals are self-interested in their own material payoffs. Recent advances in behavioral economics have relaxed the assumption and shown that people are not purely selfinterested and always exhibit social preferences in many real-life situations (Leon, 1954), such as bounded rationality, risk-seeking, loss-averse and so on (Su, 2008; Schweitzer \& Cachon, 2000; Loch \& Wu, 2007; Wang \& Webster, 2009). Researches in behavioral operation also show that people pay much attention to fairness in their daily life (Fehr \& Schmidt, 1999). Kahneman, Knetsch, and Thaler (1986) suggest that firms, like individuals, are also motivated by the concern of fairness in business relationships. In this paper, we focus on the fairness concern and investigate how it affects the supply chain with one supplier and two retailers.

There is a growing stream of literatures which study the role of fairness concern in the context of supply chain. Cui, Raju, and Zhang (2007) incorporate fairness concern into a simple dyadic supply chain and investigate the impact of fairness concern on channel coordination. The results show that the supplier can coordinate the channel by charging a simple constant wholesale price above his marginal cost. Loch and Wu (2008) provide an experimental evidence proving that status seeking induces more competitive behavior by both players and drives down individual performance and overall efficiency in supply chain. Ho and Zhang (2008) confirm the existence of fairness concern in the context of supply chain and give a descriptive utility function about fairness concern. The fairness concern between retailer and supplier has contributed significantly to the coordination failure and efficiency loss in supply chain when they are not fully informed of other member's fairness concern (Katok, Olsen, \& Pavlov, 2012; Katok \& Pavlov, 2012; Pavlov \& Katok, 2012). Du, Du, Liang, and Liu (2010) analyze how the retailer's fairness concern affects channel coordination under three different contracts
and conclude that the retailer's fairness concern will not change the state of supply chain coordination. Du, Nie, Chu, and Yu (2012) extend the model by assuming that the supplier and retailer are both fairness concerned and establishes a Nash bargaining framework to study the newsvendor problem. The results show that the channel efficiency decreases because of the fairness concern and the state of channel coordination is independent of the fairness concern. Ho and Su (2009) is the first to investigate peerinduced fairness by analyzing two independent ultimatum games theoretically and experimentally, which are played sequentially by a leader and two followers. It suggests that peer-induced fairness concern between followers is two times stronger than distributional fairness concern between leader and follower.

This paper considers a 1-supplier and 2-retailers supply chain and investigates the role of distributional and peer-induced fairness concerns on supply chain decisionmaking process. We first analyze how distributional fairness affects both wholesale price and order quantity between a supplier and a retailer. Then we extend the model by introducing peer-induced fairness in the system where the supplier must determine his wholesale price offers to two retailers sequentially. First, the supplier offers a wholesale price to the first retailer. Then, the second retailer observes the market signal and knows exactly how much the first retailer earns. Finally, the supplier makes a wholesale price offer to the second retailer. The second retailer will make his order quantity based on what the first retailer received in order not to be behind. The supplier's optimal wholesale price and the retailer's optimal order quantity may change as a result of peer-induced fairness concerns.

Fairness concerns are generally characterized by incorporating profit disparity into the utility function. Bolton (1991) and Rabin (1993) suggest that both positive and negative disparity will reduce individual's utility. De Bruyn and Bolton (2008) improve the asymmetric loss function and predict the influence of fairness concern on bargaining. Loch and Wu (2008) investigate the effects of social preferences on the performance of a supply chain and put forward a simple utility function of fairness concern: $U_{i}=\pi_{i}+\theta_{i \pi j}$. In this paper, we bring in two distributional and one peer-induced fairness parameters and suppose that the individual's utility increases when his profit is larger than the fairness reference and decreases when his profit is smaller than the fairness reference.

In this paper, Nash bargaining solution is used as distributional fairness reference to formally depict perceptively fair compromise, which is a new perspective to study fairness concerns in a supply chain. According to Nash Jr (1950), we may regard Nash bargaining solution as representing all anticipations that the two bargainers might agree upon as fair bargains. Nash bargaining solution is characterized by a set of axioms (i.e., Pareto efficiency, Symmetry, Invariant to affine transformations and Independence of irrelevant alternatives) that are
appealing in defining fairness (Nash Jr, 1950; Osborne \& Rubinstein, 1994; Touati, Altman, \& Galtier, 2006). Additionally, the first retailer's realized monetary payoff is used as peer-induced fairness reference. We assume two homogeneous retailers operating in two separate markets, which have same market demand. The unique difference of them is that the second retailer is peer-induced fairness concerned. Therefore, the second retailer regard what the first retailer get in the first game as his peer-induced fairness reference.

Under the background of newsvendor model, we establish a distributional and peer-induced fairness concern framework based Nash bargaining solutions. The general model predicts that distributional fairness results in a smaller order quantity determined by the retailer and the second retailer's optimal order quantity is larger than that of retailer 1 when the supplier provides them a same wholesale price. In addition, the model predicts that the second retailer tends to be radical in order quantity comparing with the first retailer and receives a higher wholesale price offer. The supplier increases the wholesale price offer to the second retailer, because the latter must choose an order quantity to balance the opposing forces of not being behind the supplier and not being behind the first retailer.

The rest of the paper is organized as follow. In section 2, we will give the notations of this paper and the benchmark newsvendor model, in which individuals only pay attention to the material payoffs. Section 3 will first show us how to establish the distributional and peer-induced fairness model and then analyze the two retailers' decisionmaking process. The equilibrium results of the two games are depicted in the last part of Section 3. The role of distributional and peer-induced fairness concern on the supplier's optimal wholesale price and the retailer's optimal order quantity are analyzed as well. Section 4 summarizes our main conclusions and suggests further directions.

## 1. BASIC MODEL: NO FAIRNESS

Let us begin with some notations. We consider a supply chain with one supplier (denoted by $S$ ) and two retailers (denoted as $R_{1}$ and $R_{2}$ ). The retailers face the newsvendor problem: they must choose an order quantity $q$ before the start of a selling season that has stochastic demand under a given wholesale price $w$ determined by the supplier. We assume that the two retailers operate in two separate markets and they have the same but independent demand $D$. Let $F$ be the distribution function of demand and $f$ its density function: $F$ is differentiable, strictly increasing and $F(0)=0$. Let $\bar{F}(x)=1-F(x)$ and $u=E[D]$. The retailer price is $p$ and the production cost is $c$. the distributional fairness concern parameters are $\theta_{s}, \theta_{r}$ and the peer-induced fairness concern parameter is $\lambda$.

In this section, we consider a setting where there is no fairness. That is to say, the supplier and the retailers are only interested in their own monetary payoffs. The
two retailers play two identical stackelberg games with the supplier sequentially, so they have a symmetric relationship in the supply chain. Thus, the second game is a repetition of the first game. The profits of the retailers, the supplier and the whole channel are given below respectively:
$\pi_{r i}\left(q_{i}\right)=p S\left(q_{i}\right)-w_{i} q_{i} ; \pi_{s, i}=\left(w_{i}-c\right) q_{i} ;$
$\pi_{i}=p S\left(q_{i}\right)-c q_{i}$
Where $i=1,2$ represents the two retailers and $\mathrm{S}(\mathrm{q})=\mathrm{q}-\int_{0}^{q} F(y) d y$. According to the traditional normative theories, retailer $i$ 's and channel's optimal order quantities in the decentralized and centralized cases, denoted by $q_{i}^{*}$ and $q_{i}^{0}$, satisfy the following conditions:

$$
\begin{equation*}
F\left(q_{i}^{*}\right)=\left(p-w_{i}\right) / p ; F\left(q_{i}^{0}\right)=(p-c) / p \tag{2}
\end{equation*}
$$

The left of Equation (2) is retailer $i$ 's best response function to the supplier's wholesale price decision. The supplier will take the retailer $i$ 's response into consideration and make his optimal decision. Substituting this function into the supplier's profit function and maximizing the supplier's profit, we can easily get the supplier's optimal wholesale price $w_{i}^{*}$ and the retailer $i$ 's corresponding optimal order quantity $q_{i}^{*}$ that satisfy the following conditions:

$$
\begin{equation*}
w_{i}^{*}\left(q_{i}^{*}\right)=p \bar{F}\left(q_{i}^{*}\right) ; p \bar{F}\left(q_{i}^{*}\right)\left[1-g\left(q_{i}^{*}\right)\right]=c \tag{3}
\end{equation*}
$$

Where $\mathrm{g}(x)=x f(x) / \bar{F}(x)$, namely generalized failure rate (GFR) function. We assume that the GFR function is increasing in its probability spaces and we call it increasing generalized failure rate (IGFR) function. The IGFR function captures most common distributions, such as the power, the normal and the exponential etc. therefore, it is reasonable to introduce the IGFR function into this paper.

## 2. BEHAVIORAL MODEL WITH FAIRNESS CONCERNS

### 2.1 Model Formulation

In this section, we extend the basic model by incorporating the distributional fairness concerns and peer-induced fairness concerns into the supply chain. Specifically, the supplier and the retailers are all distributional fairness concerned and the second retailer who operates behind is peer-induced fairness concerned. The order of events is as follow. First, the supplier offers retailer 1 the wholesale price $w_{1}$, and retailer 1 makes his optimal order quantity $q_{1}^{*}$. Then, the retailer 2 observes the market and knows exactly how much retailer 1 earns. Based on this information, retailer 2 makes his optimal order quantity $q_{2}^{*}$ responding to $w_{2}$ in order not to be behind.

Let us define the individual's utility functions. In game 1, a linear form is used to formulate the utility of each member in the supply chain as follows.
$u_{r 1}=\pi_{r 1}-\theta_{r}\left(\bar{\pi}_{r 1}-\pi_{r 1}\right) ; u_{s, 1}=\pi_{s, 1}-\theta_{s}\left(\bar{\pi}_{s, 1}-\pi_{s, 1}\right)$
Where $\bar{\pi}_{r 1}$ and $\bar{\pi}_{s, 1}$ denote the retailer's and supplier's Nash bargaining fairness references respectively. Obviously, $\pi_{r 1}+\pi_{s, 1}=\bar{\pi}_{r 1}+\bar{\pi}_{s, 1}$. Their utilities decrease when their real monetary payoffs are lower than the Nash bargaining fairness references and appear a converse trend otherwise. According to the Nash's axiomatic definition, Nash bargaining solution is derived by maximizing the following model.

$$
\begin{array}{ll} 
& \max u_{r 1} u_{s, 1} \\
\text { s.t. } & \pi_{r 1}+\pi_{s, 1}=\pi_{1}  \tag{5}\\
& u_{r 1}>0, u_{s, 1}>0
\end{array}
$$

We can get the supplier's utility using the right side of Equation (4).

$$
\begin{equation*}
u_{s, 1}\left(\pi_{r 1}\right)=\pi_{1}-\pi_{r 1}-\theta_{s}\left(\pi_{r 1}-\bar{\pi}_{r 1}\right) \tag{6}
\end{equation*}
$$

Thus,
$u_{r 1} u_{s, 1}\left(\pi_{r 1}\right)=\left[\pi_{r 1}-\theta_{r}\left(\bar{\pi}_{r 1}-\pi_{r 1}\right)\right]\left[\pi_{1}-\pi_{r 1}-\theta_{s}\left(\pi_{r 1}-\bar{\pi}_{r 1}\right)\right]$
Taking the second-order derivation of Equation (7) with respect to $\pi_{r 1}$, we have $\partial^{2} u_{r 1} u_{s, 1} / \partial^{2} \pi_{r 1}=-$ $2\left(1+\theta_{\mathrm{s}}+\theta r+\theta r \theta_{\mathrm{s}}\right)<0$. Hence, $u_{r 1} u_{\mathrm{s}, 1}\left(\pi_{r 1}\right)$ is strictly concave about $\pi_{r 1}$ and there exists a unique optimal solution, i.e., $\pi_{r 1}^{*}$, that satisfies the first-order condition $\partial u_{r 1} u_{s, 1} / \partial \pi_{r 1}=0$. Then, we can derive
$\bar{\pi}_{r 1}=\frac{1+\theta_{r}}{2+\theta_{s}+\theta_{r}} \pi_{1}, \bar{\pi}_{s, 1}=\frac{1+\theta_{s}}{2+\theta_{s}+\theta_{r}} \pi_{1}$
In game 2, the retailer 2 is not only distributional fairness concerned but also peer-induced fairness concerned. We use a linear form to formulate each member's utility functions in the supply chain, too. The retailer's and supplier's utility functions are given below respectively.
$u_{r 2}=\pi_{r 2}-\theta_{r}\left(\bar{\pi}_{r 2}-\pi_{r 2}\right)-\lambda\left(\pi_{r 1}-\pi_{r 2}\right)$,
$u_{s, 2}=\pi_{s, 2}-\theta_{s}\left(\bar{\pi}_{s, 2}-\pi_{s, 2}\right)$
Here, the Nash bargaining solution is used as the distributional fairness concern reference and the peer retailer 1's payoff is used as the peer-induced fairness concern reference. If retailer 2 get less than that of retailer 1 , he will feel unfair and his utility decreases.

Accordingly, the Nash bargaining solution $\left(\bar{\pi}_{r 2}, \bar{\pi}_{s, 2}\right)$ is derived by maximizing the Nash product $u_{r 2} u_{s, 2}$. Similarly to game 1, we can get the Nash bargaining solution as follow.

$$
\begin{align*}
& \bar{\pi}_{r 2}=\frac{\left(1+\theta_{r}+\lambda\right) \pi_{2}+\left(\lambda+\lambda \theta_{s}\right) \pi_{r 1}}{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda} \\
& \bar{\pi}_{s, 2}=\frac{\left(1+\theta_{s}+\lambda \theta_{s}+\lambda\right) \pi_{2}-\left(\lambda+\lambda \theta_{s}\right) \pi_{r 1}}{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda} \tag{10}
\end{align*}
$$

The two games are linked together. What the supplier offers to retailer 1 in game 1 affects the payoff of retailer 1 , the retailer 1's payoff is the peer-induced fairness concern reference in game 2, thus what the supplier offers in game 1 affects the whole game process. We can solve the game using the standard backward induction principle. In the second game, the supplier choose $w_{2}$ to maximize $u_{s, 2}$. In the first game, the supplier choose $w_{1}$ to maximize $u_{s, 1}+u_{s, 2}$.

### 2.2 Decision-Making of the Fairness Concerned Retailers

In this section, we will analyze how the retailers behave under both decentralized and centralized channel. We work backward to derive the retailers' optimal decisions. In the second game, we can derive the retailer's optimal order quantity $q_{2, \lambda}^{*}$ by maximizing his utility $u_{r 2}$ given by Equation (9); In the first game, we can derive the retailer's optimal order quantity $q_{1, \theta}^{*}$ by maximizing his utility $u_{r 1}$ given by Equation (4).

Proposition 1. The two retailers' utility functions are strictly concave about their order quantities $q_{1}$ and $q_{2}$ and there are two optimal order quantities $q_{1, \theta}^{*}$ and $q_{2, \lambda}^{*}$ that maximize their own expected utilities. The two optimal order quantities satisfy the following conditions:

$$
\begin{align*}
F\left(q_{2, \lambda}^{*}\right) & =1+\frac{\theta_{r}}{2+\theta_{s}+\lambda \theta_{s}+2 \lambda} \frac{c}{p}-\frac{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda}{2+\theta_{s}+\lambda \theta_{s}+2 \lambda} \frac{w_{2}}{p} \\
& =1-\frac{w_{2}}{p}+\frac{\theta_{r}}{2+\theta_{s}+\lambda \theta_{s}+2 \lambda} \frac{\left(c-w_{2}\right)}{p}  \tag{11}\\
F\left(q_{1, \theta}^{*}\right) & =1-\frac{2+\theta_{s}+\theta_{r}}{2+\theta_{s}} \frac{w_{1}}{p}+\frac{\theta_{r}}{2+\theta_{s}} \frac{c}{p}=1-\frac{w_{1}}{p}+\frac{\theta_{r}}{2+\theta_{s}} \frac{\left(c-w_{1}\right)}{p} \tag{12}
\end{align*}
$$

Proof. We will prove the conclusion in the second game first. Taking the first-order derivation of $u_{r 2}$ with respect to $q_{2}$, we have
$\frac{\partial u_{r 2}}{\partial q_{2}}=\left(1+\theta_{r}+\lambda\right)\left(p-w_{2}\right)-\frac{\theta_{r}\left(1+\theta_{r}+\lambda\right)}{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda}(p-c)$
$-\frac{\left(2+\theta_{s}+\lambda \theta_{s}+2 \lambda\right)\left(1+\theta_{r}+\lambda\right)}{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda} p F\left(q_{2}\right)$
Taking the second-order derivation of $u_{r 2}$ with respect to $q_{2}$, we have

$$
\begin{equation*}
\frac{\partial^{2} u_{r 2}}{\partial q_{2}^{2}}=-\frac{\left(2+\theta_{s}+\lambda \theta_{s}+2 \lambda\right)\left(1+\theta_{r}+\lambda\right)}{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda} p f\left(q_{2}\right)<0 \tag{14}
\end{equation*}
$$

Hence, the second retailer's utility function is strictly concave about $q_{2}$ and there exists a unique maximum
that satisfies the first-order condition $\left.\partial u_{r 2}\left(q_{2, \lambda}^{*}\right) / \partial q_{2}\right)=0$. Then we can get Equation (11) by solving the first-order condition. We can also prove the conclusion in the first game by maximizing $u_{r 1}$ with respect to $q_{1}$.

In the following discussion, we will show how the retailer's distributional fairness concern and peer-induced fairness concern biases his decision. That is, the difference in the optimal decisions between a fairness-neutral retailer and a fairness concerned retailer.

Proposition 2. For a fairness concerned retailer, his optimal order quantity is smaller than its fairness-neutral counterpart, which is smaller than the fairness-neutral channel's optimal order quantity.

$$
\begin{equation*}
q_{2, \lambda}^{*}<q_{2}^{*}<q_{2}^{o} ; q_{1, \theta}^{*}<q_{1}^{*}<q_{1}^{o} \tag{15}
\end{equation*}
$$

Proof. We can get this conclusion by comparing Equations (11), (12) with Equation (2) easily.

Proposition 2 tell us that a distributional fairness concerned retailer in the first game tend to be conservative in the interaction with the supplier in order not to be behind, so as the retailer with distributional fairness concern and peer-induced fairness concern in the second game. We also find that the second retailer's optimal order quantity is larger than that of retailer 1 when the supplier provides them a same wholesale price. It illustrates that the retailer's peer-induced fairness concern make him tend to be radical. The double marginalization effect and the fairness concern are the possible resources for the retailer's conservation. We can see from Equations (11) and (12) that the retailer's optimal order quantity is affected by both the distributional fairness concern parameters and the peer-induced fairness concern parameter. Proposition 3 will show us how the retailer's optimal order quantity changes with these parameters.

Proposition 3. (1) The two retailers' optimal order quantities are decreasing with their distributional fairness concern parameter $\theta_{r}$ while increasing with the supplier's distributional fairness concern parameter $\theta_{s}$. (2) The second retailer's optimal quantity is increasing with his peer-induced fairness concern parameter $\lambda$.

Proof. We can see from Equation (11) that the last term is negative, so the last term become smaller when $\theta_{r}$ become larger, that is, the retailer's optimal order quantity is decreasing with $\theta_{r}$. The rest conclusions of Proposition 3 can be revealed similarly. We can also verify the front conclusions by calculating the following equation:

$$
\begin{aligned}
& \frac{\partial q_{2, \lambda}^{*}}{\partial \lambda}=-\frac{\partial^{2} u_{r 2}\left(q_{2, \lambda}^{*}\right) / \partial q_{2} \partial \lambda}{\partial^{2} u_{r 2}\left(q_{2, \lambda}^{*}\right) / \partial q_{2}^{2}}>0 \\
& \frac{\partial q_{i, \lambda}^{*}}{\partial \theta_{r}}=-\frac{\partial^{2} u_{r i}\left(q_{i, f}^{*}\right) / \partial q_{i} \partial \theta_{r}}{\partial^{2} u_{r i}\left(q_{i, f}^{*}\right) / \partial q_{i}^{2}}<0 ; \frac{\partial q_{i, f}^{*}}{\partial \theta_{s}}= \\
& -\frac{\partial^{2} u_{r i}\left(q_{i, f}^{*}\right) / \partial q_{i} \partial \theta_{s}}{\partial^{2} u_{r i}\left(q_{i, f}^{*}\right) / \partial q_{i}^{2}}>0 . \quad i=1,2
\end{aligned}
$$

Proposition 3 indicates that the more the retailer is distributional fairness concerned, the less product he orders. And the more the retailer is peer-induced fairness concerned, the more product he orders. In other words, the distributional fairness concern make the retailer tend to be conservative and the peer-induced fairness concern make him tend to be radical.

We have analyzed the two retailers' decision-making under decentralized channel, in which they only focus on their own utilities and ignore the whole channel's utility. The decentralized decision will not usually maximize the whole channel's profit and it will cause the well-known effect "double marginalization". So we will show how the retailers make decision under centralized channel in the following part. In the second game, we can derive the centralized optimal order quantity $q_{2, \lambda}^{o}$ by maximizing the channel's utility $u_{2}=u r_{2}+u_{\mathrm{s}, 2}$; In the first game, we can derive the centralized optimal order quantity $q_{1, \theta}^{o}$ by maximizing the channel's utility $u_{1}=u r_{1}+u_{\mathrm{s}, 1}$.

Proposition 4. In the second game, the channel's utility function $u_{2}$ is concave about $q_{2}$ and there exists a unique optimal order quantity $q_{2, \lambda}^{o}$ that maximizes the whole channel's utility when $\theta_{s}\left(1+\theta_{s}\right) /\left(2+\theta_{s}\right)<1+\theta_{r}+\lambda$. The centralized optimal order quantity satisfies the following condition:
$F\left(q_{2, \lambda}^{0}\right)=1-\frac{c}{p}+\frac{\left(\lambda+\theta_{r}-\theta_{s}\right)\left(2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda\right)}{\left(1+\theta_{s}+\lambda \theta_{s}+\lambda\right)\left(1+\theta_{r}+\lambda-\theta_{s}\right)+(1+\lambda)\left(1+\theta_{r}+\lambda\right)} \frac{\left(c-w_{2}\right)}{p}$
Proof. Taking the first-order derivation of $u_{2}$ with respect to $q_{2}$, we have

$$
\begin{align*}
\frac{\partial u_{2}}{\partial q_{2}} & =\left(1+\theta_{r}\right)\left(p \bar{F}\left(q_{2}\right)-w_{2}\right)-\frac{\theta_{r}\left(1+\theta_{r}+\lambda\right)}{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda}\left(p \bar{F}\left(q_{2}\right)-c\right) \\
& -\frac{\left(1+\theta_{s}+\lambda \theta_{s}+\lambda\right) \theta_{s}}{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda}\left(p \bar{F}\left(q_{2}\right)-c\right)+\left(1+\theta_{s}\right)\left(w_{2}-c\right) \tag{17}
\end{align*}
$$

Taking the second-order derivation of $u_{2}$ with respect to $q_{2}$, we have
$\frac{\partial^{2} u_{2}}{\partial q_{2}^{2}}=-\frac{\left(1+\theta_{s}+\lambda \theta_{s}+\lambda\right)\left(1+\theta_{r}+\lambda-\theta_{s}\right)+(1+\lambda)\left(1+\theta_{r}+\lambda\right)}{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda} p f\left(q_{2}\right)$
We can see from Equation (18) that the equation will be negative when the fairness concern parameters satisfy the condition $\theta_{s}\left(1+\theta_{s}\right) /\left(2+\theta_{s}\right)<1+\theta_{r}+\lambda$, then the channel's utility function is concave about and there exists a unique optimal order quantity $q_{2, \lambda}^{o}$ that satisfies the first-order condition $\left.\partial u_{2}\left(q_{2, \lambda}^{o}\right) / \partial q_{2}\right)=0$. We can derive the centralized optimal order quantity that satisfies the condition of Equation (16).

In section 2, we have analyzed how the fairnessneutral retailer makes decisions under centralized channel. Comparing Equation (16) with Equation (2), we can get the following relationship between peerinduced fairness concerned and fairness-neutral channel's optimal order quantity: (1) there is $q_{2, \lambda}^{o}<q_{2}^{o}$ when $\theta_{s}<\lambda+\theta_{r}$; (2) there is $q_{2, \lambda}^{o}>q_{2}^{o}$ when $\theta_{s}>\lambda+\theta_{r}$ and $\theta_{s}\left(1+\theta_{s}\right) /$ $\left(2+\theta_{s}\right)<1+\theta_{r}+\lambda$; (3) the centralized optimal order quantity under fairness concerned channel doesn't exist when $\theta_{s}\left(1+\theta_{s}\right) /\left(2+\theta_{s}\right)>1+\theta_{r}+\lambda$. In other words, the fairness
concerned channel's optimal order quantity is smaller than the fairness-neutral counterpart when the supplier's distributional fairness parameter is relatively small and the opponent trend appears when the supplier's fairness parameter is relatively high.

Proposition 5. In the first game, the channel's utility function $u_{1}$ is concave about $q_{1}$ and there exists a unique optimal order quantity $q_{1, \theta}^{o}$ that maximizes the whole channel's utility when $\theta_{s}<\theta_{s}^{*}$, where $\theta_{s}^{*}$ satisfies $2\left(1+\theta_{r}\right)+\theta_{s}^{*}\left(\theta r-\theta_{s}^{*}\right)=0$. The centralized optimal order quantity $q_{1, \theta}^{o}$ satisfies the following condition:
$F\left(q_{1, \theta}^{0}\right)=1-\frac{c}{p}+\frac{\left(\theta_{r}-\theta_{s}\right)\left(2+\theta_{s}+\theta_{r}\right)}{\left(1+\theta_{r}\right)\left(2+\theta_{s}\right)-\theta_{s}\left(1+\theta_{s}\right)} \frac{\left(c-w_{1}\right)}{p}$
Proof. Taking the first and second order derivation of with respect to, we have

$$
\begin{align*}
& \frac{\partial u_{1}}{\partial q_{1}}=\frac{\left(1+\theta_{r}\right)\left(2+\theta_{s}\right)-\theta_{s}\left(1+\theta_{s}\right)}{2+\theta_{s}+\theta_{r}} p \bar{F}\left(q_{1}\right) \\
& +\frac{\theta_{r}\left(1+\theta_{r}\right)-\left(1+\theta_{s}\right)\left(2+\theta_{r}\right)}{2+\theta_{s}+\theta_{r}} c+\left(\theta_{s}-\theta_{r}\right) w_{1}  \tag{20}\\
& \frac{\partial^{2} u_{1}}{\partial q_{1}^{2}}=-\frac{\left(1+\theta_{r}\right)\left(2+\theta_{s}\right)-\theta_{s}\left(1+\theta_{s}\right)}{2+\theta_{s}+\theta_{r}} p f\left(q_{1}\right) \tag{21}
\end{align*}
$$

We can see from Equation (21) that there must be a critical point that satisfies the equation $2\left(1+\theta_{r}\right)+\theta_{s}^{*}\left(\theta r-\theta_{s}^{*}\right.$ $)=0$. Hence, the second order derivation is negative when $\theta_{s}<\theta_{s}^{*}$, and there exists a unique optimal order quantity $q_{1, \theta}^{o}$ that maximizes the fairness concerned channel's utility. Then we can derive the centralized optimal order quantity as Equation (19) showed. When $\theta_{s}<\theta_{s}^{*}$ , the centralized optimal order quantity under fairness concerned channel doesn't exist.

Comparing Equation (19) with Equation (2), we can get the following relationships between distributional fairness concerned and fairness-neutral channel's optimal order quantity: (1) there is $q_{1, \theta}^{o}<q_{1}^{o}$ when $<\theta_{s}<\theta_{r}$, that is, the distributional fairness concerned channel's optimal order quantity is smaller than the fairness-neutral counterpart; (2) there is $q_{1, \theta}^{o}>q_{1}^{o}$ when $\theta_{r}<\theta_{s}<\theta_{s}^{*}$, an opponent trend appears.

### 2.3 Equilibrium Results

In this section, we will analyze the supplier's best strategy and the corresponding results of the supply chain taking the retailer's optimal response into account. The supplier is distributional fairness concerned and he plays two stackelberg games with the retailers sequentially. We first analyze the supplier's best strategy and the retailer's corresponding optimal order quantity in the second game. The supplier chooses $w_{2}^{*}$ to maximize his own utility in the second game $u_{s, 2}$.

Equation (11) implicitly gives an optimal order quantity for the peer-induced fairness concerned retailer at a given wholesale price. The inverse function can be derived from Equation (11) as follow.
$w_{2}\left(q_{2}\right)=\frac{2+\theta_{s}+\lambda \theta_{s}+2 \lambda}{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda} p \bar{F}\left(q_{2}\right)$
$+\frac{\theta_{r}}{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda} c$
Proposition 6. Under decentralized channel with distributional and peer-induced fairness concerns, the unique equilibrium solution consisting of the supplier's optimal wholesale price $w_{2}^{*}$ and the retailer's best response $q_{2, \lambda}^{*}$ must satisfy

$$
\begin{align*}
& w_{2}^{*}\left(q_{2, \lambda}^{*}\right)=\frac{2+\theta_{s}+\lambda \theta_{s}+2 \lambda}{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda} p \bar{F}\left(q_{2, \lambda}^{*}\right) \\
& +\frac{\theta_{r}}{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda} c  \tag{23}\\
& p \bar{F}\left(q_{2, \lambda}^{*}\right)\left[1-\frac{\left(1+\theta_{s}\right)\left(2+\theta_{s}+\lambda \theta_{s}+2 \lambda\right)}{2\left(1+\theta_{s}+\lambda \theta_{s}+\lambda\right)} g\left(q_{2, \lambda}^{*}\right)\right]=c \tag{24}
\end{align*}
$$

Proof. By incorporating Equation (22) into $u_{s, 2}$, we get $u_{\mathrm{s}, 2}\left(q_{2}, w_{2}\left(q_{2}\right)\right)$. Taking the first-order derivation of $u_{\mathrm{s}, 2}$ $\left(q_{2}, w_{2}\left(q_{2}\right)\right)$ with respect to $q_{2}$, we have

$$
\begin{align*}
& \frac{\partial u_{s, 2}\left(q_{2}, w_{2}\left(q_{2}\right)\right)}{\partial q_{2}}=\frac{2\left(1+\theta_{s}+\lambda \theta_{s}+\lambda\right)}{2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda}\left\{p \bar{F}\left(q_{2}\right)\right. \\
& \left.\left[1-\frac{\left(1+\theta_{s}\right)\left(2+\theta_{s}+\lambda \theta_{s}+2 \lambda\right)}{2\left(1+\theta_{s}+\lambda \theta_{s}+\lambda\right)} g\left(q_{2}\right)\right]-c\right\} \tag{25}
\end{align*}
$$

We know that the distribution function of demand and the GRF function are both increasing with the order quantity, so Equation (25) is decreasing with $q_{2}$. There exists a unique optimal order quantity $q_{2, \lambda}^{*}$ that maximizes $u_{\mathrm{s}, 2}\left(q_{2}, w_{2}\left(q_{2}\right)\right)$ and satisfies the first order condition $\partial u_{\mathrm{s}, 2}$ $\left(q_{2}, w_{2}\left(q_{2}\right)\right) /\left(\partial q_{2}\right)=0$. Therefore, the unique quantity $q_{2, \lambda}^{*}$ sold to the retailer at the optimal wholesale price $w_{2}^{*}=w_{2}^{*}$ $\left(q_{2, \lambda}^{*}\right)$ that maximizes the supplier's utility can be given by

$$
p \bar{F}\left(q_{2, \lambda}^{*}\right)\left[1-\frac{\left(1+\theta_{s}\right)\left(2+\theta_{s}+\lambda \theta_{s}+2 \lambda\right)}{2\left(1+\theta_{s}+\lambda \theta_{s}+\lambda\right)} g\left(q_{2, \lambda}^{*}\right)\right]=c
$$

Then, substituting Equation (22) into the above expression yields Equation (23).

While in the first game, the supplier chooses to maximize his own utility in both two games $u_{\mathrm{s}}=u_{\mathrm{s}, 1}+u_{\mathrm{s}, 2}$. Equation (12) gives the distributional fairness concerned retailer's best response function to a given wholesale price. The inverse function can be derived as follow.
$w_{1}\left(q_{1}\right)=\frac{2+\theta_{s}}{2+\theta_{s}+\theta_{r}} p \bar{F}\left(q_{1}\right)+\frac{\theta_{r}}{2+\theta_{s}+\theta_{r}} c$
By incorporating Equation (26) into $u_{s}$, it is easy to derive the supplier's optimal wholesale price $w_{1}^{*}$ and the retailer's corresponding optimal order quantity $q_{1, \theta}^{o}$ that satisfy Eqquations (27) and (28) as follow.
$w_{1}^{*}\left(q_{1, \theta}^{*}\right)=\frac{2+\theta_{s}}{2+\theta_{s}+\theta_{r}} p \bar{F}\left(q_{1, \theta}^{*}\right)+\frac{\theta_{r}}{2+\theta_{s}+\theta_{r}} c$
$p \bar{F}\left(q_{1, \theta}^{*}\right)\left[1-\frac{\left(2+\theta_{s}\right)\left(2+\theta_{s}+\theta_{r}+2 \lambda\right)}{4+2 \theta_{s}+2 \theta_{r}+2 \lambda \theta_{s}+4 \lambda-\lambda \theta_{r} \theta_{s}} g\left(q_{1, \theta}^{*}\right)\right]=c$
Corollary 1. Suppose $\theta_{r}<2$, the supplier's wholesale price offer to retailer 2 is higher than the wholesale price offer to retailer 1 , that is, $w_{2}^{*}>w_{1}^{*}$; the corresponding optimal order quantity of retailer 2 is smaller than that of retailer 1, that is, $q_{2, i}^{*}<q_{1, \theta}^{*}$.

Proof. We first analyze the retailers' optimal order quantity. Let
$X=\frac{\left(1+\theta_{s}\right)\left(2+\theta_{s}+\lambda \theta_{s}+2 \lambda\right)}{2\left(1+\theta_{s}+\lambda \theta_{s}+\lambda\right)} ;$
$Y=\frac{\left(2+\theta_{s}\right)\left(2+\theta_{s}+\theta_{r}+2 \lambda\right)}{4+2 \theta_{s}+2 \theta_{r}+2 \lambda \theta_{s}+4 \lambda-\lambda \theta_{r} \theta_{s}}$
Let $Z=X-Y$, we have
$Z=\frac{\lambda \theta_{s}\left(1+\theta_{s}\right)\left(2+\theta_{s}+\lambda \theta_{s}+2 \lambda\right)\left(2-\theta_{r}\right)}{2\left(4+2 \theta_{s}+2 \theta_{r}+2 \lambda \theta_{s}+4 \lambda-\lambda \theta_{r} \theta_{s}\right)\left(1+\theta_{s}+\lambda \theta_{s}+\lambda\right)}>0$
We can see from Equation (30) that Z is positive when $\theta_{r}<2$. Then we can rewrite Equation (24) as follow.
$p \bar{F}\left(q_{2, \lambda}^{*}\right)\left[1-(Y+Z) g\left(q_{2, \lambda}^{*}\right)\right]=c>p \bar{F}\left(q_{1, \theta}^{*}\right)\left[1-(Y+Z) g\left(q_{1, \theta}^{*}\right)\right]$
Equation (24) is decreasing with $q$, so the optimal order quantity of retailer 2 is smaller than that of retailer 1. Next, we show the difference between the wholesale price offer to retailer 1 and retailer 2. Equation (23) can be rewrite as follow.
$w_{2}^{*}\left(q_{2, \lambda}^{*}\right)=w_{1}^{*}\left(q_{2, \lambda}^{*}\right)+\frac{\lambda \theta_{r}\left(2+\theta_{s}\right)}{\left(2+\theta_{s}+\theta_{r}\right)\left(2+\theta_{s}+\theta_{r}+\lambda \theta_{s}+2 \lambda\right)}\left[p \bar{F}\left(q_{2, \lambda}^{*}\right)-c\right]$
It is a common sense that the last term of Equation (32) is positive, so there is $w_{2}^{*}\left(q_{2, \lambda}^{*}\right)>w_{1}^{*}\left(q_{2, \lambda}^{*}\right)$. Equation (27) is decreasing in $q$, and we can get $w_{1}^{*}\left(q_{2, \lambda}^{*}\right)>w_{1}^{*}\left(q_{1, \theta}^{*}\right)$ as $q_{2, \lambda}^{*}$ $<q_{1, \theta}^{*}$. Therefore, we can get the conclusion $\left.w_{2}^{*}\right\rangle w_{1}^{*}$ as corollary showed.

It is well known that the larger the parameter $\theta_{r}$ is, the more the retailer concerns about fairness. However, the retailer's sensitivity to fairness is not limited in real life. It has a greater possibility that $\theta_{r}$ is less than 2 and it is more significant for managers to make decisions. Corollary 1 shows that retailer 2 tends to be conservative in order quantity and receives less favorable wholesale price offer. In other words, retailer 2 is in a worse position compared to retailer 1, as long as the retailer's distributional fairness parameter is not too larger.

## CONCLUSIONS AND LIMITATIONS

In this paper, we establish a distributional and peerinduced fairness concern framework and then build the
utility system in a 1-supplier and 2-retailer supply chain. The Nash bargaining solution is used as distributional fairness reference and the first retailer's monetary payoff is used as the second retailer's peer-induced fairness reference. On the background of newsvendor model, we investigate the role of the retailers' fairness concerns on the supply chain through behavioral analysis. The equilibrium results show that the second retailer will order less product and receive a higher wholesale price than the first retailer in the game with the supplier, as long as the retailer's distributional fairness parameter is not too larger. It is the peer-induced fairness concern that makes the second retailer tend to be conservative in order quantity and in a worse position compared to the first retailer.

We have also analyzed the decision-making processes of the fairness concerned retailers when the wholesale price is exogenous in section 3. The results show that the two retailers' optimal order quantities are both smaller than the fairness-neutral counterparts under decentralized channel. Additionally, the peer-induced fairness concerned retailer's optimal order quantity is larger than that of the distributional fairness concerned retailer when the supplier provides them a same wholesale price. We also find that the two retailers' optimal order quantities are decreasing with their distributional fairness parameter while increasing with the supplier's distributional fairness parameter and the second retailer's optimal order quantity is increasing with his peer-induced fairness parameter. Then the distributional and peer-induced fairness concerned retailers' optimal order quantity is analyzed under centralized channel. This paper enriches and develops the previous conclusions in a simple dyadic supply chain by incorporating another homogeneous retailer into the supply chain, in which the two retailers operate in two separated markets. The main contribution of this paper is that we consider a supply chain with two homogenous retailers who are distributional and peer-induced fairness concerned and we get some new and important conclusions, which will guide managers to make correct decisions in the real transactions with other firms.

Although we believe that our analysis has generated some useful new insights, it is important to point out some limitations of our model that future research can investigate. we consider two homogeneous retailers in the supply chain who operate in the same market and ignore the heterogeneity of them. Actually, retailers have different marketing strategy for themselves since they are always different from each other in scale, capability and property. Thus, we can't view the first retailer's monetary payoff as the peer-induced fairness reference simply. This paper can also be extended in several directions. First, the model can be extended with the supplier using a more complex wholesale pricing contract. Second, it may be interesting to extend the model by incorporating n-retailer in the supply chain and investigate how they divide a common pie together in the context of supply chain.

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[^0]:    SHI Yucheng, ZHU Jia'ang (2014). Game-Theoretic Analysis for a Supply Chain With Distributional and Peer-Induced Fairness Concerned Retailers. Management Science and Engineering, 8(1), 78-84. Available from: URL: http://www.cscanada.net/ index.php/mse/article/view/j.mse.1913035X20140801.4400 DOI: http://dx.doi.org/10.3968/j.mse.1913035X20140801.4400

